Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 2 (2014) 300 – 308

Transportation Research Procedia

www.elsevier.com/locate/procedia

The Conference on Pedestrian and Evacuation Dynamics 2014 (PED2014)

The many roles of the relaxation time parameter in force based models of pedestrian dynamics

Fredrik Johansson [b,a,*], Dorine Duives [c], Winnie Daamen [c], Serge Hoogendoorn [c]

[a] *Swedish National Road and Transport Research Institute (VTI) SE-581 95 Linköping, Sweden*
[b] *Linköping University, Dept of Science and Technology (ITN) SE-601 74 Norrköping, Sweden*
[c] *Department of Transport & Planning, Faculty of Civil Engineering and Geosciences, Delft University of Technology, P.O. Box 5048, NL-2600 GA Delft, The Netherlands*

Abstract

In force based models of pedestrian traffic, the relaxation time, τ, is related to the time it takes a pedestrian to adapt its motion to its preferences. An example of this is linear acceleration, but τ is also connected to how the agent adjusts to spatial variations in its preferred velocity, and affects evasive maneuvers. These many roles of τ may be a problem when calibrating force based models.

We compare linear acceleration, to new data on, and simulations of, turning movements. The results indicate that the models predict drifting of a magnitude that is not supported by the data.

Peer-review under responsibility of Department of Transport & Planning Faculty of Civil Engineering and Geosciences Delft University of Technology

Keywords: pedestrian simulation; force based models; calibration; relaxation time; social force model

1. Introduction

Microscopic models of pedestrian dynamics has been applied to a wide range of situations, from the exceptional circumstances in evacuations and mass gatherings to the daily pedestrian traffic at a public transport interchange stations. The purposes of simulating these scenarios differ significantly; when simulating extraordinary situations safety is often the main concern, while simulations of regular pedestrian traffic often aim to determine classic traffic engineering measures of the efficiency of the pedestrian facility, such as travel (walking) time, delay, and possibly also the comfort of the pedestrians. We will refer to the second type of situations as *normal conditions*, or simply *pedestrian traffic*, and assume that the subject of interest in investigations of such situations are the traffic measures mentioned above. The first class of situations will be referred to as *extreme conditions*.

The requirements on the models used to simulate these two types of situations of course differs strongly. For extreme conditions, accurate representation in the model of phenomena such as panic and physical forces are important for the reliability of the predictions. However, such effects are not important when evaluating the design of infrastructure for normal pedestrian traffic, in which panic does not occur and physical contact between pedestrians is rare. In

* Corresponding author. Tel.: +46-13-204036
E-mail address: fredrik.c.johansson@liu.se

Peer-review under responsibility of Department of Transport & Planning Faculty of Civil Engineering and Geosciences Delft University of Technology
doi:10.1016/j.trpro.2014.09.057

these situations it is instead important that the model accurately represents phenomena occurring in the wide range of densities from very sparse traffic up to the density region where safety starts becoming a concern. When safety starts becoming a concern this should of course be investigated separately, and models specially developed and calibrated for extreme conditions should be used.

The research presented in this paper is mainly important for pedestrian traffic applications under normal conditions.

As noted above, a good model of pedestrian traffic should be valid in a wide range of situations. One category of microscopic models that seems promising in this respect is the class of *force based* models. The basic idea of this type of models is to model the reactions of agents to their environment through their acceleration, under the assumption that all these reactions are additive, just like physical forces, from which the term force based derives.

One of the first models of this type was developed in Japan and presented by Hirai and Tarui (1975), inspired by a similar model of the motion of shoals of fish, presented by Suzuki and Sakai (1973). Later, a model with forces inspired by the forces between magnets was proposed by Okazaki (1979)[1].

These early models were limited in their applicability by the limited computational power available at the time, but when the Social Force Model (SFM) was proposed by Helbing and Molnár (1995), this was no longer a major obstacle for the applicability of the models, due to the rapid development of computers. Since then several force based models and variations has been proposed, such as the Centrifugal force model by proposed by Yu et al. (2005), and further developed by Chraibi et al. (2010), and a wide range of modifications to, and versions of, the SFM has been proposed, such as the inclusion of panic and physical forces by Helbing et al. (2000), a constrained interaction range (Ma and Wang, 2013; Xi et al., 2011), and limited number of other pedestrians taken into account, (Johansson, 2009). Johansson (2009); Shukla (2010); Zanlungo et al. (2011) proposed and compared different forms of the interaction force, Parisi et al. (2009) introduced a self-stopping mechanism to reduce pushing, and Moussaïd et al. (2010) introduced group formation. The SFM has also been implemented in commercial software for traffic simulation, and is routinely used by consultants in the field.

The behavioral foundations of force based models were strengthened by Hoogendoorn and Bovy (2003), who showed that models of this type of model can be derived in a subjective utility maximization framework, from reasonable behavioral assumptions. This facilitates the interpretation of the parameters of the models, and gives an alternative view of the assumptions underlaying the model structure.

With the exception of the magnetic force model by Okazaki (1979), all of these models use the same form of the *preferred force* $\mathbf{F}^p$, the force that drives the walker towards its destination, namely

$$\mathbf{F}^p = \frac{1}{\tau}\left(\mathbf{v}^p - \dot{\mathbf{x}}\right), \qquad (1)$$

where $\mathbf{v}^p$ is the preferred velocity of the walker, $\mathbf{x}$ is its position, and τ is the relaxation time. The preferred velocity is often taken to be a function of the position of the walker, $\mathbf{v}^p = \mathbf{v}^p(\mathbf{x})$, and given from some route choice model, either static or dynamic.

In contrast, in the magnetic force model the force driving the walkers towards their destination takes the form of Coulombs law, with an external limit on the maximum speed of a walker (Okazaki and Matsushita, 1993). This model thus has the drawback that the driving force increases in strength as the distance to the destination decreases.

Despite the popularity of the formulation eq. (1) of the preferred force, it has not been extensively studied. Two exceptions are the studies by Moussaïd et al. (2009), and Ma et al. (2010), that verify the form of the driving force, and fit the value of τ, by controlled experiments. Both groups report good agreement between the acceleration process predicted by eq. (1) and the observations. However, they obtain significantly different values of τ, which according to Ma et al. (2010) is due to the different populations used in the experiments; Moussaïd et al. (2009) observed French pedestrians and Ma et al. (2010) observed Chinese pedestrians. Also other factors, such as the infrastructure, the instructions given by the experiment leaders, the method used to trigger the acceleration, etc., could possibly contribute to the difference. Both studies investigate the validity of eq. (1) by only considering linear acceleration, i.e., a constant preferred direction, of a free pedestrian, i.e., a pedestrian not interacting with other pedestrians or other dynamical obstacles.

[1] The original paper is written in Japanese, but the model is briefly described in English by Okazaki and Matsushita (1993).

However, linear acceleration is only one of several walking processes that are affected by eq. (1) and the value of τ. When the preferred velocity field is non-constant in space, eq. (1) determines how precisely the walker follows its preferred path. We can not see any behavioral reason why the strength of the preference for certain path should necessarily be correlated to the linear acceleration time scale for all pedestrians.

A third behavior affected by the value of τ is evasive maneuvers, since the size of the evasive maneuvers are determined the by the resultant of the preferred force, determined by τ, and the repulsive social force. Neither this behavior seems to be obviously connected to the other two.

In this paper we investigate the possible implications of this fact, that several, seemingly independent, processes are controlled by an equation with a single parameter .

2. The problem

As noted in the introduction, the relaxation time affects several walking processes. Firstly, it determines the acceleration time scale for walkers that for some reason have an actual speed that is smaller than its preferred speed. If the walker is free the only force affecting the walker is the preferred force in eq. (1), and if the preferred velocity field constant and an initial velocity $\mathbf{v}_0$ is given, the equation can be solved for the velocity as a function of time, giving

$$\dot{\mathbf{x}}(t) = \mathbf{v}^p + (\mathbf{v}_0 - \mathbf{v}^p)\, e^{-t/\tau}, \tag{2}$$

where $v_0 = \dot{\mathbf{x}}(0)$. Moussaïd et al. (2009) and Ma et al. (2010) fitted this acceleration process to that of their observed pedestrians, obtaining $\tau = 0.54 \pm 0.05$ m/s and $\tau = 0.71 \pm 0.10$ m/s, respectively.

Secondly, the relaxation time affects how precisely the walker follows its preferred velocity when it is non-constant in space. Let's for simplicity assume that the preferred velocity field is constant in time. Since the preferred velocity field is evaluated at the walker's current position in eq. (1), the walker will apply an acceleration based on the current velocity and the preferred velocity at the current position, independent of how the preferred field varies around the current position. In this way the walker only uses local information of its preferred velocity, so when a walker rounds a corner it will at each time step have a new preferred velocity, and thus its reaction to the spatial changes in the preferred field will lag behind its optimal trajectory, corresponding to the integral curves of the preferred field. As a result the walker will under-steer, that is, drift outwards in curves. A walker with a large relaxation time will adopt its velocity over a longer time scale, and thus drift more, than a walker with a low value of the relaxation time.

Finally, the value of τ affects how inclined the walkers are to make evasive maneuvers to avoid other walkers. When a walker gets closer to another walker, the interaction force will cause it to start an evasive maneuver. But as soon as the evasive maneuver has started, the walker will no longer walk in its preferred direction, and thus the preferred force will start growing, turning the walker back towards its preferred velocity. The resulting motion of the walker will thus be given by the combination of the preferred force and the interaction force, and since the size of the preferred force is given by τ, the evasive maneuver will be smaller when the value of τ is lower. In this way the value of τ represents the aggressiveness, or indifference to others, of the walker.

From a behavioral viewpoint, there is clearly no reason to believe that all these three behavioral characteristics should always be correlated. For example, there is no reason to believe that all walkers that are slow to accelerate are unable to take curves sharply, but this is the case in the model.

This may be a problem when calibrating models of this type, since it may be desired to adjust these different behaviors independent of each other to calibrate the model. In this paper we investigate if this is a problem in practice, or if the effects are too small to have any significant importance. This is done by relating the value of τ obtained by studying linear acceleration of pedestrians by Moussaïd et al. (2009) and Ma et al. (2010), to the data from turning movements of pedestrians. We do this comparison by comparing the motion of the observed pedestrians with a simulated scenario with pedestrians rounding a corner. The observed shift in the radial distribution is then compared between the simulations and observations of real pedestrian traffic at the simulated site. If there is a significant difference between the results of the simulation and the data, we can conclude that the formulation eq. (1) of the preferred force may be problematic and in need of revision.

In this paper we restrict the investigation to only consider the potential conflict between the first two types of behavior mentioned above: linear acceleration and turning motion. The effects of τ on the evasive maneuvers are not studied in this paper, since it requires that the interaction force is specified, while the linear acceleration and turning movement can be studied for free pedestrians, without considering interactions. Thus, the results of this study is general for the large class of models that uses eq. (1) as the driving force of the walkers.

To determine how large the effect of drifting outwards in curves is, simulations of a number of versions of a simple scenario were performed. The scenario consisted of two perpendicular road segments connected with a circular bend. Figure 1 depict the paths of the walkers in the bend for different values of the radius of the curved segment, $R = 0.5, 1, 2,$ and 4 m. The walkers enters the depicted area from the left and exits at the lower edge of the figure. The dashed lines indicates the inner edge of the road, represented in the simulations by a wall with no repelling social force. For each scenario, simulation runs with different values of the relaxation time, $\tau = 0.1, 0.3, 0.5, 0.7$ s, were performed. The walkers were created one by one, with headways sufficiently large to guarantee that no interactions could take place. The simulation was performed using the simulation platform presented in Johansson (2013).

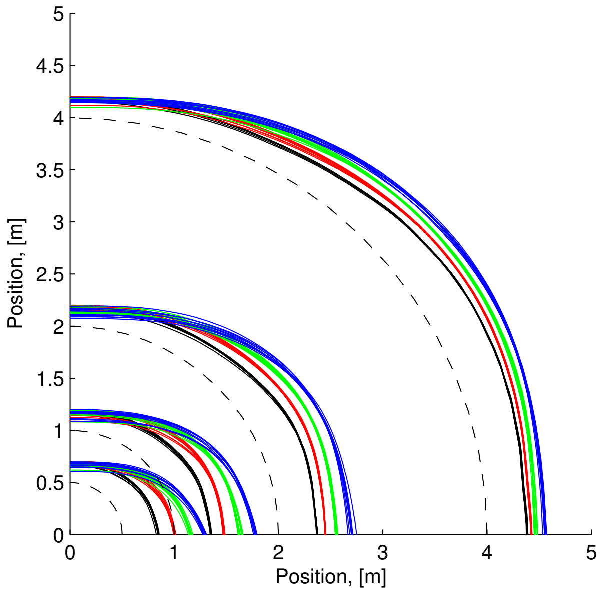

Fig. 1. The simulated paths of walkers for $R = 0.5, 1, 2,$ and 4 m, and $\tau = 0.1, 0.3, 0.5, 0.7$ s.

As can be seen in fig. 1, the absolute amount of drifting is almost independent of radius. This results in very large drift relative to the radius of curvature for sharp curves; but also for large radii the drifting is several decimeters, which may be sufficient to be observable if present in real pedestrian traffic since it is a systematic effect.

Note that, as expected, the walkers take sharper curves for lower values of the relaxation time, but that the dependence on τ becomes weaker for larger radii.

For very small radii, it is tempting to conclude that the drifting represents the inability of a pedestrian to take arbitrarily sharp curves, since this would be a reasonable property of the model. However, the presence of almost as large drifting also for curves with large radii makes this interpretation questionable.

It is also worth noticing that although the simulations presented here were performed using the Euler method, the drifting is not due to the errors of this numerical procedure. All simulations presented in this paper were performed using a time step of 0.05 s. The sufficiency of the numerical accuracy achieved by choosing this time step was guaranteed by running a couple of simulations using a time step of 0.001 s. A comparison of the result from these simulations to the ones with 0.05 s shows no significant difference, so it can be concluded that this is a sufficiently small time step to avoid any significant numerical errors.

3. Observations of turning motions in pedestrian traffic

To determine if the drifting motion predicted by the force based models appears in real pedestrian traffic, a comparison of the simulation results to empirical trajectory data was performed. In this section, the collection and processing of the empirical data used in this comparison are presented.

The data was obtained from video recordings of a march in the Netherlands, the 4daagse, that takes place of the course of four consecutive days. The approximately 42 000 pedestrians who join the march each year walk, depending on their physical ability and age, 30, 40, or 50 km per day. A mixed population joins the march, both in age, gender and physical ability. In 2013 the uni-directional flow of pedestrians was monitored at one point per route per day. In this study the video sequences of Lent and Hatert are used. In both cases, the movement of pedestrians moving through a curve was recorded. The substratum was flat and consisted of asphalt and paving brick. Where Lent was located half way through the march routes, Hatert was located near the beginning of the march routes of that day.

The observations were performed using digital video cameras with a variable frame rate, which during most the sequences used in this paper was close to $5\,\mathrm{s}^{-1}$. The variable frame rate of the camera make the frame number invalid as a parametrization of time; instead the time of each frame was extracted from a time stamp added to the video recording by the camera.

To guarantee high precision data even from the relatively low angle at which the videos were recorded, the data was manually extracted from the videos. This was done by for each pedestrian click on its head once every fifth frame, resulting in trajectories for the heads of all pedestrians in the videos, sampled approximately every second.

Using the head as the point of a pedestrian that defines its trajectory is advantageous since it is an easily identified point that is seldom occluded from the view point of the camera. On the other hand, the head of a pedestrian swings back and forth when walking, as studied by Hoogendoorn and Daamen (2005). This swinging motion is in no way represented in the models of pedestrian traffic; in these the position of the pedestrian is instead represented by its approximate center of mass.

When the trajectories have been extracted, they are in image coordinates. These need to be transformed to real world coordinates; this was done using ImageTracker (Knoppers et al., 2012). However, this method does not account for that the pedestrians have different lengths, which results in a remaining error. This error is small, since it is evident from observing the video that the differences in lengths between pedestrians are small.

Furthermore, the angle of the camera makes it difficult to directly extract the shape of the edge of the road, since this would require a projection of the pedestrian coordinates to the plane of the road, which require the unknown lengths of the pedestrians to be exact. The extraction of the road geometry was instead achieved by choosing a pedestrian that walked close to the edge by inspection of the video, and use this pedestrian's path as a proxy for the shape of the road edge.

To get velocities from this noisy and sparsely sampled data, the trajectories need to be filtered and interpolated in a suitable way. One simple way to achieve both these goals in an approximate sense was chosen: to fit cubic splines to the sampled points in a way that minimizes a convex combination of the squared distance between the spline and the data, and the curvature of the spline, measured by its second derivative. This results in trajectories guaranteed to be continuously differentiable, as required by the physics of the process, and be within a short distance of the data points.

The data extraction and processing method described above results in the paths depicted in fig. 2.

4. Analysis

If all the trajectories corresponding to the paths in fig. 2 are considered, there would be many effects to take into account, e.g., interactions, other destinations, etc. Therefore, we only consider a subset of the trajectories, those trajectories that correspond to free pedestrians, i.e., pedestrians not interacting with other pedestrians during their whole trip through the field of view of the camera. These free pedestrians are found by considering one pedestrian at a time and check if there are any other pedestrian within a circle segment of radius 1.5 m in front of the pedestrian at any time; if not, the pedestrian is considered free. The circle segment is defined by the directions $\mathbf{d}$, $|\mathbf{d}| = 1$, such that

$$\mathbf{d} \cdot \frac{\dot{\mathbf{x}}}{|\dot{\mathbf{x}}|} > -0.2, \tag{3}$$

thus only considering pedestrians not constrained in their motion by other pedestrians.

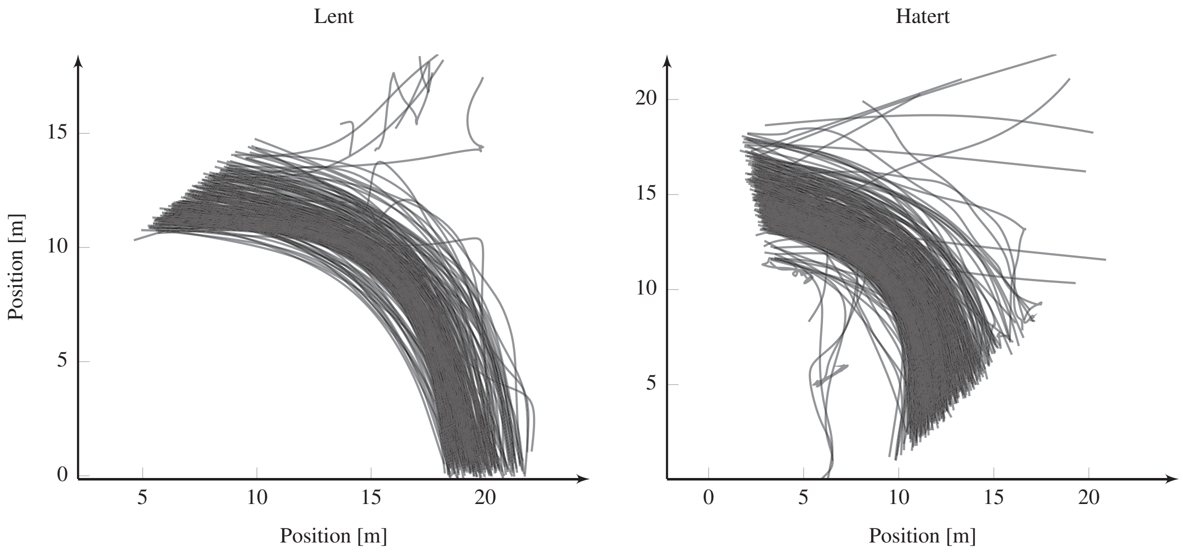

Fig. 2. Paths of all registered pedestrians at the two sites.

In addition to this we also constrain the set of considered pedestrians by requiring that they enter and exit the camera field of view within the width of the road, and furthermore that they do not stand still or suddenly change direction.

This procedure results in the pedestrians whose paths are depicted in fig. 3.

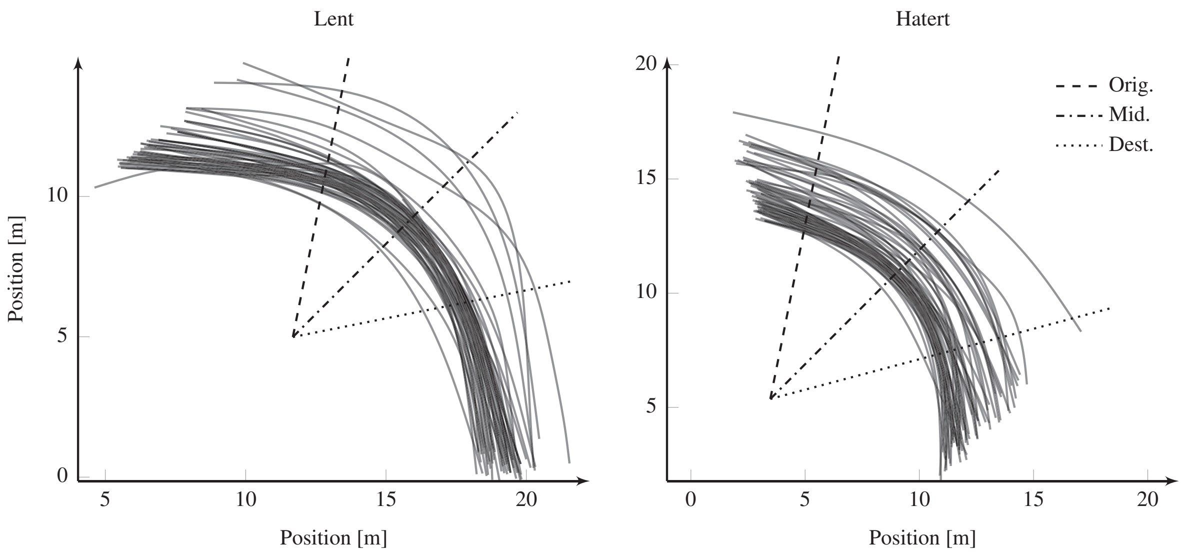

Fig. 3. Paths of free pedestrians.

As is evident from fig. 1, the model predicts drifting several decimeters outwards for a full 90° turn. Can any such drifting be detected in observed trajectories? Since we are interested in comparing the amount of drifting we can compare the radial distributions at different angles in the turning; marked in fig. 3 by the lines Orig. (dashed line), close to the beginning of the bend, Mid. (dash dotted line), in the middle of the bend, and Dest. (dotted line), close to the end of the bend. The results of the comparisons are shown in fig. 4, where the distribution close to the beginning, in the middle, and at the end of the curve are depicted.

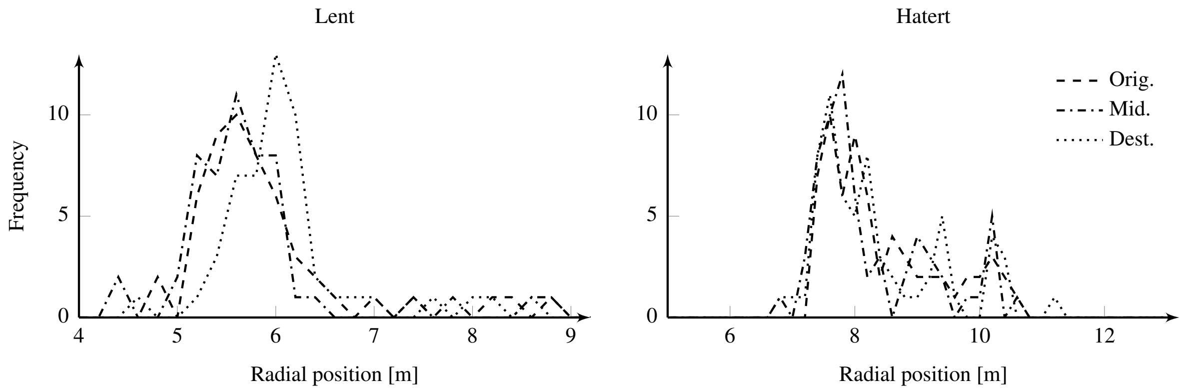

Fig. 4. The radial distributions of walkers at the three locations noted in fig. 3.

The comparison in fig. 4 is clearly inconclusive. The radial distributions in the Hatert case are almost identical, apart from random fluctuations, i.e., no indication of any drifting. In the Lent data there is a slight shift of the distribution at the end of the bend, which may indicate that some drifting takes place.

The above is a comparison of the *collective* drifting, thus it is possible that there are effects on the individual level that cancel at the collective level. To investigate if any such individual drifting occurs, the distributions of individual drifting are compared in fig. 5. Here the same positions as in fig. 4 are used, but the compared distributions are the distributions of the *difference* in radial coordinate of each pedestrian between the first and the last position, the first and the middle position and the middle and last positions, respectively.

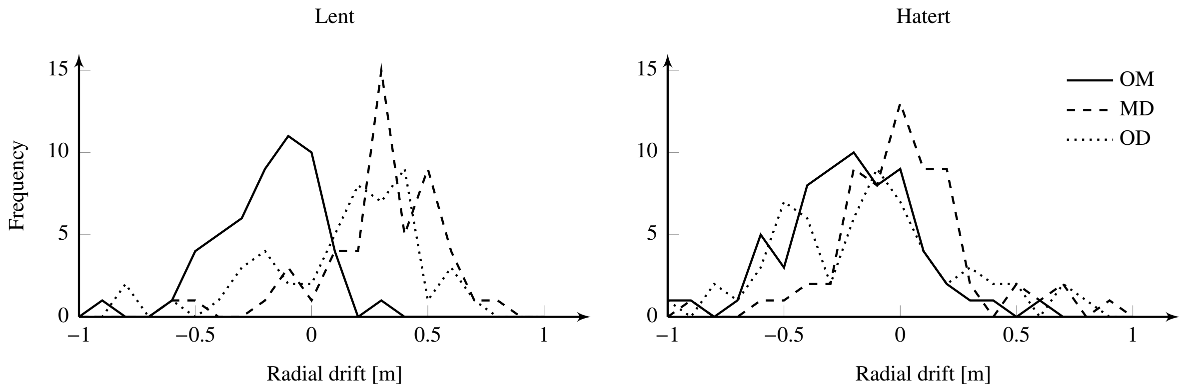

Fig. 5. Distributions of individual drifting movements between the three positions noted in fig. 3.

In the Lent case, we clearly see some negative drifting from the first position to the middle position, and then some positive drifting from the middle position to the end position. The resulting total drifting from the first to the last position is positive, with a peak around 0.4 m, but with a significant amount also on the negative side.

This drifting, first negative and then positive, gets its explanation after further observations of the videos. A significant fraction of the pedestrians are taking a shortcut through the curve, temporarily leaving the asphalt with a couple of decimeters, to return again at the end of the curve.

The Hatert case, on the other hand, does not show any clear signs of any drifting at all. However, it should be noted that the peaks are quiet wide, so several pedestrians are changing their radial position in the curve, both in the positive and the negative direction.

Thus, we can conclude that the data do not support any systematic outward drifting to the extent the model predicts. There are some hints of drifting in the Lent data, but due to the short cut behavior, it is hard to draw any clear conclusions. The Hatert data clearly supports no systematic drifting.

5. Discussion

The drifting phenomenon, clearly demonstrated in fig. 1, is in one sense a reasonable prediction of a model of pedestrian dynamics. It is clearly reasonable that pedestrians are not able to make arbitrarily sharp curves, and from the form of the preferred force, eq. (1), it is obvious that this ability is closely related to the value of the relaxation time parameter, τ.

However, from the simulation results depicted in fig. 1, where the value of τ was varied, it is clear that the walkers drift significantly for all values of τ. Furthermore, this drifting is, in absolute terms, almost constant with respect to the radius of the curve. Thus, force based models of pedestrian dynamics using eq. (1) predict a significant amount of drifting *for all values of* τ, and all radii.

One can argue, that even if there is a difference in terms of drifting between the models and real pedestrian traffic, the amount of drifting, around 0.5 m per 90° turning, is insignificant for practical applications. However, since it is a systematic effect predicted by the models, in is possible that the effects of several curves add up for some walkers, resulting in significant differences from the expected behavior.

Furthermore, the effect motivates asking a principal question regarding the relation between the preferred direction field and the actual paths of walkers. The preferred direction field is supposed to represent the preferred paths of a walker, dependent on its destination and current position. Thus, it is reasonable to expect that if a walker is not perturbed by any dynamic obstacles such as other walkers, which it could not have taken into account when planning its route, it should follow the integral curves of the preferred direction vector field. However, as we have shown in this paper, this is not the case. In fact, as soon as the preferred field is non-constant in space, no free walker will follow the integral curves. This is problematic since it makes the preferred direction field unobservable; the user of the model is forced to obtain the field through a calibration procedure, or make strong assumptions of the preferences of the modeled pedestrians. If instead the user assumes that free observed pedestrians walk according to their preferences, in the sense that they walk along the integral curves, and construct the preferred field from this assumption, the simulated free walkers, obeying eq. (1), will not move as their real counterpart.

A practical situation that exemplifies the importance of the drifting effect is when calibrating a model of this type by minimizing the difference between individual trajectories obtained from simulations and measurements. If the scenario includes a non-constant preferred direction field, and the observed pedestrians do not drift significantly, the calibration will lower the value of τ to make the curves of the walkers sharper, while other effects such as linear acceleration or interactions may demand higher values of τ. The optimal value of τ will in this case be a compromise between the optimal value for reproducing the turning behavior, and the optimal value for reproducing linear acceleration. As we have shown in this paper, these values may differ. Furthermore, the compromise will depend on the relative frequency of the behaviors in the calibration data, and the optimal parameter value becomes dependent of the local infrastructure and the portability of the calibrated model is lost.

6. Conclusions

The comparison made in this paper, between simulations of free walkers in force based models with real world observations of pedestrians rounding a smooth corner, indicates that the drifting effect is more pronounced in the simulations compared to the observations. However, more observations of pedestrians in various situations are needed, especially movement around sharp corners, to further quantify the results.

Furthermore, the values of τ obtained from observations of linear acceleration leads to significant drifting, in contrast to the observations of turning pedestrians presented in this paper. This indicates that the models may be too simple; that it may be necessary to make a distinction between longitudinal and lateral acceleration, as proposed by Hoogendoorn and Bovy (2003).

References

Chraibi, M., Seyfried, A., Schadschneider, A., 2010. Generalized centrifugal-force model for pedestrian dynamics. Physical Review E 82. doi:10.1103/PhysRevE.82.046111, arXiv:1008.4297.

Helbing, D., Farkas, I., Vicsek, T., 2000. Simulating dynamical features of escape panic. Nature 407, 487–490. doi:10.1038/35035023, arXiv:cond-mat/0009448v1.

Helbing, D., Molnár, P., 1995. Social force model for pedestrian dynamics. Physical Review E: Statistical, nonlinear and soft matter physics 51, 4282–4286. doi:10.1103/PhysRevE.51.4282, arXiv:cond-mat/9805244.

Hirai, K., Tarui, K., 1975. A simulation of the behavior of a crowd in panic, in: Proceedings of the 1975 International Conference on Cybernetics and Society, pp. 409–411.

Hoogendoorn, S., Bovy, P., 2003. Simulation of pedestrian flows by optimal control and differential games. Optimal Control Applications and Methods 24, 153–172. doi:10.1002/oca.727.

Hoogendoorn, S., Daamen, W., 2005. Pedestrian behavior at bottlenecks. Transportation science 39, 147–159. doi:10.1287/trsc.1040.0102.

Johansson, A., 2009. Data-Driven Modeling of Pedestrian Crowds. Ph.D. thesis. Technische Universität Dresden. URL: http://nbn-resolving.de/urn:nbn:de:bsz:14-qucosa-20900.

Johansson, F., 2013. Pedestrian traffic simulation platform. VTI notat 16-2013. Swedish National Road and Transport Research Institute, Linköping. URL: http://www.vti.se/en/publications/.

Knoppers, P., Van Lint, H., Hoogendoorn, S., 2012. Automatic stabilization of aerial traffic images, in: Transportation Research Board 91st Annual Meeting.

Ma, J., Song, W., Fang, Z., Lo, S., Liao, G., 2010. Experimental study on microscopic moving characteristics of pedestrians in built corridor based on digital image processing. Building and Environment 45, 2160–2169. doi:10.1016/j.buildenv.2010.03.015.

Ma, P., Wang, B., 2013. The escape of pedestrians with view radius. Physica A: Statistical Mechanics and its Applications 392, 215–220. doi:10.1016/j.physa.2012.07.053.

Moussaïd, M., Helbing, D., Garnier, S., Johansson, A., Combe, M., Theraulaz, G., 2009. Experimental study of the behavioural mechanisms underlying self-organization in human crowds. Proceedings of the Royal Society B: Biological Sciences 276, 2755–2762. doi:10.1098/rspb.2009.0405.

Moussaïd, M., Perozo, N., Garnier, S., Helbing, D., Theraulaz, G., 2010. The walking behaviour of pedestrian social groups and its impact on crowd dynamics. PLoS ONE 5, e10047. doi:10.1371/journal.pone.0010047, arXiv:1003.3894.

Okazaki, S., 1979. A study of pedestrian movement in architectural space. part 1: pedestrian movement by the application of magnetic models. Trans. of AIJ 283, 111–119.

Okazaki, S., Matsushita, S., 1993. A study of simulation model for pedestrian movement with evacuation and queuing, in: International Conference on Engineering for Crowd Safety, pp. 271–280.

Parisi, D., Gilman, M., Moldovan, H., 2009. A modification of the social force model can reproduce experimental data of pedestrian flows in normal conditions. Physica A: Statistical Mechanics and its Applications 388, 3600–3608. doi:10.1016/j.physa.2009.05.027.

Shukla, P.K., 2010. On modeling and evolutionary optimization of nonlinearly coupled pedestrian interactions, in: Applications of Evolutionary Computation. Springer, pp. 21–30. doi:10.1007/978-3-642-12239-2.

Suzuki, R., Sakai, S., 1973. Movement of a group of animals. Biophysics 13.

Xi, H., Lee, S., Son, Y.J., 2011. An integrated pedestrian behavior model based on extended decision field theory and social force model, in: Rothrock, L., Narayanan, S. (Eds.), Human-in-the-Loop Simulations. Springer, pp. 69–95. doi:10.1007/978-0-85729-883-6.

Yu, W., Chen, R., Dong, L., Dai, S., 2005. Centrifugal force model for pedestrian dynamics. Physical Review E 72. doi:10.1103/PhysRevE.72.026112.

Zanlungo, F., Ikeda, T., Kanda, T., 2011. Social force model with explicit collision prediction. Europhysics Letters 93, 68005. doi:10.1209/0295-5075/93/68005.