Not for publication



SH R17/60

NATIONAL PHYSICAL LABORATORY

Ship Division Report No. 17

# THE USE OF A COMPUTER IN SHIP HYDRODYNAMIC AND HYDROSTATIC CALCULATIONS

Compiled

By

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DEPARTMENT OF

SCIENTIFIC AND INDUSTRIAL RESEARCH

August 1960

The Ship Hydrodynamic Laboratory at Feltham was on display during the NPL Open Week from 23rd to 28th May, 1960. Because of the interest shown in the wall display 'The Use of a Computer in Ship Hydrodynamics and Hydrostatic Calculations', it was decided that a Ship Division Report, based upon this display, should be made. Each feature on the display board, as was seen during Open Week, has been reproduced in this report.

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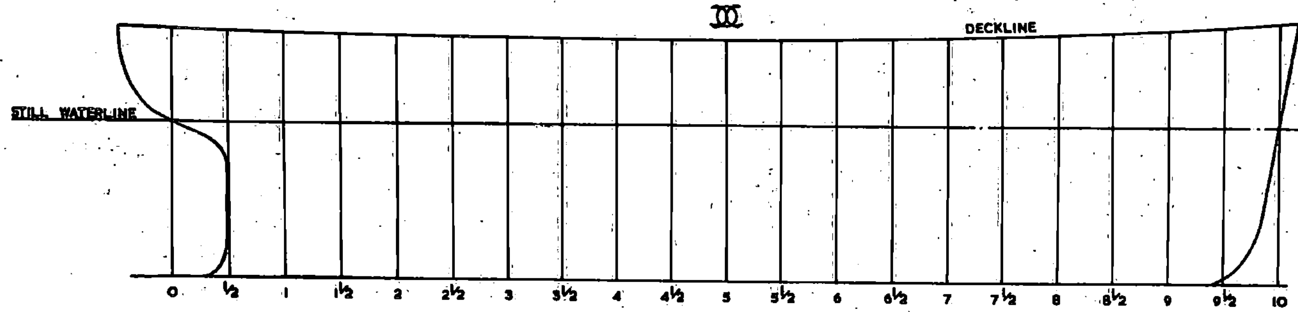
# COMPUTATUON OF BONJEAN AND HYDROSTATIC CURVE PARTICULARS

A 'DEUCE' programme was made for these calculations. The approximate computing time for this programme is 15 minutes.

The tabulation of the ship offsets from the ship's body plan and the punching of these offsets on to 'DEUCE' cards will take approximately ten man hours.

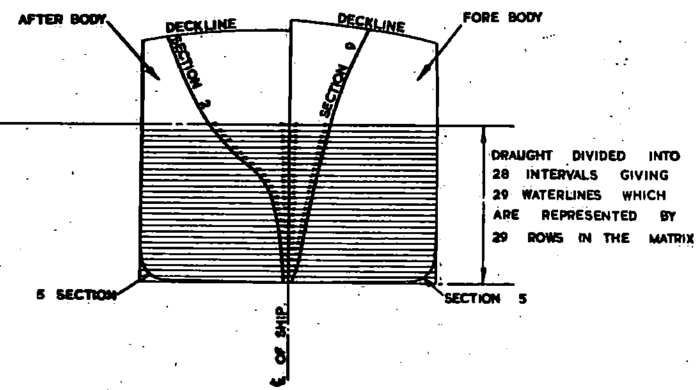
The offsets of the ship are fed into the computer in the form of a 29 (rows) x 21 (columns) matrix. The rows represent the ship's waterlines and the spacing of these waterlines can be arranged to cover the range of draughts desired.

The columns represent the ship's sections, and the spacing of these sections must be 1/20th of the ship's length.




21 - SHIP SECTIONS REPRESENTED BY 21 COLUMNS OF THE MATRIX

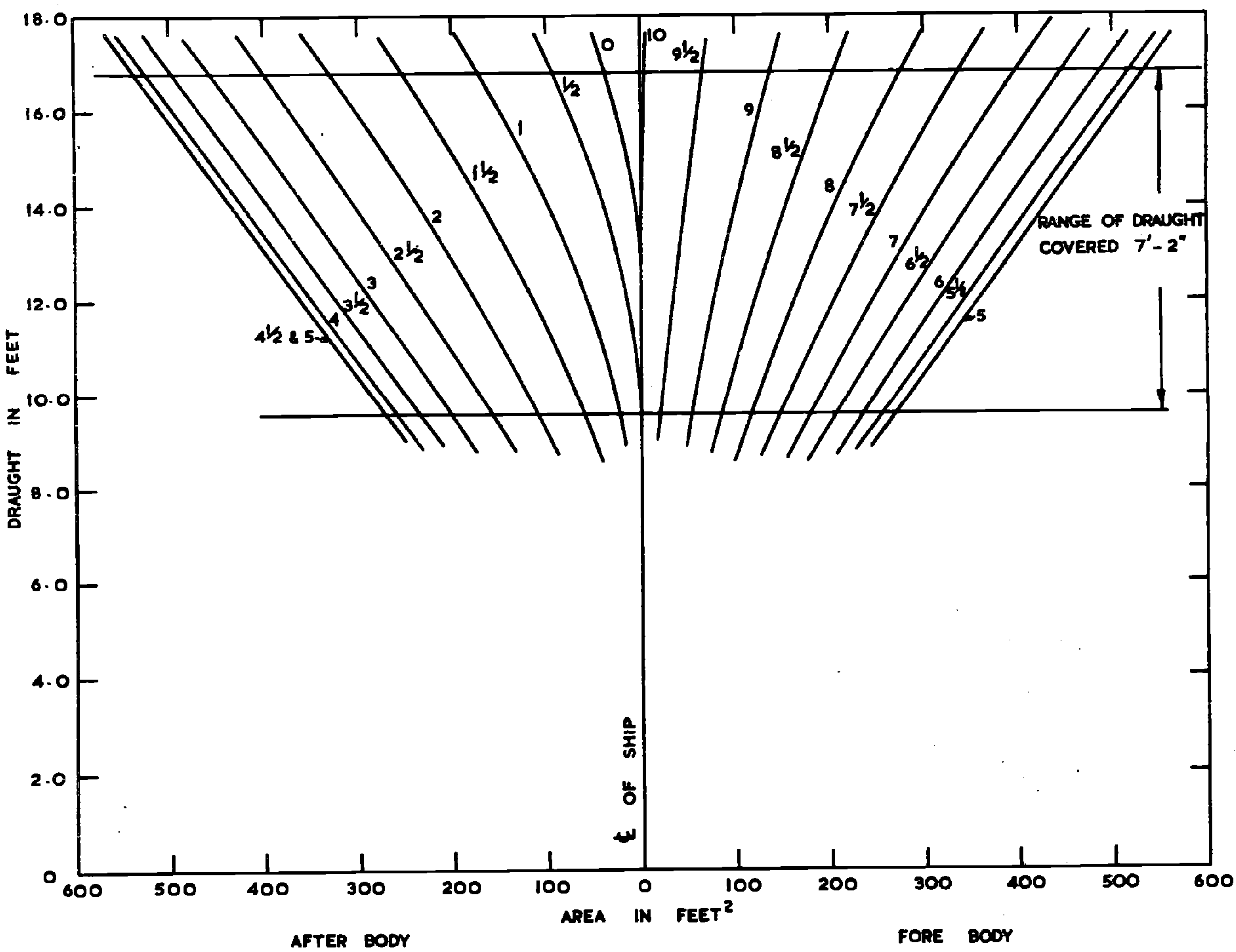
SHIP PROFILE



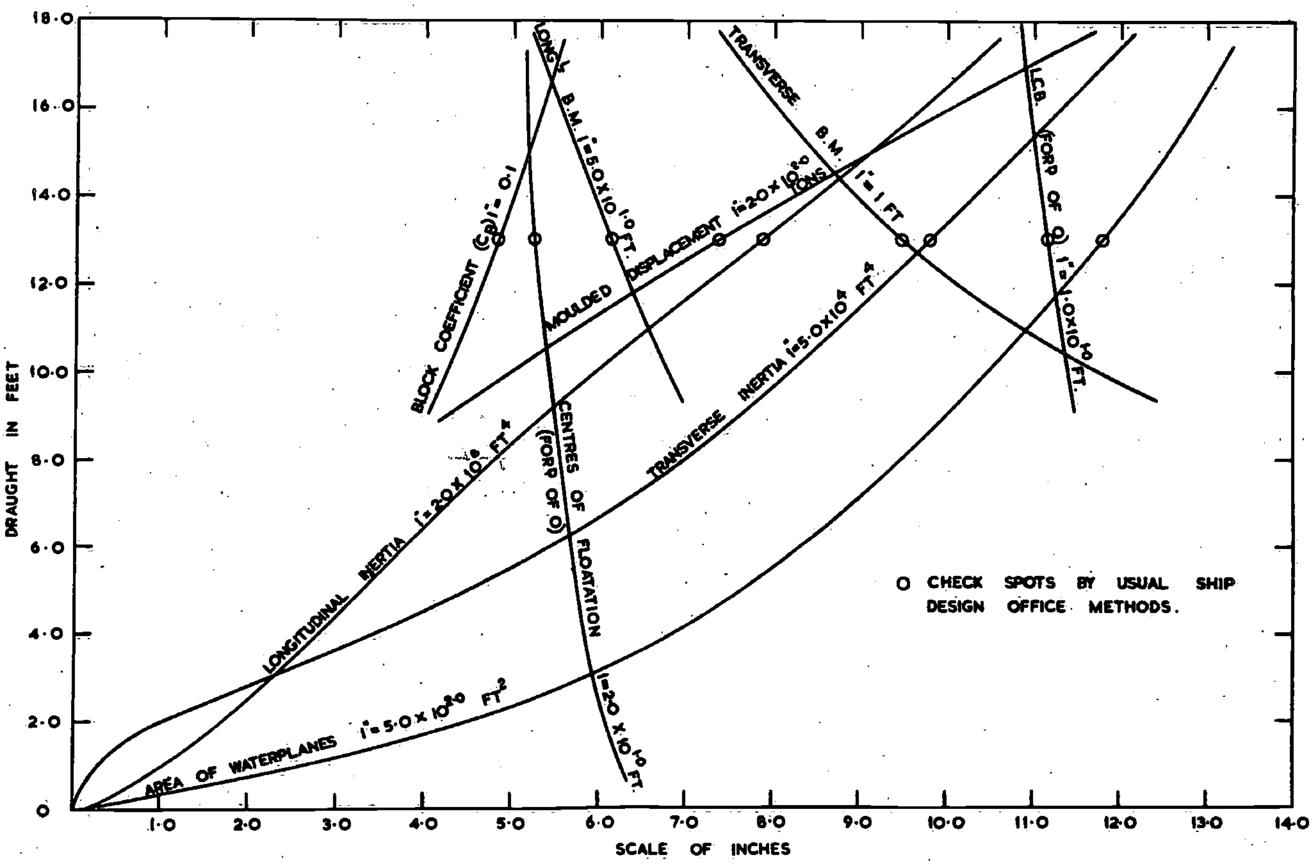

BODY PLAN

OFFSETS ARE LIFTED & TABULATED FROM THIS PLAN
OFFSETS ARE SHEWN FOR SECTIONS 2 & 9.

# BONJEAN CURVES DRAWN FROM 'DEUCE' INFORMATION



# HYDROSTATIC CURVES DRAWN FROM 'DEUCE' INFORMATION



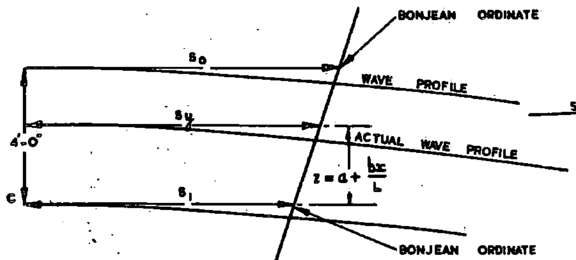
# CALCULATION OF STATIC PITCH AND HEAVE OF A SHIP

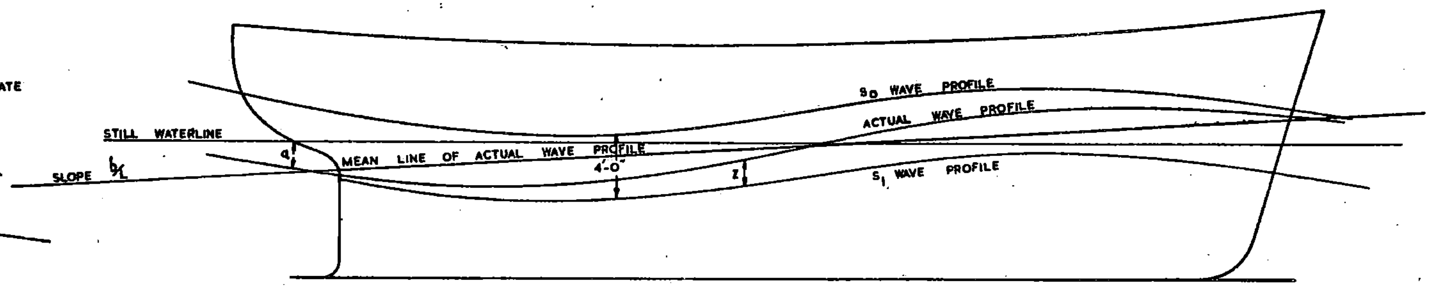
(With or without Smith Effect)

In longitudinal strength calculations only two cases of static pitch and heave of a ship are usually derived. These are the sagging and hogging conditions of equilibrium, both conditions being without Smith effect. For the analysis of ship motions, the static pitch and heave are required for many wave positions and for several different wave lengths. As these calculations are lengthy, a 'DEUCE' programme was made. The values of the wave ordinates and the Bonjean curve equations have to be calculated, tabulated and punched on to 'DEUCE' cards. The approximate time for this work is 5 man hours. The approximate computer time for four wave frequencies is one hour.

This programme is based on a method proposed by Muckle whereby the ship is statically balanced on a wave. As his calculations do not take into account the Smith correction, a method of modifying the Bonjean curves using Fernandez coefficients has been used.

For correct balancing of a ship on a wave two conditions must be satisfied. The buoyancy force must equal the ship's weight, and the longitudinal centre of buoyancy must be directly under the longitudinal centre of gravity. These conditions are fulfilled by the equations 1 and 2 shown overleaf.




AREA AT ANY HEIGHT z ABOVE C WILL BE $S_1 + z\left(\frac{S_0 - S_1}{4}\right) = S_1 + \left(a + \frac{bx}{L}\right)\left(\frac{S_0 - S_1}{4}\right)$

INTEGRATING ALONG THE SHIP'S LENGTH $\sum S_1 \delta x + a\sum\left(\frac{S_0 - S_1}{4}\right)\delta x + b\sum\frac{x}{L}\left(\frac{S_0 - S_1}{4}\right)\delta x = \nabla$ ------------ 1

MOMENT OF THE AREA ABOUT THE AFTER PERPENDICULAR = $S_1\frac{x}{L} + \left(a + \frac{bx}{L}\right)\left(\frac{S_0 - S_1}{4}\right)\frac{x}{L}$

INTERGRATING ALONG THE SHIPS LENGTH $\sum\frac{S_1}{L}\delta x + a\sum\frac{x}{L}\left(\frac{S_0 - S_1}{4}\right)\delta x + b\sum\frac{x^2}{L^2}\left(\frac{S_0 - S_1}{4}\right)\delta x = \frac{\nabla\bar{x}}{L}$ ---- 2

WHERE:
- $S_0$ $S_1$ & $S_y$ ARE BONJEAN ORDINATES AS SHEWN IN SKETCH.
- $z$ = THE AMOUNT THE WAVE HAS TO BE RAISED TO FULFILL THE CONDITIONS OF EQUILIBRIUM.
- $a$ = THE AMOUNT RAISED OR LOWERED AT THE AFTERMOST SECTION
- $b$ IS PROPORTIONAL TO THE TANGENT OF THE PITCH ANGLE, i.e. $\tan\psi = b/L$.
- $L$ IS THE SHIP'S LENGTH. $\nabla$ = THE VOLUME OF SHIP.
- $x$ IS THE ORDINATE OF LENGTH MEASURED FROM THE AFTERMOST SECTION.
- $\bar{x}$ IS THE DISTANCE OF THE LONGITUDINAL CENTRE OF GRAVITY FROM THE AFTERMOST SECTION.

FROM EQUATIONS ① & ② THE TWO UNKNOWNS "a" AND "b" CAN BE CALCULATED FROM THE FIVE SUMMATIONS TO BE MADE

IF

$$\sum\left(\frac{S_0 - S_1}{4}\right)\delta x = k_1$$
$$\sum\left(\frac{S_0 - S_1}{4}\right)\frac{x}{L} = k_2$$
$$\sum\left(\frac{S_0 - S_1}{4}\right)\frac{x^2}{L^2} = k_4$$
$$\sum(S_1)\delta x - \nabla = k_3$$
$$\sum(S_1)\frac{x}{L}\delta x - \frac{\nabla\bar{x}}{L} = k_5$$

THEN

$$a = \frac{k_3 k_4 - k_2 k_5}{(k_2)^2 - k_1 k_4}$$
$$b = \frac{k_1 k_5 - k_2 k_3}{(k_2)^2 - k_1 k_4}$$

FROM $a$ & $b$ VALUES STATIC HEAVE IS DERIVED

i.e. $z$ AT L.C.B. = $a + \frac{b\bar{x}}{L}$

FROM $b$ VALUE STATIC PITCH IS DERIVED

PITCH ANGLE = $\tan^{-1} b/L$

The Smith correction is that which is necessary to take account of the varying hydrostatic pressure in the structure of the trochoidal wave. Each sectional area is corrected by the following equation

$$\frac{S'}{S} = \lambda + \frac{0.4\ Cd}{\lambda}$$

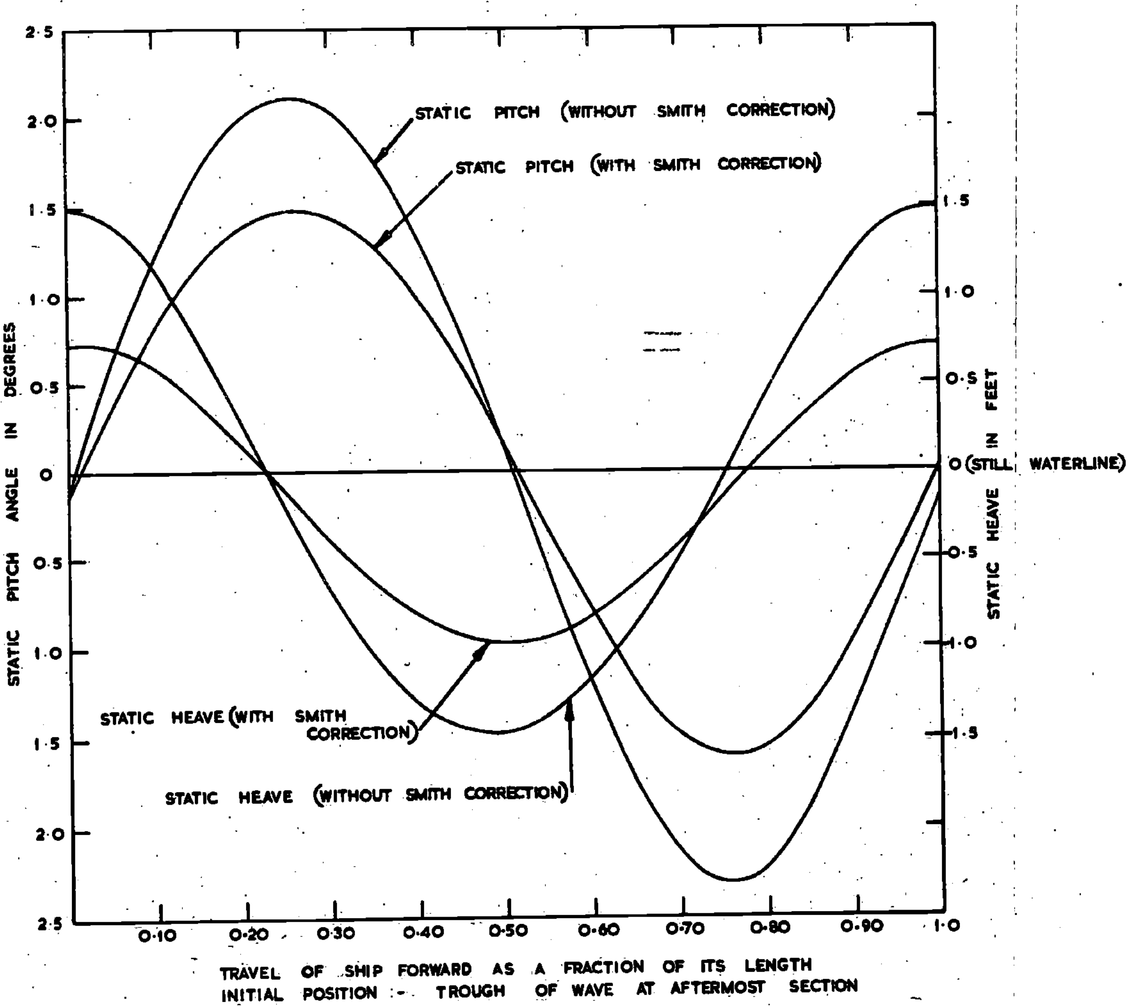
where S' = corrected sectional area at any station on the buoyancy curve

S = uncorrected sectional area at same station on the buoyancy curve

A and C = Fernandez coefficients

λ = Wave length

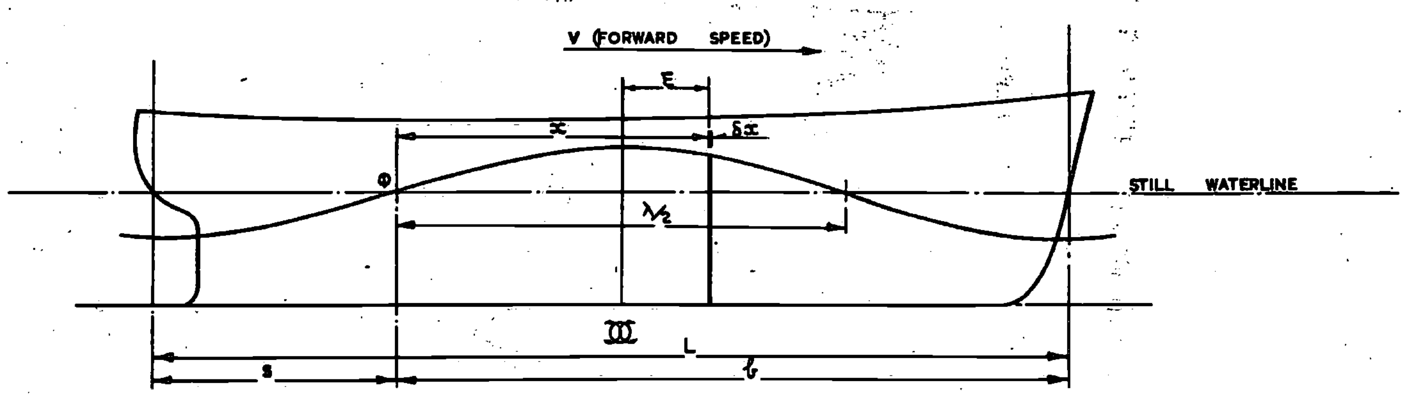
TYPICAL STATIC PITCH & HEAVE CURVES PLOTTED AGAINST VARYING WAVE POSITIONS
WAVE LENGTH = SHIP LENGTH
MODEL 3411



# COMPUTATION OF FORCES AND MOMENTS ACTING ON A SHIP IN A REGULAR SEAWAY

The gradual development of the theory of ship motions in regular waves which was started by Kriloff in 1896, has in recent years been extended and improved. This computation is based on formulae derived by Korvin Kroukovsky in a paper 'Pitching and Heaving Motions of a Ship in Regular Waves' before 'The Society of Naval Architects and Marine Engineers'.

The calculations are lengthy when several wave frequencies, and many positions of the wave relative to the ship's hull have to be computed. In the need for economy of effort a 'DEUCE' programme was made. As input to the computer the values of $x$, $\xi$, $k_2$, $k_4$, ship section areas, area curve slopes and ship's beam have to be calculated, tabulated and punched on to 'DEUCE' cards. This work takes approximately ten man hours. The approximate computing time for four wave frequencies is one hour.




TOTAL HEAVING FORCE $\quad F = \int_s^b \frac{dF}{dx}\, dx$

WHERE $\quad dF/dx = \rho\, g\, h\, B \left( K_1 \text{ SINE } 2\pi x/\lambda + K_2 \cos 2\pi x/\lambda \right)$

AND $\quad K_1 = 1 - \left(\frac{1 + k_2 k_4}{2}\right) \frac{\pi^2}{\lambda} \left(\frac{2A}{\pi}\right)^{1/2} + 4\left(1 + k_2 k_4\right) \frac{\pi}{\lambda^2} A$

$K_2 = \frac{\pi(1 + k_2 k_4)}{4} \left[ 1 - \frac{16}{3\lambda} \left(\frac{2}{\pi}\right)^{1/2} \right] \frac{V}{c} \frac{1}{(2\pi A)^{1/2}} \frac{dA}{d\xi}$

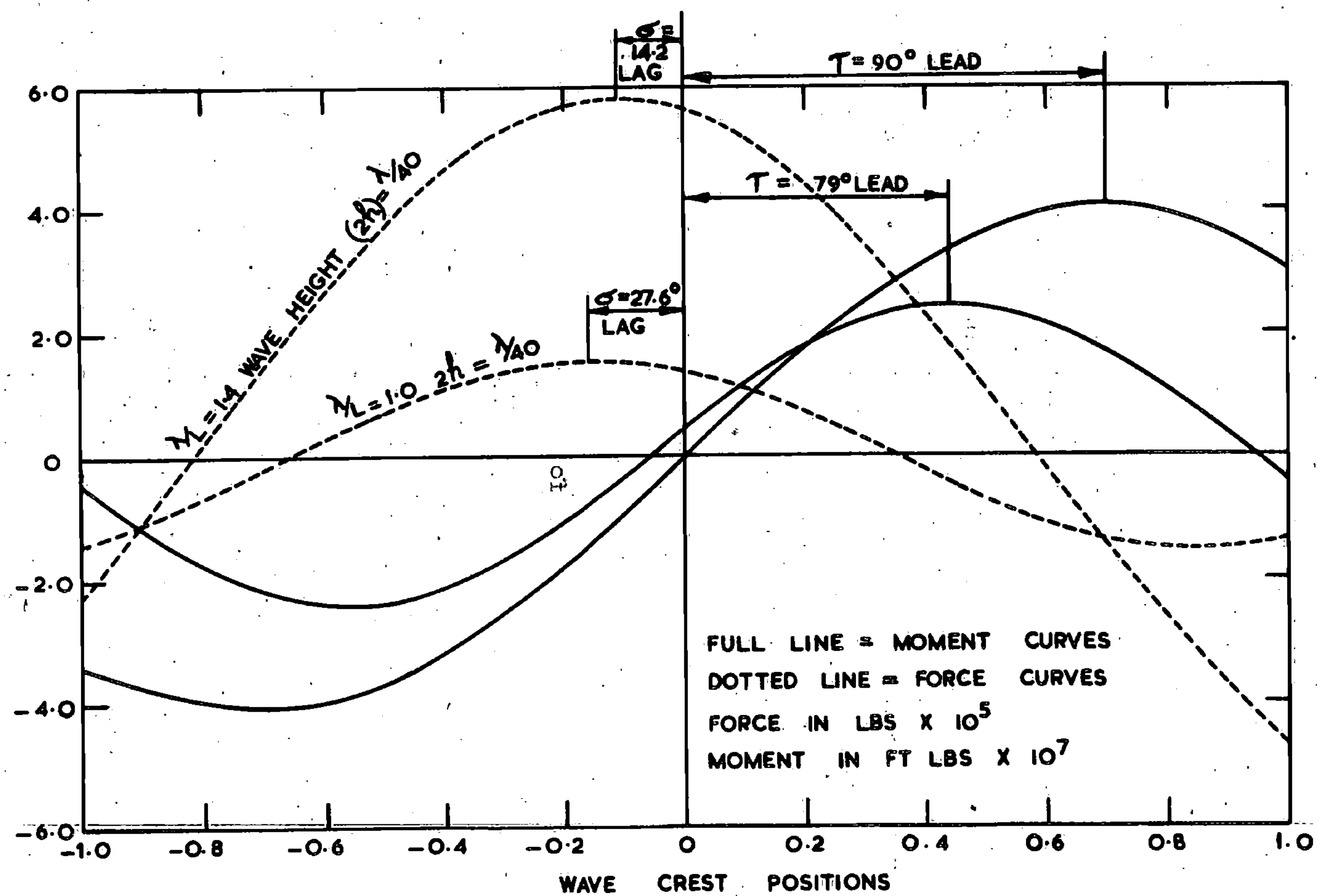
NON – DIMENSIONAL COEFFICIENTS

TOTAL PITCHING MOMENT $\quad M = \int_s^b \frac{dF}{dx} \left( \xi + \frac{1}{\pi}\, dA/d\xi \right) dx$

$k_2$ = ADDED MASS COEFFICIENT IN TWO DIMENSIONAL VERTICAL FLOW ABOUT A SHIP SECTION.
$k_4$ = CORRECTION COEFFICIENT FOR EFFECT OF FREE WATER SURFACE.
$\rho$ = WATER DENSITY
$h$ = WAVE AMPLITUDE
$V$ = SPEED OF SHIP
$A$ = AREA OF SECTION
$g$ = ACCELERATION OF GRAVITY
$\lambda$ = WAVE LENGTH
$c$ = VELOCITY OF WAVE
$B$ = BEAM OF SHIP

## FORCE AND MOMENT CURVES FOR VARIOUS WAVE LENGTHS

### SPEED 9.2 KNOTS



## COMPUTATION OF COEFFICIENTS USED IN EQUATIONS OF MOTION (COUPLED PITCH AND HEAVE)

A 'DEUCE' programme was made to calculate these coefficients. The approximate computing time for seven wave frequencies is 15 minutes. Values of ship's beam, sectional areas, coefficient of added mass, coefficient for free water correction and the damping coefficients have to be tabulated and punched on to 'DEUCE' cards. This work would take approximately ten man hours.

The coupled equations of motion for the pitching and heaving of a ship in a regular seaway are

$$a\ddot{Z} + b\dot{Z} + cZ + d\ddot{\theta} + e\dot{\theta} + g\theta = \bar{F}e^{i\omega t}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{Z} + E\dot{Z} + GZ = \bar{M}e^{i\omega t}$$

where a, b, c, d, e, g, A, B, C, D, E and G are coefficients to be calculated.

$Z$ = Heave in feet

$\theta$ = Pitch angle in radians

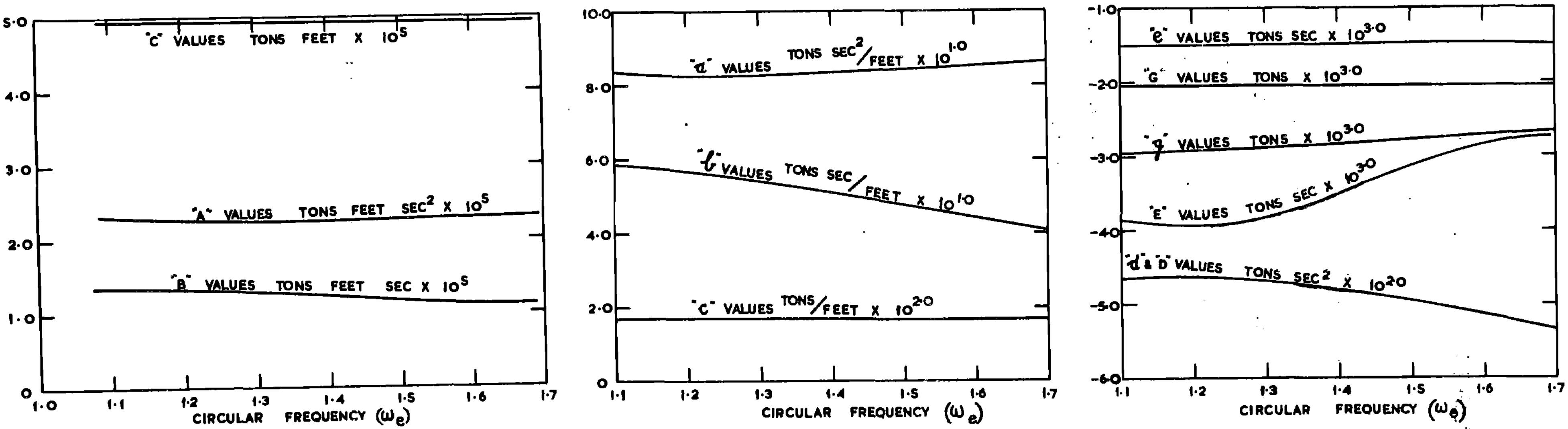
$\bar{F}$ = Heaving force imposed on the ship by waves
= $F(\cos\sigma + i\sin\sigma)$,

F and $\sigma$ having been computed in the force and moment programme

$\bar{M}$ = Pitching moment imposed on the ship by waves
= $M(\cos\tau + i\sin\tau)$,

M and $\tau$ having been computed in the force and moment programme.

# COEFFICIENT CURVES FOR VARYING FREQUENCY OF ENCOUNTER



## SOLUTION OF EQUATIONS OF MOTION

Given the values of the coefficients, forces and moments, the pitch and heave of the ship can be calculated on 'DEUCE'. Approx Time - 5 min.

If
$$P = -a\omega^2 + ib\omega + c$$
$$Q = -d\omega^2 + ie\omega + g$$
$$R = -D\omega^2 + iE\omega + G$$
$$S = -A\omega^2 + iB\omega + C$$
then
$$\bar{\theta} = \frac{\bar{F}R - \bar{M}P}{QR - SP} \qquad \bar{Z} = \frac{\bar{F}S - \bar{M}Q}{PS - RQ}$$

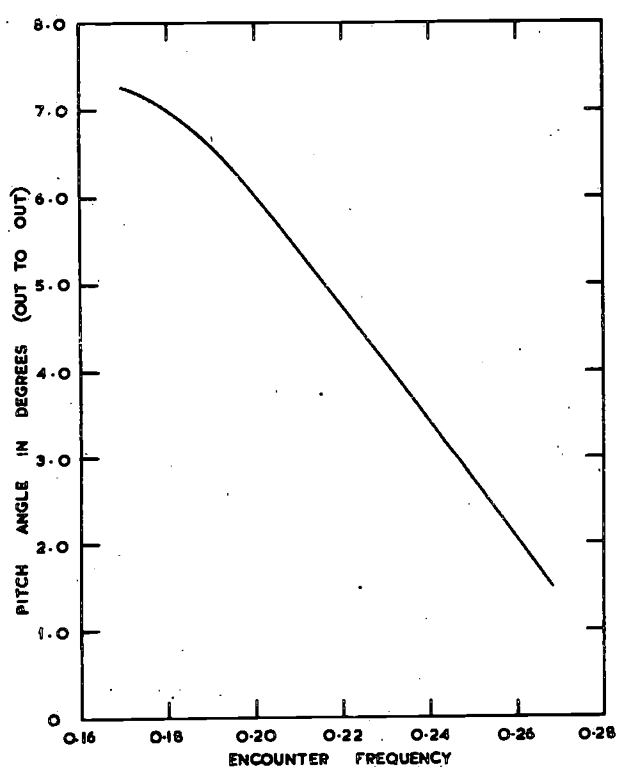
$\bar{\theta}$ & $\bar{Z}$ are complex amplitudes

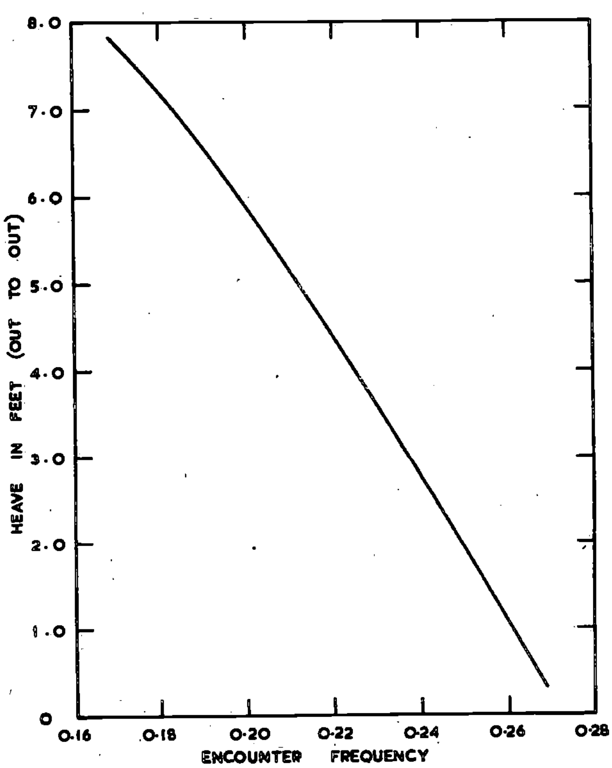
$$\bar{\theta} = \theta(\cos\epsilon + i\sin\epsilon)$$
$$\bar{Z} = Z(\cos\delta + i\sin\delta)$$

$\epsilon$ & $\delta$ are phase lags of ship motion relative to the wave crest at amidships.

CALCULATED PITCH & HEAVE CURVES PLOTTED AGAINST WAVE ENCOUNTER FREQUENCY

SPEED 9·20 KNOTS WAVE HEIGHT = WAVE LENGTH/40



# CURVE FITTING THE HULL SURFACE OF A SHIP

In designing a ship, many factors influence the final shape of the hull. The service route which the ship has to travel, the choice of speed, the amount of cargo carried, the displacement of the ship, the stability, the powering and strength of the ship are all such factors. The designer has to consider all such factors before finalizing his design which he draws on a small scale. Offsets are then lifted from this plan and are sent to the mould loft for fairing. These offsets are laid out full scale on the mould loft floor and are then faired by the loftsman. Modifications, because of scaling and drawing errors, are made by the loftsman before the exact offsets are finalized. From these faired offsets the practical ship construction work can then be started; the bending of the ship's frames to the correct faired shape, the development of the ship's hull plates and the bulkheads of the ship are some of the operations which can then be carried out.

Nearly every ship form can be represented mathematically by a series of equations which may or may not contain a large number of polynomials of a high order. The idea of having a mathematically faired ship at the initial design stage is not a new idea; several papers have been written upon this subject, but because of the length of the calculations involved, naval architects have only viewed this approach with academic interest. With the development of high-speed digital computers, the mathematical ship's lines have become a practical proposition. The problem of deriving the equations defining the surface of the hull in three dimensions is one of surface fitting. Methods are being developed in the Mathematics Division of the NPL to deal with this problem and the results obtained so far are encouraging.

Equations have been derived for two different types of ships, namely a tanker and a high-speed cargo ship. These equations are polynomials of a high order which the digital computer can handle satisfactorily, but if the equations were developed at the design stage a lower order of polynomials could be used. There are no known reasons why a mathematical shaped ship of a lower polynomial order should have a greater resistance to water than a normally designed ship which contains higher polynomial equations. The opposite could well be true.

A successful solution to this problem could have far-reaching effects. The full-scale fairing of the ship's lines on the mould loft floor would not be needed as the computer could be very easily programmed to give the offsets or slopes of any frame section or waterline. These offsets and slopes would be exact and fair as they would have been derived from the fundamental equations. Auxiliary programmes working in conjunction with this solution could do the standard calculations such as the hydrostatic, Bonjean curves, launching and strength calculations.

There are a number of possible applications of computer techniques on the ship construction side. It is not inconceivable that a machine could be computer controlled to bend ship frames to the correct shape including the correct bevel. It may also be possible to shape the hull plates by similar means. In this way all work on the scrieve board board could be eliminated. It is appreciated that many practical difficulties exist in this problem, but a solution would prove of great economic value to the industry.

# INTEGRALS FOR USE IN COMPUTING SHIP WAVE RESISTANCE

The integrals ($P_{rs}$ and $Q_{rs}$) are described in detail in the Ship Division Report:-

## SHR 9/59

"A SIMPLE APPROXIMATE METHOD OF COMPUTING WAVE RESISTANCE"

by

N. Hogben

They may be used to compute wave resistance from the formula

$$R_w = \frac{\rho g^2 A_Z^2}{\pi V^2} \sum_r \sum_s \{(Y_r Y_s)P_{rs} + (y_r y_s)Q_{rs}\}$$

where Y and y represent the non-dimensional area curve ordinates.

$P_{rs}$ and $Q_{rs}$ may be briefly described as linear combinations of the functions:-

$$G_{nm} = N_{nm} \int_0^\infty E^2 S_{n-1} S_{m-1} \, du$$

$$H_{nm} = N_{nm} \int_0^\infty E^2 C_{n-1} C_{m-1} \, du$$

where

$$N_{nm} = 2_{nm} \quad n \neq m \quad \text{or} \quad N_{nm} = nm \quad n = m$$

$$E = (e^{-gz/V^2 \cosh^2 u}) \cosh u$$

$$S_n = \int_0^1 x^n \sin(gx/V^2 \cosh u) \, dx$$

$$C_n = \int_0^1 x^n \cos(gx/V^2 \cosh u) \, dx$$

$P_{rs}$ and $Q_{rs}$ have been tabulated for

$V/\sqrt{L}$ = 0.8, 0.85, 0.9 ..... 1.3 .

$L/Z_m$ = 30, 40, 50, 60, 70 ($Z_m$ = mean draught)

# ANALYSIS OF BOUNDARY LAYER MEASUREMENTS ON A MODEL HULL

The measurements were made to determine the displacement thickness $\delta$ and the velocity profiles $(u/v)$ against $y$ at about 60 positions on the model. The numerical work is simple but the quantity of computing large.

For each position at each of 5 speeds, the following results are to be derived from measurements of total head $h$ and static pressure $p$ at varying distances $y$ from the hull.

Displacement thickness:-

$$\delta = \int_0^Y \left[ 1 - \frac{\sqrt{h - p}}{\sqrt{1 - p}} \right] dy .$$

Velocity profile:-

$$\frac{u}{v} = \frac{\sqrt{h - p}}{\sqrt{1 - p}}$$

$$v = \sqrt{1 - p_1} .$$

For the 60 holes and 5 speeds there are 300 sets of results each consisting of 1 value of $\delta$, 1 value of $v$ and about 15 values of u/v.

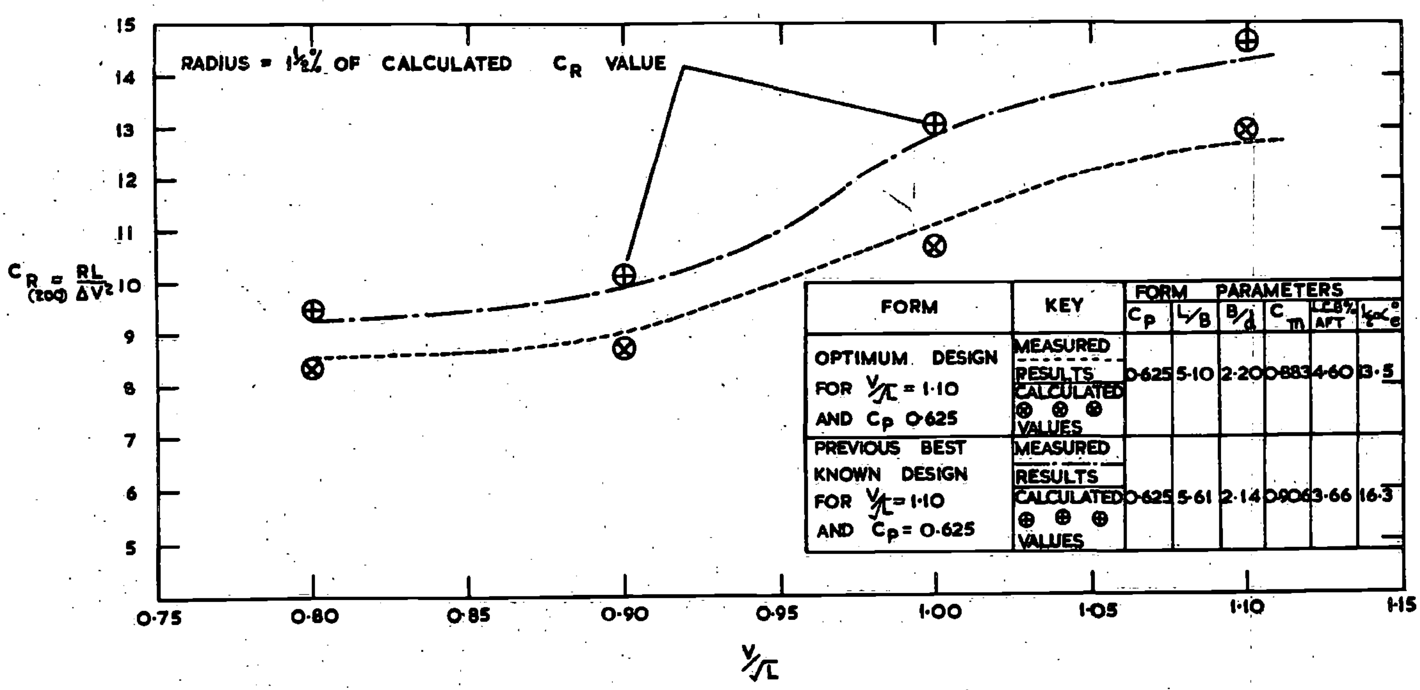
## RESISTANCE DATA FOR TRAWLERS

A statistical analysis of all resistance data for trawlers, obtained from model experiments conducted in No.1 Tank, Ship Division, NPL, has been made. From an analysis of this type, a design method has been evolved, by means of which optimum resistance characteristics can be estimated for each trawler type, together with predictions of effective horse-power for any particular form.

These EHP calculations are made by determining six parameters from the normal ship's lines plan and to facilitate the computation of results a programme has been prepared for DEUCE, so that NPL can provide an estimated EHP-speed curve very quickly for a given set of parameters.

By minimization of the regression equation obtained from the statistical analysis method, optimum combinations of form parameters have been calculated for practical design conditions. It has been found that compared with previous best known results, improvements of between 10 and 20% in total resistance per ton of displacement have been obtained.

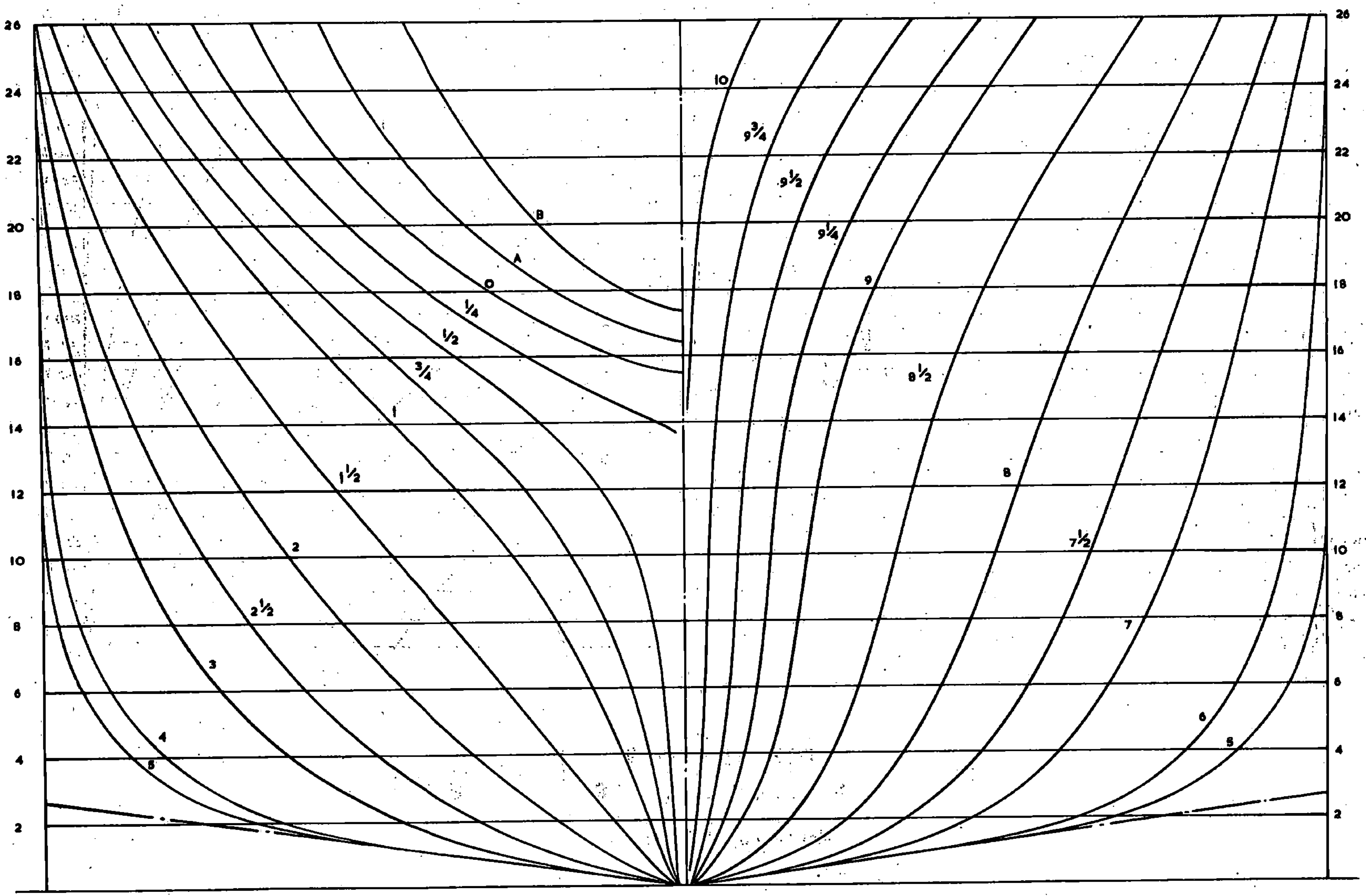
RESISTANCE DATA FOR TRAWLERS



NORMAL FORM

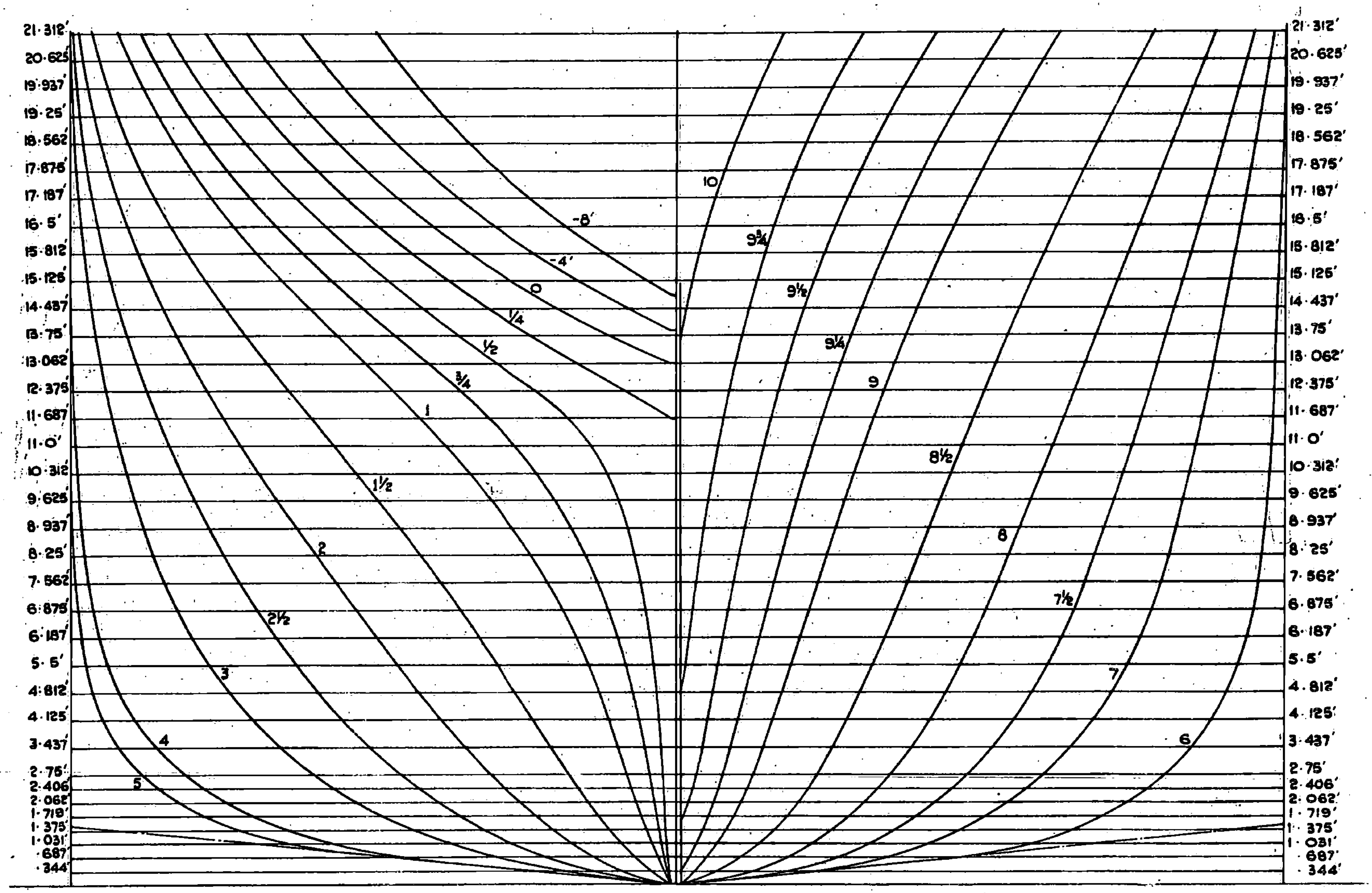
178 FT L.B.P x 31 FT BEAM x 14.52 FT DRAFT

5.53 FT TRIM



OPTIMUM DESIGN

200 FT L.B.P. x 39.25 FT BEAM x 17.833 FT DRAFT
6.67 FT TRIM



## RESEARCH INTO PROPELLER EXCITED VIBRATION

A programme of research into propeller excited ship vibration is being undertaken at NPL. In both theoretical studies and in experimental analysis extensive use is being made of digital computers.

(A) Wake traverse analysis

Knowledge of the flow conditions in way of the screw disc is essential in the determination of the thrust and torque fluctuations delivered to the shaft by a propeller operating in a ship wake. This data is obtained from pitot traverses on model hulls, using a five-holed Warden-type pitot tube. The manual analysis of this data is formidable and has therefore been programmed for a computer. From the experimental results the programme derives components of wake in the axial, radial and circumferential directions at each point in the screw disc.

(B) Calculation of thrust and torque fluctuation

Using the above wake data it is possible to calculate the thrust and torque fluctuations developed by a given propeller operating in that wake, and thus to assess its characteristics as a vibration exciter. This calculation is at present being programmed and will permit a complete harmonic analysis of the fluctuating forces transmitted to the propeller shaft.

(C) Fluctuating pressure field studies

The passage of the blades induces in the water surrounding the propeller a fluctuating pressure field. These pressures are transmitted to the adjacent hull plating and can then excite hull vibration. The magnitude of the fluctuating pressure field is being studied at NPL. Pressures are measured by means of capacitance gauges located in the propeller flow field and are recorded on film. The data obtained is transferred to punched tape (using the ARL film assessor) which is then put through a Fourier analysis programme on DEUCE.

(D) Miscellaneous

In the theoretical studies associated with propeller excited vibration, integrals and differential equations arise for which explicit solutions are laborious or even unobtainable. Numerical methods must then be employed. As an example, the integrals:-

$$\frac{1}{\pi}\int_0^{\pi} \frac{\cos n\theta}{(1 - K\cos\theta)^{\frac{1}{2}}}\, d\theta \; ; \; \frac{1-K}{\pi}\int_0^{\pi} \frac{\cos n\theta}{(1 - K\cos\theta)^{\frac{3}{2}}}\, d\theta$$

occurring in pressure field theory, have been tabulated over a range of variables K and n, by means of a DEUCE programme developed by Mathematics Division.

## PROPELLER CIRCULATION DISTRIBUTION

In designing a propeller the radial distribution of circulation is usually chosen to be the optimum. The problem of determining the optimum distribution was initially formulated in physical and mathematical terms by Betz and Goldstein. The solution was shown to be equivalent to the potential flow solution of a fluid past a rigid helical membrane of infinite extent in the axial direction, and radius equal to that of the propeller. This amounts to the solution of Laplace's differential equation subject to certain boundary conditions. The analytical solution is difficult to solve numerically due to the presence of untabulated functions whose evaluation is in terms of infinite series which are not easily calculated, and it is better to proceed in a different manner to bypass this difficulty. Ignoring the analytical solution and proceeding from the differential equation and boundary conditions in finite difference form a series of linear algebraic simultaneous equations can be evolved which can then be solved quickly on DEUCE.

Laplace's differential equation in helical co-ordinates is:-

$$\left(\mu \frac{\partial}{\partial\mu}\right)^2 \phi + (1 + \mu^2) \frac{\partial^2\phi}{\partial\zeta^2} = 0$$

and this equation is to be solved for $\phi$, the velocity potential, subject to the boundary conditions pertaining to the particular case, for example:

(1) propeller with or without boss in a fluid of infinite extent

(2) propeller with or without a boss operating in a long coaxial duct.

In all cases the presence of a boss can be represented by a coaxial cylinder of radius equal to that of the boss and extending to infinity in the axial direction.

The boundary condition applicable to all cases is that of no flow through the helical surfaces, i.e. :

$$\left(\frac{\partial\phi}{\partial\zeta}\right)_{\zeta=0,\frac{2\pi n}{p}} = \frac{\mu^2}{1+\mu^2} k \quad \text{where } k = \frac{w(v + w)}{\Omega}$$

$$n = 1, 2, 3 \ldots.$$

in the range $\mu_{hub} \leqslant \mu \leqslant \mu_0$.

The other boundary conditions simply state that there can be no flow through a fixed surface for the case of the hub and duct, whereas without the hub and duct the radial flow must be zero at the axis and the effect of the helical surface must vanish at large distances from the surface.

Solving the problem using this technique shows an appreciable saving in time as the actual computer time for such a problem amounts to a few minutes as compared with days or weeks if the computation were done by hand.