Strategies for Electrical Network Expansion

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Abstract

The main focus of the transmission expansion planning is to find the optimal structure and least cost transmission investment alternatives of the forecasted load and generation configuration. In this paper, transmission investment methodology which focuses on alleviation of transmission line congestions is proposed. The proposed methodology is based on DC power flow under constrained Lagrangian multiplier and the locational marginal price. Within this framework constraints and variables associated with the derivation of Lagrangian multiplier and the locational marginal prices are included in the formulated transmission expansion planning problem.

This expansion problem which optimize the total investment and operation cost is modeled using a single-stage and multi-stage decision framework. In the single-stage transmission expansion planning framework a single load/ generation configuration is considered and the location, type and number of extra transmission lines of the optimal network configuration are determined. In the multi-stage model, multiple dispatch in the demand and wind power generation is integrated using a number of scenarios and the optimal expansion plan which fulfills the operating condition of all scenarios is determined through a three phase selection mechanism. For illustration purpose the resulting mixed-integer nonlinear programming problem is applied on the New England 39 bus test power system. Both proposed models are implemented in AIMMS software and solved using the outer approximation algorithm provided with the optimization tool.

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I save the best for my God who destined me to go beyond deliverance into eternal land of true joyfulness to experience the fullness of His purpose and plan in my life. I thank you God for all things happened and will be happening to my life; You’re gracious, Your ultimate sacrifice made my life possible. I will never be tried of thanking You!

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Nomenclature

Acronyms:

TEP	Transmission Expansion Planning
STEP	Static Transmission Expansion Planning
DTEP	Dynamic Transmission Expansion Planning
B&B	Branch and Bound
GA	Genetic Algorithm
NC	Neuro-Computing
SA	Simulated Annealing
TS	Tabu Search
GRASP	Greedy Randomization Adaptive Search Procedure
OPF	Optimal Power flow
LMP	Locational Marginal Price
AC	Alternating Current
AC OPF	Alternating Current Optimal Power Flow
DC	Direct Current
DC OPF	Direct Current Optimal Power Flow
MW	Megawatt
MWh	Megawatt Hour
KCL	Kirchhoff's Current Law
KVL	Kirchhoff's Voltage Law
MINLP	Mixed Integer Non-linear Programming
NLP	Non-linear Programming
MIP	Mixed Integer Programming
KKT	Karush-Khun-Tacker
PTDF	Power Transfer Distribution Factor
AOA	AIMMS Outer Approximation
GMP	Generated Mathematical Programs
NPV	Net Present Value

Indices

k	index of generators
i, j	index of buses
ij	index of lines
o	index of transmission line options
t	index of stage
s	index of scenarios

Sets

B	set of all buses
CL	set of all candidate lines
EL	set of all existing lines
L	set of all lines (Existing and candidate)
O	set of candidate line options
G	set of all generators
Ω_i	set of generators located at bus i
T	set of stage
S	set of scenarios

1. Introduction

In a typical power system the supply of electricity to load centers is carried out by three main processes: generation, transmission and distribution. The power generated at the generation stations will be transferred to the distribution centers through the high voltage transmission network. At the distribution station the electrical power is reduced to lower voltage level and will be distributed to consumers. In the future, due to the growing electricity consumption and renewable energy integration, the transmission network expansion planning is required to facilitate alternative paths for power transfer from power plants to load centers. This expansion should be done in timely and proper manner. Therefore, the transmission network expansion planning (TEP) is defined as the problem of determining where to locate the new transmission line, when and how much additional new capacity must be installed over the planning horizon so that the network meet optimal operational, economical, technical and reliability criteria of the future power system.

In vertically integrated power system environment the transmission, generation and distribution operation is performed by the responsible utility. In this condition, the transmission expansion is done in such a way that the reliable operation of the power system is not compromised [1]. Hence the planner selects the optimal transmission expansion plan for the forecasted demand level. The transmission expansion planning is usually performed by a constrained optimization approach that minimizes the cost of investment and load curtailment of the system.

1.1 **Objective**

The objective of the transmission expansion planning problem is to propose least cost transmission expansion strategy while fulfilling all the operation and security constraints of the system. This can be done by adding new network components that alter power flow through the existing transmission lines and alleviate congestion or by building new transmission lines either parallel to the existing ones or new right of way. This thesis focuses on addressing an expansion planning strategy by building new transmission lines so that the growing electrical demand will be supplied in safe, secure and reliable condition. The objective of this research work is to formulate a method of transmission expansion planning that takes into account the fluctuation of load level and wind power generation by multiple dispatch scenarios. Based on the future demand and wind power generation of the power system a number of typical scenarios are defined and the optimal expansion plan which operates optimally under all dispatch condition is determined through a decision framework. The selection of candidate expansion plan is carried out in such a way that the TEP problem alleviates congestions, which are caused by a serious overloading of the transmission lines.

1.2 **Scope of the Thesis Work**

The scope of this thesis work includes developing a mathematical formulation of TEP problem with an objective function of total cost minimization of the power system. The model is formulated for both deterministic and multi-stage non-deterministic (scenario) approach. In the deterministic approach the expansion plan will be performed for a single-load condition. It also considers a single-stage planning horizon. On the other hand, in the non-deterministic approach a multi-stage expansion planning is done for a long time horizon which is divided into a number of stages. In each stage the scenarios of probable future load conditions are taken into account and the best expansion plan which operates in all scenarios selected as the optimal investment solution.

The proposed expansion approach is implemented in AIMMS optimization software. The outer-approximation algorithm (AOA), provided in the tool, is used to address the mixed-integer nonlinear programming (MINLP) expansion planning problem. In this approach, the algorithm solves an alternating sequence of the primal problem and the relaxed master problem. The primal problem is the non-linear operation problem while the relaxed master problem is the mixed integer investment problem. The New England 39 bus power system with 46 lines and 10 generators is used as the test system. The results obtained are discussed and analyzed.

1.3 **Thesis Overview**

This thesis work is organized as follows. Chapter 2 gives the review of related literature of the transmission expansion planning approaches proposed earlier. Section 2.1 gives a review on the classification of TEP problem from the viewpoint of the power system planning horizon. The classification of the TEP approach due to the uncertainties in the power is discussed in section 2.2.

Chapter 3 presents the formulation of the transmission expansion planning problem. Section 3.1 gives the derivation and formulation of the alternating current (AC) and direct current (DC) optimal power flow (OPF) models. In section 3.2 the formulation of a deterministic TEP approach is presented. In the same section, the derivation of the Lagrangian multipliers, physical meaning of the locational marginal prices (LMP) and the congestion cost are explained. Finally, description, result and discussion of the proposed TEP methodology applied on the 39 bus New England testing power system are presented.

In Chapter 4, the deterministic TEP model proposed in chapter 3 is expanded into a long-term multi-stage TEP approach. In section 4.1 the mathematical formulation of the multi-stage TEP presented. Section 4.2 discusses the solution approach followed. The minimax regret decision analysis scheme incorporated for selection of the expansion plan is presented in section 4.3. Finally in section 4.4 a case study of the multi-stage TEP problem applied on the 39 bus New England power system is presented and discussed.

Chapter 5 summarizes the conclusion and possible directions for future work.

In appendix A, the derivation of the admittance matrix for the existing and the new network topology, after including the new candidate transmission plan, is provided. Appendix B and C provides the network data of the 39 bus New England test system and the load profile used in this work. Finally in Appendix D, sample modeling AIMMS code of the proposed deterministic TEP problem is provided.

2. Literature Review

In the past, transmission expansion planning (TEP) has been carried out and different approaches have been proposed. This chapter will review and discuss most of these approaches that are helpful and pertinent to this thesis. Thus, the chapter in general is organized in two sections in which the first section revisits the transmission expansion planning models based on both static and dynamic models. The second section goes through the classification of TEP approach due to the uncertainties in power system.

2.1 Background of the Transmission Expansion Planning Methods

In a regulated power system environment, the responsible power system utility takes the task of maintaining and expanding the existing and future electric power generation, transmission and distribution. Therefore to meet the growing demand condition, the utility forecasts the future demand and performs the necessary generation and transmission expansion plan. In the common practice it is usual that the generation plan comes prior to the transmission network expansion planning is carried out. In other words the TEP is performed after the new generating units to be installed and the old decommissioning ones are determined. In this condition the main focus of the transmission expansion planning is to select the optimal and least cost transmission investment alternatives. Therefore, the transmission expansion planning is formulated as an optimization problem with a set of technical and reliability constraints. This optimization process necessitates the earlier pronouncement of cost with an optimal transmission network configuration that minimizes the total investment and operation cost.

A conventional transmission expansion planning procedure decomposed the TEP problem into three steps given in Figure 2-1[2, 3].

1. Generate possible candidate transmission expansion alternatives.
2. Perform financial and other analysis to guide the final plan selection.
3. Conduct technical impact analysis to ensure the feasibility of the plan

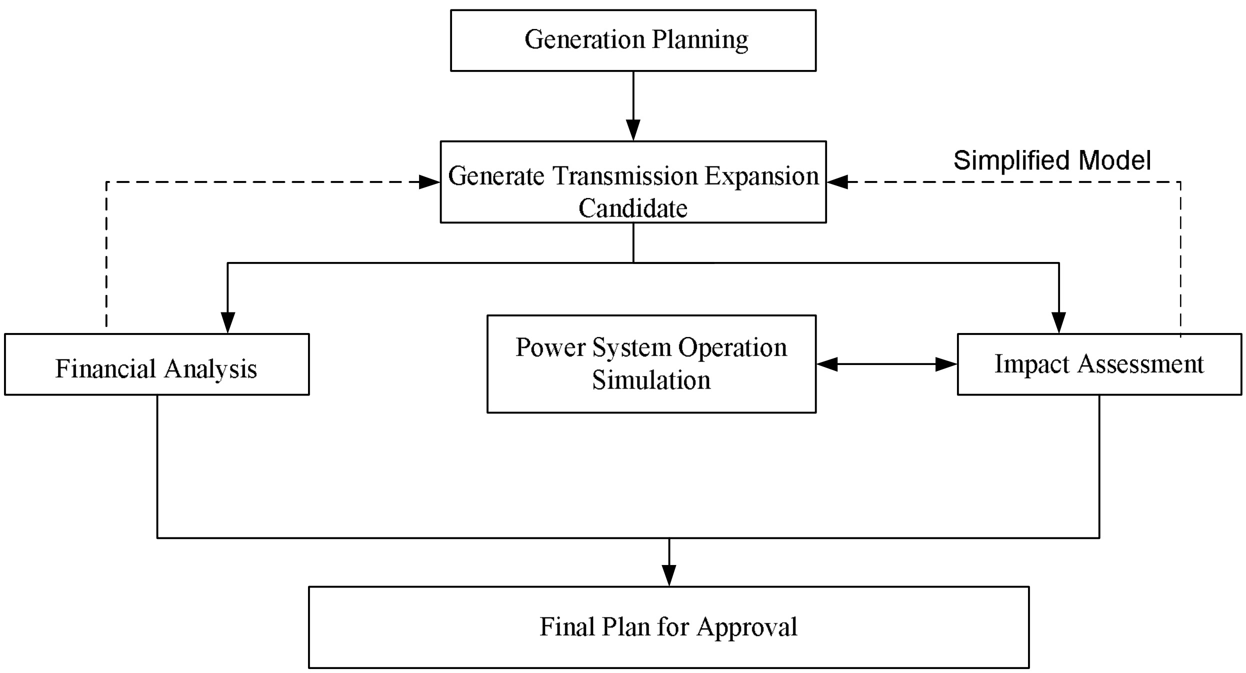

Figure 2-1 Traditional transmission expansion planning procedure

On the other hand, deregulation of power system changes the objective of transmission expansion planning. The objective of TEP under deregulated environment is different from that in the traditional power industry. In regulated environment the main concern is to maximize the total social welfare, long-term reliability and efficiency of the network. While in deregulated environment, besides maximizing the social welfare, problem formulation TEP should include maximization of the investor's or stakeholder's profit as its constraint [4]. Therefore in deregulated environment the decision of transmission expansion is made by taking the economic effect of the investment into account with the other power system investment criteria. It is a complex process as the model take the generator expansion and market related uncertainties into account.

The main objective of transmission planning in deregulated power systems is to provide a nondiscriminatory competitive environment for all stakeholders, while maintaining power system reliability [5]. The general framework of the transmission expansion planning in deregulated environment [2, 3] is shown in Figure 2:

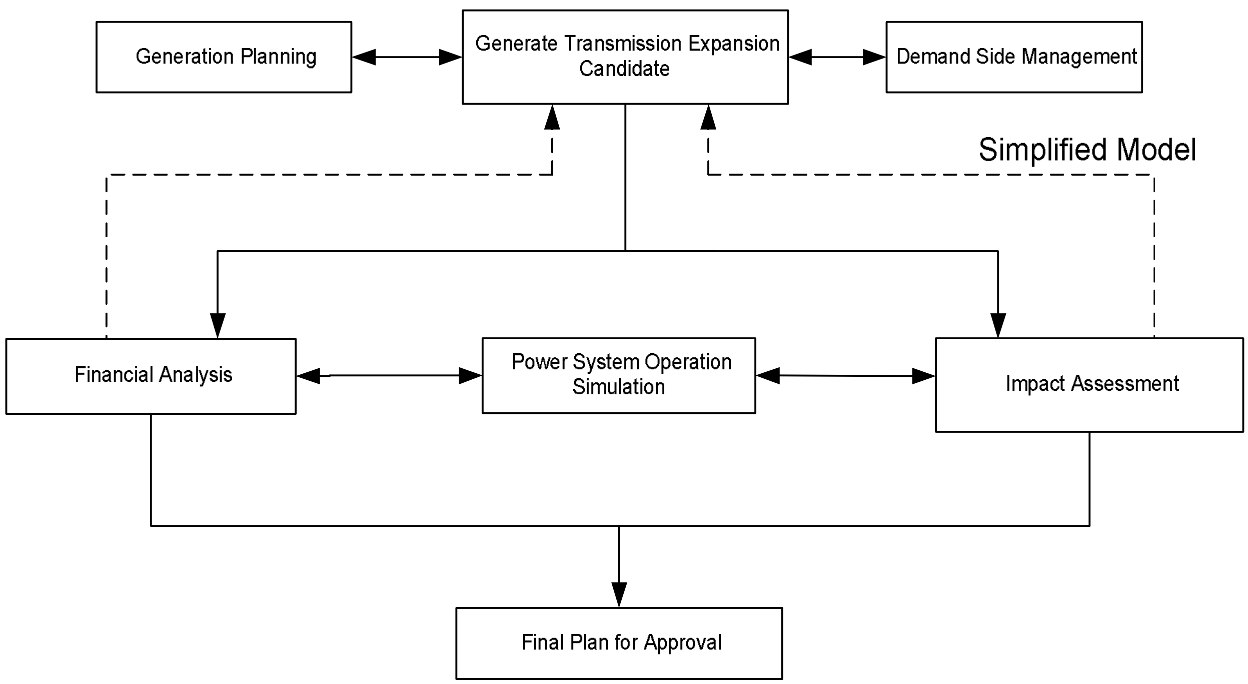

Figure 2-2 Transmission expansion planning procedure in the deregulated environment

From the perspective of power system planning horizon transmission network expansion planning can be classified as static or dynamic. Static expansion involves finding the optimal plan for a single-stage planning horizon. For example, given the network configuration of this year and the peak generation/demand of the next year one can determine the expansion plan with minimum cost. This planning method answers only what transmission facilities must be added to the system and where it must be installed. The static modeling of the transmission network planning is simpler and it allows solving problems of large size in shorter period of time than the dynamic methodology. This methodology can also extend to a multi-year context without difficulty.

Meanwhile in a dynamic planning several years or stages are considered and a year-by-year expansion plan is made that goes from the initial year through the horizon year. The dynamic planning is very complex and large because the planner needs to answer the question when the new transmission facilities must be installed in addition to the sizing and placement. This will result in large number of variables and constraints to consider and requires enormous computational effort to obtain the optimal solution.

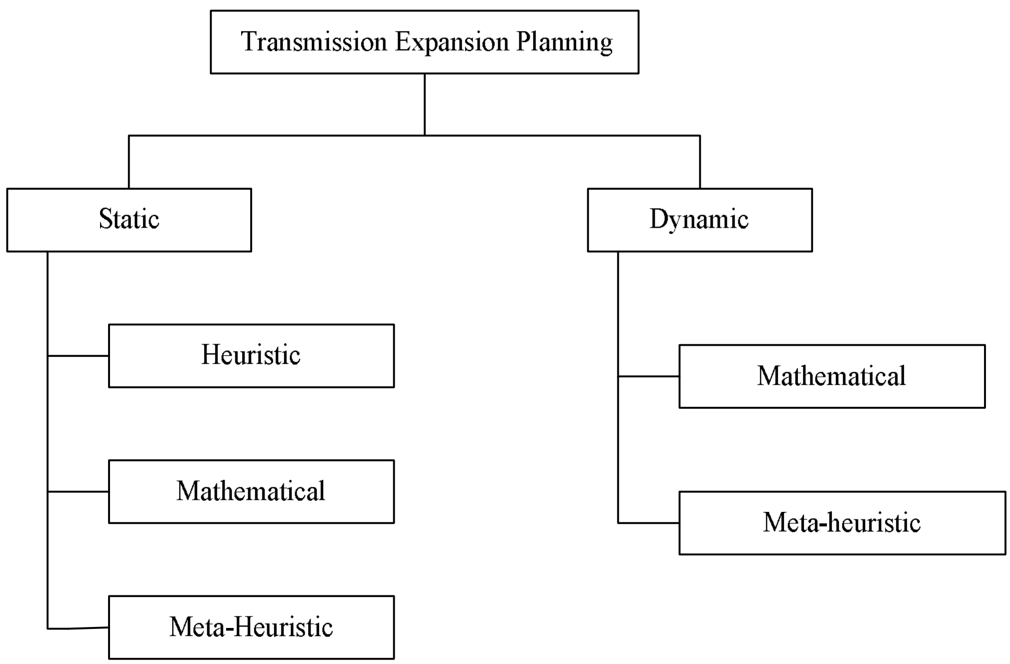

Figure 2-3 - Classification of transmission expansion planning from view point of power system horizon

To attain a reasonable computation time the dynamic problem has to be simplified. The simplest way is to solve a series of the static sub-problems (pseudo-dynamic procedure). There are two methods of applying the pseudo-dynamic transmission planning. The first one is the “forward” procedure, which solves the static expansion planning problem for all years of planning horizon sequentially (starting from the first to the last). The second way is the "backward" procedure, which first solves the static expansion planning problem for the last year and then tries to solve the intermediate years [6].

The TEP is a non-convex mixed-integer nonlinear programming problem. The presence of integer investment variable that requires the use of a combinatorial algorithm is the main difficulty of searching the optimal solution of this problem. Another difficulty of the problem arises from the large number of variables associated with many economical and operational constraints to be considered. Therefore, to overcome these associated difficulties, different algorithms have been proposed by many researchers. Brief review of the proposed approaches is presented in the next section.

2.1.1 Static Transmission Expansion Planning

From the viewpoint of algorithms applied to solve the static TEP (STEP) problem, the transmission planning approach can generally classified as: heuristic, mathematical optimizations and the meta-heuristic methods [6, 7].

2.1.1.1 Heuristics Methods

The term heuristics is used to describe all techniques that undergo a step by step generating, evaluating and selecting expansion option. A component of the solution is added at each step until good quality solution is found. It is robust and converges quickly to the optimal solution, but for large scale and complex problem it may converge to local solution that is very far away from the global optimal solution.

One of the first approaches developed to solve the transmission network expansion problem is dated of 1970 by Garver [8]. In this work, the problem was formulated as a power flow problem in which the objective function and constraints are described by linear functions that neglect the ohmic power loss. Based on the result of flow estimate new lines will be added on the largest overload network. Considering the added line, new linear flow is computed and the process continues until no overload exists in the system.

Latorre *et al.* [9] proposed a heuristic method that took the advantage of natural decomposition of the transmission expansion problem into investment and operation sub-problems. The investment sub-problem is solved by a heuristic procedure while the operation problem is solved by a well-known optimization technique.

The heuristic approach that tries to solve the same problem using sensitivity analysis was proposed [10-18]. At each step of the heuristic algorithm, the sensitivity index was used to determine the circuit to be added to the system. The sensitivity index can be built based on the algorithm that employs the electrical system performance (like minimum load shedding [10], load supplying capability [11], least effort criteria [13] the relaxed version of their own mathematical model [12], [14, 15] or optimal power flow in the circuit [16, 17]). In most of these models the interior point method is employed to solve the resulting linear or non-linear programming problem during each iteration.

2.1.1.2 Mathematical Optimization Methods

The linear programming technique is one of the first mathematical optimization method adopted to solve the transmission network expansion problem. In this case, both the objective function and the constraints are linear [19, 20]. In [19] the overall linear transmission expansion planning (TEP) problem was decomposed into two independent problems, investment and operation problems, which is defined by a linear programming model and independent Monte Carlo simulation based on DC load flow model respectively.

A TEP problem approach that involves the application of linear and dynamic programming was proposed by Kaltenbatch *et al.* [21]. In this article the minimum cost capacity increment required to ensure the change in generation and demand was computed by linear programming whereas dynamic programming is used to optimize the actual investment expense.

Nonlinear programming is the other mathematical programming tool used in solving the TEP problem [22]. In this scheme both the objective function and some of the constraints are formulated as nonlinear equations. The objective function considers the minimization of investment cost, ohmic and corona loss. The main drawbacks of this approach are that the optimal solution may fall into local optima and difficulties associated with the selection of initial value of the unknown power flow variables.

The transmission network expansion planning problem has been solved using zero-one implicit enumeration algorithm [23]. In this work a binary representation of zeros and ones are used to represent the number of new line additions to the network at the third phase linear investment sub-problem of the hierarchical bender decomposition [24]. Das *et al*. [25] also proposed a combined implicit enumeration and linear programming approach employed on a generation-transmission system expansion planning.

A mixed-integer linear programing approach that considers the transmission power losses of the system was proposed in [26] and [27]. In both approachs the transmission expansion planning problem is solved by transforming the mixed-integer nonlinear programming problem into a mixed integer linear programming problem.

The mathematical decomposition scheme is another optimization method used in solving the expansion problem. One of the first approaches is formulated by Pereira *et al.* [28]. In this work Bender decomposition was applied to decompose the main problem into two sub-problems: the master investment and operation sub-problem. The master problem decides the trial expansion plan and the second sub-problem, given the trial expansion plan, solves the operational problem and finds the additional constraints in terms of the investment variables through Bender's cuts. These new linear constraints will be added to the existing investment sub-problem and the modified problem will be re-solved until the optimal solution is obtained.

Another decomposition method that divides the overall transmission expansion into investment problem and operation problem, was proposed by Levi *et al.* [29]. The investment problem was specified as the minimum cost problem that decomposes into minimum load curtailment model which dealt with initial load flows and the marginal network model for the determination of superimposed power flow. The operation problem was solved by applying the Monte Carlo Simulation.

In 1994, due to the non-convexity of the problem, Romero *et al.* [24] used the hierarchical decomposition approach that is composed of three different levels of transmission network modeling. In the first phase the Benders decomposition is applied to solve the expansion problem considering transportation model. In the next phase the network model is represented by a hybrid model (DC model for existing branches and transportation model for new branches). Finally, in the last phase the DC power flow is used to model the operational sub-problem for all branches and the investment sub-problem is solved by using integer linear programming.

Following this work, a two phase hierarchical Benders decomposition model was proposed by Oliveira *et al*. [30]. In phase one the relaxed transportation model of the operation sub-problem is solved by the network flow model. After this, the benders feasibility cuts are used in phase two expansion problem. The investment sub-problem was solved by using the Geoffrion heuristic approach. To improve the optimality of the solution, the Gomory cut is used beside the Bender cuts.

Later in 2001 a New Bender decomposition method was proposed by Granville *et al*. [31]. To solve the expansion problem, the new approach uses a linear disjunctive model. They also use the Gomory cuts, which improve the convergence to the optimal solution of the Bender decomposition approach, instead of the Bender cuts.

The other mathematical optimization method used to solve this problem was the branch and bound (B&B) algorithm. The first method presented by Haffer *et al*. [32] was based on the transportation model. In this approach the branch and bound algorithm is used to solve the investment decision after decomposition of the original problem into two sub-problems. And the network operation problem was solved using a specialized linear programming. One year later, in their second paper, they proposed a new algorithm that applies branch and bound directly to the original problem without decomposition.

The branch and bound algorithm using the DC transmission network expansion planning modeling was presented by Rider *et al*. [33]. In this approach, the nonlinear programming TEP problem is solved at each node of the tree using an interior-point method and Pseudo-costs are used to diminish the size of the B&B tree and the processing time.

2.1.1.3 Meta- Heuristic Methods

Meta-heuristic method integrates the features of mathematical optimization and heuristic method. One of the first approaches that are widely adopted to solve TEP problem is the genetic algorithm (GA) [34],[35]. GA is based on the mechanism of evolution and natural genetics. In [34], GA is used to realize the multi-objective optimal planning of the TEP by considering the minimum investment cost, the optimum system reliability and minimum influence on the surrounding as its objective function. Silva *et al.* [36] reported the application of GA on TEP problem which implement the principle of simulated annealing (SA) for improvement of the mutation mechanism and generation of better individual. Later in 2001 [37], they proposed another approach that uses the transportation model to build the initial population and the levels of loss of load to select the best individual of the population. Combinations of GA and Neuro-computing (NC) [38], that can operate more effectively, have also been applied for solving the TEP problem.

The simulated annealing (SA) [39] is the other type of optimization method that is applied to the TEP problem. The SA tries to avoid local optima by allowing temporary, limited deterioration of the actual solution. A parallel SA algorithm [40], that greatly reduces the computational burden and improves the quality of the conventional SA solution, was adopted in solving the expansion planning problem.

A new method of solving the static TEP problem which is based on the application of tabu search (TS) was developed by Wen *et al*. [41]. They developed a tabu search-based method of solving the transmission network optimal planning problem as a zero-one integer programming problem. In addition, a tabu search approach that includes intensification and diversification phase to the main tabu search concepts was also reported in [42]. A greedy randomization adaptive search procedure (GRASP) was also proposed by Binato *et al.* [43].

2.1.2 Dynamic Transmission Expansion Planning

In dynamic trasnmission expansion planning (DTEP), instead of seeking an optimal plan for a single year, multiple years are considered and the optimal plan is searched for the whole planning horizon. Therefore, the objecive of DTEP is to minize the present value of all investment costs carried out along the planning period. In general based on the algorithm applied to solve the problem, the DTEP can be classified as mathematical optimization and meta-heruistic approach.

Traditional mathematical optimization methods, such as linear programming [44], nonlinear programming [45] and dynamic programming [46] have been applied to solve the DTEP problem. Beside a long term trasnsmission expansion model which minimizes the costs of investment and congestion over a planning horizon was proposed by S. Sozer [4]. In this approach the mixed-integer nonlinear programming (MINLP) problem is solved by herachical bender decomposition approach proposed in [24]. Due to the need of huge computational effort the use of mathematical optimization approach in solving large scale dynmaic transmission expansion planning is limited.

As an alternative to the conventional mathematical optimization method, meta-heuristic approaches that yield high quality solution with acceptable computational time was also proposed. In 2004 Escabor *et al*. [47], presented an efficient GA to solve a multi-stage and coordianted planning problem considering transnsmission and generation expansion planning as well as operation cost for the planning horizon. However, the expansion model only includes the cost of transmission investment and loss of load, ignoring the generation and operational costs. Besides other application of GA to solve dyanmic transmission expansion planning have been applied in [48] and [49].

Apart from GA, an integrated approach of genetic algorithm, tabu search and simulated annealing have been proposed by Fonseka *et al*. [50]. Though a high quality solution can be achieved by dynamic transmission expanssion planning, it is often neglected or simplified into series of static sub-problems. The reason is that the consideration of the dynamic planning is complex and acceptably negligible in the long term planning [3].

2.2 Uncertainties in the TEP Problem

The process of solving a TEP problem requires handling of certain and uncertain information. The data which are not known at the time of planning are referred to as uncertain data. The factors that cause these uncertainties are [51]:

- Demand growth

- Economic growth
- Inflation and interest rates
- Environmental regulation
- Public opinion
- Renewable energy sources
- Availability of fuels and technologies
- Individual power generating units (IPS)

These uncertainties can be classified as random and non-random. In random uncertainty the pattern of the parameters can be determined from the historical data’s and past observation. Uncertainties in load, renewable power generation, and generator costs are categorized in this group. Non-random uncertainties are not repeatable and cannot be statistically represented from past experience [5]. Transmission expansion cost, shutting down of generators, and the like are grouped in this category.

Transmission expansion planning can be done with or without considering these uncertainties. Therefore, from the perpective of uncertainities in the power system, TEP can be divided into two categories [5, 52]:

1. Deterministic
2. Non-Deterministic

2.2.1 Deterministic TEP Approach

A deterministic transmission expansion planning is formulated as a traditional optimization problem, which analyzes single or two representative scenarios. This scenarios can be worst peak load level, N-1 contingency or outage of a generating unit [53]. In this method the uncertain factors for future condition are assumed to be either perfectly known or forecasted based on the current best information. Solving a TEP problem based on this method is quite simple and requires significantly less amount of effort. Thus, the optimal investment strategy of the network for the planning horizon is known with certainty [52]. For every stage of the investment planning horizon, a new set of forecasts is assumed and decision of investment strategy will be made by recalculating the optimization problem. However, the main drawback of this method is that it tries to represent the past experience and future expectation by a single fact. Therefore, the expansion solution for the future condition becomes optimal only if it occurs as predicted. Otherwise, the solution may lead to inadequate or expensive planning decision. Besides the investment strategy of each stage is optimal for limited time of period, usually fails to provide long term investment plan [54].

2.2.2 Non-Deterministic TEP Approach

In most cases, to provide safe operation of the power system, the deterministic transmission expansion planning is adopted by using the highest demand level (the worst case scenario). Since the probability of occurrence of this situation is less, the expansion plan may result in an investment cost which is much higher than needed [1]. Therefore, to overcome the drawback of the deterministic TEP approach, a non-deterministic TEP problem is formulated by generating a set of possible scenarios of the uncertain parameters that may take place in the future. In this approach, a number of possible scenarios will be analyzed and evaluated using security and performance analysis criteria. Consideration of the uncertainties will help to identify a robust plan that is satisfactory under a range of possible outcome. In this condition, the TEP problem can be solved either by means of a stochastic optimization-based formulation, where the objective function is typically formulated in term of an expected value or by means of a decision-making framework, which encompasses a deterministic optimization plus a decision tree analysis [52]. The non-deterministic way of solving a TEP problem is a challenging task that needs an adequate treatment of different types of information. Therefore a great effort and care must be taken while solving the problem [4].

2.2.2.1 Scenario Analysis

Scenario analysis, besides the other non-deterministic TEP approaches, is one method used to solve a non-deterministic planning problem [5]. In this approach a number of possible future scenarios of uncertain parameters will be determined at first. Then all the scenarios will be analyzed and set of optimal expansion plan for each scenario is determined. Depending on these set of optimal solutions of each scenarios, decision analysis technique will be carried out and a final optimal plan which is, on average adequate for all scenarios will be selected [55]. The decision criteria used for selection of the final plan varies from planner to planner depending on their interest. The selection criteria could be based on: Expected cost criteria, Minimax regrets, Von Neumann-Morgenstern criterion, Hurwicz criterion, Robustness criterion and the like [5, 56].

2.3 Problem Formulation

In this thesis a method of transmission expansion planning for future load and generation condition is proposed. The proposed approach is based on the DC power flow model and the locational marginal price is introduced in the optimization problem to alleviate congestion. The TEP problem is formulated as an optimization problem with a set of equality and inequality operational constraints. It has an integer decision variable that indicates the number of optimal candidate investment plan. These term makes the problem to be a mixed-integer nonlinear programming (MINLP) problem with continuous and integer (discrete) variables and nonlinearities in the objective function and constraints [57].

The general formulation of MINLP problem is [58]:

$$\min_{x,y} f(\mathbf{x},\mathbf{y}) \quad \text{2-1}$$

Subjected to: $$h(\mathbf{x},\mathbf{y}) = 0 \quad \text{2-2}$$

$$g(\mathbf{x},\mathbf{y}) \le 0 \quad \text{2-3}$$

$$\mathbf{x} \in \mathbf{X}$$

$$\mathbf{y} \in \mathbf{Y}\text{, integer}$$

The Eq. (2-1) is a objective function, which includes the capital investment cost of transmission elements as well as the operating cost of the system. Eq. (2-2) represents the active power balance equality constraint and Eq. (2-3) represents the inequality constraints imposed by the generating units and the transmission network of the power system. The vector variables $\mathbf{x}$ and $\mathbf{y}$ are the control and decision variables respectively, where $\mathbf{y}$ represent a vector of integer variables whereas $\mathbf{x}$ are continues variables. $\mathbf{X}$ and $\mathbf{Y}$ impose the lower and upper bound binding restrictions on the variables. This problem is essentially finds the minimum of a real valued objective function subject to equality and inequality constraints defined by vector valued functions (*h* and *g*) in the continuous-discrete (*x*-*y*) space [58].

Besides, an additional constraint which alleviates congestion or equalizes the locational marginal price (LMP) of the system included in the optimization problem. In other words, all the transmission lines must transfer power that is lower than their maximum power transfer capacity limit, no congestion. This constraint can be introduced either by:

$$\mu_{ij} = 0 \quad \text{or}$$
$$\lambda_i = \lambda_j, \quad i \neq j \tag{2-4}$$

where λ_i, λ_j the locational marginal price (shadow price) at bus i and j

μ_{ij} the shadow price of the transmission line connecting bus i to bus j

To do so, in addition to the original equality and inequality constraints of the TEP problem, the extra constraints which are resulted from the derivation of the Lagrangian multiplier and the Karush-Khun-Tucker (KKT) optimality condition of the inequality constraints is included in the TEP optimization problem.

2.4 Summary

The literature review has provided an essential presentation on the approaches and conceptual frameworks, academic debates, scholarly writings and perspectives of the important subject matter of TEP models by categorizing them into various groups based on their techniques and approaches with an introductory backdrop. Under the static transmission expansion planning category, mathematical optimization methods, heuristics methods, meta-heuristic methods were the major approaches that are treated as one group of TEP model, whereas dynamic TEP, was treated concisely by describing it as a model that seeks an optimal plan for the whole planning horizon.

Definitions and outlooks on the process of solving a TEP problem have been discussed; meaning and implication of a TEP problem as part of the broader context of transmission expansion planning models have been presented. The causes of uncertainities in the TEP problem and the approaches, in the form of deterministic and non-deterministic with relevance to this thesis work are discussed. The final part of the chapter, presents the TEP problem formulation used in this work.

3. Transmission Expansion Planning

In an attempt to have a deeper understanding of the TEP approach this chapter presents the mathematical formulation for the transmission expansion planning modeling based on optimal power flow (OPF) and locational marginal price (LMP). In doing so, the OPF and LMP derivation will be introduced. The chapter begins with the mathematical formulation of alternating current optimal power flow (AC OPF) and the linearized simplification of the AC power flow, the direct current optimal power flow (DC OPF). Also included in this section is a formulation of a deterministic single-stage DC OPF based transmission expansion planning model. Then, the formulation of the Lagrangian multiplier associated with the binding constraints of the proposed transmission expansion model is introduced. Later in this chapter, the physical meaning of the locational marginal price and the congestion cost will be discussed. Finally, a case study of the New England 39 bus test power system of the proposed approach will be provided.

3.1 Optimal Power Flow

OPF is modeled as an optimization problem that minimizes or maximizes a given objective function subjected to a number of constraints. The most common objective function include minimum operation cost, minimum active power losses, minimum shift of generation or other control variable from an optimum operating point, etc [59]. In general OPF problem can be expresses as [60]:

$$\min\ f(\mathbf{x},\mathbf{u}) \tag{3-1}$$

Subjected to:

$$g(\mathbf{x},\mathbf{u}) = 0 \tag{3-2}$$

$$h(\mathbf{x},\mathbf{u}) \le 0 \tag{3-3}$$

where

$f(\mathbf{x},\mathbf{u})$ is objective function

$g(\mathbf{x},\mathbf{u})$ is the equality constraint

$h(\mathbf{x},\mathbf{u})$ is the inequality operating constraints

$\mathbf{x},\mathbf{u}$ is the vector of state and control variables

Depending on the selected objective function, an OPF problem can be formulated in a different way. In most cases the objective of the OPF problem is to minimize the total generator fuel cost subjected to the power balance, transmission line power flow and generators power output limits. This constraints can be represented either using AC power flow or DC power flow model. In AC OPF the AC power flow equations are utilized and both the active and the reactive power balance at all nodes of the system are considered. In DC OPF, the AC approximation, DC power flow is used and only the active power balance of the system is taken into account. Detailed simplification is provided in Section 3.1.2.

3.1.1 AC Optimal Power Flow

Given the π equivalent circuit of medium transmission line shown in Figure 3-1, the complex power that flow through the transmission line connecting bus i to j is given as:

$$S_{ij} = P_{ij} + jQ_{ij} \tag{3-4}$$

where P_{ij} is the active power through transmission line $i-j$

Q_{ij} is the reactive power through transmission line $i-j$ and

S_{ij} is the complex (apparent) power through transmission line $i-j$

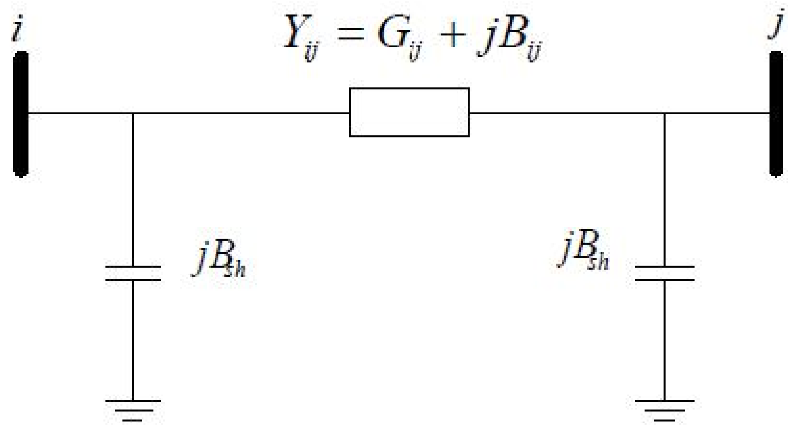

Figure 3-1 - π equivalent circuit representation of a transmission line

The relationship between the impedance and admittance of the transmission line is:

$$Y_{ij} = Z_{ij}^{-1} \quad \text{3-5}$$

where $Z_{ij} = R_{ij} + jX_{ij}$ and $Y_{ij} = G_{ij} + jB_{ij}$

The conductance (G_{ij}) and the susceptance (B_{ij}) of the transmission line are determined from its resistance (R_{ij}) and reactance (X_{ij}) as:

$$G_{ij} = \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} \quad \text{3-6}$$

$$B_{ij} = \frac{-X_{ij}}{R_{ij}^2 + X_{ij}^2} \quad \text{3-7}$$

The active power that flows through bus i to bus j is given as:

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}) \quad \text{3-8}$$

And the reactive power flow through the transmission line $i - j$ is:

$$Q_{ij} = -V_i^2 (B_{sh} + B_{ij}) - V_i V_j (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}) \quad \text{3-9}$$

The general mathematical formulation of the AC OPF problem for power system including N_b number of buses, N_g number of generating units and N_l number of transmission lines can be formulated as:

$$\min F = \sum_{k=1}^{N_g} C(P_{Gk}) = \sum_{k=1}^{N_g} a_k P_{Gk}^2 + b_k P_{Gk} + c_k \qquad \textbf{3-10}$$

Subjected to:

where F is the total generation cost

$C(P_{Gk})$ is the real power generating cost of unit k.

P_{Gk} is the real power generation of the k^{th} generator in MW

a_k, b_k and c_k are the k^{th} generator quadratic cost coefficients

Power Balance Constraints: For each node of the transmission network the power balance equation must be applicable. This is given as the total power generation minus the total power demand at each bus must be equal to the net power flow through the lines connected to it. The real and reactive power injections are expressed as:

$$P_i = P_{Gi} - P_{Di} \qquad \textbf{3-11}$$

$$Q_i = Q_{Gi} - Q_{Di} \qquad \textbf{3-12}$$

where P_{Di} is the total real power demand at bus i

P_i is the real power injection at bus i

Q_{Gi} is the reactive power generation at bus i

Q_{Di} is the reactive power load at bus i

Q_i is the reactive power injection at bus i

The equations for computing the real and reactive power injection at each bus is expressed as:

$$P_i = V_i \sum_{j=1}^{N} V_j \left[G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right] \qquad \textbf{3-13}$$

$$Q_i = V_i \sum_{j=1}^{N} V_j \left[G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right] \qquad \textbf{3-14}$$

Power Flow Constraints: This constraint specifies the apparent power flow through transmission line (from bus i to j) have to be within the upper bound of the power transfer capability limit of the line. This limit is based on the thermal consideration of the line and given as:

$$\left|S_{ij}\right| \leq \overline{S_{ij}} \qquad \textbf{3-15}$$

Where: S_{ij} is the apparent power flow through transmission line $i-j$

$\overline{S_{ij}}$ the maximum apparent power transfer capacity limit of branch $i-j$

Generators Capacity Constraint: This constraint specifies the maximum and the minimum real and reactive power generation capability of the generating units. The power generations outside these limits are inapplicable due to technical reasons.

$$\underline{P_{Gi}} \leq P_{Gi} \leq \overline{P_{Gi}} \qquad \textbf{3-16}$$

$$\underline{Q_{Gi}} \leq Q_{Gi} \leq \overline{Q_{Gi}}$$

where $\overline{P_{Gi}}$ and $\underline{P_{Gi}}$ the maximum and minimum active power generation limits of generator at bus i respectively

$\overline{Q_{Gi}}$ and $\underline{Q_{Gi}}$ the maximum and minimum reactive power generation limits of generator at bus i respectively.

Voltage Constraint: this constraint specifies limit on the maximum and minimum voltage magnitude at each bus i.

$$\underline{V_i} \leq V_i \leq \overline{V_i} \qquad \textbf{3-17}$$

$\overline{V_i}$ and $\underline{V_i}$ the minimum and maximum voltage magnitudes at bus i respectively

3.1.2 DC Power Flow

The dc power flow equations are resultants of linearization of the AC active and reactive power flow equations given in Eq.3-10 to Eq. 3-17 above. These simplifying assumptions are:

i. Neglecting the resistance of the transmission lines as it is rather small compared to the inductance. This means that the conductance of the transmission lines are zero ($G_{ij} = 0$) and admittance matrix is represented only by the line susceptance (B_{ij}). After applying this assumption the power flow equations are:

$$P_i = V_i \sum_{j=1}^{N} V_j \left[B_{ij} \sin(\delta_i - \delta_j) \right]$$

$$Q_i = -V_i \sum_{j=1}^{N} V_j \left[B_{ij} \cos(\delta_i - \delta_j) \right]$$

ii. The phase angle difference between any two buses is rather small. Therefore:

$$\sin(\delta_i - \delta_k) = \delta_i - \delta_k \text{ and } \cos(\delta_i - \delta_k) = 1$$

iii. The magnitudes of the voltages at each bus are equal to 1pu.

Therefore, after applying all assumptions to the AC OPF formulation, the DC optimal power flow equations are:

$$\min \sum_{k=1}^{N_g} C(P_{Gk}) = \sum_{k=1}^{N_g} a_k P_{Gk}^2 + b_k P_{Gk} + c_k \quad \text{3-18}$$

Subjected to constraints:

$$P_{Gi} = P_i + P_{Di} \quad \text{3-19}$$

$$P_i = \sum_{j=1}^{N_b} B_{ij} (\delta_i - \delta_j) \quad \text{3-20}$$

$$\left| P_{ij} \right| \le \overline{P_{ij}} \quad \text{3-21}$$

$$P_{ij} = \frac{1}{X_{ij}} (\delta_i - \delta_j) = B_{f,ij} (\delta_i - \delta_j) \quad \text{3-22}$$

$$\underline{P_{Gi}} \le P_{Gi} \le \overline{P_{Gi}} \quad \text{3-23}$$

where B_{ij} is the imaginary part of the ij^{th} element of bus admittance matrix

$B_{f,ij}$ is the susceptance of the transmission line connecting bus i to j

3.2 The Mathematical Formulation of the Transmission Expansion

As I have discussed in the second chapter the transmission expansion planning has been extensively studied and several mathematical modeling are employed to represent the transmission network. These are: the transportation model, the hybrid model, the disjunctive model, and the DC power flow model [61]. Recently the AC power flow model also came to application [16].

Usually the DC power flow model is the most extensively employed in solving long term TEP problem formulation as it satisfies the basic conditions stated by the operation planning studies of the power system network. It has an integer decision variable which indicates the selection and number of candidate circuits of the optimal expansion plan. The branch susceptance of each candidate circuit and the Kirchhoff's voltage law (KVL) is expressed as a function of the integer variable. These terms make the problem to be a mixed-integer nonlinear programming (MINLP) problem that its complexity increases as the size of the system increases. Furthermore, the expansion plan obtained from this model should be further investigated by using more realistic operation planning tools such as AC power flow, stability analysis, transient analysis and short circuit analysis [12]. To reduce the computational burden of the problem, the transportation and the hybrid models that are obtained by relaxing the constraint representing the Kirchhoff's second law of the DC model are also used. This relaxation results in an integer linear programming problem, which is simpler to solve than that of the DC model.

3.2.1 Transmission Network Enhancement Methods

In the future, due to the growing energy demand of the power system, the existing transmission system may become more stressed and congested. In congested power system the generation and/or demand has to be rescheduled to ensure reliability, security and normal operation condition of the system. In doing so an increased power generation from the expansive generating unit becomes mandatory as no power can transfer from the cheap generating bus to the load bus. This could cause an increase in the total operating cost and locational marginal price (LMP) at each bus. In the long run congestion of the power system can be relieved by increasing the available power transfer capability limit of the transmission system to meet the generation/load condition of the future power system. This in turn has additional system benefits such as reliability enhancement and reduction of system degradation due to operation close to capacity limit [62]. Otherwise, increase in the price of the electricity, decreases the security and reliability of the power system and increasing possibility of cascade outages of the system may result.

There are several ways in which the transmission capacity of the network can be increased. Commonly, it can be classified into two groups. The first type of transmission network enhancement is to upgrade the power transfer capability of the existing transmission system. This is done by addition of new network components or replacing components that are already in the network. These include [63]:

- New relays and switches
- New remote monitoring and control equipment
- Re-conductoring of existing links
- Operating specific transmission line at higher voltage level, with in its design limits.
- Installing new substation facilities to improve the power flow distribution among the different paths
- Transformer upgrade
- Capacitor addition

The second type of the network improvement, the one that is done in this work, is to build new transmission lines in parallel to the existing grid.

Traditionally TEP is mostly performed as cost minimization optimization problems that minimize the sum of investment cost and the cost of load curtailment caused by lack of transmission capacity, subjected to DC or AC load flow constraints. This way of the TEP approach totally neglects the explicit optimization of the power production cost and the economic effect of transmission line congestion on the network power clearing price. A TEP problem whose main objective is to minimize the investment cost of the new transmission lines and the total operation cost of the generating units is proposed in this work. The mathematical formulation of the proposed TEP problem is presented in the next section.

3.2.2 Proposed Deterministic Expansion Planning Model

Transmission expansion planning is a planning process, though it has a dynamic nature, often tackled by the simplified static transmission planning mathematical model [64]. Depending on the type of the power system and interest of the planner the objective function of the transmission expansion planning problem varies. For example in a market driven transmission investment the main purpose of the expansion strategy is to provide non-discriminatory competitive environment meanwhile maintaining power system reliability. In this case, beside the technical and economic criteria of the system, marketed based criteria must be included for measuring the goodness of the expansion plan [5]. On the other hand, if the transmission planner is a non-profitable company, then the planning problem can be tackled as the minimization of the expansion cost and the system operation cost meanwhile minimizing the congestion cost. Whereas, if the expansion is done by a profit based transmission company planner, the decision of the transmission expansion investment depend on the amount of money the company yield and year of the investment return. In this section the formulation of the transmission expansion planning, based on DC optimal power flow and LMP, used for this thesis work is presented as follows:

Traditionally, transmission expansion planning is done by assuming the new candidate transmission line to be built have the same characteristic (impedance and maximum power transfer capacity) with the existing ones [61]. But in practice this may not be the case and the planner has the chance to select a new type of circuit sets that can be installed in parallel to the existing ones or other new right of way. In this condition instead of using the index (ij) for a circuit with terminal buses of i and j, each circuit is identified by a pair of indices like (ij, o). $ij \in L$ and $o \in O$, where L is the set of all lines connecting bus i and j and O is the set of possible transmission line options.

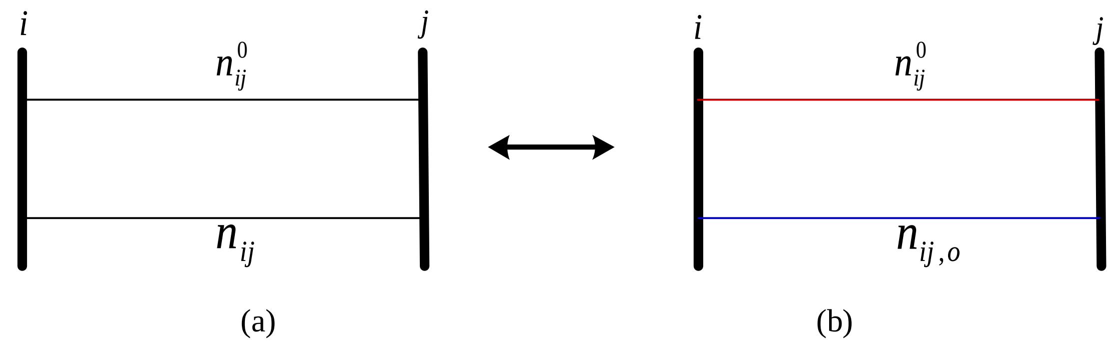

Figure 3-2 Single line representations of a parallel transmission line between bus i and j.

Figure 3-2(a) represents TEP based on new transmission line options that have the same characteristics with the existing one (n_{ij}^0). While in Figure 3-2(b) the different characteristics of the existing (n_{ij}^0) and the new transmission ($n_{ij,o}$) options is given in different color. After taking this condition into consideration the mathematical formulation of the static transmission expansion planning problem will be conversed next.

3.2.3 Objective Function

The objective function of the TEP problem consists of the minimization of the sum of the investment cost and the operational cost. The investment cost is the cost of building the proposed transmission line. It is dependent on various factors and mainly composed of installation cost, labor cost, material cost and other related costs. For a single stage static expansion strategy, it is given as:

$$C_{iv} = \sum_{o}^{N_o} \sum_{ij}^{N_l} C_{ij,o} n_{ij,o} \qquad \forall (ij) \in CL, \ \forall (o) \in O \qquad \text{3-24}$$

where $C_{ij,o}$ is the investment cost of the transmission line $i-j$ of option o in \$

$n_{ij,o}$ is a positive integer decision variable ($n_{ij,o} \neq 0$ if a line is built between bus i and j; $n_{ij,o} = 0$ if the line is not built)

N_l is the number of candidate circuit built in branch $i-j$

N_o is the possible type of the new circuits to be built in ij^{th} branch

The operating cost is the cost of power generation needed to meet the demand. It is given as the sum of the quadratic cost function of all generating units. The total operating cost of real power generation for a system with "N_g"generating units is given as:

$$C_{op,h} = \sum_{k}^{N_g} C_k(P_{Gk}) = \sum_{k=1}^{N_g} (a_k P_{Gk}^2 + b_k P_{Gk} + c_k) \qquad \forall(k) \in G \qquad \text{3-25}$$

where $C_{op,h}$ the hourly operation cost in $

P_{Gk} is the amount of power generation of generator k in MWh.

a_k, b_k and c_k - the cost coefficients of k^{th} generating unit

k is the index of the generating units

The operation cost of the generators is calculated every hour or 8760 times a year. Therefore the annualized operation cost will be given as:

$$C_{op,y} = 8760 * C_{op,h}$$

where $C_{op,y}$ is the annual operation cost.

Thus, the final objective function, which is the minimization of the total investment and annual operation cost of the expansion planning problem expressed as:

$$\min W = C_{op,y} + C_{iv} \qquad \text{3-26}$$

Given the above cost minimization objective function of the TEP problem, the mathematical model has to include the operational, physical, economical and social constraints that assure secure and reliable system condition which satisfies the transmission network requirements [65]. These constraints are based on DC OPF and are reformulated to include the effect of transmission line addition on the system variables and parameters.

3.2.4 Constraints

These constraints of the TEP problem reflect the limit on the operational and technical conditions of the power system network. These are:

- *Active power balance* - It is the linear equality constraint that models the Kirchhoff's Current Law (KCL) and represents the conservation of active power flow at each bus of the given network.

$$\sum_{k\in\Omega_i} P_{Gk} = P_i + P_{Di} \qquad (\lambda_i) \qquad \forall(k)\in\Omega_i, \forall(i)\in B \qquad \text{3-27}$$

where $P_i = \sum_{j=1}^{N_b} B_{BusNew,ij}(\delta_i - \delta_j) \qquad \forall(i,j)\in B, \forall(ij)\in L$

P_{Gk} is the active power generation of the k^{th} generator at bus i

P_{Di} is the active power demand at bus i

P_i is the net active power injection at bus i

δ_i, δ_j is the bus angle at bus i and j respectively

$B_{BusNew,ij}$ is the ij element of new susceptance matrix of the transmission system after considering the new candidate transmission line. The element of this matrix is calculated as:

$$B_{BusNew,ij} = \begin{cases} B_{Bus,ij} + \sum_{o}^{N_o} B_{ij.o} n_{ij,o} & (i \neq j) \\ B_{Bus,ij} + \sum_{o}^{N_o}\sum_{J}^{N_b} B_{ij.o} n_{,ij,o} & (i = j) \end{cases}$$

where

$B_{Bus,ij}$ is the ij^{th} element of the susceptance matrix of the existing network

$B_{ij,o}$ is the susceptance of the new circuit added in branch $i-j$ of option o.

$n_{ij,o}$ is the total number of circuit added in branch $i-j$ of option o (integer)

- *The branch power flow* - This constraint expresses the Kirchhoff's Voltage Law (KVL) of the equivalent DC network and limits the power flow at each branch of the power system. The power flow through the new transmission lines is positive if it flows from i to j, and negative otherwise. The power flow through branch $i-j$ expressed in terms of the existing and the new transmission line option of o is given as:

Through the new line of type o:

$$P_{ij,o} = B_{f,ij,o} * n_{ij,o} * (\delta_i - \delta_j) \qquad \forall(i)\in B, \forall(ij)\in CL, \forall(o)\in O \qquad \text{3-28}$$

And through the existing one are given as:

$$P_{ij} = B_{f,ij} * (\delta_i - \delta_j) \qquad \forall(i) \in B, \ \forall(ij) \in EL \qquad \text{3-29}$$

where $n_{ij,o}$ is the number of additional transmission line from bus $i-j$ of option o

$P_{ij,o}$ is the real power flow through the new transmission line $i-j$

P_{ij} is the real power flow through the existing transmission line $i-j$

$B_{f,ij,o}$ is the susceptance of the new transmission line connecting bus i to j with candidate option o.

$B_{f,ij}$ is the susceptance of the existing transmission line connecting bus i to j

The inequality constraint reflects the upper and lower bound limits on the device's physical and economical condition of the power system expansion problem. The physical devices that require enforcement of limits are the generators and transmission lines. These constraints are:

- *The transmission power flow limit* - This inequality constraint represent the limit of maximum power flow at each branch of the network based on the thermal and dynamic stability consideration. It is given for both the new and the existing transmission lines separately as:

$$\left|P_{ij,o}\right| \le n_{ij,o} * \overline{P_{ij,o}} \qquad (\mu_{ij,o}) \qquad \forall(ij) \in CL, \forall(o) \in O \qquad \text{3-30}$$

$$\left|P_{ij}\right| \le \overline{P_{ij}} \qquad (\mu_{ij}) \qquad \forall(ij) \in EL \qquad \text{3-31}$$

where $\overline{P_{ij,o}}$ the ij^{th} new transmission line real power capacity limit of option o

$\overline{P_{ij}}$ the maximum real power capacity limit of ij^{th} transmission line

It should be noticed that if a candidate branch ij with option o is selected as an expansion plan i.e. $n_{ij,o} = 1$ then Eq. (3-28) and Eq. (3-30) will become active. This will change the admittance matrix and the power transfer distribution factor (PTDF) of the power system which resulted in a different power flow pattern. On the other hand, if the candidate branch is not selected i.e. $n_{ij,o} = 0$ then $P_{ij,o}$ is zero and the PTDF and admittance matrix of the power system will remain the same.

- *The generators power output limit* - This constraint induce the minimum and maximum power generation limits of each generating units. Power generation outside this region is infeasible due to technical reasons. It is represented as:

$$\underline{P_{Gi}} \le P_{Gi} \le \overline{P_{Gi}} \qquad (\eta_i) \qquad \forall (i) \in G \qquad \text{3-32}$$

- *The right of way limit* - This constraint helps the planner to know the exact location and number of new required lines. It is included in the expansion planning problem to define the maximum number and location of new circuit that can be installed in a specified location. This is because the planners have to meet the community standards of visual impact on the environment along with the economic considerations. Mathematically it is given as [65].

$$0 \le n_{ij,o} \le \overline{n_{ij,o}} \qquad (ij) \in CL \,, \forall (o) \in O \qquad \text{3-33}$$

where $n_{ij,o}$ is the total number of circuit added in branch $i-j$ with option o (integer)

$\overline{n_{ij,o}}$ the maximum number of transmission line of option o that can be added in branch $i-j$.

- *Congestion alleviation constraint* - induce the alleviation of any transmission line congestion in the system. In other words, the LMP at every bus are equal.

$$\mu_{ij} = 0 \qquad \forall (ij) \in L$$

$$\lambda_i = \lambda_j \,, \; i \ne j \qquad \forall (i,\, j) \in B \qquad \text{3-34}$$

where μ_{ij} is the shadow price of the transmission line connecting bus $i-j$.

λ_i, λ_j the LMP at bus i and j

The hard congestion alleviation constraint given in Eq. (3-34) may result in over investment as it does not allow a little congestion in the system. This may lead to an expensive and unnecessary investment decision. To overcome this unnecessary investment due to non-severe congestion of the transmission line, the constraint which limits the maximum power flow through the existing transmission lines (Eq. 3-31) is relaxed to allow overloaded lines in the system. Then the expansion decision is made only on transmission lines that are suffering from a severe congestion, and congestion which are caused by small overloading's will be filtered out. This modification is integrated by relaxing the constraint that limits power flow through the existing lines as:

$$\left|P_{ij}\right| \le (1+\sigma * W_{ij})\overline{P_{ij}} \qquad (\mu_{ij}) \quad \forall (ij) \in EL \qquad \text{3-35}$$

where σ is the maximum percentage of allowable overloading

W_{ij} is binary variable which identify the existence of non-serious overloading in the system ($W_{ij} \in [0,1]$)

$$\sum_{ij}^{N_l} W_{ij} \le 1 \qquad \text{3-36}$$

The Eq. (3-35) shows that the optimization problem will not activate the addition of line if the overloading in the transmission line is not severe and power flow violation is not more than σ percent of the maximum power transfer capacity limit of the line. Eq. (3-36) controls the maximum number of overloaded transmission lines that can be allowed at the same time, in this case only one overloaded line is allowed. Besides the transmission line expansion and the relaxation of the power flow constraint in branch ij are mutually exclusive. This means that, if extra line is built between bus i and j then the additional line must remove the overloading in the existing transmission line.

In summary, the formulated TEP problem can be re-written as:

$$\min W = C_{op,y} + C_{iv} \qquad \text{3-37}$$

Subjected to:

$$\sum_{k\in\Omega_i} P_{Gk} = P_i - P_{Di} \qquad \text{3-38}$$

$$\text{where} \quad P_i = \sum_{j=1}^{N_b} B_{BusNew,ij}(\delta_i - \delta_j)$$

$$P_{ij,o} = B_{f,ij,o} * n_{ij,o} * (\delta_i - \delta_j) \qquad \text{3-39}$$

$$P_{ij} = B_{f,ij} * (\delta_i - \delta_j) \qquad \text{3-40}$$

$$\left|P_{ij}\right| \le (1+\sigma * W_{ij})\overline{P_{ij}} \qquad 3\text{-}41$$

$$\left|P_{ij,o}\right| \le n_{ij,o} * \overline{P_{ij,o}} \qquad 3\text{-}42$$

$$\underline{P_{Gi}} \le P_{Gi} \le \overline{P_{Gi}} \qquad 3\text{-}43$$

$$0 \le n_{ij,o} \le \overline{n_{ij,o}} \qquad 3\text{-}44$$

$$\mu_{ij} = 0 \text{ or } \lambda_i = \lambda_j \qquad 3\text{-}45$$

$$\sum_{ij}^{N_l} W_{ij} \le 1 \qquad 3\text{-}46$$

3.2.5 The Lagrangian Multiplier

The Lagrangian multipliers of the power balance equation are the locational marginal price of the power system. For the above TEP problem with objective function of Eq. (3-37) and constraints Eqs. (3-38 – 3-46), the Lagrangian function can be formulated as [66]:

$$\begin{aligned} L(P_{Gi}, \lambda, \mu_{ij}^p, \mu_{ij}^n, \mu_{ij,o}^p, \mu_{ij,o}^n \eta_i^{\max}, \eta_i^{\min}) = & \sum_{i}^{Ng} C(P_{Gi}) + \sum_{i=1}^{N} \lambda_i \Big(\sum_{j=1}^{N_b} B_{BusNew,ij}(\delta_i - \delta_j) + P_{Gi} - P_{Di}\Big) \\ & + \sum_{ij=1}^{N_l} \mu_{ij}^p \big(B_{f,ij} * (\delta_i - \delta_j) - (1+\sigma * W_{ij})\overline{P_{ij}}\big) \\ & + \sum_{ij=1}^{N_l} \mu_{ij}^n \big(-B_{f,ij} * (\delta_i - \delta_j) - (1+\sigma * W_{ij})\overline{P_{ij}}\big) \\ & + \sum_{ij=1}^{N_l} \mu_{ij,o}^p \big(B_{f,ij,o} * n_{ij,o} * (\delta_i - \delta_j) - n_{ij,o} * \overline{P_{ij,o}}\big) \\ & + \sum_{ij=1}^{N_l} \mu_{ij,o}^n \big(-B_{f,ij,o} * n_{ij,o} * (\delta_i - \delta_j) - n_{ij,o} * \overline{P_{ij,o}}\big) \\ & + \sum_{i=1}^{N_g} \eta_i^{\max} (P_{Gi} - \overline{P_{Gi}}) \\ & + \sum_{i=1}^{N_g} \eta_i^{\min} (P_{Gi} - \underline{P_{Gi}}) \end{aligned} \qquad 3\text{-}47$$

where λ_i shadow prices for the energy balance equation Eq. (3-38)

$\mu_{ij}^{p}, \mu_{ij}^{n}$ shadow prices for the branch flow constraints of existing transmission line of Eq. (3-41)

$\mu_{ij,o}^{p}, \mu_{ij,o}^{n}$ shadow prices for the branch flow constraints of new transmission line of option o Eq.(3-42)

$\eta_{i}^{\max}, \eta_{i}^{\min}$ the shadow price for the maximum and minimum generator output constraints Eq. (3-43)

Note that in the above Lagrangian function, only constraints that have direct coupling with decision variables P_{Gi} and δ_i are taken into account (Eq. 3-44 to Eq. 3-46) TEP model constraints that do not have effect on the outcome of the above shadow prices are not included.

Taking the partial derivative of the Lagrangian function, with respect to the active power generators output (P_{Gi}) and the bus angle (δ_i), equal to zero and applying the Karush-Khun-Tacker (KKT) optimality condition to the inequality constrain to Eqs. (3-41), (3-42) and (3-43) we get:

$$\frac{\partial L(P_{Gi}, \lambda, \mu, \eta)}{\partial P_{Gi}} = 2a_i P_{Gi} + b_i + \lambda_i + \eta_i^{\max} - \eta_i^{\min} = 0 \quad \text{3-48}$$

$$\frac{\partial L(P_{Gi}, \lambda, \mu, \eta)}{\partial \delta_i} = \mathbf{B}_{BusNew}\lambda + \mathbf{B}_{f,ij}^{T}\mu^{p} - \mathbf{B}_{f,ij}^{T}\mu^{n} + \mathbf{B}_{f,o}^{T}\mu_o^{p} - \mathbf{B}_{f,o}^{T}\mu_o^{n} = 0 \quad \text{3-49}$$

$$\mu_{ij}^{p}(P_{ij} - (1+\sigma * W_{ij})\overline{P_{ij}}) = 0 \quad \text{3-50}$$

$$\mu_{ij}^{n}(-P_{ij} - (1+\sigma * W_{ij})\overline{P_{ij}}) = 0 \quad \text{3-51}$$

$$\mu_{ij,o}^{p}(P_{ij,o} - n_{ij,o} * \overline{P_{ij,o}}) = 0 \quad \text{3-52}$$

$$\mu_{ij,o}^{n}(-P_{ij,o} - n_{ij,o} * \overline{P_{ij,o}}) = 0 \quad \text{3-53}$$

$$\eta_{i}^{\max}(P_{Gi} - \overline{P_{Gi}}) = 0 \quad \text{3-54}$$

$$\eta_{i}^{\min}(P_{Gi} - \underline{P_{Gi}}) = 0 \quad \text{3-55}$$

For simplicity of expression, Eq. (3-49) is given in vector form. The derivation and expressions of the vectors are given in appendix A.

Eq. (3-50) and Eq. (3-51) represents the KKT optimality condition of the transmission lines thermal power flow limit through existing line, Eq. (3-52) and Eq. (3.53) represents the new candidate transmission line and Eq. (3-54) and Eq. (3-55) represents power generation capacity limit of the generating units.

When there is no violation of the transmission system power flow limit and generating units active power generation limits, the factors in the brackets of the KKT optimality condition given in Eq. (3-50) – Eq. (3-55) will be different from zero and the respective Lagrangian multipliers become zero. In this condition the state variables (P_{Gi}, δ_i, P_{ij} and λ_i) will be calculated from the original DC OPF equations. On the other hand, when either the transmission line or the generating units are operating beyond their maximum capacity limit, the corresponding KKT optimality condition of the violated constraint becomes active. This means that the equation in the bracket will become zero meanwhile the respective Lagrangian multiplier will have certain magnitude which is dependent on the extent of the binding constraint violation.

3.2.6 Locational Marginal Prices

Locational marginal price (LMP) is a pricing system for buying and selling electric energy considering the generation marginal cost and the physical aspects of the transmission system [67]. It is the incremental cost of energy at each node (bus) of the power system. In other word it is the extra cost for supplying the next 1MW additional power at a specific bus without violating any system and operation constraints. All consumers purchase energy at the price of their load buses and all producers sell energy at the price of their generator buses. The LMP consists of three components, which are marginal cost at the reference bus, marginal cost due to transmission losses and marginal cost due to transmission system congestion. Mathematically, these components can be represented in [66]. The derivation can be found in [68]:

$$\lambda_i = \lambda + \lambda_l + \lambda_c \qquad \text{3-56}$$

Where: λ_i is the LMP at bus i in \$/hr

λ is the LMP at the reference bus in \$/hr

λ_l is the LMP due to transmission line loss

$\lambda_c = \sum_{j=1}^{N} t_{ij}\mu_{ij}^{p} - \sum_{j=1}^{N} t_{ij}\mu_{ij}^{n}$ - is the LMP due to transmission line congestion

t_{ij} is the shift factor at bus i and branch j.

The energy component of each LMP is simply the marginal cost of energy for the system at the reference bus (λ). During LMP calculation using DC OPF, the loss component of the LMP are neglected as a DC power flow model is a lossless network model which does not consider the transmission system losses. The Lagrangian multiplier μ_{ij} is the shadow price due to the binding constraint of power flow through the transmission lines. The congestion component of LMP shows the impact of each congested line on the LMP of the power system. It also denotes the increase in social welfare which could be achieved by slightly increasing the power limit of the corresponding line [66]. When there is no congestion in the system or the line flow constraints are not included in the optimization problem the congestion coefficient μ_{ij} is zero and the LMP at all nodes will be equal to the LMP at the reference bus (λ). On the other hand, when one or more of transmission line power flows is constrained, the congestion coefficient will not be zero anymore and the LMPs at each bus will vary. The differences in the locational marginal prices of the buses are dependent on the severity of the congestion. In this case two different situations can occur. First, the congestion may prevent cheap supply of energy from the serving bus to the load bus. As a result expensive unit will be committed to replace the cheaper unit and the LMP can be higher than the highest generation offer. Second, the LMP can be lower than the cheapest generator offer; in the case it is cheaper to pay customers at locations where load consumption helps to relieve congested transmission lines [66, 69].

3.2.7 Congestion Cost

When a transmission line is operating at its maximum power transfer capacity limit, it is called congested. This means that additional power transfer through this line is not allowed. Therefore during this situation, for secure and reliable operation of the power system, the congestion must be resolved by re-dispatching the generating unit outputs. This may lead in supplying the next extra load from the more expensive generating unit and different LMPs at each bus appears.

The congestion cost or rent refers to the cost difference between the total payment that the consumers pay and that of total payment that the generator receives. If the system is not congested, the cost that consumer pays will be equal with the total cost the producer earns and congestion cost will be zero. If there is congestion in one or more of the transmission line, the location marginal price (LMP) at the buses will not be the same. Therefore the cost that the consumers pay and the generator receive will not be equal [66].

Consider the l^{th} transmission line of a certain power network with end buses i and j shown below in Figure 3-3. The congestion rent of the l^{th} line is given as:

$$CR_l = (lmp_i - lmp_j) * P_{l,ij} \qquad l = 1,2,..., N_l$$

where CR_l is the congestion rent for transmission line l

lmp_i and lmp_j is the LMP at bus i and bus j respectively

$P_{l,ij}$ is the electric power transfer from bus i to j through the l^{th} line

N_l is the number of transmission lines in the network.

The total congestion rent for all transmission networks is equal to

$$TCR = \sum_{l=1}^{N_l} (lmp_i - lmp_j) * P_{l,ij} \tag{3-57}$$

where TCR is the total congestion cost in the network

The total congestion cost of the system can also be given as the sum of consumer payment minus the sum of generators income [70].

$$TCR = \sum_{i=1}^{N_b} P_{Di} * lmp_i - \sum_{i=1}^{N_b} P_{Gi} * lmp_i \tag{3-58}$$

where P_{Di} is the load at bus i

P_{Gi} is the generation at bus i

N_b is the number of buses in the network

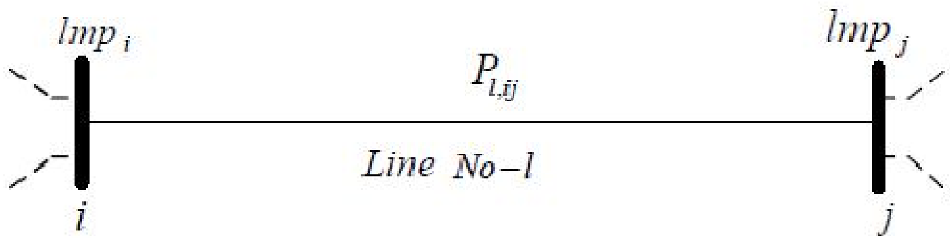

Figure 3-3 l^{th} transmission line of a power system network

3.3 Software used for Modeling the Expansion Problem

In this thesis the advanced optimization tool developed by Paragon Decision's Technology called AIMMS is used for simulation and modeling of the TEP problem formulated above whose objective function and constraints are given in section 3.2.3 and 3.2.4. AIMMS is an optimization tool used for solving mathematical problems including linear, nonlinear, mixed-integer, quadratic, mixed-integer nonlinear programming etc. It also provides links to many integrated powerful solvers that allows the user to solve all major mathematical programming problems.

The MINLP expansion planning problem is solved by using the basic AIMMS Outer Approximation (AOA) algorithm provided with the optimization tool. To solve the MINLP problem AOA uses the well-known outer approximation approach that is written in the AIMMS modeling language. The Generated Mathematical Program (GMP) library through which the user has direct access to mathematical program instances generated by AIMMS allows the user to implement advanced algorithms in an efficient manner. The basic outer approximation algorithm can be completely implemented with the model by installing the system module called "GMPOuterApproximation" and integrated using functionality provided by the GMP library. This algorithm also provides an interaction between two solvers, namely one for solving the mixed-integer programming problem and one for solving nonlinear problem [71].

The AOA solution approach is based on the decomposition technique in which the MINLP transmission expansion planning problem is decomposed into a relaxed master sub-problem and a primal sub-problem. In this algorithm, the master problem which is a mixed-integer programming (MIP) problem considers the transmission investment plan while the primal sub-problem considers the nonlinear programming (NLP) operation problem. The expansion problem is solved first by identifying the candidate transmission line options of the relaxed integer investment variable of the master sub-problem. Then the NLP operation sub-problem will be solved. This process continues alternatively until the algorithm finds the optimal expansion plan or fulfills the termination criteria.

3.4 Case Study

The above proposed expansion planning approach is applied on the New England test power system network. This system has 39 buses, 10 generators and 46 transmission lines (Figure 3-4). The different aspects of the power system are studied and the result is discussed in the next section.

3.4.1 Important Data for the Model

Transmission expansion planning is a complex problem as it is subjected to uncertainty of the future data. Some of this data's can be forecasted from past experience and future expectations. For reasonably priced transmission planning of the future operation condition, getting the exact estimation of all the required data is crucial. And the required data have to be forecasted or determined before the planning process started. Therefore great care must be given. The most important information that the planner has to know before planning includes:

- The system network topology of the base year
- Characteristics of the candidate transmission line circuits (like length and authorized right of way)
- The power generation and demand profile of the planning horizon
- Investment constraints
- Possible types of transmission line
- The cost of the transmission lines, etc.

In this paper the expansion planning is realized by assuming that the planner can select the expansion plan parallel to all the existing transmission lines without any restriction of right of way. Besides there are two different kind of transmission line options whose characteristics are assumed with respect to the existing ones. The assumption made on the candidate transmission lines are:

- The investment cost of the existing transmission line is taken to be 1000 $/km assuming an overhead 345 kV single circuit transmission line have the same cost as 380 kV line of reference [72].
- The length of all the 46 transmission line branches of the 39bus New England system is equal to 100 km.
- The transmission planner has the authority to build new lines only parallel to the existing right of way.

Then given an existing transmission line with characteristic of maximum power transfer capacity of $P_{ij}^{\max}$, reactance of X_{ij} and investment cost of C_{ij}, the characteristics (properties) of the two new transmission line option is defined as:

Option 1: A transmission line with characteristics $P_{ij,o1}^{\max} = 0.75 * P_{ij}^{\max}$, $X_{ij,o1} = 0.75 * X_{ij}$ and $C_{ij,o1} = 0.75 * C_{ij}$

Option 2**:** A transmission line with characteristics $P_{ij,o2}^{\max} = 1.25 * P_{ij}^{\max}$, $X_{ij,o2} = 1.25 * X_{ij}$ and $C_{ij,o2} = 1.25 * C_{ij}$

The quadratic operation cost functions of the 10 generating units are taken from [73]. The maximum and minimum power outputs of the generators, the demand at the load buses, the maximum power transfer capacity limit and parameter of the transmission lines are taken from MATPOWER case39.m Matlab file [74] and are given in the appendix B.

Considering the above stated conditions the problem is implemented in AIMMS version 3.11. The outer approximation module algorithm is installed in the model to solve the MINLP problem. The solver GUROBI 4.0 is used for solving the MIP investment problem and SNOPT 6.1 is used for the NLP operational problem.

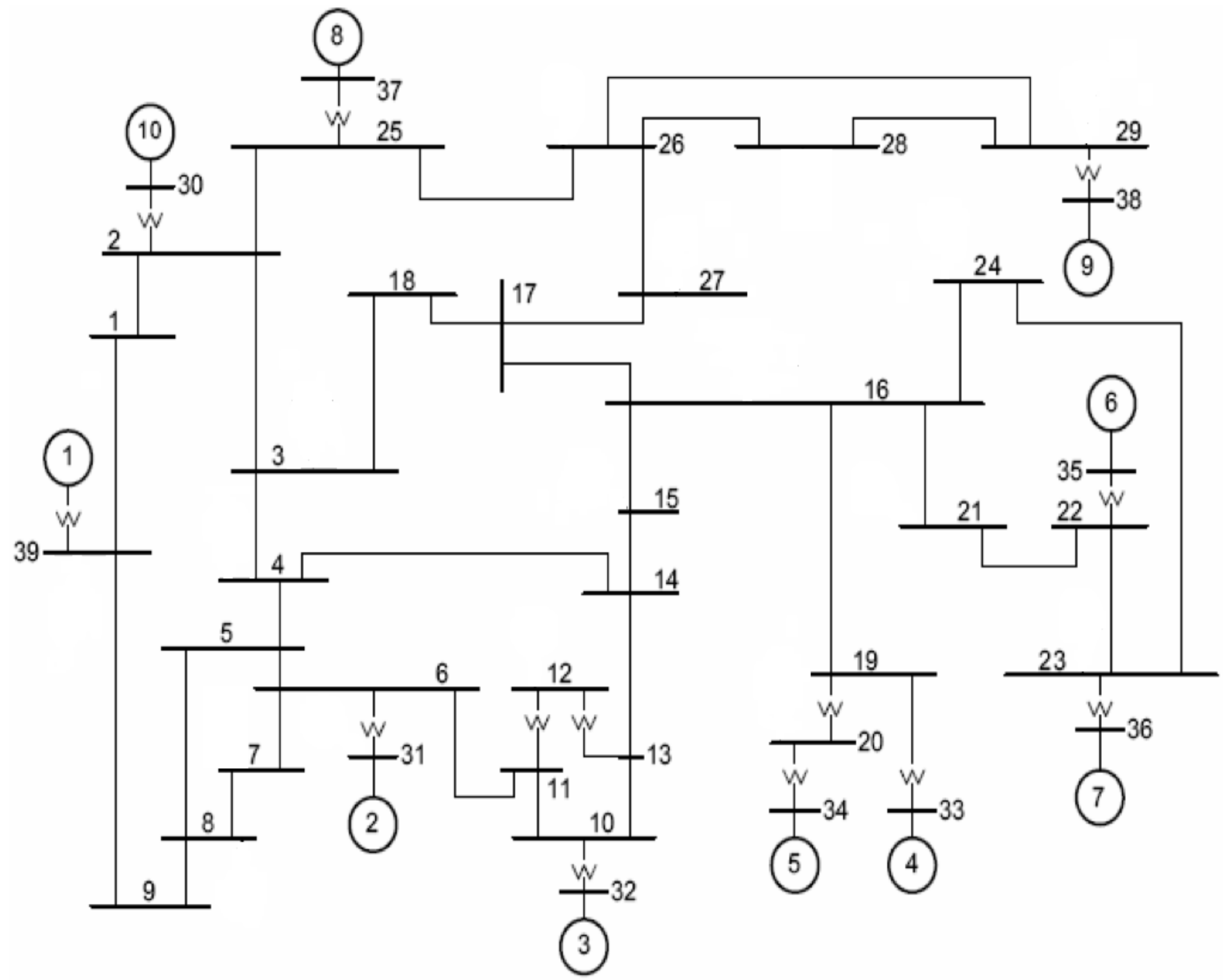

Figure 3-4 Single line diagram of the 39-bus New England test system

The transmission expansion planning can be accomplished for two specific operating condition of the system. In the first case, the power system can supply the load connected to the system under congested condition. In other word it means to keep the reliable operation of the system, the expensive generating units have to be brought online and relive the congestion on the transmission system. In the second case the system is not capable of supply the load unless some loads at certain buses are curtailed or the transmission system is expanded. Otherwise the DC OPF results in an infeasible solution. These two operating condition are realized in case-1 and case-2 respectively.

3.4.2 Case 1

The collection of data starts with the determination of the planning time frame of the network. For this case, the planning time frame is taken to be three years and the load growth per year is assumed to be 1.55% [75] which resulted in a total load increase of 4.568%. In this time frame, due to interest towards green power generation, it is assumed that the fossil fuel generator at bus 36 will be shuted down and the capacity of the nuclear generator at bus 31 doubles. The result and the discussion of the expansion plan of this system condition are presented below.

As shown in Table 3-1, before the expansion process is performed on the existing system, the system is operating under a congested condition with congestion on line 2-3. As expected the LMP at each bus are not the same and it ranges between 25.297 $/MWh and 29.58 $/MWh. Since the DC OPF seeks only the feasible generation dispatch of the existing system, the operation cost of the optimal solution is also high. The associated annual congestion cost of this condition accounts 23,480.1 k$.

After applying the expansion process given above in section 3.3.2 and 3.3.3, the optimization problem finds an optimal expansion plan with minimum cost. The overloading coefficient is taken to be 10%. This resulted in construction of new transmission line of ‘option 1’ in parallel to the existing line in the branch 2-3 without any overloaded line ($W_{ij} = 0$). With this expansion plan the multiplier of the line μ_{ij} of line 2-3 become zero and the LMP of the system converged to the same value of 27.12 $/MWh as expected. Besides this expansion plan, the annual operation cost of the system reduced by 1287.1 k$.

Table 3-1: 39 bus New England deterministic expansion plan analysis result for Case 1

	Before Expansion	After Expansion
Total Cost (k$)	935,209.2	1,008,922.1
Total Investment Cost (k$)	-	75,000
Annual Operation Cost (k$)	935,209.2	933,922.1
No. of Extra Lines Added	-	2-3 (option-1)
LMP ($/MWh)	25.297 - 29.58	27.12
μ	5.361	-
Annual Congestion Cost (k$)	23480.1	-

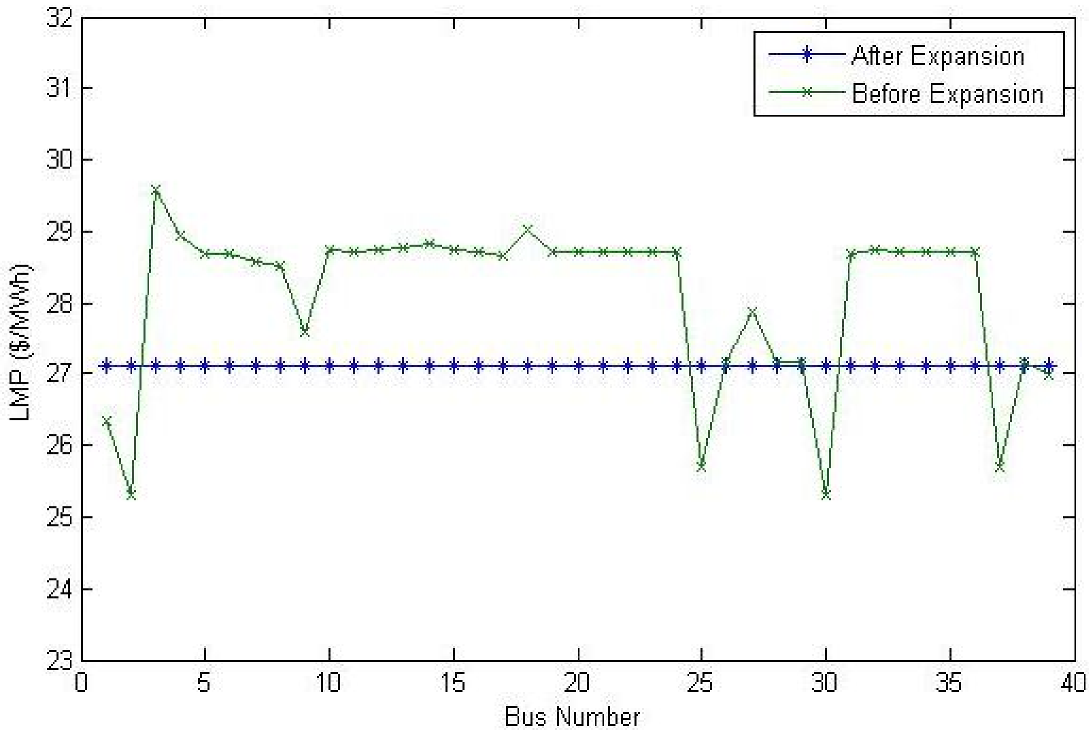

Figure 3-5 Locational marginal price (LMP) after and before expansion process on the 39 bus New England power system

3.4.3 Case 2

In this case the generators configuration, the power demand at the load bus and the yearly load growth are assumed to be the same with the first case. To achieve an infeasible operating condition, a longer planning time frame of 10 years, which resulted in a 16.9% total load increase, is taken. Like in the first case the overloading coefficient is taken to be 10%.

For the explained system condition the expansion planning problem is solved. For this case the optimization problem finds an optimal solution of building a new line with characteristic 'option 1' in parallel to line 2-3 with an overloading in line 2-30. The installation of this new line converts the system from infeasible to feasible condition. As the problem is formulated to allow one overloaded line with a maximum of 10% overloading, the optimization process doesn't activate the addition of line on branch 2-30. Whereas the transmission line connecting bus 2 to 30 operate under slightly overloaded situation. If the system operator is interested in operating the system without allowing any overloaded line ($\sum W_{ij} = 0$), only with the one line addition (in branch 2-3) of the above expansion plan, the system will operate optimally with slight congestion on branch 2-30. The simulation result with overloading in line 2-30 is presented in Table 3-2.

Table 3-2: 39 bus New England deterministic expansion plan result for Case 2

	After Expansion (with allowing overloading)	Before Expansion
Total Cost (k$)	1,218,033.1	Infeasible
Total Investment Cost (k$)	75,000	
Annual Operation Cost (k$)	1,136,733.1	
No. of Extra Lines Added	2-3 (option-1)	
Overloaded line	2-30 ($W_{ij} = 1$)	
LMP ($/Mwh)	34.41	
μ	-	

3.5 Summary

- A TEP approach based on congestion alleviation of the transmission lines is proposed. In this method in order to avoid over investment due to non-severe overloading, an overloading factor σ is introduced to the power flow limit constraint of the existing transmission lines.

- The TEP problem is modeled in such a way that there are two different types of candidate transmission line options. Depending on the minimum cost function and optimal operation condition of the power system the optimization problem selects the best expansion plan.

- The power flow and the power flow limiting constraint of candidate and the existing transmission line in the ij^{th} branch are defined separately. Therefore complete topology including the candidate and the existing grid is used for expansion to avoid loop flow.

- Deterministic expansion planning approach gives an optimal network operation condition for a snapshot future load-generation configuration. But if a load profile which is different form forecasted happens in the future, the optimal expansion strategy may result in inadequate or expensive expansion plan. Therefore, to overcome this drawback of the deterministic TEP formulation, the expansion planning strategy must be formulated in such a way that it takes the possible future scenarios of uncertain factors into account.

4. Multi-stage Transmission Expansion Planning

Based on the planning horizon there are two types of TEP problem i.e. Static TEP and Dynamic TEP. In static TEP the optimal operation and expansion plan is determined for a single stage or year while in dynamic planning a number of stages (planning horizon) are considered. The short term or yearly transmission expansion planning optimization problem explained in the previous chapter focuses in determining an optimal transmission and operation condition for a single stage future generation configuration and load profile. Its main objective is to minimize the investment and fuel cost subjected to the technical, economical and reliability constraints of the power system.

Like the static TEP, the basic objective of dynamic TEP problem is to determine the number of new transmission line that must be built in a particular planning horizon at a minimum cost subjected to reliability and operational constraint. This problem tries to answer not only the type and location of the new investment but also the most appropriate time of carrying out the investments, so that the continuously growing demand and generation operate in an optimized way [47]. Since many factors are difficult to quantify and predict, it is complex problem and difficult to formulate mathematically [76, 77]. To reduce the complexity of the dynamic TEP problem, the problem is solved by decomposing the TEP problem into sequence of one-stage problem. In the multi-stage planning, the planning horizon will be divided into several stages and the corresponding optimal expansion plan of each stage will be determined.

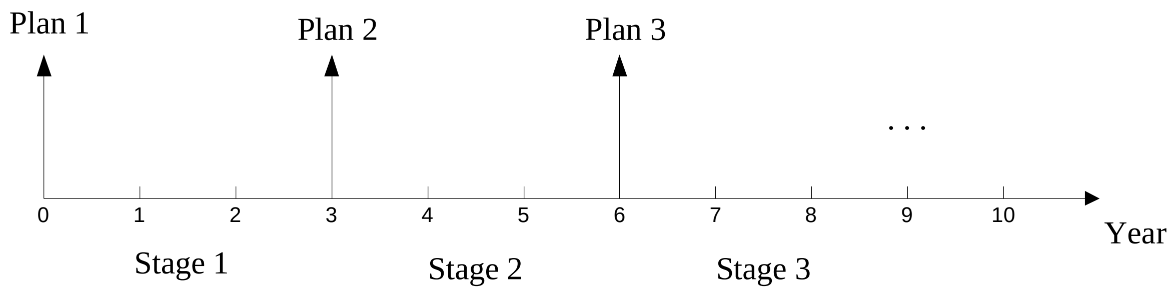

Figure 4-1 Multi-stage TEP horizons

This chapter proposes a multi-stage TEP which consider the uncertainties of load. To determine the optimal solution of each stage a three-step decision model is utilized. In the first step, a multi-stage transmission expansion problem will be solved for the typical scenarios and a set of candidate investment plans are produced. The second phase evaluates the outcome of each alternative investment plan in all the scenarios and the necessary information for the last step of the process will be calculated. Finally the best investment plan will be selected by applying the appropriate comparison procedure on the alternative expansion plans. Hence, the mathematical formulation of the multi-stage TEP model used for the identification of the alternative investment plans is explained in section 4.1. In the next section 4.2 the three phase solution approach employed is presented. The decision making criteria used to identify the best investment plan is briefly explained in section 4.3. Finally the application of the proposed model using the 39 bus New England test network is illustrated in section 4.4.

4.1 Mathematical Formulation of Multi-stage TEP

The long-term transmission network planning problem seeks a series of economical and reliable network expansion plans, which will accommodate the future generation and load growth over the planning period. In this case the planning horizon is divided into several periods. It is a non-convex, nonlinear mixed-integer problem with high complexity, especially for large-scale realistic transmission networks.

Because the times when the investment and operation occurs have an impact on the present value of the expense, the time value of money should be considered. Therefore, the cost minimization objective function and all other money related manipulations of each planning horizon are transformed into the present value of the reference year accordingly.

Consider continues transmission network expansion planning at which the installation of the candidate transmission line is performed at the start of each stage. For expansion stage of $t = 1,...,T$ and a single stage accounts m number of year, if the number of type o lines added between node i and j at previous stages is given by $n_{ij,o,t}$ then the number of existing lines at current stage t_T is given as.

$$n_{ij,t_T} = n_{ij,0} + \sum_{t=1}^{t_T} n_{ij,o,t} \qquad \text{4-1}$$

where $n_{ij,0}$ is the number of lines at the initial (start of the) planning year.

At this expansion stage, the investment cost is as a function of the lines added at the stage. For an annual depreciation rate r, the present value of the investment cost is given as:

$$C_{iv} = \frac{C_{iv,t}}{(1+r)^{m(t_T-1)}} \qquad \text{4-2}$$

$$C_{iv,t} = \sum_{o}^{N_o} \sum_{ij}^{N_l} c_{ij,o,t} n_{ij,o,t} \qquad \forall(ij) \in CL,\ \forall(o) \in O, \forall(t) \in T \qquad \text{4-3}$$

where $c_{ij,o,t}$ represent the investment cost of type o transmission line in the ij^{th} branch at stage t, $C_{iv,t}$ is the total investment cost at the beginning of stage t. C_{iv} is the net present value (NPV) expansion investment cost.

On the other hand, the production cost is a function of the number of the existing lines given by Eq. (4-1). For stage t the hourly production cost of the generating units is given as:

$$C_{op,h} = \sum_{k}^{N_g} C_k(P_{Gk}) \qquad \forall(k) \in G \qquad \text{4-4}$$

The NPV annual production cost over the planning stage is given as:

$$C_{op} = 8760 * \frac{C_{op,h}}{(1+r)^{mt_T}} \qquad \text{4-5}$$

where $C_{op,h}$ and C_{op} represents the hourly and NPV annual operational cost at stage t respectively.

Considering a multiple dispatch load level scenarios of $s = 1,2,..., S$ and planning stage $t = 1,2,..., T$ the objective function and constraints of the multi-stage TEP problem is formulated as:

$$\min W = C_{op}^{s,t} + C_{iv}^{s,t} \qquad \text{4-6}$$

- The active power balance equation at each node i of scenario s and stage t: The net power flow to each node is equal to:

$$P_{Gi}^{s,t} = P_{i}^{s,t} - P_{Di}^{s,t} \qquad (\lambda_{i}^{s,t}) \quad \forall(i) \in B, \forall(s) \in S, \forall(t) \in T \qquad \text{4-7}$$

  where

$$P_{i}^{s,t} = \sum_{j=1}^{N_b} B_{BusNew,ij}^{s,t} (\delta_{i}^{s,t} - \delta_{j}^{s,t}) \qquad \forall(i) \in B, \forall(s) \in S, \forall(t) \in T \qquad \text{4-8}$$

- Branch power flow and transmission power flow capacity limit on the ij^{th} branch of each scenario s and stage t: specify the physical limits on the amount of power flow that can be transmitted through each branches of the network.

  For the existing transmission system:

$$P_{ij}^{s,t} = B_{f,ij} * (\delta_{i}^{s,t} - \delta_{j}^{s,t}) \qquad \forall(ij) \in EL, \forall(s) \in S, \forall(t) \in T \qquad \text{4-9}$$

$$\left|P_{ij}^{s,t}\right| \le \overline{P_{ij}} \qquad (\mu_{ij}^{s,t}) \quad \forall(ij) \in EL \qquad \text{4-10}$$

  For the new candidate transmission line:

$$P_{ij,o}^{s,t} = B_{f,ij,o}^{s,t} * n_{ij,o} * (\delta_{i}^{s,t} - \delta_{j}^{s,t}) \qquad \forall(ij) \in CL, \forall(s) \in S, \forall(t) \in T, \forall(o) \in O \qquad \text{4-11}$$

$$\left|P_{ij,o}^{s,t}\right| \le n_{ij,o}^{s,t} * \overline{P_{ij,o}} \qquad (\mu_{ij,o}^{s,t}) \qquad \forall(ij) \in CL \qquad \text{4-12}$$

- Generation capacity for each unit at bus i during scenario s and planning stage t: operating limits imposed by the generating units is given as:

$$\underline{P_{Gi}} \le P_{Gi}^{s,t} \le \overline{P_{Gi}} \qquad (\eta_i) \qquad \forall(i) \in G \qquad \text{4-13}$$

- The right of way limit: impose the location and the maximum number of transmission circuit that can be built in the ij^{th} branch.

$$0 \le n_{ij,o}^{s,t} \le \overline{n_{ij,o}} \qquad \textbf{4-14}$$

- The congestion alleviation constraint: induce the alleviation of any transmission line congestion in the system. In other words, the LMP at every bus are equal.

$$\mu_{ij}^{s,t} = 0 \qquad \forall (ij) \in L$$
$$\lambda_i = \lambda_j \,,\ i \ne j \qquad \forall (i,j) \in B \qquad \textbf{4-15}$$

The subscript s,t denotes the parameters and variables when the TEP problems is solved for all scenarios s at stage t. $C_{op}^{s,t}, C_{iv}^{s,t}$ represents the NPV annual operation cost and investment cost, $n_{ij,o}^{s,t}$ is an integer variable ($n_{ij,o}^{s,t} \ne 0$ if candidate line of type o is built during scenario s and stage t otherwise 0). $P_i^{s,t}$, $P_{Gi}^{s,t}$ and $P_{Di}^{s,t}$ are the net power injection, active power generation and active power demand at bus i respectively. $P_{ij}^{s,t}$ and $P_{ij,o}^{s,t}$ represent the active power flow through the existing and type o new candidate transmission line in branch ij. $B_{BusNew,ij}^{s,t}$, $B_{f,ij,o}^{s,t}$ and $B_{f,ij}$ are the ij^{th} element of new susceptance matrix of the transmission system after considering the new candidate transmission line, susceptance of type o new and existing transmission line connecting bus i to j respectively. $\delta_i^{s,t}, \delta_j^{s,t}$ is the bus angle at node i and j respectively. $\overline{P_{ij,o}}$ is the maximum power flow capacity of the ij^{th} new candidate line of type o while $\overline{P_{ij}}$ is the power flow capacity limit of the existing line. $\underline{P_{Gi}}, \overline{P_{Gi}}$ are the maximum and minimum active power generation capacity limits of generator at bus i respectively.

For the stage under consideration, the multi-stage TEP model seeks the most economical investment plan and operational condition for each scenario. The important input information for the model consists of the forecasted load scenarios and generation facilities during the planning horizon. For each planning stage the uncertainty in the load and wind power generation is included to the model with different scenarios.

4.2 Solution Approach

This section presents the methodology adopted for selection of a robust plan under the multiple load dispatch planning problem. The selection of the best investment solution for a single stage planning time frame is conducted by three main steps. These are:

1. The candidate investment identification step
2. Operational analysis step
3. Decision analysis step

For a multi-stage planning approach the above steps will be expanded as:

- Start at the first stage
- Select a set of typical load condition that will probably happen in the future.
- Considering the network configuration of the current stage, solve the proposed deterministic TEP problem for all load conditions and generate a set of candidate expansion plans, which consists of the optimal solution of each scenario.
- Solve the normal DC OPF for all scenarios by assuming each candidate expansion plan as an optimal plan of the stage and calculate the necessary information for the decision analysis step.
- Apply a decision making criteria and select the best plan that fits all the scenarios at minimum cost.
- Check the plan selected in previous step fulfills the operating condition of all the scenarios. Otherwise update the expansion plan.
- Fix the resulting expansion plan as an existing network configuration of the current stage and initial condition for the next stage.
- Repeat the above procedures until the end of stages considered.

4.2.1 Step 1: The Candidate Investment Identification Step

The TEP problem solved using a deterministic approach will be optimal only if the electrical demand in the future occurs as predicted. But electrical demand of any power system is never constant through time [78]. It varies from hour to hour and day to day based on the activity of human kind. Therefore, to have good expansion plan which have the ability to be optimal for most of the load configuration which is expected to happen, the uncertainty caused by the fluctuation of the demand must be taken into account while performing transmission expansion planning.

In this stage, a set of possible load condition is constructed and the set of alternative candidate transmission investment plans is selected by solving the deterministic transmission expansion model given above in Eqs. (4-6) to (4-15). The flow chart of this stage is shown in Figure 4-2. The steps to be followed include:

- Using a historical data and select representative typical scenarios
- Solve the deterministic TEP problem for all representative scenarios
- Form a set of candidate expansion plans from the optimal solutions of the deterministic TEP model solved above
- Check all the sample scenarios are solved
- Go to the next step

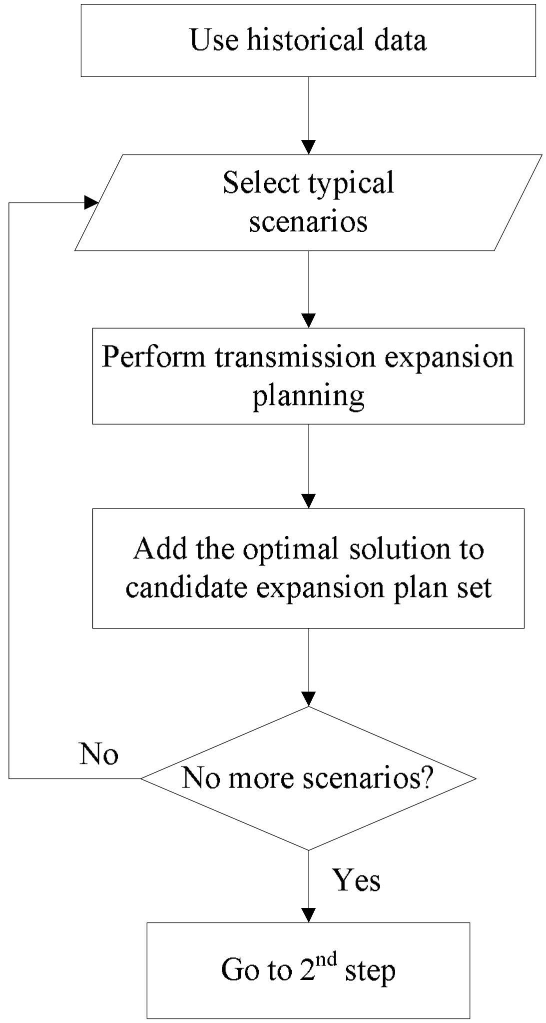

Figure 4-2 Flowchart of alternative investment plan identification

4.2.2 Step 2: Operational Analysis Step

After identifying the candidate investment plans at the first step, in this step, for all scenario-plan combination the DC OPF will be solved and the necessary costs associated with the decision analysis criteria will be calculated. The input data are the candidate expansion plans and existing system information (generation, the existing and new candidate interconnection and load scenarios). The output is the present value of the system operation cost and other important information for the decision making stage.

For load level scenarios of $s = 1,2,...,S$ and candidate expansion plans of $p = 1,2,...,P$, the mathematical formulation of the modified DC OPF for a given stage t is given as:

$$\min v = \sum_{i}^{N_G} C_i(P_{Gi}^{p,s}) \qquad \text{4-16}$$

Subjected to:

$$n_{ij,p}^{ex} = n_{ij,(t-1)} + n_{ij,p} \qquad \forall(ij) \in L\,,\ \forall(p) \in P \qquad \text{4-17}$$

$$P_{Gi}^{p,s} = P_i^{p,s} - P_{Di}^{p,s} \qquad (\lambda_i^{s,t}) \qquad \forall(i) \in B\,, \forall(s) \in S\,, \forall(p) \in P \qquad \text{4-18}$$

$$P_{ij}^{p,s} = B_{f,ij}^{p} * (\delta_i^{p,s} - \delta_j^{p,s}) \qquad \forall(ij) \in L\,, \forall(s) \in S\,, \forall(p) \in P \qquad \text{4-19}$$

$$\left|P_{ij}^{p,s}\right| \le n_{ij,p}^{ex} * \overline{P_{ij}} \qquad \forall(ij) \in L\,, \forall(s) \in S\,, \forall(p) \in P \qquad \text{4-20}$$

$$\underline{P_{Gi}} \le P_{Gi}^{p,s} \le \overline{P_{Gi}} \qquad \forall(i) \in G\,,\ \forall(s) \in S\,, \forall(p) \in P \qquad \text{4-21}$$

Where: the subscripts p,s denote the parameters and variables when the OPF problem is solved for all set of scenario s and plan p combination.

The main aim of this step is to identify the operating condition of the system when each of the candidate expansion plans is taken into account. Therefore the mathematical expression of the normal OPF model must be modified in such a way that it considers each candidate expansion plan as the optimal expansion solution of the stage. Eq. (4-17) states the existing network configuration of the system by assuming plan p as the optimal expansion plan of the stage. It is given as the sum of the existing network configuration of the pervious stage $n_{ij,(t-1)}$ and the optimal expansion plan p ($n_{ij,p}$) under consideration. Eq. (4-18) specifies the power balance at every node and Eq. (4-19) the power flow through every branch of the system, both through the existing and the new lines. The maximum power transfer capacity limit of the transmission lines as a multiplication of the existing branch is given in Eq. (4-20). And Eq. (4-21) specifies the generation capacity limits of the generating units.

4.2.3 Step 3: Decision Analysis Step

Finally using the information's calculated at the pervious step, from the candidate expansion plan the best solution is selected by applying an appropriate comparison and decision making procedure. Then the selected optimal solution will be cross checked weather it satisfies the operating condition of all the scenarios taken into consideration. If not the current solution will be updated. The procedure followed in the decision making process is shown in Figure 4-3.

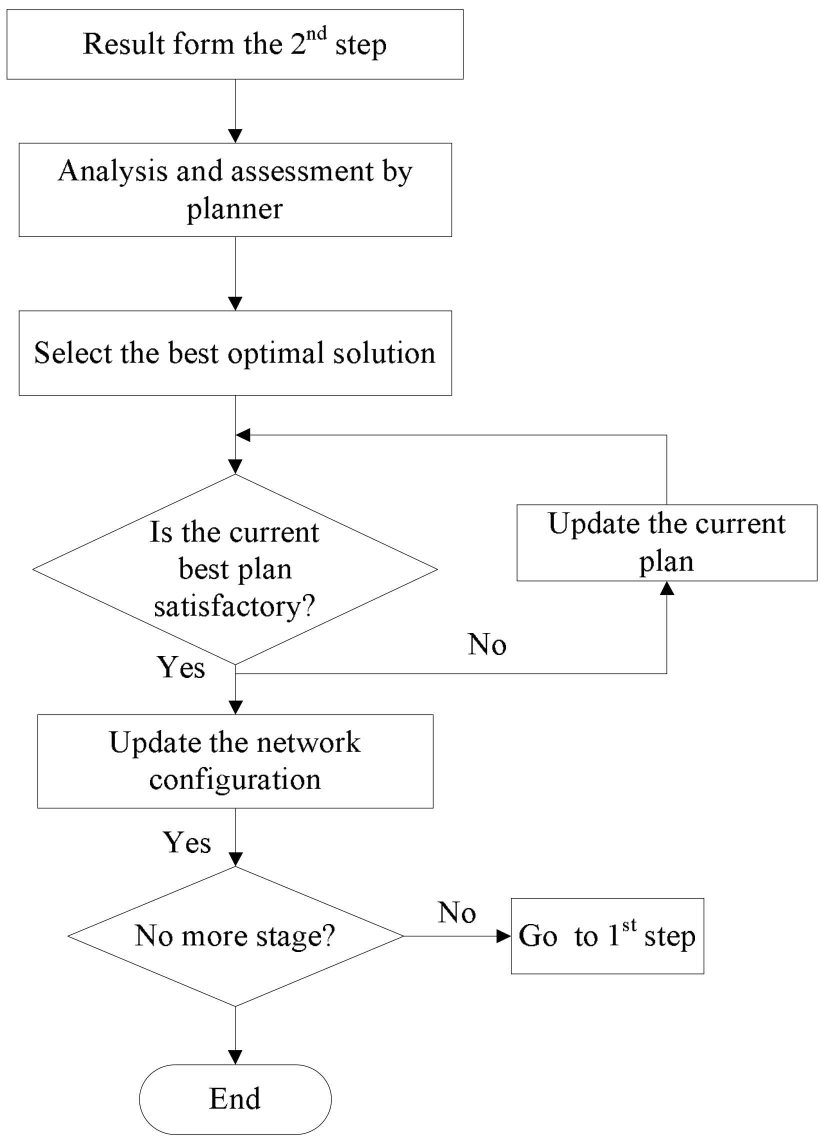

Figure 4-3 Flowchart of the decision analysis and best investment plan selection procedure

The expansion plan selected at this stage will be fixed as the best optimal solution of the stage under consideration and initial network configuration for the next stage. This planning process will be performed iteratively until the end of all planning stage.

Based on the interest and final objective of the planner the applied decision making criteria may vary. The selection of the investment plan can be seen from different angles. For example, if the investment planning is in favor of reducing the operation cost, an optimal plan that minimizes the production cost will be selected. From the perspective of the consumers the decision of the expansion plan will be made based on minimum electricity payment. On the other hand the minimum congestion cost is used to have an optimal plan that reduces the congestion in the system and maximize the social welfare. Regardless of these dimensions, the maximization of the social welfare should be the fundamental aim of the transmission expansion planning [79].

4.3 Minimax Regret Decision Analysis

To select a best plan, a decision analysis scheme must be incorporated to enable us to take into account the uncertainty of uncontrollable factors. The main goal is to determine the best plan which is robust and minimize the maximum possible economic loss caused by the uncertainty [56]. To apply a symmetrical risk analysis method we have to have a way of determining the optimal expansion plans of each scenario assuming that the future condition will occur as expected. From the result of each scenario a set of candidate expansion plan will be formed. Then for every alternative candidate plan in future scenario, the regret incurred for not having chosen this plan as the best plan for the future condition is calculated. The minimax regret criterion, often named as the criteria of min-max risk or losses tries to avoid the regret that may result from making a non-optimal solution [80]. In this framework for a particular future scenario s, the regret felt in an expansion plan p is defined as the difference between the value of the attribute[1] of the system under selected expansion plan and the value of the minimum attribute that would have been attained if the network planner had a prior knowledge that this scenario would take place. In other word regret is defined as the opportunity loss to the decision maker if a plan p is selected and scenario s happens [80]. The three main steps of the decision analysis schemes of minimax regret criteria are given as:

[1] An attribute is a measure of goodness of a plan

First for the set scenarios S and candidate expansion plans (P), the attribute value of each plan-scenario pair is calculated as [64]:

$$f^{s,p} = val(s,p) \qquad \forall s \in S \text{ , } \forall p \in P \tag{4-22}$$

Next, the regret $R(s,p)$ of the expansion plan p under future scenario s is given as [81]:

$$R(s,p) = val(s,p) - val^{\min}(s,p) \qquad \forall s \in S \text{ , } \forall p \in P \tag{4-23}$$

where $val^{\min}(s,p) = \underset{p \in P}{Min}\{val(s,p)\}$

Finally, the optimal expansion plan that minimizes the maximum regret over all future scenarios is selected as the final best plan. This means we search for each expansion plan p the maximum regret among all future scenarios, and then among all plans select the plan with the smallest maximum regret as the best plan [82]. It is formulated as:

$$\underset{p}{Min}\{\underset{s}{Max}\{R(s,p)\}\} \tag{4-24}$$

If the occurrence probability of each scenario s is known, then the minimum average regret criteria can be formulated as:

$$\underset{p}{Min}\{\underset{s}{Max}\{v^{s} * R(s,p)\}\} \tag{4-25}$$

where v^{s} is the occurrence probability of scenario s.

If the regret of a plan is zero for all future scenarios then the plan is robust otherwise among the candidate expansion plans, the best plan that operate reliably under all scenarios is chosen as the final expansion plan. Note that the selected optimal plan may not be the least cost investment plan.

4.4 Case Study and Discussion

The proposed model has been tested based on 39 bus New England test system. At the initial condition the system comprises of 39 bus, 46 lines, 10 generators and 21 load buses. This system is modified by including ten wind parks (denoted by W1-W10) each maximum power generation capacities of 100 MW. The location of these wind parks are shown in Figure 4.4. The length of the planning period is taken to be fifteen years. This planning horizon is divided into three stages, of which each stage spans five year. This means that the investment for the expansion is performed in a period of five years (in year 0, 5 and 10). The depreciation rate is set to 6% per year. The proposed multi-stage MINLP TEP problem is implemented in AIMMS optimization tool and solved by the Outer Approximation Algorithm (AOA) method. Like the deterministic problem given in chapter 3, the NLP operational problem is solved by SNOPT 6.1 and the mixed-integer investment problem is solved by GUROBI 4.0.

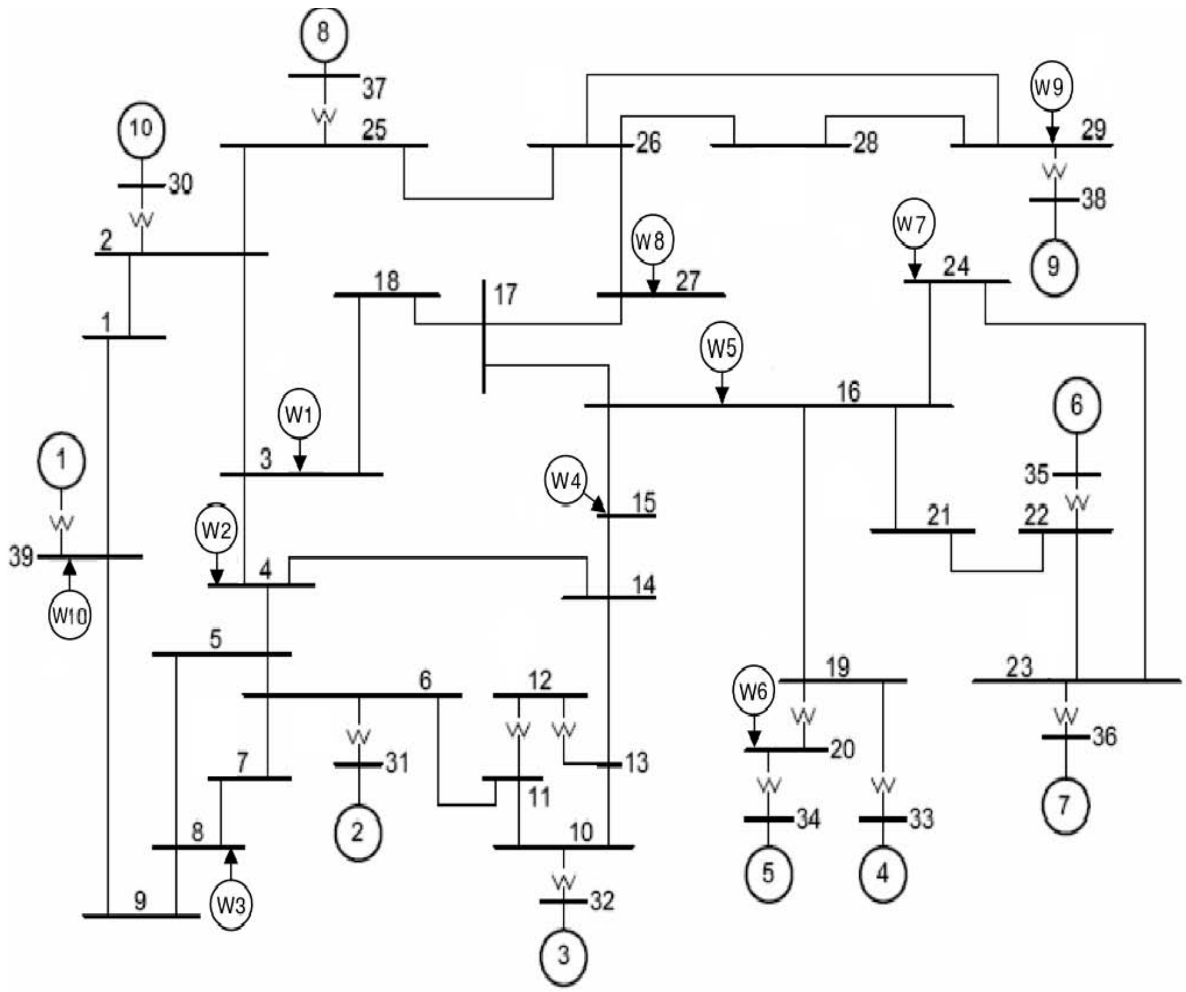

Figure 4-4 The modified single line diagram of the 39bus New England test system

4.4.1 Data Selection Criterion

The typical load scenarios that represent the possible load configuration that may occur throughout the planning year are selected by a process called clustering. Clustering is the process of grouping data into classes or clusters such that data points in clusters are more similar to each other than data points in separate clusters [83]. The similarities of the data points are mostly assessed by using distance measure. The K-medoids cluster algorithm is used to obtain K clusters of demand dataset. K-medoids algorithm finds a single representative data point called medoids, a point that is centrally located in the cluster. The remaining data sets will be clustered with respect to the representative data point to which it is most similar. The partitioning method is based on minimizing the absolute-error criterion E, defined as [83, 84]:

$$E = \sum_{j=1}^{k} \sum_{p \in C_j} \left| P - O_j \right| \quad \text{4-26}$$

Where, E is the sum of the absolute error for all objects in the data set, P is the point in space representing a given object in cluster C_j and O_j is the representative data point of cluster C_j.

The basic procedure followed includes:

- Randomly select k data points as the initial representative medoids.
- Cluster the non-medoid data points with the medoid to which it is closest to.
- Inside the clusters, search for a new medoid which have smallest average distance with the rest of the data points in the cluster. Swap the old medoid by the new medoid.
- Repeat the 2nd and 3rd step until no change in the medoid of the clusters exists.

The method is applied to the 10000 sample net load profile which is found by subtracting the wind power generation from the load profile. The number of clusters is assumed to be six, so that this method will partition the 10000 sample net load profile into six clusters first and identify the representative scenario for each cluster. These representative scenarios are used as typical load conditions of the planning year for the multi-stage TEP problem.

4.4.2 The Common Assumptions Made

- Throughout the planning horizon there are two new generators to be installed. The maximum generating capacity, quadratic cost function coefficients, the bus at which the generator will be connected and stage at which it will be operational are known and given in Table 4-1.

Table 4-1: New generation data

Stage	Bus	$a_k\,(\$/kWh^2)$	$b_k\,(\$/kWh)$	$c_k\,(\$)$	$\overline{P_{Gk}}$	$\underline{P_{Gk}}$
2nd	33	0.009	10.15	210	500	0
3rd	38	0.007	11	200	500	0

- The new expansion line options can be constructed in parallel to the existing 46 branch and there are two types of candidate line. The parameters of the these candidate transmission options are assumed with respect to the existing lines as:

  Option 1: A transmission line with characteristics

  $$P_{ij,o1}^{\max} = 0.75 * P_{ij}^{\max}\,,\ X_{ij,o1} = 0.75 * X_{ij} \text{ and } C_{ij,o1} = 0.75 * C_{ij}$$

  ***Option 2*:** A transmission line with characteristics

  $$P_{ij,o2}^{\max} = 1.25 * P_{ij}^{\max}\,,\ X_{ij,o2} = 1.25 * X_{ij} \text{ and } C_{ij,o2} = 1.25 * C_{ij}$$

- The installation of optimal expansion investment of each stage will be completed and be operational at the beginning of the stage. This means that the plan implemented at the beginning of each stage must support the demand throughout the stage (Figure 4-1).
- The maximum power transfer capacities of all transmission line are assumed to be 80% of the actual power transfer capacity given in [74].
- The parameters of the transmission lines, the minimum and maximum limits of the generators as well as the quadratic cost coefficients for the existing base case condition is given in appendix C.
- The length of all the transmission lines is assumed to be the equal (100 km) and the cost of the 345 kV transmission line have the same cost with the 380 kV transmission line given in [72].
- The annual load growth rate is taken to be 1.55% [75] and the average wind power generation growth rate is assumed to be the same with the European Union (EU) and taken to be 17.6% [85].

4.4.3 Result and Discussion

In this section the multi-stage TEP problem is tackled by using the "forward" procedure in which the static transmission expansion problem is solved starting from the first stage to the last sequentially. At the first stage, considering the current configuration of the network, the optimal expansion plan of the stage will be determined. During the remaining stages (from the second to the last) the expansion problem is solved by considering the additional lines implemented in the previous stage.

Given the above assumption and procedure, the multi-stage TEP model given in Eq. (4-6) to (4-15) is solved for six future typical load scenarios (denoted as s_1, s_2, s_3, s_4, s_5 and s_6) and six optimum transmission expansion plans (denoted as p_1, p_2, p_3, p_4, p_5 and p_6 respectively) are obtained.

Table 4-2: The candidate transmission expansion plan of the six future load conditions of each stage

Stage	Line Added (type of line)	Set of Optimal Plans					
		p_1	p_2	p_3	p_4	p_5	p_6
1st Stage	16-19 (1)		1	1			
2nd Stage	6-11 (1)		1				
	16-19 (1)		1				1
	19-33 (1)	1	1	1		1	1
3rd Stage	15-16 (1)					1	
	26-27 (1)					1	
	26-29 (1)				1	1	
	28-29 (1)			1			
	29-38 (1)	1		1		1	
	2-25 (1)			1			

The candidate expansion plans of the three stage planning horizon are given in Table 4-2. The number in the bracket indicates the type of the transmission line options. The scenarios considered in the 1st stage resulted in a candidate expansion plan of building one line in branch 16-19 (optimal expansion plan for scenario 2 and 3) and "Do-Nothing" (expansion plan for scenario 1, 4, 5, and 6). Similarly the candidate expansion plans of the other stages can be interpreted. Note that except the first stage, the candidate expansion plans of 2nd and 3rd stage are determined by considering the optimal expansion plan of the previous stages. This means the existing transmission line during stage t is derived from Eq. (4.1).

After identification of the candidate expansion plan of the stage, to select the optimal plan the operational and decision analysis step will be performed. For assessing the economic aspect of the plan the congestion cost is used as selection criteria. The congestion cost of each plan-scenario combination is computed by running the DC OPF model given in Eq. (4-16) to (4-21). Therefore it forms a $p = 1,2,...,N_p$ by $s = 1,2,...,N_s$ attribute table and the best plan is selected by applying the minimax regret decision making analysis procedure given in Eq. (2-23) to (2-25), where N_s and N_p represents the number of the scenarios and plans respectively.

1st Stage

As can be seen from Table 4-2, the main candidate expansion plans of the 1st stage are building one transmission line (option 1) in branch 16-19 and "Do-Nothing". The attribute matrix of congestion cost for these two plans is given in Table 4-3.

Table 4-3: Attribute table for scenario-candidate plan combination of 1st Stage (k$)

	s_1	s_2	s_3	s_4	s_5	s_6
16-19 (p_2 and p_3)	0	0	0	0	0	0
"Do Nothing" p_1, p_4, p_5 and p_6	0	2117.26	1409.9	0	0	0

Without further analysis it is clear that building one line in branch 16-19 is an optimal solution for all scenarios. Therefore it is selected as final expansion plan of this stage. This expansion plan is shown in Figure 4-5.

2nd Stage

Given the optimal expansion plan of the 1st stage, the TEP problem is solved for other six load scenarios taking annual load and wind power generation growth rate of 1.55% and 17.6%

respectively. It resulted in four main candidate investment plans (see Table 4-2). The attribute matrix of these candidate expansion plans are presented in Table 4-4. Similar to the 1st stage, without any further analysis plan 2 (P_2) is selected as final plan for this stage.

Table 4-4: Attribute table for scenario-candidate plan combination of 2nd Stage (k$)

	s_1	s_2	s_3	s_4	s_5	s_6
p_1, p_3 and p_5	0	3865.99	0	0	0	641.51
p_2	0	0	0	0	0	0
p_4	8096.77	23123.56	3216.42	18626.07	8308.23	16519.61
p_6	0	477.96	0	0	0	0

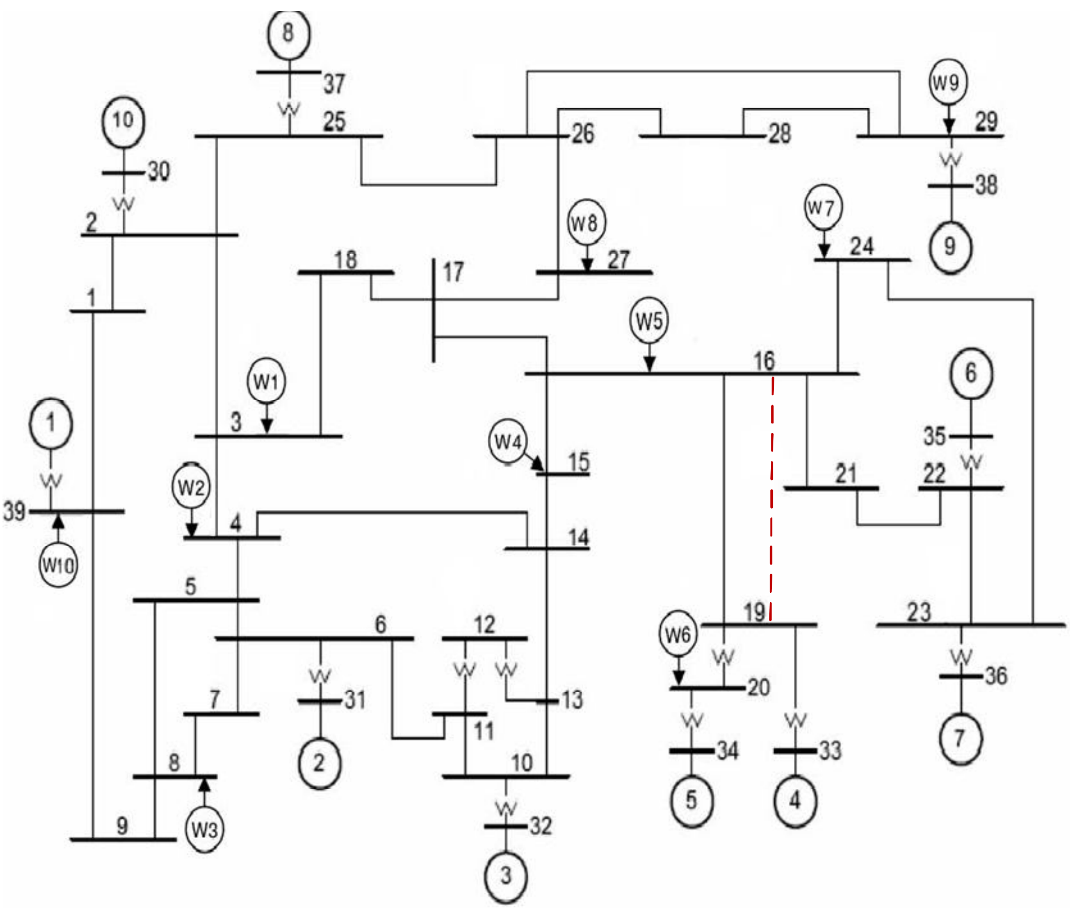

Figure 4-5 The single line diagram of the 39 bus New England after 1st stage expansion plan

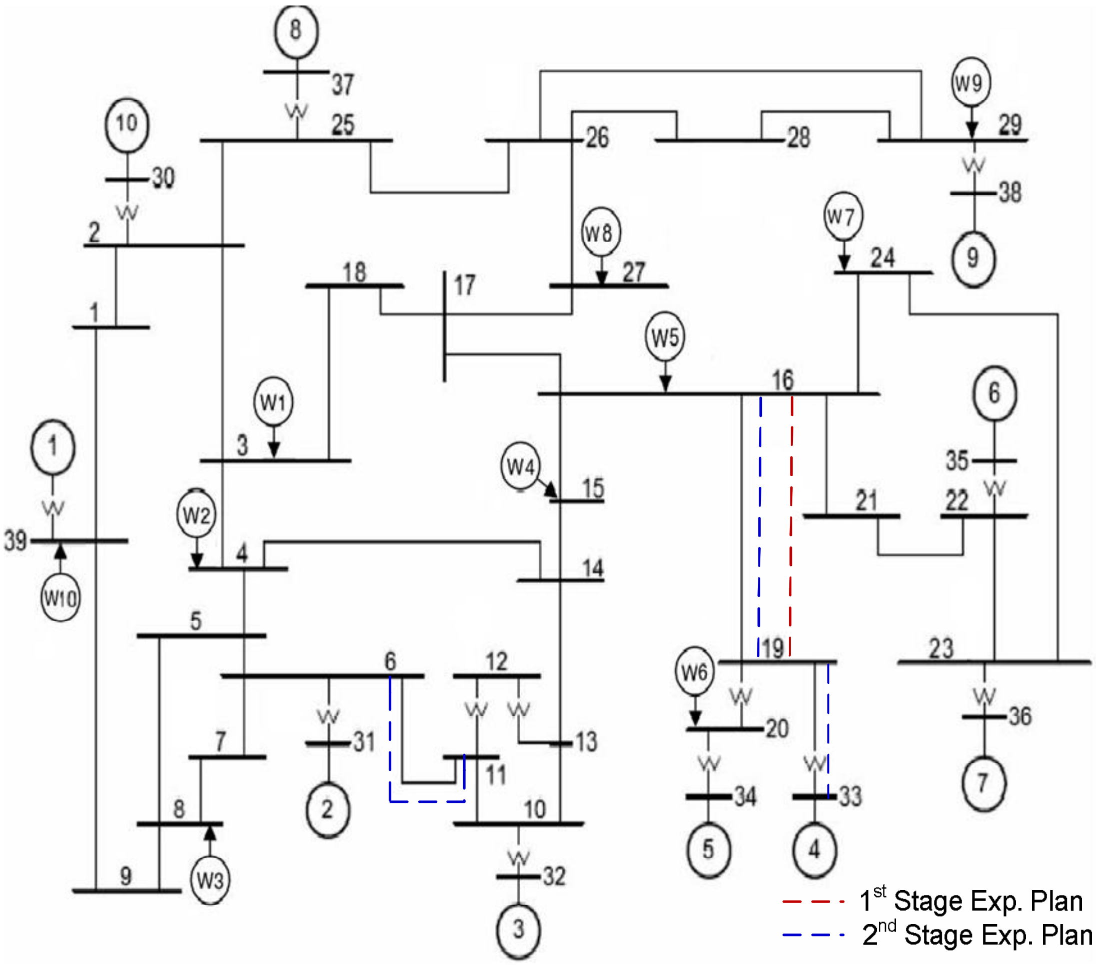

Figure 4-6 The single line diagram of the 39 bus New England after 2nd stage expansion plan

3rd Stage

At this stage of the multi-stage TEP, the simulation resulted in five main candidate expansion plans (see Table 4-2). The attribute matrix of the scenario-plan combination is given in Table 4-5. In this stage, unlike the 1st and 2nd stage, there is no plan that have zero congestion cost for all scenarios. Therefore, for further analysis the regret of each plan-scenario pair is calculated for Eq. 4-23. Table 4-6 gives the detailed regret value of each plan.

Table 4-5: Attribute table for scenario-plan combination of 3rd stage (k$)

	s_1	s_2	s_3	s_4	s_5	s_6
p_1	0	0	5478.2	823.22	9021.53	0
p_2 and p_6	439.81	0	14326.17	823.22	23202.26	0
p_3	0	0	0	0	1674.08	0
p_4	439.8	0	14326.34	0	23202.28	0
p_5	0	0	26.54	0	0	0

Table 4-6: Regret of scenario-plan combination of 3rd stage (k$)

	s_1	s_2	s_3	s_4	s_5	s_6	$\underset{s}{Max}\{R(s,p)\}$
p_1	0	0	5478.2	823.22	9021.53	0	9021.53
p_2 and p_6	439.81	0	14326.17	823.22	23202.26	0	23202.26
p_3	0	0	0	0	1674.08	0	1674.08
p_4	439.8	0	14326.34	0	23202.28	0	23202.28
p_5	0	0	26.54	0	0	0	26.54

The last column of the Table 4-6 gives the maximum regret of each plan over all scenarios. According to the minimax regret criteria, the plan that minimizes the maximum regret of all plans is selected as an optimal plan. Therefore, plan 5 (p_5) which corresponds to building four transmission line in branch 15-16, 26-27, 26-29 and 29-38 is selected as the best solution. However, the optimal expansion plan of the minimax regret criteria is optimal solution only for scenarios s_1, s_2, s_4, s_5 and s_6. This shows that the expansion plan 5 which is resulted from the highest total demand level of all considered scenarios, it is not able to remove all the congestion in the intermediate load level scenarios. This emphasizes that the transmission expansion should be able to support the multiple load and power generation configuration. Therefore the optimal solution must have to be upgraded so that it fulfill the normal operation condition of s_3, remove the congestion. The upgrading resulted in building one additional line in branch 2-25. The final optimal expansion plan of the multi-stage expansion plan and the stage at which the installation will be performed is given in Figure 4-7 and Table 4-7.

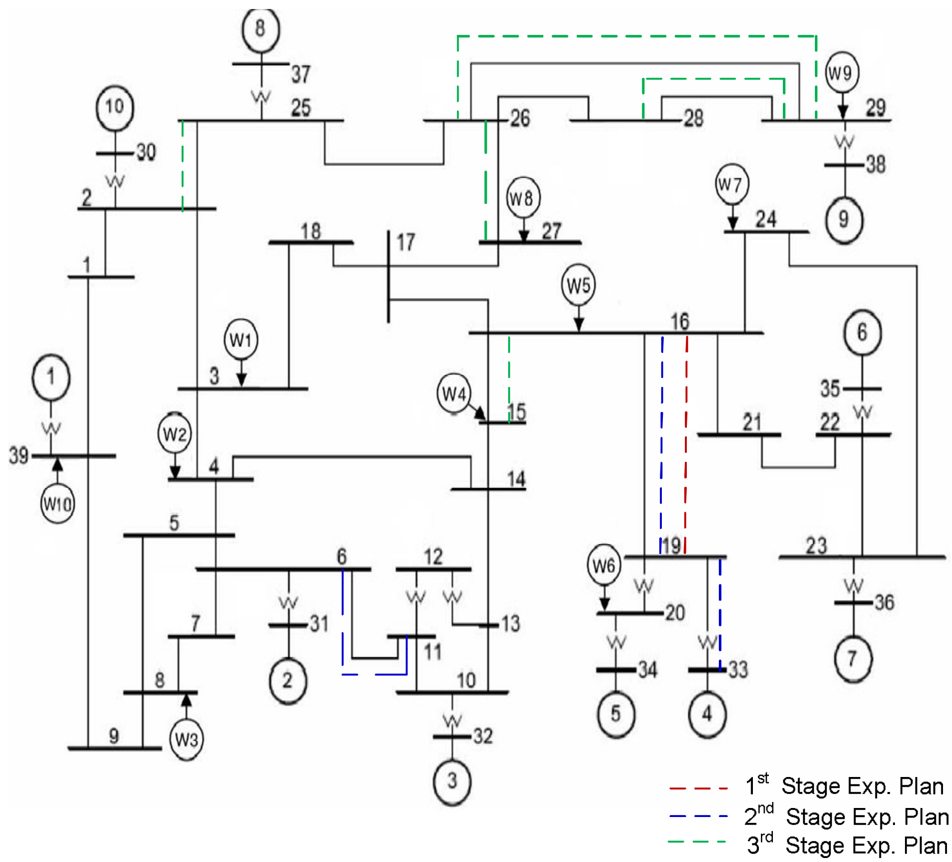

Figure 4-7 The single line diagram of the 39 bus New England after 3rd stage expansion plan

Table 4-7: The final investment plan of the multi-stage TEP

Result	Branch (from-to)	No. of line Added	Investment Cost (k$)
1st stage	16-19 (1)	1	81,300
2nd stage	6-11 (1)	1	243,900
	16-19 (1)	1	
	19-33 (1)	1	
3rd stage	15-16 (1)	1	406,500
	26-29 (1)	1	
	28-29 (1)	1	
	29-38 (1)	1	
	2-25 (1)	1	
Total Investment Cost (k$)			731,700

By the “forward” planning procedure nine new transmission line reinforcements were identified (one in the 1^{st}, three in 2^{nd} and five in the 3^{rd} stage) with a total investment cost of US$ 731.7 million.

4.5 Summary

- In this section of the thesis work a multi-stage TEP problem which evaluates the transmission line investment cost together with the operation cost is proposed. The proposed model is solved for a “forward” planning procedure. The uncertainties in the electrical demand and wind power generation are taken into account considering a representative scenario. The number of representative net load scenarios is determined by applying the K-medoid method.

- Every stage comprises of three main steps: identification of the candidate investment plans, operation analysis and decision analysis step. In the decision making analysis procedure, the congestion cost is used to measure the goodness of the solution and minimax regret criteria is used for assessing the risk associated with each candidate expansion plan. The expansion plan which minimizes the maximum regret is selected as the best plan and this plan will be updated to fulfil the operating condition of all the scenarios.

5. Conclusion

In the future, due to the growing electricity consumption, the existing transmission network may become stressed and congested. Under this condition for secure and reliable operation of the system either the load or the generation has to be rescheduled. This results increase in the price of electricity, decrease in the reliability and security of the power system and increased possibility of cascade outages. To overcome this problem transmission expansion planning has been studied and a new approach of tackling the TEP problem is proposed.

In this thesis work a transmission expansion planning approach based on single stage deterministic and multi-stage non-deterministic approached is presented. The deterministic transmission expansion planning problem seeks to find an optimal expansion plan of a single future load and generation condition with minimization of the total investment and operation cost as its objective function. The proposed methodology is based on optimal power flow under constrained locational marginal price and Lagrangian multipliers. This is done by introducing a congestion alleviation constraint in addition to the original MINLP expansion planning problem. To avoid the over-investment due to a non-severe congestion of the lines, a temporary limited overloading of the existing transmission lines is allowed. The proposed MINLP TEP problem is efficiently solved using the outer approximation algorithm under AIMMS. Finally, the methodology is applied on the New England 39 bus test system for two load/generation scenarios for validation of performance and effectiveness.

The investment plan done by the deterministic model may result in expensive or insufficient investment condition when the demand deviates from the forecasted level. To overcome this situation a multi-stage TEP model under uncertainty of the demand level and wind power generation was proposed. At each stage of the planning approach three main steps are followed. In the first step of the problem the optimal investment plan is obtained for the number of the typical demand scenarios and forms a set of candidate expansion plan. The second step of the model, evaluate the performance of each candidate investment plan over each demand level scenarios by running the DC OPF considering the plan as an optimal solution of the stage. Finally, the final optimal solution will be determined by updating the expansion plan from the risk assessment.

In conclusion, this work proposes a mathematical model of TEP based on congestion alleviation. The model is implemented in AIMMS optimization software and solved by the AOA algorithm. The performance of the model to deterministic and non-deterministic TEP examined using the 39 bus New England power system and the results are discussed.

For further development and study, important recommendations given are:

- The selection of the representative scenario is taken considering the 10000 sample load profile as a whole. Selection of representative scenarios based on the seasonal load profile corresponding to fall, winter, spring and summer is recommended.

- In this work only the random uncertainties in load and wind power generation is considered. However, in a deregulation power system, in which the generation, transmission, and distribution of power is performed by different companies, TEP become more complicated and subjected to more uncertainties. Further research on the integration of all significant non-random uncertainties caused by generation expansion, load growth, fuel costs etc. are recommended.

- In this work the number of representative scenarios ($k = 6$) is selected arbitrarily. However to find a good representative scenarios the optimal number of clusters must be determined prior employing the clustering method. Therefore it is recommended to find out the optimal number of clusters using some well-known validity measurement indices.

Appendix

A. The Admittance Matrix

For a power system with N_b number of bus and N_l number of transmission line, the Admittance matrix **B** ($N_b * N_b$) of the existing power system is given as:

$$\mathbf{B} = \begin{bmatrix} B_{11} & . & . & . & B_{N_b 1} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ B_{N_b 1} & . & . & . & B_{N_b N_b} \end{bmatrix}$$

B_{ii} = is the sum susceptance of lines connected to bus i

$B_{ij} = B_{ji}$ = is the sum of susceptance of lines connecting bus i to bus j

After the expansion process the integer variable, $n_{ij,o}$ that specify addition of a line may not be zero as the initial condition. Therefore the new **B** matrix for the new network structure with the optimal expansion plan must be re-calculated. The ij^{th} element of the new admittance matrix after candidate expansion plan with N_o number of transmission lines options will be given by:

$$B_{BusNew,ij} = \begin{cases} B_{Bus,ij} + \sum_{o}^{N_o} B_{ij.o} n_{ij,o} & (i \neq j) \\ B_{Bus,ij} + \sum_{o}^{N_o} \sum_{J}^{N_b} B_{ij.o} n_{,ij,o} & (i = j) \end{cases}$$

In the same manner, the matrix of the LMP, the Lagrangian multiplier of the positive and negative power flow and the admittance matrix $\mathbf{B}_f$ $(N_l * N_b)$ of the existing transmission lines is given as:

$$\lambda = \begin{bmatrix} \lambda_1 \\ . \\ . \\ . \\ \lambda_{N_b} \end{bmatrix} \qquad \mu^p = \begin{bmatrix} \mu_1^p \\ . \\ . \\ . \\ \mu_{N_l}^p \end{bmatrix} \qquad \mu^n = \begin{bmatrix} \mu_1^n \\ . \\ . \\ . \\ \mu_{N_l}^n \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccccc} & i & & j & \\ & \downarrow & & \downarrow & \end{array} \\ \mathbf{B}_f = & \begin{bmatrix} B_{11} & . & . & . & B_{1N_b} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ B_{N_l 1} & . & . & . & B_{N_l N_b} \end{bmatrix} \begin{array}{l} \leftarrow l_1 \\ . \\ . \\ . \\ \leftarrow N_l \end{array} \end{array}$$

where the element of the matrix $\mathbf{B}_f$ is defined as:

$$B_{li} = \begin{cases} B_{l,ij}, & \text{where i disignate the sending node of line connecting bus i to j} \\ -B_{l,ij}, & \text{where i disignate the receving node (j) of line connecting bus i to j} \\ 0, & \text{Otherwise} \end{cases}$$

Similarly the admittance matrix after the new candidate expansion transmission lines can be built separately. In this case the matrix is expressed in terms of the integer decision variable which specifies whether a line is added in branch *ij* or not. For a single transmission line expansion option (only Option 1), it is given as:

$$\begin{array}{cc} & \begin{array}{ccccc} & i & & j & \\ & \downarrow & & \downarrow & \end{array} \\ \mathbf{B}_{f,1} = n_{ij,1} & \begin{bmatrix} B_{11,1} & . & . & . & B_{1N_b,1} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ B_{N_l 1,1} & . & . & . & B_{N_l N_b,1} \end{bmatrix} \begin{array}{l} \leftarrow l_1 \\ . \\ . \\ . \\ \leftarrow N_l \end{array} \end{array}$$

Where, the subscript 1 in $\mathbf{B}_{f,1}$ represents the type of the candidate transmission line option (option 1).

The Lagrangian multipliers for both the negative and positive power flow direction of a single candidate expansion plan option can also be given as:

$$\mu_1^p = \begin{bmatrix} \mu_{1,1}^p \\ . \\ . \\ . \\ \mu_{N_l,1}^p \end{bmatrix} \qquad \mu_1^n = \begin{bmatrix} \mu_{1,1}^n \\ . \\ . \\ . \\ \mu_{N_l,1}^n \end{bmatrix}$$

For a N_o number of possible candidate transmission line options, Admittance and the Lagrangian multipliers matrix can be extended to:

$$\mathbf{B}_f = \begin{bmatrix} B_{f,1} \\ . \\ . \\ . \\ B_{f,N_o} \end{bmatrix} \qquad \mu_o^p = \begin{bmatrix} \mu_1^p \\ . \\ . \\ \mu_{N_o}^p \end{bmatrix} \qquad \mu_o^n = \begin{bmatrix} \mu_1^n \\ . \\ . \\ \mu_{N_o}^n \end{bmatrix}$$

B. 39 Bus New England System Data

Table B-1: Circuit data for 39-bus New England test power system

Line No. (l)	From bus (i)	To bus (j)	Resistance (pu) (R_{ij})	Reactance (pu) (X_{ij})	Limit (MW) ($\overline{P_{ij}}$)
1	1	2	0.0035	0.0411	600
2	1	39	0.001	0.025	1000
3	2	3	0.0013	0.0151	500
4	2	25	0.007	0.0086	500
5	2	30		0.0181	900
6	3	4	0.0013	0.0213	500
7	3	18	0.0011	0.0133	500
8	4	5	0.0008	0.0128	600
9	4	14	0.0008	0.0129	500
10	5	6	0.0002	0.0026	1200
11	5	8	0.0008	0.0112	900
12	6	7	0.0006	0.0092	900
13	6	11	0.0007	0.0082	480
14	6	31		0.025	1800
15	7	8	0.0004	0.0046	900
16	8	9	0.0023	0.0363	900
17	9	39	0.001	0.025	900
18	10	11	0.0004	0.0043	600
19	10	13	0.0004	0.0043	600
20	10	32		0.02	900
21	12	11	0.0016	0.0435	500
22	12	13	0.0016	0.0435	500
23	13	14	0.0009	0.0101	600
24	14	15	0.0018	0.0217	600
25	15	16	0.0009	0.0094	600
26	16	17	0.0007	0.0089	600
27	16	19	0.0016	0.0195	600
28	16	21	0.0008	0.0135	600
29	16	24	0.0003	0.0059	600
30	17	18	0.0007	0.008	600
31	17	27	0.0013	0.0173	600
32	19	20	0.0007	0.0138	900

33	19	33	0.0007	0.0142	900
34	20	34	0.0009	0.018	900
35	21	22	0.0008	0.014	900
36	22	23	0.0006	0.0096	600
37	22	35		0.0143	900
38	23	24	0.0022	0.035	600
39	23	36	0.0005	0.0272	900
40	25	26	0.0032	0.0323	600
41	25	37	0.0006	0.0232	900
42	26	27	0.0014	0.0147	600
43	26	28	0.0043	0.0474	600
44	26	29	0.0057	0.0625	600
45	28	29	0.0014	0.0151	600
46	29	38	0.0008	0.0156	1200

Table B-2: Generators maximum capacity and quadratic cost function coefficients data

Bus	Gen	Max. Capacity (MW) (P_{Gi})	a_i (\$/MWh^2)	b_i (\$/MWh)	c_i (MW)
30	10	1040	0.0105	15.37	240
31	2	646	0.009	11.29	200
32	3	725	0.009	8.8	220
33	4	652	0.0095	8	250
34	5	508	0.0085	11.4	220
35	6	687	0.0075	10.45	190
36	7	580	0.009	10.03	200
37	8	564	0.009	10.15	210
38	9	865	0.007	7.98	230
39	1	1100	0.006	8	220

* $S_{base} = 100MW$, $V_{base} = 345kV$

C. The Demand Scenarios

Table C-1: Typical load scenarios of each Stage

Stage	Stage 1						Stage 2						Stage 3					
Bus	S1	S2	S3	S4	S5	S6	S1	S2	S3	S4	S5	S6	S1	S2	S3	S4	S5	S6
1	102.0	16.3	36.0	67.8	58.6	80.8	65.2	45.6	71.1	46.9	67.7	115.5	51.4	72.4	111.4	81.4	51.9	58.0
3	175.3	345.4	302.7	247.7	319.2	227.2	268.7	362.1	208.5	120.9	196.1	298.7	226.3	2.9	234.4	100.9	379.2	161.5
4	271.3	482.4	440.6	388.5	435.8	364.2	295.4	511.5	332.2	248.1	401.3	458.1	329.7	89.9	495.7	297.6	546.6	225.3
7	166.0	270.5	247.8	201.4	219.7	211.5	251.1	267.6	185.0	219.8	235.6	256.0	223.6	265.6	263.0	273.6	291.3	220.8
8	302.8	484.0	426.6	380.4	420.2	320.9	331.4	458.9	326.3	233.6	419.4	441.0	364.6		459.1	187.5	475.1	259.9
9	4.1	6.7	6.6	5.3	6.5	5.9	7.3	6.3	4.8	6.0	5.8	7.0	5.2	6.0	7.1	5.4	6.8	4.9
12	5.7	9.9	9.9	7.3	8.2	7.8	9.6	10.0	6.8	8.3	8.6	8.9	7.8	9.0	10.4	8.5	11.0	7.3
15	195.2	366.7	318.1	256.1	293.1	257.3	240.2	366.3	236.4	70.1	259.7	298.7	288.6		277.3	14.8	407.3	129.7
16	158.0	351.0	314.4	224.9	272.3	240.4	211.8	317.2	204.3	73.0	215.1	286.3	228.5		320.7	133.2	348.5	120.5
18	107.8	180.8	168.9	131.7	155.9	137.6	146.4	188.3	122.8	113.9	140.6	168.9	124.3	152.3	168.5	185.4	201.1	129.9
20	384.6	673.7	557.0	505.1	644.5	520.1	554.1	705.3	440.7	423.4	556.9	611.7	485.9	191.6	645.3	378.8	713.2	338.5
21	190.9	254.6	236.6	212.8	237.4	224.0	207.2	287.6	194.9	164.8	221.0	246.6	175.3	213.4	274.7	306.4	296.1	221.5
23	146.8	263.2	234.2	171.8	237.1	221.9	225.5	280.5	166.9	176.9	203.1	245.2	176.0	257.9	260.6	262.0	291.4	174.8
24	215.3	349.3	317.6	258.7	303.9	273.5	170.1	347.3	244.2	125.7	236.6	323.6	283.0		328.8	96.8	395.0	213.3
25	137.5	248.2	246.9	161.8	220.2	185.0	218.1	242.2	147.4	145.3	192.7	228.4	177.3	202.8	230.7	287.7	286.2	183.2
26	10.2	154.7	153.0	46.1	112.8	79.9	106.7	126.6	10.2	8.2	76.1	113.8	30.0	74.9	92.9	178.0	175.7	41.9
27	175.8	263.5	210.6	213.5	248.2	177.3	144.3	254.4	183.9	67.7	185.9	221.8	201.3		275.9		309.2	80.9
28	116.5	195.7	168.1	140.1	158.2	161.7	161.1	183.5	137.0	178.0	165.9	159.1	135.7	189.9	218.9	183.6	194.3	139.1
29	150.2	306.7	269.2	202.1	230.1	235.2	188.6	289.0	206.1	122.2	218.2	250.9	221.3	5.5	256.2		315.1	118.1
31	5.7	10.3	10.2	6.8	8.7	8.2	9.3	10.6	6.8	7.6	9.1	9.2	7.9	10.2	9.9	9.3	11.1	7.4
39	656.1	1149.0	1032.9	712.5	789.5	953.8	950.1	1147.0	755.5	643.2	771.5	997.5	788.7	645.4	1094.4	927.1	1179.5	719.2

D. Sample AIMMS Code

In this section sample AIMMS code in text format is provided for reference purpose.

```
DECLARATION SECTION

  SET:
    identifier   :  Bus
    indices      :  i, j

    identifier   :  GeneratorSet
    subset of    :  Bus
    index        :  gi ;

    identifier   :  ReferenceBus
    subset of    :  Bus
    index        :  ref ;

    identifier   :  Line
    index        :  l

    identifier   :  LineBusLink
    subset of    :  (Line, Bus, Bus)
    tags         :  (lf, fb, tb)
    index        :  li
    order by     :  li.lf,li.fb,li.tb

    identifier   :  ExistingLines
    subset of    :  LineBusLink
    index        :  eli

    identifier   :  NewExpansionLines
    subset of    :  LineBusLink
    index        :  nli


  PARAMETER:
    identifier   :  Am
    index domain :  (gi) ;

   identifier   :  Bm
    index domain :  gi

    identifier   :  Cm
    index domain :  (gi) ;

    identifier   :  AngleMax
    index domain :  (i) ;

    identifier   :  AngleMin
    index domain :  (i) ;

    identifier   :  Genmax
```

```
  index domain :  gi ;

  identifier   :  Genmin
  index domain :  gi ;

  identifier   :  Demand
  index domain :  i
  range        :  nonnegative ;

  identifier   :  BranchSusc
  index domain :  (li)

  identifier   :  Branch_init
  index domain :  li
  range        :  nonnegative
  definition   :

  identifier   :  MaxLineFlow
  index domain :  li
  definition   :

  identifier   :  BranchCost
  index domain :  li
  definition   :

  identifier   :  BfNew
  index domain :  (li,i) ;

  identifier   :  Bbus
  index domain :  (i,j) ;

  identifier   :  BranchAdded
  index domain :  li
  range        :  {0..3} ;
```

VARIABLE:

```
  identifier   :  CongestionRelaxation
  index domain :  (eli)
  range        :  binary ;

  identifier   :  BBusNew
  index domain :  (i,j)
  range        :  free
  definition   :  if i=j then

 sum[li|li.fb=i, BranchAdded(li)*BranchSusc(li.lf,li.fb,li.tb)] +
 sum[li|li.tb=i, BranchAdded(li)*BranchSusc(li.lf,li.fb,li.tb)] +
 Bbus(i,j)

                  else

 sum[l, -
 BranchAdded(l,i,j)*BranchSusc(l,i,j)+Branch_Init(l,i,j)*Bbus(i,j)] +
 sum[l, -
 BranchAdded(l,j,i)*BranchSusc(l,j,i)+Branch_Init(l,j,i)*Bbus(i,j)]

                   endif ;
```

```
       identifier   :  BBfadd
       index domain :  (li,i)
       range        :  free
       definition   :  Branch_init(li)*BfNew(li,i) +
BranchAdded(li)*BfNew(li,i)  ;

       identifier   :  PowerGeneration
       index domain :  i
       range        :  [Genmin(i), Genmax(i)] ;

       identifier   :  LinePowerFlow
       index domain :  li
       range        :  free
       definition   :
                  BranchSusc(li)*(Branch_init(li)+
                  BranchAdded(li))*(BusAngle(li.fb) - BusAngle(li.tb)) ;

       identifier   :  BusAngle
       index domain :  i ;

       identifier   :  OperationCost
       range        :  free
       definition   :
                        sum[i|Bus, Am(i)*PowerGeneration(i)^2  +
                        Bm(i)*PowerGeneration(i) + Cm(i)]*8.76

       identifier   :  InvestmentCost
       range        :  free
       definition   :  sum[li, BranchAdded(li)*Branchcost(li)]*100 ;

       identifier   :  TotalCost
       range        :  free
       definition   :
                        OperationCost + InvestmentCost + sum[eli,
                        CongestionRelaxation(eli)];
       identifier   :  LMP
       index domain :  (i)
       range        :  free ;

       identifier   :  PosBranchMiw
       index domain :  (li)
       range        :  nonnegative ;

       identifier   :  NegBranchMiw
       index domain :  (li)
       range        :  nonnegative ;

       identifier   :  GeneratorEtaMax
       index domain :  (gi)
       range        :  nonnegative ;

       identifier   :  GeneratorEtaMin
       index domain :  (gi)
       range        :  nonpositive ;
```

CONSTRAINT:

```
identifier   :  PowerBlanaceConstraint
index domain :  i
property     :  ShadowPrice
definition   :

     sum[ li|li.tb = i, LinePowerFlow(li)] - sum[ li|li.fb = i,
     LinePowerFlow(li)] - 1.04568*Demand(i) + PowerGeneration(i) = 0;

identifier   :  BusAngelatReference
index domain :  ref
definition   :  BusAngle(ref)=0;

identifier   :  NewPosBranchcapacityconstraint
index domain :  nli
property     :  ShadowPrice
definition   :

             LinePowerFlow(nli) <= (Branch_Init(nli) +
             BranchAdded(nli))*MaxLineFlow(nli)  ;

identifier   :  NewNegBranchcapacityconstraint
index domain :  nli
definition   :
           LinePowerFlow(nli) >= -(Branch_Init(nli) +
           BranchAdded(nli))*MaxLineFlow(nli)  ;

identifier   :  OldPosBranchcapacityconstraint
index domain :  eli
definition   :
            LinePowerFlow(eli) <=
           (Branch_Init(eli)+CongestionRelaxation(eli)*0.1)*MaxLineFlo
           w(eli)

identifier   :  OldNegBranchcapacityconstraint
index domain :  eli
definition   :
            LinePowerFlow(eli) >= -
           (Branch_Init(eli)+CongestionRelaxation(eli)*0.1)*MaxLineFlo
           w(eli)

identifier   :  ShadowPriceConstraint1
index domain :  gi
definition   :
           2*Am(gi)*PowerGeneration(gi) + Bm(gi) - LMP(gi) +
           GeneratorEtaMax(gi) + GeneratorEtaMin(gi)=0;

identifier   :  ShadowPriceConstraint2
index domain :  i
definition   :
           sum[j, BBusNew(i,j)*LMP(j)] + sum[li|li.fb=i,
           BBfadd(li,i)*PosBranchMiw(li)] - sum[li|li.fb=i,
           BBfadd(li,i)*NegBranchMiw(li)]=0;

identifier   :  NewPosMiwKKT
index domain :  nli
definition   :
```

```
            PosBranchMiw(nli)*(LinePowerFlow(nli) - ( Branch_Init(nli)
            + BranchAdded(nli))*MaxLineFlow(nli)) =0;

identifier   :  NewNegMiwKKT
index domain :  nli
definition   :
            NegBranchMiw(nli)*(-LinePowerFlow(nli) - ( Branch_Init(nli)
            + BranchAdded(nli))*MaxLineFlow(nli)) =0;

identifier   :  OldPosMiwKKT
index domain :  eli
definition
            PosBranchMiw(eli)*(LinePowerFlow(eli) - ( Branch_Init(eli)
            + 0.1*CongestionRelaxation(eli))*MaxLineFlow(eli)) =0  ;

identifier   :  OldNegMiwKKT
index domain :  eli
definition   :
            NegBranchMiw(eli)*(-LinePowerFlow(eli) - ( Branch_Init(eli)
            + 0.1*CongestionRelaxation(eli))*MaxLineFlow(eli)) =0  ;

identifier   :  MaxGenEtaKKT
index domain :  gi
definition   :
             GeneratorEtaMax(gi)*(PowerGeneration(gi) GenMax(gi))=0;

identifier   :  MinGenEtaKKT
index domain :  (gi)
definition   :
              GeneratorEtaMin(gi)*(PowerGeneration(gi) - GenMin(gi))=0;

identifier   :  PosCongestionConstraint
index domain :  li
definition   :  PosBranchMiw(li)=0;

identifier   :  NegCongestionConstraint
index domain :  (li)
definition   :  NegBranchMiw(li)=0;

identifier   :  NoExpansionusingExistingLines
index domain :  eli
definition   :  BranchAdded(eli)=0  ;

identifier   :  RelaxationConstraint
definition   :  sum[eli, CongestionRelaxation(eli)]<=1  ;

identifier   :  MutuallyExclusiveConstarait
index domain :  (li)
definition   :  CongestionRelaxation(li)*BranchAdded(li)=0  ;
```

MATHEMATICAL PROGRAM:

```
identifier   :  Minimizecost
objective    :  TotalCost
direction    :  minimize
constraints  :  AllConstraints
variables    :  AllVariables
type         :  MINLP ;
```

```
 ENDSECTION;
```

PROCEDURE

```
   identifier :  MainInitialization
   body       :

     Empty BranchAdded;
     Empty PowerGeneration;
     Empty LinePowerFlow;
     Empty Busangle;
     Empty BBusNew;
     Empty BBfadd;
     Empty OperationCost;
     Empty InvestmentCost;
     Empty TotalCost;
     Empty LMP;
     Empty PosBranchMiw;
     Empty NegBranchMiw;
     Empty GeneratorEtaMax;

 ENDPROCEDURE;
```

PROCEDURE

```
   Identifier :  MainExecution

   DECLARATION SECTION

     ELEMENT PARAMETER:
        Identifier :  GenMP
        range      :  AllGeneratedMathematicalPrograms ;

   ENDSECTION  ;

   body       :
             GenMP := GMP::Instance::Generate( MinimizeCost ) ;
             GMPOuterApprox::DoOuterApproximation( GenMP ) ;

 ENDPROCEDURE;
```

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