Identification of Strategic Maintenance Resource Demand - A Reliability Based Approach

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Challenge the future

Identification of Strategic Maintenance Resource Demand - A Reliability Based Approach

by

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Abstract

Airline Maintenance and Engineering (M&E) organizations are faced with a number of repairs time to time for their fleet of aircraft due to accidental damages. As these damages are unpredictable in nature, the approach to repairing these damages is reactive. These type of repairs fall under the category of corrective or unscheduled maintenance policy compared to the planned preventive or scheduled maintenance. As the occurrence of these unscheduled repairs result in consumption of more maintenance resources in an untimely manner, they add to the existing costs for the organisation. Hence, it is of interest to the M&E to predict the demand for these resources for a future period so that the organisation is better prepared to handle future maintenance activities. One of the resources impacted due to unscheduled repairs is capacity, i.e maintenance hangar facility. If the capacity of a hangar facility can be divided into certain number of slots, then prediction of the demand for these slots would be beneficial to the maintenance planner. In order to identify the demand for these slots, it is first important to forecast the trend of unscheduled repairs. To achieve this goal, i.e. prediction of future repair and determination of slot capacity, a novel application for the integrated use of a reliability and inventory control model has been identified in this thesis. Here, the concepts of inventory control has been specifically applied to a maintenance application to determine the maintenance capacity by taking into account the stochastic demand of unscheduled repairs. The model used to predict the demand of unscheduled repair is a Non-homogeneous Poisson Process (NHPP) reliability model with a Power law intensity function and the inventory control model that was found to be applicable is the single-system single location Base-stock policy model. The reliability model considers the superpositioning principle through which the failure behaviour for the entire fleet of aircraft could be predicted. Certain performance measures were identified from the inventory control model, which helped in determining capacity based on optimum costs as well as service level requirements. As a proof of concept, a study is done on identifying the long-run capacity requirements for a fleet of Boeing 777 aircraft of a major European airline. Two specific structural components were identified on which the study was carried out, namely, the leading slats and the outboard flaps. The results showed the successful implementation of the model by identifying 30 slots necessary in the next 1500 flight cycles at an optimum cost for the case of leading edge slats.

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Nomenclature

α	Demand rate
β	Shape parameter
$\hat{\beta}$	Scale parameter - maximum likelihood estimate
$\hat{\beta}$	Shape parameter- maximum likelihood estimate
λ	Scale parameter for individual systems
λ_s	Scale parameter for superposed systems
$\overline{A}$	Average stockout frequency
$\overline{B}$	Average Backorders
$\bar{I}$	Average Inventory
D	Demand
b	Penalty cost for one backordered unit for one unit time
$C(s)$	Cost function
C_M^2	Cramer-von Misses test statistic
h	Cost of holding one unit inventory for one unit time
L	Leadtime
N	Number of failures
$p(u)$	Number of system under observation at time u
s	Base-stock level
T	Truncation time
t	time
T_{ij}	Global time to failure for i_{th} failure and j_{th} system
$u(t)$	Intensity function for a single system
$u^*(t)$	Intensity function for a superposed system

Abbreviations

M&E	Maintenance & Engineering
MLE	Maximum Likelihood Estimation
FC	Flight Cycle
MLTD	Mean Leadtime Demand
MLTD	Mean Leadtime Demand
HPP	Homogeneous Poisson Process
NHPP	Non-Homogeneous Poisson Process
GRP	Generalised Renewal Process
RP	Renewal Process
TTT	Total Time on Test
TBF	Time Between Failures
AGAN	As Good As New
ABAO	As Bad As Old
EPQ	Economic Production Quantity
SL	Service Level
MCS	Monte Carlo Simulation
LHS	Left-hand Side
MLTD	Right-hand Side
MTBF	Mean Time Between Failures

1 Introduction

The airline industry is one of the most maintenance intensive industries in the world. A dedicated maintenance & engineering (M&E) organisation oversees the entire maintenance activities associated with all the aircraft within the airline's fleet. The M&E faces different types of damages on aircraft structures on a regular basis, which need to be repaired quickly adhering to the regulatory requirements, to prevent any aircraft downtime [1]. One of the prominent types of damages affecting aircraft structures & its components are accidental damages. The causes of which are listed below [1] :

- Ground & cargo handling equipment, foreign objects.
- Erosion from rain, hail, lightning or runway debris.
- Spillage, freezing and thawing.
- Damages resulting from human error during aircraft operation & maintenance.

These damages are highly unpredictable and stochastic in nature compared to the damages caused by structural aging, fatigue and deterioration. Therefore a preventive maintenance action does not help in addressing this issue. Aircraft maintenance actions are broadly classified into two categories, *scheduled/preventive* and *unscheduled/corrective maintenance*. The repairs or maintenance actions due to accidental damages fall under the unscheduled category. In this thesis, we consider only accidental damage repairs as unscheduled repairs.

Based on their severity and tractability the unscheduled repairs are tackled during the routine aircraft letter checks (A,B,C & D) mainly during the A, B & C checks. Since the costs of maintenance & engineering are a significant component of overall airline costs [2], any kind of unscheduled repair activity would be an unnecessary addition to the maintenance costs because of additional usage of resources. The resources impacted directly in this case are man-power, resulting in increased man-hours, excess utilisation of hangar facilities (capacity) and material requirements. Also, there is an indirect effect caused by undesired aircraft downtime resulting in loss of revenues for the airline. Hence, when faced with these unscheduled repairs, if resources are not available at the right time, it can lead to very high costs for the M&E organisation as well as the operator [3].

Therefore, identifying resource demand for unscheduled occurrences during the maintenance production planning phase would help the M&E to better plan and prepare future maintenance activities. This leads to the question : What methods can we adopt to identify the resource demand necessary to meet future unscheduled repairs?

In current industry practise, the maintenance activities are scheduled by a dedicated team, production planning and control (PP&C) which has three main functions, *forecasting*, *planning* and *control*. The forecasting department predicts the future workload on a short-term $(1-3yrs)$, intermediate term $(3-5yrs)$ and long term $(5-10yrs)$ basis. These forecasts are mainly carried out for the scheduled maintenance activities, typically for a fleet of aircraft of each type [3]. The unscheduled maintenance repairs are carried out during these planned maintenance activities in an ad-hoc manner. This can result in unexpected and increased costs as discussed above. To overcome this problem, predicting the maintenance work-load and the capacity required, specifically for unscheduled maintenance would be of strategic importance to the M&E organisation.

The prediction of maintenance work-load can be attributed to the number of accidental failures expected in a future time period. This falls under the domain of reliability modelling. The analysis of historical failure data using reliability models to predict the inspection/ maintenance intervals is an extensively researched topic [4]. Also, capacity determination which is the objective of capacity planning problems is also a well established domain. But, there has been limited work done in integrating these two domains, solely for maintenance applications. Since the failure rate (which acts as demand) generated by the reliability model is stochastic in nature, the capacity identification model should assume a stochastic input. To tackle this problem, a unique approach is taken in this thesis by using a special class of inventory control model exclusively for maintenance application. The cross-industry application of an inventory control problem is well known but their application specifically to determine the maintenance capacity by assuming the demand based on realistic data has never been tackled before.

1.1. Problem Statement & Research Objective

This thesis addresses the problem statement outlined in the introduction from an industrial and scientific perspective:

- **Industry perspective** : This is from the perspective of the Airline M& E organization having an inadequate forecast for the unscheduled maintenance activities.
    - The occurrence of unscheduled maintenance due to accidental damages on the aircraft is inevitable for the airline which adds to the unexpected costs. Along with the scheduled maintenance base capacity, it is useful to see how much unscheduled activity would affect the maintenance slot requirements. The stochastic nature of these occurrences proves to be difficult for the M&E to predict and plan resources for such repair activities. Hence, it is beneficial for the airline from an economic point of view to have an estimate of the expected failures and resources required in the future.
- **Scientific perspective** :
    - Based on the requirements of the current thesis, a type of inventory control model, namely, the base-stock policy is used for the identification of optimum capacity required to fulfill the demand from unscheduled maintenance repairs. This is carried out from a long term and fleet level perspective. This framework aims to integrate the aspects of reliability modelling and inventory control model.

The research objective consequently is the following:

To identify the maintenance resource (capacity) demand for a fleet of aircraft, impacted by accidental damages, by integrating a reliability and inventory control model that accounts for the stochastic nature of damage occurrence.

As a contributing objective, a reliability-based approach to simulate future damage occurrence demand for a *fleet of aircraft* is adopted, to exploit an identified gap in the scientific body of work.

1.2. Scope of Research

The unavailability of particular data as well as time constraints in implementation put boundaries on the scope of the present study. These boundaries are presented below in a general overview. Motivation for individual boundaries is provided throughout the research, particularly in chapters 3 methodology and chapter 4: Case study.

- The reliability analysis is not performed with respect to a specific type of accidental damage. Rather, all damages that fall under accidental damage type category are included for analysis.
- The aim is not to analyze the capacity demand for a specific aircraft but for a fleet of aircraft of single type.
- The type of repair is not specifically looked into for the reliability model. Any damage to an aircraft component must ultimately undergo a permanent maintenance. Hence, the reliability model does not explicitly consider the effectiveness of each repair activity.
- The thesis doesn't look into the material support required to solve the maintenance repair.
- The research analyses the costs associated to the maintenance activities based on relationships with each other rather than for individual repairs.
- We have assumed that all aircraft of the fleet arrives at one base for maintenance. This may not be true in reality as it depends upon the number of bases operated by the airline.

1.3. Outline of the thesis

The structure of the report is presented as follows. This report consists of 6 chapters, beginning with the current chapter which presented the research objective for this thesis project. Chapter 2 presents a literature review on the state of the art in the present scientific domain specifically focused on the domains that aims to solve the research objective. The literature review aims to find the gaps in scientific research which would help in integrating the fields of reliability modelling and maintenance production. Chapter 3 discusses the methodology, which discusses the overarching modelling framework and also describes each of the sub-models in detail. The aim of this chapter is to show how the sub-models are integrated to finally achieve the desired result, i.e. identifying maintenance capacity. Chapter 4 presents a case study that acts as a proof of concept for the integrated model discussed in Chapter 3. The case study is conducted on the maintenance data provided by a major European airline for their fleet of Boeing-777 aircraft. The Chapter presents the results arising from each of the sub-models and finally identifying the capacity needs. Verification & validation of the reliability model is discussed in Chapter 5. This chapter mainly confirms that the chosen reliability model is indeed a suitable model for this research. Also, relevant sensitivity studies are conducted for the capacity model. Finally, the conclusions from this research work and the recommendations for future research are presented in chapter 6.

2

Literature Review

The literature was conducted keeping the main goal of project in mind. A reliability model has to be chosen to determine the failure rate and a suitable capacity planning model has to be implemented which takes the demand inputs from the reliability model and determines the a set of capacity requirements that satisfies this demand from a long-term strategic point of view.

2.1. Reliability analysis and modelling.

Determining the reliability of the given component constitutes a major part for maintenance capacity demand identification. In essence it is the reliability modelling that provides the times to future failures that would eventually act as a demand. Statistical methods used for the modelling of reliability is a well discussed topic in the literature. The focus here has been to identify suitable methods that can be implemented for the maintenance modelling of a repairable component. The aircraft structure is a complex repairable system comprising both of composites and metallic components. By analysing the life data (obtained from the maintenance organisation) of each component it is possible to predict the future failure behaviour.

Selecting a suitable reliability model that right fits our hisorical data is of utmost importance for its future synthesis with an inventory control model. To carry out this analysis [5] has presented a well-structured methodology which gives the importance to trend testing the given data through various methods and systematically converging at the right choice of reliability model. For an effective reliability analysis it is important for the analyst to have the knowledge of the following three aspects :

1. The methodology, data and information needed for model building.
2. The properties of different models.
3. The tools & techniques to determine whether a particular model is appropriate for a given data set.

It is the lack of this knowledge that can lead to using statistical models with false assumptions [6]. For repairable component the key input data is the time between failures which is extracted from the data received from maintenance organisation.

2.1.1. Data testing tools

The purpose of data testing tools is to investigate the assumptions of the nominated models for reliability analysis. The operating environment and the environmental conditions play a role in the failure time distribution. Often times, for multiple repairable components in spite of them being operated in different conditions, the assumption of identical distribution is observed, which may not be true in reality. In the case of pooling data for multiple components there are a number of conditions that has to be met as stated in [5].

It is important to realise that since we are looking at accidental damages, data pooling at a fleet level should be analysed carefully since many models require homogeneity in data. The trend behaviour can vary with a particular maintenance strategy, e.g. a perfect repair exhibits no trend in failure data, whereas for minimal repair, they show a monotonic behaviour. A set of trend analyses should be carried out for a single data set to find appropriate models. Garmabakki. et.al in their paper[7] have categorised the data analysis framework into four main categories

- Data collection
- Homogenization process
- Catergorising unit based on trend behaviours
- Reliability model selection and parameter estimates.

The data collection and homogenisation is a an important phase in reliability analysis. For repairable systems the main data are the times between failures (TBF). The data collection proposed by [7] must include : technical information concerning failures (unit id, serial numbers and operation time), description of failures and their symptoms, environmental conditions, suspected causes, repaired items, repair times and root causes. Here, homogeneous units mean a set of identical components with comparable operational and environmental stress.

The categorisation of units based on their trend using appropriate statistical test is considered the an important step because a trend in the reliability data could be monotonic or non-monotonic (or trend free). For a monotonic trend the system is said to be either improving (decreasing number of failures) or deteriorating (with increasing number of failures). Non-monotonic trends occur when the trends change in time or repeat in cycles [5]. There are various methods by which this can be measured, graphical or analytical. In [8] some graphical methods are mentioned such as cumulative failure versus time plot, Duane plots by which this categorisation can be achieved, but the drawback here is that the interpretation of these results can be subjective. In spite of this drawback [9] shows that the TTT (total time of test) plot can be modified specifically to test for a NHPP power law process. This form of TTT plots can be further adapted for a multiple system case which is shown in [5].

It is also advantageous to have analytical means by which the trends can be categorised. The characteristics of the different analytical tests and their classification based on the null hypothesis (for RP, HPP, NHPP, monotonic or non-monotonic trend) are mentioned in Ascher and Feingold [10]. The widely used statistical tests are the Mann test, Laplace test, Military Handbook (MH) test and Anderson-Darling (AD) test [5] & [7]. The Mann test has a null hypothesis of RP and an alternate hypothesis of monotonic trend. The Laplace test has a null hypothesis of HPP and an alternate hypothesis of NHPP with monotonic trend. This test is more suitable for NHPP with log-linear intensity function. Similar to the Laplace test the MH test also has HPP as its null hypothesis and NHPP with monotonic intensity as alternative hypothesis. This test is suitable for NHPP with power law process.

The reliability modeling begins with choosing an appropriate intensity function that best suits the extracted data, from which the reliability parameters are estimated. Because of its flexibility and applicability to various failure processes, the Weibull distribution is used as the distribution of time to failure to perform the analysis using various reliability models [11].

2.1.2. Reliability Models

The most commonly used models for reliability analysis are the homogeneous Poisson process (HPP), renewal process (RP), non-homogenous renewal process (NHPP) and generalised renewal process (GRP) [11]. Every model is based on certain assumptions relating to the real-world situation. The RP is a counting process where it is assumed that the time between failures are independent and identically distributed with an arbitrary life distribution and at each failure occurrence, the repair performed is a perfect one and hence restores the system to the 'as good as new' (AGAN) condition. Whereas for the

NHPP, the assumption of a minimal repair restores the system to a functional state same as the one just before its failure, i.e. 'as bad as old' (ABAO) condition.

It is to be noted that minimal repair and permanent repair are the two extremes that could occur and most repair in the real-world maintenance fall between these two extremes, and hence they fall in the imperfect repair condition. Imperfect repair models attempt to incorporate these states in their analysis. These mathematical models are much more complicated to implement the after repair states but are suitable for real operating conditions [12]. General renewal process (GRP), is one of the imperfect repair models that covers a very broad assumption concerning system repair state. This model was proposed by Kijima and Sumita [13],[14], consisting of two possible probabilistic models Kijima model I (KI) and Kijima model II (KII). The former assumes that repair is effective only for the last repair and the latter assumes that repairs can restore cumulative wear out and damage up to the present time.

Although the imperfect repair models are assumed to model conditions close to reality, [15] has shown no considerable difference in the estimates from a NHPP models and a GRP. Also, as these models are used for single systems, the disadvantage of using a GRP would arise when considering pooling of data from multiple systems. Such systems also called as a superposed or super-imposed system [10], [16] can only be modelled using a HPP or NHPP model.

Homogeneous Poisson Process (HPP)

The HPP follows the basic assumption of As good as new (AGAN), i.e. to model the repair in such a way that the component is brought back to a perfect condition as it were a new. The HPP is a Poisson process with constant intensity function. It is the simplest model used for modelling a repairable system and due to its constant intensity function, it cannot be used to model systems that deteriorate or improve [8]. The HPP has the following properties [17] :

1. $N(0) = 0$
2. It follows independent increments property, i.e. the number of failures observed in the non-overlapping time intervals are independent.
3. The number of failures observed at any time, t, has a Poisson distribution with mean λt.

Where,
N = Number of failures in an interval
λ = Rate of occurrence of failure or the failure rate.

Renewal Process (RP)

As mentioned earlier the renewal process follows the assumption of as good as new (AGAN), which means that it is used for modeling repairs that bring the component back to a perfect condition as if it were a new product. For RP, the times to failures are considered to follow an iid, i.e. independent and identically distributed random variables. Since the model represents an ideal situation, it has very limited application for modeling of repairable components. This model is more suited for non-repairable (replaceable) components [11]. RP has the advantage over HPP because of its ability to model deteriorating and improving systems. The Expected time-to-failure and the Variance associated using a Weibull distribution are presented in [18].

$$\eta = \theta\Gamma(1 + \frac{1}{\beta}) \tag{2.1}$$

where,
η = expected number of failures
Γ = gamma operator
β = Weibull shape parameter
θ = Weibull scale parameter

The probability of failure using the renewal process is given by equation below [8].

$$\lim_{t\to\infty} P(N(t) < a(t)) = \Phi(y) \tag{2.2}$$

$$a(t) = \frac{t}{\eta} + y\sigma\sqrt{\frac{t}{\eta^3}} \tag{2.3}$$

Where,
Φ = cumulative distribution function of Normal
a = expected failures in an interval
y = normal distribution test value for probability of failure
t = time or flight cycles

Non-homogeneous Poisson Process (NHPP)

The NHPP is a stochastic point process that assumes the as bad as old(ABAO) repair assumption, in which the probability of occurrence of n failures in any interval [t1,t2] is represented by a Poisson distribution with a mean [11].

$$E[N(t)] = \int_{t_1}^{t_2} \lambda(t)dt \tag{2.4}$$

NHPP is characterised by non-constant intensity function and satisfying the following 3 conditions [8]:

1. $N(0) = 0$
2. For any $a < b$, $N(a, b]\ POI(\int_a^b \lambda(t)dt$
3. The process has independent increments property (IID), i.e. for any non-overlapping intervals $(t, t + \delta t)$ and $(s, s + \delta s)$, the N in thoe intervals are independent.

Where,

N = Number of failures
t = operational time or flight cycles

The intensity function represented by $\lambda(t)$ with a power law prower law intensity is given by equation below:

$$\lambda(t) = \theta\beta t^{\beta-1} \tag{2.5}$$

and the probability of number of failure occurrence is given by :

$$P(N) = \frac{e^{t\lambda(t)}(t\lambda(t)^N}{N} \tag{2.6}$$

Where,
θ = scale factor
β = shape factor

NHPP for a superposed system

In the case of a superposed system with k systems, the power law intensity function is given by equation below [16]

$$\lambda(t) = k\theta\beta t^{\beta-1} \tag{2.7}$$

Hence, the k number of systems are multiplied to the scale factor,θ

Imperfect repair

Generalized renewal process (GRP) is the maintenance policy carried out during the assumption of imperfect repair strategy. Compared to the minimal repair NHPP and perfect renewal of RP, general repair models tries to cover a state of the component or system which lies between the assumptions of ABAO and AGAN, i.e. 'better than old but worse than new', 'better than new' or 'worse than old' states. The ubiquitous popularity achieved by GRP is due to its ability to model all the five states of a repairable system. Mathematical models that involve these repair states are much more complicated [19]. There are many models that have been introduced over the past two decades that tries to model an imperfect repair. Numerous authors have tried to develop GRP models but Kijima's models have been the most widely cited and effective among them.

The concept of virtual age was first initiated by Kijima and Sumita [13] & [14]. They modelled the imperfect repair using the GRP. Virtual age models are now one of the most researched topics for generalized repair models which has led to them having considerable portion within the imperfect repair models for repairable systems [20]. Many researchers have tried to implement GRP but the models by Kijima proved to be the most effective receiving numerous citations [11],[21], [22]. Hence, the present work would be focused on reviewing the Kijima models, specifically Kijima model I. Kijima and Sumita proposed two possible probabilistic models for general repair called the Kijima model (KI) and Kijima model II (KII) [13], [14]. In the former, the assumption is that repair is effective only for the last repair, whereas for KII, the repairs can restore the cumulative damage and wear out up to the present time. The models uses the concept of virtual age (Vn) for the repairable for the repairable system. The parameter V_n represents the calculated age achieved by the system immediately after the nth repair occurs. If $V_n = y$, then according to KI, the times between failure X_n has the following cumulative distribution function (cdf)

$$F(x|V_n = y) = \frac{F(x+y) - F(y)}{1 - F(y)} \tag{2.8}$$

Where,
$F(x)$ = cumulative distribution function of the time to first failure.
V_n = virtual age of the system.

It is assumed that the nth repair only compensates for the accumulated damage between $(n-1)_{th}$ and n_{th} failure. The virtual age of the system V_n can be expressed as

$$V_n = V_{n-1} + qX_n \qquad n = 1, 2, \dots \tag{2.9}$$

Where,
q = repair effectiveness index.

According to this model,$q = 0$ represents a perfect repair which means the component can be modelled through a perfect renewal process (RP). Whereas when the value of q = 1, the component is brought to a state similar to ABAO assumption, and hence can be modelled using NHPP model. The value of =q in the interval $0 < q < 1$, signifies a imperfect repair scenario. Therefore, q can be interpreted as an index for representing the effectiveness and quality of repair [12]. The parameter estimation of GRP through Maximum likelihood estimation is proposed by Yanez et. al [11], where they avoid the computationally intensive Monte-Carlo method of estimating the GRP parameters proposed by Kaminskiy and Krivtsov [21].

2.2. Production and Maintenance Planning models

Interdependence between production systems and maintenance

There exists a level of interdependence between these two systems, maintenance itself can be viewed as a production process with it's own planning of capacity and other resources. Also, maintenance can be viewed as process within a production system in such a way that the maintenance activities influence the outputs of the production process. Most research have been carried out in the latter. The former case would apply in capital intensive sectors like airline or ship maintenance. The following sections present the models used in these two sectors.

2.2.1. Application - Maintenance Within Production Planning

There has been significant interest in models seeking to integrate the aspects of production, quality and maintenance within an industry. The inter-dependencies between these fields have been the reason for attempting such integrated models[23]. Within the production industry, the planning refers to determination of *lot sizes* (the units of products manufactured) and computing the *capacity needs* in the case of changing demands. The production systems undergo failures from time to time, which in turn affects the outputs. It is this issue that the integrated models aim to optimize, i.e. planning the maintenance in such a way that the production system maintains the necessary outputs to meet the fluctuating demand. Economic production quantity (EPQ) models, which can be classified as a type of inventory control model have been used extensively to incorporate these failure aspects [24].

Groenvelt et. al [25] use EPQ model to study the effects of stochastic machine breakdowns and corrective maintenance on the production plant output. They assumed the conditions of both constant as well as increasing failure rates. Their results revealed that with the increase in the failure rate, the optimal stock required also increases.

Srinivasan & lee [4] uses base-stock inventory control policy to determine the invenotry level 'S' in a production plant. They showed that when the inventory of the plant goes below a certain value of 'S', a preventive maintenance is carried out. Such a maintenance is expected to bring back the production inventory back to the desired value 'S'. The failure rate they observed was a Poisson distributed with constant failure rate.

Various models have been used to illustrate the application flexibility for a preventive maintenance scheduling problem. Aghezaff et. al [26] in their paper use an inventory control model, specifically the economic lot sizing model to determine the production capacity requirements for preventive maintenance.

2.2.2. Application - Maintenance as a Production Process

Dekker in his paper[27] explains the situation of "maintenance carried out as a production process". Where the major goal is to identify the capacity needs to carry out maintenance. They mention that this activity can be carried out only for planned maintenance. The disadvantage of implementing such a model is the level of unforeseen maintenance activities that occur during the standard maintenance. Which makes it difficult to implement such a model. Hence, it would make more sense to only consider unscheduled activities and identify the capacity requirements rather combining them with the scheduled maintenance activities.

Bengu et. al [28] in their paper use a FCFS queuing model with Markovian routing to model the maintenance operations at a telecommunication work centre. The model uses failure rates derived from reactive type maintenance repair data which follows a non-homogeneous Poisson process. The purpose of the paper was to integrate the operations of two maintenance work centres into one, thereby eliminating the operational redundancies. Eventually the model aims to minimise the required manpower and the average waiting time in the system. The paper doesn't discuss on how the maintenance activities related to preventive maintenance can be included within the single work-centre, which would be the case in reality. Hence, as a result of only one work centre considered, the capacity/facility requirements as a result of a growing demand is not addressed here.

Dijkstra [29] applied maintenance planning in terms allocating personnel to certain time slots. This was done by developing a e allocation of maintenance personnel to certain time slots over the day. This allocation is based on the work demand provided by the airline. On similar lines Yan et al [30] in their paper also regards maintenance manpower as resource and builds a planning model which allocates the maintenance personnel to the demand. Both these models serve a short term need and do not provide resource requirement as a long term strategic plan.

Dufuaa [31] mentions the need for capacity planning where they have classified capacity into manpower, facility etc. and talks about the different stages needed to develop and strategically plan maintenance. The stages are, estimating a forecast, selecting a model that suits the demand, assessing the capacity requirements and finally adjusting the capacity requirements. The forecasting methods discussed in the paper are fairly simple methods based on analysing the historical data, such as the exponential smoothing, moving averages etc. The methods are extensively used to forecast the material requirements for spare parts inventory model.

Although the above section mentions maintenance as a production planning, not much research has been carried out with this regard because of the stochastic demand behaviour encountered when solely maintenance activities are considered. Hence most of the work has been devoted to "maintenance for production planning", i.e. integrating maintenance and production planning.

Although the use of a base-stock policy inventory model has been used to model inventory requirements in a production system with maintenance as a constraint, the sole application of this model for the purpose of modelling maintenance has not been discussed in the literature. The model described in Zipkin [32] can be proved usable to determine the long term capacity/facility requirements in maintenance planning especially for a corrective type repair.

Production planning and control within the airline maintenance context

Kinnison and Siddiqui [3] highlights the main functions of a production planning and control department within an airline maintenance and engineering organisation. They are briefly described below.

- Forecasting : The main function of which is estimation of the future work-load and creation of business plans for the existing fleet, all keeping in mind the future changes that might occur in the forecast period.
- Planning : The main function of which is scheduling the upcoming maintenance. This involves estimating the manpower, parts, facility and intervals required for all the maintenance checks (from daily to D checks). Broadly speaking, evaluating the capacity requirements. This is more of an idealised plan and in reality many changes could occur.
- Control : The function of control is ensure that the organisation sticks to the maintenance plan. In the case of any deviation from the actualised plan, the function of control department is to for example, increase manpower, outsource maintenance to contractors or delay the maintenance to a late check.

Dekker [27] points out the gap in applications of many maintenance optimization models in industry due mainly to it's complexity. This observation can be seen in many of the literature which only selects few sets of data or in some cases no data at all and applies an optimization model. This brings us to a situation where we need to bridge the gap between theory and application. The possibility of application in real world data is one of the aspects the current research highlights. Although this application has been simplified at multiple levels. It does not mean that the model cannot be applied. The simplification takes places due to unavailability of real data, which is not the case when working within the real industry.

2.3. Caveats in Present Literature

Although [25] considers repairs due to corrective maintenance and an inventory control model, the model used in the research was an EPQ model and applied to a production plant. Moreover methods for determination of the failure rates were not specified in their work.

The research by [4] came closest to using an base-stock policy with a Poisson demand for machine failures, but this was used in a manufacturing facility to determine production capacity, which is a direct application of the inventory control problem.

Although, [29] and [30] addressed the maintenance resource planning problem. They specifically focused on manpower planning rather than the capacity of maintenance facility. Also, the approach taken by them was to address a short term planning issue rather than a long term. Moreover, [29] employed a deterministic method for forecasting the workload demand and does not take into account the stochastic input.

As discussed above due to its adaptability to economic problems, inventory control models have been used in cross-industry applications. When it comes to application in the field of maintenance, as seen in sections above, these models have been commonly used in production/manufacturing cases. But the main objective of these models have always been to determine the manufacturing capacity subject to a maintenance issue or shutdown. Which means, the models were not directly addressing the capacity required only for a maintenance scenario.

In a maintenance intensive industry like the airline industry, which encounters far more unscheduled repairs, the prior knowledge of capacity demand needed to fulfill any future unscheduled repairs becomes important from a strategic planning point of view. Since there has been no work identified which studied the stochastic nature of demand to address the capacity issues faced by the maintenance organisation.

The possibility of adapting an inventory model (specifically the base-stock model) to solve such a problem would fill the gap in state of the art in research. This integration of an base-stock policy inventory control model and the reliability model jointly addressing only the unscheduled maintenance due to accidental damages adds to the novelty in the current research. Although the integrated reliability and inventory control model in the present thesis is applied to an airline maintenance case, in reality this model is flexible to be used in any industry that operates a fleet of valuable systems. e.g. Shipping maintenance.

3

Methodology

This chapter explains the methodology used in the present research. Section 3.1 discusses the basic model framework, which provides a visual description of the sub-models and the parameter values that are used in them. It provides an overview of the input and outputs that are expected from each model. Section 3.2 presents the global assumptions made for the reliability and the capacity identification or the inventory control model. Sections 3.3 to 3.6 discusses each of the sub-models in detail. Wherein the concepts are explained and the mathematical expressions for each of the models are presented.

3.1. Model framework

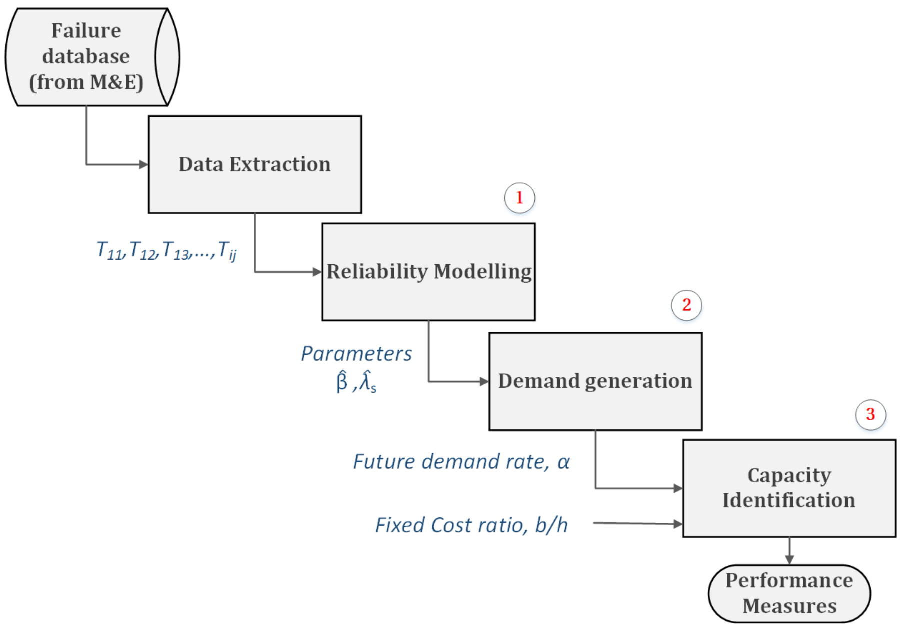


Figure 3.1: Model Framework

Figure 3.1 shows the basic modelling framework adopted for this thesis. The first step in the framework is to obtain the raw data for failures of the entire aircraft structure. This database is maintained by the airline M&E for their fleet of a type specific aircraft. Upon obtaining this raw data, data extraction needs to be carried out, which is to identify relevant parts and obtain their global failure times T_{ij}. These failure times act as input for the reliability model numbered as model 1 (in red) in the figure above. The purpose of reliability model is help estimate the parameters $\hat{\beta}$ and $lam\hat{b}da_s$ for the identified aircraft components. These parameters serve as input for the demand generation model shown as model 2, the purpose of which is to simulate the future global failure times and as a result predict the failure rate (α) for each component. Finally, these predicted demand rates along with the cost ratios are used as inputs to the capacity identification model (shown as 3), which is the inventory control model. Model 3 outputs the performance measures ($\overline{A}$, $\overline{B}$, $\overline{I}$ and $C(s)$) from which the required capacity resource is identified.

Model Parameters & Variables	
T_{ij}	Global times to failure
$\hat{\beta}$	Shape factor
$\hat{\lambda}_s$	Scale factor for superposed system
α	Failure rate/ demand rate
h	Holding cost per unit quantity per unit time
b	Penalty (Backorder) cost per unit quantity per unit time
$\bar{A}$	Stockout frequency/ Probability of stockout
$\bar{I}$	Average Inventory
$\bar{B}$	Average Backorders
$C(s)$	Cost of operating at a given base-stock.

Figure 3.2: Overview of Model parameters and variables

3.2. Model Assumptions & Motivation

The assumptions for the models used in this thesis are presented below :

Reliability Modelling

- The time taken for repairs are considered negligible.
- The model follows an independent increments property.
- The failure observation for all systems in the multi-system environment are truncated at a single time **T**.
- All the systems have their observation times starting at time 0 flight cycles.
- All systems follow identical failure intensity.

The repair time can be considered negligible in a case where the total observation period is much larger then the time taken for repair. for e.g If the total time for observation of the failures is 14 years and comparing it to the typical repair time for C-checks,which is 2 weeks, the repair time is negligible. The time taken for repair becomes significant when the observation time is small. The independent increments property would mean that any two failures occurring are independent of each other. This also means, the repair activity doesn't influence failures happening in consecutive intervals. A common truncation time can be assumed when all the system have operated for a same period of time. This also simplifies the estimation for reliability parameters. The model also assumes that for all systems, the observation time starts from 0 flight cycles. This assumption is valid because the observation starts from the year of delivery for each aircraft. All systems are assumed to be identical, this assumption can be considered for a fleet of aircraft of the same type, which have the same parts and design. Moreover, since the airline maintains a strict aircraft routing model, we can assume that the operational conditions faced by all aircraft on average will be the same.

Capacity Identification/Inventory Control Model

- The demand is stochastic in nature and follows a stationary Poisson distribution.
- The model considers that demand occurs from a single system.
- There is only one location considered for inventory storage.
- The demand occurs in a batch size of one.
- The model allows for backorders to occur.
- The leadtimes are assumed to be constant.

With respect to the maintenance application the assumption of having a demand with Poisson distribution with stationary increments holds true. As will be seen in the demand generation section, the failure times can be modelled has the demand with a constant rate Poisson process. This particular model assumes the demand to be occurring from a single system, from the thesis perspective, it would mean that the demand occurs from the components of same fleet of aircraft. The aggregative property of the Poisson process can be used in this case but there needs to be strong reasoning for accumulating demand rates from different components. Backorders are allowed to occur in this model, which means, there could be a chance that the failure is not repaired immediately and hence repair work is delayed. This plays a crucial role in the assumption of the cost ratio (which will be discussed in chapter 4). The leadtimes are assumed to be constant for the application considered in this thesis, as it can be attributed to the fixed schedule of letter checks (C-check) that are implemented by the M&E organisation. Which makes it more or less deterministic.

3.3. Data Extraction

For any mathematical model to simulate reliable results, the input data must be clearly defined based on realistic assumptions. Reliability modelling begins with data gathering and cleaning. Steps that have to be taken into account while gathering data are

1. Identification of components or parts with comparatively more number of failures. This is because the more the data points the better the reliability estimates. [5] suggests that a minimum of 5-10 data points per unit are required to carry out a meaningful analysis.
2. Having identified the relevant parts that can be studied, the next step is to order them based on the operating time. Choosing the right operating time is crucial for the analysis because this can lead to different results at the end of the analysis.
3. Based on availability of enough data points for each system, it becomes clear if a single system approach or a multiple system approach must be taken. The multiple system approach would result in super-imposed or a superposed process.

The main objective of the data extraction step is to obtain the global failure times for multiple aircraft (structural) components. These times would be the inputs for the reliability model. As for any statistical model, a data set with large number of data points would yield better parametric estimates. As discussed in the section [ref lit], the scarcity of data is one of the major limitations in modelling reliability. [5] shows that about 5-10 failures per system would be sufficient for the reliability modelling of a repairable system.

A step by step approach is taken carry out this task :

1. Classify the data, in terms of number of damage occurrences, into the main ATA chapters. Hence this would be for primary structural classification.
2. A primary structure is chosen for further analysis.
3. Further classification within the structure,in terms of number of damages, results in the identification of the most damaged components.
4. These damage occurrences are classified for each system (aircraft).
5. Since these damage occurrence is represented by a time, this would eventually be the desired global time to failures for each system.

Insufficiency in number of damage occurrences for each individual system would lead to combining the k systems into one single system. Such a system is also called as superposed system. The advantage of the superposed system is that it can model reliability for the entire fleet (of aircraft) of k systems. A superposed system is used in this research, a brief description of which is provided below.

Superposition System

An illustration of a typical superposition system is shown in figure 3.3 below. As shown in figure, each aircraft (system) is represented k. The time at which each system faces a damage is represented by T_{ij}, where i is the failure number and j is the system/aircraft number. Therefore T_{11} means time to first failure for aircraft one. Similarly T_{1k} means time to first failure for k_{th} aircraft. When the T_{ij} for all systems are combined and positioned on one single timeline, they become a superposed system. Hence, for a superposed system there exists a failure when any of the k system fails. This superposed system is obtained for each of the components identified in the section above.

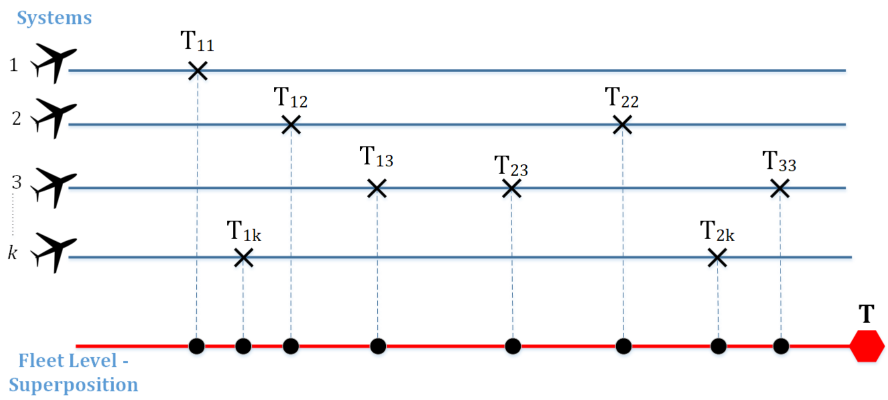


Figure 3.3: A Superposition System

3.4. Reliability model

This section discusses model 1 from figure 3.1. As we had seen in the model framework, the reliability model takes the T_i as input and generates the parameter estimates. The choice of the reliability model has been made based on two conditions: first, it must be applicable to a superposed system and secondly it should satisfy a particular trend test. There are two commonly used reliability models that satisfy the superposition condition, namely, the Homogeneous Poisson Process (HPP) & the Non-Homogeneous Poisson Process (NHPP). Based on the failure data obtained, the the most intuitive hypothesis is that the data must follow a process close to the HPP, As we are considering failures caused by accidental damages which are truly random (with constant failure intensity), failure times wouldn't exhibit an improving or deteriorating process. Even though we expect the process to be close to a HPP, there might be a possibility that the failure data experiences a non- monotonic behaviour. Hence, the model chosen for the present research is the NHPP with a power law intensity function.

Trend Test

To satisfy the second condition that the model chosen does indeed follow a NHPP power law process, we need to carry out a trend test on the extracted global times T_{ij}. The test chosen is the combined Total time on test (TTT) plot. This test adequately models the superposed system (i.e combines multiple systems) as well as tests for the acceptance of Power Law Process (PLP). This test is presented in [5] and specifically adapted for a PLP using the representation given in [9]. The plot derived from the test reveals a unit sized square with the curve representing the trend. A PLP model is accepted if the curve closely aligns with the diagonal of the unit square.

The combined TTT-test aims to identify trend for multiple-system case. Consider the case of k independent processes, with same failure intensity function (i.e. considering m identical systems). The observation intervals for k systems are contained in the interval a time interval $[0, S]$. Here, S would be the truncation time. The the total time on test statistic is calculated as follows :

$$\frac{T(S_i)}{T(S)} = \frac{\int_0^{S_k} p(u)du}{\int_0^{S} p(u)du} \tag{3.1}$$

Where, $p(u)$ is the number of processes or systems under observation at time u. S_i is the time at which the i_{th} failure takes place.

NHPP - Power Law Process

A NHPP can be modelled using two types of intensity functions :

- Power law intensity
- Log-linear intensity

Out of the two, the power law process is used in this research due to its wide acceptance and also because a log-linear intensity function is often used to model a rapidly deteriorating system which does not comply with our case of accidental damages. The NHPP- Power law process is also often called as the Crow (AMSAA) model or Weibull Process. Suppose the observation of the system starts at age 0 and observed until the time T (truncation time), the number of failures the system experiences $N(T)$ during this time is a random variable with successive times to failure T_{ij}. The intensity function for the PLP is given by [16],[33] :

$$u(t) = \lambda\beta t^{\beta-1} \qquad t > 0 \tag{3.2}$$

Where, λ, $\beta > 0$ are the scale and shape parameters respectively and t is the age of the system. This representation of the intensity function is for an individual system. Although the above representation of the intensity function is same as the failure rate of the Weibull distribution, the terminology or the interpretation of failure rate for Weibull distribution doesn't apply here. This is a common misconception in modelling repairable systems.

The expected number of failures for the intensity function above is given by differentiating it with respect to t, hence

$$E[N(t)] = \lambda t^{\beta} \qquad t > 0 \tag{3.3}$$

The above formula can be used to estimate the number of failure occurrences over any time interval. This can be applied later in the verification process.
Now, considering a superposed system with k systems under observation, the intensity function would be :

$$u^*(t) = k\lambda\beta t^{\beta-1} \qquad t > 0 \tag{3.4}$$

The $k\lambda$ expression in the above expression would be referred to as λ_s in this report, which signifies the scale parameter for a superposed system. It is to be noted that the β does not change for the superposed system.

The parameters λ_s and β are estimated by the Maximum Likelihood Estimation (MLE) method. A detailed derivation of the MLE estimators are provided in [basu] for further reference. The closed form formulas for the estimators $\hat{\lambda}$ and $\hat{\beta}$ are given by

$$\hat{\lambda} = \frac{\sum_{j=1}^{k} N_j}{kT^{\beta}} \tag{3.5}$$

$$\hat{\beta} = \frac{\sum_{j=1}^{k} N_j}{\sum_{j=1}^{k} \sum_{i=1}^{N_j} ln(\frac{T}{T_{ij}})} \tag{3.6}$$

Hence for a superposed system,

$$\hat{\lambda_s} = k\hat{\lambda}$$

These estimators are the main output from the reliability model. They are calculated for all the identified components. Following the estimation of the parameters, we calculate the intensity function of the component at various points of time using the equation 4.4. The intensity function basically tells us about the probability of failure occurring at any instant of time.

Goodness of fit

The goodness of fit test is carried out to test the compatibility between the model and data. The test used here is the Cramer-von Misses test adapted from Crow et. al [33] which is specifically used to test the data for a PLP model. The final tests statistic C_M^2 is represented by the formula below :

$$C_M^2 = \frac{1}{12M} + \sum_{j=1}^{M} (Z_j^{\overline{\beta}} - \frac{2j-1}{2M})^2 \tag{3.7}$$

Where,
M = Total number of failures for time truncated case. $\overline{\beta}$ = Unbiased estimate of the shape factor.

The C_M^2 value thus obtained is checked for their appropriate significance level by correlating with the standard critical value table provided for Cramer-von Misses test. According to Basu [8], a significance level of 95% satisfies the case for PLP model.

3.5. Demand generation

This model can be taken as an extension to the reliability model. As the parameters from the power law process are used to simulate the future demand. From figure3.1 we can see that the demand generation model takes β and λ_s parameters as the input and outputs the demand rate α.

This model essentially performs the following critical function: it simulates the failure times for an NHPP with a power law intensity, with the help of which future demand rates are derived for each component. The key parameters for generating demand for the this model comes from the λ and β values obtained from the reliability model discussed above. Here, demand is the number of failures occurring in a given unit of time and it's denoted by α. Since the demand generated is from the reliability model, it in essence follows a Poisson process. Hence we have stochastic nature in the demand generated. Which means that the occurrence of demand is probabilistic and not certain or deterministic, i.e. at any given interval of time a demand could occur or it could not.

In order to generate the failure times for the demand, a simulation technique for the power law process is required. This is done by obtaining the distribution function for the T_{ij} and with the help of the inverse transform method we can calculate the successive failure times T_i [34],[35]. The distribution function derived from power law intensity is given by :

$$F_{T_{ij}}(t) = 1 - exp(-\lambda_s[(y+t)^{\beta} - y^{\beta}] \quad (3.8)$$

The above function is used to derive the equations for the successive failure times as given below :

$$T_1 = [-\frac{1}{\hat{\lambda}_s} lnU_1]^{\frac{1}{\beta}} \quad (3.9)$$

$$T_q = [T_{q-1}^{\beta} - \frac{1}{\hat{\lambda}_s} lnU_q]^{\frac{1}{\beta}} \qquad q \geq 2 \quad (3.10)$$

Here T_1 is the time to first failure and T_q are the successive failure times after T_1. This means we would generate a series of global times T_i where, $i \geq 1$. Notice the change in global times from T_{ij} to T_i, the j is not applicable here because the failure times generated are not specific to an individual system/aircraft but for the fleet. Due to the random number U, there would exist a variation in each of the T_i generated. In order to compute the mean value for these failure times, a Monte Carlo simulation is performed. The time between failures for these successive times are computed to determine the mean time between failures (MTBF). Finally, failure rate α is computed from the MTBF, where the α signifies the number of failures per flight cycle.

3.6. Capacity Identification Model

The model 3 shown in figure 3.1 is presented in this section. The inputs to the model are the demand rates from various components generated by model 2 and the cost ratios, the outputs are a set of performance measures through which the capacity requirements can be identified. This section briefly describes the base-stock policy inventory model and its translation to a maintenance application.

Base-stock Policy Inventory Model

Within a production environment, an inventory can be considered as a buffer between the supply and demand. This could be for example a warehouse(facility) where the finished goods are stored. It is then of interest to the organisation to minimise the costs for operating this warehouse by minimising the storage quantity of the finished goods. This is where inventory control model comes into use, as it helps in estimating a certain optimum level of inventory by minimising costs while catering to the demand. There are a number of inventory control models that have been developed in the literature each having its own assumptions [32]. The model chosen for this study is the single item, single location base-stock inventory control model, as this model serves the purpose of the current study. The assumptions for this model are presented above in section 3.2.

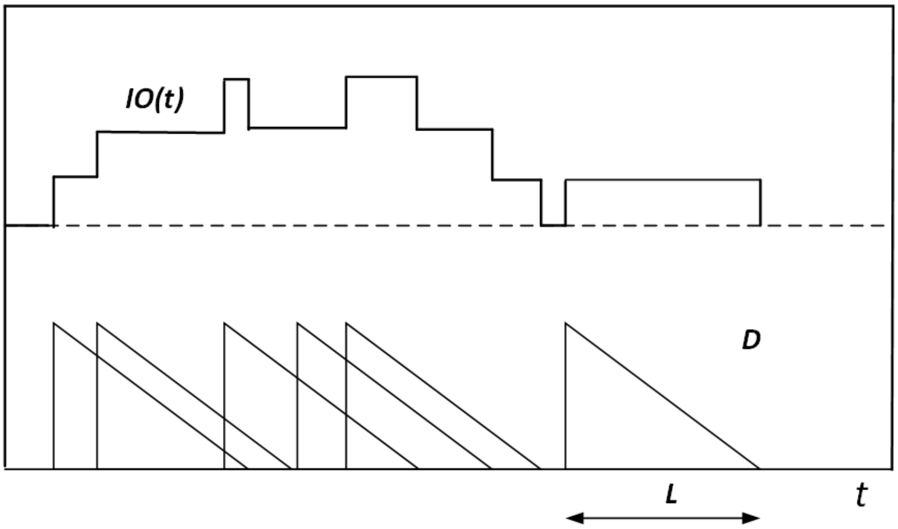


Figure 3.4: Occurrence of Demand & Orders for stochastic demand base-stock model

The Concept

- Figure 3.4 illustrates the relationship drawn between demand occurrence and orders placed. A hypothetical demand (**D**) generated by a Poisson process is shown in bottom half of the figure. As we can observe the occurrences of demand are irregular which is indicative of the stochastic nature of the demand.
- For each of these occurring demand there is a corresponding order that is being placed, instantaneously. This is shown by the step plot for inventory on order($IO(t)$) on the top half of figure 3.4.
- This relationship shows that the $IO(t)$ can be expressed as the demand within an interval $(t-L, t]$.
- Since the demand occurs in unit quantity, a unit order is placed.
- The purpose of inventory control would then be to maintain a certain inventory position, such that it adequately caters to the occurring demand. This inventory position is fixed at a value s, called the base-stock level.
- The policy then aims to keep the inventory at the constant value s. If the system starts with an inventory position less than s, then the difference is ordered immediately. If system starts with inventory position greater than s then we wait until IP reduces to s. Once IP reaches s, it remains there.
- The performance measures indicate the effect of keeping the inventory position at a given s value and based on the analysis of these measures the necessary level of s is determined.

The performance measures which helps calculate an optimum level of base-stock level are:

$\overline{A}$ = average stockout frequency
$\overline{B}$ = average backorders
$\overline{I}$ = average inventory
$C(s)$ = average cost function

These performance measures can be computed as follows [32]:

$$\overline{A} = Pr[D \geq s] = 1 - \sum_{j<s} g(j) = G^0(s-1) \tag{3.11}$$

$$\overline{B} = \alpha L - \sum_{0 \leq j < s} G^0(j) \tag{3.12}$$

$$\bar{I} = E[[IN]^+] = s - \lambda L + \overline{B} \tag{3.13}$$

$$C(s) = h\bar{I} + b\overline{B} \tag{3.14}$$

Where,
$g(j)$ = Poisson probability mass function.
α = demand rate.
L = lead time.
s = base-stock inventory level
h = cost of holding one unit of inventory for one unit of time.
b = penalty cost for one backordered unit for one unit of time.

Since obtaining the values of h and b for specific components are out of the scope of this thesis project, we have simplified the cost function by expressing h and b in relation to one another. Therefore, dividing equation 3.12 by h we get,

$$C(s) = \bar{I} + \frac{b}{h}\overline{B} \tag{3.15}$$

Model translation
Figure 3.5 shows the meaning of the performance measures and certain parameters from the maintenance perspective. If we observe aircraft maintenance as an inventory control problem, then we have the failure rate α as demand and the base-level inventory stock s that needs to satisfy the demand as the capacity requirement in terms of number of slots within a hangar facility. In that sense, the purpose of the inventory control model, which is to determine the optimal level of s for a particular demand and leadtime can be translated to the maintenance capacity required to carry out the repair work for the damaged component. The leadtime L is the time taken for order and arrival of s, which would mean it is the time between two maintenance checks, which is the time the next slot capacity would be available.

The average stockout frequency ($\overline{A}$, indicates the probability of a stockout at each value of s. A stockout scenario means there is not enough inventory (s) to match the demand. Therefore, from the maintenance perspective, this would mean that the number of slots assigned would not be able to meet the demand for repairs. $\overline{A}$ is often used as a threshold to determine the required number of s through choosing a service level. If the service level (SL) is on a scale from 0 to 1, with 1 being the highest service, then $\overline{A} \leq 1 - SL$ would be the performance target to achieve. That means, a certain s value is chosen such that the probability of stockout meets a certain performance target.

Terms	Inventory Control	Maintenance Application
s	Base-stock inventory level	Slot capacity.
L	Leadtime, time taken for order to arrive.	Leadtime, time between two consecutive C-checks.
α	Poisson distributed demand rate.	Poisson distributed failure rate.
$\bar{A}$	Probability that demand exceeds stock level.	Probability that failures exceed slot capacity.
$\bar{B}$	Average number of pending stocks.	Average number of failures for which repair is not performed.
$\bar{I}$	Average number stocks accumulated.	Average number of failures.
C(s)	Cost of holding inventory at each base-stock level.	Cost of maintenance at each level of slot capacity.

Figure 3.5: Interpretation of Inventory Control Terms in Maintenance Application

The average backorders ($\overline{B}$), is the average number items that are not delivered to the customer at each base-stock level. In terms of maintenance it translates to the average number of repairs that are not carried out when the slot capacity s takes a certain value. The effect of backorders plays a crucial role in the determining the s based on optimum cost.

The average inventory ($\overline{I}$) is the average number of stocks that needs to be maintained at each value of s. This corresponds to failures/repairs at accumulates at each slot capacity. Although, from an inventory perspective the purpose of average inventory is to control the stocks in warehouse, from an maintenance what we observe is to limit the slot capacity for a particular demand value.

$C(s)$ as expressed in equation 3.15, is the average cost function that varies with different values of $\overline{I}$ and $\overline{B}$. Since $\overline{I}$ and $\overline{B}$ are both convex functions, $C(s)$ is also a convex function. This function in turn helps in identifying the s at minimum cost. The holding cost in does convey the right message when combined with T bar because it shows so much of extra repair cost would be incurred in holding the extra capacity.

It is important to note that these performance measures are averages computed over the long term, this means these measures would convey the performance of the maintenance system over a long period of time thus providing a suitable input for strategic planning of maintenance production.

4
Case study

4.1. Description of case

The objective of the case study is to act as a proof of concept i.e. to implement the methodology developed in chapter 3 for a real world problem. This will be conducted on a fleet of aircraft from a major European airline. The historical failure data is provided by the airline M&E organisation specifically for their Boeing 777 fleet. The database provided includes the failure history data for all the Boeing 777 variants within the airline's fleet. Also, failure history for all aircraft structural parts are recorded until the date of 1st January 2016.

4.2. Input data

4.2.1. Processing raw data

The objective of data processing is to extract the time of component damage/failure for a set of structural components. This can be done by identifying components that have undergone maximum number of damages. As explained in the methodology more number of damages yields in more data points, hence delivering a good quality reliability analysis.

The analysis begins with segregating the failures based on the ATA chapters, which helps in the identification of components at the highest level. This analysis led to shortlisting the components for the fuselage and wing. Out of which the wing was chosen for further analysis because of less structural complexity and hence less number components to identify. Further analysis within the wing structure led to the identification of *Outboard flap* and *Leading edge slats* as two of the main components undergoing relatively more accidental damages. The failures obtained for these components were further sub-divided for the left and right wing. The materials used for flap and slat are composites and aluminium alloy respectively. The failure behaviour based on material properties or the physics of failure are not part of this thesis study.

Superposed system : The number of failures at an individual aircraft level typically ranged from 1 to 10 failures, with the majority of the aircraft falling below the limit of 5 failures. This led to combining the failures of all aircraft that recorded a failure for a specific component. This method of aggregating failure times is called a *super-position*, *superimposed* or a *superposed* system. This form of analysis allows us to analyse the reliability behaviour for the entire fleet. e.g to predict when any aircraft in the fleet would encounter a component failure in a given time interval in the future.

Operating time: There were three options for choosing an appropriate operating time for the reliability analysis, flight hours, flight cycles and calendar date. Out of these three, flight cycle is the operating time chosen. It is important to choose an operating time that can be closely related to the type of failure occurrence. This is best represented by a flight cycle (one cycle corresponds to a take-off and landing of an aircraft, excluding the cruise phase), because the nature of failures are such that they either happen on ground (e.g. human error) or during the take-off or landing phase (e.g. bird strikes, hail stone damages etc). Since we are considering the fleet of B777s which operate over over long ranges, the time spent on cruise phase exceeds that of take-off and landing, hence operating hours (gate to gate time) would not contribute to the right results. Similarly, calendar date does not convey the exact aircraft operation times.

Observation period: Since the data obtained is for the entire B777 fleet, this means new aircraft is added every few years and the failures of those are also recorded in the existing database.It was found from the analysis that there was some discrepancy in the failure data from aircraft delivered before the year 2002. The discrepancy owes to inconsistent data recording and inclusion of those failure times would negatively affect the reliability analysis. Hence, for our analysis aircraft delivered after the year 2002 are considered. This would make the total operational time of fleet to be 14 calendar years. Since the failures are recorded until 1st Jan 2016, this would be the observation period considered for the reliability analysis. Based on the flight cycles operated for each aircraft until 2015, the average number of flight cycles over a one year period was calculated to be approximately 500 FC. Which leads to 7000 FC on average till the end of observation i.e. 2015, and this value is taken to be the truncation time. Which means all failures recorded before 7000 FC are considered for reliability modelling.

After following the above steps, the number of failures recorded for each component and the input values for the reliability model is presented in the table below. Where, T is censored or trunctated time, N_q is the total number of failures and k is the number of independent systems (individual aircraft).

	LHS Flap	RHS Flap	LHS Slat	RHS Slat
T (FC)	7000	7000	7000	7000
N_q	64	48	61	47
k	53	30	35	35

Figure 4.1: Input conditions for reliability model

4.2.2. Capacity Identification Model

Lead times & Maintenance Checks

The lead time represented by L is the time taken between the occurrence of two maintenance checks. The base lead time for a B777 aircraft is taken as 50 flight cycles, which is equivalent to one calendar month of operation. This is an estimation made on the basis of total B777 fleet of the airline and a sample maintenance schedule.

The maintenance checks considered for this case study are C checks. The C-check is one of the major checks that falls under hangar maintenance and is carried out at approximately every 1000 FC for a given aircraft. In this study we assume that a C-check is scheduled every 50 FC in the hangar bay.

Cost data

The cost data for the planning model were mainly the holding cost *h* and the penalty cost *b*, which corresponded to the hard costs and soft costs. The hard costs included the repair costs and hangar costs. Whereas the soft cost corresponds to the costs due to delay and costs of not carrying out the maintenance at the specified time interval. Due to the difficulty in obtaining exact figures that match these costs, a more theoretical approach is taken here. The costs are expressed in ratios in relation to each other. h relates to average inventory and b relates to backorders. Allowing for a backorder to happen corresponds to a case of deferred maintenance. Which means the repair work is held off for a later period of time. Such a decision can be detrimental in terms of costs. A general observation within any industry including the airline industry is that backordered cost are higher than holding costs. In this thesis we have taken a theoretical approach and assigned the backorder cost to be 1.5 times more than the holding cost. For h= 1, b = 1.5 and therefore, b/h = 1.5. Sensitivity studies for these costs are discussed in the next chapter.

4.2.3. Case study assumptions and implications

Identical systems : All aircraft are considered as identical systems which acts as an assumption for the superposed system. Since we are considering a fleet of single type aircraft (B777s), which are used for long range operations, we can assume that all aircraft would face similar operating conditions. This can be attributed to the aircraft routing model which ensures that each tail numbers are rotated for different routes hence making sure no one aircraft flies a particular route all the time.

Negligible repair time : The repair times for each failure occurrence is considered negligible for reliability modelling. This assumption holds true because, the time taken for repair when compared to the overall observation time (of 7000 FC) is insignificant.

4.3. Results

4.3.1. TTT Trend test

The test for trend identification used in this study is the The total time on test (TTT) plot. The TTT plot tests for multiple systems are carried out for both slats and flaps which are shown in figures4.2 and4.3 respectively. The purpose of conducting a TTT plot is for reliability model identification. Ideally, the curve that wriggles around the diagonal of the unit square is expected to follow a power law process. As seen in figure4.2, the plot for slats overlaps and graces closer to the diagonal as compared to that for flap, this shows us that the slats would follow a power law process more strongly compared to that of flaps. This difference can later be seen in the parameter estimates for each component. Since, all four components, with the slight exception of left side flap, lie close to the diagonal, we can be confident in assuming a power law process for our reliability modelling.

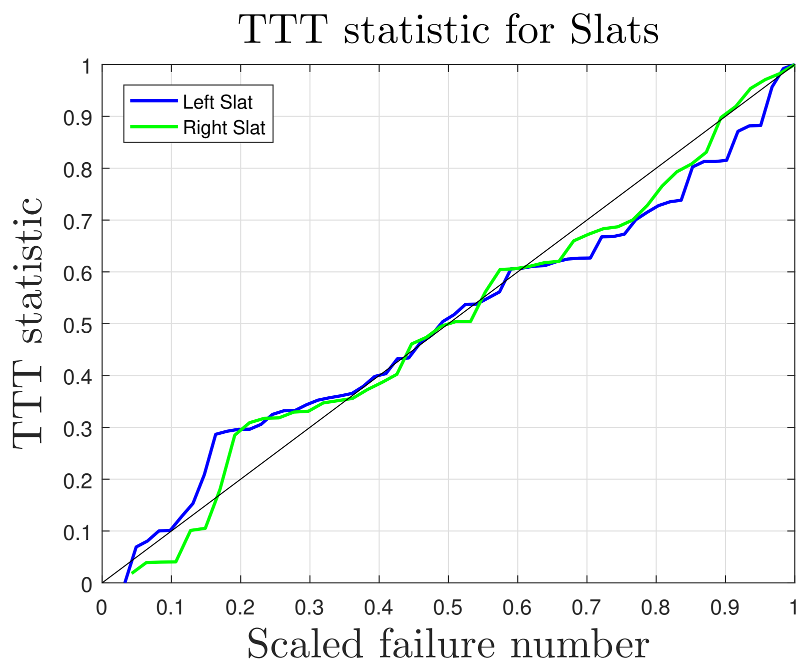


Figure 4.2: TTT plot for left and right side Slat

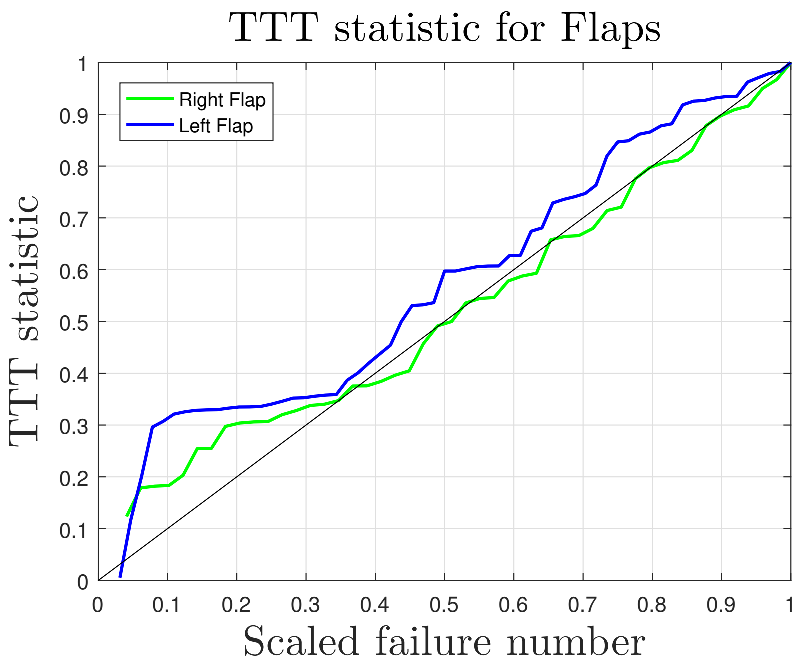


Figure 4.3: TTT plot for left and right side Outboard flap

4.3.2. Power-law Process

This section shows the results from the power law process model which was described in the methodology section. The figure below shows the the Maximum likelihood estimated parameters (MLE) computed for each component.

	Outboard Flaps		Leading edge Slats	
MLE	LHS	RHS	LHS	RHS
$\hat{\beta}$	1.108	1.045	1.311	1.236
$\hat{\lambda}_s$	0.003514	0.004593	0.000553	0.000831

Figure 4.4: Maximum likelihood estimates for PLP model

The $\hat{\beta}$ signifies the shape parameter and $\hat{\lambda}_s$ signifies the scale parameter for a superposed system. A $\beta >$ 1 signifies a deteriorating system with stochastically increasing time between successive failures and β < 1 shows an improving system. Also, a β that takes the value 1 signifies a truly random homogeneous Poisson process, which means they cannot be characterized as an improving or deteriorating system. Such a system has a constant failure rate. From the figure 4.4 we can infer that slats with β value greater than flaps observe a more deterioration characteristic. On the other hand flaps seems to be closer to a homogeneous Poisson process. Since these are accidental damages, we do not expect the component to exhibit a deterioration or improving behaviour, but we notice the slight change in slats. The higher value in β could be attributed to slats undergoing multiple damages in a short interval of time compared to flaps, thereby having shorter times between successive failures. The similarity in the β values between the left and right side wing components show us that the components have undergone similar damage history.

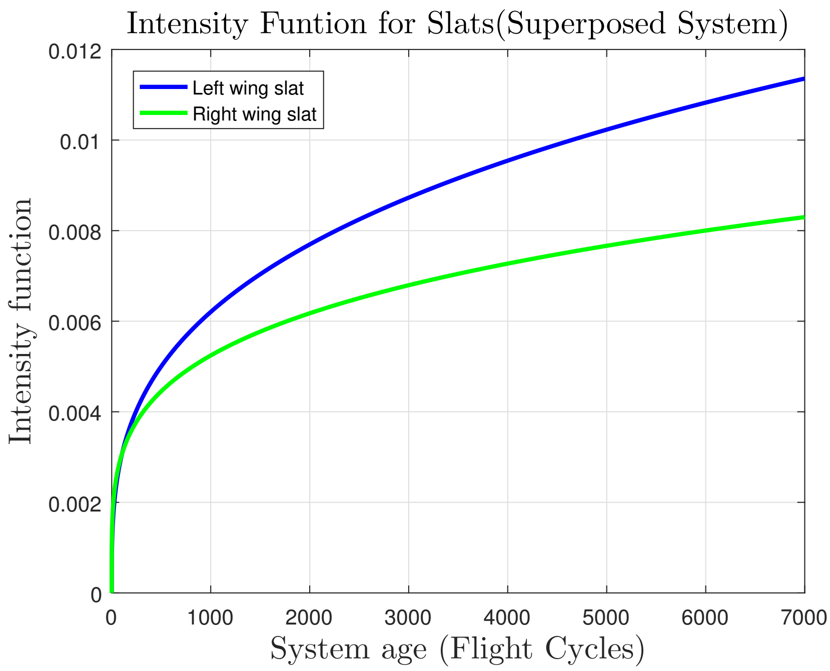


Figure 4.5: Intensity function for Slats

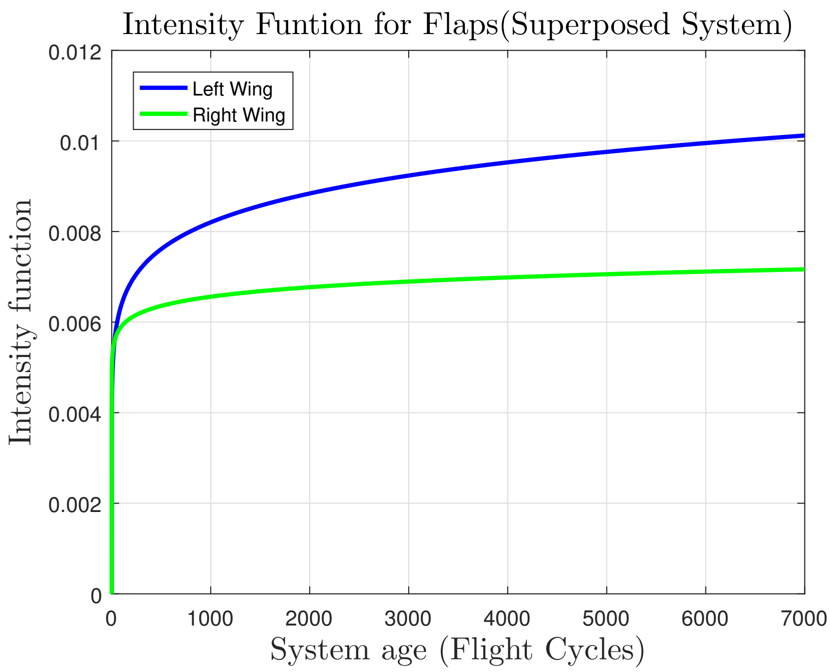


Figure 4.6: Intensity function for flaps

figures 4.5 and 4.6 shows the behaviour of the intensity function over time. The increasing the function is because of $\beta > 1$, which means the probability of failure keeps increasing with the system age. This is true for a system undergoing deterioration. As the values of β suggest slats have an increasing intensity function compared to that of flaps. For the right flap with $\beta = 1.045$ we can see that the intensity closes on achieving a constant intensity, exhibiting the properties of a HPP system. To check the validity of the model, it is important to carry out a goodness of fit test. The results from the test are presented in the chapter 6.

4.3.3. Stochastic demand generation

Simulating a Power Law Process

We simulate the failure times for the NHPP power law process and compute the average of all realisations using a Monte Carlo simulation, so that we can estimate the expected demand or the number of future failures. Each failure time is indicative of a failure occurrence. A 100 failure data points are generated so that the failure times are forecast beyond the truncation time of 7000 flight cycles and upto a period of approximately 3 years. The model used to generate these times are presented in chapter 3.

Figures 4.7 to 4.9 show the Monte Carlo simulations for 1, 15 and 5000 runs respectively compared with the real data for left side slat. The figures show the considerable variation that can occur for each realisation of the failure times. Therefore, each realisation gives a distinct set of failure times for the component. The number of runs of 5000 would mean that for each of the 100 data points generated there 5000 possible values. The mean for these MCS realisations computed is shown in figure 4.10.

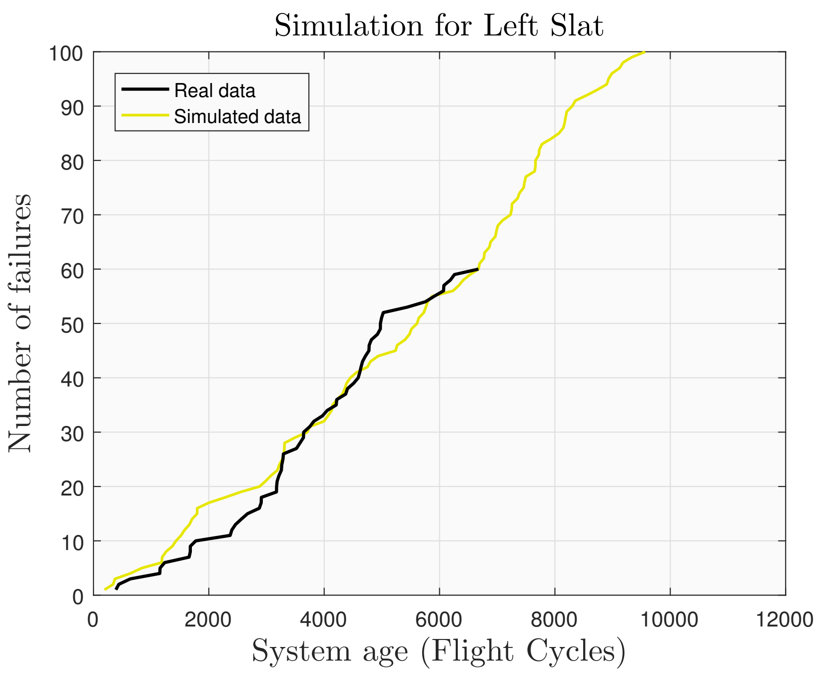


Figure 4.7: Monte Carlo simulation for 1 run

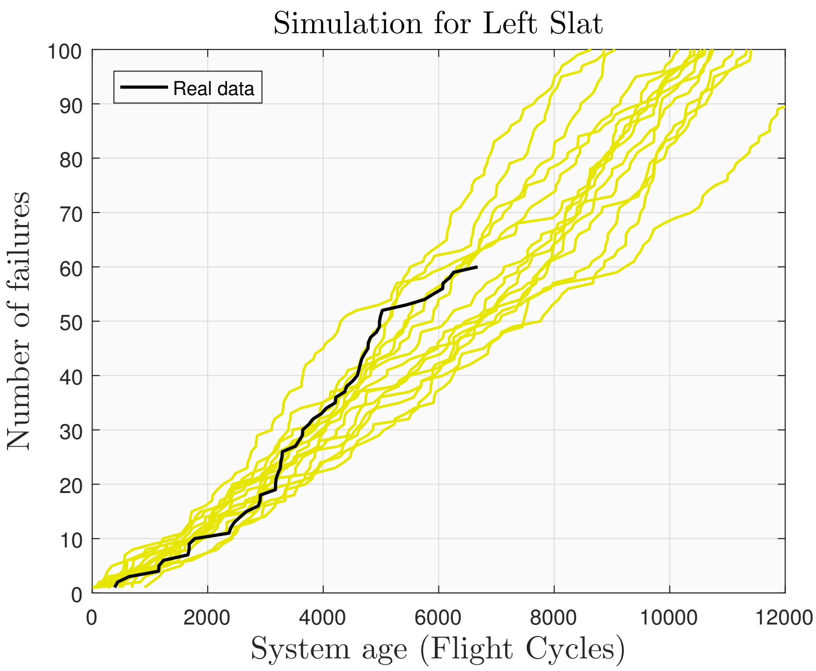


Figure 4.8: Monte Carlo simulation for 15 runs

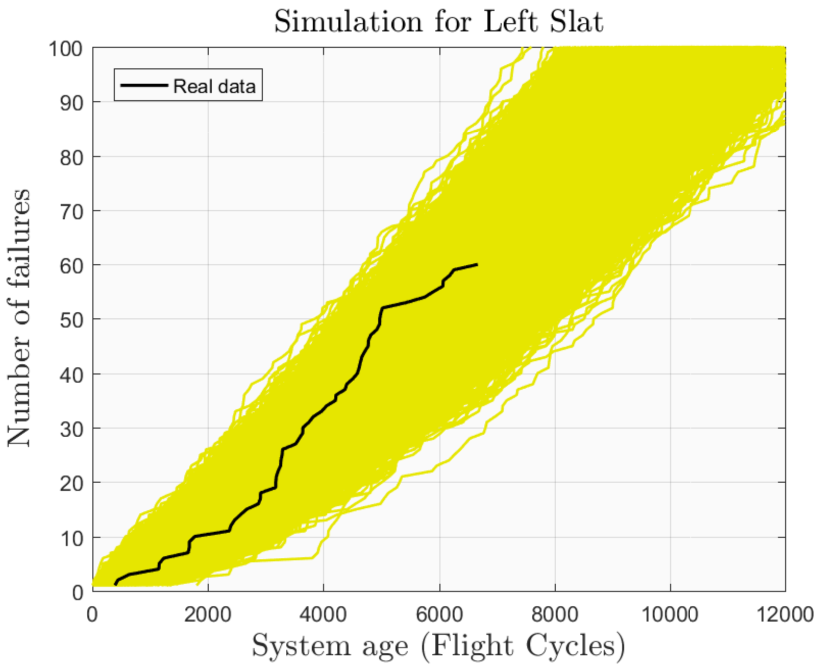


Figure 4.9: Monte Carlo simulation for 5000 runs

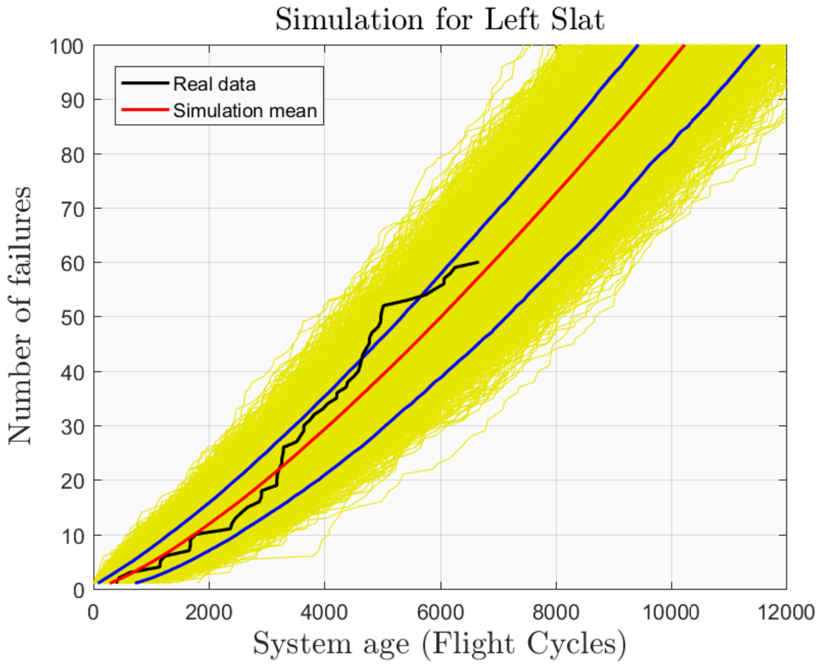


Figure 4.10: Monte Carlo simulation with quantiles

Demand as a Homogeneous Poisson Process

To calculate the future demand for the three scenarios, the failures times beyond the truncated time (representing present time) of 7000 FC are extracted. Since these times are global failure times, the time between failures would show us, between how many flight cycles a failure can occur. The main assumption for a homogeneous Poisson process is that these time between failures (TBF) do not vary with time, i.e. they remain constant. A similar conclusion can be made for our scenario. The figure below shows the mean time between failures and the standard deviation for the TBFs for each of the 4 components.

	Left Slat		Right Slat		Left Flap		Right Flap	
	MTBF	σ	MTBF	σ	MTBF	σ	MTBF	σ
α_{mean}	83	3.0	111	4.8	97	1.5	137	2.5

Figure 4.11: Mean time between failures for all components

The standard deviation shows the departure from the MTBF in terms of number of flight cycles. That means, for the case of left side slat, there is a possibility of a TBF occurring with 3 flight cycles difference from the MTBF. This is expected since the value arrives from the mean of 5000 runs. We can notice that the fluctuation of standard deviation values around MTBF are small. This means there isn't much effect of the time varying aspect of the times between failures. This type of demand can be co-related to a slow moving demand, which allows us to assume a constant rate at which the demand occurs and hence a **homogeneous Poisson demand**. Therefore, taking the MTBF for left slat, we have 1 failure occurring in 83 FC for which α_{mean} or the demand rate = 0.012 failures/FC. On similar lines, the demand rate for all components are provided in figure below. In order to account for any variation in α in the capacity identification model, sensitivity studies are conducted, the results of which are presented in chapter 5.

	Leading edge slats			**Outboard flaps**		
Demand	Left	Right	Both	Left	Right	Both
α_{mean}	0.0121	0.009	0.021	0.0103	0.0073	0.0176

Figure 4.12: Demand rate α for both components

4.3.4. Capacity Identification Model

The demand rates generated from the Monte Carlo simulations is used as the input for the planning model. The three measures that help in understanding the effects of the demand are $\overline{A}$, $\overline{B}$ and $\overline{I}$. These are functions of s, where s is the number of slots available in a hangar to carry out repair for a given component. Hence, s represents the in house repair capacity of the operator. As we had seen in section 3.6, the s increases in the increments of 1, that means, we observe the variation in the performance measures with the unit increase in the slot capacity. There are two ways by which we can identify the desired slot capacity :

1. By fixing an adequate service level through $\overline{A}$.
2. By minimisation of the cost function $C(s)$.

We will discuss results from each of these steps for the case of leading edge slats in the following paragraphs.

Capacity Identification based on service level
Figures 4.13 to 4.15 show the performance measures $\overline{I}$, $\overline{B}$ and $\overline{A}$ respectively for a fixed lead time L of 50 flight cycles, i.e. according to the maintenance schedule there is a C-check happening every 50 flight cycles. This would result in a mean lead time demand (MLTD) of 1.05 for α_{mean}. As seen in the figures the variation in the performance measures are captured for an increasing value of base capacity s from 0 to 6 slots.

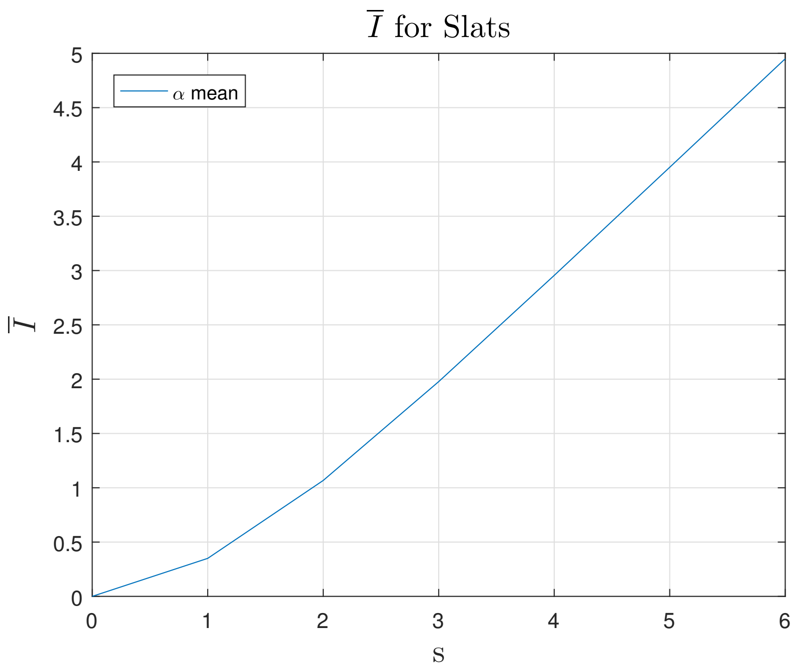


Figure 4.13: Average inventory, ($\overline{I}$)

Figure 4.13 shows the variation of $\overline{I}$ for different values of s, $\overline{I}$ is indicative of average inventory, which means this is the average number of repairs that can accumulate for the given $MLTD$ at each value of s. Hence, we can observe a proportional increase in number of repairs as the slots are increased. As expected the $\bar{I}$ at $s = 0$ is 0 because if there are no slots available to to hold the repairs then they are backlogged or backordered as seen in the next figure 4.14. The goal of the M&E would be to have minimum number of repairs and hence minimum $\bar{I}$.

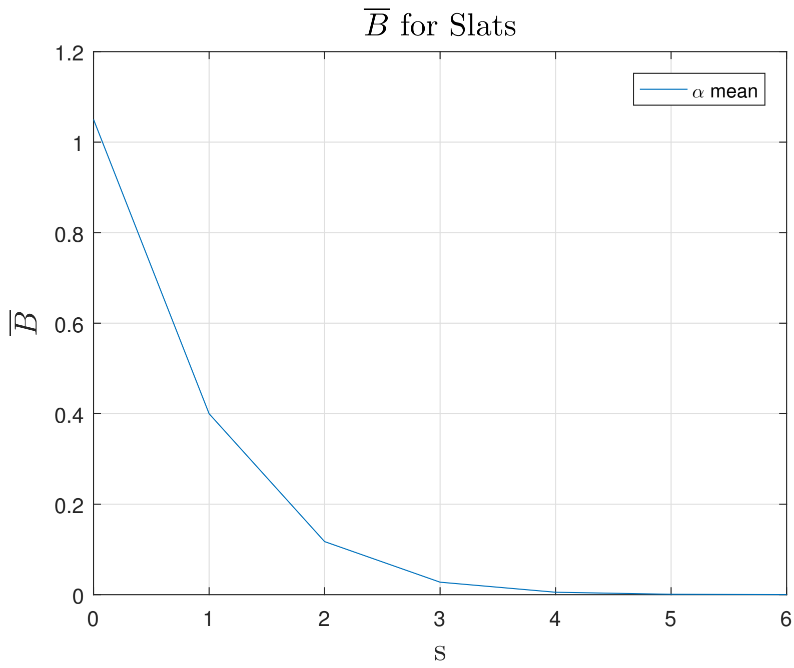


Figure 4.14: Average backorders,($\overline{B}$)

The above figure 4.14 shows the average backorders varying with s, with a lead time of 50 and s = 0, the back-order is expected to be the MLTD at 1.05, which is the expected number of failures backlogged when no capacity is available. As the slots in the maintenance hangar increases, the backorder is expected to decrease because it accommodates the demand. It might be useful for the M&E to keep a backorder less than 1, which means there are no delays in the repairs. Therefore just as required (for this particular case), with s = 1, the $\overline{B}$ is less than one. The backorder variation becomes more prominent as the α value increases.

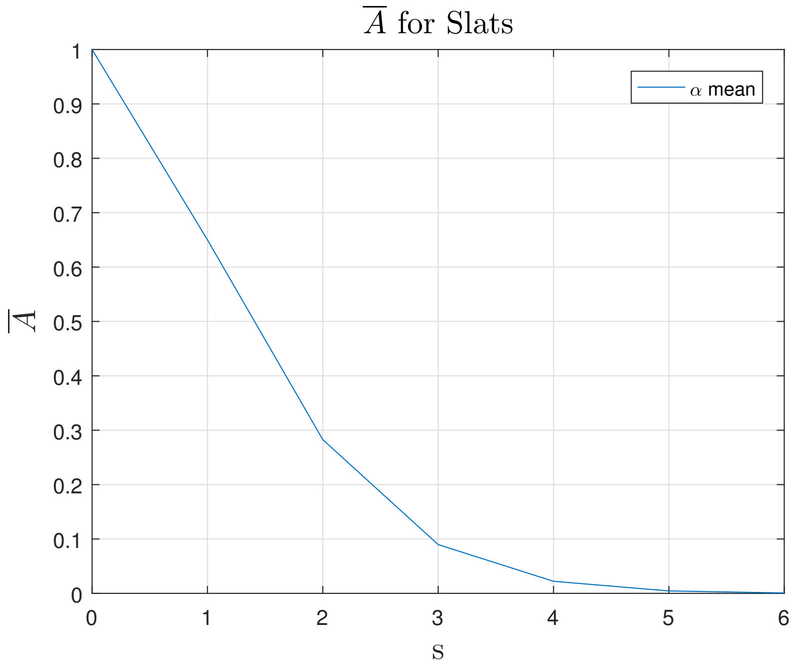


Figure 4.15: Stockout frequency, ($\overline{A}$)

Since we are faced by a stochastic demand there exists a level of uncertainty with respect to the demand being fulfilled at the given capacity, this is conveyed by the average stockout frequency. Figure 4.15 shows the average stockout frequency for the slat component. The $\overline{A}$ is a probability value, which mean its value ranges from $[0, 1]$. As shown in figure $\overline{A}$ reduces from 1 to 0 as the slot capacity is increased, which is expected because as the slots capacity increases the chances of a repair being carried out also increases. A high value of probability means that there is high chance that the demand is not fulfilled. Therefore, it would be of interest to the organisation to identify slots for lower levels of stockout frequency. Therefore, $\overline{A}$ = 0.65 at s =1 means there is 65% chance of having a backorder. Which means a delayed maintenance at that capacity level. The service level (SL) varies in a scale from 0 to 1 with 1 being the highest service that can be offered. The relationship between SL and $\overline{A}$ is that $\overline{A} \leq 1 - SL$. Therefore, for a high service level it would mean higher slot capacity requirement. For this particular case of slats, it would be a s value of 3 for $SL > 80\%$

s	$\bar{A}$	$\bar{I}$	$\bar{B}$	C(s)
0	1.00	0.000	1.050	1.575
1	0.65	0.350	0.400	**0.950**
2	0.28	1.067	0.117	1.243
3	0.09	1.978	0.028	2.019
4	0.02	2.955	0.005	2.963
5	0.00	3.951	0.001	3.952
6	0.00	4.950	0.000	4.950

Figure 4.16: Performance measures for slats

Capacity identification based on Cost function

The outcome of the performance measures is to help choose an optimal value of s, that minimizes the cost function. Since, $C(s)$ is a convex function in s, we can derive a minimum value. The average cost of operating a slot capacity as shown in equation 3.15 is dependent upon $\overline{I}$ and $\overline{B}$. The base cost ratio chosen for this study is 1.5 as explained in section 4.2.2 above. figure 4.17 shows the variation in average cost function value for different values of s. We can observe the function taking a convex form having a minimum value. It clearly shows that s takes a value of 1 for minimum value of cost which corresponds to 0.950, as highlighted in figure 4.16. The value of s at minimum cost also corresponds to the point of intersection of the curves $\overline{I}$ in figure 4.13 and $\overline{B}$ in figure 4.14. The rise in the costs beyond $s = 1$ is because the cumulative effect of $\overline{I}$ and $\overline{B}$ is increasing. Therefore, if the organisation's objective is to minimise their cost, they need to maintain one slot capacity. Nevertheless, from 4.16 we notice that at this level of capacity the stockout probability ($\overline{A}$) is 65% which corresponds to a service level of 40%. This means, an optimum level of s based on cost allows for repair delays to occur. This is the trade-off that the organisation would have to make. One of the reasons for having a low value of s at cost optimum is the cost ratio, since the cost of deferred maintenance is only 50% more than normal repair costs, the s is not affected much. Therefore, an accurate estimation of these costs could lead to accurate prediction of capacity demand. Having a $s = 1$ at cost optimum would mean that the maintenance organisation can expect at least 30 capacity slots at their base for unscheduled maintenance during the next 1500 flight cycles4.18 .

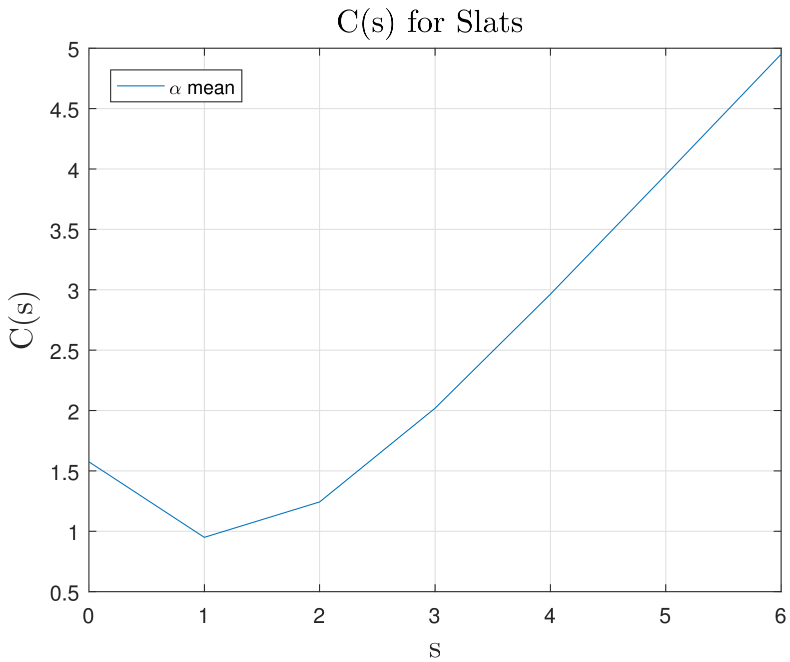


Figure 4.17: Cost function

Time Period (Flight Cycles)	Number of Slots for Slats Required (at cost optimum)
50	1
500	10
1500	30

Figure 4.18: Number of slots at cost optimum

5

Verification and Validation

This chapter comprises of verification & validation for the reliability model and sensitivity studies for the capacity identification model. The verification and validation of the reliability model are divided into two parts : first, verifying the reliability behaviour of the chosen parts and second, validating the model with respect to the number of failures observed. As for the capacity identification model, validation of the model would mean obtaining the actual maintenance slots from the airline company, due to constraints in research time this has not been carried out. Since, this model is a proof of concept, the verification is done by performing a set of sensitivity studies and observing the variation in the performance measures and whether their behaviour align with what is expected.

5.1. Reliability model - Verification and validation

5.1.1. Verification through Left & Right Components

Since the components considered are leading edge slats and outboard flaps on both left and right wing, it would be natural to assume that the parts are structurally identical and symmetrically located on either side of the aircraft. This means that the failure behaviour experienced by these components must also be similar. This was one of the reasons for splitting the reliability analysis for the left and right side components. From the reliability estimates computed in 4.4, we can confirm this similarity in the failure behaviour of the components. Both the flaps have β values close to 1 and the slats have β values close to 1.3.

5.1.2. Model Validation

Expected number of failures

As mentioned in the methodology section, the expected number of failures can be used to compute the number of failures in any given interval. Since we have estimated the reliability parameters the failure occurrences until 7000 FC, it is expected that the number of failures predicted by the model using equation 3.3 must be equal to the number of actual failures that have occurred. Figure 5.1 below shows the model estimated number of failures compared to the actual number of failures that have occurred. As we can see in the figure, the expected number of failures indeed closely match the actual data, thereby validating the model. Similarly, the results from the demand generation model are compared in 5.2. These are the averages computed from the Monte Carlo simulations compared to the history of failures during the same interval. The results show close proximity to the real values of both slats and flaps.

	Outboard Flaps		Leading edge Slats	
	LHS	RHS	LHS	RHS
Real Values	64	48	61	47
Model results	63.94	48	60.76	47.05

Figure 5.1: Comparison of number of failures for PLP model & Historical data

	Outboard Flaps		**Leading edge Slats**	
	LHS	RHS	LHS	RHS
Real Values	64	48	61	47
MCS results	64	47	60	47

Figure 5.2: Comparison of number of failures with MCS averaged results

Goodness of fit test results
Figure 5.3 presents the results from the Cramer-von Misses test performed for each component. As discussed in the methodology section, the significance level required to ensure that a Power Law Process model is compatible with the data set is 95%. Since the components do meet with this criteria, we can ensure that the PLP model is the right fit for our results.

	Flap RHS	Flap LHS	Slat RHS	Slat LHS
C_M^2	0.417	0.219	0.395	0.232
Significance Level	99.5%	95%	99.5%	95%

Figure 5.3: Number of failures comparison

5.2. Capacity Identification Model - Sensitivity Studies

The sensitivity studies observed for this model is with respect to variation in three input parameters : demand rate α, leadtime L and the cost ratio $(\frac{b}{h})$. These tests are performed to analyse the behaviour of the performance measures as a result of variation in the inputs. The studies presented here are for the leading edge slats. Similar results for flaps are presented in Appendix A. The variation in the inputs parameters is required to convey both increasing and decreasing values from the base values observed in chapter 4.

Performance measures with varying demand rate (α)
The studies performed for demand variations are for $\pm 30\%, \pm 60\%, \pm 90\%$ of the mean demand rate α. The sensitivity studies for the slats are shown below. Similar plots for the flaps are presented in the Appendix A. Figures 5.4 to 5.7 show the behaviour of the performance measures, $\overline{A}$, $\overline{B}$, $\overline{I}$ and $C(s)$ respectively, with variation in the demand α. As the demand varies from -90% to +90% of the demand value α, the mean lead time demand $(MLTD)$ also varies accordingly. These results depict

the performance measures at a L = 50 FC. It is worthwhile to notice that since these performance measures does depend on the changes in demand rate.

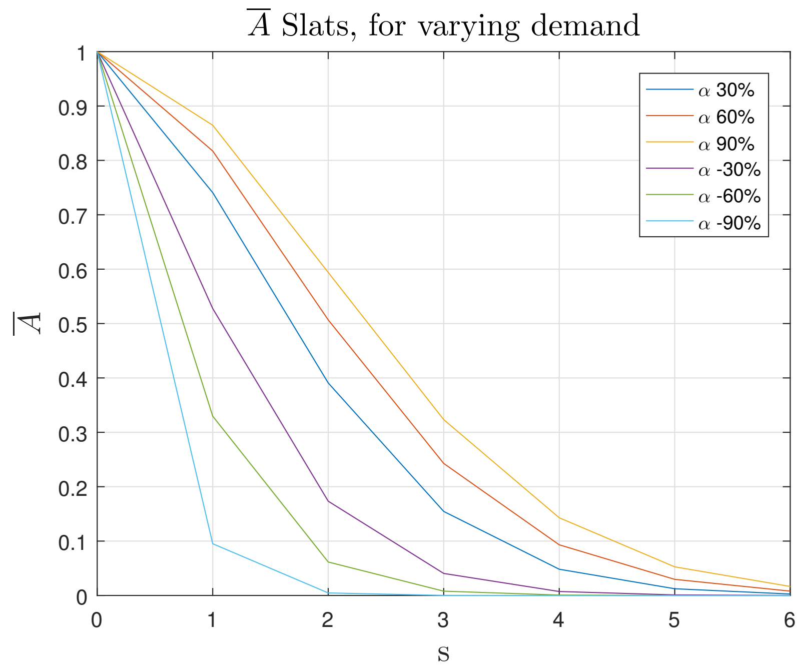


Figure 5.4: $\overline{A}$ with demand variation

The figure above shows us the variation in the stockout frequency or the probability that the number of failures is greater than the slot capacity. We notice that the trend moves in an increasing manner as the demand rate increases. That means, for a particular value of s the probability of a repair not being done increases with increase in demand. This is because for low values of α, the $MLTD$ is so low that the demand is quickly satisfied. We can notice the correlation in results with the average backorders $\overline{B}$ in the next figure. Therefore, for a component with low demand we can maintain a higher service level (SL). This also relates to the fact that for a low demand component, the slot capacity required is indeed lower (as the time taken for repair is not considered). So, having more slots for such a component would not help the organisation as it will unnecessarily increase costs.

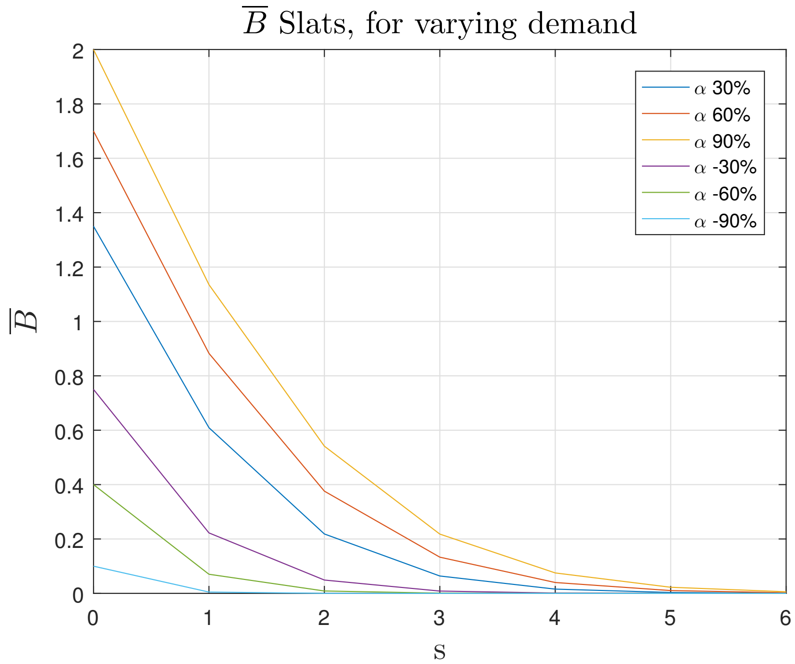


Figure 5.5: $\overline{B}$ with demand variation

As discussed above, the correlation between $\overline{A}$ and $\overline{B}$ can be noticed in the trend for average backorders in the figure above. Since at zero slot capacity $s = 0$, the $\overline{B}$ would be the $MLTD$, we see that for demand at -30%, -60%, -90%, the $MLTD$ does not even reach one failure, which means no capacity is required for those demand (on average), Therefore, having a very low demand rate could be a drawback for this model. This is also the reason why we do not get an optimum in C(s) in 5.7 for very low demand. On the other hand, from a service level perspective it still identifies slots, but from a cost perspective it wouldn't be useful.

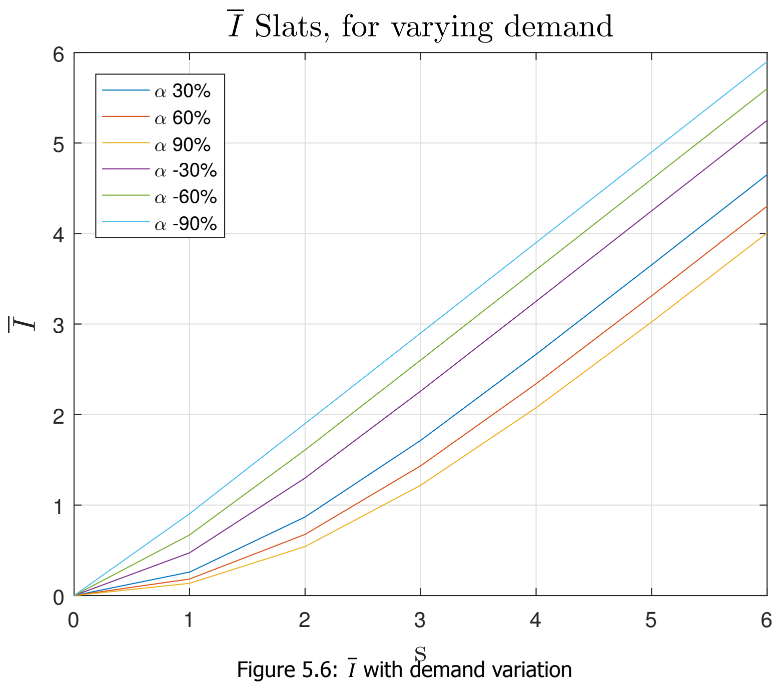


Figure 5.6: $\overline{I}$ with demand variation

Contrary to trends observed for $\overline{A}$ and $\overline{B}$, we notice that the average inventory $\overline{I}$ values for low demand rates are higher, this is because of the difference in the slot capacity and the $MLTD$ increase as the slot capacity increases. Since the inventory is the excess number of repairs accumulated at each s, we notice that due to the exceptionally low demand, the capacity overpowers the demand, which means there are more slots than the required number of repairs.

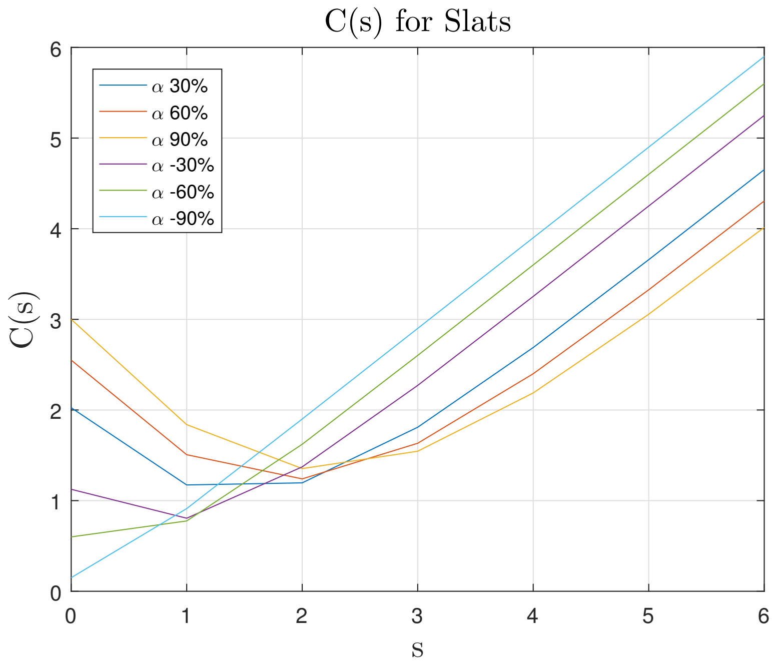


Figure 5.7: Cost variation with changing demand

We notice that for $\alpha = -90\% \& -60\%$, C(s) is not a convex function anymore and hence a minimum doesn't exist, because effect of backorder is negligible, hence it becomes a curve of I bar. At the given leadtime of 50, demand is mostly negligible hence not repairs are expected. As the backorder effect increases (due to increase in demand) the cost function achieves a minimum for s . Consequently we have a high value of s for a higher demand rate.

Performance Measures Sensitivity with Varying Leadtimes
The leadtime variations are considered for 30, 60, 90, 120 flight cycles. The performance measures with fluctuating leadtimes are shown from figures 5.8 to 5.11. Since the performance measures depend on the system only through $MLTD$, it would result in the same family of curves for leadtime variation as in the case of demand variation. Hence, even though the demand is constant an increase in the leadtime means the mean lead time demand $MLTD$ also increases. From figure 5.11 we can see that at a maximum lead time of 120 FC the minimum cost is achieved at s =2. Which means that at least 2 maintenance slots must be ready to meet the demand at 120 FC.

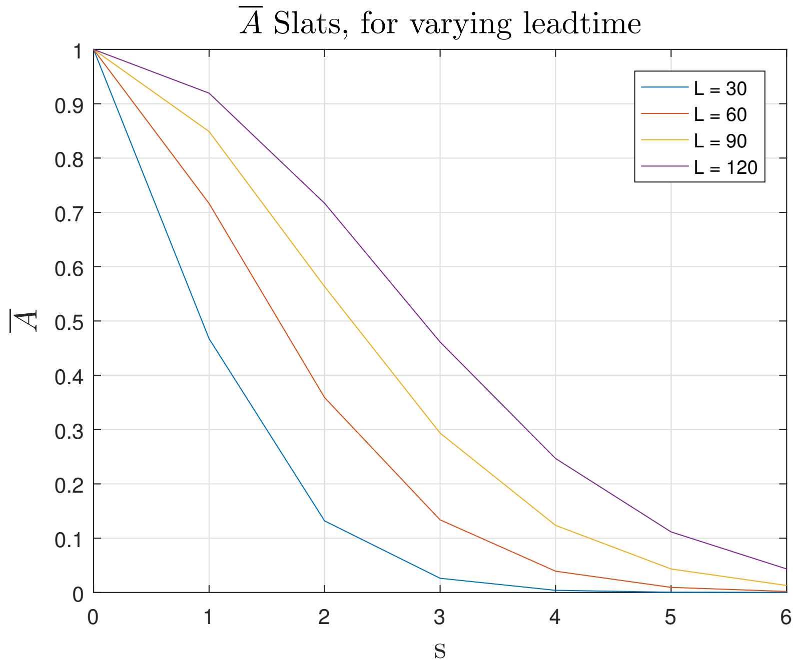


Figure 5.8: $\overline{A}$ with Leadtime variation

Figure 5.8 above shows the variation in the stockout probability as the leadtime is increased. An increased leadtime means the time between the two C-checks have increased. The trend shows that for a particular slot capacity, the probability of stockouts also increase with increase in the leadtime. In realistic terms this does not correspond to increase in demand, but having long leadtime means there is no enough slot to fulfill the demand, which means we need to increase the number of slots to achieve higher SL.

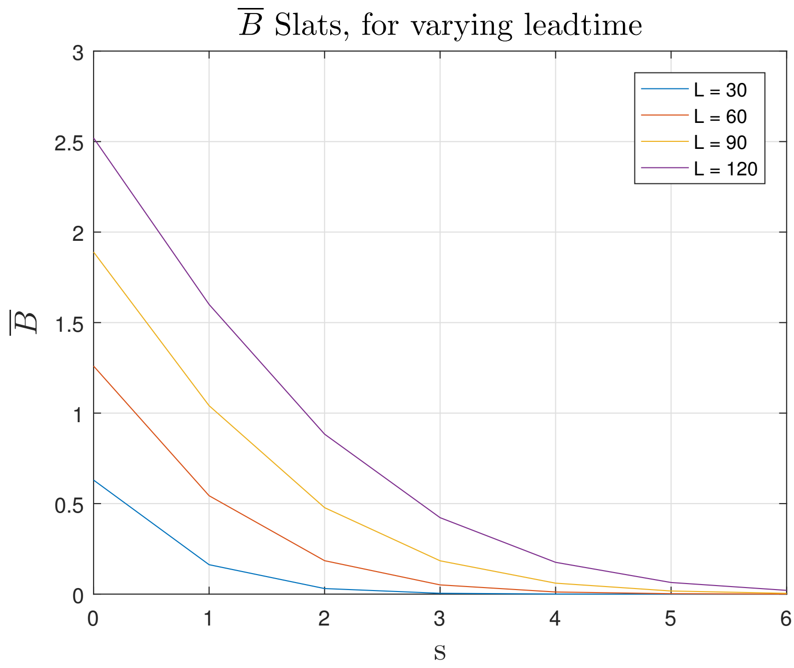


Figure 5.9: $\overline{B}$ with Leadtime variation

Figure 5.9 shows the variation in $\overline{B}$ over leadtime. We notice the decreasing trend for the backorders with increasing slot capacity because the demand is steadily being consumed. Notice that at $L = 30$ the MLTD is just above 0.5, which means it does not reach even one failure and consequently there we notice it's effects on the $\bar{I}$ and $C(s)$.

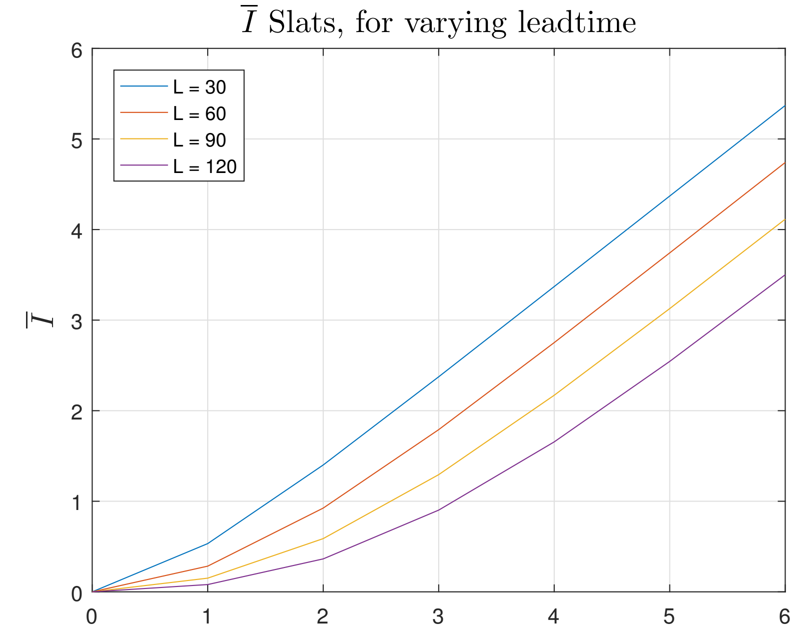


Figure 5.10: $\bar{I}$ with leadtime variation

Figure 5.10 shows the variation in average inventory $\bar{I}$ over the leadtime. Similar to demand variation, a high leadtime means higher $MLTD$, hence the organisation would have to maintain lower number of excess repairs or slots.

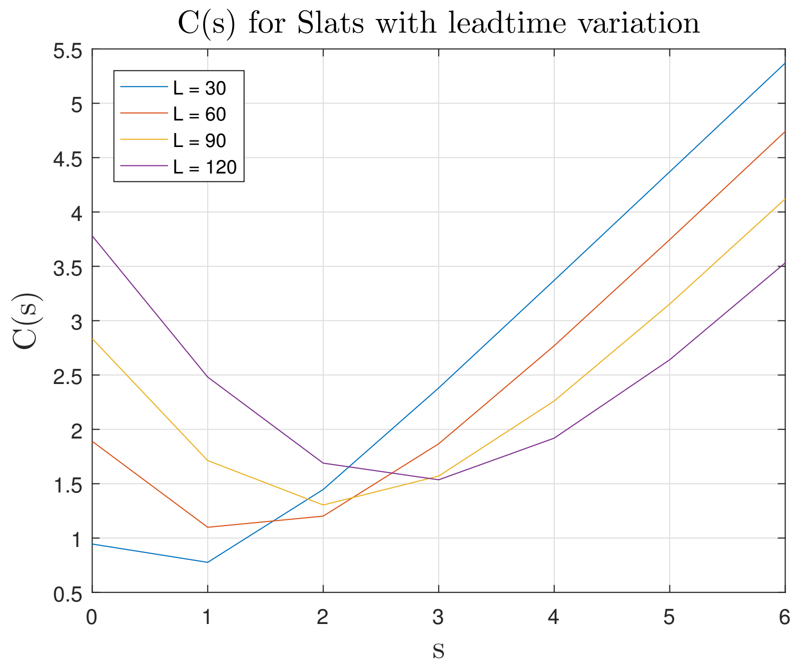
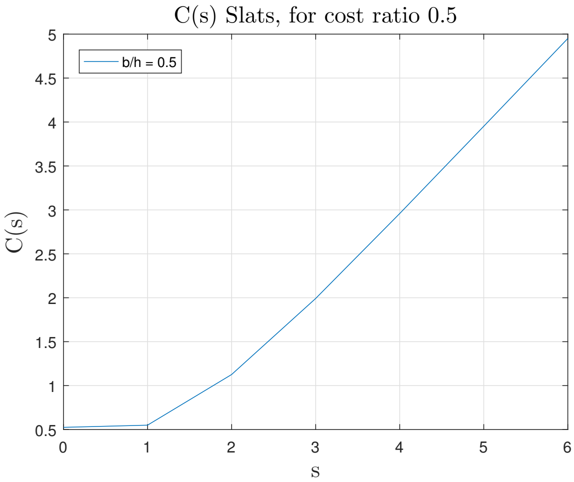


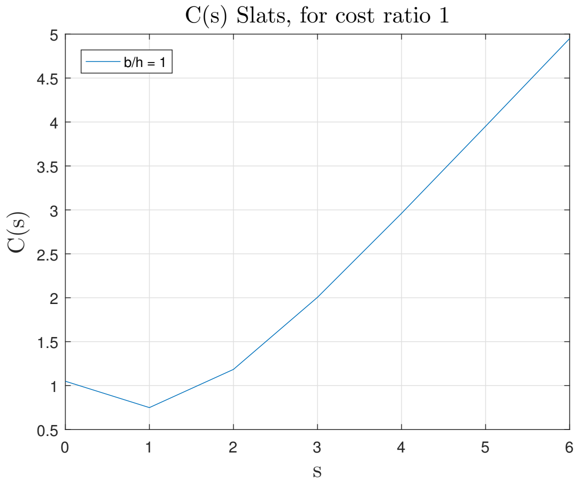
Figure 5.11: Cost variation with changing Leadtime

We can notice from figure 5.11 that as the L increases the capacity required also increases at an optimum cost. Even though $\bar{I}$ is high for $L = 30$ as shown in figure 5.10, we can notice in figure 5.11 that model optmises for a lower value of s at the same L.

Sensitivity studies for variation in cost ratio

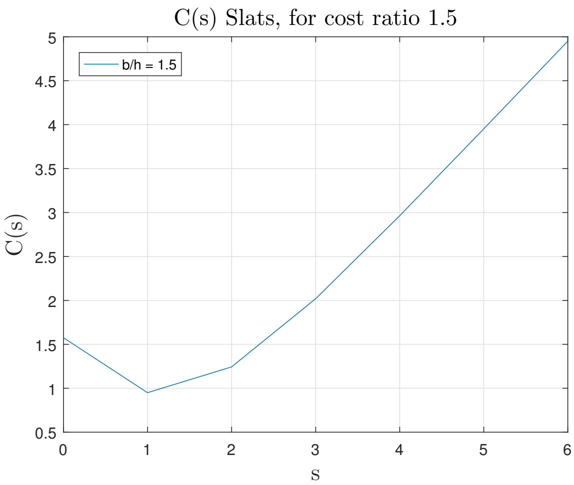


(a) Cost function for ratio = 0.5

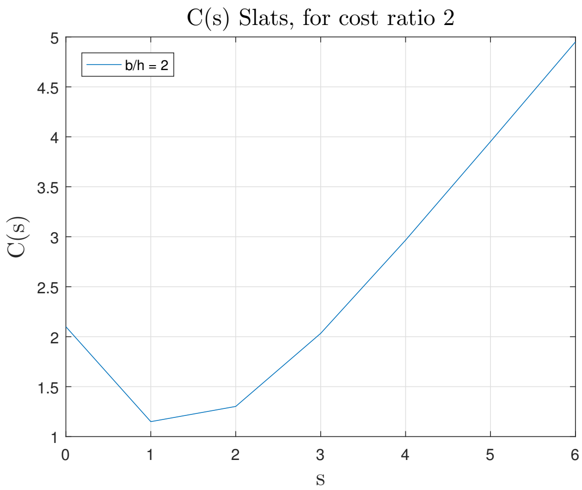


(b) Cost function for ratio = 1

Figure 5.12: Cost function



(a) Cost function for cost ratio = 1.5



(b) Cost function for cost ratio = 2

Figure 5.13: Cost function

The figures above shows the variation in the cost function with the change in the cost ratio. The cost ratios are performed for b/h ratios 0.5, 1, 1.5, 2. This reflects various scenarios wherein the penalty cost or the costs due to repair delay varies with respect to the holding cost (costs for performing one unit repair at unit time). That means b/h = 0.5 corresponds to when the penalty cost is half of that of holding cost and b/h = 2 means the penalty cost is twice the holding costs. The cost variations re performed for the base case for Slats with α_{mean} and L = 50FC. Since the cost function depends on the backorder costs, we notice that as the backorder/delay cost increases, the capacity required s also moves towards a higher value. Nevertheless, the dominance of average inventory in the cost function restricts the increase in the s even when the backorder costs are doubled.

6

Conclusions and recommendations

The following chapter presents the conclusions and the recommendations for future research work. Section 6.1 presents the conclusions, which starts with addressing the research objective followed by conclusions for reliability modelling and capacity identification model. Section 6.2 presents the recommendations for the reliability as well as capacity identification model.

6.1. Conclusions

6.1.1. Research Objective

At the start of this thesis, the negative effects of unscheduled maintenance on the resources of an airline Maintenance & Engineering organisation was established. The economic importance to determine the future resources especially in terms of the capacity brought about the need to build a model that not only predicts these unscheduled damages but also generates the required capacity at an optimum level. To realise this requirement a novel approach to capacity planning was identified. This led to the main research objective.

To identify the maintenance resource (capacity) demand for a fleet of aircraft, impacted by accidental damages, by integrating a reliability and inventory control model that accounts for the stochastic nature of damage occurrence.

6.1.2. Reliability Modelling

The reliability modelling was carried out using a Non-homogeneous Poisson Process (NHPP) model with power law intensity function. Due to limited availability of data, the superposition system was assumed by aggregating the failure times from multiple aircraft. This in turn helps in a fleet level analysis of the data. The two aircraft structural components that were identified based on the number of damage occurrences were the leading edge slats and the outboard flaps. These components were analysed for the left and right wing.

The reliability estimates computed for slats showed a deteriorating behaviour, that corresponds to an increasing failure intensity and the estimates for flaps showed a behaviour with constant failure rate. Which means that the slats are more likely to experience a damage compared to flaps. The parameter estimates were then used to determine the expected number of failures with the help of the mean value function. The predicted number of failures compared to the actual (history of) number of failures within the observation period was found to be 99.94% accurate for slats and 99.8% accurate for flaps. Moreover, the goodness of fit tests proved that the chosen PLP model was compatible with the failure data.

The estimates computed from the reliability model is then used to simulate the successive failure times for the entire fleet of aircraft. Due to the stochastic nature of the failure times, a Monte Carlo simulation is implemented to average the these times. The number of failures predicted from the MCS runs compared to the prediction from the mean valued function was found to be 99% accurate for slats and 99.6% accurate for flaps. These simulated mean failure times were assumed to represent a homogeneous Poisson process and hence used to determine the failure/demand rates for each of the components. The results showed the failure rate for slats to be 20% more than flaps, which aligns with the observation from failure intensity.

6.1.3. Capacity Identification Model

This thesis achieved to successfully adapt an inventory control model, specifically the base-stock policy model for identifying maintenance capacity resource demand. The base-stock model was used to identify the average capacity required to carry out future unscheduled maintenance for slats and flaps.

This was done by interpreting the failure rate from the reliability model as the demand and by considering the base-stock level s required to meet this demand as the in-house maintenance capacity. The quantity s was taken as the number slots required to perform an unscheduled maintenance, with one slot addressing one repair.

The performance measures used in the inventory control model were translated to apply for the maintenance application. The optimum capacity level were determined by two ways : a) based on service level, b) based on cost optimum. The required number of s based on service level was determined from the probability of delayed maintenance (backorders) whereas s from cost optimum was obtained from a cost function. Due to the limitation in obtaining real cost data, a conceptual approach was taken for deriving the cost ratios, which were necessary to determine the optimum capacity level.

As expected from the base-stock model, when the failure rate approached 1 over a certain lead time of flight cycles, a slot capacity of 1 was shown to exist. Hence, as the failure rate increases the identified capacity would also increase. It was found that the optimum number of slots based on cost function were less compared to that having a high service level. Which means the model allowed for some delayed maintenance in order to optimise the slot capacity. Based on cost optimum, it was found that the number of slots(s) required for unscheduled maintenance of slats will be 30 for the next 1500 flight cycles (at 40% service level). On the other hand for a service level of 80%, s will be three times as much, i.e. 90 slots. The means, the maintenance planner needs to account for s number of slots for unscheduled repairs in his planning. The determination of the capacity levels showed us that this inventory control model can indeed be used for such a maintenance application albeit for an intermediate or a long term strategic planning purpose.

6.2. Recommendations

The recommendations for future research aim to address the limitations of the present model by which it can be improved to for a more realistic application.

6.2.1. Reliability Model

- To improve the accuracy of the estimates from the reliability model, the observation period can be taken to be the exact operational time for each aircraft (rather than using a common truncation time) at the time of data extraction. The parameter estimates can then be computed using iterative methods. This approach becomes a time intensive task when considering multiple components and when the number of systems/aircraft is large.
- The assumption of a superposition system was made due to lack of data points per component per aircraft. Therefore, in order to study the effects on individual systems, it is possible to aggregate damage data from similar components or structures in an aircraft. This would mean the analysis is performed at a strategic level but only that it is focused on an individual aircraft rather than the fleet.

- In order to address the unscheduled maintenance specific to a failure mode, data could be segregated based on the cause and type of damages. This could be of interest to the M&E as it would help in identifying the most prominent cause or type of damage.
- This study was carried out using only one reliability model in which the repair effects are not modelled. Using other reliability models that can address the repair effects and able to adapt for multiple systems, a comparison between the data fit between the models can be made.

6.2.2. Capacity Identification Model

- **Demand rate:**
  - The inventory model adapted in this thesis holds the assumption for demand to follow a stationary Poisson process (Homogeneous Poisson Process). But in reality the demand assumes a Non-stationary (time varying) Poisson process. Which would mean that the slot capacity or base-stock level (s) varies at every time step. Hence this could be an immediate extension to the model, to incorporate NHPP failure times as the demand input.
  - This can be implemented using a discrete time formulation of the current problem and over a finite time horizon (planning period), the optimal solution can be obtained using dynamic programming method.

  .
- **Leadtime:**
  - The leadtime is assumed to be constant in this thesis, which was the time between any two consecutive C-checks. Which means that the repair time is assumed to be constant as well. In reality, due to some unscheduled repairs, the repair times can vary from the planned time. Hence, this assumption can be relaxed by considering variation in within the repair time, thus making the leadtimes stochastic.
  - Also, the model can be extended to consider other types of maintenance checks such as the A-checks and B-checks depending upon the airline company.
- **Several-systems/locations:** The present model assumes a single system and a single location, which means the demand arrives from a single system (components from an aircraft fleet) at one location (maintenance base). This can be extended to components from different structures & maintenance bases.
- **Cost data:** A more accurate estimation of cost values can be obtained from the maintenance organisation, as this plays a crucial role in determining the optimal slot capacity.

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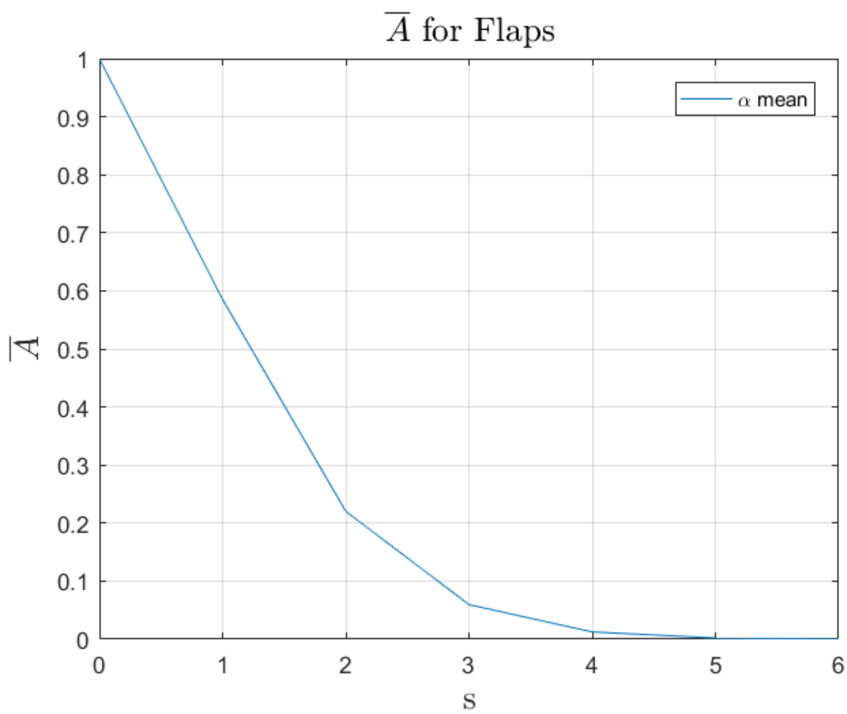


Figure A.1: Average Stockout frequency for Flaps

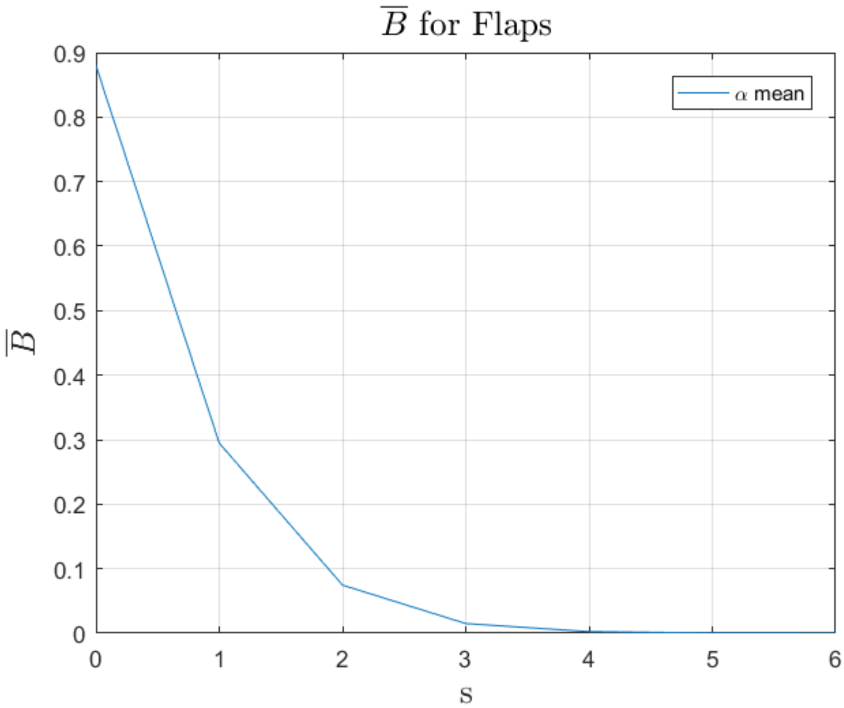


Figure A.2: Average Backorders for flaps

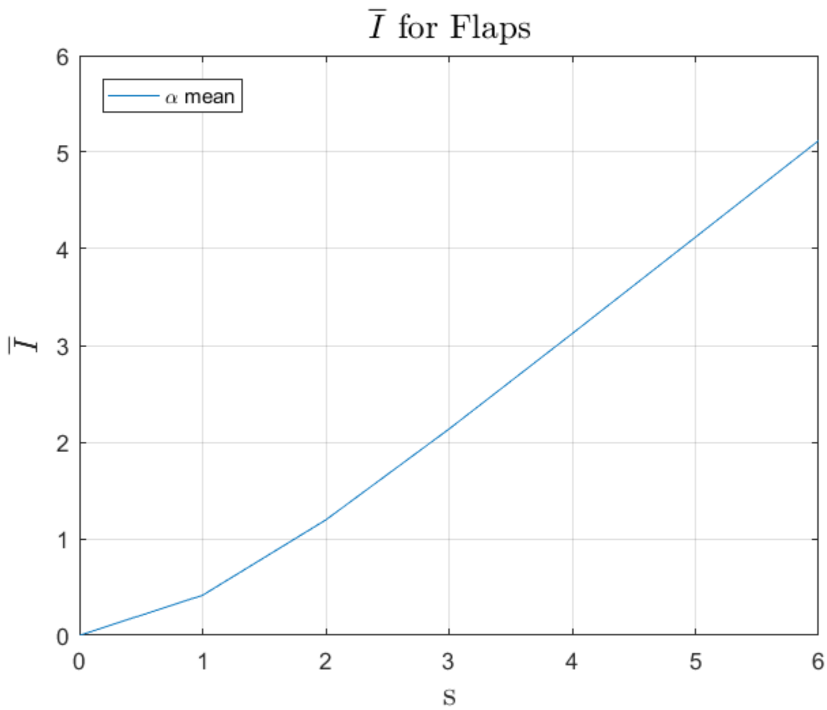


Figure A.3: Average Inventory for Flaps

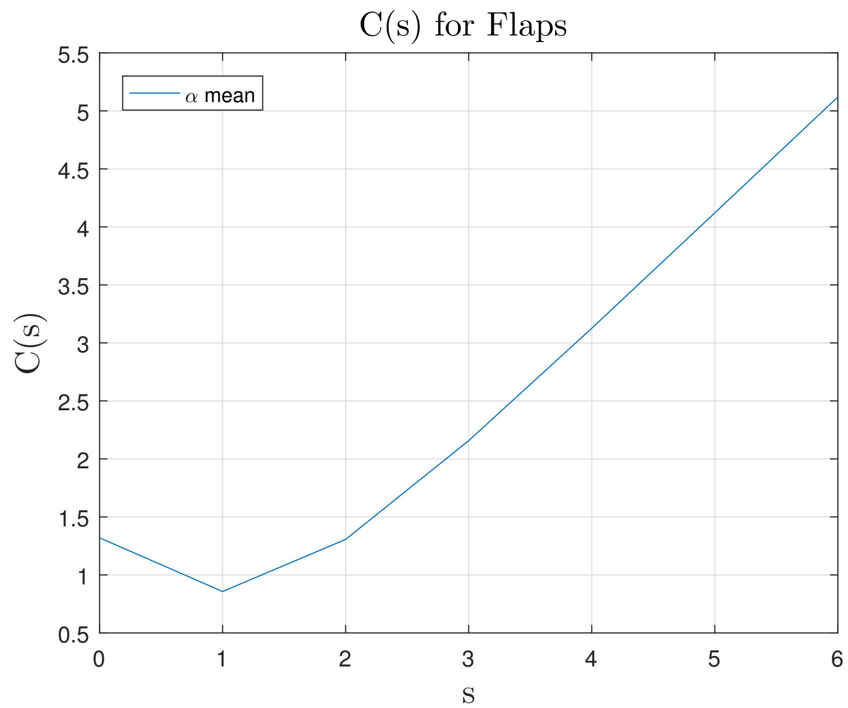


Figure A.4: Average Costs for flaps

Results of sensitivity studies for outboard flaps.

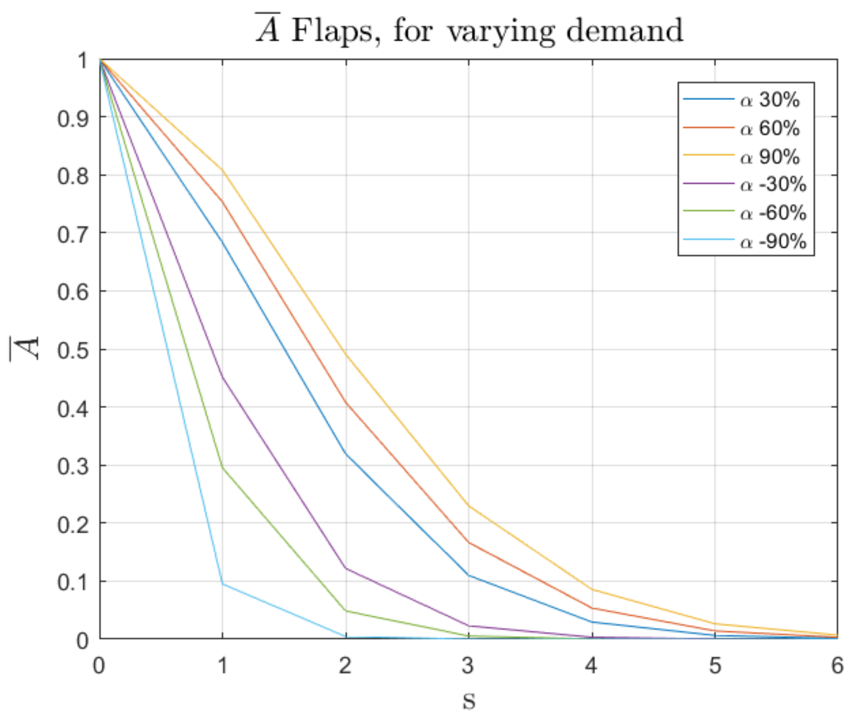


Figure A.5: Average Stockout frequency for flaps with varying demand

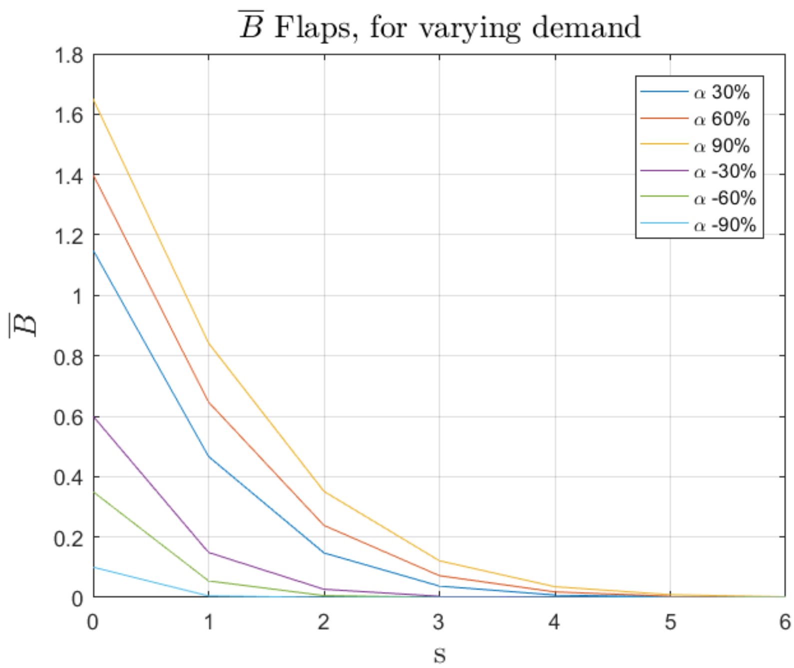


Figure A.6: Average Backorders for flaps with varying demand

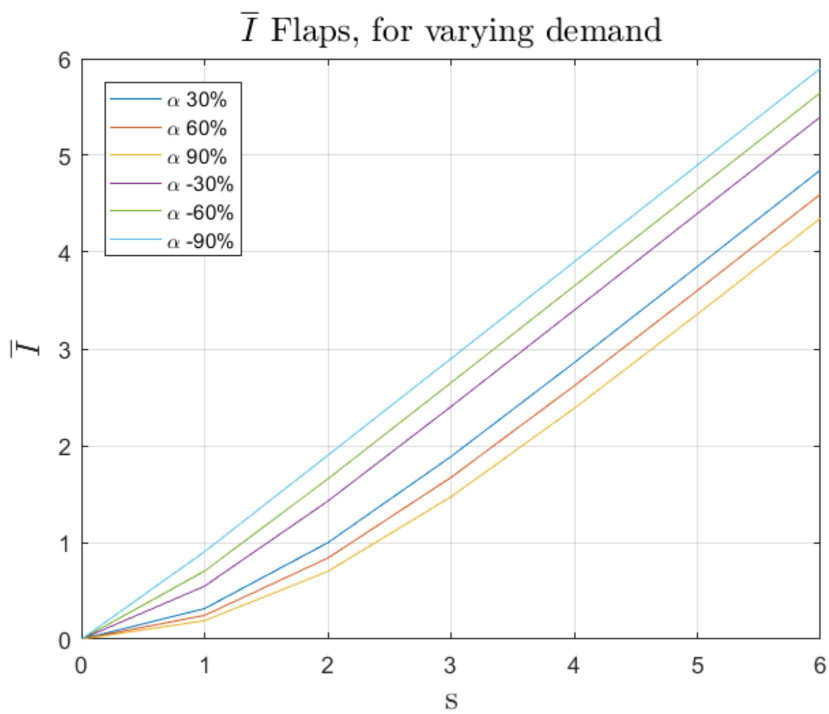


Figure A.7: Average Inventory for flaps with varying demand

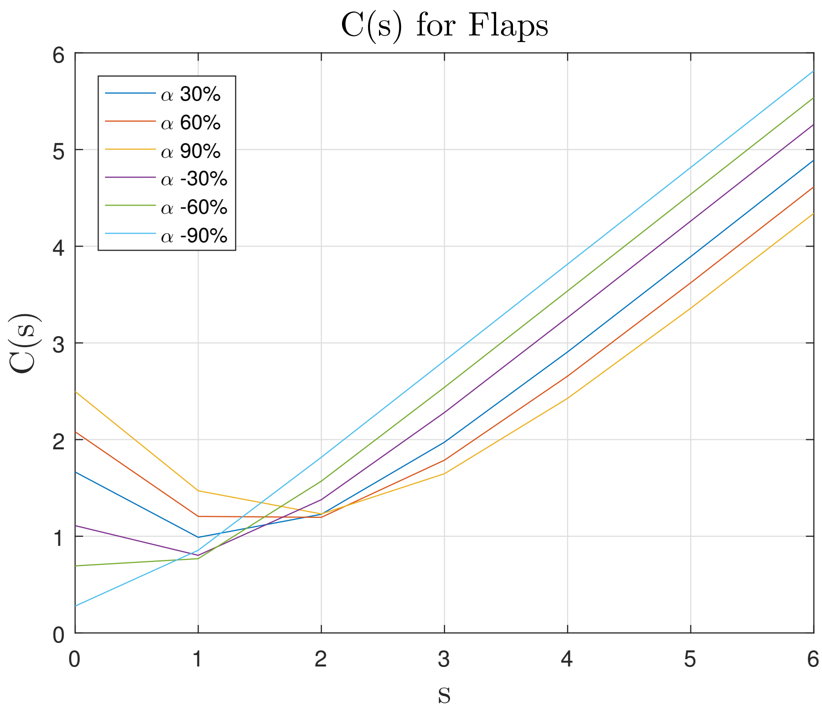


Figure A.8: Cost function for flaps with varying demand

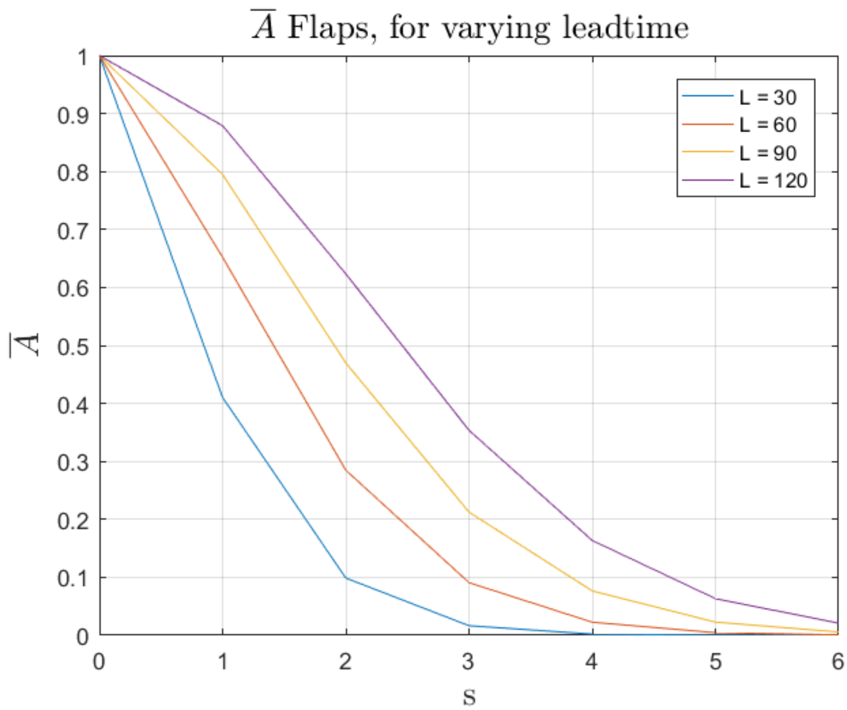


Figure A.9: Average Stockout frequency for flaps with varying leadtimes

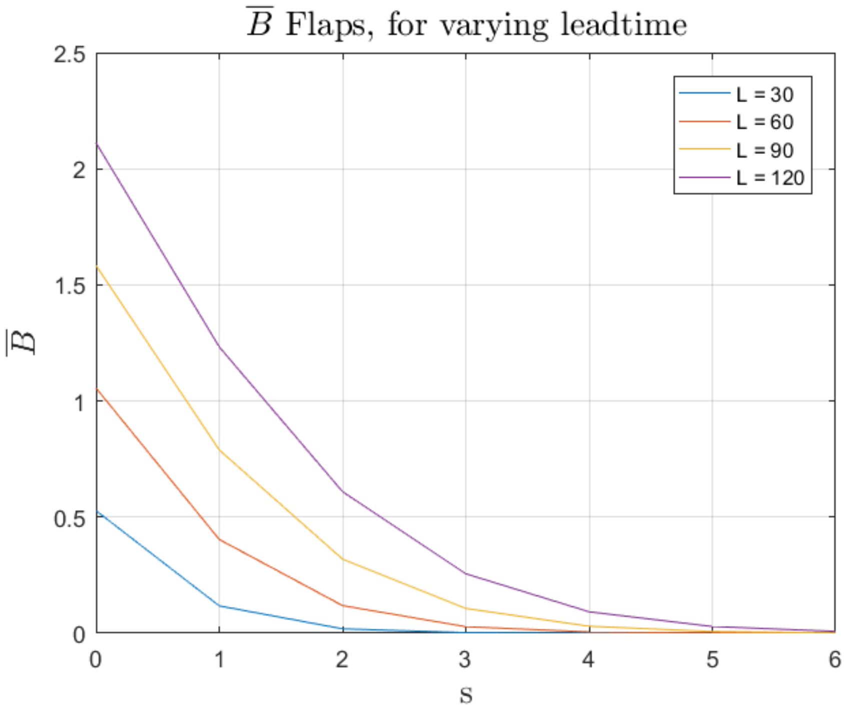


Figure A.10: Average Backorders for flaps with varying leadtimes

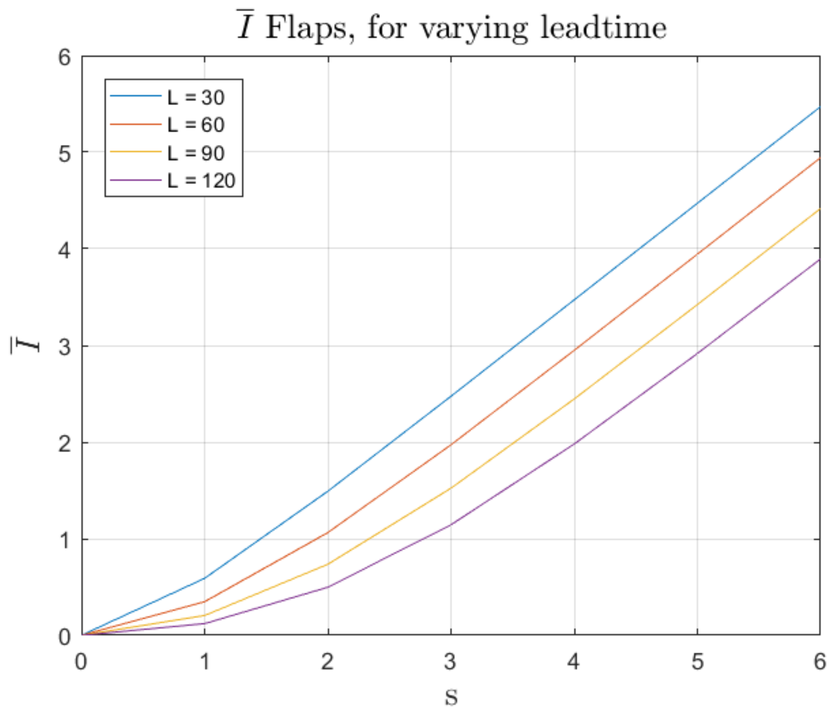


Figure A.11: Average Inventory for flaps with varying leadtimes

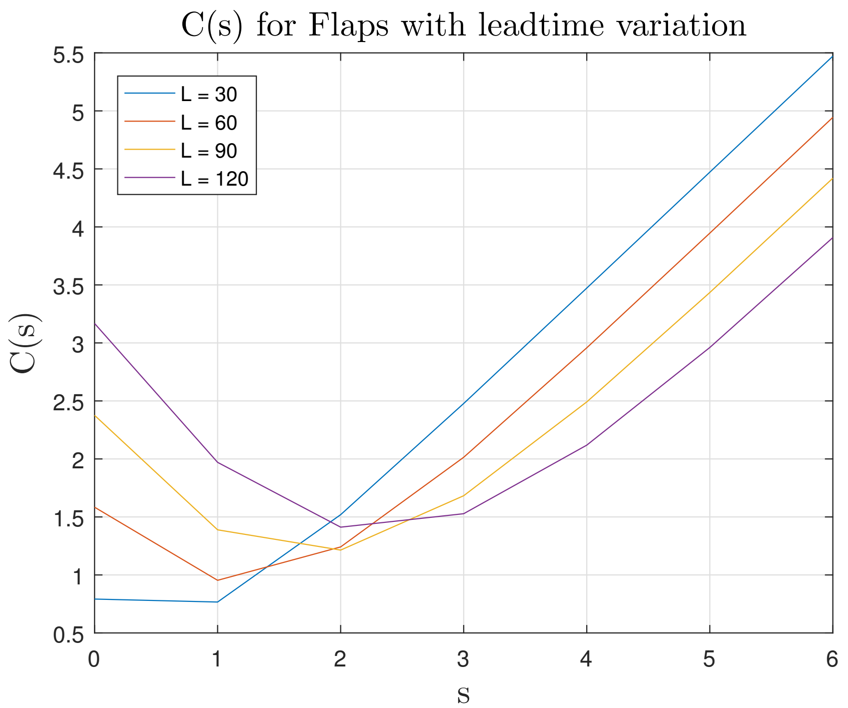
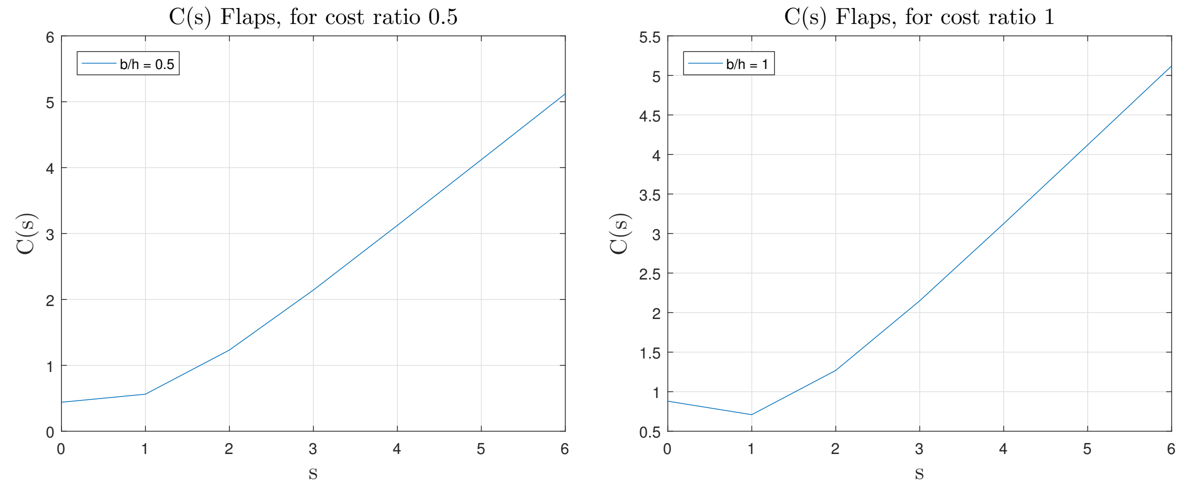


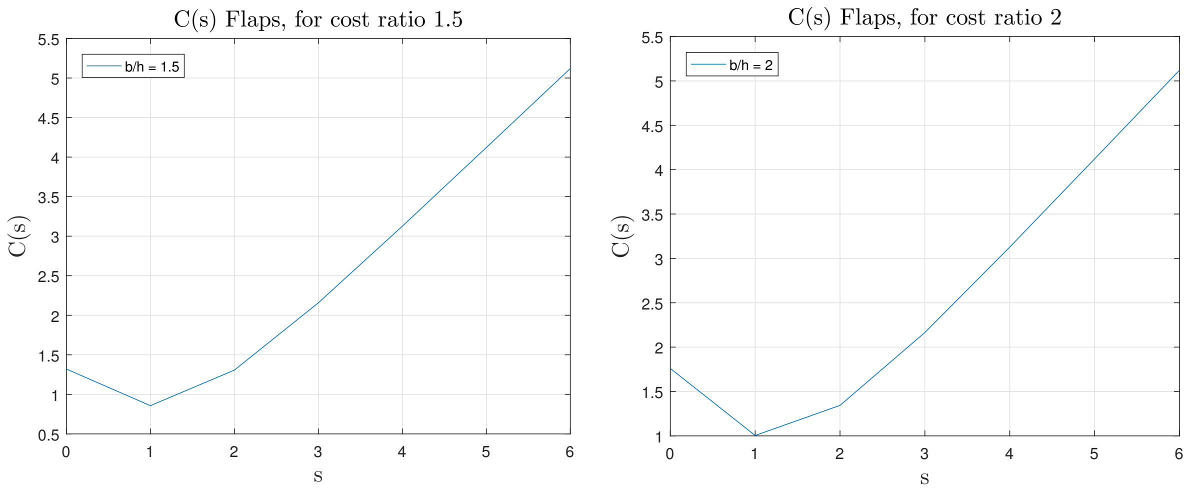
Figure A.12: Average Cost for flaps with varying leadtimes



(a) Cost function for flaps with b/h=0.5

(b) Cost function for flaps with $b/h = 1$

Figure A.13: Cost function for flaps with b/h=0.5 and b/h=1



(a) Cost function for cost ratio = 1.5

(b) Cost function for cost ratio = 2

Figure A.14: Cost function for flaps with b/h=1.5 and b/h=2