

One-Dimensional Magnetic-Field SLAM Using Handheld Smart- phone PDR

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Master of Science Thesis

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MASTER OF SCIENCE THESIS

For the degree of Master of Science in Systems and Control at Delft
University of Technology

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January 4, 2026

Abstract

Indoor localization is used in applications such as emergency response, healthcare, and navigation in large indoor spaces. Global Navigation Satellite System (GNSS) is widely used outdoors but is unreliable indoors due to signal obstructions. In addition, many indoor localization methods depend on dedicated infrastructure. As an infrastructure-free alternative, handheld smartphone Pedestrian Dead Reckoning (PDR) uses inertial sensors but accumulates drift over time. This thesis reduces that drift by integrating handheld smartphone PDR with one-dimensional magnetic-field Simultaneous Localization and Mapping (SLAM), which jointly estimates the pedestrian trajectory while building a map from sensor measurements. It uses a one-dimensional magnetic map along the walked path, which is more lightweight than constructing full two-dimensional or three-dimensional magnetic maps. The drift is reduced by detecting magnetic-field loop closures, i.e., revisits to previously traversed locations.

First, a handheld smartphone PDR trajectory is estimated from detected steps, estimated step lengths, and headings obtained from inertial sensing and orientation filtering. The PDR provides the dynamical model for the Kalman Filter (KF) position estimate update. To correct drift, three-axis magnetometer measurements are used to detect loop closures by matching magnetic sequences along the walked path. Magnetic similarity is combined with the KF positional uncertainty to score candidate loop closures. Candidates are validated using temporal constraints, a magnetic-field excitation condition, and a marginal-likelihood test based on the KF innovation. Accepted loop closures are incorporated as landmark constraints by augmenting the KF state, yielding a corrected trajectory.

The method is evaluated on three real-world handheld smartphone datasets collected inside a university building over approximately 500 m. In addition to standard walking, the evaluation includes PDR failure modes such as heading drift and step-length underestimation to test robustness, and shows that one-dimensional magnetic-field SLAM can reduce their impact. Four magnetic sequence-matching loop-closure detection methods are compared; the results indicate that using magnetic-field sequences downsampled to footsteps provides the best loop-closure detection. For the nominal walk, magnetic-field SLAM reduces the Absolute Trajectory Error from 5.5 m to 1.8 m and the final error from 11.0 m to 1.0 m.

Table of Contents

Preface	v
1 Introduction	1
1-1 Background and Motivation	1
1-2 Organization	3
2 Related Work	5
2-1 Handheld Smartphone PDR	5
2-1-1 Coordinate Frames	5
2-1-2 Position Estimation Model	6
2-1-3 Measurement Models	6
2-1-4 Step Detection Method	8
2-1-5 Step-Length Estimation	9
2-1-6 Orientation Estimation	10
2-2 Magnetic-Field SLAM	14
2-2-1 Magnetometer Measurement Model in Indoor Environments	15
2-2-2 One-Dimensional Magnetic-Field SLAM	16
3 One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR	19
3-1 Framework of One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR	19
3-2 One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR	20
3-3 Loop-Closure Detection Methods	24
3-3-1 Magnetic Gaussian Weights	26
3-3-2 Magnetic Gaussian Dynamic Time Warping	27
3-3-3 Positional Weights	29
3-4 Loop-Closure Validation Methods	29
3-4-1 Temporal Constraints	29
3-4-2 Magnetic-Field Excitation	30
3-4-3 Marginal Likelihood	30

4	Experimental Results	31
4-1	Experiment Evaluation Framework	31
4-1-1	Datasets	31
4-1-2	Evaluation Metrics	34
4-2	Sensitivity of Parameter Tuning	35
4-2-1	Sensitivity of the Loop-Closure Window Length N_{lc}	35
4-2-2	Parameters	39
4-2-3	Summary of the Results on Standard Walk	39
4-3	Robustness to PDR Failure Modes	40
4-3-1	Heading Drift under Magnetic Disturbance	40
4-3-2	Step-Length Errors under Varying Speeds	41
4-4	Comparison of Loop-Closure Detection Methods	42
4-4-1	Loop-Closure Detection in a Standard Walk	43
4-4-2	Loop-Closure Detection under Magnetic Disturbance	44
4-4-3	Loop-Closure Detection in a Varying-Speed Walk	46
5	Conclusions	49
	Bibliography	53
	Glossary	59
	List of Acronyms	59

Preface

This document contains my thesis on one-dimensional magnetic-field Simultaneous Localization and Mapping (SLAM) using handheld smartphone Pedestrian Dead Reckoning (PDR). The idea for this thesis originated from Dr. Manon Kok and continues earlier work from her study on one-dimensional magnetic-field SLAM for indoor localization. Building on this foundation, the project explores how it can be combined with handheld smartphone PDR and performs large-scale experiments.

I would like to thank my supervisors, Dr. Manon Kok and Haoyu Jia of the Delft Center for Systems and Control, for their guidance, feedback, and discussions throughout the project. I also want to thank Haoyu Jia, my daily supervisor, for the helpful conversations, and practical advice. I am very grateful to Dr. Manon Kok for her supervision and support throughout every stage of my thesis, as well as for her valuable feedback and direction.

Delft, University of Technology
January 4, 2026

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Chapter 1

Introduction

1-1 Background and Motivation

Recent studies have focused on indoor localization systems, which estimate a device or user's position within indoor spaces. Indoor localization is relevant in sectors such as healthcare, industry, and disaster management [51]. In healthcare, it can support patient tracking in hospitals, while in disaster management, it can assist emergency responders such as fire-fighters. It can also enable navigation in large shopping malls, airports, and metro stations [27, 2, 10]. Many approaches rely on pre-installed infrastructure, such as WiFi fingerprinting or Bluetooth beacons [51, 24]. However, some applications benefit from infrastructure-free methods [11, 45]. This is because deploying and maintaining infrastructure can be costly in large facilities [41], and it may fail in emergencies due to fire damage or power outages [23]. In healthcare, infrastructure-free localization can support patient monitoring while enabling privacy-preserving handling of location data [3].

The Global Navigation Satellite System (GNSS) is widely used for localization due to its high accuracy and the lack of required additional infrastructure [28]. However, specific environments can block GNSS signals, diminishing localization accuracy. Environments that disturb GNSS signals include indoor environments, areas surrounded by tall buildings, dense forests, underground mines, parking garages, and underwater settings [48].

As an alternative, this thesis investigates infrastructure-free indoor localization by combining handheld smartphone Pedestrian Dead Reckoning (PDR) with one-dimensional magnetic Simultaneous Localization and Mapping (SLAM). PDR estimates a pedestrian's position and orientation relative to a known starting point using inertial sensors [42]. However, sensor bias and noise introduce errors that accumulate with every step and lead to position drift over time [48]. The most common sensing setup is an Inertial Measurement Unit (IMU) that combines a 3D accelerometer, 3D gyroscope, and 3D magnetometer [20]. These sensors are inexpensive and embedded in most smartphones and wearable devices. This makes PDR a cost-effective solution for indoor navigation [16]. This thesis focuses on the handheld carrying mode because it is common in pedestrian navigation scenarios [14]. Therefore, this thesis

considers PDR using an IMU in a handheld smartphone. The device is assumed to be carried in a fixed orientation relative to the user for simplicity.

One-dimensional magnetic-field SLAM is introduced to reduce the drift that accumulates in PDR with loop closures. SLAM is a technique used to estimate an agent’s position while simultaneously constructing a map of the environment. Magnetic-field SLAM builds a map using spatial variations in magnetic anomalies without requiring prior knowledge of the environment [33]. These magnetic anomalies provide a location-dependent signature that can be exploited for indoor localization. They are mainly caused by ferromagnetic materials in building structures, creating a unique and stable magnetic fingerprint [28, 1]. Since the magnetic field can be measured using the magnetometer, which is typically part of the IMU, this approach does not require additional infrastructure such as WiFi or Bluetooth beacons [20, 28].

A lightweight map representation can be obtained by representing the magnetic field along one-dimensional paths [21]. One-dimensional magnetic-field SLAM represents the magnetic field along a one-dimensional path that follows the pedestrian trajectory. We assume that the motion is one-dimensional, meaning the pedestrian mainly moves through a corridor. The magnetic measurements then form a sequence along the path through the ambient magnetic landscape. Such corridor-like paths are common in indoor localization [21]. Under this assumption, it is not necessary to construct a two-dimensional or three-dimensional magnetic map. Such maps are more heavyweight and are commonly used in prior work [33, 39, 31, 40].

Loop closures are incorporated in one-dimensional magnetic-field SLAM to reduce the drift that accumulates in PDR [46]. A loop closure occurs when the system recognizes that the pedestrian has returned to a previously visited location. In the one-dimensional setting, this is detected by matching the current magnetic measurement sequence along the path to a sequence recorded earlier [21]. This exploits the location-dependent structure of indoor magnetic anomalies to improve localization accuracy [1]. Once a match is found, a loop-closure constraint is added between the corresponding path locations. This constraint can then be used to correct drift and improve the trajectory estimate [12].

Previous studies [12, 13, 46] have applied one-dimensional magnetic-field SLAM mainly for surveying, e.g., walking through an environment to collect measurements and build a magnetic map. In this setting, a dedicated surveyor repeatedly traverses trajectories and generates many loop closures that can be used to correct drift. In contrast, indoor localization without dedicated surveyors offers fewer opportunities for loop closures. This is because everyday users typically follow shorter and less repetitive routes, so loop closures are sparser than in dedicated surveying trajectories. Fewer loop closures reduce their ability to mitigate the drift that accumulates in PDR. More recently, [21] presented a proof-of-concept one-dimensional magnetic-field SLAM method for pedestrian indoor localization. However, their approach relied on artificially generated odometry rather than odometry obtained from PDR.

To address this literature gap, this thesis investigates indoor localization by integrating one-dimensional magnetic-field SLAM with PDR on a handheld smartphone, leading to the following research question:

How effectively can one-dimensional magnetic-field SLAM with loop closures reduce drift in handheld smartphone PDR for indoor localization?

To answer the main question, we divide it into three subquestions:

1. *How effectively does one-dimensional magnetic-field SLAM with loop closures reduce drift in handheld smartphone PDR during standard indoor walking?*
2. *How can one-dimensional magnetic-field SLAM with loop closures mitigate PDR failure modes, including heading drift under large magnetic-disturbances and step-length estimation errors?*
3. *How can loop-closure detection in one-dimensional magnetic-field SLAM be improved?*

The first subquestion establishes a baseline by evaluating whether one-dimensional magnetic-field SLAM with loop closures can measurably reduce the drift that accumulates in handheld smartphone PDR during typical indoor walking. The second subquestion examines robustness by considering common PDR failure modes, including heading drift under strong magnetic disturbances and step-length estimation errors, and assesses whether one-dimensional magnetic-field SLAM can still provide effective correction under these conditions. The third subquestion focuses on the core mechanism enabling drift correction in this framework, namely, loop-closure detection. The literature [46, 12, 21] proposes multiple loop-closure detection mechanisms, motivating a dedicated investigation into loop-closure detection for one-dimensional magnetic-field SLAM.

1-2 Organization

In this thesis, Chapter 2 provides background on PDR and magnetic-field SLAM. Chapter 3 introduces how one-dimensional magnetic-field SLAM with loop-closure detection is integrated into a handheld smartphone PDR system. Chapter 4 evaluates the proposed algorithm by first exploring its sensitivity to key parameters and thereby addressing the first subquestion. It then addresses the second subquestion by analyzing how magnetic SLAM with loop closures can mitigate common PDR failure modes. Finally, it addresses the third subquestion by comparing loop-closure detection methods and identifying which yields the greatest improvement. Chapter 5 discusses the implications of the findings, summarizes the contributions of this work, and outlines directions for future research.

Chapter 2

Related Work

This chapter provides the related work for the proposed algorithm in the following chapter and is divided into two main sections. Section 2-1 reviews Pedestrian Dead Reckoning (PDR) for handheld smartphones, outlining methods that form the foundation of our approach. Section 2-2 first describes magnetic-field Simultaneous Localization and Mapping (SLAM) and then reviews existing work on one-dimensional magnetic-field SLAM. Together, these sections position our work within the broader literature and highlight the key principles on which the proposed algorithm builds.

2-1 Handheld Smartphone PDR

This section presents a framework for PDR using handheld smartphones. The method relies on step detection, step-length estimation, and orientation estimation derived from accelerometer, gyroscope, and magnetometer data. These components jointly provide the position estimates that serve as odometry to the one-dimensional magnetic-field SLAM proposed in Chapter 3.

2-1-1 Coordinate Frames

This section details the coordinate frames introduced in [20] to describe the quantities measured by the Inertial Measurement Unit (IMU) from the smartphone. These frames provide the reference for the position estimation and IMU measurement models in the following sections. We introduce both the moving sensor frame and the body frame. However, for the remainder of the thesis, we assume that the moving sensor frame aligns with the body frame.

The moving sensor frame s denotes the coordinate system fixed to the IMU sensor and is aligned with its physical orientation.

The body frame b denotes a coordinate system aligned with the pedestrian being tracked, with axes in the person's forward, side, and upward directions.

The navigation frame n provides the reference frame for navigation. We generally assume that it remains stationary and follows the east-north-up (ENU) convention, where the z-axis points upward, and the y-axis aligns with the north direction.

2-1-2 Position Estimation Model

This section presents the model that estimates pedestrian trajectories from inertial measurements. Throughout this section, t denotes the discrete IMU sample time from the smartphone, whereas k denotes the step index defined by consecutive detected footsteps. We obtain the step indices k from the step-detection method detailed in Section 2-1-4.

We present the pedestrian position in the navigation frame at step k by

$$p_k = \begin{bmatrix} x_k \\ y_k \end{bmatrix}, \quad (2-1)$$

where x_k and y_k denote the position coordinates at the k -th step [17]. A widely adopted position estimation model [17] relates consecutive positions by

$$p_k = p_{k-1} + \begin{bmatrix} \cos(\phi_k) \\ \sin(\phi_k) \end{bmatrix} s_k. \quad (2-2)$$

Here, p_{k-1} denotes the position at step $k - 1$. We estimate the step length s_k at step k using Weinberg's model as in Section 2-1-5. We obtain the heading ϕ_k in the navigation frame at step k from the orientation estimate described in Section 2-1-6. Figure 2-1 illustrates the PDR framework used in this thesis. Accelerometer, gyroscope, and magnetometer measurements provide the inputs for step detection, step-length estimation, and heading estimation. These three estimates then serve as inputs to the position estimation model in (2-2), which produces the PDR trajectory. Algorithm 1 summarizes the overall procedure.

2-1-3 Measurement Models

This section introduces the gyroscope and accelerometer measurement models, which describe the inertial sensor data obtained from the IMU of the smartphone. As illustrated in Figure 2-1, the accelerometer model provides the inputs for step detection (Section 2-1-4), step-length estimation (Section 2-1-5), and inclination information used in orientation estimation (Section 2-1-6). The gyroscope model also supports the orientation estimation (Section 2-1-6).

The gyroscope measurement model follows [20]. The gyroscope measures the angular velocity ω_{ns}^s at each time instant t . These measurements are affected by a slowly varying bias $\delta_{\omega,t}^s$ and random noise $e_{\omega,t}^s$. Assuming that the navigation frame is stationary and the Earth's rotation is negligible, the gyroscope measurement model is given by

$$y_{\omega,t} = \omega_{ns,t}^s + \delta_{\omega,t}^s + e_{\omega,t}^s. \quad (2-3)$$

The bias $\delta_{\omega,t}^s$ changes gradually over time, while the noise $e_{\omega,t}^s$ follows a Gaussian distribution, $e_{\omega,t}^s \sim \mathcal{N}(0, \Sigma_\omega)$. For a properly calibrated sensor, the measurements along the three

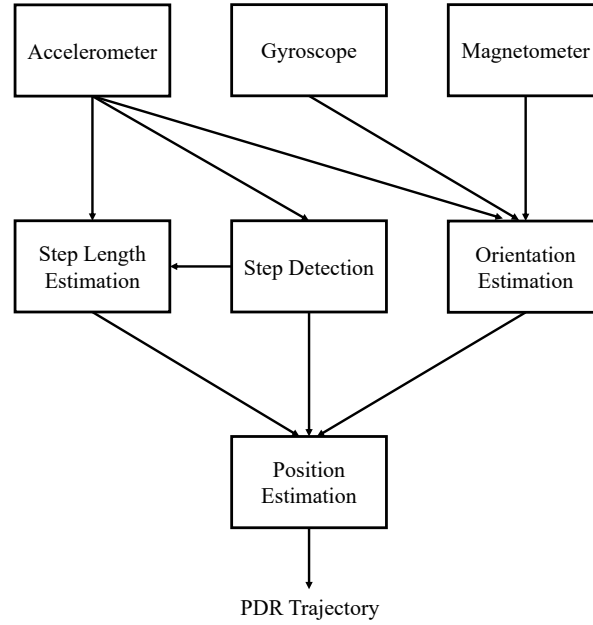


Figure 2-1: Block scheme of PDR, where accelerometer, gyroscope, and magnetometer measurements are the input for step detection, step-length estimation, and orientation estimation. These components jointly provide the input for the position estimation model that forms the PDR output.

gyroscope axes are independent, leading to

$$\Sigma_{\omega} = \begin{pmatrix} \sigma_{\omega,x}^2 & 0 & 0 \\ 0 & \sigma_{\omega,y}^2 & 0 \\ 0 & 0 & \sigma_{\omega,z}^2 \end{pmatrix}. \quad (2-4)$$

The accelerometer measurement model also follows [20]. The accelerometer measures the specific force f_t^s at each time instant t . Its measurements are influenced by a bias $\delta_{a,t}^s$ and random noise $e_{a,t}^s$, which yields the model

$$y_{a,t} = f_t^s + \delta_{a,t}^s + e_{a,t}^s. \quad (2-5)$$

The bias $\delta_{a,t}^s$ varies slowly over time. The noise $e_{a,t}^s$ is typically Gaussian and can be modeled as $e_{a,t}^s \sim \mathcal{N}(0, \Sigma_a)$. For a properly calibrated sensor, the covariance matrix Σ_a is assumed to be diagonal.

The specific force measured by the accelerometer is

$$f_t^s = R_t^{sn}(a_t^n - g^n), \quad (2-6)$$

where g denotes the gravity vector and a_t^n is the sensor's linear acceleration expressed in the navigation frame. The matrix R_t^{sn} denotes the rotation matrix that maps vectors from the navigation frame n to the sensor frame s at time instant t . Since the accelerometer measures the specific force, it captures both gravity and the sensor's linear acceleration. As a result, the

measurements contain information about both changes in motion and the sensor's inclination. For orientation estimation, the inclination information is of primary interest. This requires an assumption about the linear acceleration. We therefore assume that gravity dominates most accelerometer measurements, so that the measured specific force approximately aligns with $-g^n$ in the navigation frame.

2-1-4 Step Detection Method

Step detection identifies the occurrence of each pedestrian step, serving as a key event for determining the pedestrian position [48]. Each detected step defines a step k , which increases by one for consecutive detected footsteps [17]. At step k , the position estimation model in (2-2) uses the corresponding step length s_k and heading ϕ_k to update the position.

Time-domain methods process sensor signals over time and are widely used in PDR due to their simplicity and computational efficiency [44]. These methods analyze inertial sensor signals to identify step events. To reduce sensor measurement noise, the measurements are often filtered using a Butterworth filter [43, 15, 17].

Step detection exploits periodic patterns in the sensor data, such as peaks and valleys associated with foot motion [47]. Many approaches use vertical (z-axis) acceleration for this purpose [47, 25, 6]. The preference for z-axis acceleration follows from its stronger periodicity, symmetry, and stability compared with measurements from the other axes [25].

Standard time-domain techniques for step detection include peak detection and zero-crossing detection [2]. Peak detection identifies peaks and valleys in the measured acceleration, where the interval between consecutive peaks indicates a step [19]. Zero-crossing detection identifies time instants at which the signal passes through zero acceleration, either from positive to negative or vice versa [29, 49]. To improve accuracy, many works combine peak detection with zero-crossing detection [34, 15, 49]. A slight performance improvement has been reported when integrating these techniques rather than using them separately [35]. Therefore, this thesis uses peak detection integrated with zero-crossing detection.

Peak Detection Integrated with Zero-Crossing Detection

Peak detection is often combined with zero-crossing information to improve step detection accuracy [35]. Accelerometer measurements are affected by bias and measurement noise [20]. Introducing a zero-crossing condition as an additional threshold can therefore increase the robustness of peak detection [34]. This thesis follows the implementation of peak detection integrated with zero-crossing detection described in [15].

Based on the accelerometer measurement model (2-5), the vertical (z-axis) accelerometer measurement used for step detection is defined as

$$y_{a,z,t} := e_z^\top y_{a,t}, \quad e_z = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top. \quad (2-7)$$

The vertical acceleration $y_{a,z,t}$ is filtered using a bandpass Butterworth filter. We define

$$\tilde{y}_{a,z,t} := \mathcal{F}\{y_{a,z,t}\}, \quad (2-8)$$

where $\mathcal{F}\{\cdot\}$ denotes the Butterworth bandpass filtering operation and $\tilde{y}_{a,z,t}$ denotes the filtered vertical acceleration measurement.

Next, step detection identifies peak/valley pairs in $\tilde{y}_{a,z,t}$. Due to sensor noise, two consecutive peaks or valleys may occasionally occur. The bandpass filter reduces the likelihood of such events, and a fail-safe further removes remaining unwanted spikes. The zero-crossing information implements this fail-safe via a time threshold. After detecting a peak or valley, the algorithm ignores any additional peaks or valleys outside the defined replacement zone. This yields the detection time instants t_{peak} and t_{valley} and their interval,

$$t_{\text{peak}} = \arg \max_t \tilde{y}_{a,z,t}, \quad t \in [t_c^+, t_c^-], \quad (2-9a)$$

$$t_{\text{valley}} = \arg \min_t \tilde{y}_{a,z,t}, \quad t \in [t_c^-, t_{c+1}^+]. \quad (2-9b)$$

The time instants t_c^+ and t_c^- , with $c \in \mathbb{N}$, denote the instants at which the acceleration crosses zero from negative to positive and from positive to negative, respectively.

2-1-5 Step-Length Estimation

This section gives an overview of methods to estimate the step length s_k used in the position estimation model in (2-2) in Section 2-1-2. Step-length estimation determines the distance covered in each step and thus the pedestrian displacement [9, 25, 30]. Existing step-length models fall into two categories, namely direct and indirect methods [9]. Direct methods estimate step length by integrating measured accelerations, whereas indirect methods use models to infer step length. This study focuses on indirect methods because direct methods suffer from cumulative integration errors [6]. Indirect methods further divide into biomechanical models and statistical regression techniques [9].

Biomechanical models exploit the mechanical characteristics of the human body to estimate step length. These models enable step-length estimation through analytical expressions based on geometric relationships between body segments [30, 43, 13, 19]. However, due to the complexity of the human body, these expressions rely on approximations and simplifications. Moreover, pedestrian-specific differences require model adjustments for each individual [19].

The models differ in complexity and in the assumed sensor placement. The most straightforward approach uses a fixed step length [13]. Some models use wrist-worn sensors to estimate step length from arm-swing motion [9]. However, most approaches rely on the inverted pendulum model [6, 47, 34, 17, 42, 25, 5, 15]. This model approximates human walking by representing the body as a pendulum that pivots on the stance leg, with the pedestrian's center of mass following an arc-like trajectory. The inverted pendulum model also supports step-length estimation using handheld smartphone sensors because the estimate is based on the measured motion signals [6].

Weinberg's Model

Weinberg [47] proposes a method to determine step length using a biomechanical model. Since step length can vary depending on leg length and walking speed, it cannot be accurately

modeled as a constant. Consequently, knowing the leg length is insufficient to estimate the step length. The proposed inverted pendulum model treats the leg as a lever of fixed length while the foot remains in contact with the ground, as the knee only bends when the foot is lifted. In the geometric construction underlying the model, α denotes the angular displacement of the hip relative to the body's central axis during walking, and θ denotes the corresponding angular displacement of the hip in the vertical plane. Using the geometry of similar angles in this construction, these angles satisfy $\theta = \alpha$, which yields the step-length estimate

$$s_k = \frac{2r_k}{\alpha}. \quad (2-10)$$

At step k , r_k represents the vertical displacement.

The vertical displacement r_k can be obtained by double-integrating the filtered vertical acceleration signal $\tilde{y}_{a,z,t}$ in (2-8) over the interval defined by the peak and valley time instants t_{peak} and t_{valley} identified in (2-9). Under the assumptions in [47], this leads to a displacement estimate that is proportional to the square root of the peak-to-valley amplitude of $\tilde{y}_{a,z,t}$. Since α is a small angle with a lot of noise in the axes while walking, it can be replaced by a constant K . Substituting the resulting expressions for r_k and α into (2-10) then yields the step length model

$$s_k = K \sqrt[4]{\tilde{y}_{a,z,t_{\text{peak}}} - \tilde{y}_{a,z,t_{\text{valley}}}}, \quad (2-11)$$

where K is the Weinberg model coefficient, and $\tilde{y}_{a,z,t_{\text{peak}}}$ and $\tilde{y}_{a,z,t_{\text{valley}}}$ denote the maximum and minimum filtered vertical acceleration within one step, respectively. In practice, K is determined empirically for a given sensor setup and can be tuned to account for different users and walking styles. The coefficient K depends on pedestrian characteristics such as gender and walking speed, and it generally increases with walking speed [19]. Consequently, if walking speed increases while K remains fixed, the model underestimates the step length s_k , which leads to position errors in PDR. Several refinements have been proposed in [25], primarily to improve the estimation of K .

2-1-6 Orientation Estimation

This section reviews the literature on determining the heading ϕ_k used for position estimation in (2-2) in Section 2-1-2. Heading refers to the direction in which a person is oriented or moving relative to the navigation frame [17]. Gyroscope measurements provide heading information but suffer from cumulative drift over time. In outdoor environments without magnetic disturbances, magnetometers provide orientation information based on the direction of the magnetic field. Therefore, heading estimation typically combines gyroscope and magnetometer measurements, since gyroscope measurements alone do not provide a reliable long-term heading reference [20]. Accelerometer measurements also support orientation estimation by providing inclination information, as the gravity component dominates the measured specific force and thus defines the vertical direction.

In indoor environments, distortions in the magnetic field render magnetometer-based heading estimation unreliable [43]. Versatile Quaternion-based Filtering (VQF) can deal with the presence of magnetic disturbances. A comprehensive evaluation against eight state-of-the-art

orientation methods reported superior accuracy across multiple datasets [22]. For this reason, this section focuses exclusively on VQF.

This section first introduces methods for parametrizing orientation, which support the explanation of VQF and its relation to the position estimation model. It then introduces two additional coordinate frames that are essential for describing VQF. Finally, it presents VQF in detail.

Parametrizing Orientation

Describing orientations in three-dimensional space is required to estimate and represent pedestrian motion. This section discusses Euler angles and quaternions.

Euler Angles are a common representation of the orientation of a rigid body in three-dimensional space [7]. They parameterize orientation through three sequential rotations about specified coordinate axes, which describe how the body is oriented relative to a reference frame. The three angles yaw, pitch, and roll correspond to rotations about the z-axis, y-axis, and x-axis, respectively [20]. In the position estimation model, the heading ϕ corresponds to the yaw Euler angle.

The disadvantage of using Euler angles for orientation is that they are susceptible to gimbal lock, where two rotation axes become aligned, reducing the system's degrees of freedom and making specific orientations challenging to represent accurately [38].

Unit Quaternions provide an efficient representation of orientation and avoid the singularities inherent in Euler-angle parametrizations. They also simplify vector rotations, since these transformations can be computed using quaternion multiplication [38]. A unit quaternion provides a four-dimensional representation of orientation in three-dimensional space [20], and it is defined as

$$q = \begin{pmatrix} q_0 & q_1 & q_2 & q_3 \end{pmatrix}^T = \begin{pmatrix} q_0 \\ q_v \end{pmatrix}, \quad q \in \mathbb{R}^4, \quad \|q\|_2 = 1, \quad (2-12)$$

where q_0 denotes the scalar (real) part and $q_v = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^T$ denotes the vector (imaginary) part. The constraint $\|q\|_2 = 1$ ensures that q represents a valid rotation in three-dimensional space [20].

The relation between the yaw Euler angle ϕ , which represents the heading, and the quaternion q is

$$\phi = \text{atan2}\left(2(q_0q_3 + q_1q_2), 1 - 2(q_2^2 + q_3^2)\right). \quad (2-13)$$

Two additional definitions support the use of quaternions in VQF. First, quaternion multiplication, which combines rotations, is defined as follows. Given two quaternions p and q , their multiplication $p \otimes q$ is expressed as

$$p \otimes q = \begin{pmatrix} p_0q_0 - p_v \cdot q_v \\ p_0q_v + q_0p_v + p_v \times q_v \end{pmatrix}, \quad (2-14)$$

where $\cdot$ and $\times$ denote the dot and cross products, respectively [20]. Second, a quaternion that represents a rotation by an angle $\alpha \in \mathbb{R}$ around an axis $u \in \mathbb{R}^3$ takes the form [22]

$$(\alpha @ u) := \begin{bmatrix} \cos \frac{\alpha}{2} \\ \frac{u}{\|u\|} \sin \frac{\alpha}{2} \end{bmatrix}. \quad (2-15)$$

Coordinate Frames for Versatile Quaternion-based Filtering

Two additional coordinate frames are introduced to describe VQF. The almost-inertial frame and an auxiliary coordinate frame are used in deriving the orientation estimation quaternion [22].

Almost-inertial reference frame e . The almost-inertial frame is similar to the navigation frame but experiences gradual drift around the z -axis over time. This drift can be corrected using magnetometer measurements, which realign the frame with the Earth's magnetic field to maintain orientation accuracy.

Auxiliary coordinate frame i . The auxiliary coordinate frame is an introduced almost-inertial frame that slowly drifts around arbitrary axes due to errors in gyroscope integration.

VQF

VQF is introduced in [22] to estimate orientation. It consists of three stages. First, it predicts the orientation using gyroscope measurements. Next, it applies an inclination correction using accelerometer measurements to align the estimate with the vertical direction. The basic VQF performs gyro strapdown integration and inclination correction (6D estimate), as well as an optional heading correction (9D estimate). The full VQF additionally estimates gyroscope bias and performs magnetic disturbance detection/rejection; this is the variant used in this thesis.

Local magnetic-field distortions indoors can make magnetometer-based heading corrections unreliable. The full VQF addresses this by detecting magnetic disturbances and suppressing magnetometer updates, effectively reverting to the 6D estimate for heading during disturbed intervals. In this thesis, such distortions are expected because the spatial magnetic variability is exploited for localization.

Gyroscope prediction is achieved by updating the previous estimate through quaternion multiplication, detailed in (2-14) in Section 2-1-6, with a quaternion derived from the norm and direction of the (bias-compensated) angular velocity. Using the gyroscope measurements $y_{\omega,t}$ in (2-3) at time instant t , the bias-compensated angular rate is defined as

$$\tilde{y}_{\omega,t} = y_{\omega,t} - \hat{b}_{\omega,t}, \quad (2-16)$$

where $\hat{b}_{\omega,t}$ denotes the estimated gyroscope bias provided by VQF. The prediction is then given by

$$q_t^{is} = q_{t-1}^{is} \otimes (T_s \|\tilde{y}_{\omega,t}\| @ \tilde{y}_{\omega,t}), \quad (2-17)$$

where T_s is the sampling interval. The quaternion q_t^{is} represents the orientation of the sensor frame relative to the auxiliary coordinate frame at time instant t . The rotation operator $@$ is defined in (2-15).

To correct the inclination, VQF uses the accelerometer measurements $y_{a,t}$ in (2-5) expressed in the sensor frame and rotates them into the auxiliary frame i . The accelerometer measurement in frame i is obtained as

$$y_{a,t}^i = R(q_t^{is}) y_{a,t}. \quad (2-18)$$

The rotation matrix $R(q_t^{is})$ corresponds to the quaternion q_t^{is} , used to rotate the vector from the sensor frame s into the auxiliary frame i at time instant t . A low-pass filter is applied to $y_{a,t}^i$, and the filtered signal is rotated into the almost-inertial frame e and normalized to obtain

$$y_{a,t}^e = R(q_t^{ei}) \text{LPF}(y_{a,t}^i), \quad (2-19)$$

$$a_t^e = \frac{y_{a,t}^e}{\|y_{a,t}^e\|} = \begin{bmatrix} a_{x,t}^e & a_{y,t}^e & a_{z,t}^e \end{bmatrix}^\top. \quad (2-20)$$

The filtered, normalized acceleration $[a_{x,t}^e, a_{y,t}^e, a_{z,t}^e]^\top$ is used to calculate the shortest inclination correction. The correction quaternion is expressed as

$$q_{\text{corr},t} = \left[q_{w,t}, \frac{a_{y,t}^e}{2q_{w,t}}, \frac{-a_{x,t}^e}{2q_{w,t}}, 0 \right]^\top, \quad (2-21)$$

where $q_{w,t} = \sqrt{\frac{a_{z,t}^e + 1}{2}}$. The quaternion that corrects the accelerometer inclination is defined as

$$q_t^{ei} = q_{\text{corr},t} \otimes q_{t-1}^{ei}, \quad (2-22)$$

where q_t^{ei} denotes the orientation of the auxiliary coordinate frame to the almost inertial frame at time instant t .

Finally, magnetometer measurements are incorporated to correct heading drift by adjusting the orientation about the vertical axis. The magnetometer measurement at time instant t , expressed in the sensor frame, is denoted by $y_{m,t}$ (see Section 2-2-1). Using the 6D estimate

$$q_t^{es} := q_t^{ei} \otimes q_t^{is}, \quad (2-23)$$

the magnetometer measurement is rotated into the almost-inertial frame

$$y_{m,t}^e = R(q_t^{es}) y_{m,t}. \quad (2-24)$$

Heading corresponds to a rotation about the vertical axis; therefore, the heading information is extracted from the horizontal component of $y_{m,t}^e$. The heading correction is tracked using a scalar state δ_t , which represents the vertical rotation required to align frame e with the navigation frame n . In VQF, δ_t is updated by low-pass filtering the difference between the current heading estimate and the magnetometer-derived heading. In full VQF, magnetic disturbance detection is used to gate this update: when the magnetic field is classified as disturbed, magnetometer-based heading corrections are suppressed (i.e., δ_t is held constant), and the filter effectively falls back to the 6D estimate for heading.

The magnetometer-based heading correction quaternion q_t^{ne} represents the orientation from frame e to frame n and is expressed as

$$q_t^{ne} = \delta_t @ \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T. \quad (2-25)$$

The 6D estimate uses only gyroscope and accelerometer measurements and is given by (2-23), where q_t^{is} is obtained by (bias-compensated) gyroscope integration and q_t^{ei} applies the inclination correction from the accelerometer. The 9D estimate additionally incorporates magnetometer measurements through the heading correction q_t^{ne} . The resulting 9D estimate q_t^{ns} , describing the orientation from the sensor frame to the navigation frame at time instant t , is obtained by composing the heading correction with the 6D estimate, which can be represented as

$$\underbrace{q_t^{ns}}_{\text{9D estimate}} = \underbrace{q_t^{ne}}_{\substack{\text{heading correction} \\ \text{(magnetometer)}}} \otimes \underbrace{q_t^{ei}}_{\substack{\text{inclination correction} \\ \text{(accelerometer)}}} \otimes \underbrace{q_t^{is}}_{\substack{\text{integration} \\ \text{(gyroscope)}}}. \quad (2-26)$$

In full VQF, magnetic disturbance detection gates the magnetometer update; during disturbed intervals the heading correction q_t^{ne} is held constant, so yaw is effectively propagated inertially until magnetometer updates are re-enabled.

For the position estimation model (2-2), the stepwise heading ϕ_k is obtained by sampling the VQF orientation estimate at the detected step times t_k , i.e., $q_k^{ns} := q_{t_k}^{ns}$, and computing the corresponding yaw angle from q_k^{ns} using (2-13). The resulting yaw angle is used as the heading. When magnetometer updates are suppressed due to magnetic disturbance detection, this heading can still drift over time because it is propagated from gyroscope integration.

Algorithm 1 Handheld Smartphone PDR

Require: IMU measurements $\{y_{a,t}, y_{\omega,t}, y_{m,t}\}_{t=1}^{N_t}$, initial position p_0
parameters K (Weinberg constant), $e_z^\top = [0 \ 0 \ 1]$, $\mathcal{F}\{\cdot\}$ (Butterworth bandpass filter in (2-8))
Ensure: Step time instants $\{t_k\}_{k=1}^{N_k}$, step lengths $\{s_k\}_{k=1}^{N_k}$, headings $\{\phi_k\}_{k=1}^{N_k}$, and PDR trajectory $\{p_k\}_{k=0}^{N_k}$

- 1: Initialize $k = 0$ and set $p_k = p_0$
- 2: Initialize the VQF state
- 3: Initialize zero-crossing buffers for $\{t_c^+, t_c^-\}$
- 4: **for** $t = 1, \dots, N_t$ **do**
- 5: Update the VQF orientation estimate with $(y_{a,t}, y_{\omega,t}, y_{m,t})$ and obtain q_t^{ns} using (2-26)
- 6: Compute vertical acceleration $y_{a,z,t}$ in (2-7)
- 7: Filter $\tilde{y}_{a,z,t}$ in (2-8)
- 8: Update zero-crossing time instants t_c^+ and t_c^- from $\tilde{y}_{a,z,t}$ (used in (2-9))
- 9: **if** a complete step interval $[t_c^+, t_c^-]$ and $[t_c^-, t_{c+1}^+]$ is available **then**
- 10: Compute t_{peak} and t_{valley} in (2-9)
- 11: $k \leftarrow k + 1$ and set $t_k \leftarrow t_{\text{peak}}$
- 12: Compute step length s_k using (2-11)
- 13: Sample orientation at the step time instant $q_k^{ns} \leftarrow q_{t_k}^{ns}$ and compute heading ϕ_k using (2-13)
- 14: Update position p_k using (2-2)
- 15: **end if**
- 16: **end for**

2-2 Magnetic-Field SLAM

This section presents magnetic-field SLAM as an approach to correct drift in PDR [12]. In this context, SLAM combines odometry (PDR) with map observations. It constructs a map from

magnetic-field measurements and uses this map for localization. In outdoor environments, the Earth's magnetic field is relatively uniform on local spatial scales. Indoor environments, however, often exhibit location-dependent magnetic anomalies caused by surrounding structures [1]. These anomalies can serve as observations for localization and can be used to build a magnetic map within magnetic-field SLAM [28]. This section introduces magnetic-field SLAM, presents the indoor magnetometer measurement model, and reviews the literature on one-dimensional magnetic-field SLAM.

SLAM is a concept originally introduced in robotics. It describes the process by which an agent constructs a map of its environment while simultaneously using that map to localize, without requiring a prior map [8]. SLAM for pedestrian navigation is introduced in [32], where PDR with foot-mounted IMUs is combined with the SLAM approach used in robotics. Subsequently, [33] explores the use of magnetic field anomalies for SLAM in indoor environments, where the map used for localization is based on these anomalies.

Magnetic-field SLAM approaches use different representations of the magnetic map. One approach constructs the map using hexagonal grids with varying-sized cells, where each cell stores statistical summaries of the magnetic field strength [33]. Another approach employs Gaussian processes (GPs) for map representation [39]. GPs are a statistical tool for regression and prediction tasks, defining a distribution over functions that provides predictions with associated uncertainties [31]. This makes GPs suitable for magnetic field mapping, as they can model complex spatial variations while providing confidence intervals for predictions [40]. However, the GP approach can be computationally demanding: a direct implementation scales cubically with the number of measurements, making it impractical for large datasets. Although reduced-rank approximations can alleviate this issue, obtaining an accurate GP approximation often remains computationally intensive, limiting scalability and efficiency [36]. A lightweight alternative represents the magnetic map along a one-dimensional trajectory [21], and the remainder of this section further explores one-dimensional magnetic-field SLAM. Before introducing these one-dimensional approaches, the next section presents the indoor magnetometer measurement model.

2-2-1 Magnetometer Measurement Model in Indoor Environments

The magnetic field measurement model for indoor environments differs from that for outdoor environments, since magnetic materials in buildings and furniture can significantly distort magnetometer measurements [20]. This section follows the indoor magnetometer measurement model in [28]. The magnetometer measures the Earth's magnetic field, denoted m^n in the navigation frame. In indoor environments, environmental magnetic disturbances and measurement noise affect the readings, which can be modeled as

$$y_{m,t} = A_t R_t^{sn} m^n + b_t + e_{m,t}^s, \quad (2-27)$$

where $y_{m,t}$ is the magnetometer measurement expressed in the sensor frame s . The distortion matrix A_t jointly captures scale factor, misalignment, and soft-iron effects, whereas the offset vector b_t captures the hard-iron effect together with sensor bias. These terms, therefore, combine together sensor calibration effects and environment-induced distortions. If A_t and b_t are constant over time, they can be estimated once using a standard magnetometer

calibration procedure. In indoor environments, however, these terms also depend on surrounding ferromagnetic structures and are therefore position-dependent. In the remainder, this position-dependent distortion is treated as part of the local magnetic signature used for mapping and localization. This spatial dependence is precisely what enables magnetic-field-based indoor localization. The rotation matrix R_t^{sn} maps vectors from the navigation frame n to the sensor frame s , thereby expressing m^n in the sensor frame. The term $e_{m,t}^s$ represents Gaussian measurement noise, modeled as $e_{m,t}^s \sim \mathcal{N}(0, \Sigma_m)$.

2-2-2 One-Dimensional Magnetic-Field SLAM

Magnetic-field SLAM using one-dimensional maps provides a lightweight alternative to approaches that use two- or three-dimensional maps [21]. A one-dimensional map represents the magnetic field strength along one-dimensional trajectories, such as the paths pedestrians follow while moving through building corridors [21]. In this setting, revisits can be incorporated as loop closures, i.e., additional constraints introduced when the trajectory returns to a previously visited location, which helps reduce accumulated PDR drift [12]. This thesis focuses on one-dimensional magnetic-field SLAM. Although one-dimensional magnetic-field SLAM for pedestrian navigation remains relatively underexplored, several studies have investigated its potential. The reviewed works use one-dimensional magnetic-field SLAM for two main purposes. Some focus on constructing magnetic maps, while others focus on localization. The works also differ in the way they detect loop closures and how they incorporate the resulting loop-closure constraints into one-dimensional magnetic-field SLAM.

For surveying and map construction, [12] proposes one-dimensional magnetic-field SLAM in which pedestrians with smartphones generate signal maps during surveys. The method obtains the trajectory through PDR and detects revisits using Dynamic Time Warping (DTW) on three-axis magnetic-field measurement sequences. It then incorporates the resulting loop closures using pose-graph optimization. This formulation represents the trajectory as a graph of poses connected by relative constraints, which enables loop closures to be incorporated through global graph optimization. In this setting, DTW matches magnetic measurement sequences while allowing the time axis to stretch or compress. This makes the matching robust to variations in walking speed and stride timing. Building on this approach, [13] introduces a particle-filter-based extension. Similarly, [46] presents a method for surveying and map construction that also relies on pose-graph optimization. The method forms keyframes from sequences of the magnetic-field magnitude downsampled to footsteps and correlated with motion patterns. It then detects loop closures by matching these keyframes. The method uses PDR as odometry and incorporates the resulting loop-closure constraints using pose-graph optimization.

In contrast, [21] introduces a proof-of-concept one-dimensional magnetic-field SLAM for pedestrian indoor localization. The method targets localization rather than magnetic map construction and uses an Extended Kalman Filter (EKF). It detects loop closures by comparing three-axis magnetic-field measurements and using the Gaussian measurement model to assess the loop-closure likelihood. Since the method does not include PDR, it generates synthetic odometry by perturbing trajectories from visual-inertial odometry.

Among the approaches reviewed above, [46] is the only published one-dimensional magnetic-field SLAM combined with PDR that reports comparable position errors from large-scale

experimental walks. They evaluate on a 0.75 km walk in an office building and a 1.2 km walk in a museum, reporting mean/maximum position errors of 0.37/1.52 m (office) and 0.60/3.09 m (museum) as shown in Table 2-1. The revisit percentages in Table 2-1 are not reported in [46] and were instead estimated manually from the trajectory figures, and should therefore be interpreted only as rough approximations.

Method	Dataset	Mean Error (m)	Max Error (m)	% Revisited
[46]	Office (0.75 km)	0.37	1.52	97
[46]	Museum (1.2 km)	0.60	3.09	95

Table 2-1: Reported mean and maximum position errors from [46] on large-scale experimental trajectories, with the estimated fraction of each trajectory that is revisited.

In the next chapter, this thesis proposes a new one-dimensional magnetic-field SLAM algorithm inspired by [21] that targets indoor localization. In addition, the algorithm uses PDR to provide odometry. Loop-closure detection follows [21] by matching magnetic-field sequences, and it is extended with DTW and step-based magnetic sequence matching inspired by [46, 12]. Accepted loop closures are then incorporated in SLAM using a Kalman Filter (KF) to obtain a drift-reduced trajectory.

One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR

This chapter presents a one-dimensional magnetic-field Simultaneous Localization and Mapping (SLAM) approach with loop closures, using handheld smartphone Pedestrian Dead Reckoning (PDR) as the odometry input. We first outline the overall framework for one-dimensional magnetic-field SLAM with handheld PDR. We then describe the Kalman Filter (KF) formulation in Section 3-2, where loop-closure information is fused with PDR to reduce drift and improve position estimates. Next, we introduce various strategies for loop-closure detection in Section 3-3. Finally, the loop-closure validation techniques are presented in Section 3-4.

3-1 Framework of One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR

This section provides an overview of the proposed algorithm and its main components, illustrated in Figure 3-1. We first briefly recap the handheld smartphone PDR approach from Chapter 2 and describe how its output is used as input to the KF in the proposed framework. We then introduce the loop-closure detection and validation modules proposed in this chapter, and explain how validated loop closures are incorporated into the KF.

The PDR described in Section 2-1 provides the step times, step-length estimates, and orientations used for the KF time update in the proposed framework. The smartphone measures acceleration, angular velocity, and the magnetic field. The gyroscope and accelerometer measurement models follow (2-3) and (2-5) (Section 2-1-3), and the magnetometer measurements are modeled as in (2-27) (Section 2-2-1).

The PDR module detects steps from the accelerometer signal using peak detection with zero-crossing integration, see (2-9) (Section 2-1-4). It then estimates step length using Weinberg's model in (2-11), based on the accelerometer measurements and detected steps (Section 2-1-5). In parallel, it estimates orientation using the Versatile Quaternion-based Filtering (VQF) in (2-26) from all smartphone measurements (Section 2-1-6). The resulting step times, step lengths, and orientations form the inputs to the KF time update.

To mitigate accumulated PDR drift, loop closures are detected, validated, and incorporated into the KF. Loop-closure detection relies on magnetic and positional weights (Section 3-3). Magnetic weights are computed from magnetometer measurements (Sections 3-3-1 and 3-3-2), while positional weights are obtained from the KF position estimates and their uncertainty, which quantify the likelihood of a loop closure (Section 3-3-3). Detected loop closures are subsequently validated (Section 3-4). Accepted loop closures are incorporated as measurement updates. The KF then outputs the one-dimensional magnetic-field SLAM trajectory by combining loop-closure constraints with PDR; the KF is discussed in more detail in the following section.

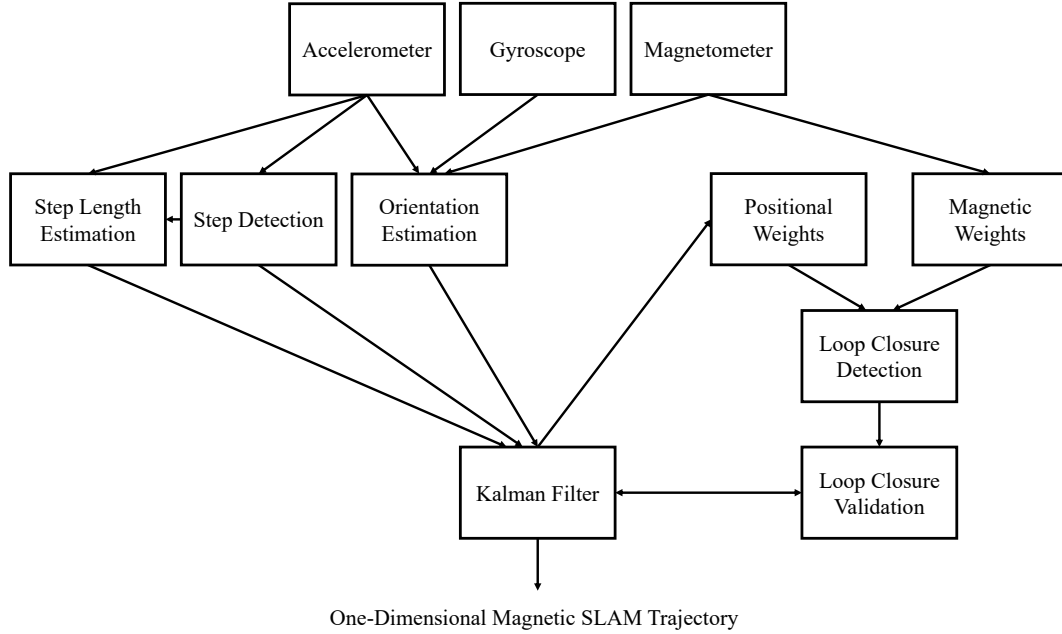


Figure 3-1: Block scheme of one-dimensional magnetic-field SLAM using handheld smartphone PDR detailing all its main components.

3-2 One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR

We are interested in jointly estimating the pedestrian trajectory $\{\hat{p}_{k|k}\}_{k=0}^{N_k}$ and the landmark locations $\{\hat{l}_{i,k|k}\}_{i=1}^M$ over the interval $k = 0, \dots, N_k$. The trajectory is obtained by recursively estimating the current pedestrian position $\hat{p}_{k|k}$ at each time step k and storing the sequence.

The pedestrian position at step k is denoted by $p_k \in \mathbb{R}^2$. To mitigate the drift that accumulates in PDR, revisiting information is incorporated through loop closures. A loop closure relates pedestrian positions at two different time instants that correspond to (approximately) the same physical location (e.g., p_k and $p_{k-\ell}$). Following [21], each detected loop closure is represented by introducing an artificial landmark state $l_{i,k} \in \mathbb{R}^2$, where i denotes the landmark index. Each loop closure is thus associated with a landmark that represents the revisited location and provides a constraint to correct the trajectory.

The augmented state x_k at step k is defined as the concatenation of the current pedestrian position and all landmarks

$$x_k = [p_k^T, l_{1,k}^T, \dots, l_{M,k}^T]^T, \quad (3-1)$$

where M denotes the number of detected loop closures and thus the number of introduced landmarks up to step k . The state dimension, therefore, increases whenever a new loop closure is detected.

In the dynamical model, the state transition of the pedestrian position p_k follows the PDR motion model in (2-2). All previously introduced landmark locations are assumed static and are therefore modeled as

$$l_{i,k} = l_{i,k-1}. \quad (3-2)$$

The measurement model incorporates information from magnetic loop-closure detection. When a loop closure is declared between step k and a previous step $k - \ell$. The i th loop closure models the pedestrian positions p_k and $p_{k-\ell}$ as (approximately) equal to the corresponding landmark location l_i as

$$p_k = l_{i,k} + e_{lc,k}, \quad (3-3a)$$

$$p_{k-\ell} = l_{i,k-\ell} + e_{lc,k-\ell}, \quad (3-3b)$$

where $e_{lc,k}$ denotes the loop-closure measurement noise, modeled as independent, zero-mean Gaussian, $e_{lc,k} \sim \mathcal{N}(0, \sigma_{lc}^2 I_2)$, with σ_{lc}^2 the corresponding noise variance. Although the landmark location is constant, i.e., $l_{i,k} = l_{i,k-\ell}$, the notation $l_{i,k}$ and $l_{i,k-\ell}$ is retained to preserve the Markov property of the system.

Before running the KF, we apply the PDR described in Section 2-1 offline to compute the sequence of time steps $k = 1, 2, \dots, N_k$, the corresponding step lengths $\{s_k\}_{k=1}^{N_k}$, and headings $\{\phi_k\}_{k=1}^{N_k}$ and store the resulting trajectory. We do not include the pedestrian heading in the state x_k ; instead, we treat the step length s_k and heading ϕ_k as known inputs obtained from PDR. This choice preserves the linearity of the state-transition model.

We initialize at $k = 0$ then run a KF to recursively estimate the pedestrian position $\hat{p}_{k|k}$ and landmark locations $\hat{l}_{i,k|k}$ for $k = 1, 2, \dots, N_k$. We initialize the KF with $M = 0$ and define the initial state $\hat{x}_{0|0}$ and covariance $P_{0|0}$ as

$$\hat{x}_{0|0} = [\hat{p}_{0|0}^T, \hat{l}_{1,0|0}^T, \dots, \hat{l}_{M,0|0}^T]^T \in \mathbb{R}^{2+2M}, \quad (3-4a)$$

$$P_{0|0} = \text{blkdiag}(P_{p,0}, \underbrace{P_{l,0}, \dots, P_{l,0}}_M) \in \mathbb{R}^{(2+2M) \times (2+2M)}. \quad (3-4b)$$

If a loop closure is accepted at time k , we augment the state by introducing a new landmark and set $M \leftarrow M + 1$; the KF is then rerun from $\tau = 0$ to $\tau = k$ with the augmented initialization (3-4). We set $\hat{p}_{0|0} = (0 \ 0)^T$ and, when landmarks are present, $\hat{l}_{i,0|0} = (0 \ 0)^T$ for $i = 1, \dots, M$. The initial position covariance $P_{p,0} = 10^{-8} I_2 \text{ m}^2$ reflects the uncertainty in the chosen initial pedestrian position and accounts for the unobservability of the algorithm. We use this tight initial position covariance to anchor the origin at the initial position. The initial landmark covariance $P_{l,0} = 10^4 I_2 \text{ m}^2$ is set large to represent the high initial uncertainty of a newly introduced landmark.

For $k = 1, 2, \dots, N_k$, we perform the KF time update to obtain the predicted state estimate $\hat{x}_{k|k-1}$ and covariance $P_{k|k-1}$ as

$$\hat{x}_{k|k-1} = \begin{bmatrix} \hat{p}_{k|k-1} \\ \hat{l}_{1,k|k-1} \\ \vdots \\ \hat{l}_{M,k|k-1} \end{bmatrix} = \begin{bmatrix} \hat{p}_{k-1|k-1} + s_k \begin{bmatrix} \cos \phi_k & \sin \phi_k \end{bmatrix}^T \\ \hat{l}_{1,k-1|k-1} \\ \vdots \\ \hat{l}_{M,k-1|k-1} \end{bmatrix}, \quad (3-5a)$$

$$P_{k|k-1} = P_{k-1|k-1} + G_k Q G_k^T. \quad (3-5b)$$

Here, $\hat{x}_{k-1|k-1}$ and $P_{k-1|k-1}$ denote the posterior state estimate and covariance at step $k-1$. The propagation of $\hat{p}_{k|k-1}$ follows the PDR motion model in (2-2), while the landmark states are propagated according to (3-2) for all $i = 1, \dots, M$. If $M = 0$, no landmark states are present and only the pedestrian position is propagated. The matrix Q models the process noise acting on the pedestrian position, and G_k maps this noise to the full augmented state. They are defined as

$$Q = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_p^2 \end{bmatrix}, \quad G_k = \begin{bmatrix} I_2 \\ 0_{2M \times 2} \end{bmatrix}, \quad (3-6)$$

where σ_p^2 denotes the variance of the process noise.

At each time step k , we compute the loop-closure detection weights described in Section 3-3 for $j = 1, \dots, k - N_{\text{lag}}$. A loop closure is declared when the weights exceed a threshold and the validation criteria in Section 3-4 are satisfied. The time update in (3-5) is applied at every step k , after which the loop-closure criteria are evaluated until a loop closure is detected.

When a loop closure is detected between the time steps k and $k - \ell$, we augment the state with a new landmark l_{M+1} . To accommodate the increased state dimension, we augment the initial state $\hat{x}_{0|0}$ and covariance $P_{0|0}$ in (3-4). The KF is then rerun from $\tau = 0$ to $\tau = k$ using the augmented state. This rerun applies all previously accepted loop-closure updates at their respective time instants, and additionally includes the two measurement updates associated with the new landmark at $\tau = k - \ell$ and $\tau = k$.

At each loop-closure update step τ , the KF incorporates the measurement and corrects the prior state estimate $\hat{x}_{\tau|\tau-1}$ and covariance $P_{\tau|\tau-1}$ according to

$$\hat{x}_{\tau|\tau} = \hat{x}_{\tau|\tau-1} + K_{\tau} \nu_{\tau}, \quad (3-7a)$$

$$P_{\tau|\tau} = (I - K_{\tau} H_{\tau}) P_{\tau|\tau-1} (I - K_{\tau} H_{\tau})^{\top} + K_{\tau} R K_{\tau}^{\top}. \quad (3-7b)$$

Here, the innovation ν_τ , its covariance S_τ , and the Kalman gain K_τ are given by

$$\nu_\tau = z_\tau - H_\tau \hat{x}_{\tau|\tau-1}, \quad (3-8a)$$

$$S_\tau = H_\tau P_{\tau|\tau-1} H_\tau^\top + R, \quad (3-8b)$$

$$K_\tau = P_{\tau|\tau-1} H_\tau^\top S_\tau^{-1}. \quad (3-8c)$$

The measurement update is based on the loop-closure model in (3-3), which can be rewritten as

$$p_\tau - l_{i,\tau} = e_{lc,\tau}, \quad e_{lc,\tau} \sim \mathcal{N}(0, R), \quad R = \sigma_{lc}^2 I_2. \quad (3-9)$$

This relation can be expressed in the standard linear KF form

$$z_\tau = H_\tau x_\tau + e_{lc,\tau}. \quad (3-10)$$

Since a valid loop closure implies that $p_\tau - l_{i,\tau} \approx 0$, we use the pseudo-measurement

$$z_\tau = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (3-11)$$

and construct the measurement matrix H_τ such that it extracts the difference between the pedestrian position and the landmark from the augmented state

$$H_\tau x_\tau = p_\tau - l_{i,\tau}. \quad (3-12)$$

With the state x_τ defined as in (3-1) (with k replaced by τ), the matrix $H_\tau \in \mathbb{R}^{2 \times (2+2M)}$ has the block structure

$$H_\tau = [H_\tau^{(1)} \ H_\tau^{(2)} \ \dots \ H_\tau^{(M+1)}], \quad (3-13)$$

where each block $H_\tau^{(b)} \in \mathbb{R}^{2 \times 2}$ is defined as

$$H_\tau^{(b)} = \begin{cases} I_2, & b = 1 \quad (\text{pedestrian position}), \\ -I_2, & b = i + 1 \quad (\text{landmark } i), \\ 0_{2 \times 2}, & \text{otherwise.} \end{cases} \quad (3-14)$$

The measurement noise covariance R determines how strongly the filter enforces the constraint $p_\tau - l_{i,\tau} \approx 0$ in the measurement update. A smaller loop-closure variance σ_{lc}^2 (i.e., a smaller R) gives a higher weight to the pseudo-measurement and pulls the posterior estimates $\hat{p}_{\tau|\tau}$ and $\hat{l}_{i,\tau|\tau}$ closer together, while a larger σ_{lc}^2 weakens this coupling.

In summary, the KF propagates the pedestrian position using PDR odometry and incorporates accepted loop closures as pseudo-measurements that couple the pedestrian position to landmark states. The overall algorithm is summarized in Algorithm 2.

Algorithm 2 One-Dimensional Magnetic-Field SLAM Using Handheld Smartphone PDR

Require: Inertial Measurement Unit (IMU) measurements $\{y_{a,t}, y_{\omega,t}, y_{m,t}\}_{t=1}^{N_t}$
parameters $\sigma_p^2, \sigma_{lc}^2, \sigma_m^2, \gamma, N_{lc}, N_{lag}, N_{dis}, \gamma_{mag}, \gamma_{ml}$
initial covariances $P_{p,0}, P_{l,0}$

Ensure: Estimated pedestrian trajectory $\{\hat{p}_{k|k}\}_{k=0}^{N_k}$ and landmark estimates $\{\hat{l}_{i,N_k|N_k}\}_{i=1}^M$

- 1: Run PDR Algorithm 1 on IMU measurements $\{y_{a,t}, y_{\omega,t}, y_{m,t}\}_{t=1}^{N_t}$ to compute step time instants $\{t_k\}_{k=1}^{N_k}$, step lengths $\{s_k\}_{k=1}^{N_k}$, and headings $\{\phi_k\}_{k=1}^{N_k}$.
- 2: Set the number of landmarks $M = 0$
- 3: Initialize $\hat{x}_{0|0}$ and $P_{0|0}$ as in (3-4)
- 4: **for** $k = 1, 2, \dots, N_k$ **do**
- 5: Update $\hat{x}_{k|k-1}$ and $P_{k|k-1}$ using the KF time update (3-5)
- 6: Compute the weights using (3-15) for $j = 1, \dots, k - N_{lag}$
- 7: A loop closure is detected if (3-16) holds and if the conditions in Sections 3-4-1 and 3-4-2 hold
- 8: **if** a loop closure is detected **then**
- 9: Let $j^* \leftarrow \arg \max_j w(k, j)$
- 10: Augment the state vector with a new landmark l_{M+1}
- 11: Rerun the filter from step 3 with $M + 1$ from $\tau = 0$ to $\tau = k$ using the augmented state and covariance, applying measurement updates (3-7) at $\tau = j^*$ and $\tau = k$ for the new landmark (and all previously accepted loop closures).
- 12: Reject the new loop closure if the marginal likelihood validation in (3-30) holds
- 13: **end if**
- 14: **end for**

3-3 Loop-Closure Detection Methods

Loop-closure detection aims to identify revisited locations by matching magnetic-field measurements collected at different time instants [12]. For magnetic matching, we use the magnetometer measurements $y_{m,t}$ defined in (2-27). The magnetometer readings are expressed in the sensor frame. Since the smartphone is held in a fixed orientation, we assume that the sensor frame remains aligned with the body frame, and we therefore use the measurements directly in body-frame coordinates.

A loop closure is detected when the magnetic field observed at the current time is sufficiently similar to magnetic-field measurements recorded previously [21]. Indoor magnetic fields are often spatially distinctive due to local disturbances, which makes them useful for recognizing revisited locations [1]. However, the indoor magnetic field does not provide a perfectly unique fingerprint. Magnetometer measurements are affected by sensor noise and bias, and similar magnetic field values may occur at different locations, leading to limited discernibility [28]. Therefore, we need robust loop-closure detection methods with effective loop closure validation.

To perform magnetic matching, we must specify how measurements are represented and compared to one another. Each measurement is represented by the three-dimensional magnetic-field vector, which retains more information than the magnetic-field magnitude alone [12]. Moreover, we compare sequences of the magnetic field rather than individual samples, since sequence-based matching exploits the more distinctive information contained in magnetic field patterns [18]. Using three-dimensional vectors together with sequence matching thus provides a more discriminative basis for detecting loop closures.

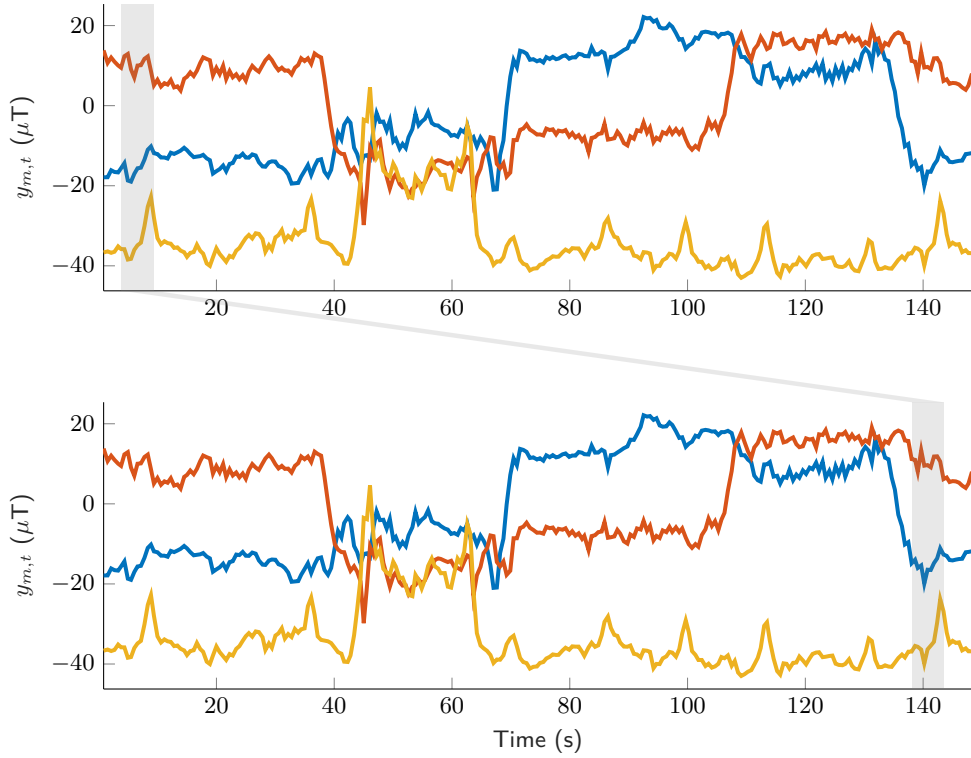


Figure 3-2: Magnetometer measurements for the walk described in Section 4-1-1, shown up to the first detected loop closure. The three magnetic field components are plotted over time. The shaded gray windows indicate the two time intervals that are matched by the magnetic sequence-matching and are identified as a loop closure.

Figure 3-2, inspired by [21], shows the three-axis magnetometer measurements from the walk described in Section 4-1-1. For clarity, the measurements are shown only up to the first detected loop closure. The figure illustrates the principle of magnetic sequence matching: the shaded gray windows indicate the two time intervals that are identified as a loop closure, i.e., intervals for which the measured magnetic-field sequences are sufficiently similar. The most recent time instants of the two intervals are taken as the corresponding matching times.

For sequence-based detection, one magnetic sequence is held fixed while the other is shifted forward and backward along the pedestrian trajectory. For each candidate match between the current step k and a past step j , a magnetic similarity weight $w_m(k, j)$ is computed. In addition, we exploit the KF position estimate and covariance to quantify how likely it is that the current location coincides with a past one, resulting in a positional weight $w_p(k, j)$ [21].

Magnetic and positional information are combined into a single weight

$$w(k, j) = \max(w_m^{\text{fwd}}(k, j) w_p(k, j), w_m^{\text{bwd}}(k, j) w_p(k, j)), \quad (3-15)$$

that indicates the likelihood that the pedestrian revisits at step k the location visited at step

j . A loop closure is declared whenever

$$\max_j w(k, j) > \gamma. \quad (3-16)$$

Here, γ denotes the detection threshold. The weights $w_m^{\text{fwd}}(k, j)$ and $w_m^{\text{bwd}}(k, j)$ represent magnetic similarity for traversals in the same direction and in opposite directions, respectively.

In the following sections, we introduce four strategies to compute the magnetic weights. Section 3-3-1 presents two magnetic weight formulations: the first follows [21], while the second replaces the full-rate magnetometer sequence by a step-based magnetic sequence, following [46]. Section 3-3-2 introduces Dynamic Time Warping (DTW) and applies it to both magnetic weight formulations. This results in four magnetic weight definitions: Sample Magnetic Gaussian Weights (Sample-MGW), Step Magnetic Gaussian Weights (Step-MGW), Sample Magnetic Gaussian Dynamic Time Warping (Sample-MGDTW), and Step Magnetic Gaussian Dynamic Time Warping (Step-MGDTW), which are described in their respective sections. Finally, the positional weight is introduced in Section 3-3-3.

3-3-1 Magnetic Gaussian Weights

The study [21] introduces Magnetic Gaussian Weights (MGW) to quantify the similarity between two sequences of magnetometer measurements for probabilistic loop-closure detection. This section introduces the corresponding magnetic similarity weights. In addition to the original formulation based on full-rate magnetometer samples, we consider a step-based variant that constructs the magnetic sequences at detected footstep instants.

To define these weights, we consider two discrete time series: the footstep sequence indexed by $k = 1, \dots, N_k$ and the magnetometer sample sequence indexed by $t = 1, \dots, N_t$, acquired at 50 Hz. Let t_k denote the magnetometer sample index t at which the k th step occurs; accordingly, t_{k-n} denotes the sample index of step $(k - n)$. We consider two windowing strategies that differ only in how the magnetic sequences are constructed. In Sample-MGW, the sequences are defined on the full-rate sample index t and contain N_{lc} consecutive magnetometer samples. In Step-MGW, the sequences are defined on the step index k and contain N_{lc} samples taken at footstep instants, i.e., one magnetometer sample per step at t_k . For the same N_{lc} , the step-based window spans a longer time interval because footsteps are sparser in time than the raw magnetometer samples.

Sample-MGW. For a candidate loop closure between footsteps (k, j) with $1 \leq j < k$, we assign a magnetic similarity weight by comparing two local magnetic sequences of length N_{lc} . The weight is computed as a product of N_{lc} Gaussian terms that penalize the squared Euclidean distance between pairs of three-dimensional magnetometer vectors. In the sample-based variant, the sequences are extracted from the full-rate magnetometer signal (indexed by t) using consecutive sample indices around t_j and t_k , i.e., the pairs $(t_j - n, t_k - n)$ for $n = 0, \dots, N_{\text{lc}} - 1$. The forward (same travel direction) and backward (opposite travel direction)

weights are defined as

$$w_m^{\text{fwd}}(k, j) = \prod_{n=0}^{N_{\text{lc}}-1} \exp\left(-\frac{1}{12\sigma_m^2} \|y_{m,t_j-n} - y_{m,t_k-n}\|_2^2\right), \quad (3-17a)$$

$$w_m^{\text{bwd}}(k, j) = \prod_{n=0}^{N_{\text{lc}}-1} \exp\left(-\frac{1}{12\sigma_m^2} \|y_{m,t_j+N_{\text{lc}}-1-n} - \tilde{R}(\pi) y_{m,t_k-n}\|_2^2\right), \quad (3-17b)$$

$y_{m,t} \in \mathbb{R}^3$ denotes the three-axis body-frame magnetometer measurement and $\|\cdot\|_2$ the Euclidean norm. The backward weights include $\tilde{R}(\pi) \in \mathbb{R}^{3 \times 3}$, a fixed 180° rotation about the body z -axis to account for traversing the same path in the opposite direction. The parameter σ_m^2 denotes the per-axis magnetometer noise variance. The factor $1/(12\sigma_m^2)$ follows from the assumed zero-mean Gaussian noise model: the difference of two independent three-dimensional measurements has variance $2\sigma_m^2$ per axis, and the squared three-dimensional norm sums the contributions across the three axes. Consequently, matched sequences yield weights close to one, whereas mismatched sequences produce exponentially small weights.

Step-MGW. The step-based formulation uses the same Gaussian product structure, but forms sequences only at detected footstep instants. Compared to Sample-MGW, the only difference is the indexing: consecutive offsets correspond to previous steps rather than previous magnetometer samples. Concretely, we use the magnetometer measurements at indices (t_{j-n}, t_{k-n}) for $n = 0, \dots, N_{\text{lc}} - 1$. The corresponding forward (same travel direction) and backward (opposite travel direction) weights are

$$w_{m,\text{step}}^{\text{fwd}}(k, j) = \prod_{n=0}^{N_{\text{lc}}-1} \exp\left(-\frac{1}{12\sigma_m^2} \|y_{m,t_{j-n}} - y_{m,t_{k-n}}\|_2^2\right), \quad (3-18a)$$

$$w_{m,\text{step}}^{\text{bwd}}(k, j) = \prod_{n=0}^{N_{\text{lc}}-1} \exp\left(-\frac{1}{12\sigma_m^2} \|y_{m,t_{j+N_{\text{lc}}-1-n}} - \tilde{R}(\pi) y_{m,t_{k-n}}\|_2^2\right). \quad (3-18b)$$

3-3-2 Magnetic Gaussian Dynamic Time Warping

We extend MGW by replacing the fixed one-to-one (index-matched) alignment with DTW. In MGW, the alignment is fixed by construction: each element at offset n in the j -window is compared only to the element at the same offset n in the k -window, i.e., the index-matched pairs $(t_j - n, t_k - n)$ in (3-17) for Sample-MGW or (t_{j-n}, t_{k-n}) in (3-18) for Step-MGW. DTW aligns two sequences by allowing local stretching and compression in time, which makes it robust to variations in walking speed [12].

As in Section 3-3-1, we assume that each three-axis body-frame magnetometer measurement $y_{m,t} \in \mathbb{R}^3$ is corrupted by independent, zero-mean Gaussian noise with per-axis variance σ_m^2 . Accordingly, we use the same Gaussian kernel with factor $1/(12\sigma_m^2)$. We consider both sample-based and step-based variants that combine this Gaussian noise model with DTW.

Sample-MGDTW. As in MGW, we first construct pairwise squared-distance matrices between the two sequences. Let $p, q = 0, \dots, N_{lc} - 1$ index offsets within the two windows, with $p = 0$ and $q = 0$ anchored at the respective footstep instants. The forward and backward squared-distance matrices are defined as

$$d_{k,j}^{\text{fwd}}(p, q) = \|y_{m,t_j-p} - y_{m,t_k-q}\|_2^2, \quad (3-19a)$$

$$d_{k,j}^{\text{bwd}}(p, q) = \|y_{m,t_j+N_{lc}-1-p} - \tilde{R}(\pi) y_{m,t_k-q}\|_2^2, \quad (3-19b)$$

where the backward distance reverses the j -window in time and applies a 180° rotation to the k -window to account for opposite-direction traversals.

Following [50] for the use of DTW and [26] for the DTW formulation, given a distance matrix $d_{k,j} \in \{d_{k,j}^{\text{fwd}}, d_{k,j}^{\text{bwd}}\}$, DTW determines an alignment between the two sequences by finding a minimum-cost warping path through this matrix. Each entry $d_{k,j}(p, q)$ is the mismatch cost of pairing the p th element of the j -window with the q th element of the k -window. The cumulative cost $g_{k,j}(p, q)$ is defined as the minimum total cost over all monotone paths from $(0, 0)$ to (p, q) , where each step can move right, up, or diagonally. This yields the recursion

$$g_{k,j}(p, q) = d_{k,j}(p, q) + \min\{g_{k,j}(p-1, q), g_{k,j}(p, q-1), g_{k,j}(p-1, q-1)\}, \quad (3-20)$$

with boundary conditions

$$g_{k,j}(0, 0) = d_{k,j}(0, 0), \quad (3-21a)$$

$$g_{k,j}(p, 0) = d_{k,j}(p, 0) + g_{k,j}(p-1, 0), \quad p = 1, \dots, N_{lc} - 1, \quad (3-21b)$$

$$g_{k,j}(0, q) = d_{k,j}(0, q) + g_{k,j}(0, q-1), \quad q = 1, \dots, N_{lc} - 1. \quad (3-21c)$$

The DTW distance is the terminal cumulative cost,

$$\text{DTW}_{\text{fwd}}(k, j) = g_{k,j}(N_{lc} - 1, N_{lc} - 1), \quad (3-22)$$

when using $d_{k,j}^{\text{fwd}}$, and analogously $\text{DTW}_{\text{bwd}}(k, j)$ when using $d_{k,j}^{\text{bwd}}$. Substituting into the Gaussian kernel gives the magnetic forward and backward weights

$$w_{\text{m}}^{\text{fwd},\text{dtw}}(k, j) = \exp\left(-\frac{1}{12\sigma_m^2} \text{DTW}_{\text{fwd}}(k, j)\right), \quad (3-23a)$$

$$w_{\text{m}}^{\text{bwd},\text{dtw}}(k, j) = \exp\left(-\frac{1}{12\sigma_m^2} \text{DTW}_{\text{bwd}}(k, j)\right). \quad (3-23b)$$

Step-MGDTW. In the step-based variant, the sequences are downsampled to footstep instants, such that p and q index step offsets rather than sample offsets. The corresponding forward and backward distance matrices are

$$d_{\text{step},k,j}^{\text{fwd}}(p, q) = \|y_{m,t_j-p} - y_{m,t_k-q}\|_2^2, \quad (3-24a)$$

$$d_{\text{step},k,j}^{\text{bwd}}(p, q) = \|y_{m,t_j+N_{lc}-1-p} - \tilde{R}(\pi) y_{m,t_k-q}\|_2^2, \quad (3-24b)$$

and the same DTW recursion and boundary conditions as in (3-20)–(3-21) apply (with $d_{k,j}$ replaced by $d_{\text{step},k,j}$). This yields the step-based DTW distances $\text{DTW}_{\text{fwd}}^{\text{step}}(k, j)$ and $\text{DTW}_{\text{bwd}}^{\text{step}}(k, j)$, and the corresponding weights

$$w_{\text{m,step}}^{\text{fwd,dtw}}(k, j) = \exp\left(-\frac{1}{12\sigma_m^2} \text{DTW}_{\text{fwd}}^{\text{step}}(k, j)\right), \quad (3-25a)$$

$$w_{\text{m,step}}^{\text{bwd,dtw}}(k, j) = \exp\left(-\frac{1}{12\sigma_m^2} \text{DTW}_{\text{bwd}}^{\text{step}}(k, j)\right). \quad (3-25b)$$

3-3-3 Positional Weights

Since the indoor magnetic field does not provide a perfectly unique fingerprint, the magnetic weights are complemented by positional weights. The positional weights $w_p(k, j)$ in (3-15) are adopted from [21] and quantify how likely the current position coincides with a past estimate using the KF covariance. At each time step k , the KF provides a position estimate $\hat{p}_{k|k}$ together with its covariance $P_{p,k|k}$. For any $j < k$, the positional weight is defined as

$$w_p(k, j) = \exp\left(-\frac{1}{8\sigma_{w_p}^2} \|\hat{p}_{k|k} - \hat{p}_{j|j}\|_2^2\right), \quad j < k, \quad (3-26)$$

where σ_{w_p} is taken as the mean of the square roots of the diagonal elements of $P_{p,k|k}$. The form of the weight follows from treating the estimation error as Gaussian. To keep the computation lightweight, we assume that the errors in $\hat{p}_{k|k}$ and $\hat{p}_{j|j}$ are independent and that their individual components are uncorrelated.

3-4 Loop-Closure Validation Methods

Loop-closure detection can generate many plausible candidates, but it can also detect false loop closures. Reliably identifying true revisits remains challenging. False loop closures introduce erroneous constraints, which can degrade the position estimates [52]. We therefore apply loop-closure validation to screen detected loop closures.

This section adopts the loop-closure validation strategy of [21]. In particular, three rejection criteria are considered: temporal constraints, magnetic field excitation, and marginal likelihood.

3-4-1 Temporal Constraints

To keep the computational complexity of the filtering process manageable, we use a selective strategy for incorporating loop closures. In particular, a detected loop closure is considered for inclusion only if it is separated by at least N_{dis} footsteps from the most recently accepted loop closure. This constraint follows [21] and limits the number of incorporated loop closures. Thereby, it preserves computational efficiency.

In addition, we introduce a lag parameter N_{lag} to prevent the filter from closing loops over recent time steps. Due to the weighting design, the weights tend to be disproportionately large for candidate matches close to the current time, which increases the risk of false loop closures in this region. Incorporating such false positives can degrade performance unless the user is truly stationary.

3-4-2 Magnetic-Field Excitation

If the magnetic field is locally homogeneous, the magnetic sequences within the loop-closure window are insufficiently distinctive. In such segments, the magnetic weights can become uniformly high across many candidate matches, increasing the risk of spurious loop closures. To reduce this effect, we impose a minimum excitation condition on the most recent window of magnetic measurements, following [21]. Since the magnetometer output is three-dimensional, the $\max(\cdot)$ and $\min(\cdot)$ operators below are applied componentwise over the window.

For the sample-based methods, with t the 50 Hz sample index and t_k the sample index at which step k occurs, we require

$$\left\| \max_{0 \leq n \leq N_{\text{lc}}-1} y_{m,t_k-n} - \min_{0 \leq n \leq N_{\text{lc}}-1} y_{m,t_k-n} \right\|_2 > \gamma_{\text{mag}}, \quad (3-27)$$

otherwise the candidate loop closure is rejected.

For the step-based methods, the excitation is evaluated on the downsampled sequence at footstep instants, and we require

$$\left\| \max_{0 \leq n \leq N_{\text{lc}}-1} y_{m,t_k-n} - \min_{0 \leq n \leq N_{\text{lc}}-1} y_{m,t_k-n} \right\|_2 > \gamma_{\text{mag}}, \quad (3-28)$$

otherwise the candidate loop closure is rejected. This ensures that loop closures are considered only when the magnetic signal exhibits sufficient variation within the corresponding window.

3-4-3 Marginal Likelihood

Even when the magnetic excitation exceeds γ_{mag} , false loop closures may still occur. Following [21], we therefore introduce a marginal-likelihood validation weight that evaluates how consistent a candidate loop-closure constraint is with the KF uncertainty. At a candidate loop-closure step k , we compute the marginal likelihood of the pseudo-measurement z_k under the KF measurement-update model,

$$p(z_k) = \frac{1}{2\pi \sqrt{|S_k|}} \exp\left(-\frac{1}{2} \nu_k^T S_k^{-1} \nu_k\right), \quad (3-29)$$

where ν_k is the innovation and S_k its covariance. The loop closure is rejected if

$$p(z_k) < \gamma_{\text{ml}}, \quad (3-30)$$

such that only loop closures with sufficiently high likelihood are retained.

Experimental Results

This chapter presents the experimental results. Section 4-1 introduces the datasets and evaluation methods. Overall, the sections are structured according to the three research questions. Section 4-2 assesses parameter tuning under standard conditions and addresses the first subquestion. Section 4-3 examines how Simultaneous Localization and Mapping (SLAM) addresses limitations of Pedestrian Dead Reckoning (PDR) and answers the second subquestion. Finally, Section 4-4 compares the four loop-closure detection methods in both standard and PDR failure mode scenarios, thereby answering the third subquestion.

4-1 Experiment Evaluation Framework

This section presents the three datasets and evaluation metrics used to compare the performance of one-dimensional magnetic-field SLAM with handheld smartphone PDR, introduced in the previous chapter. For readability, the remainder of this chapter refers to this method simply as SLAM.

4-1-1 Datasets

We collected three datasets by conducting walks inside the Physics building at TU Delft. In each trial, the pedestrian held the smartphone in a fixed orientation, with the screen facing upward and the top pointing in the walking direction, while the Phyphox app recorded the sensor data at 50 Hz. The first dataset represents a standard walk at a constant speed, the second captures walks with varying speeds, and the third includes a strong magnetic disturbance during one instance. Table 4-1 shows the total walking distance for each dataset and the part of the trajectory that was revisited. The revisited part represents where SLAM has the opportunity to detect loop closures. In this work, about 50% of the trajectory was revisited, unlike related work [46, 12], where almost the entire path was revisited. This setup allows testing SLAM under more challenging conditions, due to the presence of PDR failure modes and the limited portion of the trajectory where loop closures can be detected.

Dataset	Total (m)	Revisited (m)	% Revisited
Standard walk	421	216	51
Varying-speed walk	514	259	50
Large magnetic-disturbance walk	516	262	51

Table 4-1: Ground-truth trajectory lengths and revisited portions for the three evaluated datasets.

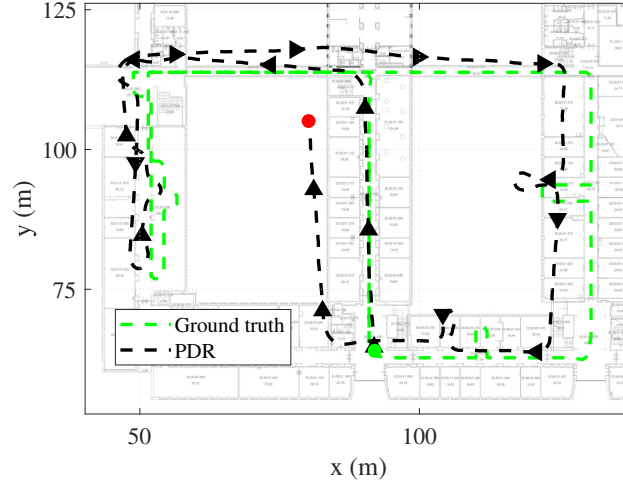


Figure 4-1: The PDR and ground-truth trajectories during the standard walk, overlaid on the map of the building. The plot shows how PDR drifts away from the ground truth over time.

Standard Walk

The first dataset represents a standard walk through the Physics building. The trajectory primarily follows corridors, with occasional segments inside rooms. The pedestrian maintains a constant walking speed throughout the trial. The total walking distance is 421 m, and 51% of the trajectory is revisited. PDR trajectory accumulates substantial position drift over the course of the walk, resulting in a last-step error of approximately 11 m as shown in Figure 4-1.

Varying-Speed Walk

The second dataset is a walk with varying speeds inside the Physics building. During this experiment, the pedestrian initially walks at the same pace as in the first dataset. At the vertical segment around $x \approx 90$, the walking speed increases and remains higher along the long horizontal corridor at $y \approx 115$. This change is illustrated in Figure 4-2b, where the higher-speed section is highlighted in green. Since the Weinberg constant K in the step-length model in (2-11) is calibrated for the original walking speed, PDR underestimates the step length when the pedestrian moves faster. Consequently, the computed step lengths are systematically shorter than the actual ones. Figure 4-2a illustrates how PDR trajectory starts the vertical downward segment near $x \approx 190$ m—significantly earlier than the ground truth, which begins near $x \approx 210$ m—and overshoots at the end of the walk. The total walking

distance is 514 m, and the pedestrian revisits approximately 50% of the trajectory.

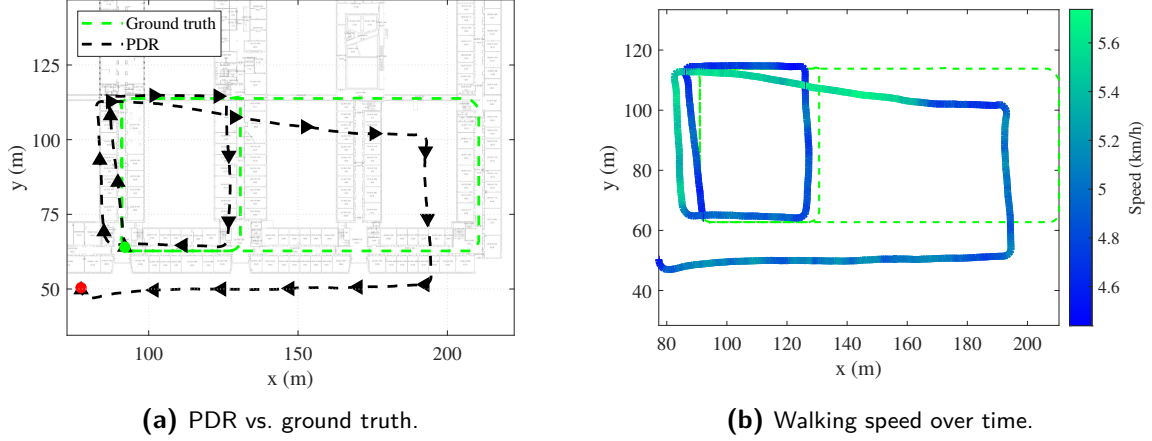


Figure 4-2: Analysis of the varying walking speed dataset. (a) PDR and ground-truth trajectories for the varying-speed walk, overlaid on the building map. The PDR trajectory underestimates the step length, causing the estimate to drift away from the ground truth over time. (b) Stepwise speed indicator over time for the varying-speed walk, showing an increase along the straight corridors $x \approx 90$ and $y \approx 115$.

The stepwise walking speed is approximated from the estimated step length s_k and the inter-step time $\Delta t_k := t_k - t_{k-1}$ as

$$v_k \approx \frac{s_k}{\Delta t_k}, \quad (4-1a)$$

$$v_k^{\text{km/h}} := 3.6 v_k. \quad (4-1b)$$

Since s_k is obtained from the Weinberg step-length model, v_k should be interpreted as an indicator of relative speed changes rather than an accurate absolute speed estimate. Consequently, the true walking speed is expected to be higher than the values reported in Figure 4-2b.

Large Magnetic-Disturbance Walk

The third dataset represents a walk at constant speed through the Physics building, characterized by a peak in the magnetic field measurements. During the second pass near the point $(x \approx 100, y \approx 115)$, the pedestrian deviates slightly toward a magnetic beacon, a steel doorknob, which generates a pronounced spike in the measured magnetic field (Figure 4-3b, yellow marker). This magnetic anomaly corrupts the heading estimate produced by the Versatile Quaternion-based Filtering (VQF) within PDR, causing the estimated trajectory to drift away from the ground-truth trajectory. The resulting heading drift is visible in Figure 4-3a, where PDR trajectory deviates vertically downward instead of continuing horizontally along $y \approx 115$. The total walking distance is 516 m, and the participant revisits 51% of the trajectory.

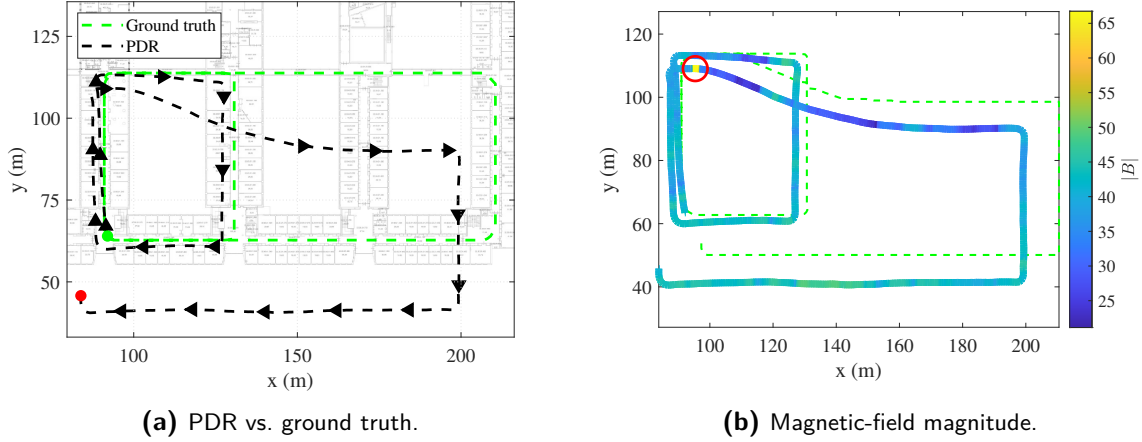


Figure 4-3: Analysis of the large magnetic disturbance walk. (a) The PDR and ground-truth trajectories during the large magnetic-disturbance walk, overlaid on the map of the building. The plot shows the heading drift of PDR due to a large magnetic disturbance near a steel doorknob. (b) Magnetic-field magnitude during the large magnetic-disturbance walk, showing a strong spike (red circle) near the magnetic beacon.

4-1-2 Evaluation Metrics

We evaluate SLAM performance using the per-step position error between the estimated trajectory and the manually refined ground truth. For each footstep k , we define

$$\varepsilon_k = \|p_{k,\text{gt}} - \hat{p}_k\|_2, \quad (4-2)$$

where N_k is the total number of footsteps, $p_{k,\text{gt}}$ denotes the manually refined ground-truth position at step k , and $\hat{p}_k$ is the corresponding SLAM/PDR estimate. Because the ground truth is obtained by manually correcting the PDR trajectory using a floor-plan map (rather than an independent high-precision reference), all reported errors should be interpreted as approximate indicators of performance.

SLAM performance is evaluated using three complementary metrics: Absolute Trajectory Error (ATE), maximum error, and last-step error. We report ATE because its squared formulation increases sensitivity to occasional large deviations, which the mean error can mask. Nevertheless, we also report the mean error for the standard walk to enable comparison with existing literature. The same metrics are used for both standard walking and scenarios that induce PDR failure modes, allowing us to quantify how effectively the proposed loop-closure strategies mitigate drift.

The mean error [4] is defined as the average Euclidean deviation over all N_k footsteps as

$$\text{Mean Error} = \frac{1}{N_k} \sum_{k=1}^{N_k} \varepsilon_k. \quad (4-3)$$

The ATE [37] provides a global measure of accuracy by computing the root-mean-square position error between estimated and ground-truth positions over all N_k footsteps and is

given by

$$\text{ATE} = \sqrt{\frac{1}{N_k} \sum_{k=1}^{N_k} \varepsilon_k^2}. \quad (4-4)$$

The maximum error captures the worst-case deviation encountered at any step and follows

$$\text{Max Error} = \max_{1 \leq k \leq N_k} \varepsilon_k, \quad (4-5)$$

and the last-step error measures the drift remaining at the final footstep and is defined as

$$\text{Last-Step Error} = \varepsilon_{N_k}. \quad (4-6)$$

We evaluate performance using two visualizations. First, reconstructed trajectories with annotated loop-closure events show where SLAM corrects drift. Second, an error-over-time curve plots the per-step position error ε_k from (4-2) at each footstep k , revealing how drift accumulates and how it decreases after accepted loop closures. Throughout this chapter, we use a fixed color scheme for the ground truth and PDR trajectories. From Section 4-3 onwards, the same colors are also used consistently for the SLAM methods. In Section 4-2, some colors are reused for different SLAM methods, since assigning a unique color to each method would reduce contrast and make the figures difficult to interpret.

4-2 Sensitivity of Parameter Tuning

This sensitivity analysis examines how each tuning parameter influences loop-closure detection and overall position estimation. By evaluating these parameters across a range of values, we aim to strike a balance in which as many genuine loop closures as possible are detected, while avoiding false loop closures. The parameters optimized on the standard walk in this section are subsequently applied to all other datasets to assess their generality. In doing so, the analysis not only identifies optimal parameter settings but also delineates the operational boundaries of SLAM, highlighting conditions under which performance may degrade.

Concretely, we evaluate SLAM performance on the standard walk dataset, determine its optimal parameter settings, and use the tuning process to investigate the limits of its reliability. As an illustrative example, the tuning procedure for the matching sequence length N_{lc} is presented for both the step-based and sample-based methods in the following section. A complete overview of all parameter values is provided in Section 4-2-2. We summarise the results in Section 4-2-3. We additionally report the mean error to facilitate comparison with prior work.

4-2-1 Sensitivity of the Loop-Closure Window Length N_{lc}

The parameter N_{lc} controls how many consecutive magnetic field samples are used when comparing magnetic-field sequences for loop-closure detection, as explained in Sections 3-3-1 and 3-3-2. Increasing N_{lc} extends the magnetic sequence over which each candidate loop closure is evaluated. The value of N_{lc} is tuned for the step-based methods using Step Magnetic Gaussian Dynamic Time Warping (Step-MGDTW), and for the sample-based methods using Sample Magnetic Gaussian Dynamic Time Warping (Sample-MGDTW).

Step-Based Loop-Closure Detection Methods

Figure 4-4 shows the effect of varying N_{lc} on Step-MGDTW. We plot the corresponding position error over time in Figure 4-4a and mark accepted loop closures. We compare the estimated SLAM and PDR trajectories against the ground truth in Figure 4-4b. Figure 4-1 indicates the walking direction.

In our experiments, varying the loop closure window length N_{lc} directly changes the number of detected closures and, consequently, the overall localization accuracy. Small windows ($N_{lc} = 3$) produce many detections, including false closures, which increase position errors. As shown in Figure 4-5, false loop closures occur near $(x \approx 50, y \approx 85)$ and $(x \approx 110, y \approx 65)$, where the trajectories come close but do not overlap. In contrast, large windows ($N_{lc} = 12$) are too restrictive, missing valid closures and reducing the effectiveness of drift correction. A window length of $N_{lc} = 5$ provides the most balanced performance in this scenario, as shown in Table 4-3, detecting a sufficient number of loop closures while effectively rejecting false matches.

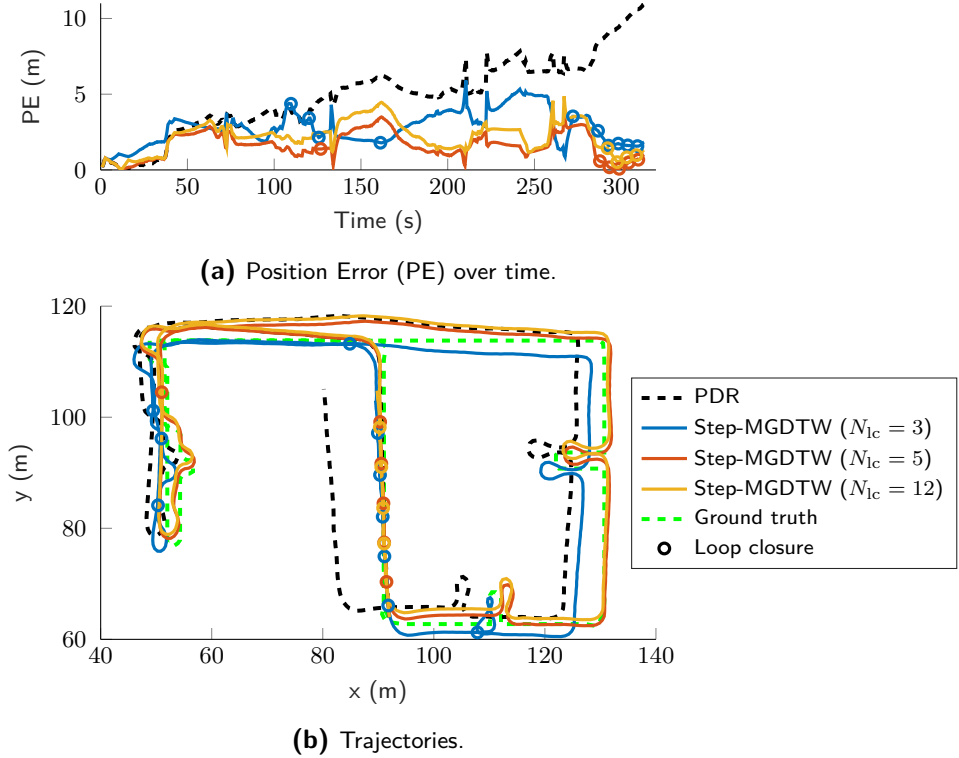


Figure 4-4: Sensitivity analysis for Step-MGDTW on the standard-walk dataset. The cases $N_{lc} = 3$ and $N_{lc} = 12$ are highlighted, as well as the selected setting $N_{lc} = 5$.

Figure 4-6 shows the position error metrics for values of the loop closure window length N_{lc} ranging from 1 to 20. As N_{lc} increases, SLAM accepts fewer closures, except at $N_{lc} = 1$, where the validation step rejects most candidates. Position errors grow large for $N_{lc} < 2$ and again at $N_{lc} = 20$, since SLAM fails to detect any closures. Within the range $N_{lc} = 4$ to 19, the errors remain relatively stable, with both the ATE and last-step error averaging around 2 m and the maximum error around 4 m.

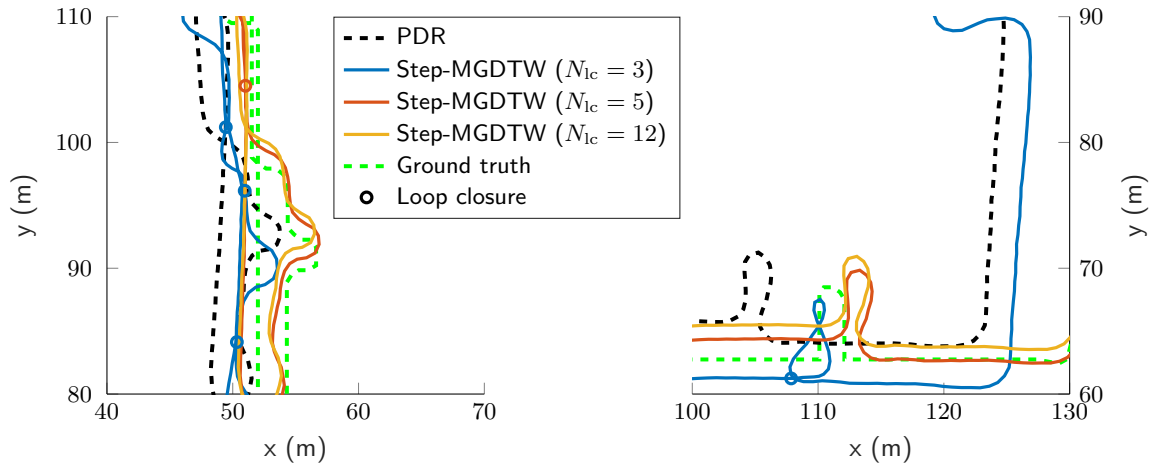


Figure 4-5: Effect of N_{lc} on loop-closure detection on two zoomed segments of the standard walk: false matches for $N_{lc} = 3$, and a valid closure detected at $N_{lc} = 5$ but missed for $N_{lc} = 12$.

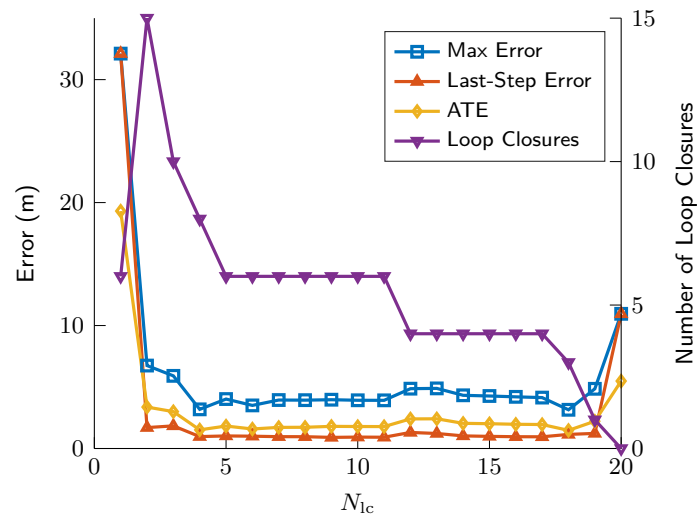


Figure 4-6: Sensitivity analysis on the standard walk showing maximum error, last-step error, ATE, and number of detected loop closures with respect to the loop closure window length N_{lc} for Step-MGDTW.

Sample-Based Loop-Closure Detection Methods

We apply the same sensitivity-based tuning procedure used for the step-based methods to Sample-MGDTW to select N_{lc} . Figure 4-7 shows the resulting trajectories and the corresponding error-over-time curves for three different values of N_{lc} .

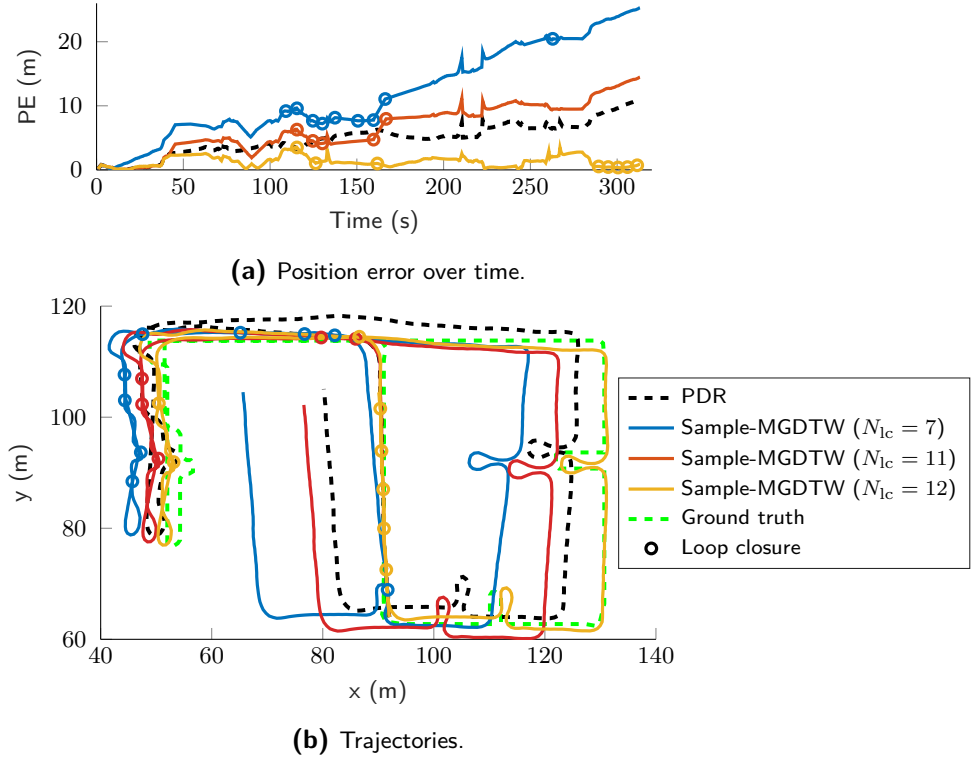


Figure 4-7: Sensitivity analysis for Sample-MGDTW on the standard-walk dataset. The cases $N_{lc} = 7$ and $N_{lc} = 11$ are highlighted, as well as the selected setting $N_{lc} = 12$.

Figure 4-8 shows that the sample-based approach produces larger fluctuations in error across the different N_{lc} values. Increasing N_{lc} consistently reduces the number of detected loop closures for Step-MGDTW. This trend does not hold for Sample-MGDTW, which makes selecting an appropriate value of N_{lc} more challenging. For N_{lc} between 2 and 13, the error remains relatively low. However, clear increases appear at $N_{lc} = 7$ and $N_{lc} = 11$. Figure 4-7 shows that these increases result from false loop closures. These false detections increase the position error and push the trajectory too far away from the true path, which causes future loop closures to be missed. This explains the drop in detected loop closures in Figure 4-8. For $N_{lc} > 13$ the position error grows again because only four loop closures are detected. Based on this analysis, the best choice is $N_{lc} = 12$, which yields the lowest position error. The maximum error is 3.5 m, the last-step error is 0.9 m, and the ATE is 1.5 m as reported in Table 4-3 with eight detected loop closures. Using the selected parameter settings, Sample-MGDTW slightly outperforms Step-MGDTW on the standard-walk dataset, with lower mean, maximum, last-step, and ATE errors.

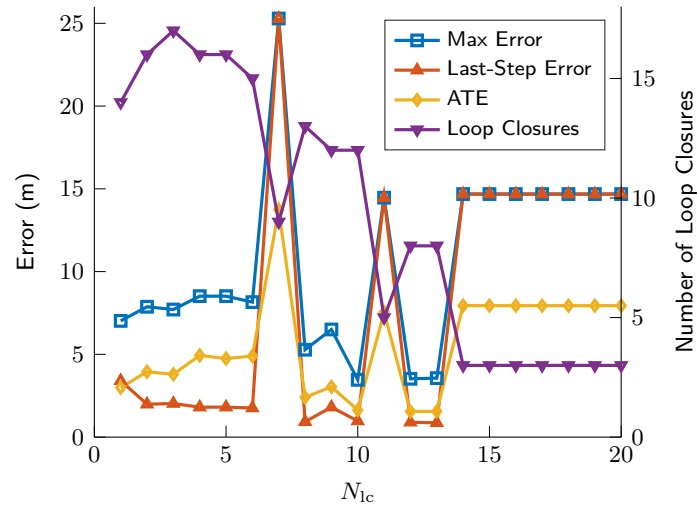


Figure 4-8: Sensitivity analysis on the standard walk showing maximum error, last-step error, ATE, and number of detected loop closures with respect to the loop closure window length N_{lc} for Sample-MGDTW.

4-2-2 Parameters

The approach described in the previous section for tuning the loop-closure window length N_{lc} is also applied to the magnetometer noise standard deviation σ_m , the loop closure weight threshold γ , and the process noise standard deviation σ_p . This leads to the parameter values shown in Table 4-2. Throughout this chapter, these values are used unless stated otherwise.

Parameter	Value	Unit
σ_m	2.50	μT
σ_{lc}	$\sqrt{0.025}$	m
σ_p	$\sqrt{0.05}$	m
γ	0.25	–
γ_{ml}	1×10^{-16}	–
γ_{mag}	2.50	μT
N_{dis}	10	steps
N_{lag}	25	steps
N_{lc} (step)	5	steps
N_{lc} (sample)	12	samples

Table 4-2: List of parameters. For step-based methods, we set $N_{lc} = 5$; for sample-based methods, we set $N_{lc} = 12$.

4-2-3 Summary of the Results on Standard Walk

This section summarizes the results for the two methods on the standard walk using the parameter settings selected during the sensitivity-based tuning procedure. To facilitate comparison with prior work, we additionally report the mean error on the standard walk. The

mean error is 1.6 m for Step-MGDTW and 1.3 m for Sample-MGDTW. Table 4-3 lists these values together with the maximum error. The standard walk is the closest match to the datasets considered in the literature.

Method	Mean Error (m)	Max Error (m)	Last-step Error (m)	ATE (m)
Step-MGDTW	1.6	4.0	1.0	1.8
Sample-MGDTW	1.3	3.5	0.9	1.5

Table 4-3: Mean, maximum, last-step error, and ATE during the standard walk.

4-3 Robustness to PDR Failure Modes

In this section, we examine two failure modes of PDR and demonstrate how SLAM can recover from each, thereby addressing the second subquestion. The previous section analyzed the performance of SLAM under standard conditions; this section analyzes its performance in more challenging scenarios. First, we consider a case in which heading estimates become corrupted by a local magnetic disturbance. Second, we analyze a scenario in which varying walking speeds cause PDR to underestimate the step length. We use the method Step-MGDTW throughout this section.

4-3-1 Heading Drift under Magnetic Disturbance

In this section, we assess the robustness of SLAM on the first PDR failure mode by leveraging the dataset with a large magnetic disturbance introduced in Section 4-1-1.

For this dataset, we start with the parameters from Table 4-2. However, when we apply the parameters identified as optimal for a standard walk in Section 4-2, SLAM fails to recover. By the end of the walk, the pedestrian traverses previously visited areas, but the system does not detect any loop closures to correct this drift, as shown in Figure 4-9b. This behavior is also evident in the error plot of Figure 4-9a, where the line corresponding to the process noise variance $\sigma_p^2 = 0.05$ maintains a position error of about 19 m after 350 s. For $\sigma_p^2 = 0.05$, SLAM only slightly reduces the maximum error, last-step error, and ATE, as shown in Table 4-4.

To address this, we increase the process noise variance on the position, σ_p^2 , from 0.05 to 0.1. In this scenario, heading errors cause the PDR position estimates to become increasingly uncertain over time. To reflect this higher uncertainty in the Kalman Filter (KF) time update, we model it by increasing σ_p^2 . A larger σ_p^2 makes the filter less confident in the predicted position and inflates the process covariance, which in turn makes the positional weight $w_p(k, j)$ in Section 3-3-3 less strict when evaluating candidate loop closures. We did not choose a larger value for the process noise covariance because it would be too permissive and allow false loop closures.

As shown in Figure 4-9a, with $\sigma_p^2 = 0.1$ SLAM successfully detects loop closures after 350 s, which substantially reduces the last-step error from 25.6 m for PDR to around 0.5 m. The rise in position error after the magnetic disturbance at around 170 s is also reduced. However, the trajectory still exhibits considerable errors in the segment affected by heading drift, with a maximum error of 14.2 m. As a result, the ATE values remain relatively large with around 6.4

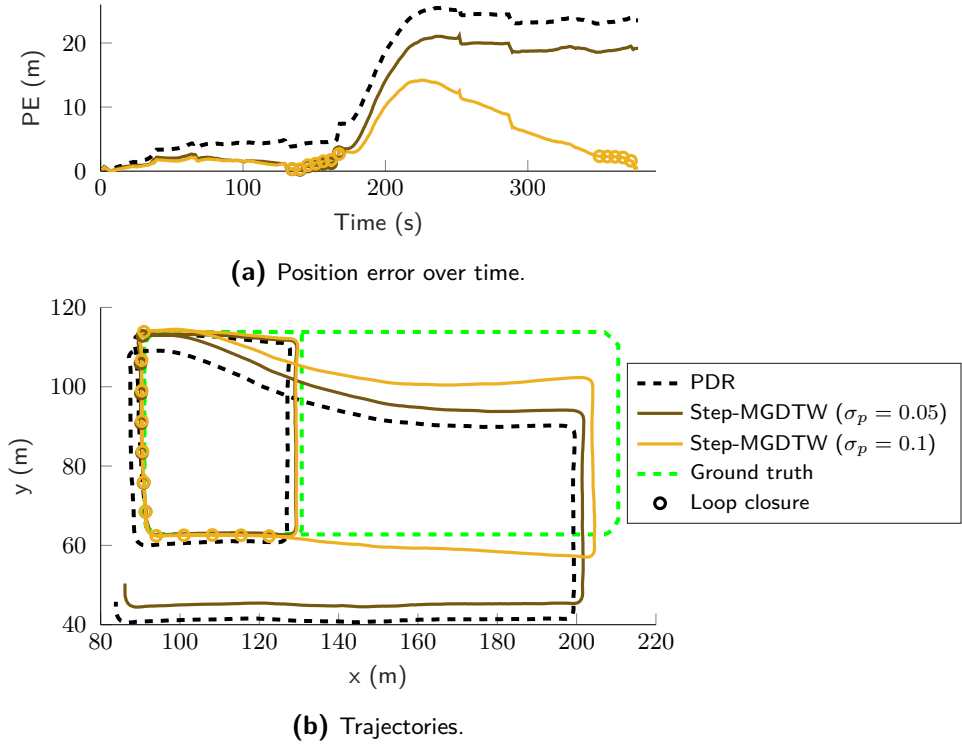


Figure 4-9: Comparison of SLAM (using $\sigma_p^2 = 0.05$ and $\sigma_p^2 = 0.1$) and PDR, evaluated against ground truth on the large magnetic-disturbance walk.

m. The corresponding quantitative results, including the number of detected loop closures, are summarized in Table 4-4.

Method	Max Error (m)	Last-Step Error (m)	ATE (m)	LC (count)
PDR	25.6	23.5	16.9	0
SLAM ($\sigma_p^2 = 0.05$)	21.1	19.2	13.5	7
SLAM ($\sigma_p^2 = 0.1$)	14.2	0.5	6.4	13

Table 4-4: Maximum error, last-step error, ATE, and number of for SLAM with different σ_p^2 values and for PDR during the large magnetic-disturbance walk.

4-3-2 Step-Length Errors under Varying Speeds

In this section, we assess how well SLAM performs when the walking speed varies, using the dataset introduced in Section 4-1-1. We use the baseline settings from Table 4-2, but increase the process noise variance from $\sigma_p^2 = 0.05$ to $\sigma_p^2 = 0.1$, as motivated in Section 4-3-1. This larger process noise captures the increased uncertainty in the PDR-based position increments under speed changes (e.g., due to step-length errors), and makes the filter more permissive when enforcing loop closures across segments with different walking speeds.

Around $t \approx 150$ s, the pedestrian accelerates on the vertical segment at $x \approx 90$. Because the step-length estimator underestimates stride length, both PDR and SLAM show an increase in position error (Figure 4-10a). While SLAM also detects loop closures on the vertical straight,

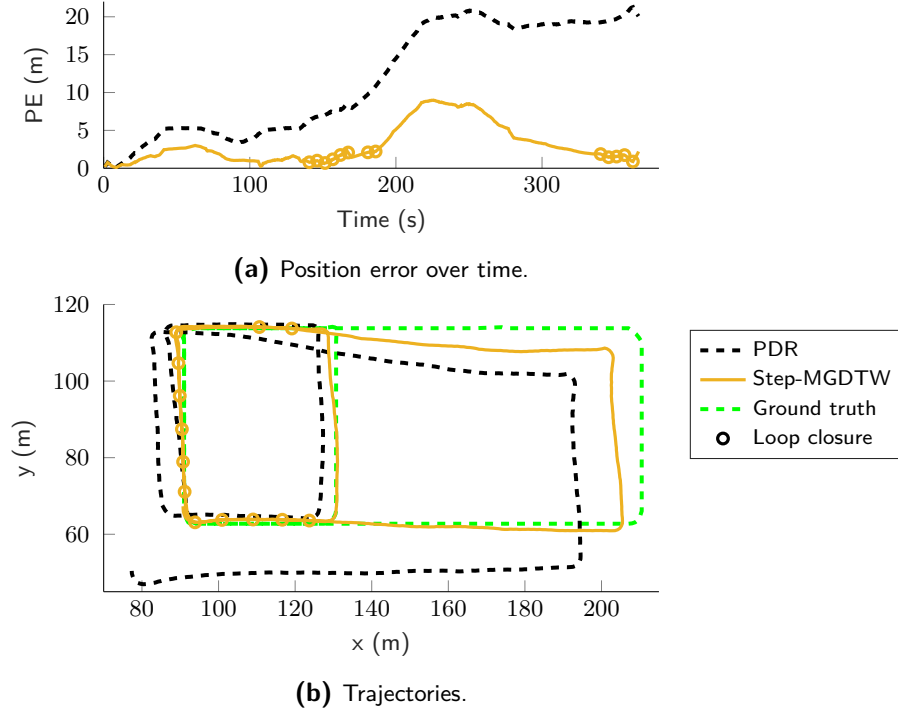


Figure 4-10: Comparison of SLAM and PDR, evaluated against ground truth on the varying-speed walk.

the two loop closures on the horizontal corridor at $y \approx 115$ around $t \approx 190$ s are particularly important, as they prevent a large drift. As illustrated in Figure 4-10b, the vertical component of SLAM aligns more closely with the ground-truth vertical trajectory around $x = 210$ m than PDR, indicating that SLAM partially corrects the step-length underestimation observed in PDR. Table 4-5 shows that SLAM reduces the maximum error by more than half from 21.3 m to 9.0 m and improves the ATE from 13.8 m to 4.0 m. However, these values are still large, indicating that SLAM exhibits large position errors in segments where no loop closures are detected. Later in the trajectory, loop closures detected around 350 s bring the estimated path back toward the ground truth (Figure 4-10a), reducing the last-step error from 20.1 m to 2.2 m.

	Max Error (m)	Last-Step Error (m)	ATE (m)	LC (count)
PDR	21.3	20.1	13.8	0
SLAM	9.0	2.2	4.0	13

Table 4-5: Maximum error, last-step error, ATE, and number of loop closures for SLAM with $\sigma_p^2 = 0.1$ and for PDR during the varying-speed walk.

4-4 Comparison of Loop-Closure Detection Methods

In this section, we compare the four loop-closure detection methods introduced in Section 3-3, thereby addressing the third subquestion. We evaluate the four strategies: Sample Mag-

netic Gaussian Weights (Sample-MGW), Step Magnetic Gaussian Weights (Step-MGW), Sample-MGDTW, and Step-MGDTW. The objective is to compare their performance along two aspects: (i) step-based versus sample-based and (ii) Magnetic Gaussian Weights (MGW) versus Magnetic Gaussian Dynamic Time Warping (MGDTW). We first evaluate the methods under standard walking conditions and subsequently consider the two challenging scenarios introduced earlier, namely a large magnetic disturbance and varying walking speeds.

4-4-1 Loop-Closure Detection in a Standard Walk

First, we compare the performance of the different loop closure methods on the standard-walk dataset introduced in Section 4-1-1. We consider the parameters from Table 4-2.

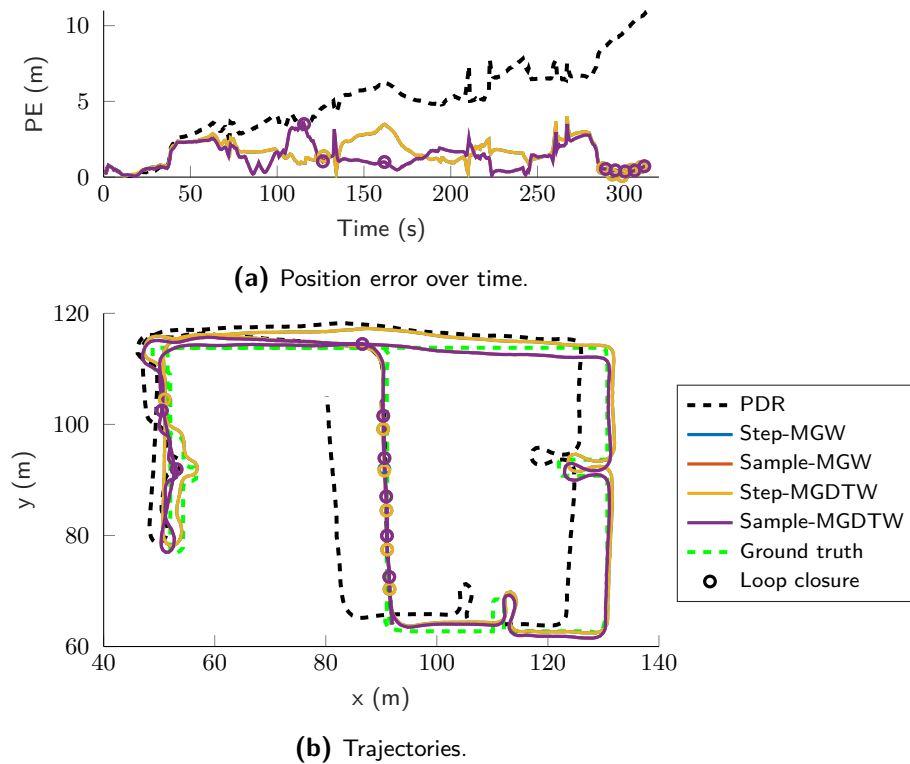


Figure 4-11: Comparison of loop-closure detection methods and PDR, evaluated against ground truth during the standard walk.

For the standard walk, the methods using MGDTW and those using MGW yield identical results, as reported in Table 4-6. This is also evident in Figure 4-11: the corresponding curves lie on top of each other, so only the MGDTW-based methods are visible. However, we observe a difference between the step-based and sample-based methods. The step-based methods produce fewer loop closures in the left segment of the trajectory than the sample-based methods. The sample-based methods, however, generate a false loop closure, which causes a rise in position error at around 115 s, as shown in Figure 4-12a. Figure 4-12b also indicates that the step-based methods follow the ground-truth trajectory more closely in the left segment. However, at around 160 s, the sample-based methods detect a loop closure that the step-based methods miss. This detection results in a decrease in position error and

slightly improved overall error values for the sample-based methods in Table 4-6.

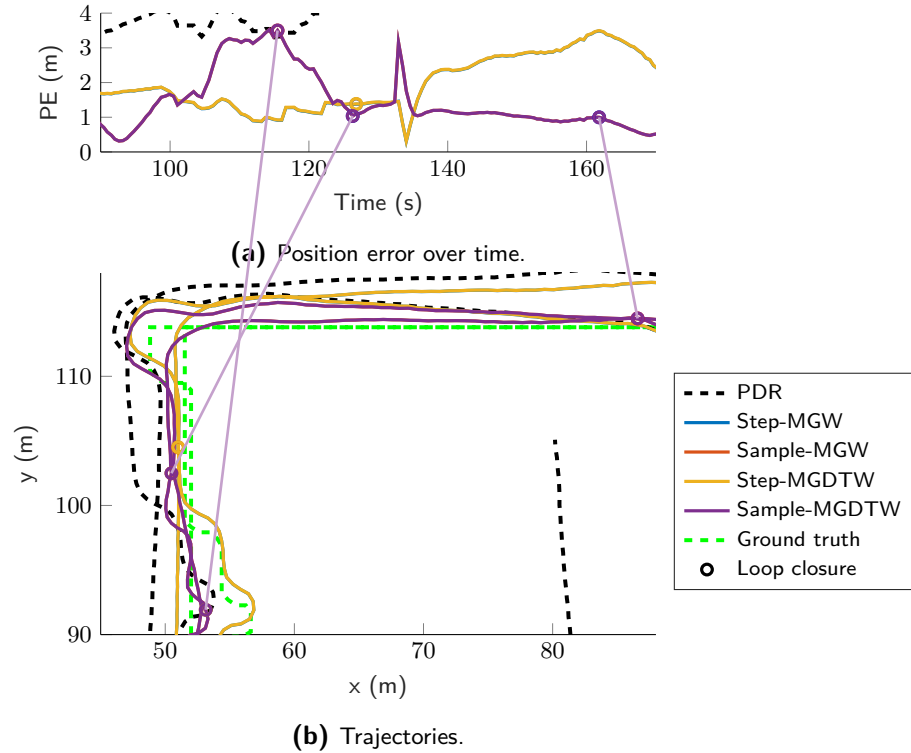


Figure 4-12: Comparison of loop-closure detection methods and PDR against ground truth during the standard walk, with an enlarged view of the left segment highlighting a false loop closure by the sample-based methods (~ 115 s) and a missed loop closure by the step-based methods (~ 160 s).

Method	Max Error (m)	Last-Step Error (m)	ATE (m)	LC (count)
PDR	11.0	11.0	5.5	0
Step-MGW	4.0	1.0	1.8	6
Sample-MGW	3.5	0.9	1.5	8
Step-MGDTW	4.0	1.0	1.8	6
Sample-MGDTW	3.5	0.9	1.5	8

Table 4-6: Maximum error, last-step error, ATE, and number of loop closures for SLAM with different loop-closure detection methods and PDR during the standard walk.

4-4-2 Loop-Closure Detection under Magnetic Disturbance

In this section, we compare the performance of the different loop-closure detection methods on the large magnetic-disturbance dataset introduced in Section 4-1-1. We use the parameters listed in Table 4-2. For the step-based methods, we set the process noise variance to $\sigma_p^2 = 0.1$, as motivated in Section 4-3. For the sample-based methods, we tune σ_p^2 on the large magnetic-disturbance walk using the procedure described in Section 4-2, resulting in $\sigma_p^2 = 0.15$.

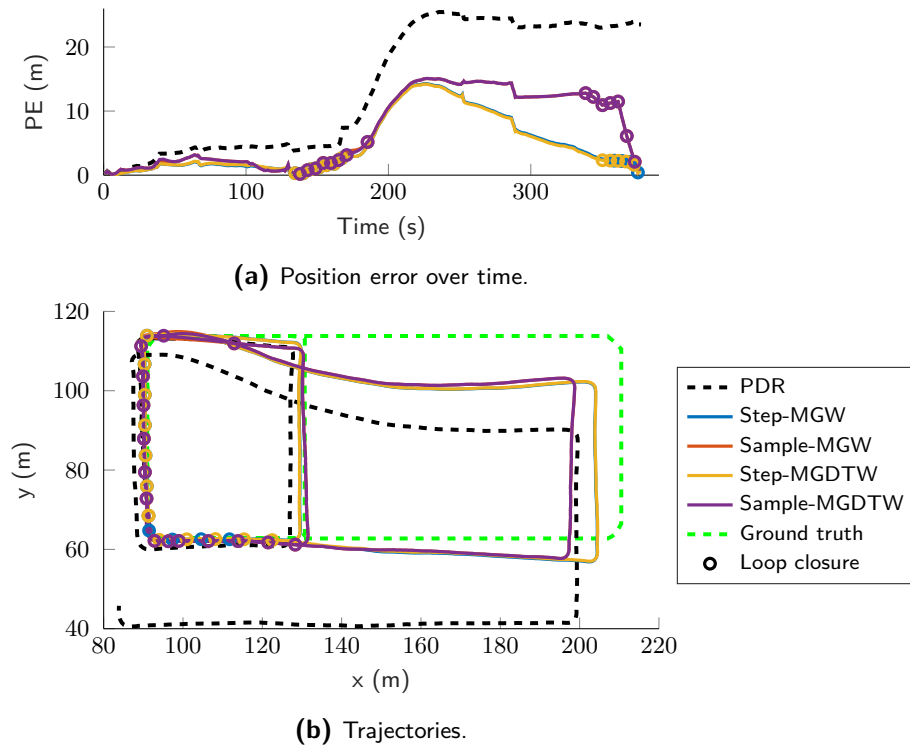


Figure 4-13: Comparison of loop-closure detection methods and PDR, evaluated against ground truth on the large magnetic-disturbance walk.

In Figure 4-13, the sample-based methods register a loop closure on the disturbed straightaway at $y \approx 115$ around $t \approx 190$ s, whereas the step-based methods do not. However, only the step-based approaches subsequently recover the correct path. In contrast, the false loop closures detected by the sample-based methods around 350 s lead to large position errors. Table 4-7 confirms the superior performance of the step-based approaches: both substantially reduce the errors compared to PDR, lowering the maximum error from 25.6 m to about 14.3 m and the last-step error from 23.5 m to below 0.6 m. The ATE likewise improves from 16.9 m under PDR to 6.5 m.

The choice between MGW and MGDTW has little influence on the error metrics, as both yield nearly identical values. Nevertheless, MGDTW detects a larger number of loop closures (Table 4-7).

Method	Max Error (m)	Last-Step Error (m)	ATE (m)	LC (count)
PDR	25.6	23.5	16.9	0
Step-MGW	14.3	0.6	6.5	11
Sample-MGW	15.1	1.0	9.1	14
Step-MGDTW	14.2	0.5	6.4	13
Sample-MGDTW	15.1	1.0	9.1	15

Table 4-7: Maximum error, last-step error, ATE, and number of loop closures for SLAM with different loop-closure detection methods and PDR during the large magnetic-disturbance walk.

4-4-3 Loop-Closure Detection in a Varying-Speed Walk

In this section, we evaluate the loop-closure detection methods on the varying-speed dataset introduced in Section 4-1-1. We use the base parameter set from Table 4-2. For the step-based methods, the process noise variance on the position is set to $\sigma_p^2 = 0.1$, as motivated in Section 4-3. For the sample-based methods, we also use the value $\sigma_p^2 = 0.1$, obtained by applying the tuning procedure from Section 4-2 to the varying-speed walk.

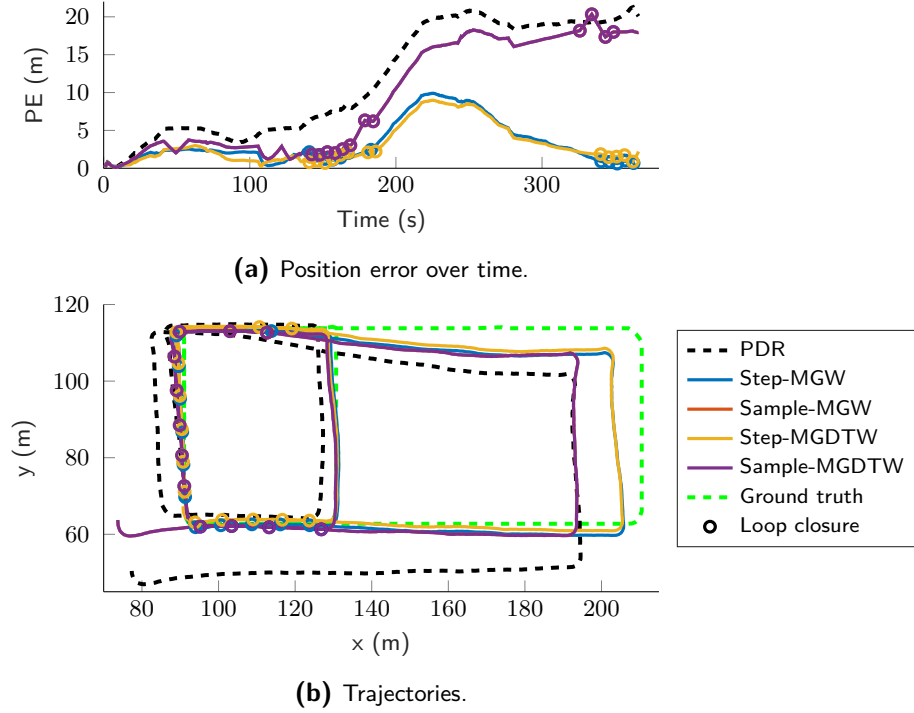


Figure 4-14: Comparison of loop-closure detection methods and PDR, evaluated against ground truth on the varying-speed walk.

From Table 4-8, it is clear that the sample-based methods incur large position errors. This degradation stems from the incorporation of false loop closures: as shown in Figure 4-14a, loop closures are detected at 350 s, yet the position error remains above 15 m, confirming that the detection was spurious. By contrast, the step-based methods achieve better accuracy.

We then analyze the influence of using MGDTW for loop-closure detection. When comparing the two sample-based methods, their behavior is identical. In contrast, between the two step-based methods, Step-MGDTW detects one additional loop closure. In the segment revisits at increased walking speed around $y \approx 115$ m, Step-MGDTW correctly identifies two loop closures, whereas Step-MGW detects only one, as shown in Figure 4-15b. However, in the final part of the trajectory, Step-MGW achieves more accurate loop closures and yields a lower last-step error than Step-MGDTW (Table 4-8).

A closer look at Table 4-8 shows that both step-based methods substantially reduce the position errors compared to PDR. The maximum error decreases from 21.3 m to below 9.9 m, and the last-step error drops from 20.1 m to below 2.2 m. The ATE also improves from 13.8 m for PDR to below 4.4 m for both step-based methods. Between the two, Step-MGDTW

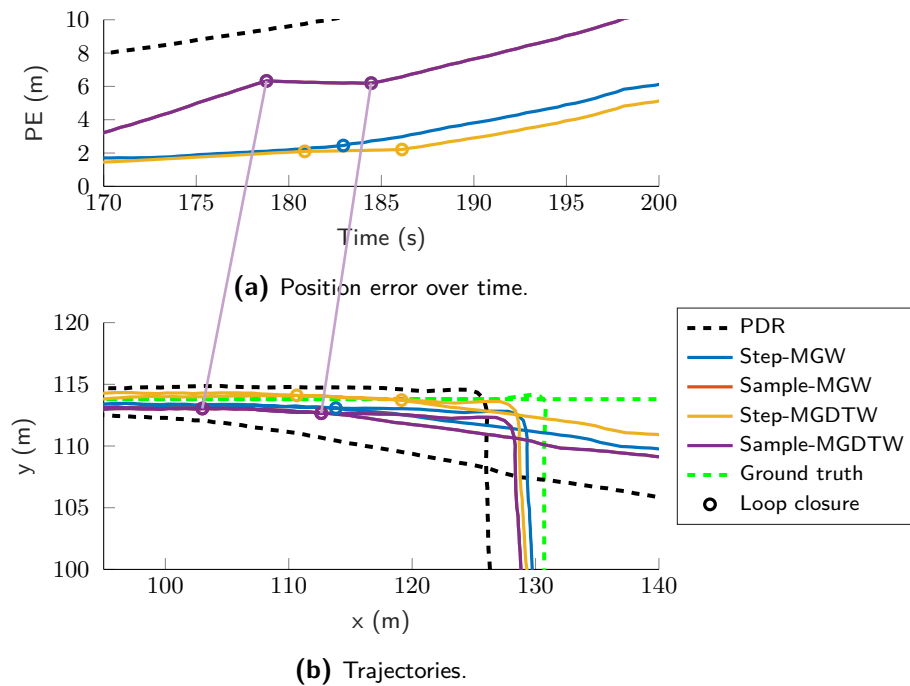


Figure 4-15: Comparison of loop-closure detection methods and PDR against ground truth on the varying-speed walk, highlighting the speed-up revisit around $y \approx 115$ m, where Step-MGDTW detects an additional loop closure compared to Step-MGW.

provides slightly better overall performance across the entire trajectory, achieving the lowest ATE error, whereas Step-MGW attains a lower last-step error.

Method	Max Error (m)	Last-Step Error (m)	ATE (m)	LC (count)
PDR	21.3	20.1	13.8	0
Step-MGW	9.9	0.7	4.4	12
Sample-MGW	20.3	17.9	11.8	12
Step-MGDTW	9.0	2.2	4.0	13
Sample-MGDTW	20.3	17.9	11.8	12

Table 4-8: Maximum error, last-step error, ATE, and number of loop closures for SLAM with different loop-closure detection methods and PDR during the varying-speed walk.

Chapter 5

Conclusions

We have presented a method for one-dimensional magnetic-field Simultaneous Localization and Mapping (SLAM), in which magnetic loop closures are detected and integrated into a Kalman Filter (KF) to mitigate Pedestrian Dead Reckoning (PDR) drift for indoor localization with a handheld smartphone. The method was evaluated under nominal walking conditions and under induced PDR failure modes, using data collected with an iPhone 15 held in a fixed orientation.

To answer subquestion 1, we assessed whether one-dimensional magnetic-field SLAM can reduce PDR drift under standard walking in Section 4-2. On the standard walk dataset, incorporating step-based magnetic loop closures using Step Magnetic Gaussian Dynamic Time Warping (Step-MGDTW) reduced the Absolute Trajectory Error (ATE) from 5.5 m to 1.8 m, reduced the maximum error from 11.0 m to 4.0 m, and reduced the last-step error from 11.0 m to 1.0 m (Table 4-3). These results indicate that repeatable magnetic signatures can yield informative loop closures and limit accumulated PDR drift when revisits occur. The performance, however, depends on reliable loop-closure decisions: false loop closures can distort the trajectory, and residual errors can persist between loop closures.

The results on the standard walk were in a similar range to [46], although our position errors were higher (Table 5-1). We use [46] as the primary reference point because it is the only one-dimensional magnetic-field SLAM approach that reports quantitative position errors on large-scale experimental walks. Overall, our mean errors were roughly a few times larger than those reported in [46], which is still promising given our localization focus: our standard walk revisited about half the trajectory, whereas [46] targets surveying and induces dense revisits by repeating the walks in reverse. However, the comparison is only indicative, since performance depends on loop-closure density, the dataset, and PDR/ground-truth quality.

For subquestion 2, we analyzed how one-dimensional magnetic-field SLAM mitigated the effects of induced PDR failure modes in Section 4-3. Under large magnetic disturbances, incorporating loop closures reduced the last-step error from 23.5 m to 0.5 m and reduced the ATE from 16.9 m to 6.4 m (Table 4-4). For varying walking speeds, where PDR underestimated stride length, loop closures reduced the last-step error from approximately 20.1 m to

Method	Dataset (total)	Mean Error (m)	Max Error (m)
[46]	Office (0.75 km)	0.4	1.5
[46]	Museum (1.20 km)	0.6	3.1
Sample-MGDTW	Standard walk (0.42 km)	1.3	3.5
Step-MGDTW	Standard walk (0.42 km)	1.6	4.0

Table 5-1: Summary comparison of position errors against [46] on large-scale experimental trajectories.

2.2 m and reduced the ATE from 13.8 m to 4.0 m (Table 4-5). These results suggest that loop closures can partially compensate for systematic PDR errors such as heading drift and step-length underestimation. However, the remaining ATE shows that substantial errors can persist between loop closures. This is consistent with the maximum errors, which remained 14.2 m for the large magnetic-disturbance walk and 9.0 m for the varying-speed walk. Large drift between loop closures can become problematic when the user later revisits such a region. In that case, a loop closure there, which could be correct, could still introduce a wrong correction and degrade the trajectory.

Finally, to answer subquestion 3, we compared multiple loop-closure detection variants, including sample-based versus step-based methods and Magnetic Gaussian Weights (MGW) versus Magnetic Gaussian Dynamic Time Warping (MGDTW) in Section 4-4. Overall, the step-based approaches were more robust across datasets, whereas the sample-based method occasionally produced large errors consistent with false loop closures. On the varying-speed walk, the sample-based method yielded a maximum error of approximately 24.4 m, a last-step error of roughly 10 m, and an ATE of about 9–10 m, while the step-based methods reduced these to below 9.9 m, under 3 m, and around 4 m, respectively (Table 4-8). A similar trend was observed on the large magnetic-disturbance walk (Table 4-7). Comparing MGW and MGDTW, the overall behavior was similar; however, on the varying-speed walk, MGDTW detected one additional valid loop closure, which suggest improved loop-closure detection under varying walking speeds.

A limitation of this work is that the experiments were performed under controlled carrying conditions: the smartphone was held in a fixed orientation that was approximately aligned with the pedestrian’s body frame. In realistic use, changes in device pose are likely and can degrade PDR. In our position estimation model, the estimated heading is assumed to coincide with the walking direction. When the phone rotates with respect to the body, this assumption can be violated, causing step updates to be applied in an incorrect direction. Moreover, step detection and step-length estimation may become less reliable because they rely on axis-specific characteristics (e.g., periodicity along the device z -axis) that change when the phone tilts or rotates. Finally, device pose changes also affect the magnetic measurements: the 3-axis magnetic signature at the same location depends on sensor orientation, which can reduce loop-closure repeatability and thereby degrade loop-closure detection.

Future work related to the methodology could therefore consider PDR methods that are robust to device pose changes, use orientation-robust step detection, and adopt adaptive or learned step-length models. The latter is particularly relevant under varying walking speeds, where fixed-parameter step-length models could systematically underestimate or overestimate stride length. In addition, magnetic loop-closure matching could be improved using orientation-invariant magnetic features. We also deliberately adopted a linear KF back-end

instead of Extended Kalman Filter (EKF) or pose-graph optimizers. By omitting the heading from the state and using PDR only as odometry input, the dynamical model remained strictly linear, making the KF an exact and computationally efficient choice. Future work could explore an EKF or pose-graph back-end to incorporate richer state representations and improve scalability to larger and denser maps.

Regarding subquestion 1, future work should study parameter tuning more systematically. Our sensitivity analysis showed that several parameters strongly influence the trade-off between detecting true loop closures and rejecting false positives, and that certain settings can trigger false loop closures. This is critical because even a single false loop closure can pull the trajectory away from the true path and may reduce the likelihood that later valid closures are accepted. An important next step would therefore be a tuning procedure that is validated across multiple datasets, for example via systematic sweeps or automated tuning strategies.

Regarding subquestion 2, future work could reduce reliance on manual tuning by introducing adaptive uncertainty modeling. A limitation of the current approach is that the KF uses a constant process noise variance, and therefore does not reflect the increased PDR uncertainty during failure modes. In our experiments, manually increasing the process noise made the PDR propagation less trusted and thereby promoted loop-closure acceptance. Future work could estimate PDR reliability and adjust the process noise accordingly. Similarly, the currently constant loop-closure measurement-noise variance could be made adaptive by assigning each detected loop closure an uncertainty based on match confidence, for example, using the combined loop-closure detection weight.

Regarding subquestion 3, future work should clarify why the step-based methods outperformed the sample-based methods in our experiments. One plausible explanation is that sampling magnetometer measurements at consistent points within each step may reduce gait-phase-dependent variation, but this hypothesis requires targeted evaluation. Gait-phase-dependent variation refers to the systematic, within-step changes in a sensor measurement as the foot progresses through the phases of a step cycle. Another possibility is that the step-based representation effectively compares measurements over longer distance intervals due to its sparser sampling. Finally, the choice between MGW and MGDTW had a limited impact on the evaluated datasets; whether this also holds for other trajectories and environments remains to be investigated.

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Glossary

List of Acronyms

GNSS	Global Navigation Satellite System
IMU	Inertial Measurement Unit
DTW	Dynamic Time Warping
PDR	Pedestrian Dead Reckoning
SLAM	Simultaneous Localization and Mapping
GP	Gaussian Process
VQF	Versatile Quaternion-based Filtering
Step-MGW	Step Magnetic Gaussian Weights
Sample-MGW	Sample Magnetic Gaussian Weights
Step-MGDTW	Step Magnetic Gaussian Dynamic Time Warping
Sample-MGDTW	Sample Magnetic Gaussian Dynamic Time Warping
KF	Kalman Filter
EKF	Extended Kalman Filter
MGDTW	Magnetic Gaussian Dynamic Time Warping
MGW	Magnetic Gaussian Weights
ATE	Absolute Trajectory Error
LC	Loop Closure
PE	Position Error

