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Transmit and Receive Sensor Selection Using the Multiplicity in the Virtual Array

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***Abstract*—The main focus of this paper is an active sensing application that involves selecting transmit and receive sensors to optimize the Cramér-Rao bound (CRB) on target parameters. Although the CRB is non-convex in the transmit and receive selection, we demonstrate that it is convex in the virtual array weight vector, which describes the multiplicity of the virtual array elements. Based on this finding, we propose a novel algorithm that optimizes the virtual array weight vector first and then finds a matching transceiver array. This greatly enhances the efficiency of the transmit and receive sensor selection problem.**

***Index Terms*—Active sensing, Cramér-Rao lower bound (CRB), multiplicity, redundancy, sensor selection.**

I. Introduction

With advances in hardware capabilities, the feasibility of complex digital processing has increased, resulting in the use of larger sensor arrays. Despite this progress, specific applications remain constrained by limitations such as power constraints and budgetary restrictions. Consequently, there is a motivation to explore sparse arrays that can deliver comparable performance while respecting resource constraints.

The utilization of sparse arrays is a topic that has received significant attention for various applications such as automotive radar [1] and wireless communications networks [2]. However, it introduces the crucial challenge of waveform design and sensor selection, both on transmit and receive.

The interplay between the transmit and receive sensors is of great importance. It is well known that the identifiability (number of identifiable targets) of a transceiver array is upper bounded by the size of the virtual array (also named sum co-array) [3]. Moreover, when the rank of the transmitted waveform (often called the waveform rank) is smaller than the number of transmitting sensors, the identifiability of the transceiver array—even for full sum co-arrays—is affected by the redundancy of the transceiver sensor positions [4]. This highlights the importance of selecting suitable transceiver sensors.

The research conducted in [5] demonstrated that the problem of choosing a group of sensors (specifically, receiving sensors) to enhance estimation performance, although inherently non-convex and NP-hard, can be effectively addressed by using its convex relaxation. Since then, several studies have concentrated on improving the selection of receive sensors (see, e.g., [6]–[10]). The work in [6] and [7] discussed sparsity-aware sensor selection in a centralized and decentralized scenario, while [8]–[10] discuss greedy methods, where sensors are chosen one by one. However, these papers discuss the selection of only the receive sensors, whereas, for many active sensing applications, the interplay between the transmit and receive sensors is crucial.

The work in [11] discussed a multiple-input and multiple-output (MIMO) radar scenario in which the number of deployed sensors is minimized under a localization estimation mean squared error (MSE) constraint and considers both the transmit and receive selection. In [1], a genetic algorithm originally proposed in [12], was used. In each iteration, the "genes", i.e., a set of transceiver selections, are ranked based on the ambiguity function; only the ones ranked high are used to generate a new set of genes for the next iteration. Nevertheless, the research mentioned above has not considered exploiting the interplay between the transmit and receive selection, specifically the virtual array, which is ultimately responsible for the estimation performance. Although the authors of [13] proposed an approach to find a transceiver selection that matches a desired virtual array, it does not cover optimizing the virtual array with respect to a transmit/receive objective. It also mainly considers virtual arrays for which an exactly matching transceiver selection is known to exist.

This paper presents an approach that exploits the knowledge of the virtual array for transmit and receive sensor selection. This leads to a substantial improvement in complexity compared to conventional approaches to address the sensor selection problem. The enhancement is rooted in the understanding that although the performance measure typically exhibits non-linearity in the selection of transmit and receive elements, leading to a non-convex optimization problem, it is linear in the multiplicity vector, resulting in a convex optimization problem.

II. Signal Model and Problem Statement

Consider an active sensing scenario with K targets in the far field, which have reflection coefficients $\boldsymbol{\alpha} \in \mathbb{R}^K$ and incident angles $\boldsymbol{\theta} \in [-\pi/2, \pi/2)^K$, as visualized in Fig. 1.[1]

The work is part of a project funded by the Netherlands Organisation for Applied Scientific Research (TNO) and the Netherlands Defence Academy (NLDA).

[1]The model and following derivations can be extended to $\boldsymbol{\alpha} \in \mathbb{C}^K$ trivially as discussed in [14, Section 15.7].

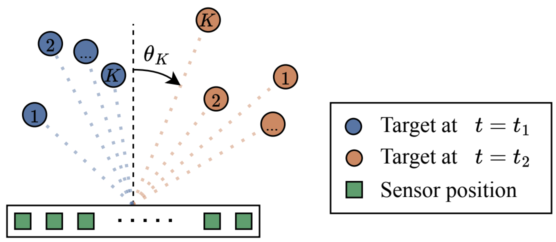


Fig. 1: Possible use-case of K moving targets.

The objective is to estimate certain parameters of the targets using a uniform linear array (ULA) of transceivers. In this work, we consider the reflection coefficients of the targets as the parameters of interest, to improve the tracking performance [15], and we assume the angles of incidence (AoIs) known.

Suppose that a linear array of N_t and N_r colocated transmitters and receivers is used, and the N_r received signals are collected for T time instances. Then, the data matrix $\boldsymbol{X} \in \mathbb{C}^{N_\mathrm{r} \times T}$, under the narrowband assumption, can be modelled by

$$\begin{aligned} \boldsymbol{X} &= \sum_{k=1}^{K} \alpha_k \boldsymbol{a}_\mathrm{r}(\theta_k) \boldsymbol{a}_\mathrm{t}^\mathsf{T}(\theta_k) \boldsymbol{S}^\mathsf{T} + \boldsymbol{N}\,, \\ &= \boldsymbol{A}_\mathrm{r} \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{A}_\mathrm{t}^\mathsf{T} \boldsymbol{S}^\mathsf{T} + \boldsymbol{N}\,, \end{aligned} \tag{1}$$

where α_k is the reflection coefficient of the kth target, $\left[\boldsymbol{a}_{(\cdot)}(\theta_k)\right]_i = \exp(\mathrm{j}\pi d_i \sin(\theta_k))$ denotes the steering vector at the AoI θ_k of the kth target, and $\boldsymbol{A}_{(\cdot)} = \left[\boldsymbol{a}_{(\cdot)}(\theta_1) \quad \cdots \quad \boldsymbol{a}_{(\cdot)}(\theta_K)\right]$. The distance from the ith array element to the reference element, here chosen as the center of the array, is denoted by d_i, $\boldsymbol{S} \in \mathbb{C}^{T \times N_\mathrm{t}}$ contains the N_t transmitted narrowband signals, and $\boldsymbol{N}$ contains additive Gaussian noise.

Constrained by budget and/or limited processing resources, our objective is to choose a subset of transmit and receive sensors from the set of candidates that performs better than any other selection of the same number of sensors for estimation of the target reflection coefficients. It should be noted that the complexity of the algorithm is important here, as there is a limited amount of time available before the situation, i.e., the AoIs, changes. Let $\boldsymbol{p}_\mathrm{t}$ and $\boldsymbol{p}_\mathrm{r}$ denote the selection vectors for transmit and receive, respectively, which indicate with a one (zero) when a sensor of the array should be selected (omitted). The selection-dependent data model is then described by

$$\tilde{\boldsymbol{X}} = \boldsymbol{\Phi}(\boldsymbol{p}_\mathrm{r}) \boldsymbol{A}_\mathrm{r} \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{A}_\mathrm{t}^\mathsf{T} \boldsymbol{\Phi}^\mathsf{T}(\boldsymbol{p}_\mathrm{t}) \boldsymbol{S}^\mathsf{T} + \boldsymbol{N}\,, \tag{2}$$

where the wide matrix $\boldsymbol{\Phi}(\boldsymbol{p})$ denotes a selection matrix that contains only the rows of the identity matrix corresponding to the indices where $\boldsymbol{p}$ has elements equal to one.

Finally, given a cost function $f(\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r})$, our problem is to solve

$$\min_{\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r}} \quad f(\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r})\,, \qquad \text{s.t.} \quad \boldsymbol{p}_\mathrm{t} \in \mathcal{B}_\mathrm{t}\,,\ \boldsymbol{p}_\mathrm{r} \in \mathcal{B}_\mathrm{r}\,, \tag{3}$$

where $\mathcal{B}_{(\cdot)} = \left\{\boldsymbol{p} \in \{0,1\}^{N_{(\cdot)}} \mid \mathbf{1}^\mathsf{T}\boldsymbol{p} = M_{(\cdot)}\right\}$, and M_t and M_r are the desired number of selected sensors.

III. Performance Metric

We need a selection-dependent metric that describes the array performance to compare different sensor selections. As stated in Section II, the array will be used for estimating the reflection coefficients of K targets given known AoIs. We will use a selection dependent Cramér-Rao Bound (CRB) on the estimation of $\boldsymbol{\alpha}$ as the basis for our cost function. To simplify notation, we refer to the selection matrix $\boldsymbol{\Phi}(\boldsymbol{p}_{(\cdot)}) \in \{0,1\}^{M_{(\cdot)} \times N_{(\cdot)}}$ as $\boldsymbol{\Phi}_{(\cdot)}$.

We first vectorize the data matrix $\tilde{\boldsymbol{X}}$ in (2) to obtain

$$\operatorname{vec}\!\left(\tilde{\boldsymbol{X}}\right) = (\boldsymbol{S} \otimes \boldsymbol{I}_{M_\mathrm{r}}) \overbrace{(\boldsymbol{\Phi}_\mathrm{t}\boldsymbol{A}_\mathrm{t} \odot \boldsymbol{\Phi}_\mathrm{r}\boldsymbol{A}_\mathrm{r})}^{\text{sum co-array response}} \boldsymbol{\alpha} + \boldsymbol{n}\,, \tag{4}$$

where $\otimes$ and $\odot$ represent the Kronecker and Khatri-Rao products, respectively. The sum co-array response could have repeated rows, indicating these virtual elements have some multiplicity. This notion of multiplicity can be used to re-write the sum co-array response as

$$\boldsymbol{\Phi}_\mathrm{t}\boldsymbol{A}_\mathrm{t} \odot \boldsymbol{\Phi}_\mathrm{r}\boldsymbol{A}_\mathrm{r} = (\boldsymbol{\Phi}_\mathrm{t} \otimes \boldsymbol{\Phi}_\mathrm{r}) \boldsymbol{\Upsilon} \boldsymbol{A}_\Sigma\,, \tag{5}$$

where $\boldsymbol{\Upsilon} \in \{0,1\}^{N_\mathrm{t}N_\mathrm{r} \times N_\Sigma}$ is the redundancy pattern matrix with $N_\Sigma = N_\mathrm{t} + N_\mathrm{r} - 1$ the number of virtual array elements, and $\boldsymbol{A}_\Sigma \in \mathbb{C}^{N_\Sigma \times K}$ is the virtual steering matrix containing all steering vectors of the virtual array, without repeated rows [16]. Note that in the formulation of (5), $\boldsymbol{\Upsilon}$ and $\boldsymbol{A}_\Sigma$ refer to the redundancy and steering matrices of the virtual array when using all transceivers for transmit and receive.

Let $\boldsymbol{E}(\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r})$ be the CRB matrix (CRBM), then

$$\operatorname{Var}(\hat{\boldsymbol{\alpha}}) \geq \operatorname{diag}(\boldsymbol{E}(\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r})) = \operatorname{diag}\!\left(\boldsymbol{\mathcal{I}}^{-1}\right), \tag{6}$$

where $\hat{\boldsymbol{\alpha}}$ is an estimate of $\boldsymbol{\alpha}$, and $\boldsymbol{\mathcal{I}}$ is the Fisher information matrix (FIM). Using (4) and (5), we find that the FIM is [14],

$$\boldsymbol{\mathcal{I}} = 2\operatorname{Re}\!\left\{\boldsymbol{A}_\Sigma^\mathsf{H} \tilde{\boldsymbol{\Upsilon}}^\mathsf{T} (\boldsymbol{S} \otimes \boldsymbol{I}_{M_\mathrm{r}})^\mathsf{H} \boldsymbol{R}_n^{-1} (\boldsymbol{S} \otimes \boldsymbol{I}_{M_\mathrm{r}}) \tilde{\boldsymbol{\Upsilon}} \boldsymbol{A}_\Sigma\right\}, \tag{7}$$

where $\boldsymbol{R}_n$ is the covariance matrix of the noise and $\tilde{\boldsymbol{\Upsilon}} = (\boldsymbol{\Phi}_\mathrm{t} \otimes \boldsymbol{\Phi}_\mathrm{r})\boldsymbol{\Upsilon}$. We make two assumptions: The noise is white, i.e. $\boldsymbol{R}_n = \sigma_n^2 \boldsymbol{I}_{M_\mathrm{r}}$,[2] and the transmitting sensors send orthogonal sequences (in time), i.e. $\boldsymbol{S}^\mathsf{H}\boldsymbol{S} = \boldsymbol{I}_{M_\mathrm{t}}$. Under these assumptions, the FIM in (7) simplifies to

$$\boldsymbol{\mathcal{I}} = 2\operatorname{Re}\!\left\{\boldsymbol{A}_\Sigma^\mathsf{H} \tilde{\boldsymbol{\Upsilon}}^\mathsf{T} \tilde{\boldsymbol{\Upsilon}} \boldsymbol{A}_\Sigma\right\}. \tag{8}$$

Before we continue to derive our sensor selection optimization methods, let us investigate how $\boldsymbol{\mathcal{I}}$ depends on $\boldsymbol{p}_\mathrm{t}$ and $\boldsymbol{p}_\mathrm{r}$.

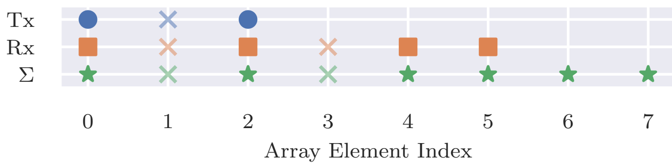
Since $\tilde{\boldsymbol{\Upsilon}}^\mathsf{T}\tilde{\boldsymbol{\Upsilon}} = \operatorname{diag}(\boldsymbol{v})$, where $\boldsymbol{v} = \boldsymbol{p}_\mathrm{t} * \boldsymbol{p}_\mathrm{r}$ is the virtual array weight (VAW) vector, indicating the multiplicities of the virtual array elements,[3] where $*$ represents linear convolution. With this, we can rewrite (8) as

$$\boldsymbol{\mathcal{I}} = 2\operatorname{Re}\{\boldsymbol{A}_\Sigma^\mathsf{H} \operatorname{diag}(\boldsymbol{v}) \boldsymbol{A}_\Sigma\}\,. \tag{9}$$

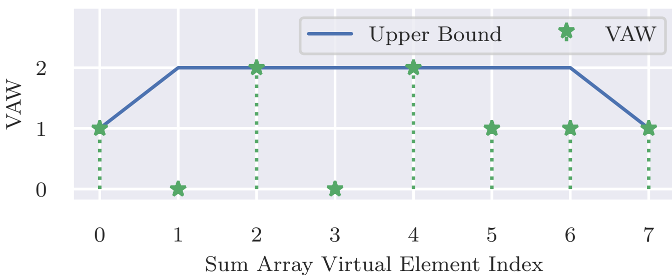
So, the FIM is bilinear in the selection vectors $\boldsymbol{p}_\mathrm{t}$ and $\boldsymbol{p}_\mathrm{r}$, and *linear* in the VAW vector.

[2] We acknowledge that this might not be realistic for every application, as it implies that noise is uncorrelated over the receiving sensors.

[3] The multiplicity vector $\boldsymbol{v}_\Sigma$ from [17], [18] is obtained by removing all zero elements from $\boldsymbol{v}$. A proof of this relation will be published in the extension of this work.



(a) A transceiver array, and their resulting sum array. ×s indicate unselected sensors and empty virtual array positions.



(b) The resulting VAW vector and VAW bounds for the example in Fig. 2a.

Fig. 2: Example of a transceiver array, and the resulting VAW. Here, $(N_\Sigma, N_\mathrm{t}, N_\mathrm{r}, M_\mathrm{t}, M_\mathrm{r}) = (8, 3, 6, 2, 4)$.

In Fig. 2, an example of a sensor selection and the resulting VAW vector are shown. Note that by construction via the convolution, we have $\mathbf{1}^\mathsf{T}\boldsymbol{v} = M_\mathrm{t}M_\mathrm{r}$, and

$$v_i \leq \min(i, N_\mathrm{t} + N_\mathrm{r} - i, M_\mathrm{t}, M_\mathrm{r}), \quad \forall i \in \mathcal{N}, \tag{10}$$

where $\mathcal{N} = \{0, 1, \ldots, N_\Sigma\}$. The constraint in (10) results in an upper bound on the vector $\boldsymbol{v}$, shown in Fig. 2.

Although we now have a metric to evaluate the quality of sensor selections, we do not yet have a convex scalar cost function. There are several ways to obtain scalar cost functions from a matrix. We focus on minimization of the worst-case error, represented by the maximum eigenvalue of $\boldsymbol{E}(\boldsymbol{v})$, commonly called the E-optimality. Analogous methods involve considering the trace (A-optimality) or the logarithm of the determinant (D-optimality) of $\boldsymbol{E}(\boldsymbol{v})$, as outlined in [19]. The problem at hand can now be formulated as

$$\min_{\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r}} \quad \lambda_\mathrm{max}(\boldsymbol{E}(\boldsymbol{v})), \qquad \text{s.t.} \quad \boldsymbol{p}_\mathrm{t} \in \mathcal{B}_\mathrm{t}, \ \boldsymbol{p}_\mathrm{r} \in \mathcal{B}_\mathrm{r}. \tag{11}$$

IV. Joint Transmit and Receive Sensor Selection

We discuss two methods of approximately solving the non-convex program, as given in (11), using the properties of the FIM noted in Section III.

A. Alternating between Transmit and Receive Selection

In Section III, we saw that the FIM is bilinear in the selection vectors. To use this property, we first relax the binary constraints on the selection vectors by introducing box constraints on new continuous variables $\tilde{\boldsymbol{p}}_\mathrm{t} \in \tilde{\mathcal{B}}_\mathrm{t}$ and $\tilde{\boldsymbol{p}}_\mathrm{r} \in \tilde{\mathcal{B}}_\mathrm{r}$, where $\tilde{\mathcal{B}}_{(\cdot)} = \left\{\boldsymbol{p} \in \mathbb{R}^{N_{(\cdot)}} \mid \mathbf{0} \leq \boldsymbol{p} \leq \mathbf{1} \wedge \mathbf{1}^\mathsf{T}\boldsymbol{p} = M_{(\cdot)}\right\}$. Additionally, we express the problem in (11) in terms of the FIM instead

Algorithm 1 The "Alternating" algorithm.

Input: $\boldsymbol{A}_\mathrm{t}, \boldsymbol{A}_\mathrm{r}, M_\mathrm{t}, M_\mathrm{r}$, Stopping condition
Output: $\boldsymbol{p}_\mathrm{t}, \boldsymbol{p}_\mathrm{r}$

$j \leftarrow 0$
$\tilde{\boldsymbol{p}}_\mathrm{t}^{(0)} \leftarrow \mathrm{shuffle}\left(\left[\mathbf{1}_{M_\mathrm{t}}^\mathsf{T} \quad \mathbf{0}_{N_\mathrm{t}-M_\mathrm{t}}^\mathsf{T}\right]^\mathsf{T}\right)$
while Stopping condition is not met **do**
  $j \leftarrow j + 1$
  $\tilde{\boldsymbol{p}}_\mathrm{t}^{(j)} \leftarrow$ Solution of (12) for $\tilde{\boldsymbol{p}}_\mathrm{r} = \tilde{\boldsymbol{p}}_\mathrm{r}^{(j-1)}$
  $\tilde{\boldsymbol{p}}_\mathrm{r}^{(j)} \leftarrow$ Solution of (12) for $\tilde{\boldsymbol{p}}_\mathrm{t} = \tilde{\boldsymbol{p}}_\mathrm{t}^{(j)}$
end while
$\boldsymbol{p}_\mathrm{t} \leftarrow \mathrm{randround}\left(\tilde{\boldsymbol{p}}_\mathrm{t}^{(j)}\right), \ \boldsymbol{p}_\mathrm{r} \leftarrow \mathrm{randround}\left(\tilde{\boldsymbol{p}}_\mathrm{r}^{(j)}\right)$

of the CRBM. Ignoring constant factors, this results in the following optimization problem,

$$\begin{aligned} \max_{\lambda, \tilde{\boldsymbol{p}}_\mathrm{t}, \tilde{\boldsymbol{p}}_\mathrm{r}} \quad & \lambda \\ \text{s.t.} \quad & \mathrm{Re}\left\{\boldsymbol{A}_\Sigma^\mathsf{H} \operatorname{diag}(\tilde{\boldsymbol{p}}_\mathrm{t} * \tilde{\boldsymbol{p}}_\mathrm{r}) \boldsymbol{A}_\Sigma\right\} - \lambda \boldsymbol{I} \succeq \mathbf{0} \\ & \tilde{\boldsymbol{p}}_\mathrm{t} \in \tilde{\mathcal{B}}_\mathrm{t}, \ \tilde{\boldsymbol{p}}_\mathrm{r} \in \tilde{\mathcal{B}}_\mathrm{r}. \end{aligned} \tag{12}$$

The problem is bilinear in the selection vectors and, hence, non-convex. Therefore, an iterative algorithm that alternates between solving only for $\tilde{\boldsymbol{p}}_\mathrm{t}$ or $\tilde{\boldsymbol{p}}_\mathrm{r}$, keeping the other selection vector fixed, can be applied. Once convergence to a local minimum is achieved, the sensor selection is obtained from the continuous vectors $\tilde{\boldsymbol{p}}_\mathrm{t}$ and $\tilde{\boldsymbol{p}}_\mathrm{r}$ by a randomized rounding procedure, where the continuous values in $\tilde{\boldsymbol{p}}_\mathrm{t}$ and $\tilde{\boldsymbol{p}}_\mathrm{r}$ are used as probabilities [20]. Alg. 1 describes this procedure, which we will refer to as the "Alternating" method, where $\mathrm{randround}(\cdot)$ is the randomized rounding procedure [20] and $\mathrm{shuffle}(\cdot)$ returns the input vector with its elements randomly ordered.

Note that in each iteration of Alg. 1 a semi-definite program (SDP) of size K must be solved. The complexity of solving an SDP with an interior point method depends mainly on the complexity of calculating and inverting the Hessian of the log barrier function in each iteration. This leads to a complexity of about $\mathcal{O}(K^3)$, assuming the number of iterations of the interior point method is constant [21]–[23]. For more details, the interested reader is referred to [23] and references therein.

This complexity introduces a problem for sensor selection. The complexity of Alg. 1 is $\mathcal{O}(IK^3)$, where I denotes the number of iterations that the alternating approach needs to converge. To tackle this complexity problem, we propose a new method that uses the VAW vector.

B. Using Multiplicity

As shown in Section III, the FIM is linear in the vector $\boldsymbol{v}$. So, instead of directly trying to find the selection vectors, we could first optimize for the VAW, using a new set of constraints on $\boldsymbol{v}$ as derived in Section III. The optimization of the CRB

Algorithm 2 The "Multiplicity" algorithm.

Input: $\boldsymbol{A}_{\mathrm{t}}, \boldsymbol{A}_{\mathrm{r}}, M_{\mathrm{t}}, M_{\mathrm{r}}$, Stopping condition
Output: $\boldsymbol{p}_{\mathrm{t}}, \boldsymbol{p}_{\mathrm{r}}$
1: $\tilde{\boldsymbol{v}} \leftarrow$ Solution of (13)
2: $j \leftarrow 0$
3: $\tilde{\boldsymbol{p}}_{\mathrm{t}}^{(0)} \leftarrow \mathrm{shuffle}\left(\begin{bmatrix}\mathbf{1}_{M_{\mathrm{t}}}^{\mathsf{T}} & \mathbf{0}_{N_{\mathrm{t}}-M_{\mathrm{t}}}^{\mathsf{T}}\end{bmatrix}^{\mathsf{T}}\right)$
4: $\tilde{\boldsymbol{p}}_{\mathrm{r}}^{(0)} \leftarrow \mathrm{shuffle}\left(\begin{bmatrix}\mathbf{1}_{M_{\mathrm{r}}}^{\mathsf{T}} & \mathbf{0}_{N_{\mathrm{r}}-M_{\mathrm{r}}}^{\mathsf{T}}\end{bmatrix}^{\mathsf{T}}\right)$
5: **while** Stopping condition is not met **do**
6: $\quad j \leftarrow j+1$
7: $\quad \tilde{\boldsymbol{p}}_{\mathrm{t}}^{(j)} \leftarrow$ Solution of (14) for $\tilde{\boldsymbol{p}}_{\mathrm{r}} = \tilde{\boldsymbol{p}}_{\mathrm{r}}^{(j-1)}$
8: $\quad \tilde{\boldsymbol{p}}_{\mathrm{r}}^{(j)} \leftarrow$ Solution of (14) for $\tilde{\boldsymbol{p}}_{\mathrm{t}} = \tilde{\boldsymbol{p}}_{\mathrm{t}}^{(j)}$
9: **end while**
10: $\boldsymbol{p}_{\mathrm{t}} \leftarrow \mathrm{randround}\left(\tilde{\boldsymbol{p}}_{\mathrm{t}}^{(j)}\right), \ \boldsymbol{p}_{\mathrm{r}} \leftarrow \mathrm{randround}\left(\tilde{\boldsymbol{p}}_{\mathrm{r}}^{(j)}\right)$

over $\boldsymbol{v}$ can be described by the mixed-integer SDP,

$$\begin{aligned}\max_{\boldsymbol{v}\in\mathbb{Z}^{N_\Sigma},\lambda} \quad & \lambda \\ \text{s.t.} \quad & \mathrm{Re}\left\{\boldsymbol{A}_\Sigma^{\mathsf{H}}\,\mathrm{diag}(\boldsymbol{v})\boldsymbol{A}_\Sigma\right\} - \lambda\boldsymbol{I} \succeq 0 \\ & 0 \le v_i \le \min(i, N_{\mathrm{t}}+N_{\mathrm{r}}-i, M_{\mathrm{t}}, M_{\mathrm{r}})\,, \ \forall i \in \mathcal{N} \\ & \mathbf{1}^{\mathsf{T}}\boldsymbol{v} = M_{\mathrm{t}}M_{\mathrm{r}}\,.\end{aligned}$$

Note that this embodies the fact that solely the combination of transmitter *and* receiver selection determines the performance. This motivates the following approach. We first solve the SDP for $\boldsymbol{v}$, followed by an iterative approach that, from $\boldsymbol{v}$, calculates the sensor selection. We relax the integer constraint to box constraints on a new continuous vector variable $\tilde{\boldsymbol{v}}$ to obtain the convex problem

$$\begin{aligned}\max_{\tilde{\boldsymbol{v}}\in\mathbb{R}^{N_\Sigma},\lambda} \quad & \lambda \\ \text{s.t.} \quad & \mathrm{Re}\left\{\boldsymbol{A}_\Sigma^{\mathsf{H}}\,\mathrm{diag}(\tilde{\boldsymbol{v}})\boldsymbol{A}_\Sigma\right\} - \lambda\boldsymbol{I} \succeq 0 \qquad (13) \\ & 0 \le \tilde{v}_i \le \min(i, -i+1+N_\Sigma, M_{\mathrm{t}}, M_{\mathrm{r}})\,, \ \forall i \in \mathcal{N} \\ & \mathbf{1}^{\mathsf{T}}\tilde{\boldsymbol{v}} = M_{\mathrm{t}}M_{\mathrm{r}}\,.\end{aligned}$$

Having solved for $\tilde{\boldsymbol{v}}$, in the second step, we still have to use an alternating approach to find the transmitter and receiver selections that constitute $\tilde{\boldsymbol{v}}$. Unlike the previous alternating approach, this will be independent of K. Note, however, that $\tilde{\boldsymbol{v}}$ obtained from solving (13) contains continuous elements instead of integer ones. Therefore, a binary selection exactly matching $\tilde{\boldsymbol{v}}$ probably does not exist. Additionally, the binary constraints are non-convex.

Like before, we relax the problem by introducing box constraints on the selection vectors and minimizing the l_2-norm. We end up with the following quadratic program (QP)

$$\min_{\tilde{\boldsymbol{p}}_{\mathrm{t}},\tilde{\boldsymbol{p}}_{\mathrm{r}}} \quad \|(\tilde{\boldsymbol{p}}_{\mathrm{t}} * \tilde{\boldsymbol{p}}_{\mathrm{r}}) - \tilde{\boldsymbol{v}}\|_2^2\,, \qquad \text{s.t.} \quad \tilde{\boldsymbol{p}}_{\mathrm{t}} \in \tilde{\mathcal{B}}_{\mathrm{t}}\,, \ \tilde{\boldsymbol{p}}_{\mathrm{r}} \in \tilde{\mathcal{B}}_{\mathrm{r}}\,, \quad (14)$$

which is initialized by random selection vectors in $\mathcal{B}_{(\cdot)}$. The complete procedure, which we will refer to as the "Multiplicity" method, is summarized by Alg. 2.

Note that in Alg. 2 we only need to solve an SDP once. Subsequently, we proceed with iterations using a quadratic program, rather than repeating the process with an SDP as in Alg. 1. This drastically reduces the complexity of the sensor selection problem, which we will corroborate with simulations in the following section.

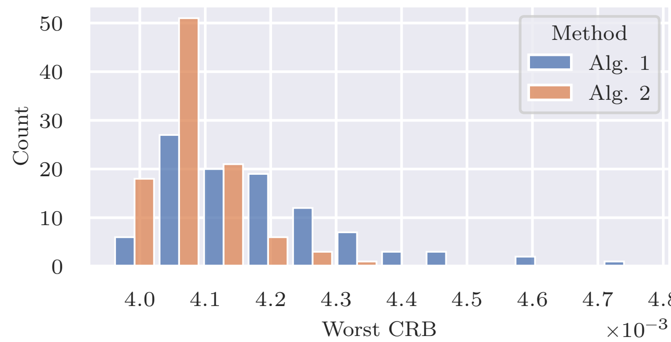


Fig. 3: The worst CRBs for different initializations when $K = 28$.

V. Numerical Results

To verify our proposed method, we performed Monte Carlo trials to investigate the performance of the discussed methods in terms of solve-time and the achieved CRBs. We take $N_{\mathrm{t}} = N_{\mathrm{r}} = 32$, $M_{\mathrm{t}} = M_{\mathrm{r}} = 16$, and a half wavelength spaced candidate ULA. We performed 20 trials, each for different values of K. For each value of K, the targets are equally spaced in their AoI. For each trial, both methods use the same $\tilde{\boldsymbol{p}}_{\mathrm{t}}^{(0)}$ and $\tilde{\boldsymbol{p}}_{\mathrm{r}}^{(0)}$. The stopping condition for both methods is $\lambda_{\min}(\boldsymbol{\mathcal{I}})$ not improving by 10^{-3} for three consecutive iterations.

We compare the methods using three different metrics. First, we evaluate the methods by comparing the worst-case of their resulting sensor selections. The worst-case CRB is given by $\mathrm{CRB_w} = \max\left(\mathrm{diag}\left(\boldsymbol{\mathcal{I}}^{-1}\right)\right)$. This will indicate whether the methods produce selections that perform similarly or that one is decidedly better than the other. Second, we compare the total time needed to perform the methods on equal computer platforms using off-the-shelf convex solvers. Details of the implementation and comparison are available in a supplementary code notebook.[4] Last, the time taken to solve one iteration of alternating optimization on equal computer platforms using off-the-shelf convex solvers between each algorithm is compared. To be precise, one iteration refers to executing lines 4 through 6 of Alg. 1, and lines 6 through 8 of Alg. 2.

The results of the CRB comparison over multiple random initializations are presented in Fig. 3. It shows that the difference in outcomes of both methods is small, with Alg. 2 being slightly more consistent. The total solve-time of the methods as the number of targets changes is given in Fig. 4a. We see that our hypothesis is confirmed: The solve-time of Alg. 1 climbs much faster. There is some fluctuation in the solve-time of Alg. 2, which we found to be due to the stopping condition. As Fig. 4b shows, Alg. 2 typically requires more iterations before the stopping condition is met. The stopping

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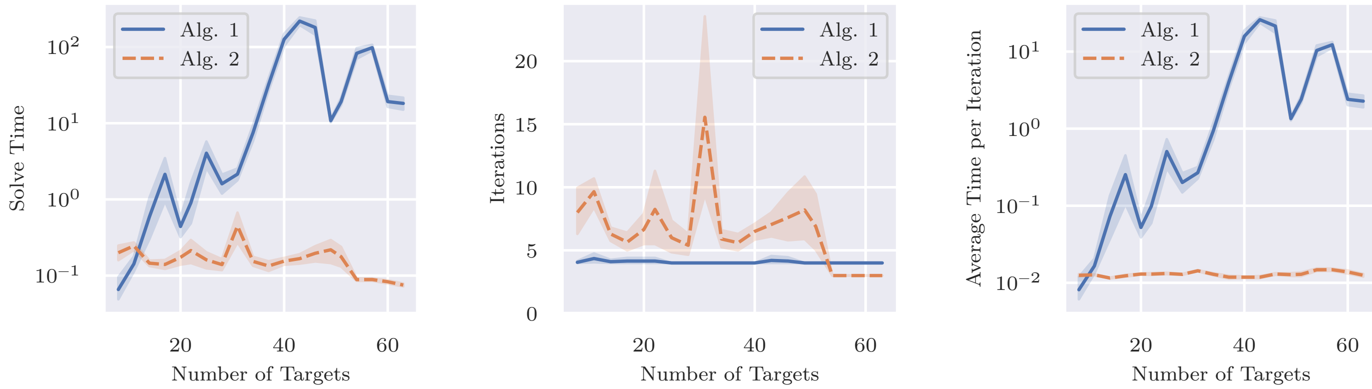

(a) Total solve-time of the algorithms.

(b) The number of iterations of the alternating section of the algorithms.

(c) Average solve-time per iteration of the alternating sections of the algorithms.

Fig. 4: The Monte-Carlo simulation results, for $N_\text{t} = N_\text{r} = 32$ and $M_\text{t} = M_\text{r} = 16$. In these figures, DA refers to Alg. 1, and UM refers to Alg. 2. The shaded regions in Figs. 4a and 4c are the $95\,\%$ confidence intervals.

condition is based on the FIM, which is not part of (14). We chose this stopping condition for a fair comparison between both methods, but in practice, a condition tailored to the optimization problem would be more effective. In Fig. 4c, we see the QP solve-time per iteration is much lower than that of the SDP for a larger number of targets. As expected, the solve-time per iteration of Alg. 2 does not scale with the number of targets, only the number of available transceiver positions.

VI. Conclusion

We have shown a joint transmit and receive sensor selection method using the VAW vector. Compared to solving directly the bilinear problem for the selection vectors, our method has the benefit of lower complexity in the presence of many targets. Additionally, the solve-time of our method fluctuates less with the addition or removal of targets from the scene, leading to better predictability and consistency of the execution time. For future work, we will extend this method to unknown AoIs, estimating AoIs, and non-co-located transmit and receive arrays. We would also like to improve the solve-time by reducing the number of iterations needed for convergence.

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