Deeper piping erosion

Finding how to model it and what contributes to the emergence of a deeper pipe

Master Thesis

Steven Coevert

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Finding how to model it and what contributes to the emergence of a deeper pipe

by

Steven Coevert

In partial fulfillment to obtain the degree of Master of Science at Delft University of Technology.

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Cover Image: Especially steep dike along the Schie canal between Delft and Rotterdam. Own image.

Abstract

The Netherlands is a country that is being threatened by water, both from the rivers and from the sea. The Dutch have built dikes to keep their lands from inundation. To ensure the strength and stability of these dikes, they are being assessed on the basis of several failure mechanisms. One of these failure mechanisms is Backward Erosion Piping, or piping for short.

In piping, the current underneath a dike is strong enough to take soil particles with it. Tests on piping in tidal subsoil were conducted in the summer of 2021, where a pipe was found to have grown at greater depth than expected The occurrence of this deeper piping has rarely been seen before, let alone described. This lack of knowledge poses a potential safety risk, as it may underestimate the vulnerability of certain subsoil configurations. Therefore, the objective of this thesis is to develop a comprehensive understanding of deeper piping and identify the key parameters influencing its formation.

To achieve this objective, a definition of deeper piping and its differentiation from conventional piping is established. Sub-mechanisms governing deeper piping are examined by analysing the forces responsible for grain movement and the forces that maintain grain stability. A Finite Element Model of the subsoil is constructed to quantify the driving forces within the subsoil, which, when combined with resisting forces, enables the determination of whether deeper piping can occur in a given subsoil configuration.

To investigate the factors contributing to deeper piping, a series of simulations are conducted using this Finite Element Model. By varying the parameter values while keeping other factors constant, the influence of each parameter on the occurrence of deeper piping was examined. The analysis revealed that several key parameters significantly affect deeper piping formation, including cohesion force (c), cohesion anisotropy (α_c), permeability and thickness of the top layer (k_0 and D_0, respectively), permeability of underlying layer (k_1), permeability anisotropy (α_k) and representative grain diameter d_{rep}. Also, it was found that the entrance configuration plays a large role in deeper pipe formation.
These findings provide valuable insights into the mechanisms underlying deeper piping and enhance our ability to identify subsoil configurations that are prone to this phenomenon. These findings enhance the identification of subsoil configurations prone to deeper piping, thereby improving risk assessment and mitigation strategies associated with this failure mechanism.

Preface

Dear reader,

Before you lies my master thesis, which partially fulfills the requirements for obtaining a master's degree in Civil Engineering at the Delft University of Technology. It is the result of a long, informative and educational journey. Writing a master thesis has been the most evident example of the fact that getting a message across can be very challenging. One can have a clear picture in your head of what to tell, but if one fails to convey that message to the reader, the objective is not reached. In real life, if someone does not understand what you're saying, you can rephrase the explanation until they get it. In a thesis, you're stuck with the words you choose. This has become all too clear in the past few months, which is something to learn from.

Luckily, I've had a great help from my committee members, pointing out things that were formulated in an unclear way. Also, you provided me with valuable insights and feedback, and made me see that piping is a very complex mechanism to describe from scratch. Wim, Bram, Stef and Lisa, I would like to thank you for your continuous constructive feedback.
Special thanks goes out to Juan, my daily supervisor and chairman, for having our weekly talks. You kept me on track when I tended to wander off and helped me become a COMSOL ninja, as you often said ;)

Someone else I would like to thank from the bottom of my heart is Romy, who is reading the final version of my thesis right now, as I'm writing this. You've read my thesis so often and talked to me so much about backward erosion piping, that I think you're an expert on this subject yourself now ;) Also, the much-needed emotional support was very much appreciated during the hard times. I'm going to have a hard time ever repaying you for that.

I would also like to thank my family for the continued interest in my progress and the emotional support throughout the process. Unfortunately, both of my grandfathers did not live to see me graduate university, which is something I really would have wanted them to be able to see, but some things are beyond our control. Opa Fred, opa Coevert, ik heb het eindelijk gered! Ik hoop dat jullie trots op me zijn.

The final 'thank you' goes out to my parents specifically. Thank you for supporting me emotionally and financially throughout the last nine years. As soon as I've paid off my student loan (or at least nearly so), I hope to have saved enough money to send you on a nice holiday together.

Steven Coevert,
21st of June, 2023

Contents

List of symbols

Symbol	Definition	Unit
$A_{c,hor}$	Contact area to which cohesion in horizontal direction applies	m^2
$A_{c,ver}$	Contact area to which cohesion in vertical direction applies	m^2
a_{max}	Pipe width used in model Geometry	m
c_{hor}	Cohesion in vertical direction	kPa
c_{ver}	Cohesion in horizontal direction	kPa
D_0	Thickness of Layer 0	m
D_1	Thickness of Layer 1	m
D_2	Thickness of Layer 2	m
D_3	Thickness of Layer 3	m
D_h	Thickness of erosion channel in fictitious permeability formula	m
$d_{50,0}$	Grain diameter in Layer 0 for which 50% is smaller	m
d_{rep}	Representative grain diameter	m
$d_{rep,0}$	Representative grain diameter of Layer 0	m
$d_{rep,1}$	Representative grain diameter of Layer 1	m
$d_{pipe\ top}$	Distance between top of pipe and ground level	m
F_B	Buoyancy force	N
$F_{c,hor}$	Cohesion force in horizontal direction	N
$F_{c,ver}$	Cohesion force in vertical direction	N
F_{down}	Downwards directed force	N
$F_{flow,x}$	Flow force in x direction	N
$F_{flow,y}$	Flow force in y direction	N
$F_{forcing,h}$	Driving force in horizontal direction	N
$F_{forcing,v}$	Driving force in vertical direction	N
F_G	Gravity force	N
$F_{resisting,h,max}$	Maximum resisting force in horizontal direction	N
$F_{resisting,v,max}$	Maximum resisting force in vertical direction	N
F_{up}	Upwards directed force	N
H	Hydraulic head level	m
k_0	Permeability of Layer 0	m^2
k_1	Permeability of Layer 1	m^2
k_2	Permeability of Layer 2	m^2
k_3	Permeability of Layer 3	m^2
k_{pipe}	Fictitious permeability of a pipe	m^2
k_{soil}	Permeability of the soil	m^2
$L_{Cut\ line}$	Length of the Cut line	m
n_g	Number of grains	$-$
x	Coordinate	m
$x_{vertical\ pipe}$	X-coordinate of the left side of the pipe	m
y	Coordinate	m
α_{coh}	Cohesion anisotropy factor	$-$
α_k	Permeability anisotropy factor	$-$
γ_s	Effective weight of soil	N/m^2
γ_s'	Weight of soil	N/m^2
γ_w	Weight of water	N/m^2
π	Approximately 3.14159265358	$-$
ϕ	Friction angle of the soil	$^\circ$

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Glossary

Anisotropy
When a material has different properties depending on direction. In the case of hydraulic conductivity, this means that soils are more conductive in horizontal direction than in vertical direction.

Aquifer
Permeable, water-containing layer of soil, through which water can flow easily.

Cohesion
Attraction between small particles, allowing them to 'cluster' together.

Conventional pipe
A pipe located on the interface between water-retaining structure and subsoil.

Darcy's law
A relation between the hydraulic head gradient and the flow velocities, through the permeability. Only applicable to laminar flows.

Dike
A water-retaining, earthen embankment.

Deeper pipe
A pipe located deeper than a conventional pipe.

Erosion
The removal of material as a result of, in this case, flow of water.

Erosion, Primary
The removal of material at the tip of a pipe, resulting in an increased pipe length.

Erosion, Secondary
The removal of material along the bed of the pipe, transporting away the material that was removed during primary erosion. Secondary erosion might also contribute to the widening of the pipe.

Fictitious Permeability
A way of describing a water-filled pipe as a soil domain with very large permeability.

Gradient
How quickly a certain value changes in space.

Heave
The mechanism where the hydraulic head gradient inside the soil is large enough to overcome the weight of the grains, resulting in the grains losing contact and the soil liquefying.

Hinterland
The area on the protected side of a water-retaining structure.

Hydraulic Conductivity
Hydraulic conductivity can be described as how easily water can pass through a medium. It is not the same as permeability, though is it related to it. However, next to permeability, the hydraulic conductivity

also contains information about density and viscosity.

Hydraulic Head
How high water would rise if one were to apply an observation well in the ground at a certain depth.

Impervious
Something is impervious when nothing can pass through it.

Interface
The plane or line along which something ends and something else begins.

No-flow boundary
A boundary in a Finite Element Model through and along which no flow occurs.

Permeability
The ability of a medium to convey a fluid. This is not the same is hydraulic conductivity, although it is related to it.

Sensitivity analysis
The process of modelling something, then changing a value slightly and observing the difference in the outcome caused by this change.

Sub-mechanism
A process that is part of a larger process, in this case, a failure mechanism.

Subsoil
The soil beneath ground level.

Uplift
The mechanism where the pressures underneath an impermeable blanket are such, that it gets lifted up. It is one of the first sub-mechanisms to occur in conventional and deeper piping.

1
Introduction

The Netherlands is a country that has been threatened by water, both from the sea and from rivers, for centuries. The Dutch have built dikes to prevent their lands from being inundated. Dikes can fail in several ways, one of them being known as Backward Erosion Piping, or piping for short. Piping is a phenomenon where sand is eroded from underneath the dike due to groundwater flow. To make this possible, first, the impermeable soil layer on the surface (also termed the blanket layer) bursts due to high pressures underneath (this is termed uplift), after which water starts flowing due to a head gradient. If this flux of water is strong enough, the water starts eroding sand particles away from under the dike. See Figure 1.1 below.

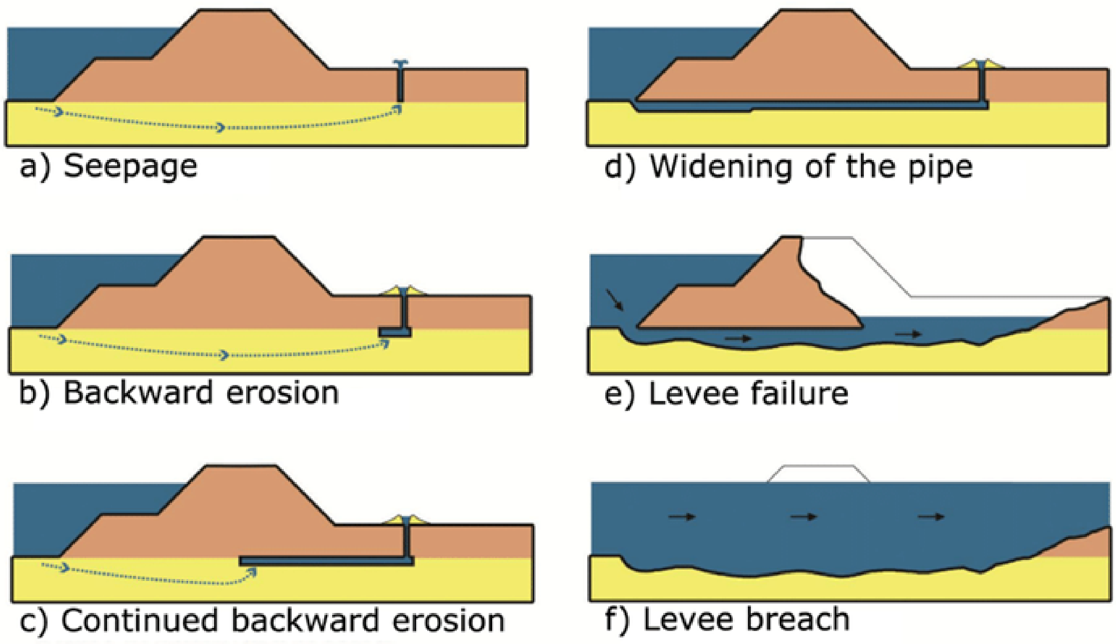


Figure 1.1: Piping mechanism explained (Van Beek, 2015[26])

Piping has been the subject of research for many years. One of the first researchers to describe an equation on piping was Bligh (Bligh, 1910[6]; Bligh, 1916[7]). He related the head difference and seepage length to the piping phenomenon through a soil constant. After Bligh, many researchers have continued working on this topic, one of whom being Sellmeijer (Sellmeijer, 1988[20]; Sellmeijer, 2011[19]), who conceptualised the groundwater system during piping and solved it analytically. His model is used a lot in daily practice nowadays. Later still, Hoffmans and Van Rijn (Hoffmans, Van Rijn, 2018[12]) proposed yet another approach, which is being criticised, but contains an interesting view on sand transport (Pol, 2020[18]). All of these models are based on the assumption that the pipe forms at the interface between an impervious body (a dike, concrete structure, etc.) and a permeable subsoil (also termed the aquifer).

This assumption turned out to not always be true (Van der Linde, personal communication). In section 1.1, the problem with this assumption is stated and an observation from a series of two experiments from 2021 is explained. This is converted into the objective of this thesis in section 1.2. Also, the research sub-questions used to reach this objective is presented there. Next, the research method is elaborated on in section 1.3. It describes how the research sub-questions are answered, ultimately leading to fulfilling the objective of the thesis. Lastly, in section 1.4 the outline of the thesis is presented.

1.1. Problem description

In 2021, Fugro, Deltares and Waterschap Hollandse Delta conducted full-scale piping experiments in the Hertogin Hedwigepolder (HHP). This is an area where the tide has played a big role in the subsoil formation. These tidal deposits contain much smaller grains than deposits along rivers. This is because the larger grains have already been deposited farther upstream (Van den Berg, Boersma, Van Gelder, 2007[3]; Wiersma, 2018[14]).
During these piping experiments, it was observed that a pipe had grown at a greater depth than assumed by the current knowledge and models: the pipe had not formed at the interface between the dike and the aquifer, but deeper: under a sandy clay layer (see figure 1.2). Whether or not this layer can be counted as another part of the blanket, is out of the scope of this thesis, and is, therefore, not be treated in-depth.
A pipe growing at a greater depth than this dike-aquifer interface has rarely been observed before. It is yet to be well-understood how a deeper pipe comes into being, or what subsoil configurations (the combination of dimensions and properties) contribute to the formation of a deeper pipe.
This makes it harder to apply proper piping measures: contractors would simply apply their solution to the piping problem at the depth of a 'conventional' pipe, whereas apparently, this may not be sufficient in some cases.

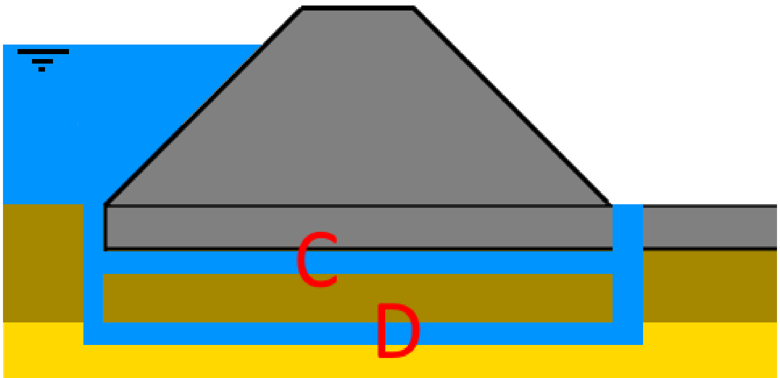


Figure 1.2: Conventional (C) and Deeper pipe (D) visualised

1.2. Objective and Research Questions

The objective of this thesis is:

"To identify the key factors influencing the formation of deeper pipes."

For this objective to be reached, it is useful to split it up in parts. The questions answered in this thesis are:

1. *What is the difference between conventional piping and deeper piping?*
2. *How can a deeper pipe be modelled?*
3. *Can deeper piping in the Hedwigepolder be explained by the model?*

4. *What are the most important parameters contributing to a deeper pipe?*

Answering the first sub-question gives insight in the process of deeper piping. It also sets the framework for the thesis, namely, what the difference between conventional piping and deeper piping is. It also requires the description of a way to describe deeper piping. After answering it, an idea of what might be the mechanisms behind deeper piping is established.

Answering the second sub-question defines how a Finite Element-based subsurface flow model can be built and how it can be used to describe whether or not deeper piping is to be expected. The similarities between conventional piping and deeper piping are not treated: The focus lies exclusively on the differences.

The third question acts as a verification of the model. As there was little data from the Hedwigepolder experiment, the only way of verifying the model is to implement the Hedwigepolder experimental set-up and confirming that deeper piping indeed takes place in that set-up, according to the model.

The answer to the last question helps in understanding which properties are important for the deeper piping mechanism. This helps identifying in which cases deeper piping is to be expected.

1.3. Methodology

The first question is answered by studying conventional piping and how this is modelled. A literature study suffices to achieve this. Next, the deeper piping process is described in a similar way as found for conventional piping in literature. The concept of the 'Force Ratio Parameter' (FRP) is introduced here as being the maximum resisting forces divided by the maximum driving forces. These are described more elaborately in the next phase. Mind that these are not equal to the commonly used Factor of Safety: though they are calculated identically, the FRP does not automatically imply safety.

The second question aims at finding a way to model the subsurface, such that it describes the flow pattern in the soil. This is done by means of a literature study. Upon modelling the subsoil, the obtained flow pattern is converted to flow forces, driving the erosion mechanism. Also, the maximum resisting forces are quantified. The FRP values can now be quantified more thoroughly. These FRP values are then used to determine whether or not deeper piping takes place. This cannot be done by means of a literature study, as this is new.

The third part is carried out by setting up the model as described in the previous part, and implementing the geometry and boundary conditions from the Hedwigepolder. By running the model and verifying whether or not deeper piping occurs for this model using the FRP values defined earlier, the model is verified. **Important note 1**: The Hedwigepolder experiment serves as a reference case only: it is not the aim of this thesis to develop a detailed model of the Hedwigepolder experiment. However, a deeper pipe was observed there, so it is expected that if the model runs correctly, it indeed returns a deeper pipe when the Hedwigepolder set-up is adopted.
Important note 2: it is not the goal to develop a fully functional model for deeper piping to be generically applied, the goal of the model is help verify different hypotheses about deeper piping.

The last question is answered by conducting a sensitivity analysis. By changing the parameters one by one and keeping the other parameters the same, the influence of this parameter on the deeper piping process is quantified. The parameters which influence the FRP values the most are deemed the most important.

1.4. Thesis Outline

In Chapter 2, a deeper piping description is proposed. Next, in Chapter 3, a model to describe deeper piping is described. After that, in Chapter 4, the model is verified. Next, in chapter 5, the most influential parameters are identified. The discussion of the approach and results are found in Chapter 6, after which the conclusion is drawn in chapter 7. The thesis is concluded by a set of recommendations in Chapter 8.

2

Difference between conventional piping and deeper piping

In this chapter, the conceptual difference between conventional piping and deeper piping is established. In section 2.1, the conventional piping process is briefly treated and compared to the proposed description of a deeper pipe. In section 2.2, the sub-mechanisms involved in deeper piping are treated. Lastly, in section 2.3, possible causes for deeper piping are stated, forming a hypothesis on what is expected to influence deeper piping.

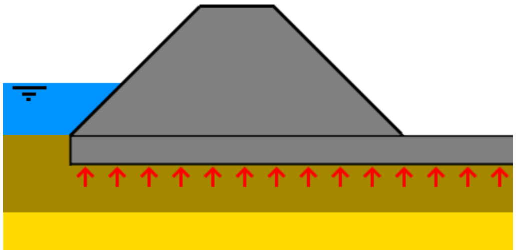
2.1. Conceptual difference

In this section, the conceptual difference between conventional piping and deeper piping is discussed. This is done by first explaining the concept on conventional piping. After that, the differences between this form of piping and the deeper variant is treated.

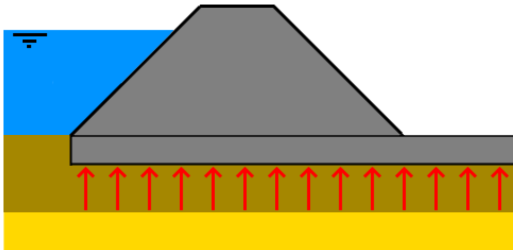
2.1.1. Conventional piping

The conventional piping process is visualised in Figure 2.1 below. Piping only occurs in case of a high water event (Figure 2.1b). This high water results in an increased pressure in the subsoil (also Figure 2.1b). This increased pressure might get large enough to lift up the blanket layer (hence the name 'uplift', Figure 2.1c). If the blanket is deformed too much, it is expected to burst (Tao, 2020[23]) (Figure 2.1d). This is the first requirement of piping to take place. Now there is a hole in the blanket layer. If the head gradients inside the subsoil are large enough to overcome the weight of the soil, the grains lose contact (the effective stresses reduce to zero), which means they are brought into suspension (Deltares, 2012[10]) (also termed 'heave', Figure 2.1e).
Now that the soil has been fluidised, the flow moves the grains out of the aquifer (Figure 2.1f). The removal of sediment at the tip of the pipe is called primary erosion. Primary erosion leads to lengthening of the pipe (Figures 2.1f, 2.1g and 2.1h). As the pipe gets longer, the grains that are loosened at the tip need to be able to be transported to the exit hole. This is termed secondary erosion. The process of primary erosion and secondary erosion repeats until the pipe has grown all the way to the upstream water body (Figure 2.1h). Once this has occurred, the pipe flow velocities increase, accelerating secondary erosion, which then leads to widening of the pipe (Figure 2.1i). At some point, so much soil has been removed from the subsoil that the dike on top of it is not properly supported any more, leading to settlements and ultimately, collapse of the dike (Figure 2.1j).

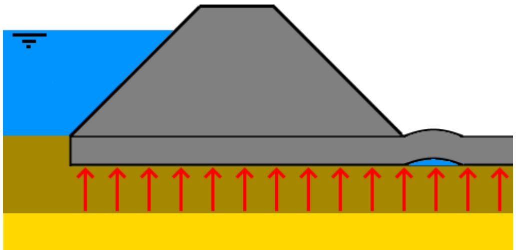
In Figure 2.2, the process is shown in a flow chart. It can be seen that after the uplift and heave phases, horizontal erosion is initiated. After this primary erosion and secondary erosion make the pipe longer and wider, until it has reached the upstream water body.



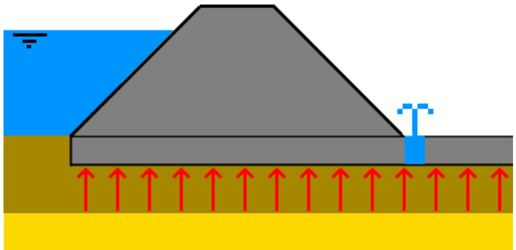
(a) Normal situation



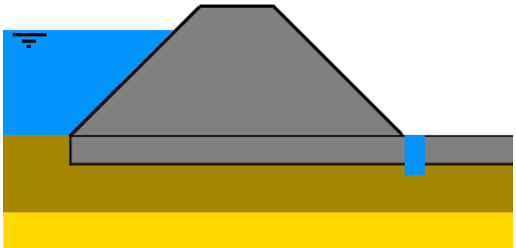
(b) High water, pressures increase



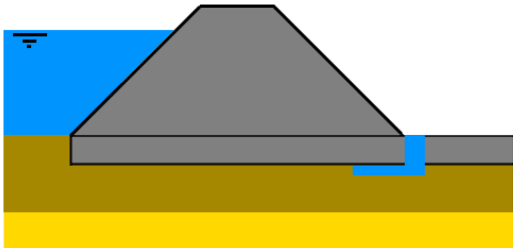
(c) Blanket gets lifted up



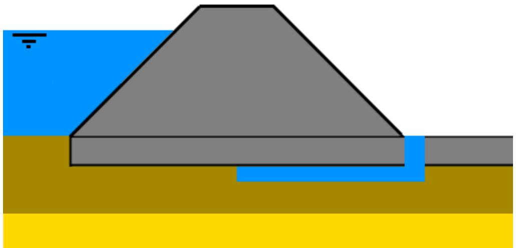
(d) Blanket bursts



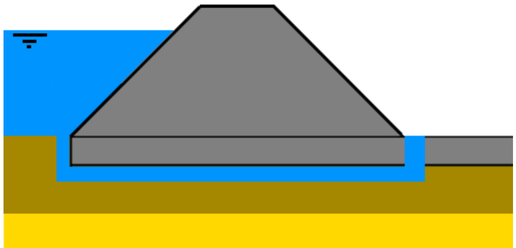
(e) Heave loosens up the soil



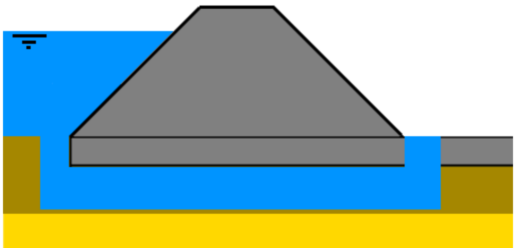
(f) Loosened soil gets flushed away, a pipe forms



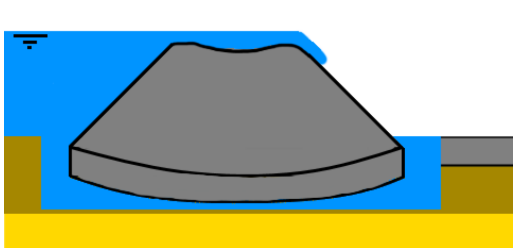
(g) The pipe gets longer



(h) The pipe reaches the upstream water body



(i) The pipe widens rapidly



(j) The dike collapses

Figure 2.1: Conventional piping process. Blue: water. Grey: impermeable dike/blanket. Brown: sand layer 0. Yellow: sand layer 1

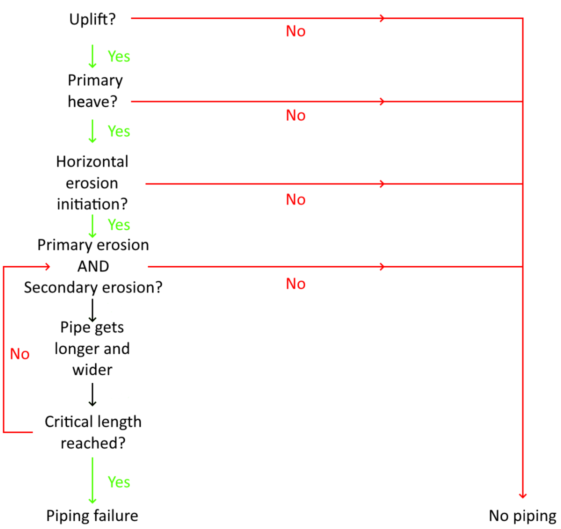
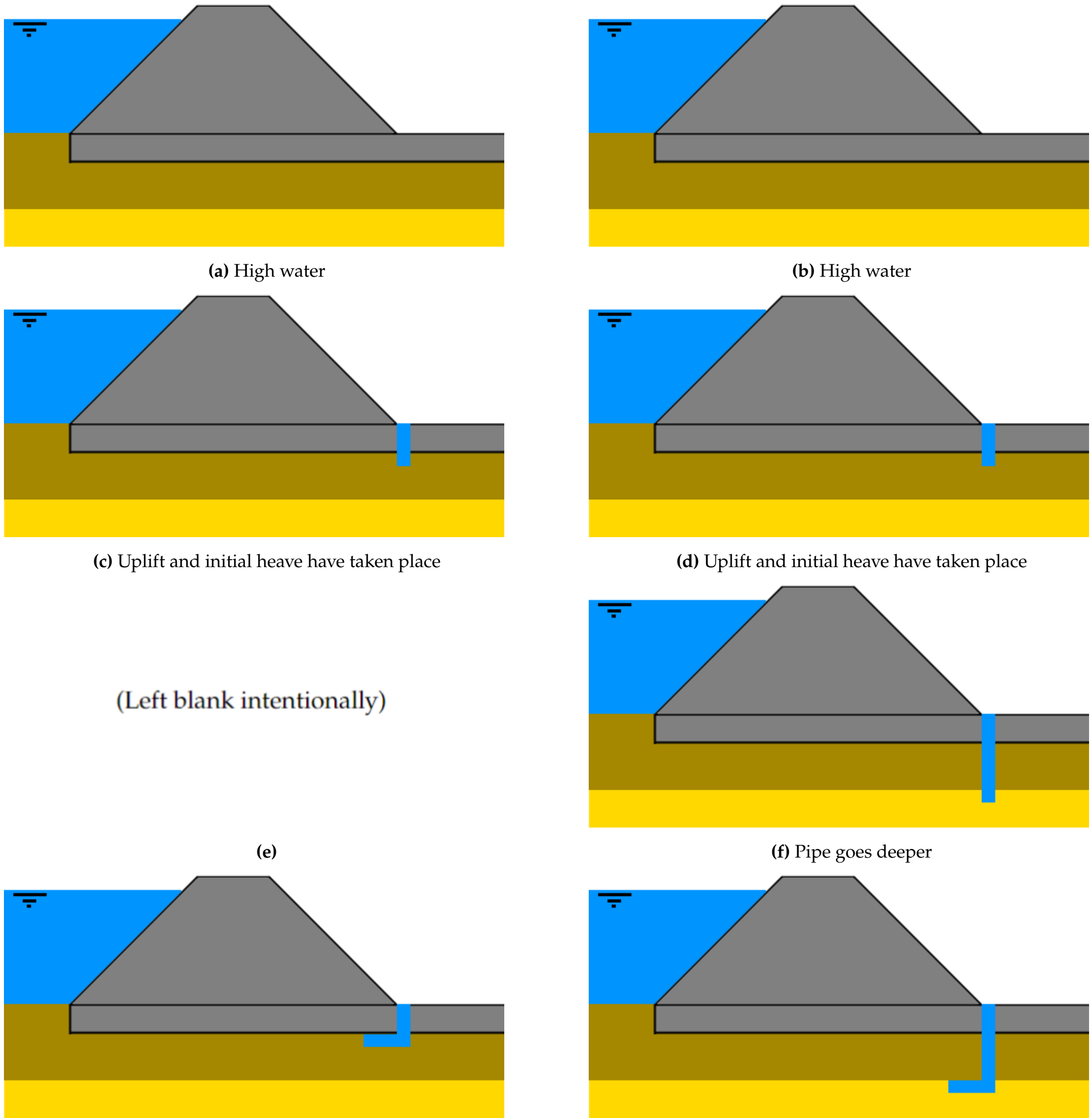


Figure 2.2: Conventional piping flow chart

2.1.2. Deeper piping

In deeper piping, the horizontal erosion phase is not initiated until after the pipe has first developed in vertical direction. This is shown in Figure 2.3. Contrary to the conventional piping case, the upper layer (the brown in these figures) depicts a sandy clay layer. Four sub-mechanisms need to be satisfied. First, horizontal erosion should *not* be initiated after the initial heave phase. Secondly, the heave process should continue below the part of soil where the initial heave has taken place. Third, the heaved soil at the bottom of the newly forming (deeper) pipe should be eroded and, lastly, this eroded soil needs to be transported away. These four criteria are shown in Figure 2.4 and are explained in subsections 2.2.2 through 2.2.5. As soon as the pipe has reached a depth where the horizontal erosion *is* initiated, the deeper piping process continues like a conventional piping process: primary and secondary erosion take place until the pipe has reached the upstream water body.



(a) High water

(b) High water

(c) Uplift and initial heave have taken place

(d) Uplift and initial heave have taken place

(e)

(f) Pipe goes deeper

(g) Horizontal pipe growth

(h) Horizontal pipe growth

Figure 2.3: Difference conventional and deeper piping. Left column: conventional piping. Right column: deeper piping

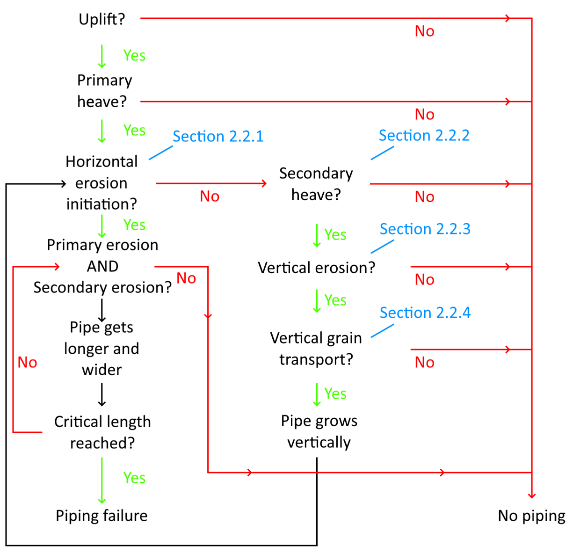


Figure 2.4: Deeper piping flow chart

2.2. Sub-mechanisms of deeper piping

Four sub-mechanisms are decisive in the formation of a *deeper* pipe, rather than a *conventional* pipe. A deeper pipe emerges when four criteria are met. First of all, horizontal erosion should not take place right after the 'initial' heave phase. This mechanism is treated in subsection 2.2.2. Next, more heave below the already-heaved part of the subsoil is necessary. This is treated in subsection 2.2.3. Then, the newly-heaved soil should be eroded from the soil skeleton as treated in subsection 2.2.4. Lastly, this soil should be transported away through the erosion channel as shown in subsection 2.2.5. However, first of all, the concept of the Force Ratio Parameter (FRP) is introduced. With the FRP, it is determined whether or not a sub-mechanism occurs.

2.2.1. Force Ratio Parameter

Several sub-mechanisms need to occur at certain points in time, while others do not. It is therefore useful to introduce a parameter that determines whether or not a sub-mechanism takes place. A sub-mechanism is considered to take place if the forces starting the mechanism are larger than the forces trying to keep the mechanism from happening. This happens if the driving forces surpass the maximum resisting forces, breaking the particle equilibrium. The ratio of these forces (the driving force divided by the maximum resisting force) is named the FRP. This is not the same as a Factor of Safety (FoS), as the FoS determines whether something happens or not and if it does not happen, the situation is safe. The FRP, however, is also used to determine whether conventional piping occurs or not, which does not guarantee safety if one of the mechanisms does not happen.

$$FRP = \frac{Resisting\ force}{Driving\ force} \tag{2.1}$$

If the driving forces are larger than the resisting forces, the value of FRP is smaller than 1. If this is the case, the mechanism occurs. Conversely, if the maximum resisting force is larger than the driving force, FRP is larger than 1 and the mechanism is not activated. In case the FRP of multiple sub-mechanisms are smaller than 1 at the same time, it is expected that the mechanism with the lowest FRP has higher chances to progress than the one with the highest FRP. For example, if $FRP_{horizontal\ erosion} = 0.75$ and $FRP_{vertical\ erosion} = 0.5$, it is assumed that vertical erosion takes place, and horizontal erosion does not.

2.2.2. Horizontal erosion initiation

The first mechanism under consideration is the initiation of horizontal primary erosion. This mechanism is checked when the initial heave mechanism has just taken place. The situation looks as in Figure 2.5a. In this Figure, the driving and resisting forces (red and black arrows) acting on a grain (the yellow dot) are shown. For a conventional pipe, the horizontal erosion stage starts at this moment. However, for deeper piping, the horizontal erosion is not initiated at this moment yet. In terms of the FRP:

$$FRP_{horizontal\ erosion} > 1 \tag{2.2}$$

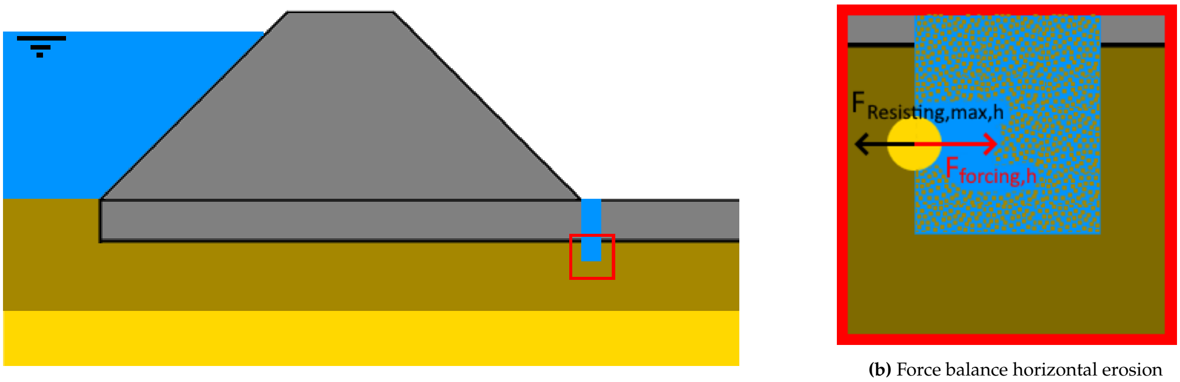
Or, if both $FRP_{horizontal\ erosion}$ and $FRP_{vertical\ erosion}$ are smaller than 1:

$$FRP_{horizontal\ erosion} > FRP_{vertical\ erosion} \tag{2.3}$$

Where:

$$FRP_{horizontal\ erosion} = \frac{F_{Resisting,max,h}}{F_{Forcing,h}} \tag{2.4}$$

as shown in Figure 2.5b, and $FRP_{vertical\ erosion}$ as described in subsection 2.2.4. This formula is elaborated on in chapter 3.



(a) Location under consideration with respect to dike geometry

(b) Force balance horizontal erosion initiation

Figure 2.5: Horizontal erosion initiation. With grain under consideration (yellow dot), rest of the soil (brown) and the beginning pipe (blue)

2.2.3. Continued heave below already-heaved soil

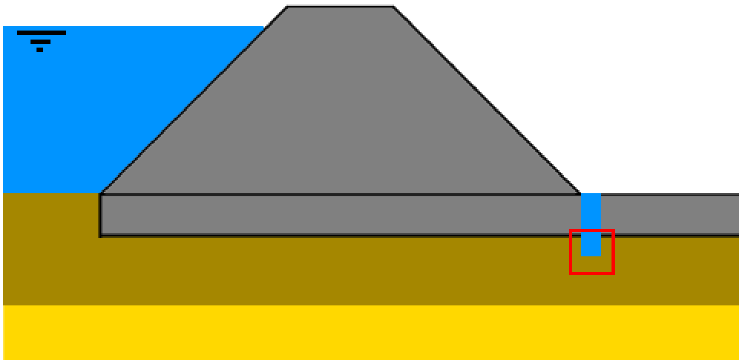
Before the flow can erode more particles from the soil bed, the soil at the bottom of the erosion channel first needs to be loosened as shown in Figure 2.6b. This heave process is identical to the 'initial' heave process, but in deeper piping, it keeps occurring underneath the pipe tip for as long as horizontal erosion does not take place. The FRP for this continued heave sub-mechanism is shown in equation 2.5, where i and i_{cr} are the occurring hydraulic gradient (the change in pressure head in space) and the critical hydraulic gradient (the maximum hydraulic gradient that can be resisted without fluidisation), respectively. These are further elaborated on in chapter 3. Now that the upper layer consists of sandy clay, a cohesion term needs to be taken into account. This is done by including cohesion in the erosion sub-mechanisms (both horizontal and vertical). The 'continued heave' phase is to show the 'willingness' of the soil to fluidise, if not for cohesion.

$$FRP_{heave} = \frac{i_{cr}}{i} \tag{2.5}$$

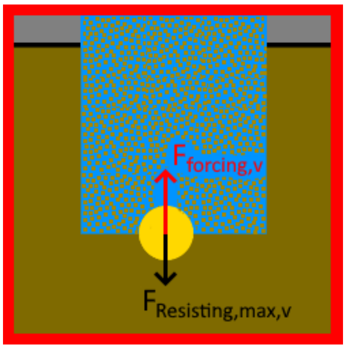
2.2.4. Vertical erosion

Because of the soil being heaved below the pipe bottom, the grains on the bottom lose contact and the friction between grains diminishes. This makes the grain more susceptible to being removed. Vertical erosion makes the pipe longer (in vertical direction) and the $FRP_{vertical\ erosion}$ is given by equation 2.6. This formula is elaborated on in chapter 3. The terms in the numerator and the denominator are as shown in Figure 2.6b.

$$FRP_{vertical\ erosion} = \frac{F_{Resisting,max,v}}{F_{Forcing,v}} \tag{2.6}$$



(a) Location under consideration with respect to dike geometry



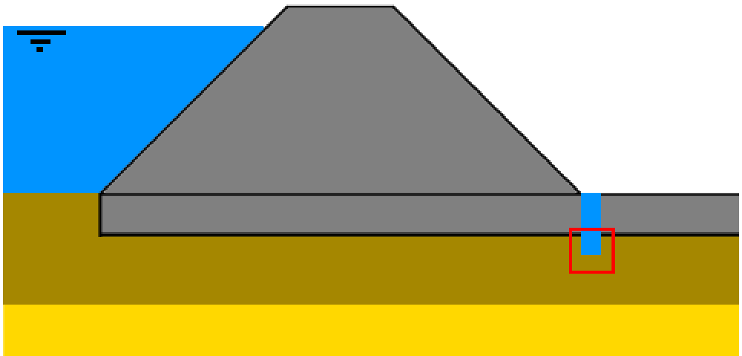
(b) Force balance vertical erosion

Figure 2.6: Vertical erosion initiation

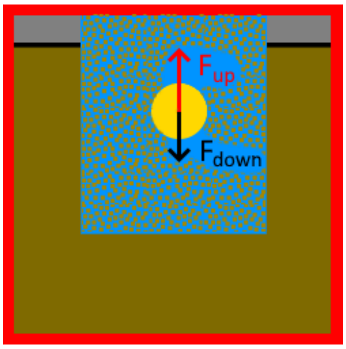
2.2.5. Vertical grain transport

After the grain has been eroded, it needs to be transported out of the system through the erosion channel (much like secondary erosion, but vertical). In order for this to happen, the vertical forces acting on a grain should be large enough to move it vertically. This is described by equation 2.7 and Figure 2.7.

$$FRP_{ver\ tra} = \frac{F_{down}}{F_{up}} \tag{2.7}$$



(a) Location under consideration with respect to dike geometry



(b) Force balance vertical transport

Figure 2.7: Vertical grain transport

2.2.6. When does deeper piping occur?

A sub-mechanism takes place if its FRP is below 1. For deeper piping to occur, specifically, the following FRP's should be satisfied:

1. Heave should take place, so $FRP_{heave} < 1$
2. Horizontal erosion does not take place in the top layer ($FRP_{horizontal\ erosion} < FRP_{vertical\ erosion}$) and it *should* take place in a layer below the top layer ($FRP_{horizontal\ erosion} > FRP_{vertical\ erosion}$)
3. Vertical erosion takes place until a layer is found where horizontal erosion takes is initiated, $FRP_{vertical\ erosion} < 1$
4. In all occasions, vertical transport takes place $FRP_{vertical\ transport} < 1$

2.3. Possible physical causes of deeper piping

In this section, some properties and subsoil configurations expected to contribute to deeper piping are treated.

2.3.1. Properties

Conventional piping is assumed to take place in a cohesionless, isotropic, homogeneous subsoil. However, deeper piping is found in cohesive, anisotropic, layered soils. Therefore, it is hypothesised that cohesion and the subsoil configuration are of large impact on the formation of a deeper pipe. These are explained briefly in this section. In chapters 3 and 4, these properties are elaborated on.

Cohesion is a soil property where lutum particles (clay particles and particles smaller than clay) of the subsoil stick together due to electrostatic forces. Clay particles are 'glued' to larger (sand) grains by means of capillary forces. This makes the soil mixture in a sandy clay more resistant against erosion. As is seen in chapter 4, the piping in the Hedwigepolder took place below such a cohesive sandy clay layer. Clay particles are shaped like plates. As cohesion acts along the surface area of the clay particles, cohesion has a directional effect (Stockton, Leshchinsky, Olsen, Evans, 2019[22]): in some directions it is stronger than in other directions. This is henceforth referred to as 'cohesion anisotropy', or α_{coh}. This may play a role in why erosion in vertical direction occurs, while it does not in horizontal direction, hence contributing to the formation of a deeper pipe over a conventional one.

The force on a grain and its direction are determined by the magnitude and direction of the groundwater flow. These are, in turn, influenced by the subsoil configuration. Subsoil configurations describe how the subsoil is built up. This means a specific combination of for example the amount of layers, the layer thicknesses, the permeability of each layer, and anisotropy of this permeability. The subsoil configurations treated in this thesis are a combination of layer thicknesses, permeability and anisotropy of the permeability.

Cohesion contributes to the resisting term of two out of four of the sub-mechanisms described in section 2.2, as is described in chapter 3. The flow determines the forcing term of all sub-mechanisms, as is seen in the same chapter. This underlines the importance of these properties. It is expected that these have a large influence on the deeper piping process.

2.3.2. Subsoil configurations assisting the formation of a deeper pipe

It is expected that deeper piping only occurs in very specific circumstances, otherwise it would have been seen much more often. For deeper piping to occur, the water must flow into the ditch vertically as much as possible. This is made easier if the water is allowed to travel horizontally though a deeper layer, until it has reached below the point where it needs to flow up and out. It, therefore, is expected that subsoil configurations where two (or more) layers of distinctly different permeabilities are on top on one another, with the more permeable one below the less permeable one, as shown in Figure 2.8.
It is also expected to be beneficial for deeper piping if the water is allowed to enter the more permeable layer directly, without first having to penetrate the less permeable layer. This is, for example, be the case if the less permeable layer was excavated away during dredging of the riverbed, as shown in Figure 2.9.

In this thesis, the subsoil configuration from the Hedwigepolder is assumed. This subsoil configuration is described in chapter 4.

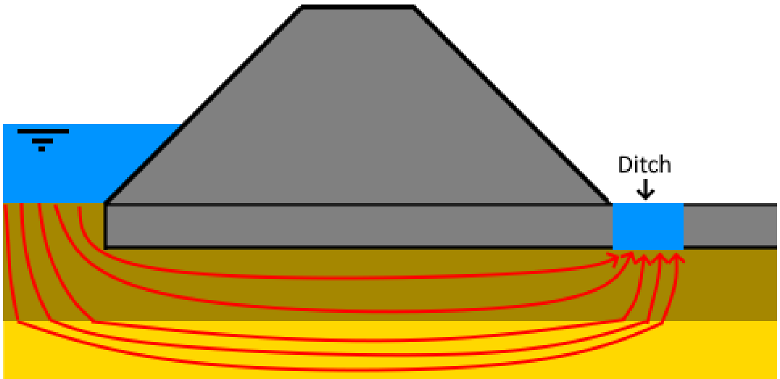


Figure 2.8: More permeable layer (yellow) below a less permeable layer (brown), below an impermeable layer (grey)

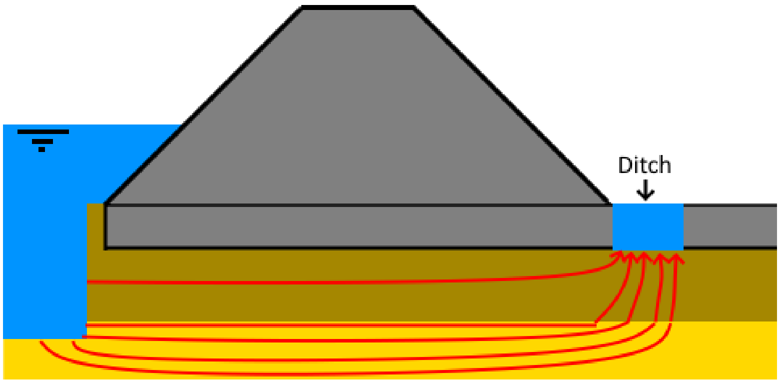


Figure 2.9: Water can enter the more permeable layer (yellow) easily

3
Model set-up

In this chapter, it is described how a 2D Finite Element Model, thatis used to describe deeper piping, can be built. The model as presented in this chapter is based on approaches of Aguilar-López et al (Aguilar-López, Warmink, Schielen, Hulscher, 2018[2]) and Van Dijk (Van Dijk, 2023[27]). In section 3.1, the physics making up the groundwater and pipe flow are shown. Then, in section 3.2, it is shown how a groundwater model is built in COMSOL Multiphysics. In section 3.3, a way of modelling the effects of cohesion is proposed, and in section 3.4, the sub-mechanisms mentioned in chapter 2 are elaborated on and formulated in such a way that the model can be used to determine whether or not deeper piping occurs.

3.1. Model physics

The subsurface flow can be described using the combined concepts of Darcy's law and the fictitious permeability, which is treated in this section.

3.1.1. Darcy's law

Groundwater flow is described by Darcy's law. It describes subsurface flow. Darcy's law assumes laminar flow, which is reasonable in soil and small pipes. If the pipe, however, gets larger, turbulent flow can develop, resulting in Darcy's law being inapplicable. Darcy's law is formulated as shown in equation 3.1. From this equation it can be seen that the permeability ($k[m^2]$) is one of the factors that couples head gradients ($\nabla p[N/m^2]$) and flow velocities ($u[m/s]$). The others are the dynamic viscosity ($\mu[Pa * s]$), the density of the water ($\rho[kg/m^3]$) and gravitational acceleration ($g[N/kg]$), but these are considered to be constant.

$$u = -\frac{k}{\mu} * (\nabla p + \rho g) \tag{3.1}$$

The head gradients are the driving force behind the sub-mechanisms, as the gradients are what makes water flow. It is, therefore necessary to know the head (gradient) profile in the subsoil. Darcy's law can be used for this: in combination with a geometry and boundary conditions, the entire flow pattern in the soil can be calculated. The boundary conditions are either a flow velocity (Neumann Boundary condition) at the boundary or a head level (Dirichlet Boundary condition). These are related through equation 3.1. In this thesis, the boundary conditions are head levels, because in the Hedwigepolder, the head levels at the boundaries were controlled. These boundary conditions, along with the geometry that are used in this thesis, are further shown in chapter 4.

3.1.2. Flow regime

As mentioned in the previous subsection, Darcy's law assumes laminar flow at all times. In subsurface flow modelling, on pore scale, laminar flow means that the Reynolds number is below 10 (Dybbs, Edwards, 1984[9]). This number is calculated using equation 3.2.

$$Re = \frac{\rho * u * L}{\eta} \tag{3.2}$$

Where ρ is the density of the fluid $[kg/m^3]$, u is the flow velocity $[m/s]$, L is the characteristic length $[m]$ (in this case, the channel width) and η is the dynamic viscosity $[N * s/m^2]\ or\ [Pa * s]$.

3.1.3. Fictitious permeability

The erosion channel is a part of the domain where the soil has been removed due to erosion. There is a part of the geometry that is still soil, and there is a part that is a cavity as the result of erosion, through which water flows. Whereas the flow in soil is described using Darcy's law, other equations govern the flow in the pipe. As long as the flow in the pipe is laminar (and thus, head loss is only caused by cross-sectional dimensions and not friction), the 'fictitious permeability' model can be used to describe this flow. This model treats the erosion channel as a domain of soil with very high permeability. Based on fracture flow (the flow of water through cracks in rock formations in two dimensions) and the cubic law (that relates the discharge through a crack to the width of that crack to the power 3), Muzychka & Yovanovich (Muzychka, Yovanovich, 2009[15]) derived equation 3.3 which can be used to describe the fictitious permeability of the erosion channel. In this equation, $k_{pipe}[m^2]$ is the fictitious permeability of the pipe and $D_h[m]$ denotes the hydraulic diameter of the conduit. In case of an infinitely wide pipe (as is the case in the 2D model), this diameter is equal to the pipe height.
Using the fictitious permeability model allows the description of the entire soil-pipe domain with Darcy's law. The denominator value of 64 is based on the assumption of an infinitely wide pipe, which is reasonable for a 2D model, but might be less realistic in a 3D model.

$$k_{pipe} = \frac{2 * D_h^2}{64} \tag{3.3}$$

3.2. Implementation in COMSOL Multiphysics

For the modelling of deeper piping, the software of COMSOL Multiphysics (COMSOL Multiphysics[1]) is used. COMSOL is a Finite Element Modelling software that solves models in one, two or three dimensions. The model presented in this thesis is a 2D one. In this section, several aspects about COMSOL modelling are treated.

3.2.1. Quasi-steady modelling

While COMSOL is capable of calculating time-dependant models (for example, with boundary conditions changing in time), the model presented in this thesis is quasi-steady state. This implies that for several points in time, the flow is modelled as a steady-state process. In the model presented in this thesis, this is done for four moments in time: at the beginning stage of pipe formation (when the pipe is only a few grain diameters wide) and when the pipe has penetrated through $1/3$, $2/3$ and $3/3$ of the top layer.

3.2.2. Geometry, permeability zone modelling and properties

One of the first things to be put in COMSOL is the geometry of the model. In the case of (deeper) piping modelling, this geometry represents the subsoil. The flow in the dike body is not calculated, so the dike body itself does not have to be included in the model geometry. However, the pipe *is* an important part of the (deeper) piping mechanism, as the flow in and around it determine whether or not deeper piping occurs. A problem, though, is that the size of the erosion channel (pipe) in the beginning stages of pipe formation is not known. A solution to this problem is has been found by Van Dijk (Van Dijk, 2023[27]). She calls this the 'permeability zone method'. The essence is that when inserting the geometry in COMSOL, an 'outline' of the pipe is entered. This outline, of width a_{max}, is wider than the pipe width (D_h) is ever expected to become. See Figure 3.1. By using this method, the model can be run for several erosion channel sizes without changing the mesh. Changing the mesh is very time-inefficient, the permeability zone method helps the efficiency of the process.
When the geometry domains are being assigned their properties (this is explained below Figure 3.1), the actual pipe (the blue in Figure 3.1) can be given the fictitious permeability properties mentioned in section 3.1.3. The part of the soil that *is* within the outline but is *not* actually pipe domain then gets 'regular soil' properties. This is shown in equation 3.4. The meaning of k_{soil} is explained in equation 3.5.

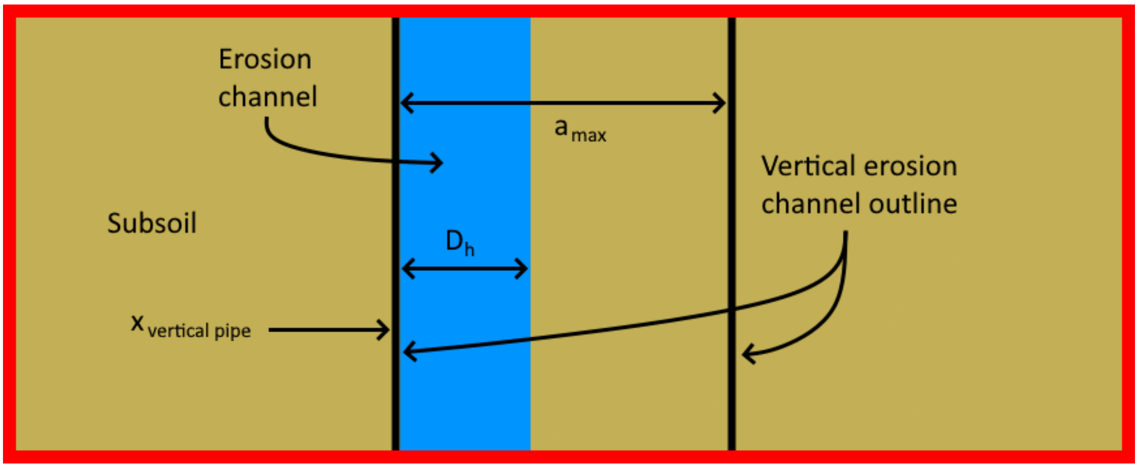


Figure 3.1: Permeability zone model

$$k = \begin{cases} k_{soil}, & \text{if } x < x_{vertical\ pipe}\ or\ x > x_{vertical\ pipe} + D_h \\ k_{pipe}, & \text{if } x_{vertical\ pipe} \leq x \leq x_{vertical\ pipe} + D_h \end{cases} \quad (3.4)$$

As was seen in chapter 2, deeper piping requires a layered subsoil. One aspect about this layering that has been investigated in chapter 5 is the thickness of the layer. It is possible to put multiple layers (with their corresponding properties) as separate layers into the COMSOL geometry. However, this implies that each time the thickness of a layer is changed in a parametric sweep (explained in section 3.2.4), a new calculation mesh needs to be created. This remeshing leads to significantly longer calculation times (if a new mesh needs to be built for every parameter under consideration, this takes much time), and induces inaccuracies and errors. This problem can be tackled by modelling the subsoil as one single entity with permeability $k_{soil}[m^2]$, and give it properties depending on the y-coordinate. This is shown in Figure 3.2 and equation 3.5. In this equation, $k_0[m^2]$ is the permeability of Layer 0, $k_1[m^2]$ is the permeability of Layer 1, et cetera. $D_0[m]$ is the thickness of layer 0, $D_1[m]$ is the thickness of layer 1, et cetera.

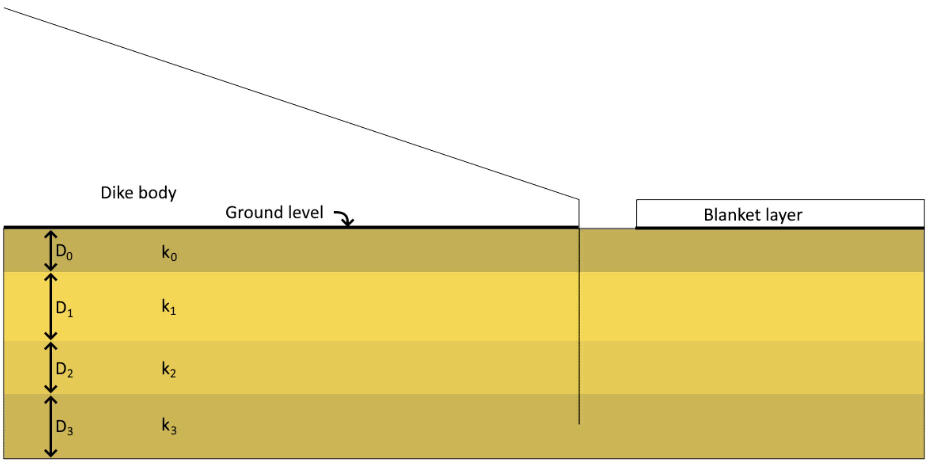


Figure 3.2: Conditional layering of the soil

$$k_{soil} = \begin{cases} k_0, & \text{if } Ground\ level - D0 \leq y \leq Ground\ level \\ k_1, & \text{if } Ground\ level - D0 - D1 \leq y \leq Ground\ level - D0 \\ k_2, & \text{if } Ground\ level - D0 - D1 - D2 \leq y \leq Ground\ level - D0 - D1 \\ k_3, & \text{if } Ground\ level - D0 - D1 - D2 - D3 \leq y \leq Ground\ level - D0 - D1 - D2 \end{cases} \tag{3.5}$$

3.2.3. Meshing

Every Finite Element Model requires a calculation mesh. Because the flow inside the pipe works on a very small scale, the mesh needs to be very fine there. The flow around the pipe is important for knowing if a sub-mechanism occurs or not. For this reason, close to the pipe, the mesh is also fine. Farther away from the pipe, the accuracy is not as important, so a coarser mesh is allowed. COMSOL gives the opportunity for the user to define the mesh manually. How this is used is shown in section 4.2.2. A fine mesh has more elements than a coarse mesh. The more elements a mesh has, the longer it takes to run the model, as for each node in the mesh, all values need to be calculated. Creating the mesh itself also takes a considerable amount of time, so when running the model for different parameters (as is explained in the next subsection), remeshing is to be avoided as much as possible.

3.2.4. Parametric sweeps

COMSOL possesses the ability to run a parametric sweep. This means that the model is run for different combinations of parameter values (for example, D_0 = 1, 2 or 3 metres). Parametric sweeping is used in chapter 5 to find the parameters that most contribute to the formation of a deeper pipe. If a parameter included in the sweep influences the geometry, a new mesh is built for each parameter value. This takes much time, so nifty tricks like the 'conditional properties' or permeability zone modelling are useful to avoid this.

3.2.5. Model output

After running, the COMSOL model returns the results from the Darcy's law calculation. For every point in the geometry, the head is returned. The head gradients can, therefore, also be determined, as these are the spatial derivatives of the head levels. The driving forces behind the sub-mechanisms described in chapter 2 depend on the head gradient, as is described in section 3.4.

3.3. Modelling the effect of cohesion

In this section, the effect of cohesion, which is an important factor in the resistance terms, is included in the model. Cohesion is typically represented by a cohesion value (in kPa) multiplied by a surface area on which the cohesion acts.

The force exerted by cohesion can be modelled separately from the COMSOL simulation. This separate modelling is done to account for the cohesive forces acting on individual grains and their interactions within the soil. By incorporating cohesion into the model, its influence on the occurrence of deeper piping can be evaluated.

To model the effects of cohesion, various approaches can be used depending on the specific characteristics of the soil being studied. The approach to model cohesion used in this thesis is to assign a cohesive force to each grain based on the cohesion value and a fraction of the grain's surface area (SA_{grain}). The cohesive force can be calculated using equations 3.6 through 3.9, which take into account the contact area and the cohesion value.

It is important to note that the modelling of cohesion is an additional step that complements the COMSOL simulation. The cohesive forces obtained from the separate modelling can be incorporated into the COMSOL model as additional resisting forces acting on the grains.

By including cohesion in the model, it is investigated how cohesive forces contribute to the stability of the soil and the potential for deeper piping. This allows us to gain insights into the mechanisms and conditions under which cohesive forces play a significant role in the occurrence of piping.

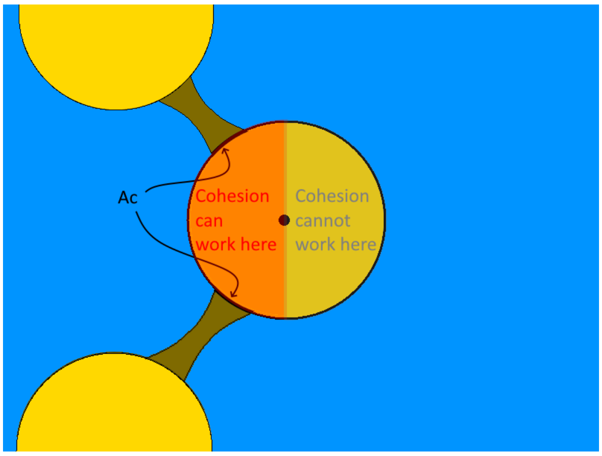


Figure 3.3: Area where cohesion is assumed to be able to act on

As mentioned in the introduction of this section, the effect of cohesion is included in the model by multiplying the cohesive strength $c[kPa]$ by an area. This are is a fraction of the total surface area of a sand grain, an shown in Figure 3.3 and equations 3.6 and 3.7. The sand grain is assumed to be a perfect sphere.

$$SA_{half\ grain} = \frac{1}{2} * \frac{\partial V_{grain}}{\partial d_{rep}} = \frac{\partial \left(\frac{1}{2} * \frac{1}{6} * \pi * d_{rep}^3 \right)}{\partial d_{rep}} = \frac{1}{4} * \pi * d_{rep}^2 \tag{3.6}$$

So the total area on which cohesion works is:

$$A_{coh} = A_{coh,fraction} * SA_{half\ grain} \tag{3.7}$$

Where $SA_{half\ grain}$ is one half of the total surface area of the grain $[m^2]$, V_{grain} is the volume of the grain $[m^3]$, d_{rep} is the representative grain diameter $[m]$ and $A_{c,fraction}$ is the fraction of $SA_{half\ grain}$ on which cohesion works $[-]$, which makes A_{coh} the total area on which cohesion acts $[m^2]$. A_{coh} is one of the parameters of which the influence is investigated in chapter 5.
As mentioned in section 2.3, it is very well possible that cohesion is higher in one direction than in the other. This is named 'cohesion anisotropy', or α_{coh}. This means that the cohesion force in horizontal direction are different from the one in vertical direction. They are expressed by equations 3.8 and 3.9, respectively.

$$F_{coh,hor} = A_{coh} * c_{hor} = A_{coh,fraction} * \frac{1}{4} * \pi * d_{rep}^2 * c_{hor} \tag{3.8}$$

$$F_{coh,ver} = \frac{A_{coh} * c_{hor}}{\alpha_{coh}} = \frac{A_{coh,fraction} * \frac{1}{4} * \pi * d_{rep}^2 * c_{hor}}{\alpha_{coh}} \tag{3.9}$$

The cohesion force is the only force resisting grain erosion once heave has taken place at the location under consideration. This force is, therefore, very important for the mechanisms described in section 3.4.

3.4. When does deeper piping occur?

A sub-mechanism as described in chapter 2 takes place if its FRP is below 1. In that chapter, the FRP functions have been defined as the driving forces divided by the resisting forces. However, to be able to use COMSOL to determine whether or not deeper piping occurs, a more detailed description is required. That is done in this section. The COMSOL output is read long so-called 'cut lines', which are the red lines in the figures included in the description of each sub-mechanism. These are further explained in 3.4.7, but for now, it is important to know that the cut line takes the mean of the output data along that line.

3.4.1. From head gradient to force

A sub-mechanism occurs when the driving force is larger than the resisting force. COMSOL returns head gradients, and not forces. For this reason, the head gradients returned by COMSOL need to be transformed into a force. For the horizontal direction, this is done as follows:
Firstly, the pressure head ($H_p[m]$) on either side of the grain is transformed to a pressure($p[Pa]$). This is done by multiplying by the weight of water ($\gamma_w[N/m^3]$), as shown by equation 3.10.

$$p = H_p * \gamma_w \tag{3.10}$$

The pressure head gradient $\frac{\partial H_p}{\partial x}[-]$ is defined as the amount the head changes in a certain direction, in this case the horizontal one. This means that the pressure head at $x = 1$ is $\frac{\partial H_p}{\partial x}$ higher than the pressure head at $x = 0$. The left and right sides of a grain are one representative grain diameter ($d_{rep}[m]$) metres apart, making the difference in pressure head between either side:

$$\Delta p = \frac{\partial H_p}{\partial x} * d_{rep} * \gamma_w \tag{3.11}$$

To transform this difference in pressures to a net horizontal force, the pressure needs to be multiplied by the area on which it works ($A_{frontal,grain}[m^2]$): the frontal area of the grain, which is equal to:

$$A_{frontal,grain} = \frac{1}{4} * \pi * d_{rep}^2 \tag{3.12}$$

By combining equations 3.10, 3.11 and 3.12, the total net horizontal force ($F_{forcing,horizontal}[N]$) is arrived at:

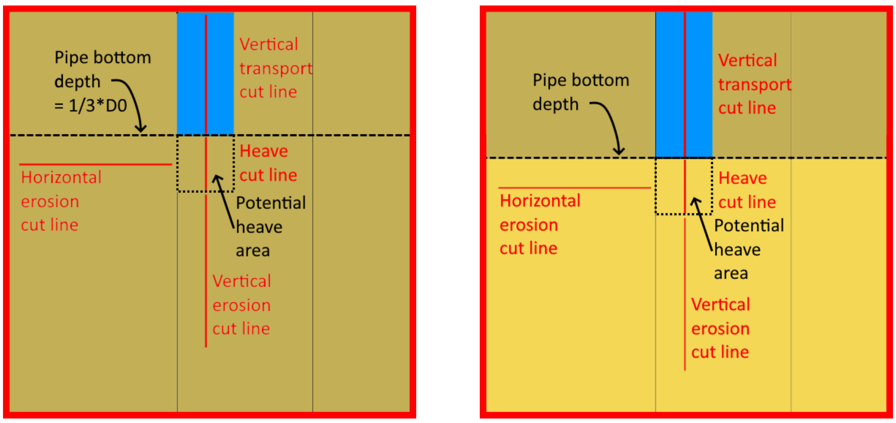
$$F_{forcing,horizontal} = \frac{1}{4} * \pi * d_{rep}^3 * \gamma_w * \frac{\partial H_p}{\partial x} \tag{3.13}$$

Similarly, the force in the vertical direction ($F_{forcing,vertical}[N]$) is derived to be as shown in equation 3.14. Mind that here, the vertical gradient is used.

$$F_{forcing,vertical} = \frac{1}{4} * \pi * d_{rep}^3 * \gamma_w * \frac{\partial H_p}{\partial y} \tag{3.14}$$

3.4.2. Heave

As explained in chapter 2, the first mechanism that needs to occur in deeper piping is secondary heave. Before heave has taken place, it is checked if $FRP_{heave} < 1$ for the situation depicted in Figure 3.4a (for heave in the top layer) or Figure 3.4b (for heave just below the first layer). If over this heave cut line the average head gradient exceeds the critical head gradient as shown in equation 3.15, heave takes place. This is the Terzaghi (Terzaghi, 1922[24]) heave criterion.



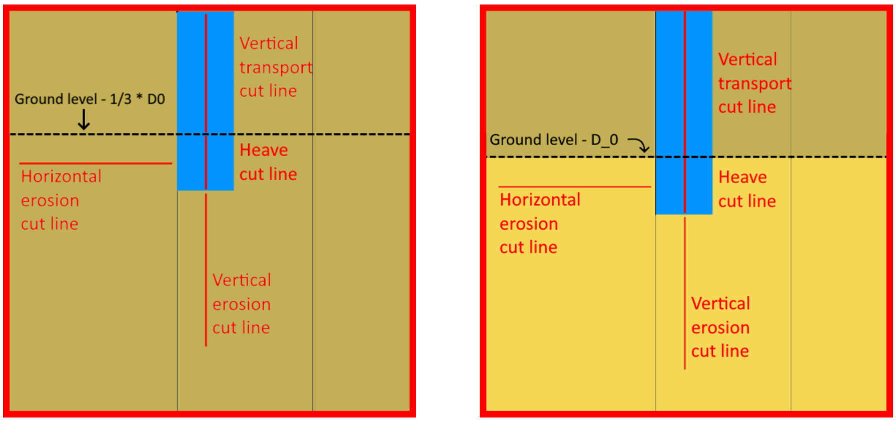
(a) Pipe tip before heave. Pipe tip is somewhere in the top layer

(b) Pipe tip before heave. Pipe has penetrated top layer

Figure 3.4: Pre-heave situations for two different pipe depths

$$FRP_{heave} = \frac{i_{crit}}{i} = \frac{1.65 * (1 - n)}{\frac{\partial H}{\partial y}} < 1 \tag{3.15}$$

After heave has occurred, the situation becomes as shown in Figure 3.5. The next step is to check if horizontal erosion is initiated. This can only happen if the condition in 3.15 is satisfied.



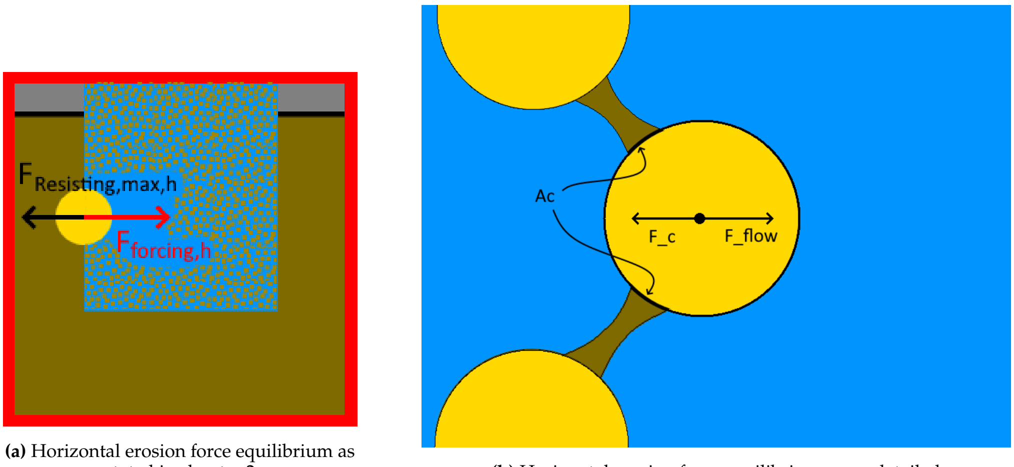
(a) Pipe tip after heave. Pipe tip is somewhere in the top layer

(b) Pipe tip after heave. Pipe has penetrated top layer

Figure 3.5: Post-heave situations for two different pipe depths

3.4.3. Horizontal erosion

After heave has occurred in the soil block under consideration, the question is whether or not horizontal erosion occurs. In order to determine this, the simplified $FRP_{horizontal\ erosion}$ from chapter 2 needs to be defined in a more detailed manner. This is done by considering the situation as shown in Figure 3.6b.



(a) Horizontal erosion force equilibrium as stated in chapter 2

(b) Horizontal erosion force equilibrium more detailed

Figure 3.6: Horizontal erosion force equilibrium

It can be seen that if $F_{flow} > F_{coh}$, horizontal erosion is considered to initiate. The determination of the flow force F_{flow} is done according to equation 3.13. The cohesion force is calculated by multiplying the cohesion force (c) by the area over which the cohesion force works (A_{coh}). The $FRP_{horizontal\ erosion}$ is then formulated as follows:

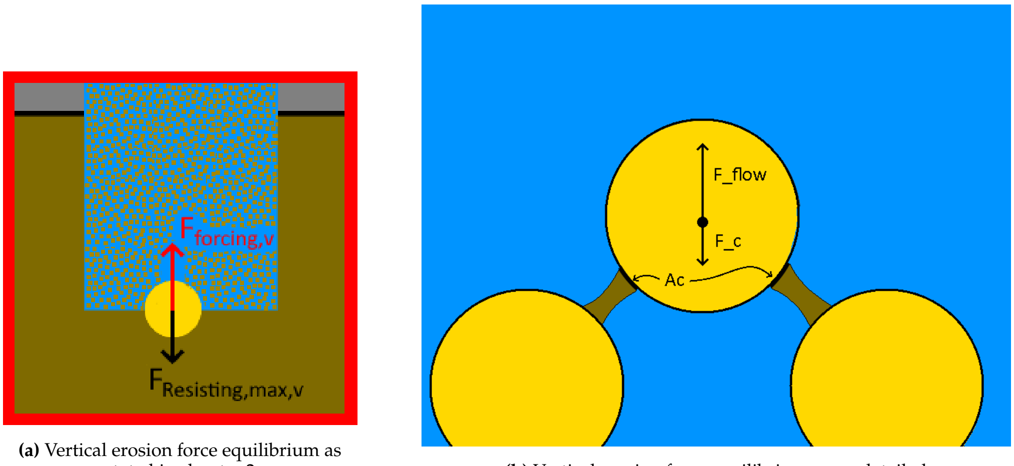
$$FRP_{horizontal\ erosion} = \frac{Ac * c_{hor}}{\frac{1}{4} * \pi * d_{rep}^{3} * \frac{\partial H}{\partial x} * \gamma_w} \tag{3.16}$$

Horizontal erosion is considered to occur when $FRP_{horizontal\ erosion} < 1$ *and* $FRP_{horizontal\ erosion} <$

$FRP_{vertical\,erosion}$. For deeper piping to occur, horizontal erosion should *not* take place in Layer 0. It *should*, however, do so in a deeper layer. In the case of the Hedwigepolder, this was Layer 1.

3.4.4. Vertical erosion

Vertical erosion in the post-heave situation is in principle identical to the horizontal erosion: cohesion forces need to be exceeded by the flow forces. There are two differences. The first is that in vertical erosion, the flow forcing depends on the *vertical* head gradient, whereas the horizontal erosion depends on the horizontal gradient. Secondly, the cohesion in vertical direction might be different from the cohesion in horizontal direction (cohesion anisotropy, as explained in section 2.3)



(a) Vertical erosion force equilibrium as stated in chapter 2

(b) Vertical erosion force equilibrium more detailed

Figure 3.7: Vertical erosion force equilibrium

$$FRP_{vertical\,erosion} = \frac{Ac * c_{ver}}{\frac{1}{4} * \pi * d_{rep}^3 * \frac{\partial H}{\partial y} * \gamma_w} \tag{3.17}$$

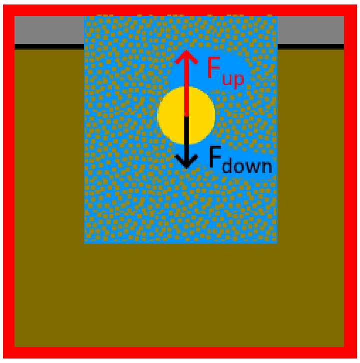
Vertical erosion, and therefore deeper piping, only takes place if $FRP_{vertical\,erosion} > FRP_{horizontal\,erosion}$ *and* $FRP_{ver,ero<1}$ for as long as the pipe has not penetrated the first layer yet. Once it has done so, in deeper piping, $FRP_{horizontal\,erosion} < FRP_{vertical\,erosion}$ and the pipe develops horizontally.

3.4.5. Horizontal erosion or vertical erosion

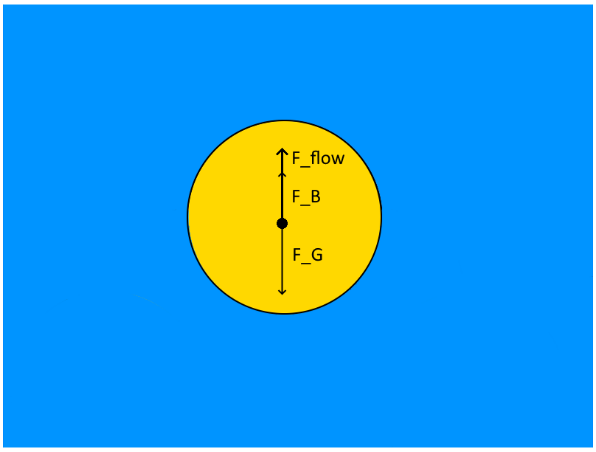
To determine whether deeper piping occurs or not, it is necessary to know whether horizontal erosion or vertical erosion occurs. It is assumed that the erosion takes place in vertical direction if $FRP_{vertical\,erosion}$ is smaller than $FRP_{horizontal\,erosion}$. That means vertical erosion is not strictly limited to situations where $FRP_{horizontal\,erosion} > 1$: it could also happen simultaneously with vertical erosion, and it is assumed that vertical erosion takes place when this sub-mechanism is dominant.

3.4.6. Vertical grain transport

After the grain has been eroded, it needs to be transported away. This is the case if the flow forces F_{flow} are larger than the gravity force working on a grain minus the buoyancy force acting on the grain, as shown in equation 3.18.



(a) Vertical transport force equilibrium as stated in chapter 2



(b) Vertical transport force equilibrium more detailed

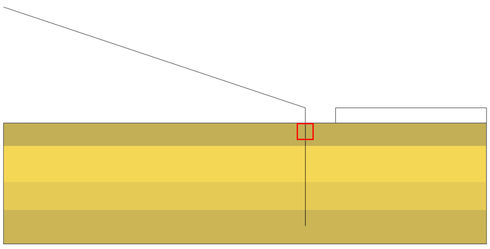
Figure 3.8: Vertical transport force equilibrium

$$FRP_{vertical\ transport} = \frac{F_G - F_B}{F_{flow}} = \frac{\frac{1}{6} * \pi * d_{rep}^3 * \gamma_s - \frac{1}{6} * \pi * d_{rep}^3 * \gamma_w}{\frac{1}{4} * \pi * d_{rep}^3 * \frac{\partial H}{\partial y} * \gamma_w} = \frac{2 * (\gamma_s - \gamma_w)}{3 * \frac{\partial H}{\partial y} * \gamma_w} \tag{3.18}$$

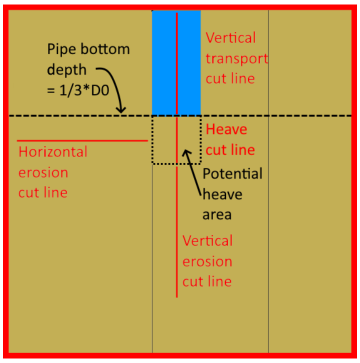
Where $F_G[N]$ is the gravity force acting on a grain, $F_B[N]$ is its buoyancy force and $\gamma_s[N/m^3]$ is the weight of soil. The rest of the parameters have already been explained previously.

3.4.7. Cut lines

The driving forces in the FRP formulae 3.15, 3.16, 3.17 and 3.18 contain a head gradient term. The value of the head gradient as inserted in these driving force equations is equal to the mean value of the head gradient along a measurement line. These measurement lines are termed cut lines. In Figure 3.9, the locations of the cut lines with respect to the pipe tip are shown for the situation where the pipe has penetrated through 1/3 of the top layer. If the pipe goes deeper, the cut lines move along downwards with it.
In this figure, four cut lines can be seen: a horizontal erosion cut line to the left of the pipe tip, a vertical erosion cut line below the pipe tip, a vertical cut line in the tip of the pipe and a vertical transport cut line inside the rest of the vertical erosion channel. The average head gradient over the length of these cut lines is used in the calculation of the horizontal erosion FRP, the vertical erosion FRP, the heave FRP and the vertical transport FRP, respectively.



(a) Location pipe tip at a depth of 1/3 times D_0



(b) Zoomed in on location pipe tip at a depth of 1/3 times D_0

Figure 3.9: Pipe tip and cut line locations

4 Case study: Hedwigepolder experiments

In this chapter, the Hedwigepolder is modelled in COMSOL Multiphysics. To do so, the geometry and boundary conditions as they were in the Hedwigepolder experiments are shown in section 4.1. These are used as the geometry and boundary conditions in the COMSOL model in section 4.2.
Additionally, it is checked if the model presented in chapter 3 succeeds in modelling deeper piping in a Hedwigepolder-type set-up.

4.1. Hedwigepolder experiments

In the summer of 2021, Fugro, Deltares and Waterschap Hollandse Delta conducted a set of experiments on piping in a tidal subsoil. Two sections of ground were enclosed with sheet pile walls, and pumping wells were installed in the crest of the dike down to the bottom of the aquifer to introduce water with a controlled head level. On the hinterland, a thick clay layer was applied to ensure impermeability. The combination of the sheet pile walls and this blanket forced the flow out through an excavated ditch. After the experiment, cement was injected into the formed pipe, allowing for research into its final shape, size and trajectory.
In this section, the situation as it was at the Hedwigepolder is presented. Doing so, it allows for insertion into a COMSOL model in the next section. First, the geometry is treated, and then, the boundary conditions are shown.

4.1.1. Hedwigepolder geometry

The geometry of the Hedwigepolder experiments is shown in Figure 4.1. It can be seen that the distance between the inlet and the ditch was 10 metres. The ditch itself was 1 metre wide. The hinterland was 5 metres long. The entire geometry was 4 metres thick, consisting of 4 layers numbered Layer 0 through Layer 3. The respective thicknesses were 0.6m, 1.4m, 0.8m, 1.2m.
Note: after excavation, it was found that the top layer, Layer 0, was cohesive. The value of this cohesion was, however, not measured. A value of $1kPa$ is assumed. This value is an estimate based on the knowledge that the soil is a sandy clay. These dimensions are based on the design of the piping experiments, deviations between design and the actual situation after construction are considered negligible in this thesis.

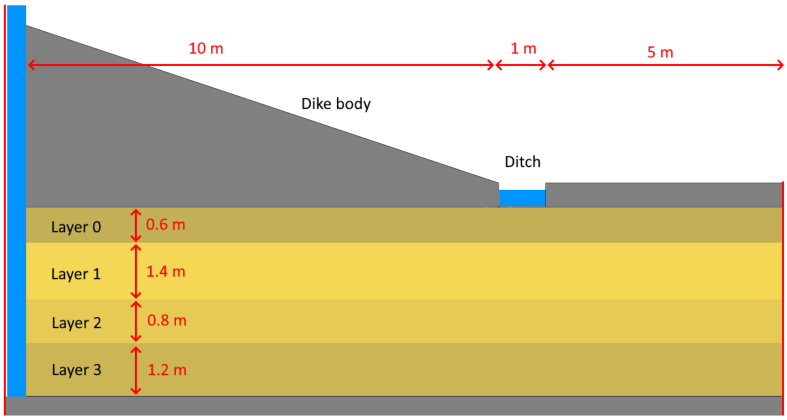


Figure 4.1: Hedwigepolder situation

4.1.2. Hedwigepolder boundary conditions

At the Hedwigepolder, the head level on the right was kept constant at NAP + 1.3 metres, whereas on the left, water was injected, of which the head was controlled. The head on the left was increased from NAP + 1.3 metres, with increments of 30 centimetres, until piping was initiated. The rest of the boundaries was either an impermeable layer or sheet pile walls. This can be seen in Figure 4.2. Due to the analysis being a quasi-steady state model, it is assumed that a constant head level difference of 2 metres is maintained.

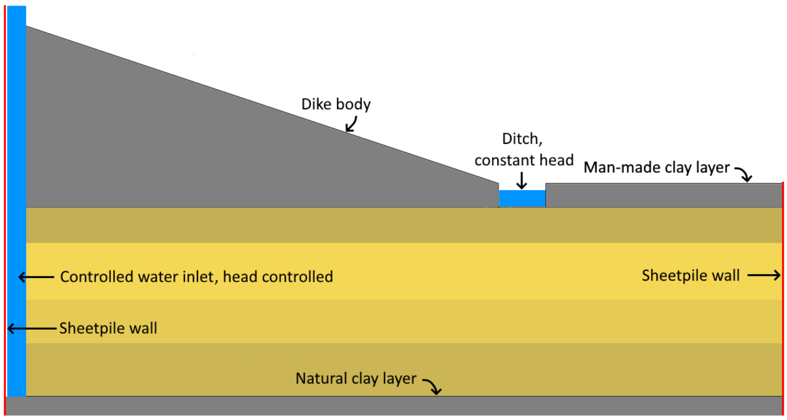


Figure 4.2: Hedwigepolder boundary conditions

4.2. Hedwigepolder in COMSOL

In this section, it is shown how the situation in the Hedwigepolder is converted to a COMSOL model. In section 4.2.1, the geometry is treated. In section 4.2.2, the calculation mesh is elaborated on, and in 4.2.3, the boundary conditions are shown. Then, in section 4.2.4, the model input parameters are treated.

4.2.1. COMSOL geometry

In Figure 4.3, the geometry of the model as put into COMSOL is shown. The geometry consists of two components (or domains): the subsoil and the vertical erosion channel outline.

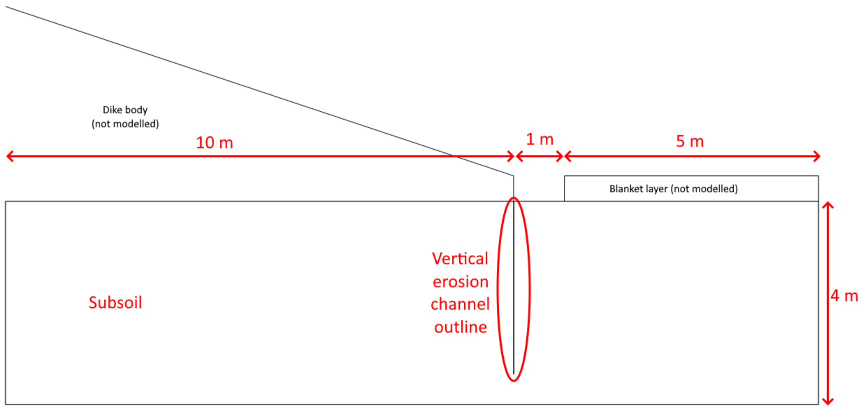


Figure 4.3: Geometry of COMSOL model

COMSOL subsoil

As can be seen in Figure 4.3, the subsoil is modelled as a single domain, so no distinct layers can be seen. However, in the Hedwigepolder, four layers could be distinguished. This has been taken into account by describing the subsoil as one entity with conditional properties, as explained in 3.2.2. Using this, COMSOL treats the subsoil as one entity with one permeability k_{soil}, but the value of k_{soil} depends on the y-coordinate. This permeability is used to describe the groundwater flow. This approach is useful when in chapter 5, the layer thicknesses are treated as a variable.

Vertical erosion channel outline

The vertical erosion channel is a part of the domain where the soil has been eroded by the flow. Using the fictitious permeability model as described in chapter 3, this part of the subsoil is described as a soil domain with high permeability. Using the permeability zone principle as described in chapter 3, the length and width (D_h) of the pipe can be altered. According to this principle, the 'actual pipe' is not allowed to outgrow the outlines: the pipe width ($D_h[m]$) always has to be smaller than the width of the outline ($a_{max}[m]$). The location of the vertical pipe outline with respect to the dike body and the ditch is shown in Figure 4.4.

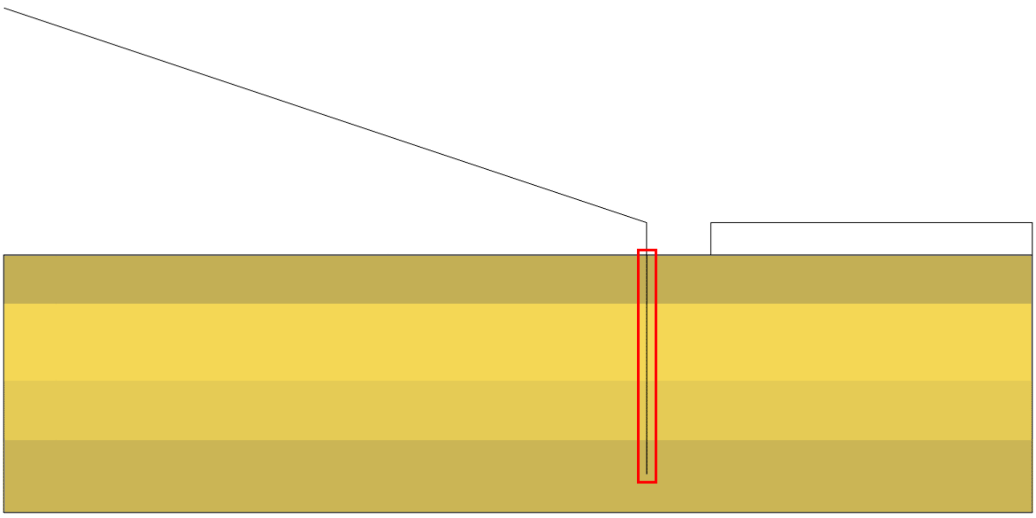


Figure 4.4: Vertical erosion channel location

4.2.2. COMSOL mesh

A finite element method like COMSOL requires a mesh to do calculations. In the model, the mesh inside the pipe is chosen manually, with a grid size of maximum $a_{max}/20$ (remember: a_{max} is the widest the erosion channel is expected to become), see Figure 4.6. The rest of the mesh is automatically generated by COMSOL, see Figure 4.5. Close to the pipe, the mesh is extremely fine, as it improves the quality of the results. Farther away from the pipe, computational accuracy is not as important. For this reason, the mesh can be set to be coarser to save calculation time, as explained in section 3.2.3.

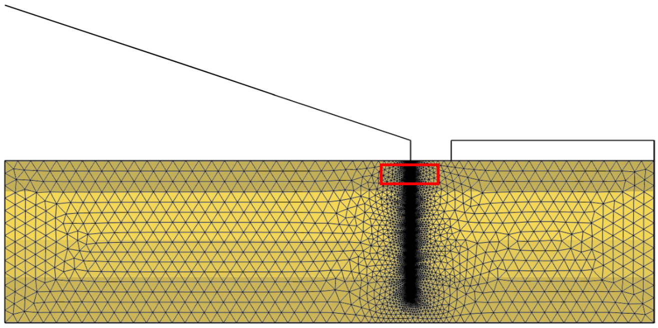


Figure 4.5: Full mesh

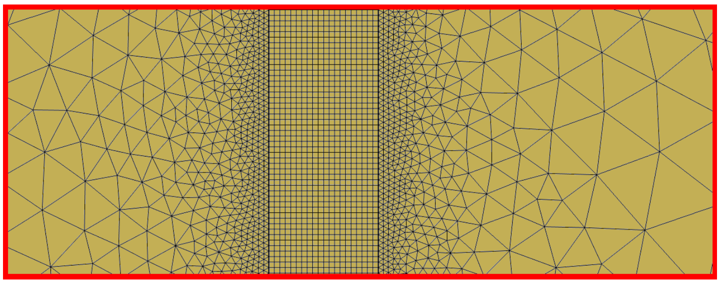


Figure 4.6: Mesh inside and around vertical erosion channel

4.2.3. COMSOL boundary conditions

The boundary conditions inserted in the COMSOL model are shown in Figure 4.7 below. These boundary conditions are used by the Darcy's law module to calculate the flow in the soil, and with that, the magnitude and direction of the flow forces, which act as driving force to the (deeper) piping mechanism. The 'Head left' and 'Head right' boundary conditions coincide with the Head boundary conditions described in section 4.1.2. As mentioned before, the head level difference between both head boundaries is kept constant at 2 metres.

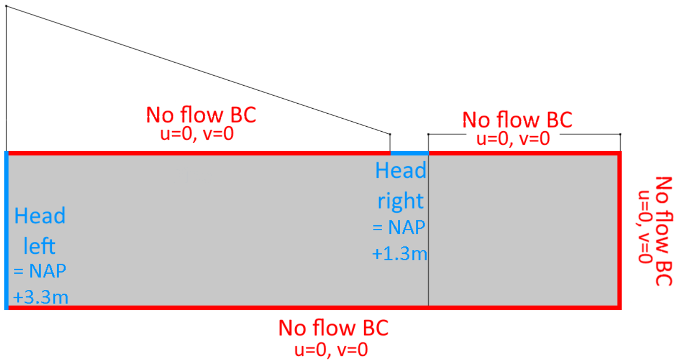


Figure 4.7: Boundary conditions Hedwigepolder

4.2.4. Parameters

The parameters as inserted in the COMSOL model of the Hedwigepolder are as shown in Table 4.1:

Parameter	Symbol	Value	Unit
Weight of the soil	γ_s	26.5	kN/m^3
Weight of the water	γ_w	10.0	kN/m^3
Representative grain diameter of Layer 0	$d_{rep,0}$	$8.2 * 10^{-5}$	m
Representative grain diameter of Layer 1	$d_{rep,1}$	$1.15 * 10^{-4}$	m
Permeability of Layer 0	k_0	$5.90 * 10^{-13}$	m^2
Permeability of Layer 1	k_1	$1.77 * 10^{-12}$	m^2
Permeability of Layer 2	k_2	$4.60 * 10^{-12}$	m^2
Permeability of Layer 3	k_3	$3.54 * 10^{-12}$	m^2
Porosity of the soil	n	0.4	$-$
Permeability anisotropy factor	α_k	3	$-$
Cohesion anisotropy factor of Layer 0	α_{coh}	1.5	$-$
Cohesion anisotropy factor of Layer 1	$-$	0.66	$-$
Vertical erosion channel outline width	a_{max}	$50 * 1.15 * 10^{-4} m = 5.75 * 10^{-3}$	m
Cohesion value Layer 0	c_0	1	kPa
Cohesion value Layer 1	c_1	0.01	kPa

Table 4.1: COMSOL input parameters

Using these parameters with the geometry from section 4.2.1, the mesh from section 4.2.2 and the boundary conditions from section 4.2.3, the model is ready to be run. Combining the results with the FRP equations from chapter 3, it is determined if deeper piping occurs in a Hedwigepolder-type set-up.

4.3. Deeper piping in COMSOL model

In this section, it is first shown how the flow in a standard piping case (isotropic, homogeneous subsoil) looks. The flow lines and FRP values are identified, and it is checked if deeper piping could also occur in such a subsoil, according to the model. After this, it is checked how the flow lines and FRP values change when a geometry like the Hedwigepolder is adopted. It is also checked whether the FRP values describe deeper piping.

As a reference case, a standard piping case is shown. The subsoil in this standard case is homogeneous (no layers) and isotropic (same properties in all directions). The parameters used in this model are the same as the parameters described above, except for the following:

- The permeability anisotropy factor (α_k) was switched from 3 to 1

- All layers have the permeability of Layer 1: $k_{soil} = k_1 = 1.77 * 10^{-12} m^2$

Henceforth, the vertical erosion channel width ($D_h[m]$) will be described as a number of grains ($n_g[-]$) times the representative grain diameter ($d_{rep}[m]$). The model is run for a chosen (arbitrary) channel width of 5 times the representative grain diameter ($n_g = 5$) After running, the flow lines look as shown in Figure 4.8.

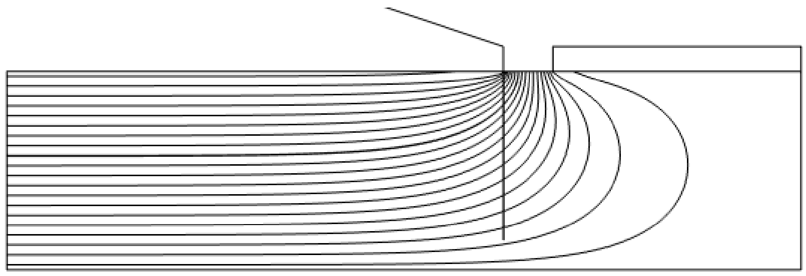
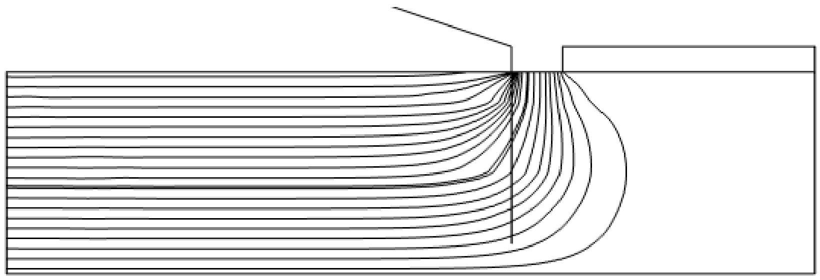
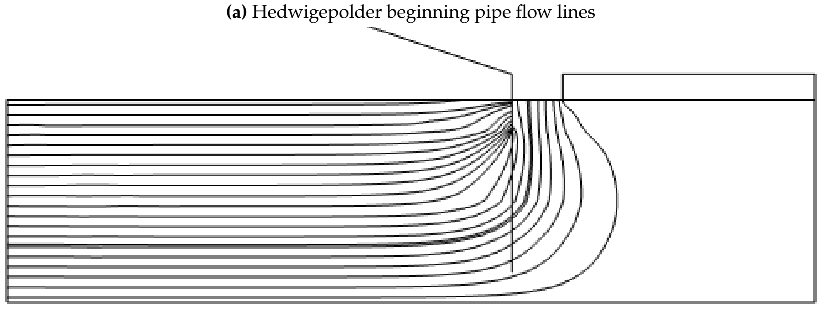


Figure 4.8: Flow lines standard piping case

Considering the Hedwigepolder case like described in the previous sections, the flow lines were found to be different, see Figure 4.9. It can already be seen that the flow lines are more vertically oriented near the exit. This implies that something in the Hedwigepolder subsoil configuration makes the flow lines more vertical, and therefore, deeper piping more likely.



(a) Hedwigepolder beginning pipe flow lines



(b) Hedwigepolder flow lines when pipe has penetrated Layer 0

Figure 4.9: Flow lines Hedwigepolder

The FRP values of all four sub-mechanisms are shown in Table 4.2. For every depth considered (recall that this was a beginning pipe (0/3), and when the pipe has penetrated through 1/3, 2/3 and the entire (3/3) Layer 0). For clarity, only the case where $n_g = 5$ is shown.

FRP	depth = $0/3 * D_0$	depth = $1/3 * D_0$	depth = $2/3 * D_0$	depth = $3/3 * D_0$
FRP_{heave}	0.0203	0.2223	0.0219	0.0191
$FRP_{horizontal\ erosion}$	0.1553	0.2789	0.2929	0.0016
$FRP_{vertical\ erosion}$	0.1085	0.1100	0.1126	0.0103
$FRP_{vertical\ transport}$	0.7353	0.8105	0.7414	0.8223

Table 4.2: FRP values for all sub-mechanisms and for every pipe depth, for a pipe width of 5 representative grain diameters

For the beginning pipe (where the pipe depth is equal to $0/3$ times the thickness of the top layer D_0), it can be seen in Table 4.2 that heave (primary heave in this case) takes place, because the FRP_{heave} is below 1. Having met this criterion, it is now checked whether horizontal or vertical erosion occurs. It can be seen that the vertical erosion $FRP_{vertical\ erosion}$ is smaller than 1 and smaller than $FRP_{horizontal\ erosion}$. It is, therefore concluded that vertical erosion takes place. Also, the vertical transport criterion is met (the value of $FRP_{vertical\ transport}$ is below 1 for all stages), so deeper piping is shown to occur. In the last column, the one that represents the situation once the pipe has vertically penetrated through the top layer entirely, it can be seen that the erosion takes place horizontally because the value of $FRP_{horizontal\ erosion}$ is smaller than $FRP_{vertical\ erosion}$. The model has, therefore, been shown to be able to return deeper piping in a Hedwigepolder-type set-up.

It is hypothesised that cohesion anisotropy and the permeability of the two top layers play important roles in deeper piping. This is tested in chapter 5.

5 Sensitivity Analysis

In this chapter, the most important parameters for the formation of a deeper pipe are listed. This is done by conducting a sensitivity analysis. In section 5.1, the method is treated. Then, in section 5.2, the parameters included in the sensitivity analysis are introduced. After that, in section 5.3, the results are shown.

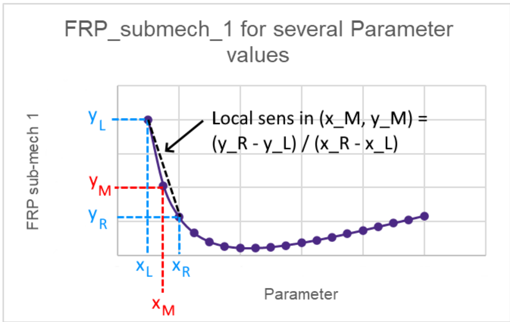
5.1. Method

As was described in chapter 2, each of the four sub-mechanisms (heave, horizontal erosion, vertical erosion and vertical transport) have a forcing term and a resisting term. The forcing term was found to depend on the parameters that influence the flow lines inside the soil domain. These are: α_k, k_0, k_1, k_2, k_3, D_0, D_1, D_2 and D_3. The resisting terms were found to mostly depend on cohesion and the size of a grain. The parameters contributing to this are A_c, c, α_c and d_{rep}. These parameters are all be included in the analysis.

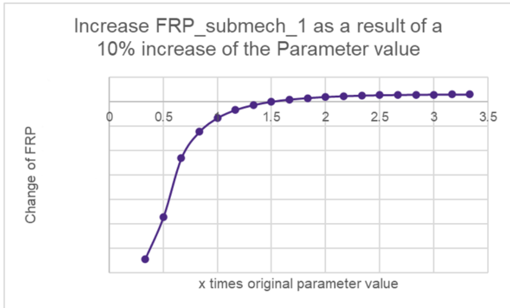
The procedure is as follows. First, the parameters mentioned in the intro of this section are assigned a 'default value'. This is the value they have when they are not being considered. The default value is defined to be the value of a parameter as in the Hedwigepolder. Then, the first parameter is considered. Using a parametric sweep as defined in chapter 3, the FRP values of all four sub-mechanisms is recorded for a range of parameter values. This produces a graph in which the FRP is plotted against the parameter value, like described in Figure 5.1. Next, the influence of the parameter on the FRP is determined by finding the slope of this graph, around the default Hedwigepolder value of that parameter (indicated in Figure 5.1 as (x_M, y_M)). This is done by considering the points to the left (x_L, y_L) and to the right (x_R, y_R), and applying equation 5.1. This slope is then used to determine how much the FRP changes if the parameter is increased by 10% of its original value. The parameter for which the absolute value of this change in FRP is largest, is considered to be the most influential parameter.

$$Slope = \frac{y_R - y_L}{x_R - x_L} \tag{5.1}$$

This procedure is conducted for every stage of vertical pipe formation described in section 3.2.1: the beginning pipe (where the vertical pipe is only a few grain diameters long), the 1/3 pipe (where it has penetrated one third of Layer 0), the 2/3 pipe and the 3/3 pipe. The FRP values vary with chosen pipe width D_h. To keep the amount of graphs and tables presented in this chapter limited, only the ones belonging to a pipe width of four grain diameters are presented.



(a) Determining local sensitivity



(b) Influence of parameter on FRP for several parameter values (derivative of image on the left)

Figure 5.1: Determining local influence (left) and example influence graph (right)

5.2. Parameters

In this section, the default parameters are treated, and it is shown which parameter values have been used in the parametric sweep.

The default parameters are shown in Table 5.1. These are the values a parameter is set to if it is not considered, and also, this is the value around which the sensitivity of an FRP to that parameter is determined. The values presented in this Table are the ones as they were in the Hedwigepolder.

Parameter	Symbol	Value	Unit
Fraction of grain surface area on which cohesion works	A_c	0.001	–
Cohesion strength	c	1	kPa
Cohesion anisotropy	α_c	1.5	–
Permeability anisotropy	α_k	3	–
Permeability of Layer 0	k_0	$5.90 * 10^{-13}$	m^2
Permeability of Layer 1	k_1	$1.77 * 10^{-12}$	m^2
Permeability of Layer 2	k_2	$5.60 * 10^{-12}$	m^2
Permeability of Layer 3	k_3	$3.54 * 10^{-12}$	m^2
Thickness of Layer 0	D_0	0.6	m
Thickness of Layer 1	D_1	1.4	m
Representative grain diameter	d_{rep}	$8.2 * 10^{-5}$ (Layer 0), or $1.15 * 10^{-4}$ (Layer 1)	m
Cut line length	$L_{Cutline}$	$200 * d_{rep}$	m

Table 5.1: Default parameters

It can be seen that from the parameters mentioned in section 5.1, two parameters are missing: D_2 and D_3. These have been left out of the analysis, because these parameters were used to ensure that the total thickness of the subsoil remains at 4 metres. This is done to avoid remeshing, as explained in section 3.2.3. Also, it can be seen that the cut line length is included as a variable, as it was found that the length over which the average head is measured matters as well.

For each parameter, a range of values is put through the analysis. For each parameter, the lowest value, the highest value and the step size are defined in Table 5.2.

Parameter	Lowest value	Highest value	Step size	Unit
A_c	0.001	0.01	0.001	−
c	0	2	0.1	kPa
α_c	1	10	0.5	−
α_k	1	10	0.5	−
k_0	$0.1 * 5.90 * 10^{-13}$	$4.9 * 5.90 * 10^{-13}$	$0.3 * 5.90 * 10^{-13}$	m^2
k_1	$0.1 * 1.77 * 10^{-12}$	$4.9 * 1.77 * 10^{-12}$	$0.3 * 1.77 * 10^{-12}$	m^2
k_2	$0.1 * 5.60 * 10^{-12}$	$4.9 * 5.60 * 10^{-12}$	$0.3 * 5.60 * 10^{-12}$	m^2
k_3	$0.1 * 3.54 * 10^{-12}$	$4.9 * 3.54 * 10^{-12}$	$0.3 * 3.54 * 10^{-12}$	m^2
D_0	0.25	2.0	0.25	m
D_1	0.25	2.0	0.25	m
d_{rep}	$0.1 * 8.2 * 10^{-5}$	$4.9 * 8.2 * 10^{-5}$	$0.3 * 8.2 * 10^{-5}$	m
$L_{Cutline}$	$50 * d_{rep}$	$500 * d_{rep}$	$50 * d_{rep}$	m

Table 5.2: Parameter value range and step size

5.3. Results

The results of the analysis are shown in Tables 5.3 through 5.6. These tables show, for every stage of vertical pipe formation described in section 5.1, the influence of a 10% increase of the parameter value on the FRP values of the four sub-mechanisms. That is: using the slope of the graph (of which an example is given in Figure 5.1a), the change in FRP as a result of a 10% increase of the parameter is estimated. If, for example, the value in the table is 20.0, it means that the FRP changes by 20% upon a 10% increase of the parameter. Bear in mind that only the results for a pipe width of 4 grain diameters is taken, to limit the amount of tables shown. The rest of the tables describing % change of the FRP values can be found in Appendix A.1. Appendix A.2 shows the local influence around the Hedwigepolder values and Appendix A.3 show how these behave at values 'x times' larger than the Hedwigepolder value.

Sub-mech	Most important parameter	Value	More or less deeper piping	Second most important parameter	Value	More or less deeper piping	Third most important parameter	Value	More or less deeper piping
Heave	d_{rep}	5.14	*Less*	k_0	3.74	*More*	α_k	3.06	*Less*
Hor. ero	d_{rep}	94.8	*Less*	A_c & c	85.2	*More*			
Ver. ero	d_{rep}	69.7	*Less*	α_c	66.5	*More*	A_c & c	59.1	*Less*
Ver. tra	d_{rep}	363.7	*More*	α_k	13.3	*More*	k_0	11.9	*More*

Table 5.3: % change of FRP as a result of a 10% increase of the most influential parameters, for a depth of 0/3 times the thickness of Layer 0

Sub-mech	Most important parameter	Value	More or less deeper piping	Second most important parameter	Value	More or less deeper piping	Third most important parameter	Value	More or less deeper piping
Heave	D_0	65.7	*More*	k_0	9.38	*Less*			
Hor. ero	d_{rep}	22.3	*Less*	A_c & c	11.0	*More*	k_0	6.54	*More*
Ver. ero	d_{rep}	8.31	*More*	α_c	4.79	*More*	A_c & c	4.26	*Less*
Ver. tra	d_{rep}	19.8	*Less*	α_k	6.16	*More*	k_0	3.35	*More*

Table 5.4: % change of FRP as a result of a 10% increase of the most influential parameters, for a depth of 1/3 times the thickness of Layer 0

Sub-mech	Most important parameter	Value	More or less deeper piping	Second most important parameter	Value	More or less deeper piping	Third most important parameter	Value	More or less deeper piping
Heave	D_0	103.6	$More$	k_0	8.17	$Less$	d_{rep}	8.00	$More$
Hor. ero	d_{rep}	398.6	$Less$	A_c & c	143.3	$More$	k_0	114.9	$More$
Ver. ero	d_{rep}	96.8	$More$	α_c	59.7	$More$	A_c & c	53.1	$Less$
Ver. tra	d_{rep}	190.8	$Less$	α_k	70.4	$More$	k_1	31.4	$More$

Table 5.5: % change of FRP as a result of a 10% increase of the most influential parameters, for a depth of 2/3 times the thickness of Layer 0

Sub-mech	Most important parameter	Value	More or less deeper piping	Second most important parameter	Value	More or less deeper piping	Third most important parameter	Value	More or less deeper piping
Heave	D_0	37.6	$More$	k_0	10.4	$Less$	d_{rep}	10.0	$More$
Hor. ero	d_{rep}	1.69	$More$	A_c & c	0.72	$Less$			
Ver. ero	A_c, c & α_c	3.61	$More$						
Ver. tra	d_{rep}	190.0	$Less$	α_k	67.2	$More$	k_1	46.1	$More$

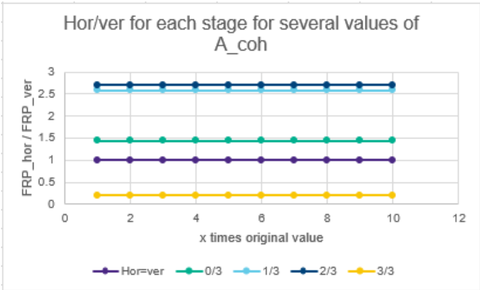
Table 5.6: % change of FRP as a result of a 10% increase of the most influential parameters, for a depth of 3/3 times the thickness of Layer 0

5.3.1. Results: comparing horizontal and vertical erosion

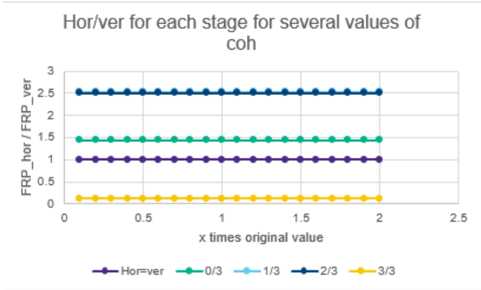
As deeper piping is a phenomenon that occurs when vertical erosion takes place rather than horizontal erosion, is it useful to plot the $\frac{FRP_{horizontal\ erosion}}{FRP_{vertical\ erosion}}$ graph. If the graph is above 1, vertical erosion is dominant and if the line is below 1, horizontal erosion is dominant. These graphs are shown in Figure 5.2. From these Figures it can be seen that deeper piping is most likely in the following situations:

- Large α_c
- Large α_k
- Large k_0
- Small k_1
- Large D_0
- Small d_{rep}

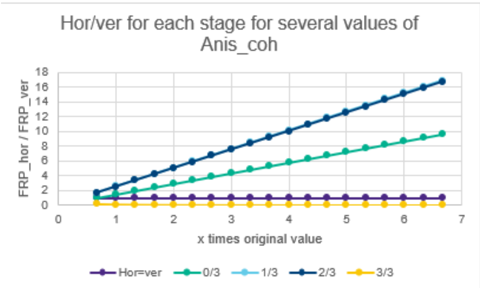
It can furthermore be seen from Figures 5.2a and 5.2b, respectively, that an increase of A_{coh} and c do not affect the ratio of horizontal and vertical erosion $FRP's$, because they change the value of the former just as quickly as the latter. From Figures 5.3a, 5.3b and 5.3d it can be seen that k_2, k_3 and D_1 do not influence the ratio of horizontal/vertical much.



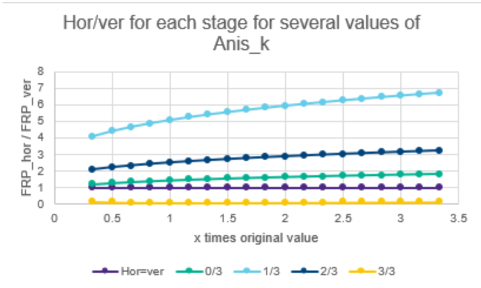
(a) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of A_{coh}



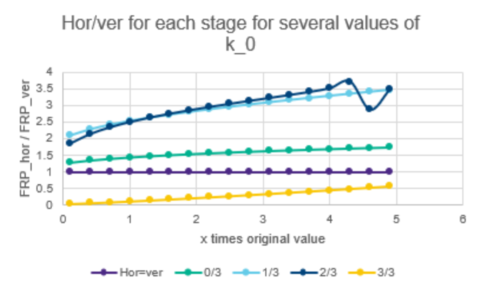
(b) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of c



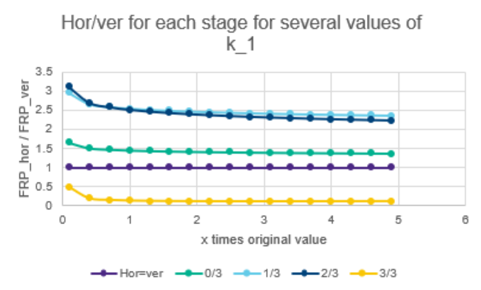
(c) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of α_{coh}



(d) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of α_k

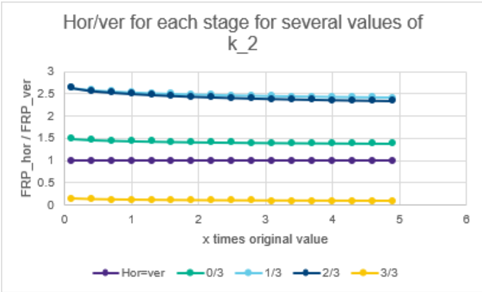


(e) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of k_0

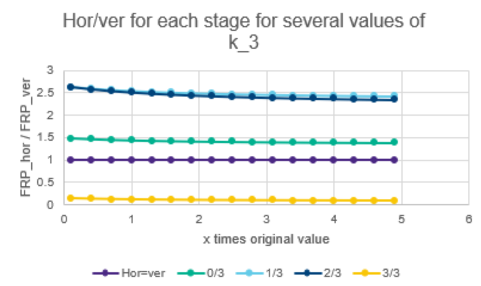


(f) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of k_1

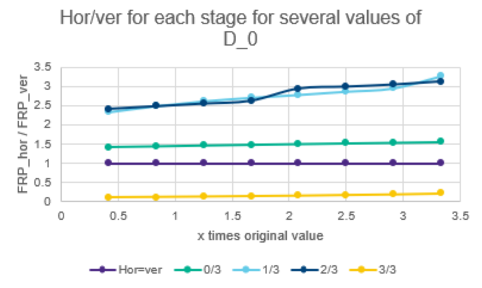
Figure 5.2: $FRP_{horizontal}/FRP_{vertical}$ graphs



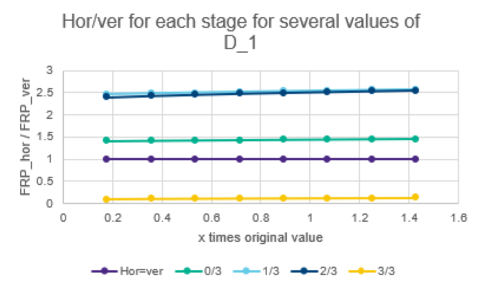
(a) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of k_2



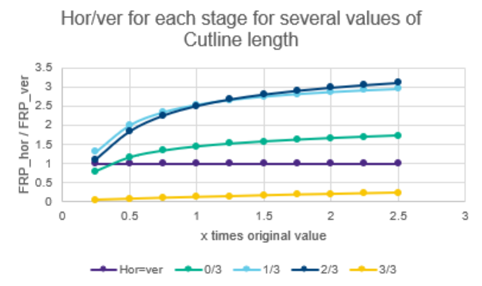
(b) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of k_3



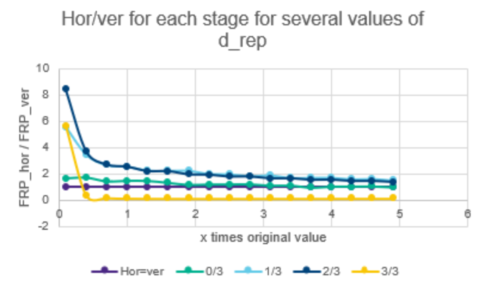
(c) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of D_0



(d) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of D_1



(e) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of $L_{cut\ line}$



(f) $FRP_{horizontal}/FRP_{vertical}$ graph for x times the original value of d_{rep}

Figure 5.3: $FRP_{horizontal}/FRP_{vertical}$ graphs (continued)

6 Discussion

In this chapter the model and results are discussed. In section 6.1, the modelling simplifications are discussed, and in section 6.2, the results are discussed.

6.1. Modelling simplifications

Because models are a representation of reality, they come with simplifications and limitations. The assumptions, simplifications and limitations and the effects of these are mentioned in this section.

Linear behaviour in FRP

It was assumed in chapter 2 that if for a given set of conditions, both $FRP_{horizontal\,erosion}$ and $FRP_{vertical\,erosion}$ are smaller than 1, the smallest of the two occurs. This has been done assuming that for a smaller water level difference, there would be a point where the largest of the two would be *above* 1, and the smallest *below* 1. However, this assumes linear behaviour, whereas such a complicated flow problem is likely to be nonlinear. This has likely influenced the results. To counter this, the analysis could be run for a range of water level differences, to see for which one occurs and the other does not. This also automatically gives insight in the critical head difference. This has not been done in this thesis, because of storage space limitations and computation time considerations.

Fictitious permeability model

The fictitious permeability model used in this model has the drawback that it can only be used in laminar flow. The model was checked to comply with this criterion. In this thesis, the size of the vertical erosion channel was assumed to be small (this assumption is treated later), but in reality it could be larger. Larger pipe widths cause the pipe width parameter to go up, but the flow velocities to decrease, as a result of the pressure head decreasing. The Reynolds number is expected to increase as a result of widening of the pipe (Nur, Afrianita, Ramli, 2019[16]).
If the Reynolds number exceeds a value of 10 at any point, the model as described in this thesis cannot be applied anymore and a different approach must be sought for. This should be taken into account when doing research into the vertical erosion channel.

2D flow model versus 3D reality

The fracture flow on which the fictitious permeability model as used in this thesis is based, uses an infinitely wide pipe. This is reasonable for a 2D model, which means the width of the model is much bigger than the height. In reality, however, piping problems are three-dimensional: water flows into the pipe from all sides, and the pipe develops in an all but straight line (tortuosity). This makes the approach used in this thesis less applicable in 3-dimensional path finding models.
Aguilar-López et al (Aguilar-López, Warmink, Schielen, Hulscher, 2018[2]) proposes not to use the parallel plate approach, as in the IJkdijk experiment, this approach does not lead to pipe progression. However, Bersan et al (Bersan, Jommi, Koelewijn, Simonini, 2013[4]) and Sellmeijer (Sellmeijer, 1988[20]) argue that the parallel plate approach *can* be used.

Quasi-steady state model versus transient reality
In this thesis, it was assumed that the boundary conditions remain constant over time. However, usually, the water level on the upstream side is determined by either tide or a river water level. These usually vary in time, which means the boundary conditions are not constant. This also implies that the flow in the subsoil is not constant in time. Taking into account the typical development rate of pipes (order of hours to days), the boundary conditions may in reality be different between the 'beginning pipe' stage and the 'penetrated into second layer' stage. This effect, however, has been disregarded in this thesis for the sake of simplicity. Taking this into account would imply running the model for every stage of pipe formation, for numerous pipe diameters, for a set of parameter values as well as for multiple water levels, which quickly becomes unfeasible. For this simplification, it needs to be assumed that the sub-mechanism (horizontal erosion or vertical erosion) with the lowest FRP is the one that takes place, which in turn assumes linear behaviour, which is not necessarily the case.

Boundary conditions
An important point to discuss is the fact that the boundary conditions applied during the Hedwigepolder experiment (and, therefore, in this thesis) were rather unique. A pumping well was installed on the upstream side (left side of Figure 6.1) of the dike. This well penetrated all layers, which means the water could already start in a given layer, as can be seen in Figure 6.1. Also, the sheetpile wall at the downstream side of the geometry does not allow for landward flow, forcing the water out through the ditch.
In more realistic situations, the water would first have to penetrate from the river into layer 0, then to layer 1, then 2 then 3, as shown in Figure 6.2. This would (especially with a high permeability anisotropy) experience much resistance, making the water less likely to even reach Layer 1. Also, the absence of a sheetpile wall allows for water 'escaping' the geometry and pressures dissipating as a result of leakage (Bezuijen, 1970[5]).

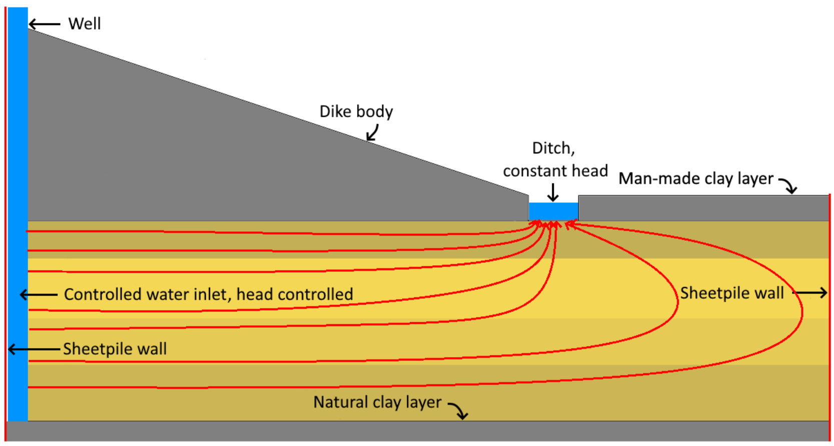


Figure 6.1: Hedwigepolder Boundary Conditions

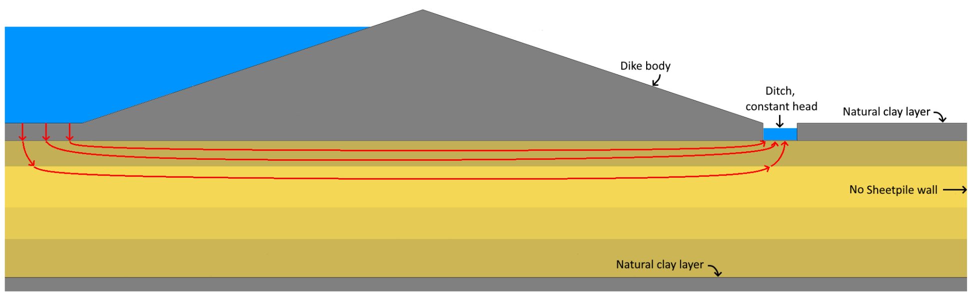


Figure 6.2: More realistic Boundary Conditions

Erosion after heave

Throughout the thesis it is assumed that for the erosion sub-mechanisms (in both horizontal and vertical erosion), cohesion is the only resisting mechanism. In other words, no factors such as friction are present. Friction is absent if the grains are not in contact, which is true in the case of heave. For this reason, it is checked if heave occurs, before erosion of any kind is evaluated. It could, however, also be argued that cohesion is a part of the heave mechanism. This has, however, not been done for the reason that in this case, there would be no mechanism keeping the grains in place, and erosion would always happen once heave has taken place. In other words, $FRP_{horizontale\ erosion}$ and $FRP_{vertical\ erosion}$ would both be zero, which would not give any results the way the process is modelled in this thesis. It could, however be possible to model vertical as heave in vertical direction, in which case the cohesion term should be included in the heave mechanism and the erosion mechanisms would become obsolete.

Cohesion and -anisotropy
Cohesion takes place between small grains (silt and smaller, also called 'lutum' (Visser, 1976[28])) only. In the Hedwigepolder, Layer 0 was a mixture of clay and sand. To take the increased strength due to cohesion into account, it was modelled as if cohesion works on a fraction of the surface area of a sand grain, whereas in reality, this is not the case: cohesion works between the smaller particles, which in turn stick to sand grains (Gu, Huang, Liu, Zhang, Gao, Xue, 2019[11]). Furthermore, it is assumed that a certain kind of directionality is experienced by the cohesion. This cohesion anisotropy is assumed to differ per layer. It was assumed that although having the same value, the direction of the anisotropy is different between Layers 0 and 1. In Layer 0, a cohesion anisotropy of 1.5 means that cohesion is 1.5 times stronger in horizontal direction than in vertical direction, whereas in Layer 1, it means that the cohesion in vertical direction is 1.5 times stronger than in horizontal direction. It was found that this was critical in the ability of the model to describe a deeper pipe.

Cohesion value in Layer 1
Layer 1 consists of sand. Therefore, no cohesion is to be expected. However, the FRP for the horizontal erosion and vertical erosion sub-mechanisms depend on cohesion for their resisting terms. To avoid getting FRP values of 0, Layer 1 needs to have at least some cohesion. This was assumed to have a value of $1/100$ times the cohesion of Layer 0. This could be reasonable, because usually in the subsoil, layers do not change abruptly. It is, therefore, not unthinkable that there is still some cohesion in the top part of Layer 1, which is where the pipe was observed in the Hedwigepolder.

Erosion channel shape and size
The shape and size of the vertical erosion channel were found to have a large influence on the FRP values. However, the development of the vertical erosion channel in the early stages of pipe development was not recorded in the Hedwigepolder (as no deeper pipe was expected initially), so its size and shape remain unknown. It is recommended to do more research into these aspects of the vertical erosion channel. After the experiments, the deeper pipe *was* filled with cement and excavated, so the final shape of the channels is known. However, the final size suggests that there may have been turbulent flow, so the final shape could not be used in this model, as Darcy's law only works for laminar flow. Instead, this model assumes a pipe with a constant pipe width.

Local sensitivity versus global sensitivity
In this thesis, the influence of parameters was investigated using the local sensitivity analysis. This is the sensitivity of a parameter using a reference point. In this case, the reference point are the Hedwigepolder values. However, the value of one parameter dictates the influence of another. For example, the contact area on which cohesion works, A_{coh}, depends on the surface area of the grain, which in turn depends on its diameter d_{rep}. Also, the permeability of a layer is related to its grain size distribution, and, thus, d_{rep} (Shepherd, 1989[21]; Chilingar, 1964[8]; Onur, 2014[17]) . These intercorrelations were ignored in this thesis. Instead, measured values from the Hedwigepolder were used, which should agree with the relations posted by these papers.
A global sensitivity analysis are often seen as more sophisticated sensitivity analyses. They do not only take intercorrelations between parameters into account, but also accounts for the stochastic nature of these variables (Iooss, Lemaitre, 2015[13]). The stochastic nature of elements was outside of the scope of this thesis.

Aquifer thickness
In this thesis, it was assumed that the total thickness of the subsoil domain was four metres at all times, like it was in the Hedwigepolder. It is likely that this has influenced the calculation of the influence of layers 2 and 3, as a change in D_0 or D_1 influences D_2 and D_3 and the ability of these layers to convey water. The model could be improved by keeping the transmissivity $T_n = k_n * D_n$ of Layers 2 and 3 constant, rather than their permeabilities. The fact that D_2 and D_3 change when D_0 or D_1 changes makes the results of the model concerning these parameters unreliable.

Blanket layer definition
In the Hedwigepolder, a very thick and hardly permeable (order of $1 * 10^{-16} m^2$ (Geotechdata, 2023[25])) blanket layer was applied on the hinterland side of the dike (except at the location of the ditch), to ensure outflow through the ditch. Below this layer was Layer 0, with a permeability of order $1 * 10^{-13} m^2$. This significant difference in permeability justifies calling the former the blanket layer, and the latter the aquifer. However, between the permeabilities of Layers 0 and 1 is another order of magnitude ($1 * 10^{-13}$ versus $1 * 10^{-12}$, respectively). It could, therefore, be argued that Layer 0 is an extra (albeit more permeable) blanket layer. In this case, the deeper piping process would be reduced to conventional piping with an unconventionally thick, multiple-layered blanket layer.

Disregarded processes
An aspect that has not been treated in this thesis is the ability of the soil to withstand collapse. The focus of this report has been to identify how a deeper pipe can come to be, but after coming to be, the newly formed pipe has to be able to remain intact. Soil grains in the 'walls' of the vertical erosion channel and 'roof' of the horizontal erosion channel have not been investigated. However, these are subject to all kinds of seepage and soil pressure forces, which could lead to collapse of the pipe. This is an interesting subject for further research.

6.2. Discussion of results

The results of the sensitivity analysis show some interesting findings. These are discussed here.

Impact d_{rep}
In chapter 5, it was found that the representative grain diameter (d_{rep}) had quite a large influence on several sub-mechanisms. This can be explained by the fact that the flow force on a grain is expressed as a pressure gradient timed the grain diameter. However, the representative grain diameter of a layer was simply arbitrarily taken to be the d_{50} of that layer (for example, Sellmeijer (Sellmeijer, 1988[20]) used d_{70} as representative grain diameter, because fit the results better, so d_{50} is not necessarily the best value for d_{rep}). As this thesis is not calibrated to results regarding pressure or discharges in the Hedwigepolder, it is not certain if d_{50} is the right choice for the representative grain diameter: a if the d_{70} were to be used, the forces on grains would be larger, the contact area of cohesion would be larger, and so, the weight of the representative particle. All of these influence the FRP values, which means that upon taking a different grain size fractile, these values may vary significantly.

Impact A_{coh} and c
It was also found that the cohesion contact area A_{coh} and the cohesion value c have a large influence in the horizontal and vertical erosion processes. This is logical, because for these sub-mechanisms, cohesion is the only resisting force present. If there are other parameters contributing to the resistance against erosion, that have not been addressed in this thesis, the effect of cohesion is already different. Furthermore, as the impact of cohesion is found to be very large, it is required to have a better understanding of how cohesion can best be modelled in a sub-mechanism (the way of modelling cohesion was chosen to be relatively arbitrarily for the impact A_{coh} turns out to have). More research into how to include cohesion in a sub-mechanism is recommended.

Impact α_{coh}
It was found that the cohesion anisotropy is an important parameter in deeper piping. It dictates the directionality of the cohesive strength, which could be explained by the fact that clay particles are shaped like plates. The fact that the cohesion anisotropy is used in this thesis to work in favor

of deeper piping (stronger horizontal cohesion in Layer 0, stronger vertical cohesion in Layer 1) can be explained by (Stockton, Leshchinsky, Olsen, Evans, 2019[22]), who described situations where the cohesion anisotropy aligns with the layering of the soil, which in this case would mean Stronger effects in horizontal direction than in vertical direction. It is likely (though not researched) that this effect takes place differently in Layer 1 than in Layer 0.

Impact α_k
The anisotropy of the permeability is found to work in favour of deeper piping. This can be explained by the fact that the flow prefers to flow farther in horizontal direction, because in the vertical direction, it experiences more resistance. This means that the water is tempted to travel its horizontal distance in layers with higher transmissivity, and then goes straight up towards the exit. This makes the flow lines more vertical, and deeper piping more likely.
Mind that this is only the case for the boundary conditions as they were in the Hedwigepolder (see Figure 6.1). In more realistic scenarios, the water would first have to enter Layer 1 through Layer 0 (see Figure 6.2), which would experience a lot of resistance if α_k is high: the water would much rather flow horizontally.

Impact k_0 and k_1
It was found that k_0 and k_1 have a large influence on the deeper piping mechanism. A small value of k_0 results in the flow 'avoiding' that layer, and choosing a more permeable layer instead. Similarly, a high value of k_1 attracts flow. The flow is expected to follow the path of least resistance, which means that if travelling vertically through Layer 0 twice and flowing horizontally through Layer 1 poses less resistance that only flowing through Layer 0 horizontally, this path is preferred. This does mean, though, that the water that 'decided' to go through Layer 1 has to flow vertically up towards the exit. In conclusion, this would mean a combination of a small permeability in Layer 0 and a high permeability of Layer 1 (relative to Layer 0) is favourable for deeper piping. However, Figures 5.2e and 5.2f show that the $FRP_{horizontal\ erosion}/FRP_{vertical\ erosion}$ is largest (in other words: vertical erosion most is most likely) for large values of k_0 and small values of k_1.

Impact D_0
A large value of D_0 makes it harder for the flow to 'find' Layer 1, and, therefore, reduces the probability for deeper piping. Also, if D_0 is larger, there is more soil to erode through before finding the layer where horizontal erosion starts.
In Appendix A.3 it can be seen that FRP_{heave} behaves rather strangely as a result of changes in D_0. This might be explained by the fact that a cell in the mesh is $a_{max}/20$ large, whereas a_{max} was taken to be equal to 50 grains. This means a cell is 2.5 by 2.5 grains large, while the vertical erosion channel sizes under consideration (being 4, 5, 6, 7, 8, 10 and 20 grains) operate on that same scale. As a cell is assigned only one value, a mean value is taken for each cell. If the channel is 1.5 cells big for example, the 0.5 cell that is not entirely covered by erosion channel properties might behave strangely for this reason. This could explain the strange behaviour of D_0.

Cut line length
It was found that the cut line length does play a role in whether or not a pipe develops horizontally or vertically. From Figure 5.3e in chapter 5, it can be seen that for smaller cut line lengths, the values are much closer to unity than for higher values, implying that one is more likely to find conventional piping when a short cut line is chosen. Conversely, higher values tend to return a larger likelihood for a conventional pipe. As the cut line length is to be chosen by the modeller, knowledge on the impact of this choice is required.

7 Conclusion

During two piping experiments in the Hertogin Hedwigepolder, a pipe was found at a greater depth than expected. This 'deeper piping' phenomenon had not yet been identified, described or modelled before. This problem was addressed in this thesis.

What is the difference between conventional piping and deeper piping?
The difference between conventional piping and deeper piping was defined, highlighting the presence of a vertical erosion channel in the latter. This can only occur if the initial heave sub-mechanism is not followed by horizontal erosion, but by vertical erosion. This vertical erosion phase consists of more heave below the already-formed pipe, the erosion of material in vertical direction and the transport of this material. After the vertical pipe has penetrated through the top subsoil layer, horizontal erosion takes place like it would have in conventional piping. To assess whether or not these processes take place, the concept of the Force Ratio Parameter (FRP) was introduced, which is equal to the maximum resisting term divided by the occurring forcing term. Deeper piping has been defined to take place if $FRP_{heave} < 1$, $FRP_{horizontal\ erosion} > FR_{ver,ero}$ and $FRP_{vertical\ transport} < 1$ for as long as the pipe tip is located in the top subsoil layer, and $FRP_{heave} < 1$, $FRP_{horizontal\ erosion} < FR_{ver,ero}$ and $FRP_{vertical\ transport} < 1$ once the pipe tip has penetrated entirely through this layer. It was assumed that cohesion could be included in the mechanism description by including it in the 'erosion' phase, rather than in the 'more heave' phase. This assumption might have been wrong, but had this not been done, heave would imply direct onset of erosion.

How can a deeper pipe be modelled?
To give more meaning to the FRP values, the forcing and resisting terms were elaborated on. The resistance term for heave was modeled as a critical gradient, while the one for vertical transport was represented by the submerged weight of a sand grain. Cohesion c multiplied by a cohesion contact area A_{coh} was used to model resistance forces for horizontal and vertical erosion, and cohesion anisotropy was introduced to take account of directionality of cohesion. A Finite Element Model, made in COMSOL Multiphysics, was employed to calculate driving forces by describing groundwater flows using Darcy's law. To avoid iterative boundary conditions at the pipe-to-soil interface, the fictitious permeability model was applied. Uncertainty regarding the size of the vertical pipe was accounted for using the permeability zone model. These models have their ranges of applicability, which have not been exceeded in this model. Lastly, the conditional permeability approach facilitated the variation of geometric properties of the subsurface without the need for constructing a new mesh.
Mind that a proper characterisation of the term 'blanket layer' might result in deeper piping to be conventional piping under an unusually thick blanket layer.

Can deeper piping in the Hedwigepolder be explained by the model?
By utilizing these modeling techniques, a model was developed to simulate the conditions in the Hedwigepolder test site. The Hedwigepolder had four layers on soil, of which the top one had low permeability compared to the other three. It could be argued that this layer is a kind of 'second blanket layer', but for this, a description of the definition of a blanket layer is needed. Furthermore, the boundary

conditions were such that the water was injected in every layer directly, bypassing the necessity of the water to penetrate the upper layers first before entering the lower layers.
It was found that the model is able to return a deeper pipe, and with this, the proposed model has been verified to be able to return a deeper pipe.

What are the most important parameters contributing to a deeper pipe?
Subsequently, the influence of specific parameters on the mechanism of deeper piping was examined. It was found that numerous parameters play a role in deeper piping, as different sub-mechanisms often have distinct 'influential parameters'. The following aspects and parameters were identified as significant contributors to deeper piping. Subsoil configurations that posses many of these traits are found to be more susceptible to deeper piping, given the boundary conditions and parameters as in the Hedwigepolder:

- Anisotropy of cohesion (α_{coh}): Larger values increase the chance of deeper piping.
- Anisotropy of permeability (α_k): Larger values increase the chance of deeper piping.
- Permeability of Layer 0 (k_0): Larger values increase the chance of deeper piping.
- Permeability of Layer 1 (k_1): Smaller values increase the chance of deeper piping.
- Thickness of Layer 0 (D_0): Larger values increase the chance of deeper piping.
- Representative grain diameter d_{rep}: Smaller values increase the chance of deeper piping.

To identify the key factors influencing the formation of deeper pipes
The objective of this thesis, has therefore been achieved. This research successfully addressed the challenges associated with defining and predicting deeper piping. By developing a model and identifying key influencing parameters, valuable insights into the underlying mechanisms of this phenomenon were gained. The findings contribute to an understanding of deeper piping.

8 Recommendations

It is recommended to do more research into the following subjects:

Vertical erosion channel shape and size
It was found that the size of the vertical erosion channel can have a large influence on deeper piping. As the shape (straight) and size (between 3 and 20 grain diameters) of the vertical erosion channel were simply assumed in this thesis, this forms a big factor of uncertainty. It is therefore strongly recommended to do more research into the shape and size of the vertical erosion channel in deeper piping.

Turbulence
If the channel size gets larger due to widening of the pipe, the flow might become turbulent. Until that point, the fictitious permeability model can be used. However, if Reynolds numbers exceed 10, the turbulent influences become too large to ignore. It is recommended to do research into other ways of modelling the pipe flow, which do allow for turbulence.

Internal instability
The model can be extended with a suffusion module. This can be done by inserting a grain size distribution. If this grain size distribution is gap-graded, suffusion is possible. It does, however, need to be known what the fictitious permeability of the soil is after suffusion. This is an interesting topic for further research.

Horizontal erosion channel
A small addition to the model could be the horizontal erosion channel. This has not been done in this thesis, because of the large computational effort and storage limitations, but it would make for a complete description of the deeper piping process as a whole. Finding an even more efficient way of modelling might overcome these difficulties. If a horizontal pipe part were to be included in this model, its size and shape also have to be determined.

Global sensitivity
It is also recommended to do a global sensitivity analysis, to get a comprehensive understanding of the sensitivity of the sub-mechanism to the parameters, combined with the probabilities of parameters having certain values.

Wall and roof stability criteria
This thesis has shown how a deeper pipe can come to be. However, after being formed, a pipe also has to be able to remain intact. Lateral soil pressures on the 'walls' of the vertical erosion channel and 'roof' of the horizontal erosion channel are subject to forces like earth pressure and seepage flow forces. These could potentially make the formed pipe collapse. Aspects like soil arching and cohesion might play a role in this process.

Include heave in cohesion term
In this thesis, the following order was used: first show the 'willingness' of the soil to be heaved, after which the actual erosion takes place. The former was assumed not to be affected by cohesion, whereas the latter was. For future research, it might be interesting to include cohesion in the heave term, and then, either include it in the erosion terms as well, or skip the erosion terms altogether. The argument for the latter would be that as soon as inter-particle contact is lost (in other words: heave has occurred), so are all the mechanisms keeping a grain in place.

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A

Appendices

A.1. Parametric sweep results

In this Appendix, the % change in FRP values for a 10% increase in parameter value for every parameter in the parametric sweep is shown for each of the stages of pipe development.

$0/3 * D_0$

A_coh

	%change FRP heave						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	-3E-16	0	0
4	0	0	0	0	-3E-16	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	2.8E-16	0	0
7	0	0	0	0	2.8E-16	0	0
8	0	0	0	0	0	0	0
9	0	0	0	0	-3E-16	0	0
10	0	0	0	0	-6E-16	0	0

	%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
2	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
3	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
4	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
5	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
6	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
7	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
8	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
9	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
10	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676

	%change FRP ver ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
2	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
3	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
4	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
5	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
6	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
7	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
8	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
9	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
10	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608

	%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	2.7E-15	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	-3E-15	0	0
6	0	0	0	0	2.7E-15	0	0
7	0	0	0	0	0	0	0
8	0	0	0	0	-3E-15	0	0
9	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0

c

	%change FRP heave						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0
0.2	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0
0.4	0	0	2.8E-15	0	0	0	0
0.5	0	0	2.8E-15	0	0	0	0
0.6	0	0	-3E-15	0	0	0	0
0.7	0	0	-3E-15	0	0	0	0
0.8	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	0	-3E-15	0	0
1.8	0	0	2.8E-15	0	0	0	0
1.9	0	0	0	0	2.7E-15	0	0
2	0	0	-6E-15	0	0	0	0

	%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.1	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.2	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.3	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.4	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.5	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.6	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.7	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.8	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
0.9	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.1	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.2	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.3	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.4	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.5	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.6	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.7	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.8	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
1.9	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676
2	0.85189	0.85227	0.84333	0.84376	0.83886	0.83397	0.82676

	%change FRP ver ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.1	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.2	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.3	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.4	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.5	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.6	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.7	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.8	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
0.9	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.1	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.2	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.3	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.4	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.5	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.6	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.7	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.8	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
1.9	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608
2	0.59101	0.59532	0.65849	0.66192	0.60363	0.72323	0.89608

	%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0
0.2	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0
0.6	0	0	0	0	0	0	0
0.7	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0
1.8	0	0	0	0	0	0	0
1.9	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

Figure A.1: A_c and c

Alpha_coh

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6.333333333	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6.666666667	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	-0.8865	-0.893	-0.9877	-0.9929	-0.9054	-1.0848	-1.3441	0	0	0	0	0	0	0
1	-0.6649	-0.6697	-0.7408	-0.7447	-0.6791	-0.8136	-1.0081	-8E-15	0	0	0	0	0	0
1.333333333	-0.3546	-0.3572	-0.3951	-0.3972	-0.3622	-0.4339	-0.5376	0	0	0	0	0	0	0
1.666666667	-0.2216	-0.2232	-0.2469	-0.2482	-0.2264	-0.2712	-0.336	0	0	0	0	0	0	0
2	-0.152	-0.1531	-0.1693	-0.1702	-0.1552	-0.186	-0.2304	-8E-15	0	0	0	0	0	0
2.333333333	-0.1108	-0.1116	-0.1235	-0.1241	-0.1132	-0.1356	-0.168	0	0	0	0	0	0	0
2.666666667	-0.0844	-0.085	-0.0941	-0.0946	-0.0862	-0.1033	-0.128	0	0	0	0	0	0	0
3	-0.0665	-0.067	-0.0741	-0.0745	-0.0679	-0.0814	-0.1008	0	0	0	0	0	0	0
3.333333333	-0.0537	-0.0541	-0.0599	-0.0602	-0.0549	-0.0657	-0.0815	8.2E-15	0	0	0	0	0	0
3.666666667	-0.0443	-0.0446	-0.0494	-0.0496	-0.0453	-0.0542	-0.0672	8.2E-15	0	0	0	0	0	0
4	-0.0372	-0.0375	-0.0414	-0.0417	-0.038	-0.0455	-0.0564	-8E-15	0	0	0	0	0	0
4.333333333	-0.0317	-0.0319	-0.0353	-0.0355	-0.0323	-0.0387	-0.048	-8E-15	0	0	0	0	0	0
4.666666667	-0.0273	-0.0275	-0.0304	-0.0306	-0.0279	-0.0334	-0.0414	0	0	0	0	0	0	0
5	-0.0237	-0.0239	-0.0265	-0.0266	-0.0243	-0.0291	-0.036	0	0	0	0	0	0	0
5.333333333	-0.0209	-0.021	-0.0232	-0.0234	-0.0213	-0.0255	-0.0316	8.2E-15	0	0	0	0	0	0
5.666666667	-0.0185	-0.0186	-0.0206	-0.0207	-0.0189	-0.0226	-0.028	0	0	0	0	0	0	0
6	-0.0165	-0.0166	-0.0183	-0.0184	-0.0168	-0.0202	-0.025	-8E-15	0	0	0	0	0	0
6.333333333	-0.0148	-0.0149	-0.0165	-0.0165	-0.0151	-0.0181	-0.0224	0	0	0	0	0	0	0
6.666666667	-0.014	-0.0141	-0.0156	-0.0157	-0.0143	-0.0171	-0.0212	0	0	0	0	0	0	0

Alpha_k

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.1539	-0.1686	-0.1827	-0.196	-0.2079	-0.2293	-0.3106	-0.3215	-0.3212	-0.3227	-0.3224	-0.3224	-0.3213	-0.3045
0.5	-0.1174	-0.1288	-0.1396	-0.1499	-0.159	-0.1755	-0.2381	-0.2363	-0.2359	-0.2373	-0.237	-0.237	-0.2359	-0.22
0.666666667	-0.0659	-0.0724	-0.0786	-0.0846	-0.0898	-0.0992	-0.1351	-0.1153	-0.1149	-0.1161	-0.1158	-0.1157	-0.1146	-0.1
0.833333333	-0.0429	-0.0474	-0.0515	-0.0555	-0.059	-0.0651	-0.0889	-0.0615	-0.0612	-0.0621	-0.0618	-0.0617	-0.0605	-0.0466
1	-0.0306	-0.0338	-0.0368	-0.0397	-0.0422	-0.0467	-0.0638	-0.0333	-0.033	-0.0337	-0.0334	-0.0332	-0.0319	-0.0184
1.166666667	-0.0231	-0.0256	-0.0279	-0.0301	-0.032	-0.0354	-0.0484	-0.0168	-0.0165	-0.017	-0.0168	-0.0165	-0.0152	-0.0021
1.333333333	-0.0181	-0.0202	-0.022	-0.0238	-0.0253	-0.028	-0.0383	-0.0066	-0.0063	-0.0067	-0.0064	-0.006	-0.0047	0.00814
1.5	-0.0147	-0.0165	-0.0179	-0.0194	-0.0206	-0.0228	-0.0312	0.00013	0.0004	0.00016	0.00041	0.00084	0.00222	0.01478
1.666666667	-0.0122	-0.0137	-0.015	-0.0162	-0.0172	-0.019	-0.0261	0.00466	0.00492	0.00479	0.00503	0.00551	0.00692	0.01924
1.833333333	-0.0103	-0.0117	-0.0127	-0.0138	-0.0146	-0.0162	-0.0222	0.0078	0.00805	0.00802	0.00825	0.00877	0.0102	0.0223
2	-0.0089	-0.0101	-0.011	-0.0119	-0.0127	-0.014	-0.0192	0.01001	0.01026	0.01031	0.01053	0.01108	0.01254	0.02442
2.166666667	-0.0077	-0.0088	-0.0096	-0.0104	-0.0111	-0.0122	-0.0168	0.01159	0.01183	0.01195	0.01217	0.01275	0.01422	0.0259
2.333333333	-0.0068	-0.0078	-0.0085	-0.0092	-0.0098	-0.0108	-0.0149	0.01271	0.01295	0.01313	0.01335	0.01395	0.01544	0.02693
2.5	-0.0061	-0.0069	-0.0076	-0.0082	-0.0087	-0.0097	-0.0133	0.01351	0.01375	0.01399	0.01419	0.01482	0.01632	0.02763
2.666666667	-0.0054	-0.0062	-0.0068	-0.0074	-0.0079	-0.0087	-0.012	0.01408	0.01431	0.0146	0.0148	0.01545	0.01695	0.02809
2.833333333	-0.0049	-0.0056	-0.0062	-0.0067	-0.0071	-0.0079	-0.0108	0.01447	0.01469	0.01504	0.01523	0.01589	0.0174	0.02837
3	-0.0045	-0.0051	-0.0056	-0.0061	-0.0065	-0.0072	-0.0099	0.01473	0.01495	0.01533	0.01552	0.0162	0.01771	0.02851
3.166666667	-0.0041	-0.0047	-0.0052	-0.0056	-0.006	-0.0066	-0.0091	0.01489	0.0151	0.01552	0.01571	0.0164	0.01791	0.02856
3.333333333	-0.0039	-0.0045	-0.005	-0.0054	-0.0057	-0.0063	-0.0087	0.01495	0.01516	0.0156	0.01578	0.01648	0.01799	0.02856

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.5071	-0.5088	-0.4984	-0.4961	1.22715	-0.4821	-0.4507	0.0865	-0.2617	-0.1035	-0.0532	-0.0754	-0.0223	-0.0041
0.5	-0.4003	-0.4015	-0.3929	-0.3911	-1.1195	-0.38	-0.3548	0.00179	-0.2308	-0.0919	-0.0499	-0.0722	-0.022	-0.0038
0.666666667	-0.2449	-0.2453	-0.2395	-0.2386	-1.5456	-0.2317	-0.2156	-0.0997	-0.18	-0.0738	-0.0443	-0.0635	-0.0211	-0.0034
0.833333333	-0.1697	-0.1698	-0.1654	-0.165	0.19249	-0.1601	-0.1486	-0.1187	-0.1469	-0.063	-0.0403	-0.054	-0.0199	-0.0031
1	-0.1266	-0.1266	-0.1232	-0.1229	-0.0136	-0.1193	-0.1105	-0.1193	-0.1246	-0.0559	-0.0371	-0.0468	-0.0188	-0.0029
1.166666667	-0.0993	-0.0992	-0.0964	-0.0963	-0.0411	-0.0934	-0.0864	-0.1148	-0.1085	-0.0508	-0.0347	-0.0413	-0.0178	-0.0027
1.333333333	-0.0807	-0.0806	-0.0782	-0.0782	-0.0443	-0.0758	-0.07	-0.1091	-0.0965	-0.047	-0.0326	-0.0372	-0.0168	-0.0025
1.5	-0.0674	-0.0672	-0.0652	-0.0652	-0.042	-0.0632	-0.0583	-0.1034	-0.0872	-0.0441	-0.0309	-0.0339	-0.016	-0.0024
1.666666667	-0.0574	-0.0573	-0.0555	-0.0555	-0.0386	-0.0538	-0.0496	-0.0981	-0.0798	-0.0417	-0.0295	-0.0313	-0.0152	-0.0023
1.833333333	-0.0498	-0.0496	-0.048	-0.0481	-0.0351	-0.0466	-0.0429	-0.0933	-0.0737	-0.0397	-0.0282	-0.0291	-0.0146	-0.0022
2	-0.0437	-0.0436	-0.0422	-0.0422	-0.0319	-0.0409	-0.0376	-0.089	-0.0687	-0.0381	-0.0271	-0.0273	-0.014	-0.0021
2.166666667	-0.0389	-0.0387	-0.0374	-0.0375	-0.029	-0.0363	-0.0333	-0.0851	-0.0644	-0.0367	-0.0261	-0.0258	-0.0134	-0.002
2.333333333	-0.0349	-0.0347	-0.0336	-0.0336	-0.0266	-0.0325	-0.0299	-0.0817	-0.0607	-0.0355	-0.0252	-0.0245	-0.0129	-0.002
2.5	-0.0316	-0.0314	-0.0304	-0.0304	-0.0244	-0.0294	-0.027	-0.0785	-0.0576	-0.0344	-0.0244	-0.0233	-0.0125	-0.0019
2.666666667	-0.0288	-0.0286	-0.0277	-0.0277	-0.0225	-0.0268	-0.0245	-0.0757	-0.0548	-0.0335	-0.0237	-0.0223	-0.0121	-0.0018
2.833333333	-0.0264	-0.0262	-0.0253	-0.0254	-0.0209	-0.0245	-0.0225	-0.0732	-0.0523	-0.0326	-0.0231	-0.0214	-0.0117	-0.0018
3	-0.0243	-0.0241	-0.0234	-0.0234	-0.0194	-0.0226	-0.0207	-0.0708	-0.0501	-0.0319	-0.0225	-0.0206	-0.0113	-0.0017
3.166666667	-0.0225	-0.0224	-0.0216	-0.0216	-0.0181	-0.0209	-0.0191	-0.0687	-0.0482	-0.0312	-0.0219	-0.0198	-0.011	-0.0017
3.333333333	-0.0217	-0.0215	-0.0208	-0.0208	-0.0175	-0.0201	-0.0184	-0.0677	-0.0473	-0.0308	-0.0217	-0.0195	-0.0109	-0.0017

Figure A.2: $Anis_c$ and $Anis_k$

k_0

Times original value	%change FRP heave							%change FRP hor ero						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.04521	0.05029	0.05473	0.05907	0.06297	0.06991	0.09695	0.38792	0.38813	0.3834	0.38357	0.38188	0.37922	0.3762
0.4	0.04333	0.04818	0.05243	0.05657	0.06029	0.06691	0.09265	0.37232	0.37253	0.36819	0.36835	0.36664	0.36417	0.36128
0.7	0.04004	0.0445	0.04839	0.05219	0.0556	0.06166	0.08512	0.3452	0.3454	0.34161	0.34175	0.34007	0.33789	0.33523
1	0.03744	0.04159	0.04521	0.04874	0.0519	0.05751	0.07918	0.32398	0.32416	0.32068	0.3208	0.31919	0.31718	0.3147
1.3	0.03521	0.0391	0.04248	0.04578	0.04873	0.05397	0.0741	0.3058	0.30596	0.30271	0.30282	0.30128	0.29941	0.29708
1.6	0.03325	0.0369	0.04008	0.04317	0.04594	0.05085	0.06965	0.28975	0.2899	0.28684	0.28693	0.28547	0.28371	0.28151
1.9	0.0315	0.03494	0.03794	0.04085	0.04345	0.04806	0.06568	0.27535	0.27549	0.2726	0.27268	0.27129	0.26962	0.26754
2.2	0.02991	0.03316	0.036	0.03874	0.0412	0.04555	0.06211	0.26229	0.26242	0.25968	0.25975	0.25842	0.25684	0.25487
2.5	0.02846	0.03155	0.03423	0.03683	0.03916	0.04326	0.05888	0.25036	0.25048	0.24787	0.24794	0.24667	0.24517	0.24329
2.8	0.02714	0.03007	0.03262	0.03508	0.03728	0.04118	0.05593	0.2394	0.23951	0.23702	0.23708	0.23586	0.23443	0.23264
3.1	0.02592	0.02871	0.03113	0.03347	0.03556	0.03926	0.05322	0.22927	0.22937	0.227	0.22705	0.22588	0.22451	0.22281
3.4	0.02479	0.02745	0.02975	0.03198	0.03398	0.03749	0.05073	0.21987	0.21997	0.2177	0.21774	0.21662	0.21532	0.21368
3.7	0.02374	0.02628	0.02848	0.03061	0.0325	0.03585	0.04843	0.21113	0.21122	0.20905	0.20908	0.20801	0.20675	0.20519
4	0.02277	0.02519	0.0273	0.02933	0.03114	0.03432	0.0463	0.20296	0.20305	0.20097	0.20099	0.19996	0.19876	0.19726
4.3	0.02186	0.02418	0.02619	0.02813	0.02986	0.03291	0.04432	0.19532	0.1954	0.1934	0.19342	0.19243	0.19128	0.18983
4.6	0.02101	0.02323	0.02516	0.02702	0.02867	0.03158	0.04247	0.18815	0.18823	0.1863	0.18632	0.18537	0.18425	0.18287
4.9	0.0206	0.02277	0.02466	0.02648	0.02809	0.03094	0.04158	0.18468	0.18475	0.18286	0.18288	0.18194	0.18085	0.17949

Times original value	%change FRP ver ero							%change FRP ver tra						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.22046	0.22356	0.21478	0.21964	0.19341	0.21563	0.2031	-0.3007	-0.0672	-0.1576	-0.0657	-0.0184	-0.0231	-0.005
0.4	0.20903	0.21166	0.2038	0.20775	0.17526	0.2039	0.19209	-0.2363	-0.0387	-0.1318	-0.0563	-0.0039	-0.02	-0.0043
0.7	0.18859	0.19051	0.18434	0.1869	0.13568	0.18335	0.17279	-0.1563	-0.0088	-0.0921	-0.0417	0.00948	-0.015	-0.0032
1	0.17207	0.17363	0.16869	0.17049	0.07633	0.16721	0.15764	-0.1334	-0.0072	-0.0697	-0.0332	0.00656	-0.0121	-0.0026
1.3	0.1581	0.15945	0.15535	0.15668	-0.0842	0.15365	0.14491	-0.1221	-0.0073	-0.0557	-0.0277	0.00341	-0.0103	-0.0022
1.6	0.14604	0.14726	0.14375	0.14477	-5.7128	0.14197	0.13395	-0.1157	-0.008	-0.0463	-0.024	0.00081	-0.009	-0.0019
1.9	0.13548	0.1366	0.13355	0.13436	0.91989	0.13176	0.12437	-0.1119	-0.0088	-0.0397	-0.0213	-0.0013	-0.0081	-0.0017
2.2	0.12616	0.1272	0.12451	0.12516	6.37303	0.12275	0.1159	-0.1097	-0.0098	-0.0348	-0.0192	-0.003	-0.0075	-0.0015
2.5	0.11785	0.11883	0.11644	0.11697	-0.169	0.11473	0.10836	-0.1084	-0.0108	-0.0312	-0.0177	-0.0045	-0.007	-0.0014
2.8	0.1104	0.11133	0.10919	0.10963	0.00611	0.10754	0.10161	-0.1079	-0.0117	-0.0283	-0.0165	-0.0057	-0.0066	-0.0013
3.1	0.10369	0.10458	0.10265	0.10302	0.04831	0.10106	0.09551	-0.1079	-0.0126	-0.0261	-0.0155	-0.0068	-0.0062	-0.0012
3.4	0.09762	0.09847	0.09671	0.09703	0.06289	0.09519	0.09	-0.1083	-0.0135	-0.0242	-0.0147	-0.0077	-0.006	-0.0012
3.7	0.09209	0.09291	0.09131	0.09158	0.06812	0.08986	0.08498	-0.1091	-0.0143	-0.0228	-0.014	-0.0086	-0.0058	-0.0011
4	0.08704	0.08784	0.08637	0.08661	0.06943	0.08499	0.08039	-0.1101	-0.0151	-0.0215	-0.0135	-0.0093	-0.0056	-0.0011
4.3	0.08242	0.08319	0.08185	0.08205	0.0689	0.08052	0.07619	-0.1114	-0.0158	-0.0205	-0.013	-0.0099	-0.0054	-0.001
4.6	0.07817	0.07892	0.07768	0.07786	0.06744	0.07642	0.07233	-0.1129	-0.0164	-0.0196	-0.0126	-0.0105	-0.0053	-0.001
4.9	0.07613	0.07688	0.07569	0.07586	0.06656	0.07446	0.07048	-0.1137	-0.0167	-0.0192	-0.0124	-0.0108	-0.0052	-0.001

k_1

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.1798	-0.1993	-0.2163	-0.2327	-0.2474	-0.2734	-0.3724	-1.5761	-1.5768	-1.5608	-1.5616	-1.5529	-1.5443	-1.533
0.4	-0.1112	-0.1232	-0.1338	-0.144	-0.1531	-0.1693	-0.2309	-0.9722	-0.9726	-0.9627	-0.9632	-0.9578	-0.9525	-0.9456
0.7	-0.0324	-0.036	-0.0392	-0.0422	-0.0449	-0.0497	-0.0683	-0.2807	-0.2809	-0.278	-0.2781	-0.2766	-0.2751	-0.273
1	-0.0187	-0.0207	-0.0225	-0.0243	-0.0259	-0.0287	-0.0394	-0.1611	-0.1611	-0.1595	-0.1596	-0.1587	-0.1578	-0.1566
1.3	-0.013	-0.0145	-0.0158	-0.017	-0.0181	-0.0201	-0.0276	-0.1125	-0.1125	-0.1114	-0.1114	-0.1108	-0.1102	-0.1094
1.6	-0.01	-0.0111	-0.0121	-0.013	-0.0139	-0.0154	-0.0212	-0.0863	-0.0863	-0.0854	-0.0855	-0.085	-0.0845	-0.0838
1.9	-0.0081	-0.009	-0.0098	-0.0106	-0.0112	-0.0125	-0.0172	-0.0697	-0.0698	-0.0691	-0.0691	-0.0687	-0.0683	-0.0678
2.2	-0.0068	-0.0075	-0.0082	-0.0088	-0.0094	-0.0104	-0.0144	-0.0583	-0.0584	-0.0578	-0.0578	-0.0575	-0.0571	-0.0567
2.5	-0.0058	-0.0065	-0.007	-0.0076	-0.0081	-0.0089	-0.0124	-0.0499	-0.05	-0.0494	-0.0495	-0.0492	-0.0489	-0.0485
2.8	-0.0051	-0.0056	-0.0061	-0.0066	-0.007	-0.0078	-0.0108	-0.0435	-0.0435	-0.0431	-0.0431	-0.0428	-0.0426	-0.0423
3.1	-0.0045	-0.005	-0.0054	-0.0058	-0.0062	-0.0069	-0.0095	-0.0384	-0.0384	-0.038	-0.038	-0.0378	-0.0376	-0.0373
3.4	-0.004	-0.0044	-0.0048	-0.0052	-0.0055	-0.0061	-0.0085	-0.0342	-0.0342	-0.0338	-0.0339	-0.0337	-0.0335	-0.0332
3.7	-0.0036	-0.004	-0.0043	-0.0047	-0.005	-0.0055	-0.0076	-0.0307	-0.0307	-0.0304	-0.0304	-0.0303	-0.0301	-0.0298
4	-0.0032	-0.0036	-0.0039	-0.0042	-0.0045	-0.005	-0.0069	-0.0278	-0.0278	-0.0275	-0.0275	-0.0274	-0.0272	-0.027
4.3	-0.0029	-0.0033	-0.0036	-0.0038	-0.0041	-0.0045	-0.0063	-0.0253	-0.0253	-0.0251	-0.0251	-0.0249	-0.0248	-0.0246
4.6	-0.0027	-0.003	-0.0033	-0.0035	-0.0037	-0.0042	-0.0058	-0.0232	-0.0232	-0.0229	-0.0229	-0.0228	-0.0227	-0.0225
4.9	-0.0026	0.07647	-0.0031	0.09281	-0.0036	0.10702	-0.0055	-0.0222	0.64931	-0.0219	0.64274	-0.0218	0.63931	-0.0215

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.7592	-0.7624	-0.7511	-0.749	-1.5135	-0.7326	-0.6913	0.06877	-0.2539	-0.1082	-0.0471	-0.0815	-0.0168	-0.0036
0.4	-0.4764	-0.4783	-0.4712	-0.4697	-0.7099	-0.4593	-0.4333	0.01104	-0.172	-0.0746	-0.0326	-0.0577	-0.0116	-0.0025
0.7	-0.1488	-0.1493	-0.1471	-0.1463	0.02637	-0.143	-0.1347	-0.0422	-0.0722	-0.0333	-0.0147	-0.0274	-0.0052	-0.0011
1	-0.0872	-0.0875	-0.0862	-0.0857	-0.0407	-0.0837	-0.0788	-0.0344	-0.0469	-0.0223	-0.0099	-0.0181	-0.0035	-0.0008
1.3	-0.0617	-0.0618	-0.0609	-0.0605	-0.0374	-0.0591	-0.0556	-0.0289	-0.0354	-0.0171	-0.0076	-0.0138	-0.0027	-0.0006
1.6	-0.0477	-0.0478	-0.0471	-0.0468	-0.0321	-0.0457	-0.043	-0.0251	-0.0287	-0.0141	-0.0063	-0.0113	-0.0023	-0.0005
1.9	-0.0388	-0.0389	-0.0383	-0.0381	-0.0277	-0.0372	-0.035	-0.0223	-0.0243	-0.0121	-0.0054	-0.0096	-0.0019	-0.0004
2.2	-0.0327	-0.0327	-0.0322	-0.032	-0.0241	-0.0312	-0.0294	-0.0201	-0.0211	-0.0106	-0.0048	-0.0083	-0.0017	-0.0004
2.5	-0.0281	-0.0281	-0.0277	-0.0275	-0.0213	-0.0268	-0.0253	-0.0183	-0.0187	-0.0095	-0.0043	-0.0074	-0.0015	-0.0003
2.8	-0.0246	-0.0246	-0.0242	-0.024	-0.019	-0.0235	-0.0221	-0.0168	-0.0168	-0.0086	-0.0039	-0.0066	-0.0014	-0.0003
3.1	-0.0217	-0.0218	-0.0215	-0.0213	-0.017	-0.0208	-0.0195	-0.0155	-0.0152	-0.0078	-0.0035	-0.006	-0.0013	-0.0003
3.4	-0.0194	-0.0195	-0.0192	-0.019	-0.0154	-0.0186	-0.0174	-0.0144	-0.0139	-0.0072	-0.0032	-0.0055	-0.0012	-0.0003
3.7	-0.0175	-0.0175	-0.0173	-0.0171	-0.014	-0.0167	-0.0157	-0.0135	-0.0127	-0.0066	-0.003	-0.0051	-0.0011	-0.0002
4	-0.0159	-0.0159	-0.0157	-0.0155	-0.0128	-0.0152	-0.0143	-0.0126	-0.0117	-0.0062	-0.0028	-0.0047	-0.001	-0.0002
4.3	-0.0145	-0.0145	-0.0143	-0.0142	-0.0118	-0.0138	-0.013	-0.0118	-0.0109	-0.0057	-0.0026	-0.0043	-0.0009	-0.0002
4.6	-0.0133	-0.0133	-0.0131	-0.013	-0.0109	-0.0127	-0.0119	-0.0111	-0.0101	-0.0054	-0.0024	-0.004	-0.0009	-0.0002
4.9	-0.0127	0.47779	-0.0126	0.54668	-0.0105	0.52292	-0.0114	-0.0108	4.87134	-0.0052	5.87496	-0.0039	5.28532	-0.0002

Figure A.3: k_0 and k_1

k_2

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0279	-0.031	-0.0337	-0.0363	-0.0386	-0.0428	-0.0587	-0.2431	-0.2432	-0.2407	-0.2408	-0.2394	-0.2381	-0.2362
0.4	-0.0213	-0.0236	-0.0257	-0.0277	-0.0295	-0.0326	-0.0448	-0.1853	-0.1853	-0.1834	-0.1835	-0.1825	-0.1814	-0.18
0.7	-0.0128	-0.0142	-0.0155	-0.0167	-0.0177	-0.0197	-0.027	-0.1112	-0.1113	-0.1101	-0.1102	-0.1095	-0.1089	-0.108
1	-0.0098	-0.0109	-0.0119	-0.0128	-0.0136	-0.0151	-0.0208	-0.0853	-0.0853	-0.0844	-0.0845	-0.084	-0.0835	-0.0828
1.3	-0.0079	-0.0088	-0.0096	-0.0103	-0.011	-0.0122	-0.0168	-0.0688	-0.0688	-0.0681	-0.0682	-0.0678	-0.0674	-0.0668
1.6	-0.0066	-0.0073	-0.008	-0.0086	-0.0091	-0.0101	-0.014	-0.057	-0.0571	-0.0565	-0.0565	-0.0562	-0.0559	-0.0554
1.9	-0.0056	-0.0062	-0.0067	-0.0073	-0.0077	-0.0086	-0.0118	-0.0482	-0.0482	-0.0477	-0.0477	-0.0475	-0.0472	-0.0468
2.2	-0.0048	-0.0053	-0.0058	-0.0062	-0.0066	-0.0073	-0.0101	-0.0413	-0.0413	-0.0409	-0.0409	-0.0407	-0.0404	-0.0401
2.5	-0.0041	-0.0046	-0.005	-0.0054	-0.0058	-0.0064	-0.0088	-0.0358	-0.0358	-0.0355	-0.0355	-0.0353	-0.0351	-0.0348
2.8	-0.0036	-0.004	-0.0044	-0.0047	-0.005	-0.0056	-0.0077	-0.0314	-0.0314	-0.0311	-0.0311	-0.0309	-0.0307	-0.0305
3.1	-0.0032	-0.0036	-0.0039	-0.0042	-0.0045	-0.0049	-0.0068	-0.0277	-0.0277	-0.0274	-0.0275	-0.0273	-0.0271	-0.0269
3.4	-0.0029	-0.0032	-0.0035	-0.0037	-0.004	-0.0044	-0.0061	-0.0247	-0.0247	-0.0244	-0.0244	-0.0243	-0.0242	-0.024
3.7	-0.0026	-0.0028	-0.0031	-0.0033	-0.0036	-0.0039	-0.0054	-0.0221	-0.0221	-0.0219	-0.0219	-0.0218	-0.0216	-0.0215
4	-0.0023	-0.0026	-0.0028	-0.003	-0.0032	-0.0036	-0.0049	-0.0199	-0.0199	-0.0197	-0.0197	-0.0196	-0.0195	-0.0194
4.3	-0.0021	-0.0023	-0.0025	-0.0027	-0.0029	-0.0032	-0.0045	-0.0181	-0.0181	-0.0179	-0.0179	-0.0178	-0.0177	-0.0175
4.6	-0.0019	-0.0021	-0.0023	-0.0025	-0.0026	-0.0029	-0.0041	-0.0164	-0.0164	-0.0163	-0.0163	-0.0162	-0.0161	-0.016
4.9	-0.0018	-0.002	-0.0022	-0.0024	-0.0025	-0.0028	-0.0039	-0.0157	-0.0157	-0.0155	-0.0155	-0.0154	-0.0154	-0.0152

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.1263	-0.1267	-0.1248	-0.1242	0.03017	-0.1214	-0.1143	-0.0325	-0.0601	-0.0274	-0.0121	-0.0227	-0.0043	-0.0009
0.4	-0.0969	-0.0972	-0.0957	-0.0952	0.00511	-0.093	-0.0876	-0.0276	-0.0473	-0.0218	-0.0096	-0.0179	-0.0034	-0.0007
0.7	-0.0591	-0.0592	-0.0584	-0.058	-0.0216	-0.0567	-0.0533	-0.0212	-0.0309	-0.0145	-0.0064	-0.0119	-0.0023	-0.0005
1	-0.0457	-0.0458	-0.0452	-0.0449	-0.0228	-0.0438	-0.0413	-0.0186	-0.025	-0.0119	-0.0053	-0.0097	-0.0019	-0.0004
1.3	-0.0372	-0.0373	-0.0367	-0.0365	-0.0213	-0.0356	-0.0335	-0.0166	-0.021	-0.0101	-0.0045	-0.0082	-0.0016	-0.0003
1.6	-0.031	-0.0311	-0.0306	-0.0304	-0.0193	-0.0297	-0.0279	-0.0149	-0.018	-0.0088	-0.0039	-0.007	-0.0014	-0.0003
1.9	-0.0263	-0.0264	-0.026	-0.0258	-0.0172	-0.0252	-0.0237	-0.0135	-0.0157	-0.0077	-0.0034	-0.0062	-0.0012	-0.0003
2.2	-0.0227	-0.0227	-0.0224	-0.0222	-0.0154	-0.0217	-0.0204	-0.0122	-0.0138	-0.0068	-0.003	-0.0054	-0.0011	-0.0002
2.5	-0.0197	-0.0198	-0.0195	-0.0193	-0.0138	-0.0189	-0.0178	-0.0111	-0.0122	-0.0061	-0.0027	-0.0048	-0.001	-0.0002
2.8	-0.0173	-0.0174	-0.0171	-0.017	-0.0124	-0.0166	-0.0156	-0.0101	-0.0109	-0.0055	-0.0024	-0.0043	-0.0009	-0.0002
3.1	-0.0154	-0.0154	-0.0152	-0.0151	-0.0112	-0.0147	-0.0138	-0.0092	-0.0098	-0.0049	-0.0022	-0.0039	-0.0008	-0.0002
3.4	-0.0137	-0.0137	-0.0135	-0.0134	-0.0101	-0.0131	-0.0123	-0.0085	-0.0089	-0.0045	-0.002	-0.0035	-0.0007	-0.0002
3.7	-0.0123	-0.0123	-0.0122	-0.0121	-0.0092	-0.0118	-0.0111	-0.0078	-0.0081	-0.0041	-0.0018	-0.0032	-0.0007	-0.0001
4	-0.0111	-0.0111	-0.011	-0.0109	-0.0084	-0.0106	-0.01	-0.0072	-0.0074	-0.0037	-0.0017	-0.0029	-0.0006	-0.0001
4.3	-0.0101	-0.0101	-0.01	-0.0099	-0.0077	-0.0096	-0.0091	-0.0067	-0.0068	-0.0034	-0.0015	-0.0027	-0.0006	-0.0001
4.6	-0.0092	-0.0092	-0.0091	-0.009	-0.007	-0.0088	-0.0083	-0.0062	-0.0062	-0.0032	-0.0014	-0.0025	-0.0005	-0.0001
4.9	-0.0088	-0.0088	-0.0087	-0.0086	-0.0068	-0.0084	-0.0079	-0.006	-0.006	-0.0031	-0.0014	-0.0024	-0.0005	-0.0001

k_3

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.02	-0.0222	-0.0241	-0.026	-0.0276	-0.0306	-0.042	-0.1737	-0.1738	-0.172	-0.1721	-0.1711	-0.1701	-0.1688
0.4	-0.0173	-0.0192	-0.0209	-0.0225	-0.024	-0.0266	-0.0365	-0.1506	-0.1507	-0.1491	-0.1492	-0.1484	-0.1475	-0.1463
0.7	-0.013	-0.0144	-0.0157	-0.0169	-0.018	-0.0199	-0.0274	-0.1127	-0.1127	-0.1116	-0.1116	-0.111	-0.1104	-0.1095
1	-0.0101	-0.0112	-0.0122	-0.0132	-0.014	-0.0155	-0.0214	-0.0876	-0.0877	-0.0868	-0.0868	-0.0863	-0.0858	-0.0851
1.3	-0.0081	-0.009	-0.0098	-0.0105	-0.0112	-0.0125	-0.0171	-0.0702	-0.0702	-0.0695	-0.0695	-0.0691	-0.0687	-0.0682
1.6	-0.0066	-0.0074	-0.008	-0.0086	-0.0092	-0.0102	-0.0141	-0.0575	-0.0575	-0.0569	-0.0569	-0.0566	-0.0563	-0.0558
1.9	-0.0055	-0.0062	-0.0067	-0.0072	-0.0077	-0.0085	-0.0118	-0.048	-0.048	-0.0475	-0.0475	-0.0472	-0.047	-0.0466
2.2	-0.0047	-0.0052	-0.0057	-0.0061	-0.0065	-0.0072	-0.01	-0.0406	-0.0406	-0.0402	-0.0402	-0.04	-0.0398	-0.0395
2.5	-0.004	-0.0045	-0.0049	-0.0053	-0.0056	-0.0062	-0.0086	-0.0349	-0.0349	-0.0345	-0.0345	-0.0343	-0.0341	-0.0339
2.8	-0.0035	-0.0039	-0.0042	-0.0046	-0.0049	-0.0054	-0.0074	-0.0302	-0.0303	-0.0299	-0.03	-0.0298	-0.0296	-0.0294
3.1	-0.0031	-0.0034	-0.0037	-0.004	-0.0043	-0.0047	-0.0065	-0.0265	-0.0265	-0.0262	-0.0262	-0.0261	-0.0259	-0.0257
3.4	-0.0027	-0.003	-0.0033	-0.0035	-0.0038	-0.0042	-0.0058	-0.0234	-0.0234	-0.0232	-0.0232	-0.023	-0.0229	-0.0227
3.7	-0.0024	-0.0027	-0.0029	-0.0031	-0.0034	-0.0037	-0.0051	-0.0208	-0.0208	-0.0206	-0.0206	-0.0205	-0.0204	-0.0202
4	-0.0022	-0.0024	-0.0026	-0.0028	-0.003	-0.0033	-0.0046	-0.0186	-0.0187	-0.0185	-0.0185	-0.0184	-0.0183	-0.0181
4.3	-0.0019	-0.0022	-0.0024	-0.0025	-0.0027	-0.003	-0.0041	-0.0168	-0.0168	-0.0166	-0.0166	-0.0165	-0.0164	-0.0163
4.6	-0.0018	-0.002	-0.0021	-0.0023	-0.0024	-0.0027	-0.0038	-0.0152	-0.0152	-0.0151	-0.0151	-0.015	-0.0149	-0.0148
4.9	-0.0017	-0.0019	-0.002	-0.0022	-0.0023	-0.0026	-0.0036	-0.0145	-0.0145	-0.0143	-0.0143	-0.0143	-0.0142	-0.0141

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0906	-0.0908	-0.0895	-0.089	0.00858	-0.087	-0.0819	-0.0244	-0.0436	-0.0199	-0.0088	-0.0165	-0.0031	-0.0007
0.4	-0.079	-0.0792	-0.078	-0.0776	-0.0055	-0.0758	-0.0714	-0.0235	-0.039	-0.018	-0.008	-0.0149	-0.0028	-0.0006
0.7	-0.0598	-0.06	-0.0591	-0.0588	-0.0218	-0.0574	-0.054	-0.0214	-0.0313	-0.0147	-0.0065	-0.012	-0.0023	-0.0005
1	-0.047	-0.0471	-0.0464	-0.0461	-0.0234	-0.0451	-0.0424	-0.0191	-0.0256	-0.0122	-0.0054	-0.0099	-0.0019	-0.0004
1.3	-0.0379	-0.038	-0.0374	-0.0372	-0.0218	-0.0363	-0.0342	-0.017	-0.0214	-0.0103	-0.0046	-0.0083	-0.0016	-0.0004
1.6	-0.0312	-0.0313	-0.0309	-0.0307	-0.0195	-0.0299	-0.0282	-0.0151	-0.0182	-0.0088	-0.0039	-0.0071	-0.0014	-0.0003
1.9	-0.0262	-0.0263	-0.0259	-0.0257	-0.0172	-0.0251	-0.0236	-0.0134	-0.0156	-0.0077	-0.0034	-0.0061	-0.0012	-0.0003
2.2	-0.0223	-0.0223	-0.022	-0.0219	-0.0152	-0.0213	-0.0201	-0.012	-0.0136	-0.0067	-0.003	-0.0053	-0.0011	-0.0002
2.5	-0.0192	-0.0192	-0.019	-0.0188	-0.0134	-0.0184	-0.0173	-0.0108	-0.0119	-0.0059	-0.0026	-0.0047	-0.0009	-0.0002
2.8	-0.0167	-0.0167	-0.0165	-0.0164	-0.012	-0.016	-0.015	-0.0097	-0.0106	-0.0053	-0.0024	-0.0042	-0.0008	-0.0002
3.1	-0.0147	-0.0147	-0.0145	-0.0144	-0.0107	-0.014	-0.0132	-0.0088	-0.0094	-0.0047	-0.0021	-0.0037	-0.0008	-0.0002
3.4	-0.013	-0.013	-0.0128	-0.0127	-0.0096	-0.0124	-0.0117	-0.008	-0.0084	-0.0042	-0.0019	-0.0033	-0.0007	-0.0001
3.7	-0.0116	-0.0116	-0.0114	-0.0114	-0.0086	-0.0111	-0.0104	-0.0073	-0.0076	-0.0038	-0.0017	-0.003	-0.0006	-0.0001
4	-0.0104	-0.0104	-0.0103	-0.0102	-0.0078	-0.0099	-0.0094	-0.0067	-0.0069	-0.0035	-0.0016	-0.0027	-0.0006	-0.0001
4.3	-0.0094	-0.0094	-0.0093	-0.0092	-0.0071	-0.009	-0.0084	-0.0062	-0.0063	-0.0032	-0.0014	-0.0025	-0.0005	-0.0001
4.6	-0.0085	-0.0085	-0.0084	-0.0083	-0.0065	-0.0081	-0.0076	-0.0057	-0.0057	-0.0029	-0.0013	-0.0023	-0.0005	-0.0001
4.9	-0.0081	-0.0081	-0.008	-0.0079	-0.0062	-0.0077	-0.0073	-0.0054	-0.0055	-0.0028	-0.0013	-0.0022	-0.0004	-1E-04

D_0

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	0.02293	0.02549	0.02772	0.02991	0.03187	0.03535	0.04892	0.19112	0.19121	0.18941	0.1895	0.18854	0.18763	0.18685
0.833333333	0.02214	0.02461	0.02676	0.02886	0.03074	0.03409	0.04706	0.1871	0.18718	0.18536	0.18545	0.18447	0.18352	0.18252
1.25	0.01977	0.02195	0.02386	0.02572	0.02739	0.03035	0.04175	0.17011	0.17019	0.16846	0.16855	0.16761	0.16669	0.16551
1.666666667	0.01738	0.01929	0.02096	0.02259	0.02404	0.02663	0.03655	0.15061	0.15068	0.14914	0.14921	0.14837	0.14755	0.14644
2.083333333	0.01643	0.01823	0.01981	0.02133	0.0227	0.02513	0.03443	0.14302	0.14308	0.14161	0.14168	0.14088	0.14009	0.13902
2.5	0.01615	0.01792	0.01946	0.02095	0.02229	0.02466	0.03373	0.14107	0.14113	0.13968	0.13975	0.13896	0.13818	0.13711
2.916666667	0.01643	0.01823	0.01979	0.0213	0.02266	0.02506	0.03422	0.14395	0.14401	0.14253	0.1426	0.1418	0.141	0.1399
3.333333333	0.01684	0.01868	0.02027	0.02182	0.02321	0.02566	0.03501	0.14773	0.14779	0.14627	0.14634	0.14551	0.14469	0.14357

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	0.1153	0.11557	0.11395	0.11324	0.0806	0.11069	0.10457	0.05945	0.06701	0.03312	0.01479	0.02639	0.00528	0.00116
0.833333333	0.10818	0.10845	0.1069	0.10627	0.06482	0.10385	0.09799	0.04959	0.06043	0.02936	0.01308	0.02362	0.00467	0.00102
1.25	0.0923	0.09256	0.09119	0.09071	0.02679	0.08863	0.08351	0.03189	0.04724	0.02216	0.00981	0.01815	0.0035	0.00076
1.666666667	0.07909	0.07934	0.07815	0.07779	-0.0355	0.07602	0.07162	0.01962	0.03709	0.0169	0.00745	0.01397	0.00266	0.00058
2.083333333	0.07329	0.07354	0.07244	0.07214	-0.2356	0.0705	0.06643	0.01225	0.03182	0.01417	0.00622	0.01171	0.00222	0.00048
2.5	0.07082	0.07108	0.07001	0.06975	6.51724	0.06818	0.06425	0.00676	0.02864	0.01251	0.00548	0.01027	0.00195	0.00042
2.916666667	0.07091	0.07119	0.07012	0.06988	0.8895	0.06832	0.0644	0.00188	0.02672	0.01149	0.00502	0.00927	0.00179	0.00039
3.333333333	0.07211	0.0724	0.07131	0.07108	-11.651	0.0695	0.06552	-0.0005	0.02625	0.0112	0.00489	0.00895	0.00174	0.00038

Figure A.4: k_2, k_3 and D_0

D_1

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.01471	0.01635	0.01778	0.01917	0.02042	0.02265	0.03125	0.12655	0.1266	0.12531	0.12537	0.12466	0.12397	0.12303
0.357142857	0.01398	0.01554	0.01689	0.01822	0.0194	0.02151	0.02968	0.12044	0.12049	0.11925	0.11931	0.11864	0.11797	0.11707
0.535714286	0.01274	0.01415	0.01539	0.01659	0.01767	0.01959	0.027	0.11001	0.11006	0.10892	0.10898	0.10836	0.10774	0.1069
0.714285714	0.01166	0.01295	0.01408	0.01518	0.01617	0.01792	0.02469	0.10088	0.10093	0.09989	0.09994	0.09937	0.0988	0.09802
0.892857143	0.01071	0.0119	0.01294	0.01395	0.01485	0.01646	0.02266	0.09279	0.09283	0.09187	0.09191	0.09139	0.09087	0.09015
1.071428571	0.01058	0.01175	0.01277	0.01377	0.01466	0.01625	0.02236	0.09171	0.09176	0.09081	0.09085	0.09033	0.08982	0.0891
1.25	0.01072	0.0119	0.01294	0.01394	0.01485	0.01645	0.02263	0.09297	0.09301	0.09205	0.0921	0.09157	0.09105	0.09032
1.428571429	0.0106	0.01177	0.0128	0.01379	0.01469	0.01627	0.02237	0.09201	0.09205	0.0911	0.09114	0.09062	0.09011	0.08939

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.07052	0.07066	0.06964	0.06916	0.04924	0.06752	0.06354	0.03892	0.04332	0.02147	0.0096	0.01704	0.00343	0.00075
0.357142857	0.0667	0.06684	0.06587	0.06542	0.04536	0.06387	0.06011	0.03555	0.04038	0.01991	0.0089	0.01586	0.00318	0.00069
0.535714286	0.06022	0.06035	0.05947	0.05909	0.03851	0.05769	0.05428	0.0298	0.03538	0.01726	0.0077	0.01384	0.00275	0.0006
0.714285714	0.05476	0.05489	0.05408	0.05375	0.03263	0.05248	0.04938	0.02527	0.0313	0.01513	0.00674	0.01221	0.0024	0.00052
0.892857143	0.05005	0.05017	0.04943	0.04913	0.02742	0.04798	0.04515	0.02158	0.02787	0.01336	0.00594	0.01083	0.00212	0.00046
1.071428571	0.04917	0.0493	0.04857	0.04828	0.0242	0.04715	0.04437	0.01981	0.02672	0.01271	0.00564	0.01035	0.00201	0.00044
1.25	0.04957	0.0497	0.04896	0.04869	0.02122	0.04755	0.04475	0.01868	0.02632	0.01243	0.00551	0.01016	0.00197	0.00043
1.428571429	0.04892	0.04905	0.04832	0.04806	0.01917	0.04693	0.04417	0.01778	0.02567	0.01207	0.00535	0.00988	0.00191	0.00041

L_cutline

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	0	0	0	0	0	0	0	0.69822	0.69695	0.66836	0.66784	0.64684	0.6166	0.48897
0.5	0	0	0	0	0	0	0	0.62924	0.62822	0.609	0.60839	0.59385	0.57197	0.47319
0.75	0	0	0	0	0	0	0	0.51998	0.51943	0.51247	0.51194	0.50607	0.49587	0.44019
1	0	0	0	0	0	0	0	0.45294	0.45264	0.44968	0.44936	0.44655	0.44112	0.40709
1.25	0	0	0	0	0	0	0	0.40682	0.40657	0.40521	0.40494	0.40343	0.40017	0.37774
1.5	0	0	0	0	0	0	0	0.37294	0.37272	0.37215	0.37192	0.37107	0.36898	0.35331
1.75	0	0	0	0	0	0	0	0.34666	0.34647	0.34631	0.34611	0.34563	0.34425	0.33285
2	0	0	0	0	0	0	0	0.3253	0.32523	0.32538	0.32528	0.32511	0.32433	0.31647
2.25	0	0	0	0	0	0	0	0.30812	0.30805	0.30833	0.30825	0.30821	0.3077	0.30179
2.5	0	0	0	0	0	0	0	0.30051	0.30036	0.3006	0.30043	0.30036	0.2998	0.29399

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	-0.0562	-0.075	-0.5443	-0.5452	-0.1632	-0.9365	-1.747	0	0	0	0	2.2E-14	0	0
0.5	0.04536	0.03149	-0.2872	-0.29	-0.0301	-0.5631	-1.1562	1.1E-14	0	0	0	1.1E-14	0	0
0.75	0.15804	0.15103	0.02406	0.0201	0.12465	-0.0984	-0.3924	0	0	0	0	0	0	0
1	0.16655	0.16241	0.09348	0.09069	0.14822	0.02436	-0.1494	0	0	0	0	0	0	0
1.25	0.15872	0.15599	0.11264	0.11065	0.14732	0.06831	-0.046	1.1E-14	0	0	0	0	0	0
1.5	0.14802	0.14609	0.11632	0.11484	0.14035	0.08553	0.00482	-1E-14	0	0	0	0	0	0
1.75	0.13753	0.13608	0.11439	0.11326	0.13207	0.0918	0.0319	-1E-14	0	0	0	0	0	0
2	0.12796	0.12684	0.11034	0.10945	0.12392	0.09309	0.04695	0	0	0	0	-1E-14	0	0
2.25	0.11943	0.11854	0.10558	0.10485	0.11634	0.09199	0.05541	1.1E-14	0	0	0	0	0	0
2.5	0.11542	0.11462	0.10312	0.10247	0.11272	0.09105	0.05845	2.2E-14	0	0	0	2.2E-14	0	0

d_rep

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.05128	0.0668	0.08301	0.09959	0.11615	0.14534	0.23128	-21.736	-21.838	-21.829	-21.823	-21.821	-21.882	-21.521
0.4	0.05567	0.07172	0.08499	0.09604	0.10582	0.12427	0.18698	-12.452	-12.444	-12.458	-12.455	-12.474	-12.471	-12.273
0.7	0.05915	0.06834	0.07482	0.08072	0.08497	0.09214	0.12772	-2.2072	-2.1475	-2.1624	-2.1619	-2.1694	-2.1457	-2.1044
1	0.05141	0.0546	0.05931	0.06364	0.06819	0.07523	0.10419	-0.9481	-0.9595	-0.9472	-0.9546	-0.9323	-0.9412	-0.9041
1.3	0.04183	0.0473	0.0517	0.05514	0.0585	0.0654	0.08984	-0.5378	-0.5434	-0.5339	-0.5376	-0.5323	-0.5266	-0.5011
1.6	0.03825	0.04242	0.04568	0.0493	0.05261	0.05838	0.07986	-0.3551	-0.3482	-0.3444	-0.3399	-0.3419	-0.3354	-0.3182
1.9	0.03504	0.03823	0.04187	0.04497	0.04796	0.05309	0.07244	-0.2461	-0.2431	-0.2396	-0.2376	-0.2352	-0.2327	-0.2191
2.2	0.0316	0.03554	0.03869	0.04164	0.04414	0.04898	0.06663	-0.1788	-0.1784	-0.1767	-0.1748	-0.174	-0.1711	-0.161
2.5	0.02987	0.03311	0.03617	0.03873	0.04124	0.04561	0.06191	-0.136	-0.1354	-0.1352	-0.1346	-0.1335	-0.1306	-0.125
2.8	0.02816	0.0313	0.03393	0.03648	0.03878	0.04284	0.05799	-0.1104	-0.1067	-0.1069	-0.1063	-0.1048	-0.1028	-0.0961
3.1	0.02652	0.02953	0.03214	0.0345	0.03662	0.04049	0.05466	-0.088	-0.088	-0.0864	-0.0853	-0.0844	-0.0825	-0.0782
3.4	0.02544	0.0281	0.03057	0.03277	0.03482	0.03846	0.05178	-0.0723	-0.0726	-0.0714	-0.0706	-0.07	-0.0677	-0.0662
3.7	0.02428	0.02685	0.0292	0.0313	0.03324	0.03668	0.04926	-0.062	-0.0607	-0.0598	-0.0592	-0.0584	-0.0579	-0.0539
4	0.02316	0.02575	0.02795	0.02997	0.0318	0.0351	0.04704	-0.052	-0.0516	-0.0509	-0.0497	-0.0492	-0.0493	-0.0445
4.3	0.02235	0.02477	0.02687	0.0288	0.03055	0.03369	0.04506	-0.0447	-0.0443	-0.0438	-0.043	-0.0432	-0.0411	-0.0389
4.6	0.02154	0.02383	0.0259	0.02773	0.02943	0.03241	0.04327	-0.0394	-0.0384	-0.038	-0.0377	-0.037	-0.0366	-0.0361
4.9	0.02109	0.02342	0.02546	0.02721	0.02888	0.03181	0.04242	-0.0366	-0.036	-0.0357	-0.0348	-0.0334	-0.0351	-0.0352

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-13.505	-12.557	-12.58	-12.804	-13.019	-13.395	-16.106	-3.7995	18.6981	18.9754	19.2248	17.4457	15.7018	-0.2708
0.4	-7.4478	-7.4979	-7.528	-7.5447	-7.4269	-7.5842	-8.7062	8.77128	9.9283	5.22454	6.19124	9.13854	9.58769	1.05228
0.7	-1.1726	-1.6499	-1.5498	-1.4555	-1.4806	-1.2572	-1.0217	8.39803	-1.993	-0.4414	-0.5362	-0.6016	1.79422	1.19662
1	-0.697	-0.5794	-0.9362	-0.4889	-0.6421	-0.5731	-0.6495	-3.6375	0.20982	3.36325	2.92668	0.153	0.06774	-0.5011
1.3	-0.2996	-0.3634	-0.3441	-0.3558	-0.1911	-0.33	-0.4027	1.56504	1.85759	0.16946	-2.1699	1.18191	0.01985	-0.1485
1.6	-0.1092	-0.209	0.15686	-0.2411	-0.2923	-0.2694	-0.2016	2.08604	0.23929	0.00516	0.02603	-0.7288	-0.698	0.51553
1.9	-0.1375	-0.0856	-0.1608	-0.1816	-0.179	0.01875	-0.1454	0.20779	0.68785	0.01663	1.34776	-0.0268	-0.3765	0.15439
2.2	-0.1738	-0.1128	-0.2396	-0.1265	-0.0372	-0.1046	-0.1265	1.07266	0.01744	0.06669	0.01225	0.75515	0.5127	-0.1501
2.5	-0.1208	-0.0776	-0.0926	-0.0322	-0.1344	-0.2976	-0.1267	-2.2059	0.49982	0.01069	0.90412	-0.5688	0.38553	0.00042
2.8	-0.0614	-0.1171	0.05865	-0.0538	-0.0871	-0.0622	-0.0997	0.00908	-0.9612	0.04963	-1.8898	-0.0989	0.20434	0.15102
3.1	-0.0437	-0.0539	-0.0609	-0.0678	-0.0098	-0.0549	-0.088	2.23723	0.00549	0.0039	0.00268	0.61406	0.00102	-0.1879
3.4	-0.0094	-0.0286	-0.1017	-0.08	-0.0796	-0.0467	-0.0776	-0.2617	0.5127	0.03519	1.40741	-0.4435	0.00085	-0.0485
3.7	-0.0332	-0.0383	-0.0433	-0.0482	-0.0554	-0.0635	-0.045	0.00996	0.00346	0.0023	0.00162	-0.1522	-0.3268	0.1882
4	-0.0859	-0.0178	0.01916	-0.0168	-0.0027	-0.0375	-0.0414	0.27481	0.22606	0.04418	0.48908	0.54751	-0.1797	0.04874
4.3	-0.0478	-0.0288	-0.0325	-0.038	-0.0512	-0.0295	-0.049	-0.3678	0.00213	0.00128	-1.5612	-0.36	0.23084	0.12563
4.6	-0.0206	-0.0352	-0.0549	-0.0334	-0.0383	-0.0439	-0.0504	0.00237	0.24081	-0.0005	0.00065	-1.2667	0.18043	5.1E-05
4.9	0.00514	-0.0432	-0.1036	-0.0078	0.00911	-0.02	-0.0426	0.74498	0.00163	-0.9248	3.12449	-1.5063	0.19415	-0.2511

Figure A.5: D_1, $L_{Cut\ line}$ and d_{rep}

$1/3 * D_0$

A_coh

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	0	0	0	0	0	3.6E-17
2	0	0	0	-1E-14	0	0	1.8E-17
3	0	0	0	0	0	0	0
4	0	0	0	1.2E-14	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	-4E-17
10	0	0	0	0	0	0	-8E-17

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
2	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
3	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
4	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
5	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
6	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
7	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
8	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
9	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
10	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
2	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
3	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
4	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
5	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
6	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
7	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
8	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
9	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
10	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	0	0	0	0	0	0
2	0	0	0	-4E-16	0	0	0
3	0	3.7E-16	0	0	0	0	0
4	0	3.7E-16	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	3.7E-16	0	0	0
7	0	-4E-16	0	0	0	0	1.9E-15
8	0	0	0	0	0	0	0
9	0	3.7E-16	0	0	0	0	-2E-15
10	0	0	0	0	0	0	0

c

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0
0.2	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0
0.6	0	0	0	0	0	0	0
0.7	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0
1.8	0	0	0	0	0	0	0
1.9	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.1	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.2	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.3	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.4	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.5	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.6	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.7	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.8	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
0.9	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.1	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.2	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.3	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.4	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.5	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.6	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.7	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.8	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
1.9	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025
2	0.11008	0.10339	0.0981	0.09519	0.09413	0.09204	0.09025

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.1	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.2	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.3	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.4	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.5	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.6	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.7	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.8	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
0.9	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.1	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.2	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.3	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.4	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.5	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.6	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.7	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.8	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
1.9	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405
2	0.04259	0.04077	0.0393	0.0432	0.04283	0.04683	0.05405

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0
0.2	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0
0.6	0	0	0	0	0	0	0
0.7	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0
1.8	0	0	0	0	0	0	0
1.9	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0

Figure A.6: A_c and c

Alpha_coh

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.333333333	0	0	0	0	0	0	0
1.666666667	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2.333333333	0	0	0	0	0	0	0
2.666666667	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
3.333333333	0	0	0	0	0	0	0
3.666666667	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
4.333333333	0	0	0	0	0	0	0
4.666666667	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
5.333333333	0	0	0	0	0	0	0
5.666666667	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
6.333333333	0	0	0	0	0	0	0
6.666666667	0	0	0	0	0	0	0

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.333333333	0	0	0	0	0	0	0
1.666666667	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2.333333333	0	0	0	0	0	0	0
2.666666667	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
3.333333333	0	0	0	0	0	0	0
3.666666667	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
4.333333333	0	0	0	0	0	0	0
4.666666667	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
5.333333333	0	0	0	0	0	0	0
5.666666667	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
6.333333333	0	0	0	0	0	0	0
6.666666667	0	0	0	0	0	0	0

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	-0.0639	-0.0612	-0.059	-0.0648	-0.0642	-0.0702	-0.0811
1	-0.0479	-0.0459	-0.0442	-0.0486	-0.0482	-0.0527	-0.0608
1.333333333	-0.0256	-0.0245	-0.0236	-0.0259	-0.0257	-0.0281	-0.0324
1.666666667	-0.016	-0.0153	-0.0147	-0.0162	-0.0161	-0.0176	-0.0203
2	-0.011	-0.0105	-0.0101	-0.0111	-0.011	-0.012	-0.0139
2.333333333	-0.008	-0.0076	-0.0074	-0.0081	-0.008	-0.0088	-0.0101
2.666666667	-0.0061	-0.0058	-0.0056	-0.0062	-0.0061	-0.0067	-0.0077
3	-0.0048	-0.0046	-0.0044	-0.0049	-0.0048	-0.0053	-0.0061
3.333333333	-0.0039	-0.0037	-0.0036	-0.0039	-0.0039	-0.0043	-0.0049
3.666666667	-0.0032	-0.0031	-0.0029	-0.0032	-0.0032	-0.0035	-0.0041
4	-0.0027	-0.0026	-0.0025	-0.0027	-0.0027	-0.0029	-0.0034
4.333333333	-0.0023	-0.0022	-0.0021	-0.0023	-0.0023	-0.0025	-0.0029
4.666666667	-0.002	-0.0019	-0.0018	-0.002	-0.002	-0.0022	-0.0025
5	-0.0017	-0.0016	-0.0016	-0.0017	-0.0017	-0.0019	-0.0022
5.333333333	-0.0015	-0.0014	-0.0014	-0.0015	-0.0015	-0.0017	-0.0019
5.666666667	-0.0013	-0.0013	-0.0012	-0.0013	-0.0013	-0.0015	-0.0017
6	-0.0012	-0.0011	-0.0011	-0.0012	-0.0012	-0.0013	-0.0015
6.333333333	-0.0011	-0.001	-0.001	-0.0011	-0.0011	-0.0012	-0.0014
6.666666667	-0.001	-0.001	-0.0009	-0.001	-0.001	-0.0011	-0.0013

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.333333333	0	0	0	0	0	0	0
1.666666667	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2.333333333	0	0	0	0	0	0	0
2.666666667	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
3.333333333	0	0	0	0	0	0	0
3.666666667	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
4.333333333	0	0	0	0	0	0	0
4.666666667	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
5.333333333	0	0	0	0	0	0	0
5.666666667	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
6.333333333	0	0	0	0	0	0	0
6.666666667	0	0	0	0	0	0	0

Alpha_k

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.007	-0.0085	-0.0148	-0.0135	-0.013	-0.0145	-0.021
0.5	-0.0048	-0.0066	-0.0109	-0.0105	-0.0099	-0.0112	-0.0162
0.666666667	-0.002	-0.004	-0.0055	-0.0063	-0.0056	-0.0065	-0.0092
0.833333333	-0.0009	-0.0029	-0.003	-0.0043	-0.0037	-0.0044	-0.0061
1	-0.0006	-0.0023	-0.0017	-0.0031	-0.0026	-0.0033	-0.0044
1.166666667	-0.0004	-0.002	-0.0009	-0.0024	-0.002	-0.0025	-0.0033
1.333333333	-0.0005	-0.0018	-0.0005	-0.0019	-0.0016	-0.0021	-0.0026
1.5	-0.0006	-0.0017	-0.0002	-0.0015	-0.0013	-0.0017	-0.0021
1.666666667	-0.0007	-0.0016	7.4E-06	-0.0013	-0.0011	-0.0015	-0.0018
1.833333333	-0.0008	-0.0015	0.00013	-0.0011	-0.0009	-0.0013	-0.0015
2	-0.0009	-0.0014	0.00022	-0.0009	-0.0008	-0.0011	-0.0013
2.166666667	-0.001	-0.0014	0.00027	-0.0008	-0.0007	-0.001	-0.0011
2.333333333	-0.0011	-0.0014	0.00031	-0.0006	-0.0006	-0.0009	-0.001
2.5	-0.0011	-0.0013	0.00033	-0.0005	-0.0005	-0.0008	-0.0009
2.666666667	-0.0012	-0.0013	0.00034	-0.0005	-0.0005	-0.0007	-0.0008
2.833333333	-0.0013	-0.0013	0.00034	-0.0004	-0.0004	-0.0006	-0.0007
3	-0.0013	-0.0012	0.00034	-0.0004	-0.0004	-0.0006	-0.0006
3.166666667	-0.0014	-0.0012	0.00033	-0.0003	-0.0004	-0.0005	-0.0006
3.333333333	-0.0014	-0.0012	0.00033	-0.0003	-0.0003	-0.0005	-0.0006

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.0154	-0.0198	-0.0233	-0.0251	-0.0258	-0.0269	-0.0276
0.5	-0.0085	-0.0127	-0.016	-0.0177	-0.0183	-0.0194	-0.02
0.666666667	0.00123	-0.0026	-0.0056	-0.0072	-0.0078	-0.0087	-0.0092
0.833333333	0.00528	0.00175	-0.0011	-0.0025	-0.0031	-0.0039	-0.0044
1	0.00723	0.00391	0.00125	-9E-05	-0.0006	-0.0014	-0.0018
1.166666667	0.00824	0.00509	0.00255	0.00129	0.00075	4.5E-05	-0.0003
1.333333333	0.00877	0.00576	0.00332	0.00212	0.0016	0.00094	0.0006
1.5	0.00904	0.00615	0.0038	0.00265	0.00214	0.00153	0.00121
1.666666667	0.00916	0.00637	0.00409	0.00299	0.0025	0.00191	0.00163
1.833333333	0.00918	0.00649	0.00428	0.00321	0.00273	0.00218	0.00191
2	0.00916	0.00654	0.00439	0.00336	0.00289	0.00236	0.00212
2.166666667	0.0091	0.00655	0.00445	0.00346	0.003	0.00249	0.00226
2.333333333	0.00901	0.00653	0.00448	0.00351	0.00306	0.00258	0.00236
2.5	0.00892	0.0065	0.00449	0.00355	0.0031	0.00264	0.00244
2.666666667	0.00882	0.00645	0.00448	0.00356	0.00312	0.00267	0.00249
2.833333333	0.00871	0.00639	0.00445	0.00356	0.00313	0.0027	0.00252
3	0.0086	0.00633	0.00443	0.00355	0.00313	0.00271	0.00254
3.166666667	0.00849	0.00626	0.00439	0.00354	0.00312	0.00271	0.00255
3.333333333	0.00844	0.00623	0.00437	0.00353	0.00311	0.00271	0.00256

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.0155	-0.0158	-0.0157	-0.0159	-0.0159	-0.0155	-0.0139
0.5	-0.0121	-0.0124	-0.0124	-0.0125	-0.0125	-0.0123	-0.0111
0.666666667	-0.0072	-0.0075	-0.0075	-0.0076	-0.0076	-0.0075	-0.007
0.833333333	-0.0048	-0.0051	-0.0052	-0.0052	-0.0053	-0.0052	-0.0048
1	-0.0035	-0.0037	-0.0038	-0.0039	-0.0039	-0.0038	-0.0036
1.166666667	-0.0026	-0.0028	-0.003	-0.003	-0.003	-0.003	-0.0028
1.333333333	-0.0021	-0.0023	-0.0024	-0.0024	-0.0024	-0.0024	-0.0023
1.5	-0.0017	-0.0018	-0.0019	-0.002	-0.002	-0.002	-0.0019
1.666666667	-0.0014	-0.0015	-0.0016	-0.0017	-0.0017	-0.0017	-0.0016
1.833333333	-0.0011	-0.0013	-0.0014	-0.0014	-0.0015	-0.0015	-0.0014
2	-0.001	-0.0011	-0.0012	-0.0012	-0.0013	-0.0013	-0.0012
2.166666667	-0.0008	-0.0009	-0.001	-0.0011	-0.0011	-0.0011	-0.0011
2.333333333	-0.0007	-0.0008	-0.0009	-0.001	-0.001	-0.001	-0.0009
2.5	-0.0006	-0.0007	-0.0008	-0.0009	-0.0009	-0.0009	-0.0008
2.666666667	-0.0005	-0.0006	-0.0007	-0.0008	-0.0008	-0.0008	-0.0008
2.833333333	-0.0005	-0.0006	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007
3	-0.0004	-0.0005	-0.0006	-0.0006	-0.0006	-0.0007	-0.0006
3.166666667	-0.0004	-0.0004	-0.0005	-0.0006	-0.0006	-0.0006	-0.0006
3.333333333	-0.0003	-0.0004	-0.0005	-0.0005	-0.0006	-0.0006	-0.0006

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.1322	-0.1064	-0.0689	-0.0482	-0.0344	-0.0195	-0.0022
0.5	-0.1166	-0.0958	-0.0635	-0.045	-0.0323	-0.0185	-0.0022
0.666666667	-0.0909	-0.0778	-0.0542	-0.0393	-0.0287	-0.0167	-0.0021
0.833333333	-0.0738	-0.0652	-0.0473	-0.035	-0.0259	-0.0153	-0.002
1	-0.0616	-0.0559	-0.042	-0.0316	-0.0236	-0.0141	-0.0019
1.166666667	-0.0525	-0.0488	-0.0377	-0.0289	-0.0218	-0.0132	-0.0018
1.333333333	-0.0456	-0.0432	-0.0342	-0.0266	-0.0203	-0.0124	-0.0017
1.5	-0.0401	-0.0386	-0.0313	-0.0247	-0.019	-0.0117	-0.0016
1.666666667	-0.0357	-0.0348	-0.0289	-0.023	-0.0179	-0.0111	-0.0016
1.833333333	-0.032	-0.0317	-0.0267	-0.0215	-0.0169	-0.0106	-0.0015
2	-0.029	-0.029	-0.0249	-0.0203	-0.016	-0.0101	-0.0015
2.166666667	-0.0264	-0.0267	-0.0233	-0.0191	-0.0152	-0.0097	-0.0014
2.333333333	-0.0242	-0.0247	-0.0219	-0.0181	-0.0145	-0.0093	-0.0014
2.5	-0.0222	-0.0229	-0.0206	-0.0172	-0.0139	-0.009	-0.0013
2.666666667	-0.0206	-0.0214	-0.0195	-0.0164	-0.0133	-0.0087	-0.0013
2.833333333	-0.0191	-0.02	-0.0184	-0.0156	-0.0127	-0.0084	-0.0013
3	-0.0178	-0.0188	-0.0175	-0.015	-0.0122	-0.0081	-0.0012
3.166666667	-0.0167	-0.0177	-0.0166	-0.0143	-0.0118	-0.0079	-0.0012
3.333333333	-0.0161	-0.0172	-0.0162	-0.014	-0.0116	-0.0077	-0.0012

Figure A.7: $Anis_c$ and $Anis_k$

k_0

Times original value	%change FRP heave							%change FRP hor ero						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.00071	0.0092	-0.0088	0.00101	0.00259	0.00508	0.0066	0.06295	0.05755	0.05316	0.051	0.05012	0.04869	0.04747
0.4	0.00107	0.00807	-0.0057	0.00145	0.0026	0.00484	0.00639	0.0634	0.05734	0.05233	0.04992	0.04888	0.04733	0.04598
0.7	0.00175	0.00628	-0.0013	0.0021	0.00271	0.00445	0.00602	0.06442	0.05713	0.05096	0.04809	0.04676	0.04498	0.04337
1	0.00228	0.00526	0.00066	0.00242	0.00286	0.00419	0.00571	0.06547	0.05717	0.04999	0.04671	0.04512	0.04312	0.04127
1.3	0.00261	0.00466	0.00172	0.00258	0.00293	0.00398	0.00544	0.06636	0.05722	0.04917	0.04552	0.0437	0.04149	0.03942
1.6	0.00285	0.00428	0.00234	0.00267	0.00295	0.0038	0.00519	0.06705	0.05724	0.04843	0.04445	0.04242	0.04003	0.03775
1.9	0.00302	0.00402	0.00272	0.00271	0.00294	0.00365	0.00496	0.06756	0.05721	0.04774	0.04346	0.04125	0.03868	0.03622
2.2	0.00314	0.00385	0.00296	0.00273	0.00291	0.00352	0.00475	0.06791	0.05712	0.0471	0.04255	0.04016	0.03744	0.0348
2.5	0.00324	0.00372	0.00312	0.00273	0.00288	0.00339	0.00456	0.06811	0.05699	0.04648	0.0417	0.03916	0.03629	0.03348
2.8	0.00331	0.00362	0.00322	0.00272	0.00284	0.00328	0.00437	0.06817	0.05681	0.04589	0.0409	0.03821	0.03521	0.03225
3.1	0.00337	0.00355	0.00328	0.00271	0.0028	0.00318	0.0042	0.06811	0.05659	0.04533	0.04015	0.03733	0.0342	0.0311
3.4	0.00341	0.0035	0.00331	0.00269	0.00275	0.00308	0.00404	0.06795	0.05633	0.04478	0.03943	0.03649	0.03325	0.03001
3.7	0.00344	0.00345	0.00333	0.00267	0.00271	0.00299	0.00389	0.06769	0.05603	0.04425	0.03875	0.0357	0.03235	0.02899
4	0.00347	0.00341	0.00333	0.00264	0.00267	0.0029	0.00375	0.06735	0.0557	0.04373	0.0381	0.03495	0.03151	0.02803
4.3	0.00348	0.00338	0.00332	0.00262	0.00262	0.00282	0.00361	0.06694	0.05534	0.04322	0.03748	0.03424	0.03071	0.02712
4.6	0.00349	0.00335	0.00331	0.00259	0.00258	0.00275	0.00349	0.06647	0.05495	0.04273	0.03689	0.03357	0.02995	0.02626
4.9	0.0035	0.00333	0.0033	0.00258	0.00256	0.00271	0.00343	0.06622	0.05475	0.04248	0.0366	0.03324	0.02958	0.02584

Times original value	%change FRP ver ero							%change FRP ver tra						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.01997	0.01849	0.01727	0.01637	0.01615	0.01563	0.01472	-0.1957	-0.1502	-0.0922	-0.0629	-0.0441	-0.0247	-0.0031
0.4	0.01976	0.01814	0.01672	0.01582	0.01559	0.015	0.01407	-0.1385	-0.1107	-0.0713	-0.0497	-0.0354	-0.0201	-0.0026
0.7	0.01932	0.01748	0.01575	0.01484	0.01457	0.01388	0.0129	-0.0618	-0.0561	-0.0412	-0.0305	-0.0225	-0.0133	-0.0018
1	0.01885	0.01688	0.01496	0.01402	0.01368	0.01296	0.01194	-0.0335	-0.0334	-0.0271	-0.021	-0.016	-0.0097	-0.0013
1.3	0.01835	0.0163	0.01426	0.0133	0.0129	0.01216	0.01111	-0.0199	-0.0218	-0.0193	-0.0156	-0.0121	-0.0076	-0.0011
1.6	0.01781	0.01574	0.01361	0.01265	0.01221	0.01145	0.01036	-0.0123	-0.015	-0.0145	-0.0121	-0.0097	-0.0062	-0.0009
1.9	0.01725	0.01519	0.01301	0.01205	0.01157	0.01081	0.00967	-0.0078	-0.0107	-0.0113	-0.0098	-0.008	-0.0052	-0.0008
2.2	0.01668	0.01466	0.01245	0.0115	0.011	0.01023	0.00903	-0.0048	-0.0078	-0.0091	-0.0082	-0.0068	-0.0045	-0.0007
2.5	0.01611	0.01414	0.01192	0.01099	0.01047	0.0097	0.00843	-0.0028	-0.0058	-0.0075	-0.0069	-0.0059	-0.004	-0.0007
2.8	0.01554	0.01364	0.01141	0.01052	0.00999	0.00922	0.00785	-0.0015	-0.0044	-0.0063	-0.006	-0.0052	-0.0036	-0.0006
3.1	0.01498	0.01316	0.01092	0.01009	0.00954	0.00877	0.00727	-0.0005	-0.0033	-0.0053	-0.0053	-0.0047	-0.0033	-0.0006
3.4	0.01443	0.01269	0.01044	0.00968	0.00913	0.00836	0.00665	0.00018	-0.0025	-0.0046	-0.0047	-0.0042	-0.003	-0.0006
3.7	0.01389	0.01224	0.00996	0.00931	0.00875	0.00798	0.00591	0.00068	-0.0019	-0.004	-0.0042	-0.0039	-0.0028	-0.0006
4	0.01337	0.01181	0.00945	0.00895	0.00839	0.00763	0.00488	0.00105	-0.0014	-0.0036	-0.0038	-0.0036	-0.0026	-0.0007
4.3	0.01287	0.01139	0.00889	0.00862	0.00806	0.00731	0.00299	0.00132	-0.001	-0.0032	-0.0035	-0.0033	-0.0025	-0.0008
4.6	0.01238	0.01099	0.00818	0.00831	0.00775	0.007	-0.0027	0.00152	-0.0007	-0.0029	-0.0032	-0.0031	-0.0024	-0.0015
4.9	0.01215	0.01079	0.00778	0.00816	0.0076	0.00686	-0.0072	0.0016	-0.0006	-0.0028	-0.0031	-0.003	-0.0023	-0.002

k_1

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0124	-0.0131	-0.0242	-0.019	-0.0189	-0.0207	-0.0307	-0.2603	-0.2476	-0.2377	-0.232	-0.23	-0.2259	-0.2227
0.4	-0.0076	-0.0079	-0.0156	-0.0119	-0.0117	-0.0127	-0.0189	-0.16	-0.1522	-0.1461	-0.1426	-0.1414	-0.1388	-0.1368
0.7	-0.0021	-0.0021	-0.0055	-0.0036	-0.0034	-0.0035	-0.0054	-0.0452	-0.043	-0.0412	-0.0402	-0.0399	-0.0391	-0.0385
1	-0.0012	-0.0011	-0.0033	-0.0021	-0.0019	-0.002	-0.003	-0.0254	-0.0241	-0.0231	-0.0225	-0.0223	-0.0219	-0.0216
1.3	-0.0008	-0.0007	-0.0024	-0.0015	-0.0013	-0.0013	-0.0021	-0.0174	-0.0166	-0.0159	-0.0155	-0.0153	-0.015	-0.0148
1.6	-0.0006	-0.0005	-0.0019	-0.0011	-0.001	-0.001	-0.0016	-0.0132	-0.0125	-0.012	-0.0117	-0.0116	-0.0114	-0.0112
1.9	-0.0005	-0.0004	-0.0016	-0.0009	-0.0008	-0.0008	-0.0013	-0.0106	-0.01	-0.0096	-0.0093	-0.0093	-0.0091	-0.0089
2.2	-0.0004	-0.0003	-0.0014	-0.0008	-0.0007	-0.0006	-0.001	-0.0088	-0.0083	-0.0079	-0.0077	-0.0077	-0.0075	-0.0074
2.5	-0.0003	-0.0003	-0.0012	-0.0007	-0.0006	-0.0005	-0.0009	-0.0074	-0.0071	-0.0068	-0.0066	-0.0065	-0.0064	-0.0063
2.8	-0.0003	-0.0002	-0.0011	-0.0006	-0.0005	-0.0005	-0.0008	-0.0064	-0.0061	-0.0058	-0.0057	-0.0056	-0.0055	-0.0054
3.1	-0.0002	-0.0002	-0.0009	-0.0005	-0.0004	-0.0004	-0.0007	-0.0057	-0.0054	-0.0051	-0.005	-0.0049	-0.0049	-0.0048
3.4	-0.0002	-0.0002	-0.0009	-0.0005	-0.0004	-0.0004	-0.0006	-0.005	-0.0048	-0.0046	-0.0044	-0.0044	-0.0043	-0.0042
3.7	-0.0002	-0.0002	-0.0008	-0.0004	-0.0003	-0.0003	-0.0005	-0.0045	-0.0043	-0.0041	-0.004	-0.0039	-0.0039	-0.0038
4	-0.0002	-0.0001	-0.0007	-0.0004	-0.0003	-0.0003	-0.0005	-0.0041	-0.0038	-0.0037	-0.0036	-0.0035	-0.0035	-0.0034
4.3	-0.0002	-0.0001	-0.0007	-0.0004	-0.0003	-0.0003	-0.0004	-0.0037	-0.0035	-0.0033	-0.0032	-0.0032	-0.0032	-0.0031
4.6	-0.0001	-0.0001	-0.0006	-0.0003	-0.0003	-0.0002	-0.0004	-0.0034	-0.0032	-0.003	-0.003	-0.0029	-0.0029	-0.0028
4.9	-0.0001	0.09867	-0.0006	0.05609	-0.0002	0.05204	-0.0004	-0.0032	0.07953	-0.0029	0.07039	-0.0028	0.06737	-0.0027

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0702	-0.0679	-0.0652	-0.0644	-0.0639	-0.0621	-0.0569	-0.1129	-0.0928	-0.0621	-0.0443	-0.0321	-0.0185	-0.0021
0.4	-0.0439	-0.0425	-0.0408	-0.0402	-0.04	-0.0388	-0.0358	-0.0763	-0.0635	-0.0431	-0.031	-0.0225	-0.013	-0.0016
0.7	-0.0135	-0.0131	-0.0125	-0.0123	-0.0122	-0.0118	-0.0111	-0.0318	-0.0274	-0.0195	-0.0143	-0.0106	-0.0062	-0.0008
1	-0.0077	-0.0075	-0.0072	-0.007	-0.007	-0.0067	-0.0063	-0.0204	-0.0179	-0.013	-0.0096	-0.0071	-0.0042	-0.0006
1.3	-0.0054	-0.0052	-0.0049	-0.0048	-0.0048	-0.0046	-0.0044	-0.0152	-0.0134	-0.0099	-0.0074	-0.0055	-0.0033	-0.0004
1.6	-0.0041	-0.0039	-0.0038	-0.0037	-0.0037	-0.0035	-0.0033	-0.0122	-0.0109	-0.0081	-0.0061	-0.0045	-0.0027	-0.0004
1.9	-0.0033	-0.0032	-0.003	-0.003	-0.0029	-0.0028	-0.0027	-0.0102	-0.0091	-0.0068	-0.0052	-0.0039	-0.0023	-0.0003
2.2	-0.0027	-0.0026	-0.0025	-0.0025	-0.0024	-0.0024	-0.0022	-0.0088	-0.0079	-0.006	-0.0045	-0.0034	-0.0021	-0.0003
2.5	-0.0023	-0.0022	-0.0021	-0.0021	-0.0021	-0.002	-0.0019	-0.0077	-0.007	-0.0053	-0.004	-0.003	-0.0018	-0.0002
2.8	-0.002	-0.0019	-0.0019	-0.0018	-0.0018	-0.0017	-0.0016	-0.0069	-0.0062	-0.0048	-0.0036	-0.0027	-0.0017	-0.0002
3.1	-0.0018	-0.0017	-0.0016	-0.0016	-0.0016	-0.0015	-0.0014	-0.0062	-0.0056	-0.0043	-0.0033	-0.0025	-0.0015	-0.0002
3.4	-0.0016	-0.0015	-0.0014	-0.0014	-0.0014	-0.0014	-0.0013	-0.0056	-0.0051	-0.0039	-0.003	-0.0023	-0.0014	-0.0002
3.7	-0.0014	-0.0014	-0.0013	-0.0013	-0.0013	-0.0012	-0.0011	-0.0051	-0.0047	-0.0036	-0.0028	-0.0021	-0.0013	-0.0002
4	-0.0013	-0.0012	-0.0012	-0.0012	-0.0011	-0.0011	-0.001	-0.0047	-0.0043	-0.0034	-0.0026	-0.002	-0.0012	-0.0002
4.3	-0.0012	-0.0011	-0.0011	-0.001	-0.001	-0.001	-0.0009	-0.0043	-0.004	-0.0031	-0.0024	-0.0018	-0.0011	-0.0002
4.6	-0.0011	-0.001	-0.001	-0.001	-0.0009	-0.0009	-0.0009	-0.004	-0.0037	-0.0029	-0.0022	-0.0017	-0.001	-0.0001
4.9	-0.001	0.0331	-0.0009	0.03057	-0.0009	0.03433	-0.0008	-0.0039	0.23917	-0.0028	0.31904	-0.0017	0.36235	-0.0001

Figure A.8: k_0 and k_1

k_2

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0016	-0.0016	-0.004	-0.0027	-0.0025	-0.0026	-0.0041	-0.0354	-0.0335	-0.032	-0.0312	-0.0309	-0.0303	-0.0298
0.4	-0.0012	-0.0012	-0.0031	-0.0021	-0.0019	-0.002	-0.0031	-0.0269	-0.0255	-0.0244	-0.0237	-0.0235	-0.023	-0.0227
0.7	-0.0007	-0.0007	-0.002	-0.0013	-0.0012	-0.0012	-0.0019	-0.0161	-0.0152	-0.0146	-0.0142	-0.014	-0.0138	-0.0135
1	-0.0005	-0.0005	-0.0016	-0.001	-0.0009	-0.0009	-0.0014	-0.0123	-0.0116	-0.0111	-0.0108	-0.0107	-0.0105	-0.0103
1.3	-0.0004	-0.0004	-0.0013	-0.0008	-0.0007	-0.0007	-0.0011	-0.0099	-0.0094	-0.009	-0.0087	-0.0086	-0.0085	-0.0083
1.6	-0.0004	-0.0003	-0.0011	-0.0007	-0.0006	-0.0006	-0.0009	-0.0082	-0.0078	-0.0074	-0.0072	-0.0072	-0.007	-0.0069
1.9	-0.0003	-0.0003	-0.001	-0.0006	-0.0005	-0.0005	-0.0008	-0.0069	-0.0066	-0.0063	-0.0061	-0.006	-0.0059	-0.0058
2.2	-0.0003	-0.0002	-0.0008	-0.0005	-0.0004	-0.0004	-0.0007	-0.0059	-0.0056	-0.0054	-0.0052	-0.0052	-0.0051	-0.005
2.5	-0.0002	-0.0002	-0.0007	-0.0004	-0.0004	-0.0004	-0.0006	-0.0051	-0.0049	-0.0047	-0.0045	-0.0045	-0.0044	-0.0043
2.8	-0.0002	-0.0002	-0.0007	-0.0004	-0.0003	-0.0003	-0.0005	-0.0045	-0.0043	-0.0041	-0.004	-0.0039	-0.0038	-0.0038
3.1	-0.0002	-0.0002	-0.0006	-0.0003	-0.0003	-0.0003	-0.0005	-0.004	-0.0038	-0.0036	-0.0035	-0.0035	-0.0034	-0.0033
3.4	-0.0002	-0.0001	-0.0005	-0.0003	-0.0003	-0.0003	-0.0004	-0.0035	-0.0033	-0.0032	-0.0031	-0.0031	-0.003	-0.003
3.7	-0.0001	-0.0001	-0.0005	-0.0003	-0.0002	-0.0002	-0.0004	-0.0032	-0.003	-0.0029	-0.0028	-0.0028	-0.0027	-0.0027
4	-0.0001	-0.0001	-0.0004	-0.0003	-0.0002	-0.0002	-0.0003	-0.0029	-0.0027	-0.0026	-0.0025	-0.0025	-0.0024	-0.0024
4.3	-0.0001	-1E-04	-0.0004	-0.0002	-0.0002	-0.0002	-0.0003	-0.0026	-0.0024	-0.0023	-0.0023	-0.0023	-0.0022	-0.0022
4.6	-1E-04	-9E-05	-0.0004	-0.0002	-0.0002	-0.0002	-0.0003	-0.0024	-0.0022	-0.0021	-0.0021	-0.0021	-0.002	-0.002
4.9	-9E-05	-8E-05	-0.0004	-0.0002	-0.0002	-0.0002	-0.0003	-0.0022	-0.0021	-0.002	-0.002	-0.002	-0.0019	-0.0019

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0101	-0.0097	-0.0093	-0.0091	-0.0091	-0.0088	-0.0082	-0.0243	-0.0209	-0.0148	-0.0108	-0.008	-0.0047	-0.0006
0.4	-0.0077	-0.0074	-0.0071	-0.007	-0.0069	-0.0067	-0.0063	-0.019	-0.0164	-0.0117	-0.0086	-0.0063	-0.0037	-0.0005
0.7	-0.0047	-0.0045	-0.0043	-0.0042	-0.0042	-0.0041	-0.0038	-0.0123	-0.0108	-0.0077	-0.0057	-0.0042	-0.0025	-0.0003
1	-0.0036	-0.0035	-0.0033	-0.0033	-0.0032	-0.0031	-0.0029	-0.0099	-0.0087	-0.0063	-0.0047	-0.0035	-0.0021	-0.0003
1.3	-0.0029	-0.0028	-0.0027	-0.0026	-0.0026	-0.0025	-0.0024	-0.0083	-0.0073	-0.0054	-0.004	-0.003	-0.0018	-0.0002
1.6	-0.0024	-0.0024	-0.0022	-0.0022	-0.0022	-0.0021	-0.002	-0.0071	-0.0063	-0.0046	-0.0035	-0.0026	-0.0015	-0.0002
1.9	-0.0021	-0.002	-0.0019	-0.0019	-0.0019	-0.0018	-0.0017	-0.0062	-0.0055	-0.0041	-0.003	-0.0023	-0.0014	-0.0002
2.2	-0.0018	-0.0017	-0.0016	-0.0016	-0.0016	-0.0015	-0.0014	-0.0054	-0.0048	-0.0036	-0.0027	-0.002	-0.0012	-0.0002
2.5	-0.0015	-0.0015	-0.0014	-0.0014	-0.0014	-0.0013	-0.0013	-0.0048	-0.0043	-0.0032	-0.0024	-0.0018	-0.0011	-0.0001
2.8	-0.0014	-0.0013	-0.0013	-0.0012	-0.0012	-0.0012	-0.0011	-0.0043	-0.0038	-0.0029	-0.0022	-0.0016	-0.001	-0.0001
3.1	-0.0012	-0.0012	-0.0011	-0.0011	-0.0011	-0.001	-0.001	-0.0038	-0.0035	-0.0026	-0.002	-0.0015	-0.0009	-0.0001
3.4	-0.0011	-0.001	-0.001	-0.001	-0.001	-0.0009	-0.0009	-0.0035	-0.0031	-0.0023	-0.0018	-0.0013	-0.0008	-0.0001
3.7	-0.001	-0.0009	-0.0009	-0.0009	-0.0009	-0.0008	-0.0008	-0.0032	-0.0028	-0.0021	-0.0016	-0.0012	-0.0007	-1E-04
4	-0.0009	-0.0008	-0.0008	-0.0008	-0.0008	-0.0007	-0.0007	-0.0029	-0.0026	-0.002	-0.0015	-0.0011	-0.0007	-9E-05
4.3	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0006	-0.0026	-0.0024	-0.0018	-0.0014	-0.001	-0.0006	-8E-05
4.6	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0024	-0.0022	-0.0017	-0.0013	-0.001	-0.0006	-8E-05
4.9	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006	-0.0023	-0.0021	-0.0016	-0.0012	-0.0009	-0.0006	-7E-05

k_3

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0011	-0.0011	-0.0029	-0.0019	-0.0018	-0.0019	-0.0029	-0.0251	-0.0237	-0.0227	-0.0221	-0.0219	-0.0214	-0.0211
0.4	-0.001	-0.0009	-0.0026	-0.0017	-0.0015	-0.0016	-0.0025	-0.0217	-0.0206	-0.0196	-0.0191	-0.0189	-0.0186	-0.0183
0.7	-0.0007	-0.0007	-0.002	-0.0013	-0.0012	-0.0012	-0.0019	-0.0162	-0.0154	-0.0147	-0.0143	-0.0142	-0.0139	-0.0136
1	-0.0006	-0.0005	-0.0016	-0.001	-0.0009	-0.0009	-0.0015	-0.0126	-0.0119	-0.0114	-0.0111	-0.011	-0.0108	-0.0106
1.3	-0.0004	-0.0004	-0.0013	-0.0008	-0.0007	-0.0007	-0.0012	-0.0101	-0.0095	-0.0091	-0.0089	-0.0088	-0.0086	-0.0085
1.6	-0.0004	-0.0003	-0.0011	-0.0007	-0.0006	-0.0006	-0.001	-0.0083	-0.0078	-0.0075	-0.0073	-0.0072	-0.0071	-0.0069
1.9	-0.0003	-0.0003	-0.001	-0.0006	-0.0005	-0.0005	-0.0008	-0.0069	-0.0065	-0.0062	-0.0061	-0.006	-0.0059	-0.0058
2.2	-0.0003	-0.0002	-0.0008	-0.0005	-0.0004	-0.0004	-0.0007	-0.0058	-0.0055	-0.0053	-0.0051	-0.0051	-0.005	-0.0049
2.5	-0.0002	-0.0002	-0.0007	-0.0004	-0.0004	-0.0004	-0.0006	-0.005	-0.0047	-0.0045	-0.0044	-0.0044	-0.0043	-0.0042
2.8	-0.0002	-0.0002	-0.0006	-0.0004	-0.0003	-0.0003	-0.0005	-0.0043	-0.0041	-0.0039	-0.0038	-0.0038	-0.0037	-0.0036
3.1	-0.0002	-0.0001	-0.0006	-0.0003	-0.0003	-0.0003	-0.0004	-0.0038	-0.0036	-0.0034	-0.0033	-0.0033	-0.0032	-0.0032
3.4	-0.0001	-0.0001	-0.0005	-0.0003	-0.0002	-0.0002	-0.0004	-0.0034	-0.0032	-0.003	-0.0029	-0.0029	-0.0029	-0.0028
3.7	-0.0001	-0.0001	-0.0004	-0.0003	-0.0002	-0.0002	-0.0003	-0.003	-0.0028	-0.0027	-0.0026	-0.0026	-0.0025	-0.0025
4	-0.0001	-0.0001	-0.0004	-0.0002	-0.0002	-0.0002	-0.0003	-0.0027	-0.0025	-0.0024	-0.0023	-0.0023	-0.0023	-0.0022
4.3	-0.0001	-9E-05	-0.0004	-0.0002	-0.0002	-0.0002	-0.0003	-0.0024	-0.0023	-0.0022	-0.0021	-0.0021	-0.0021	-0.002
4.6	-9E-05	-8E-05	-0.0003	-0.0002	-0.0002	-0.0002	-0.0003	-0.0022	-0.0021	-0.002	-0.0019	-0.0019	-0.0019	-0.0018
4.9	-9E-05	-8E-05	-0.0003	-0.0002	-0.0002	-0.0001	-0.0002	-0.0021	-0.002	-0.0019	-0.0018	-0.0018	-0.0018	-0.0017

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0072	-0.0069	-0.0066	-0.0065	-0.0064	-0.0062	-0.0059	-0.0175	-0.0151	-0.0107	-0.0078	-0.0058	-0.0034	-0.0004
0.4	-0.0062	-0.006	-0.0058	-0.0056	-0.0056	-0.0054	-0.0051	-0.0156	-0.0135	-0.0096	-0.0071	-0.0052	-0.0031	-0.0004
0.7	-0.0047	-0.0045	-0.0044	-0.0043	-0.0042	-0.0041	-0.0039	-0.0125	-0.0108	-0.0078	-0.0058	-0.0043	-0.0025	-0.0003
1	-0.0037	-0.0036	-0.0034	-0.0033	-0.0033	-0.0032	-0.003	-0.0102	-0.0089	-0.0065	-0.0048	-0.0036	-0.0021	-0.0003
1.3	-0.003	-0.0029	-0.0027	-0.0027	-0.0027	-0.0026	-0.0024	-0.0085	-0.0075	-0.0055	-0.0041	-0.003	-0.0018	-0.0002
1.6	-0.0025	-0.0024	-0.0023	-0.0022	-0.0022	-0.0021	-0.002	-0.0072	-0.0064	-0.0047	-0.0035	-0.0026	-0.0016	-0.0002
1.9	-0.0021	-0.002	-0.0019	-0.0019	-0.0018	-0.0018	-0.0017	-0.0062	-0.0055	-0.004	-0.003	-0.0023	-0.0014	-0.0002
2.2	-0.0017	-0.0017	-0.0016	-0.0016	-0.0016	-0.0015	-0.0014	-0.0053	-0.0048	-0.0035	-0.0026	-0.002	-0.0012	-0.0002
2.5	-0.0015	-0.0015	-0.0014	-0.0014	-0.0013	-0.0013	-0.0012	-0.0047	-0.0042	-0.0031	-0.0023	-0.0018	-0.0011	-0.0001
2.8	-0.0013	-0.0013	-0.0012	-0.0012	-0.0012	-0.0011	-0.0011	-0.0041	-0.0037	-0.0028	-0.0021	-0.0016	-0.0009	-0.0001
3.1	-0.0011	-0.0011	-0.0011	-0.001	-0.001	-0.001	-0.0009	-0.0037	-0.0033	-0.0025	-0.0019	-0.0014	-0.0008	-0.0001
3.4	-0.001	-0.001	-0.0009	-0.0009	-0.0009	-0.0009	-0.0008	-0.0033	-0.003	-0.0022	-0.0017	-0.0013	-0.0008	-0.0001
3.7	-0.0009	-0.0009	-0.0008	-0.0008	-0.0008	-0.0008	-0.0007	-0.003	-0.0027	-0.002	-0.0015	-0.0011	-0.0007	-9E-05
4	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0027	-0.0024	-0.0018	-0.0014	-0.001	-0.0006	-8E-05
4.3	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0024	-0.0022	-0.0017	-0.0013	-0.001	-0.0006	-8E-05
4.6	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006	-0.0005	-0.0022	-0.002	-0.0015	-0.0012	-0.0009	-0.0005	-7E-05
4.9	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006	-0.0005	-0.0005	-0.0021	-0.0019	-0.0015	-0.0011	-0.0008	-0.0005	-7E-05

D_0

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	0.09439	0.07641	0.05595	0.05272	0.04749	0.03904	0.02088	0.04851	0.04152	0.03741	0.03291	0.0327	0.03001	0.02903
0.833333333	0.09376	0.07573	0.05723	0.05291	0.04737	0.03882	0.02076	0.04838	0.04188	0.03523	0.03146	0.03056	0.02923	0.02702
1.25	0.09264	0.07454	0.05942	0.05318	0.0471	0.03838	0.02046	0.04827	0.04088	0.03221	0.029	0.02732	0.02642	0.02374
1.666666667	-0.1339	-0.1079	-0.0772	-0.073	-0.0653	-0.0525	-0.0238	0.04941	0.0402	0.0328	0.02953	0.02617	0.02427	0.02264
2.083333333	-0.1764	-0.1423	-0.1052	-0.0978	-0.0871	-0.07	-0.0322	0.05542	0.04263	0.03436	0.03072	0.02705	0.02494	0.02314
2.5	0.01328	0.00969	0.00711	0.00577	0.00562	0.00523	0.00439	0.06034	0.0463	0.03593	0.03158	0.03065	0.02763	0.02431
2.916666667	0.00134	0.00184	0.0015	0.00164	0.00174	0.00193	0.00297	0.06407	0.0541	0.03971	0.03625	0.03371	0.0304	0.02676
3.333333333	-0.0167	-0.0106	-0.0069	-0.0047	-0.0043	-0.0034	0.00048	0.06777	0.05998	0.04206	0.03972	0.0341	0.03126	0.02837

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	0.0134	0.01153	-0.0003	0.00911	0.01332	0.00819	-0.003	-0.0313	-0.031	-0.0255	-0.0191	-0.0137	-0.0085	-0.0019
0.833333333	0.01355	0.01156	0.00459	0.00889	0.01079	0.00231	0.00198	-0.0191	-0.0205	-0.018	-0.0141	-0.0104	-0.0067	-0.0011
1.25	0.01329	0.01129	0.00922	0.00842	0.00802	0.00178	0.00664	-0.0032	-0.0067	-0.0079	-0.0071	-0.0057	-0.0038	-0.0002
1.666666667	0.013	0.01128	0.00947	0.00449	0.00777	0.00714	0.0068	0.00278	-0.0012	-0.0036	-0.0039	-0.0033	-0.002	-0.0002
2.083333333	0.0082	0.01164	0.00983	0.00481	0.00796	0.00734	0.007	0.00639	0.00251	-0.0006	-0.0015	-0.0016	-0.0011	-1E-04
2.5	0.00887	0.01203	0.00978	0.00906	0.04598	0.00859	0.00696	0.00888	0.00513	0.00153	0.00038	-0.0003	-0.0003	-5E-05
2.916666667	0.01496	0.00807	0.01056	0.00872	0.01629	0.00932	0.00752	0.01076	0.00712	0.00333	0.00185	0.00095	0.00039	1.7E-05
3.333333333	0.01548	0.00379	0.01126	0.00812	-0.0512	0.00898	0.00789	0.01161	0.00807	0.00424	0.00261	0.00165	0.00073	6.8E-05

Figure A.9: k_2, k_3 and D_0

D_1

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.00088	0.0008	0.00295	0.00175	0.00149	0.0015	0.00238	0.01982	0.01881	0.01802	0.01756	0.01741	0.01708	0.01682	0.00621	0.00599	0.00572	0.0056	0.00556	0.00537	0.00507	0.01865	0.01668	0.01242	0.00936	0.00702	0.00421	0.00057
0.357142857	0.00082	0.00075	0.0027	0.00162	0.00138	0.00139	0.00221	0.01855	0.01759	0.01684	0.01641	0.01627	0.01595	0.0157	0.00574	0.00553	0.00529	0.00518	0.00514	0.00496	0.00468	0.01709	0.01524	0.01131	0.00851	0.00637	0.00381	0.00051
0.535714286	0.00072	0.00067	0.00228	0.00139	0.00121	0.00122	0.00193	0.01647	0.0156	0.01493	0.01454	0.0144	0.01412	0.01389	0.00497	0.0048	0.00459	0.00449	0.00446	0.00431	0.00406	0.01453	0.01289	0.0095	0.00712	0.00531	0.00317	0.00042
0.714285714	0.00065	0.00061	0.00199	0.00123	0.00108	0.0011	0.00173	0.01488	0.01409	0.01347	0.01311	0.01299	0.01273	0.01253	0.00443	0.00427	0.00409	0.004	0.00397	0.00384	0.00362	0.01267	0.01118	0.00819	0.00612	0.00456	0.00271	0.00036
0.892857143	0.00059	0.00056	0.00177	0.00111	0.00098	0.00101	0.00157	0.01357	0.01284	0.01228	0.01195	0.01184	0.01161	0.01141	0.00401	0.00387	0.0037	0.00362	0.0036	0.00347	0.00327	0.01121	0.00986	0.00718	0.00535	0.00398	0.00236	0.00031
1.071428571	0.00058	0.00056	0.0017	0.00108	0.00096	0.00099	0.00154	0.01334	0.01262	0.01207	0.01175	0.01164	0.0114	0.01122	0.00391	0.00377	0.00361	0.00354	0.00351	0.00339	0.0032	0.01071	0.00938	0.0068	0.00505	0.00375	0.00222	0.00029
1.25	0.00059	0.00057	0.00168	0.00108	0.00097	0.001	0.00155	0.01347	0.01275	0.01218	0.01186	0.01175	0.01151	0.01132	0.00393	0.00379	0.00362	0.00355	0.00352	0.00341	0.00321	0.01053	0.00919	0.00663	0.00491	0.00364	0.00216	0.00028
1.428571429	0.00058	0.00057	0.00165	0.00106	0.00095	0.00099	0.00153	0.01332	0.0126	0.01204	0.01172	0.01161	0.01138	0.01119	0.00387	0.00373	0.00357	0.0035	0.00347	0.00336	0.00316	0.01026	0.00894	0.00644	0.00476	0.00353	0.00209	0.00027

L_cutline

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	0	0	0	0	0	0	0	0.08903	0.08336	0.07888	0.0754	0.07464	0.06993	0.0619	-0.0153	-0.0156	-0.0164	-0.0485	-0.0471	-0.0754	-0.1041	0	-3E-15	-3E-15	0	0	0	0
0.5	-9E-14	0	0	0	0	0	0	0.07807	0.07342	0.0695	0.06693	0.06618	0.06287	0.05702	-0.0048	-0.0052	-0.006	-0.0279	-0.0271	-0.0469	-0.0688	0	0	-1E-15	0	0	0	0
0.75	0	0	0	0	0	0	0	0.06116	0.05796	0.05493	0.05355	0.05286	0.05138	0.04852	0.00746	0.00701	0.00636	-0.0026	-0.0026	-0.011	-0.0232	0	0	1.5E-15	0	0	0	0
1	9E-14	0	0	0	0	0	1.7E-14	0.05154	0.04901	0.04654	0.04557	0.04497	0.04409	0.0423	0.00936	0.00902	0.00856	0.00361	0.00357	-0.0012	-0.0086	0	0	1.5E-15	0	0	0	0
1.25	0	0	0	0	0	0	1.7E-14	0.04491	0.04281	0.04073	0.03997	0.03946	0.03886	0.03762	0.0094	0.00915	0.00882	0.00567	0.00562	0.00258	-0.0024	0	1.5E-15	-1E-15	0	0	0	0
1.5	0	0	0	0	0	0	0	0.03999	0.0382	0.03641	0.0358	0.03536	0.03494	0.03409	0.00897	0.0088	0.00855	0.00637	0.00632	0.00421	0.00063	0	-1E-15	0	0	0	0	0
1.75	0	0	0	0	0	0	0	0.03635	0.03479	0.03322	0.0327	0.03231	0.03198	0.03132	0.00844	0.00832	0.00813	0.00653	0.00649	0.00494	0.00226	0	-1E-15	0	0	0	0	0
2	0	0	0	0	0	0	-2E-14	0.03344	0.03205	0.03066	0.0302	0.02985	0.02957	0.02902	0.00792	0.00783	0.00768	0.00646	0.00643	0.00525	0.00317	0	1.5E-15	-7E-16	0	0	0	0
2.25	0	0	0	0	0	0	0	0.03094	0.02971	0.02846	0.02805	0.02774	0.02751	0.02706	0.00743	0.00736	0.00725	0.00629	0.00627	0.00533	0.00368	0	0	1.5E-15	0	0	0	0
2.5	0	0	0	0	0	0	3.3E-14	0.02968	0.02852	0.02735	0.02696	0.02666	0.02645	0.02606	0.00719	0.00714	0.00704	0.00619	0.00617	0.00534	0.00386	0	-3E-15	1.5E-15	0	0	0	0

d_rep

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.1119	29.6446	-0.3585	0.96099	-0.2169	-0.3153	-30.024	-18.402	-18.409	-17.489	-16.583	-15.796	-14.181	-5.6316	-3.0025	-3.155	-3.1116	-2.9943	-2.9927	-2.8587	-1.5089	0.02883	0.11988	0.16485	0.21199	0.32607	0.48161	0.71907
0.4	0.52918	-0.087	-0.1365	-0.1696	-0.2075	-0.2263	-15.041	-10.245	-9.72	-9.1853	-8.6637	-8.1863	-7.3212	-2.9908	-1.7153	-1.7023	-1.6694	-1.6362	-1.6074	-1.5039	-0.8123	0.10765	0.18514	0.25608	0.29083	0.34695	0.39259	0.40384
0.7	-0.0383	-14.965	-0.0162	-0.6921	-0.1193	-0.0888	-0.038	-1.2165	-0.6309	-0.5354	-0.4624	-0.3664	-0.3035	-0.2404	-0.2782	-0.1653	-0.1481	-0.1646	-0.1236	-0.0994	-0.0884	0.23106	0.23655	0.27909	0.28962	0.25432	0.1985	0.05142
1	-0.658	-0.1017	-0.0683	-0.0563	-0.0361	-0.0334	-0.0137	-0.2238	-0.1637	-0.137	-0.1294	-0.117	-0.109	-0.1003	-0.0831	-0.053	-0.0451	-0.0438	-0.0239	-0.0436	-0.0413	0.19769	0.19033	0.16032	0.14521	0.09934	0.06376	0.00956
1.3	-0.0733	-0.0468	-0.0209	-0.0268	-0.0252	-0.0199	-0.0059	-0.0826	-0.0739	-0.0665	-0.0627	-0.0615	-0.0587	-0.0563	-0.0259	-0.0233	-0.0217	-0.0235	-0.0235	-0.0264	-0.0181	0.12792	0.12298	0.07866	0.05692	0.04072	0.02394	0.00344
1.6	-0.0385	-0.0105	-0.0217	-0.02	-0.0153	-0.0112	-0.0024	-0.0486	-0.0419	-0.0401	-0.0388	-0.0379	-0.0378	-0.0358	-0.0145	-0.0186	-0.0141	-0.0133	-0.0156	-0.0117	-0.0126	0.10927	0.0646	0.03758	0.02384	0.01765	0.00974	0.00139
1.9	-0.0076	-0.0184	-0.0164	-0.0123	-0.0101	-0.0073	-0.0006	-0.03	-0.0284	-0.0274	-0.0266	-0.0266	-0.026	-0.0249	-0.0135	-0.0093	-0.0084	-0.0103	-0.0066	-0.0075	-0.0134	0.06213	0.03358	0.0204	0.01244	0.0092	0.00473	0.00041
2.2	-0.0157	-0.0147	-0.0106	-0.0083	-0.0069	-0.0046	0.00035	-0.0213	-0.0205	-0.0198	-0.0198	-0.0194	-0.019	-0.0184	-0.0064	-0.0055	-0.0072	-0.0041	-0.0083	-0.0063	-0.0099	0.02937	0.01971	0.01134	0.00817	0.00484	0.0029	0.00034
2.5	-0.0145	-0.0102	-0.0075	-0.0061	-0.0049	-0.0032	0.00084	-0.0157	-0.0154	-0.0152	-0.0151	-0.0147	-0.0147	-0.014	-0.0039	-0.0052	-0.0057	-0.0051	-0.0067	-0.005	-0.0042	0.01866	0.01072	0.00685	0.00493	0.00289	0.00157	0.00039
2.8	-0.0112	-0.0076	-0.0059	-0.0044	-0.0036	-0.0021	0.00133	-0.0122	-0.0121	-0.0119	-0.0117	-0.0118	-0.0116	-0.011	-0.0061	-0.005	-0.0044	-0.005	-0.0027	-0.0041	-0.0068	0.0159	0.00655	0.00441	0.00286	0.00227	0.00094	-0.0003
3.1	-0.0084	-0.0058	-0.0045	-0.0034	-0.0025	-0.0012	0.00154	-0.0101	-0.0098	-0.0098	-0.0095	-0.0095	-0.0093	-0.009	-0.0029	-0.0034	-0.0011	-0.005	-0.0044	-0.0055	-0.0057	0.00841	0.00483	0.00349	0.00123	0.00102	0.00025	-1E-05
3.4	-0.0065	-0.0047	-0.0034	-0.0026	-0.0018	-0.0007	0.00173	-0.0081	-0.0079	-0.008	-0.0079	-0.0078	-0.0077	-0.0073	-0.0023	-0.0018	-0.003	-0.0034	-0.0048	-0.0053	-0.0029	0.0061	0.00389	0.00203	0.00114	0.00069	0.00028	0.00052
3.7	-0.0048	-0.0037	-0.0027	-0.002	-0.0013	-0.0005	0.00181	-0.0067	-0.0066	-0.0066	-0.0067	-0.0066	-0.0065	-0.006	-0.0042	-0.0042	-0.0027	-0.0008	-0.0015	-0.0021	-0.0029	0.00532	0.00229	0.00148	0.00145	0.00103	0.00064	0.00014
4	-0.0044	-0.003	-0.0021	-0.0013	-0.0008	-0.0002	0.00181	-0.0059	-0.0058	-0.0057	-0.0057	-0.0056	-0.0055	-0.0052	-0.0016	-0.0019	-0.0022	-0.0029	-0.0004	-0.0013	-0.0034	0.00351	0.00191	0.00105	0.00057	0.00066	0.00042	-0.0012
4.3	-0.0038	-0.0024	-0.0017	-0.001	-0.0007	0.0002	0.00197	-0.005	-0.005	-0.0049	-0.0048	-0.0048	-0.0047	-0.0046	-0.0009	-0.0018	-0.0021	-0.0023	-0.0032	-0.0017	-0.0036	0.00263	0.00161	0.00082	0.00051	0.00036	0.00026	2.2E-05
4.6	-0.0029	-0.0021	-0.0013	-0.0008	-0.0003	0.00028	0.00194	-0.0043	-0.0043	-0.0043	-0.0042	-0.0042	-0.0041	-0.004	-0.0022	-0.0015	-0.0017	-0.0007	-0.0024	-0.0016	-0.0026	0.00205	0.00099	0.00059	0.00075	0.00027	0.00014	0.0012
4.9	-0.0026	-0.0021	-0.0012	-0.0006	-5E-05	0.00026	0.00186	-0.0041	-0.0041	-0.0039	-0.004	-0.0039	-0.0038	-0.0035	-0.0015	0.00084	-0.0031	-0.0015	0.00043	-0.0015	-0.002	0.002	0.00123	0.00038	0.00059	0.00018	0.00011	1.5E-05

Figure A.10: D_1, $L_{Cut\ line}$ and d_{rep}

$2/3 * D_0$

A_coh

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	-5E-15	0	0	0	0	-2E-15
2	0	0	0	0	0	6.6E-16	-1E-15
3	0	2.6E-15	0	0	0	6.6E-16	1.1E-15
4	0	-2E-15	0	0	0	-7E-16	1.1E-15
5	0	0	6.6E-16	0	0	-7E-16	-1E-15
6	0	-7E-16	0	0	0	0	-1E-15
7	0	0	0	0	0	6.6E-16	0
8	0	2.6E-15	6.6E-16	0	0	6.6E-16	1.1E-15
9	0	0	0	0	0	-7E-16	0
10	0	0	0	0	0	-1E-15	-2E-15

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
2	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
3	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
4	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
5	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
6	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
7	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
8	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
9	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
10	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
2	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
3	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
4	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
5	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
6	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
7	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
8	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
9	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
10	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	-9E-15	0	-1E-14	0	0	0	0
2	-4E-15	0	0	0	0	0	0
3	4.4E-15	0	6.6E-15	0	0	0	0
4	0	0	-7E-15	0	0	0	0
5	-4E-15	0	2.2E-15	0	0	0	0
6	0	0	6.6E-15	0	0	0	2.2E-14
7	0	0	-9E-15	0	0	0	0
8	0	0	2.2E-15	0	0	0	-2E-14
9	0	0	8.8E-15	0	0	0	2.2E-14
10	0	0	0	0	0	0	4.4E-14

c

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	-1E-14	0	0
0.1	0	0	0	0	-2E-14	0	0
0.2	0	0	0	0	6.6E-15	0	0
0.3	0	0	0	0	1.5E-14	0	0
0.4	0	0	0	8.9E-15	-7E-15	0	0
0.5	0	0	0	0	-7E-15	0	0
0.6	0	0	0	-9E-15	6.6E-15	0	0
0.7	0	0	0	0	6.6E-15	0	0
0.8	0	0	0	8.9E-15	0	0	0
0.9	0	0	0	8.9E-15	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	-9E-15	0	0	0
1.2	0	0	0	-9E-15	0	0	0
1.3	0	0	0	0	-7E-15	0	0
1.4	0	0	0	0	0	0	0
1.5	0	0	0	0	-9E-15	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	0	0	0	0
1.8	0	0	0	0	-7E-15	0	0
1.9	0	0	0	0	8.8E-15	0	0
2	0	0	0	0	0	0	0

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.1	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.2	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.3	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.4	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.5	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.6	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.7	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.8	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
0.9	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.1	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.2	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.3	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.4	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.5	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.6	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.7	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.8	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
1.9	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797
2	1.43259	1.28915	1.15026	1.10044	1.06162	1.03068	0.98797

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.1	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.2	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.3	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.4	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.5	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.6	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.7	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.8	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
0.9	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.1	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.2	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.3	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.4	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.5	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.6	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.7	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.8	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
1.9	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183
2	0.53081	0.49571	0.45911	0.46014	0.49085	0.47907	0.6183

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	-4E-14	0	0	-4E-14	0	0
0.1	0	-2E-14	0	0	0	0	0
0.2	0	2.2E-14	0	0	2.2E-14	0	0
0.3	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0
0.5	0	2.2E-14	0	1.3E-13	0	0	-2E-13
0.6	0	-2E-14	0	0	0	0	-2E-13
0.7	0	0	0	-1E-13	0	0	2.2E-13
0.8	0	2.2E-14	0	1.3E-13	0	0	2.2E-13
0.9	0	-1E-13	0	0	0	0	0
1	0	-2E-14	0	-1E-13	0	0	0
1.1	0	1.3E-13	0	0	0	0	0
1.2	6.6E-14	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	-7E-14	2.2E-14	0	0	-2E-14	0	0
1.5	0	0	0	0	-2E-14	0	0
1.6	0	0	0	0	2.2E-14	0	0
1.7	0	0	0	0	2.2E-14	0	0
1.8	0	0	0	0	-4E-14	0	0
1.9	0	-2E-14	0	0	-4E-14	0	0
2	0	-4E-14	0	0	0	0	0

Figure A.11: A_c and c

Alpha_coh

	%change FRP heave						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	1.2E-14	1.6E-14	0	0	0	0
1	0	6.6E-15	7.9E-15	0	0	0	0
1.333333333	0	-6E-15	0	0	0	0	0
1.666666667	0	-7E-16	-8E-15	0	0	0	2E-15
2	0	0	0	0	0	0	0
2.333333333	0	0	7.9E-15	0	0	0	0
2.666666667	0	6.6E-15	0	0	0	0	0
3	0	6.6E-16	0	0	0	0	-2E-15
3.333333333	0	-7E-15	-8E-15	0	0	0	0
3.666666667	0	-7E-15	-8E-15	0	0	0	2E-15
4	0	6.6E-15	7.9E-15	0	0	0	0
4.333333333	-1E-14	6.6E-15	7.9E-15	0	0	0	-2E-15
4.666666667	0	-7E-16	-8E-15	0	0	0	0
5	0	-7E-15	0	0	0	0	0
5.333333333	0	-6E-15	7.9E-15	0	0	0	2E-15
5.666666667	1.4E-14	5.9E-15	0	0	0	0	0
6	0	6.6E-15	0	0	0	0	-2E-15
6.333333333	0	6.6E-16	-8E-15	0	0	0	0
6.666666667	0	0	-2E-14	0	0	0	0

	%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	2.6E-14	0	0
1	0	0	0	0	0	0	0
1.333333333	0	0	6.6E-15	0	-1E-14	0	0
1.666666667	0	0	0	0	6.6E-15	0	0
2	-7E-15	0	-7E-15	0	1.3E-14	0	0
2.333333333	0	0	0	0	6.6E-15	0	0
2.666666667	6.6E-15	0	0	0	-1E-14	0	0
3	0	0	6.6E-15	0	-1E-14	0	0
3.333333333	0	0	0	0	0	-7E-15	0
3.666666667	0	0	0	0	1.3E-14	0	0
4	0	0	0	0	0	6.8E-15	0
4.333333333	0	0	-7E-15	0	0	0	0
4.666666667	0	0	0	0	1.3E-14	0	0
5	0	0	6.6E-15	0	0	-7E-15	0
5.333333333	0	0	0	0	-1E-14	0	0
5.666666667	0	0	-7E-15	0	0	0	0
6	0	0	6.6E-15	0	1.3E-14	0	0
6.333333333	0	0	0	0	-7E-15	6.8E-15	0
6.666666667	0	0	-1E-14	0	-1E-14	0	0

	%change FRP ver ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	-0.7962	-0.7436	-0.6887	-0.6902	-0.7363	-0.7186	-0.9274
1	-0.5972	-0.5577	-0.5165	-0.5177	-0.5522	-0.539	-0.6956
1.333333333	-0.3185	-0.2974	-0.2755	-0.2761	-0.2945	-0.2874	-0.371
1.666666667	-0.1991	-0.1859	-0.1722	-0.1726	-0.1841	-0.1797	-0.2319
2	-0.1365	-0.1275	-0.1181	-0.1183	-0.1262	-0.1232	-0.159
2.333333333	-0.0995	-0.0929	-0.0861	-0.0863	-0.092	-0.0898	-0.1159
2.666666667	-0.0758	-0.0708	-0.0656	-0.0657	-0.0701	-0.0684	-0.0883
3	-0.0597	-0.0558	-0.0516	-0.0518	-0.0552	-0.0539	-0.0696
3.333333333	-0.0483	-0.0451	-0.0417	-0.0418	-0.0446	-0.0436	-0.0562
3.666666667	-0.0398	-0.0372	-0.0344	-0.0345	-0.0368	-0.0359	-0.0464
4	-0.0334	-0.0312	-0.0289	-0.029	-0.0309	-0.0302	-0.0389
4.333333333	-0.0284	-0.0266	-0.0246	-0.0247	-0.0263	-0.0257	-0.0331
4.666666667	-0.0245	-0.0229	-0.0212	-0.0212	-0.0227	-0.0221	-0.0285
5	-0.0213	-0.0199	-0.0184	-0.0185	-0.0197	-0.0192	-0.0248
5.333333333	-0.0187	-0.0175	-0.0162	-0.0162	-0.0173	-0.0169	-0.0218
5.666666667	-0.0166	-0.0155	-0.0143	-0.0144	-0.0153	-0.015	-0.0193
6	-0.0148	-0.0138	-0.0128	-0.0128	-0.0137	-0.0133	-0.0172
6.333333333	-0.0133	-0.0124	-0.0115	-0.0115	-0.0123	-0.012	-0.0155
6.666666667	-0.0126	-0.0117	-0.0109	-0.0109	-0.0116	-0.0113	-0.0146

	%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	2.6E-14	0	0
1	0	0	0	0	0	0	0
1.333333333	0	0	0	0	-1E-14	0	0
1.666666667	0	0	0	0	0	0	0
2	-2E-14	0	0	0	1.3E-14	0	0
2.333333333	0	0	0	0	1.3E-14	0	0
2.666666667	2E-14	0	0	0	-1E-14	0	0
3	0	0	0	0	-1E-14	0	-7E-14
3.333333333	0	0	0	0	0	0	-7E-14
3.666666667	0	0	0	0	1.3E-14	0	0
4	0	0	0	0	0	0	6.6E-14
4.333333333	0	0	0	0	0	0	6.6E-14
4.666666667	0	0	0	0	1.3E-14	0	0
5	0	0	0	0	6.6E-15	0	0
5.333333333	0	0	0	0	-1E-14	0	0
5.666666667	0	0	0	0	-7E-15	0	0
6	-2E-14	0	0	0	1.3E-14	0	0
6.333333333	0	0	0	0	-1E-14	0	0
6.666666667	4E-14	0	0	0	-3E-14	0	0

Alpha_k

	%change FRP heave						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.1307	-0.129	-0.1486	-0.1497	-0.1616	-0.1777	-0.2407
0.5	-0.0986	-0.0977	-0.1132	-0.1144	-0.1236	-0.1362	-0.1849
0.666666667	-0.0531	-0.0533	-0.0629	-0.0642	-0.0695	-0.0771	-0.1054
0.833333333	-0.0328	-0.0334	-0.0404	-0.0416	-0.0452	-0.0504	-0.0693
1	-0.0218	-0.0227	-0.0281	-0.0293	-0.0319	-0.0358	-0.0496
1.166666667	-0.0152	-0.0162	-0.0207	-0.0218	-0.0238	-0.0269	-0.0375
1.333333333	-0.0109	-0.0119	-0.0158	-0.0168	-0.0185	-0.021	-0.0294
1.5	-0.0079	-0.009	-0.0124	-0.0134	-0.0148	-0.0169	-0.0238
1.666666667	-0.0058	-0.0069	-0.0099	-0.0109	-0.0121	-0.0139	-0.0197
1.833333333	-0.0042	-0.0053	-0.0081	-0.009	-0.0101	-0.0117	-0.0166
2	-0.003	-0.0041	-0.0067	-0.0075	-0.0085	-0.0099	-0.0142
2.166666667	-0.0021	-0.0032	-0.0055	-0.0064	-0.0072	-0.0085	-0.0123
2.333333333	-0.0014	-0.0024	-0.0046	-0.0054	-0.0062	-0.0074	-0.0108
2.5	-0.0008	-0.0018	-0.0039	-0.0047	-0.0054	-0.0065	-0.0095
2.666666667	-0.0003	-0.0013	-0.0033	-0.0041	-0.0047	-0.0057	-0.0085
2.833333333	7.1E-05	-0.0009	-0.0028	-0.0035	-0.0042	-0.0051	-0.0076
3	0.00039	-0.0006	-0.0024	-0.0031	-0.0037	-0.0046	-0.0068
3.166666667	0.00066	-0.0003	-0.0021	-0.0027	-0.0033	-0.0041	-0.0062
3.333333333	0.00078	-0.0002	-0.0019	-0.0025	-0.0031	-0.0039	-0.0059

	%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.1031	-0.192	-0.2767	-0.3087	-0.3307	-0.3487	-0.3657
0.5	-0.0196	-0.1044	-0.1854	-0.2161	-0.237	-0.2543	-0.2705
0.666666667	0.09717	0.01923	-0.0555	-0.0839	-0.1031	-0.1192	-0.1338
0.833333333	0.14587	0.07277	0.00252	-0.0242	-0.0422	-0.0573	-0.0707
1	0.16911	0.09968	0.03291	0.00742	-0.0096	-0.024	-0.0363
1.166666667	0.18081	0.11433	0.05037	0.02587	0.0097	-0.0042	-0.0157
1.333333333	0.18667	0.12262	0.06098	0.0373	0.02181	0.00845	-0.0024
1.5	0.18931	0.12732	0.06767	0.04469	0.02977	0.01683	0.00663
1.666666667	0.1901	0.12989	0.07196	0.04959	0.03517	0.02259	0.01291
1.833333333	0.18979	0.13114	0.07472	0.05287	0.0389	0.02664	0.01741
2	0.18879	0.13153	0.07646	0.05508	0.0415	0.02954	0.02071
2.166666667	0.18739	0.13137	0.0775	0.05656	0.04332	0.03162	0.02316
2.333333333	0.18573	0.13083	0.07807	0.0575	0.04459	0.03312	0.02499
2.5	0.18392	0.13005	0.07829	0.05807	0.04546	0.0342	0.02636
2.666666667	0.18202	0.1291	0.07826	0.05837	0.04602	0.03496	0.02739
2.833333333	0.18009	0.12804	0.07806	0.05847	0.04637	0.03549	0.02817
3	0.17814	0.12691	0.07773	0.05841	0.04654	0.03583	0.02874
3.166666667	0.17621	0.12573	0.0773	0.05823	0.04658	0.03603	0.02915
3.333333333	0.17525	0.12514	0.07706	0.05812	0.04658	0.0361	0.02932

	%change FRP ver ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.3173	-0.3302	-0.3344	-0.3461	-0.342	-0.3314	-0.3103
0.5	-0.2441	-0.2561	-0.2617	-0.2713	-0.2684	-0.2612	-0.2459
0.666666667	-0.138	-0.1485	-0.1558	-0.1625	-0.1613	-0.1587	-0.1512
0.833333333	-0.0875	-0.0969	-0.1044	-0.1098	-0.1095	-0.1086	-0.1041
1	-0.059	-0.0677	-0.0751	-0.0797	-0.0799	-0.0798	-0.0769
1.166666667	-0.0413	-0.0493	-0.0565	-0.0607	-0.0611	-0.0615	-0.0595
1.333333333	-0.0296	-0.037	-0.044	-0.0478	-0.0484	-0.049	-0.0477
1.5	-0.0214	-0.0284	-0.035	-0.0385	-0.0393	-0.0401	-0.0393
1.666666667	-0.0154	-0.022	-0.0284	-0.0317	-0.0325	-0.0335	-0.033
1.833333333	-0.0109	-0.0172	-0.0234	-0.0265	-0.0274	-0.0284	-0.0281
2	-0.0075	-0.0135	-0.0194	-0.0224	-0.0233	-0.0244	-0.0243
2.166666667	-0.0048	-0.0105	-0.0163	-0.0191	-0.0201	-0.0212	-0.0213
2.333333333	-0.0027	-0.0082	-0.0138	-0.0165	-0.0175	-0.0186	-0.0188
2.5	-0.001	-0.0063	-0.0117	-0.0143	-0.0153	-0.0164	-0.0167
2.666666667	0.00036	-0.0047	-0.0099	-0.0125	-0.0135	-0.0146	-0.015
2.833333333	0.00149	-0.0034	-0.0085	-0.0109	-0.012	-0.0131	-0.0135
3	0.00242	-0.0023	-0.0073	-0.0096	-0.0107	-0.0118	-0.0123
3.166666667	0.00319	-0.0013	-0.0062	-0.0085	-0.0096	-0.0107	-0.0112
3.333333333	0.00355	-0.0009	-0.0057	-0.008	-0.009	-0.0102	-0.0107

	%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-1.7262	-1.5109	-1.0862	-0.8001	-0.5907	-0.3477	-0.0453
0.5	-1.4901	-1.3324	-0.9862	-0.7382	-0.5509	-0.3281	-0.0434
0.666666667	-1.1092	-1.0373	-0.8153	-0.6307	-0.4811	-0.2935	-0.04
0.833333333	-0.868	-0.8394	-0.692	-0.5501	-0.4276	-0.2664	-0.0371
1	-0.7036	-0.6986	-0.5989	-0.4873	-0.3851	-0.2445	-0.0346
1.166666667	-0.5854	-0.594	-0.5262	-0.437	-0.3504	-0.2263	-0.0324
1.333333333	-0.497	-0.5134	-0.4679	-0.3956	-0.3215	-0.2109	-0.03
1.5	-0.4288	-0.4498	-0.4202	-0.3611	-0.297	-0.1976	-0.0263
1.666666667	-0.3747	-0.3984	-0.3805	-0.3317	-0.2758	-0.1861	-0.008
1.833333333	-0.3311	-0.3562	-0.3469	-0.3065	-0.2574	-0.1758	-0.0493
2	-0.2953	-0.321	-0.3182	-0.2845	-0.2412	-0.1667	-0.0575
2.166666667	-0.2654	-0.2912	-0.2934	-0.2653	-0.2269	-0.1586	-0.0149
2.333333333	-0.2401	-0.2658	-0.2717	-0.2483	-0.2141	-0.1512	-0.0221
2.5	-0.2186	-0.2439	-0.2527	-0.2331	-0.2026	-0.1445	-0.0229
2.666666667	-0.2001	-0.2248	-0.2359	-0.2196	-0.1922	-0.1384	-0.0228
2.833333333	-0.184	-0.2081	-0.2209	-0.2074	-0.1828	-0.1328	-0.0225
3	-0.1699	-0.1934	-0.2075	-0.1964	-0.1742	-0.1276	-0.022
3.166666667	-0.1575	-0.1803	-0.1955	-0.1863	-0.1663	-0.1228	-0.0216
3.333333333	-0.1517	-0.1741	-0.1898	-0.1816	-0.1626	-0.1206	-0.0213

Figure A.12: $Anis_c$ and $Anis_k$

k_0

Times original value	%change FRP heave						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.06726	0.06328	0.06157	0.06141	0.0631	0.0676	0.08998
0.4	0.07041	0.06527	0.06242	0.06161	0.06296	0.06704	0.08849
0.7	0.07638	0.06913	0.06412	0.06212	0.06279	0.06607	0.08578
1	0.08169	0.07271	0.06573	0.06272	0.06275	0.06524	0.08336
1.3	0.08638	0.07593	0.06717	0.06325	0.06267	0.06442	0.08103
1.6	0.0905	0.07879	0.06844	0.06367	0.06252	0.06358	0.07874
1.9	0.09408	0.08132	0.06954	0.064	0.06231	0.06271	0.07649
2.2	0.09717	0.08353	0.07049	0.06424	0.06206	0.06183	0.07429
2.5	0.09981	0.08546	0.0713	0.0644	0.06176	0.06095	0.07214
2.8	0.10203	0.08712	0.07198	0.0645	0.06142	0.06006	0.07005
3.1	0.10387	0.08853	0.07254	0.06454	0.06105	0.05917	0.06803
3.4	0.10537	0.08972	0.073	0.06452	0.06066	0.05829	0.06607
3.7	0.10655	0.0907	0.07336	0.06445	0.06025	0.05742	0.06417
4	0.10745	0.09148	0.07363	0.06434	0.05983	0.05657	0.06235
4.3	0.10808	0.09209	0.07382	0.0642	0.05939	0.05573	0.06058
4.6	0.10849	0.09254	0.07394	0.06402	0.05894	0.0549	0.05889
4.9	0.10864	0.09272	0.07398	0.06392	0.05871	0.0545	0.05805

Times original value	%change FRP hor ero						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.9735	0.85341	0.74142	0.70083	0.67148	0.64731	0.61655
0.4	1.01228	0.87619	0.74876	0.70169	0.66861	0.64089	0.60661
0.7	1.08538	0.92031	0.76407	0.7047	0.66452	0.62993	0.58881
1	1.14864	0.96023	0.77913	0.70872	0.66207	0.62108	0.57353
1.3	1.20146	0.99465	0.79227	0.71195	0.65941	0.61248	0.55891
1.6	1.24467	1.02377	0.80337	0.71419	0.6563	0.60387	0.54464
1.9	1.27925	1.04803	0.81257	0.7155	0.65274	0.59523	0.53067
2.2	1.30614	1.06786	0.82003	0.71597	0.64879	0.58658	0.51703
2.5	1.32624	1.0837	0.82592	0.7157	0.64453	0.57797	0.50372
2.8	1.34034	1.09593	0.83038	0.71478	0.64001	0.56944	0.49078
3.1	1.3492	1.10494	0.83357	0.71328	0.63528	0.56102	0.47821
3.4	1.35347	1.11104	0.83559	0.71129	0.6304	0.55273	0.46602
3.7	1.35374	1.11455	0.83657	0.70885	0.62539	0.54459	0.45422
4	1.35057	1.11574	0.83659	0.70603	0.62028	0.53662	0.4428
4.3	1.34441	1.11487	0.83577	0.70285	0.61511	0.52883	0.43177
4.6	1.3357	1.11218	0.83416	0.69938	0.60989	0.52121	0.4211
4.9	1.33075	1.1104	0.83318	0.69757	0.60727	0.51744	0.41586

Times original value	%change FRP ver ero						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.29507	0.26652	0.23207	0.22511	0.20919	0.20215	0.18562
0.4	0.29941	0.26754	0.23014	0.22116	0.20597	0.19738	0.18028
0.7	0.30507	0.26831	0.2264	0.21404	0.1996	0.18855	0.17039
1	0.30564	0.26684	0.22246	0.20784	0.19327	0.18063	0.16158
1.3	0.3026	0.26361	0.21783	0.2018	0.18695	0.17305	0.15325
1.6	0.29689	0.25893	0.21258	0.19579	0.18067	0.16573	0.14527
1.9	0.28927	0.25314	0.20682	0.18982	0.17452	0.15866	0.13756
2.2	0.28031	0.24651	0.20059	0.18392	0.16852	0.15182	0.13004
2.5	0.27048	0.23929	0.19387	0.17812	0.1627	0.14521	0.12254
2.8	0.26014	0.23169	0.18649	0.17243	0.15709	0.1388	0.11476
3.1	0.24957	0.22386	0.17796	0.16689	0.15169	0.13255	0.10612
3.4	0.23896	0.21593	0.16686	0.1615	0.1465	0.12639	0.09511
3.7	0.22847	0.20801	0.14806	0.15628	0.14152	0.12024	0.07707
4	0.2182	0.20018	0.08803	0.15122	0.13676	0.11397	0.02808
4.3	0.20825	0.19248	0.67568	0.14635	0.13221	0.10739	-1.3171
4.6	0.19865	0.18498	0.34738	0.14165	0.12785	0.10016	0.33778
4.9	0.19394	0.18127	-0.6163	0.13934	0.12572	0.09634	3.30051

Times original value	%change FRP ver tra						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-2.6176	-2.2243	-1.5311	-1.0987	-0.7978	-0.4615	-0.0613
0.4	-1.676	-1.4913	-1.0874	-0.8049	-0.5964	-0.3527	-0.0444
0.7	-0.5027	-0.548	-0.4922	-0.4015	-0.3159	-0.1987	-0.0266
1	-0.1821	-0.2506	-0.2711	-0.2392	-0.1974	-0.1305	-0.0214
1.3	-0.0524	-0.12	-0.1643	-0.1571	-0.1358	-0.094	-0.0153
1.6	0.00976	-0.0527	-0.105	-0.1098	-0.0995	-0.072	-0.0122
1.9	0.04206	-0.0147	-0.069	-0.0801	-0.0763	-0.0578	-0.0102
2.2	0.05943	0.0079	-0.0457	-0.0604	-0.0605	-0.048	-0.0089
2.5	0.06866	0.02186	-0.0299	-0.0466	-0.0494	-0.0409	-0.0079
2.8	0.07323	0.03059	-0.019	-0.0366	-0.0411	-0.0357	-0.0072
3.1	0.075	0.03604	-0.0111	-0.0292	-0.0349	-0.0316	-0.0066
3.4	0.07508	0.03935	-0.0054	-0.0235	-0.0301	-0.0285	-0.0062
3.7	0.0741	0.04125	-0.0012	-0.0192	-0.0263	-0.0259	-0.0059
4	0.07248	0.04218	0.00187	-0.0157	-0.0232	-0.0238	-0.0057
4.3	0.07049	0.04246	0.00408	-0.013	-0.0207	-0.0221	-0.0055
4.6	0.06827	0.04229	0.0055	-0.0108	-0.0186	-0.0206	-0.0053
4.9	0.06713	0.04211	0.006	-0.0098	-0.0176	-0.02	-0.0053

k_1

	%change FRP heave						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.36	-0.3536	-0.3609	-0.3674	-0.3807	-0.4115	-0.5415
0.4	-0.2188	-0.215	-0.2196	-0.2236	-0.2317	-0.2506	-0.3306
0.7	-0.0578	-0.0569	-0.0582	-0.0594	-0.0617	-0.0669	-0.0892
1	-0.0309	-0.0304	-0.0312	-0.0318	-0.033	-0.0358	-0.0479
1.3	-0.0204	-0.0201	-0.0206	-0.021	-0.0218	-0.0237	-0.0317
1.6	-0.015	-0.0148	-0.0151	-0.0154	-0.016	-0.0174	-0.0234
1.9	-0.0118	-0.0116	-0.0118	-0.0121	-0.0125	-0.0136	-0.0183
2.2	-0.0096	-0.0094	-0.0096	-0.0098	-0.0102	-0.0111	-0.0149
2.5	-0.008	-0.0079	-0.0081	-0.0082	-0.0086	-0.0093	-0.0125
2.8	-0.0069	-0.0068	-0.0069	-0.007	-0.0073	-0.0079	-0.0107
3.1	-0.006	-0.0059	-0.006	-0.0061	-0.0064	-0.0069	-0.0093
3.4	-0.0053	-0.0052	-0.0053	-0.0054	-0.0056	-0.0061	-0.0082
3.7	-0.0047	-0.0046	-0.0047	-0.0048	-0.005	-0.0054	-0.0073
4	-0.0042	-0.0041	-0.0042	-0.0043	-0.0045	-0.0048	-0.0065
4.3	-0.0038	-0.0037	-0.0038	-0.0039	-0.004	-0.0044	-0.0059
4.6	-0.0034	-0.0034	-0.0035	-0.0035	-0.0037	-0.004	-0.0053
4.9	-0.0033	0.06624	-0.0033	0.06216	-0.0035	0.06404	-0.0051

	%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-4.7697	-4.4429	-4.1139	-3.9983	-3.8998	-3.8255	-3.7226
0.4	-2.9078	-2.7079	-2.5066	-2.436	-2.3757	-2.3303	-2.2671
0.7	-0.7814	-0.7265	-0.6713	-0.652	-0.6354	-0.623	-0.6053
1	-0.4225	-0.3923	-0.3619	-0.3513	-0.3422	-0.3353	-0.3256
1.3	-0.2818	-0.2613	-0.2407	-0.2336	-0.2274	-0.2228	-0.2161
1.6	-0.2084	-0.1931	-0.1777	-0.1723	-0.1677	-0.1642	-0.1592
1.9	-0.1638	-0.1516	-0.1394	-0.1351	-0.1315	-0.1287	-0.1248
2.2	-0.134	-0.1239	-0.1138	-0.1103	-0.1073	-0.105	-0.1018
2.5	-0.1126	-0.1041	-0.0956	-0.0926	-0.09	-0.0881	-0.0853
2.8	-0.0966	-0.0892	-0.0819	-0.0793	-0.0771	-0.0755	-0.0731
3.1	-0.0842	-0.0777	-0.0713	-0.069	-0.0671	-0.0656	-0.0635
3.4	-0.0742	-0.0685	-0.0628	-0.0608	-0.0591	-0.0578	-0.0559
3.7	-0.0661	-0.061	-0.0559	-0.0541	-0.0526	-0.0514	-0.0498
4	-0.0594	-0.0547	-0.0501	-0.0485	-0.0472	-0.0461	-0.0446
4.3	-0.0537	-0.0495	-0.0453	-0.0438	-0.0426	-0.0417	-0.0403
4.6	-0.0488	-0.045	-0.0412	-0.0399	-0.0387	-0.0379	-0.0366
4.9	-0.0466	0.96131	-0.0393	0.74961	-0.0369	0.68382	-0.0349

	%change FRP ver ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-1.1812	-1.1331	-1.0607	-1.0592	-1.0313	-0.9879	-0.8889
0.4	-0.7364	-0.7054	-0.6605	-0.6583	-0.6405	-0.6147	-0.5572
0.7	-0.2196	-0.209	-0.1958	-0.1934	-0.1878	-0.1816	-0.1694
1	-0.1212	-0.1151	-0.1076	-0.1062	-0.103	-0.0996	-0.0929
1.3	-0.0815	-0.0773	-0.0722	-0.0713	-0.0691	-0.0668	-0.0622
1.6	-0.0607	-0.0574	-0.0536	-0.0529	-0.0512	-0.0495	-0.0461
1.9	-0.0479	-0.0453	-0.0422	-0.0417	-0.0403	-0.0389	-0.0363
2.2	-0.0393	-0.0371	-0.0346	-0.0341	-0.033	-0.0319	-0.0297
2.5	-0.0331	-0.0313	-0.0291	-0.0287	-0.0278	-0.0268	-0.025
2.8	-0.0285	-0.0269	-0.025	-0.0247	-0.0239	-0.023	-0.0214
3.1	-0.0249	-0.0235	-0.0218	-0.0215	-0.0208	-0.0201	-0.0187
3.4	-0.022	-0.0207	-0.0193	-0.019	-0.0184	-0.0177	-0.0165
3.7	-0.0196	-0.0185	-0.0172	-0.0169	-0.0164	-0.0158	-0.0147
4	-0.0176	-0.0166	-0.0154	-0.0152	-0.0147	-0.0142	-0.0132
4.3	-0.016	-0.0151	-0.014	-0.0138	-0.0133	-0.0128	-0.0119
4.6	-0.0146	-0.0137	-0.0127	-0.0126	-0.0121	-0.0117	-0.0108
4.9	-0.0139	0.39283	-0.0121	0.33751	-0.0116	0.37471	-0.0104

	%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-1.6933	-1.4772	-1.076	-0.8029	-0.5993	-0.3572	-0.047
0.4	-1.1554	-1.0225	-0.7603	-0.5742	-0.4322	-0.26	-0.0346
0.7	-0.4924	-0.4564	-0.3619	-0.2832	-0.2185	-0.1351	-0.0186
1	-0.3138	-0.2966	-0.242	-0.1926	-0.1504	-0.0943	-0.013
1.3	-0.2313	-0.2213	-0.184	-0.1482	-0.1167	-0.0738	-0.01
1.6	-0.1838	-0.1775	-0.1497	-0.1216	-0.0964	-0.0615	-0.0079
1.9	-0.1528	-0.1486	-0.1268	-0.1038	-0.0827	-0.053	-0.0056
2.2	-0.1308	-0.128	-0.1102	-0.0908	-0.0726	-0.0469	0.00237
2.5	-0.1142	-0.1123	-0.0976	-0.0808	-0.0649	-0.0421	-0.0215
2.8	-0.1012	-0.1	-0.0875	-0.0728	-0.0587	-0.0382	-0.0212
3.1	-0.0907	-0.09	-0.0793	-0.0662	-0.0536	-0.035	0.00383
3.4	-0.082	-0.0817	-0.0724	-0.0607	-0.0493	-0.0323	-0.0027
3.7	-0.0747	-0.0747	-0.0665	-0.056	-0.0456	-0.03	-0.0035
4	-0.0684	-0.0686	-0.0614	-0.0519	-0.0423	-0.028	-0.0037
4.3	-0.063	-0.0634	-0.057	-0.0483	-0.0395	-0.0262	-0.0036
4.6	-0.0583	-0.0588	-0.0531	-0.0451	-0.037	-0.0246	-0.0035
4.9	-0.0561	2.46922	-0.0512	3.37903	-0.0358	4.01344	-0.0034

Figure A.13: k_0 and k_1

k_2

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0363	-0.0356	-0.0363	-0.0369	-0.0383	-0.0415	-0.0553	-0.5155	-0.4746	-0.4341	-0.4199	-0.408	-0.3989	-0.3858	-0.1363	-0.1293	-0.1207	-0.1191	-0.1154	-0.1115	-0.1038	-0.3413	-0.3145	-0.2476	-0.1931	-0.1486	-0.0917	-0.0126
0.4	-0.0275	-0.027	-0.0276	-0.028	-0.0291	-0.0315	-0.042	-0.3913	-0.3602	-0.3294	-0.3186	-0.3095	-0.3026	-0.2927	-0.1042	-0.0988	-0.0922	-0.0909	-0.0881	-0.0851	-0.0792	-0.2676	-0.2476	-0.1962	-0.1536	-0.1185	-0.0733	-0.0101
0.7	-0.0164	-0.0161	-0.0164	-0.0167	-0.0173	-0.0187	-0.025	-0.2324	-0.2138	-0.1955	-0.189	-0.1837	-0.1795	-0.1736	-0.063	-0.0597	-0.0556	-0.0548	-0.0531	-0.0513	-0.0477	-0.173	-0.1617	-0.1302	-0.1029	-0.0799	-0.0498	-0.0069
1	-0.0125	-0.0122	-0.0125	-0.0127	-0.0132	-0.0143	-0.0191	-0.1772	-0.1631	-0.149	-0.1441	-0.14	-0.1368	-0.1323	-0.0485	-0.0459	-0.0428	-0.0422	-0.0408	-0.0394	-0.0367	-0.139	-0.1308	-0.1064	-0.0846	-0.066	-0.0414	-0.0057
1.3	-0.01	-0.0098	-0.01	-0.0102	-0.0106	-0.0115	-0.0154	-0.1425	-0.1311	-0.1198	-0.1158	-0.1125	-0.11	-0.1063	-0.0393	-0.0372	-0.0346	-0.0341	-0.033	-0.0319	-0.0297	-0.1164	-0.1102	-0.0904	-0.0722	-0.0566	-0.0356	-0.0049
1.6	-0.0083	-0.0081	-0.0083	-0.0084	-0.0088	-0.0095	-0.0127	-0.1179	-0.1084	-0.0991	-0.0958	-0.093	-0.0909	-0.0879	-0.0327	-0.031	-0.0288	-0.0284	-0.0275	-0.0265	-0.0247	-0.0996	-0.0947	-0.0782	-0.0628	-0.0493	-0.0312	-0.0043
1.9	-0.007	-0.0069	-0.007	-0.0071	-0.0074	-0.008	-0.0107	-0.0994	-0.0914	-0.0835	-0.0807	-0.0784	-0.0767	-0.0741	-0.0278	-0.0262	-0.0244	-0.024	-0.0233	-0.0224	-0.0209	-0.0864	-0.0825	-0.0686	-0.0553	-0.0435	-0.0276	-0.0038
2.2	-0.006	-0.0059	-0.006	-0.0061	-0.0063	-0.0069	-0.0092	-0.085	-0.0782	-0.0715	-0.0691	-0.0671	-0.0656	-0.0634	-0.0239	-0.0225	-0.021	-0.0206	-0.02	-0.0193	-0.0179	-0.0757	-0.0726	-0.0607	-0.0491	-0.0387	-0.0246	-0.0033
2.5	-0.0052	-0.0051	-0.0052	-0.0053	-0.0055	-0.0059	-0.0079	-0.0737	-0.0677	-0.0619	-0.0598	-0.0581	-0.0568	-0.0549	-0.0208	-0.0196	-0.0182	-0.0179	-0.0174	-0.0168	-0.0156	-0.067	-0.0644	-0.0541	-0.0439	-0.0347	-0.0221	-0.0029
2.8	-0.0045	-0.0044	-0.0045	-0.0046	-0.0048	-0.0052	-0.007	-0.0645	-0.0593	-0.0541	-0.0524	-0.0508	-0.0497	-0.048	-0.0182	-0.0172	-0.016	-0.0157	-0.0152	-0.0147	-0.0137	-0.0597	-0.0576	-0.0486	-0.0395	-0.0313	-0.02	-0.0026
3.1	-0.004	-0.0039	-0.004	-0.0041	-0.0042	-0.0046	-0.0061	-0.0569	-0.0523	-0.0478	-0.0462	-0.0449	-0.0439	-0.0424	-0.0161	-0.0152	-0.0142	-0.0139	-0.0135	-0.013	-0.0121	-0.0536	-0.0518	-0.0439	-0.0358	-0.0284	-0.0182	-0.0023
3.4	-0.0036	-0.0035	-0.0036	-0.0036	-0.0038	-0.0041	-0.0055	-0.0506	-0.0465	-0.0425	-0.0411	-0.0399	-0.039	-0.0377	-0.0144	-0.0136	-0.0126	-0.0124	-0.012	-0.0116	-0.0108	-0.0484	-0.0469	-0.0398	-0.0325	-0.0259	-0.0166	-0.002
3.7	-0.0032	-0.0031	-0.0032	-0.0032	-0.0034	-0.0037	-0.0049	-0.0453	-0.0417	-0.0381	-0.0368	-0.0357	-0.0349	-0.0338	-0.0129	-0.0122	-0.0113	-0.0111	-0.0108	-0.0104	-0.0097	-0.0439	-0.0426	-0.0363	-0.0297	-0.0237	-0.0152	-0.0017
4	-0.0029	-0.0028	-0.0029	-0.0029	-0.003	-0.0033	-0.0044	-0.0408	-0.0375	-0.0343	-0.0331	-0.0322	-0.0315	-0.0304	-0.0117	-0.011	-0.0102	-0.0101	-0.0097	-0.0094	-0.0087	-0.04	-0.0389	-0.0333	-0.0273	-0.0218	-0.014	-0.0014
4.3	-0.0026	-0.0025	-0.0026	-0.0026	-0.0027	-0.003	-0.004	-0.037	-0.034	-0.031	-0.03	-0.0291	-0.0285	-0.0275	-0.0106	-0.01	-0.0093	-0.0091	-0.0088	-0.0085	-0.0079	-0.0366	-0.0357	-0.0306	-0.0251	-0.0201	-0.0129	-0.0011
4.6	-0.0024	-0.0023	-0.0024	-0.0024	-0.0025	-0.0027	-0.0036	-0.0336	-0.0309	-0.0282	-0.0273	-0.0265	-0.0259	-0.025	-0.0096	-0.0091	-0.0084	-0.0083	-0.008	-0.0078	-0.0072	-0.0337	-0.0328	-0.0282	-0.0232	-0.0186	-0.012	-0.0007
4.9	-0.0023	-0.0022	-0.0023	-0.0023	-0.0024	-0.0026	-0.0035	-0.0321	-0.0295	-0.0269	-0.026	-0.0253	-0.0247	-0.0239	-0.0092	-0.0087	-0.0081	-0.0079	-0.0077	-0.0074	-0.0069	-0.0323	-0.0315	-0.0271	-0.0223	-0.0179	-0.0115	-0.0005

k_3

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0255	-0.025	-0.0255	-0.0259	-0.0269	-0.0291	-0.0388	-0.3608	-0.3321	-0.3036	-0.2936	-0.2853	-0.2788	-0.2697	-0.0961	-0.0911	-0.085	-0.0838	-0.0812	-0.0784	-0.073	-0.2447	-0.2257	-0.1781	-0.1392	-0.1072	-0.0662	-0.0091
0.4	-0.022	-0.0216	-0.0221	-0.0224	-0.0233	-0.0252	-0.0336	-0.3124	-0.2875	-0.2628	-0.2542	-0.2469	-0.2414	-0.2334	-0.0837	-0.0793	-0.074	-0.0729	-0.0706	-0.0682	-0.0635	-0.2185	-0.2025	-0.1608	-0.1261	-0.0974	-0.0603	-0.0083
0.7	-0.0164	-0.0161	-0.0164	-0.0167	-0.0173	-0.0188	-0.0251	-0.233	-0.2144	-0.1959	-0.1894	-0.184	-0.1799	-0.1739	-0.0632	-0.0599	-0.0558	-0.055	-0.0532	-0.0514	-0.0478	-0.174	-0.1626	-0.1308	-0.1034	-0.0803	-0.05	-0.0069
1	-0.0127	-0.0125	-0.0127	-0.013	-0.0135	-0.0146	-0.0195	-0.1808	-0.1663	-0.152	-0.1469	-0.1427	-0.1395	-0.1349	-0.0495	-0.0469	-0.0437	-0.043	-0.0416	-0.0402	-0.0374	-0.1421	-0.1337	-0.1087	-0.0864	-0.0674	-0.0422	-0.0058
1.3	-0.0102	-0.01	-0.0102	-0.0104	-0.0107	-0.0117	-0.0156	-0.1445	-0.1329	-0.1214	-0.1174	-0.114	-0.1115	-0.1078	-0.0399	-0.0377	-0.0351	-0.0346	-0.0335	-0.0323	-0.0301	-0.1183	-0.112	-0.0918	-0.0734	-0.0575	-0.0362	-0.005
1.6	-0.0083	-0.0082	-0.0083	-0.0085	-0.0088	-0.0095	-0.0127	-0.1182	-0.1087	-0.0993	-0.096	-0.0933	-0.0911	-0.0881	-0.0328	-0.031	-0.0289	-0.0285	-0.0275	-0.0266	-0.0247	-0.1001	-0.0952	-0.0786	-0.0631	-0.0496	-0.0313	-0.0043
1.9	-0.0069	-0.0068	-0.0069	-0.0071	-0.0073	-0.0079	-0.0106	-0.0985	-0.0906	-0.0827	-0.08	-0.0777	-0.0759	-0.0734	-0.0275	-0.026	-0.0242	-0.0238	-0.0231	-0.0222	-0.0207	-0.0858	-0.0819	-0.0681	-0.0549	-0.0432	-0.0274	-0.0037
2.2	-0.0059	-0.0057	-0.0059	-0.006	-0.0062	-0.0067	-0.009	-0.0833	-0.0766	-0.07	-0.0677	-0.0657	-0.0643	-0.0621	-0.0234	-0.0221	-0.0205	-0.0202	-0.0196	-0.0189	-0.0176	-0.0743	-0.0713	-0.0595	-0.0482	-0.038	-0.0242	-0.0032
2.5	-0.005	-0.0049	-0.005	-0.0051	-0.0053	-0.0058	-0.0077	-0.0715	-0.0657	-0.06	-0.058	-0.0564	-0.0551	-0.0532	-0.0201	-0.019	-0.0177	-0.0174	-0.0168	-0.0162	-0.0151	-0.0651	-0.0626	-0.0525	-0.0426	-0.0337	-0.0215	-0.0028
2.8	-0.0044	-0.0043	-0.0044	-0.0044	-0.0046	-0.005	-0.0067	-0.0619	-0.057	-0.052	-0.0503	-0.0489	-0.0477	-0.0462	-0.0175	-0.0165	-0.0154	-0.0151	-0.0146	-0.0141	-0.0131	-0.0574	-0.0554	-0.0467	-0.038	-0.0301	-0.0192	-0.0025
3.1	-0.0038	-0.0037	-0.0038	-0.0039	-0.004	-0.0044	-0.0059	-0.0542	-0.0499	-0.0455	-0.044	-0.0428	-0.0418	-0.0404	-0.0154	-0.0145	-0.0135	-0.0133	-0.0128	-0.0124	-0.0115	-0.0511	-0.0494	-0.0418	-0.034	-0.027	-0.0173	-0.0022
3.4	-0.0034	-0.0033	-0.0034	-0.0034	-0.0036	-0.0039	-0.0052	-0.0479	-0.044	-0.0402	-0.0389	-0.0377	-0.0369	-0.0357	-0.0136	-0.0128	-0.0119	-0.0117	-0.0114	-0.011	-0.0102	-0.0457	-0.0443	-0.0376	-0.0307	-0.0244	-0.0157	-0.0019
3.7	-0.003	-0.0029	-0.003	-0.003	-0.0032	-0.0034	-0.0046	-0.0426	-0.0391	-0.0357	-0.0346	-0.0336	-0.0328	-0.0317	-0.0121	-0.0114	-0.0106	-0.0105	-0.0101	-0.0098	-0.0091	-0.0412	-0.0399	-0.034	-0.0278	-0.0222	-0.0142	-0.0016
4	-0.0027	-0.0026	-0.0027	-0.0027	-0.0028	-0.0031	-0.0041	-0.0381	-0.035	-0.032	-0.0309	-0.03	-0.0294	-0.0284	-0.0109	-0.0103	-0.0095	-0.0094	-0.0091	-0.0088	-0.0081	-0.0373	-0.0362	-0.0309	-0.0253	-0.0202	-0.013	-0.0013
4.3	-0.0024	-0.0024	-0.0024	-0.0025	-0.0025	-0.0028	-0.0037	-0.0343	-0.0315	-0.0288	-0.0278	-0.027	-0.0264	-0.0255	-0.0098	-0.0093	-0.0086	-0.0085	-0.0082	-0.0079	-0.0073	-0.0339	-0.033	-0.0282	-0.0232	-0.0185	-0.0119	-0.0011
4.6	-0.0022	-0.0021	-0.0022	-0.0022	-0.0023	-0.0025	-0.0034	-0.0311	-0.0285	-0.0261	-0.0252	-0.0245	-0.0239	-0.0231	-0.0089	-0.0084	-0.0078	-0.0077	-0.0074	-0.0072	-0.0066	-0.0309	-0.0302	-0.0259	-0.0213	-0.017	-0.011	-0.0008
4.9	-0.0021	-0.002	-0.0021	-0.0021	-0.0022	-0.0024	-0.0032	-0.0295	-0.0272	-0.0248	-0.024	-0.0233	-0.0228	-0.022	-0.0085	-0.008	-0.0074	-0.0073	-0.0071	-0.0068	-0.0063	-0.0296	-0.0289	-0.0248	-0.0204	-0.0163	-0.0105	-0.0007

D_0

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	2.18931	1.76316	1.36565	1.24562	1.10694	0.90149	0.46279	0.73531	0.59943	0.43856	0.38079	0.34949	0.33523	0.28786	0.23406	0.19129	0.15232	0.13683	0.12943	-0.0171	0.10497	-0.1988	-0.29	-0.3108	-0.2733	-0.2202	-0.1452	-0.0138
0.833333333	-1.0356	-0.8438	-0.5497	-0.553	-0.5031	-0.407	-0.1749	0.80082	0.61853	0.46484	0.40405	0.35579	0.33006	0.29125	0.17265	0.19792	0.15861	0.09264	0.12978	0.05086	0.10792	-0.0535	-0.1474	-0.1957	-0.1844	-0.1539	-0.1024	-0.0109
1.25	-2.1259	-1.7164	-1.2267	-1.169	-1.0493	-0.8495	-0.3919	0.88573	0.68913	0.49525	0.43488	0.3817	0.33561	0.29095	0.17867	0.14398	0.1605	0.08272	0.01194	0.12603	0.10719	0.13055	0.03974	-0.0399	-0.0598	-0.0609	-0.0447	-0.0068
1.666666667	1.02232	0.83328	0.61243	0.57313	0.5164	0.42442	0.22883	0.9751	0.75162	0.54282	0.44722	0.41233	0.34835	0.30005	0.2587	0.1496	0.10365	0.13293	0.0751	0.1288	0.04345	0.1864	0.10915	0.02509	-0.0036	-0.0157	-0.0185	-0.0049
2.083333333	2.10609	1.70002	1.30457	1.18903	1.0603	0.86377	0.44639	1.08701	0.83398	0.5981	0.47888	0.43518	0.38425	0.32233	0.27099	0.22488	0.11576	0.15399	0.20191	0.06124	0.04777	0.21149	0.14648	0.06675	0.032	0.01481	0.00073	-0.0015
2.5	-0.8718	-0.7123	-0.5161	-0.4888	-0.4372	-0.3479	-0.1402	1.24216	0.93297	0.66461	0.55763	0.47203	0.42549	0.35579	0.21407	0.23837	0.19383	0.12258	0.15175	0.06902	0.12251	0.2224	0.16646	0.0945	0.05675	0.03536	0.01563	0.00213
2.916666667	-1.9488	-1.5687	-1.2002	-1.0964	-0.9727	-0.7789	-0.3476	1.40693	1.0861	0.75899	0.65586	0.56393	0.48462	0.40787	0.22555	0.19576	0.20771	0.12523	0.19109	0.1591	0.13361	0.23033	0.18078	0.11364	0.07619	0.05167	0.02813	0.00349
3.333333333	0.02224	0.03856	0.02784	0.03029	0.03145	0.0332	0.04913	1.45847	1.2116	0.79867	0.70224	0.63082	0.53641	0.43599	0.29365	0.14872	0.20809	0.16442	0.22605	0.17833	0.13435	0.23539	0.18758	0.12251	0.0859	0.05948	0.03371	0.00397

Figure A.14: k_2, k_3 and D_0

D_1

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.02348	0.02308	0.02361	0.02406	0.02498	0.02712	0.03637
0.357142857	0.02123	0.02085	0.02132	0.02173	0.02255	0.02448	0.0328
0.535714286	0.01776	0.01743	0.01781	0.01813	0.01882	0.02041	0.02732
0.714285714	0.01558	0.01529	0.01561	0.0159	0.01649	0.01789	0.02391
0.892857143	0.01402	0.01375	0.01404	0.01429	0.01483	0.01608	0.02148
1.071428571	0.01366	0.0134	0.01368	0.01392	0.01444	0.01565	0.0209
1.25	0.01371	0.01345	0.01373	0.01397	0.01449	0.01571	0.02097
1.428571429	0.01353	0.01327	0.01354	0.01378	0.01429	0.01549	0.02067

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.32321	0.29923	0.27528	0.26692	0.25979	0.2544	0.24669
0.357142857	0.29454	0.27226	0.25008	0.24233	0.23575	0.23076	0.22361
0.535714286	0.2501	0.23057	0.2112	0.20443	0.19872	0.19438	0.18813
0.714285714	0.22086	0.2034	0.1861	0.18004	0.17495	0.17107	0.16549
0.892857143	0.1989	0.18311	0.16747	0.16199	0.15739	0.15388	0.14883
1.071428571	0.19397	0.17852	0.16323	0.15787	0.15338	0.14995	0.14501
1.25	0.19488	0.17934	0.16395	0.15855	0.15404	0.15058	0.14561
1.428571429	0.19222	0.17689	0.16171	0.15638	0.15193	0.14852	0.14361

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.09533	0.0901	0.08393	0.08272	0.08011	0.07734	0.07205
0.357142857	0.08565	0.08095	0.07538	0.07429	0.07194	0.06945	0.06467
0.535714286	0.07074	0.06687	0.06227	0.06134	0.0594	0.05734	0.05338
0.714285714	0.06149	0.05815	0.05416	0.05335	0.05166	0.04988	0.04642
0.892857143	0.05492	0.05195	0.0484	0.04768	0.04617	0.04458	0.04149
1.071428571	0.05313	0.05028	0.04686	0.04616	0.04471	0.04317	0.04018
1.25	0.05301	0.05019	0.04679	0.0461	0.04465	0.04312	0.04013
1.428571429	0.05214	0.04938	0.04603	0.04535	0.04393	0.04242	0.03948

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.28638	0.27706	0.23429	0.19073	0.15131	0.09665	0.01246
0.357142857	0.25713	0.24778	0.20836	0.16907	0.13381	0.08524	0.01123
0.535714286	0.21106	0.20177	0.16774	0.13518	0.10646	0.06744	0.00919
0.714285714	0.18085	0.17186	0.14162	0.11354	0.08908	0.05618	0.00774
0.892857143	0.15869	0.15004	0.12273	0.09796	0.07661	0.04814	0.00665
1.071428571	0.15071	0.14186	0.11527	0.09163	0.07146	0.04476	0.0062
1.25	0.14761	0.13838	0.11177	0.08854	0.06887	0.04301	0.00595
1.428571429	0.14371	0.13444	0.10826	0.0856	0.06649	0.04146	0.00574

L_cutline

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	0	0	0	0	0	-1E-14	0
0.5	0	0	0	0	0	0	0
0.75	0	0	0	0	0	5.3E-15	0
1	0	0	0	0	0	-5E-15	0
1.25	-3E-15	0	0	0	0	0	0
1.5	-2E-15	0	0	0	0	5.3E-15	0
1.75	2.6E-15	0	0	0	0	0	0
2	1.8E-15	0	0	0	0	0	-5E-15
2.25	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	1.1E-14

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	1.16738	1.0545	0.92777	0.8916	0.84076	0.80646	0.67202
0.5	1.03899	0.9373	0.82951	0.7957	0.75674	0.72822	0.62795
0.75	0.83868	0.75504	0.67471	0.64524	0.62212	0.60183	0.54832
1	0.71876	0.64682	0.58022	0.55429	0.53712	0.52073	0.4861
1.25	0.63768	0.5742	0.51637	0.49325	0.47924	0.46538	0.44048
1.5	0.57848	0.52136	0.46968	0.44873	0.43663	0.42438	0.40444
1.75	0.5305	0.47846	0.43161	0.4124	0.40168	0.39061	0.37416
2	0.49185	0.4439	0.40089	0.38306	0.37334	0.36319	0.34921
2.25	0.45996	0.41561	0.37583	0.35929	0.35043	0.34119	0.32966
2.5	0.44485	0.40234	0.36421	0.3484	0.34004	0.33144	0.32189

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	-0.29	-0.3115	-0.3311	-0.4271	-0.702	-0.6573	-1.3609
0.5	-0.1284	-0.1484	-0.1662	-0.2348	-0.4244	-0.3982	-0.9106
0.75	0.06226	0.04635	0.03232	6.7E-05	-0.0796	-0.0749	-0.3263
1	0.09714	0.08538	0.07494	0.05552	0.01092	0.01186	-0.137
1.25	0.10259	0.09365	0.08562	0.07242	0.04372	0.04362	-0.0548
1.5	0.1	0.09302	0.08667	0.07703	0.05695	0.05653	-0.0133
1.75	0.09504	0.08949	0.08436	0.07697	0.06212	0.06161	0.00948
2	0.08956	0.08507	0.08088	0.07502	0.06359	0.06307	0.02271
2.25	0.08421	0.08054	0.07707	0.07231	0.06323	0.06275	0.03059
2.5	0.08161	0.0783	0.07515	0.07086	0.06276	0.06229	0.03356

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	-2E-14	7E-14	0	0	-5E-14	0	1.8E-13
0.5	8.8E-15	0	0	0	-3E-14	0	8.8E-14
0.75	8.8E-15	-4E-14	0	0	-9E-15	0	-9E-14
1	-2E-14	-9E-15	0	0	0	0	-9E-14
1.25	-9E-15	-9E-15	8.8E-15	0	8.8E-15	0	8.8E-14
1.5	0	8.8E-15	0	0	0	0	8.8E-14
1.75	0	1.8E-14	-9E-15	0	0	0	0
2	0	0	1.8E-14	0	0	0	0
2.25	1.8E-14	-9E-15	8.8E-15	0	0	0	0
2.5	3.5E-14	0	-2E-14	0	0	0	0

d_rep

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-3.2103	-3.1944	-2.801	-2.4556	-2.2157	-1.7426	-0.3018
0.4	-1.9654	-1.7319	-1.5065	-1.3076	-1.1395	-0.8763	-0.1123
0.7	-0.4339	-0.1573	-0.1101	-0.0787	-0.0246	0.0134	0.07422
1	-0.0801	-0.0213	0.00143	0.01504	0.02252	0.03839	0.06473
1.3	-0.0054	0.00644	0.01774	0.02594	0.03225	0.03895	0.05706
1.6	0.00472	0.01596	0.02274	0.026	0.03009	0.03399	0.05237
1.9	0.01272	0.01979	0.02297	0.02566	0.02772	0.03163	0.0476
2.2	0.0151	0.01998	0.02271	0.02477	0.02872	0.03221	0.04318
2.5	0.01357	0.01874	0.0217	0.02605	0.02556	0.02933	0.03952
2.8	0.01693	0.0162	0.02093	0.02315	0.02358	0.02736	0.0385
3.1	0.01544	0.01834	0.01997	0.02105	0.02447	0.02674	0.03558
3.4	0.01334	0.01808	0.01924	0.02022	0.02201	0.0257	0.03695
3.7	0.01602	0.01735	0.01843	0.01966	0.02129	0.02274	0.03706
4	0.01457	0.01682	0.01779	0.0208	0.02147	0.02176	0.03372
4.3	0.01232	0.01618	0.01712	0.01874	0.01872	0.02269	0.03427
4.6	0.01437	0.01479	0.01659	0.01727	0.01898	0.0212	0.03354
4.9	0.01587	0.01245	0.01525	0.01798	0.0205	0.02014	0.03246

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-434.58	-422.01	-391.43	-363.18	-339.4	-295.09	-100.78
0.4	-238.36	-220.67	-203.87	-188.31	-174.49	-150.87	-52.411
0.7	-24.3	-11.578	-9.5505	-7.9757	-5.8208	-4.2287	-2.7605
1	-3.9858	-2.6114	-1.9503	-1.7506	-1.4833	-1.3247	-1.1205
1.3	-1.2018	-1.0386	-0.8375	-0.7741	-0.7139	-0.6758	-0.617
1.6	-0.6893	-0.5409	-0.4733	-0.4524	-0.4389	-0.4177	-0.394
1.9	-0.3891	-0.336	-0.3132	-0.2994	-0.2977	-0.2902	-0.273
2.2	-0.2481	-0.2321	-0.2233	-0.2179	-0.2151	-0.2112	-0.1995
2.5	-0.1868	-0.1703	-0.169	-0.1651	-0.1651	-0.1603	-0.1537
2.8	-0.1436	-0.1367	-0.1321	-0.1319	-0.1281	-0.1264	-0.1216
3.1	-0.1107	-0.11	-0.1067	-0.1063	-0.1042	-0.1023	-0.0966
3.4	-0.0937	-0.0901	-0.0877	-0.0863	-0.0864	-0.0843	-0.0782
3.7	-0.0768	-0.0753	-0.0736	-0.0726	-0.0719	-0.0712	-0.066
4	-0.0639	-0.0636	-0.0625	-0.0614	-0.0613	-0.0604	-0.0583
4.3	-0.0568	-0.0538	-0.0538	-0.0533	-0.0529	-0.0514	-0.0494
4.6	-0.048	-0.0467	-0.0468	-0.0467	-0.0456	-0.0456	-0.0415
4.9	-0.043	-0.0448	-0.0437	-0.0432	-0.0422	-0.0436	-0.0393

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-41.456	-43.329	-43.078	-42.73	-42.573	-40.764	-23.878
0.4	-24.041	-23.879	-23.421	-22.967	-22.523	-21.498	-12.704
0.7	-4.0218	-2.6699	-2.3602	-2.0219	-1.5309	-1.4133	-1.0147
1	-0.9676	-0.6982	-0.6129	-0.5645	-0.4983	-0.4213	-0.3681
1.3	-0.4105	-0.3149	-0.2665	-0.214	-0.2675	-0.1993	-0.1996
1.6	-0.1942	-0.1635	-0.1648	-0.1622	-0.1083	-0.1287	-0.185
1.9	-0.106	-0.109	-0.0941	-0.1165	-0.1183	-0.0933	-0.1384
2.2	-0.1124	-0.0674	-0.0812	-0.0797	-0.0932	-0.0706	-0.0672
2.5	-0.0522	-0.0822	-0.0657	-0.0666	-0.035	-0.0789	-0.0818
2.8	-0.0351	-0.0429	-0.0496	-0.0212	-0.0564	-0.0733	-0.0738
3.1	-0.0574	-0.0408	-0.0357	-0.0462	-0.0489	-0.0374	-0.0434
3.4	-0.0247	-0.0293	-0.0339	-0.0386	-0.016	-0.0251	-0.0357
3.7	-0.0189	-0.0216	-0.0279	-0.0248	-0.016	-0.0244	-0.035
4	-0.0387	-0.0214	-0.0249	-0.0284	-0.033	-0.0216	-0.0448
4.3	-0.0141	-0.0161	-0.0183	-0.01	-0.0293	-0.0194	-0.0388
4.6	-0.0096	-0.034	-0.0192	-0.0288	-0.0082	-0.0302	-0.0327
4.9	-0.0284	-0.0331	-0.0006	-0.0512	-0.0089	-0.042	-0.0413

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.15545	0.75793	1.08411	1.44125	2.38349	3.88622	7.64824
0.4	0.72711	1.37874	2.08262	2.4743	3.1886	3.83321	4.5705
0.7	1.88691	2.15291	2.81751	3.12762	3.00208	2.63488	0.8852
1	1.90751	2.16442	2.07817	2.01845	1.48177	1.04861	0.17834
1.3	1.54708	1.6712	1.19462	0.92154	0.6913	0.42553	0.05631
1.6	1.48708	1.00243	0.62457	0.41428	0.31413	0.1715	0.02271
1.9	0.94778	0.5588	0.35332	0.22297	0.15677	0.08264	0.0112
2.2	0.47781	0.33683	0.20185	0.13786	0.09453	0.05138	0.00744
2.5	0.31678	0.18759	0.13252	0.08733	0.05725	0.02662	0.0013
2.8	0.27987	0.11762	0.0804	0.05475	0.0318	0.01609	0.00181
3.1	0.14697	0.08852	0.04808	0.0411	0.02356	0.01378	0.0038
3.4	0.10668	0.06874	0.03766	0.02267	0.01891	0.00998	0.00159
3.7	0.1018	0.05029	0.02703	0.00662	0.01264	0.00638	-0.0006
4	0.06209	0.03502	0.01984	0.01344	0.00764	0.00414	0.00055
4.3	0.04399	0.02631	0.01419	0.01178	0.00577	0.00344	0.00183
4.6	0.03842	0.01587	0.01141	0.00369	0.00617	0.00021	-0.0001
4.9	0.03509	0.01014	0.01441	-0.0003	0.00489	-0.0025	-0.0006

Figure A.15: D_1, $L_{Cut\ line}$ and d_{rep}

$3/3 * D_0$

A_coh

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	7.9E-16	0	0	2.4E-15	0	0	4E-16
2	4E-16	0	0	0	0	0	0
3	0	0	0	-1E-15	0	0	-2E-16
4	-4E-16	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	4E-16	0	0	0	0	0	0
7	-4E-16	0	0	1.2E-15	0	0	-1E-15
8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	1.6E-15
10	-8E-16	0	0	2.4E-15	0	0	4E-16

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
2	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
3	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
4	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
5	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
6	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
7	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
8	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
9	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
10	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
2	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
3	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
4	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
5	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
6	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
7	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
8	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
9	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
10	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
1	0	0	-4E-15	0	0	0	0
2	-2E-15	0	-2E-15	0	0	0	0
3	-4E-15	0	2E-15	0	0	0	0
4	-2E-15	0	0	0	0	0	0
5	3.9E-15	0	0	0	0	0	0
6	0	0	0	5.9E-15	0	0	-2E-14
7	-2E-15	0	-2E-15	0	0	0	0
8	0	0	0	0	0	0	2E-14
9	-2E-15	0	0	3.9E-15	0	0	0
10	0	0	0	-4E-15	0	0	0

c

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	-4E-15
0.1	0	0	0	2E-15	0	1.9E-15	0
0.2	0	0	0	0	0	0	2E-15
0.3	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0
0.6	0	0	0	0	0	1.9E-15	-2E-15
0.7	0	0	0	-2E-15	0	-2E-15	1.6E-14
0.8	0	0	0	0	0	-2E-15	0
0.9	0	0	0	2E-15	0	1.9E-15	-2E-14
1	0	0	0	2E-15	0	0	0
1.1	0	0	0	-2E-15	0	-2E-15	0
1.2	0	0	0	0	0	1.9E-15	1.8E-14
1.3	0	0	0	2E-15	0	0	0
1.4	0	0	0	-2E-15	0	0	0
1.5	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	0	0	-2E-15	0	0	-2E-15
1.8	6.1E-14	0	0	2E-15	0	-2E-15	-2E-14
1.9	0	0	0	0	0	0	1.8E-14
2	-1E-13	0	0	-4E-15	0	0	3.1E-14

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.1	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.2	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.3	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.4	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.5	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.6	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.7	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.8	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
0.9	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.1	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.2	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.3	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.4	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.5	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.6	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.7	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.8	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
1.9	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527
2	0.00717	0.00628	0.00583	0.0056	0.00543	0.00528	0.00527

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.1	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.2	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.3	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.4	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.5	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.6	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.7	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.8	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
0.9	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.1	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.2	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.3	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.4	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.5	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.6	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.7	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.8	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.9	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
2	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0	0	0	0	0	0	0	0
0.1	0	0	0	0	0	0	0
0.2	0	0	0	0	0	0	0
0.3	0	0	0	2E-14	0	0	0
0.4	0	0	0	2E-14	0	0	0
0.5	0	0	0	-2E-14	0	0	0
0.6	0	0	0	0	0	0	-2E-13
0.7	0	0	0	0	0	0	0
0.8	0	0	0	-2E-14	0	0	2E-13
0.9	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.1	0	0	0	0	0	0	0
1.2	0	0	-2E-14	0	0	0	0
1.3	0	0	0	0	0	0	0
1.4	0	0	2E-14	0	0	0	0
1.5	0	0	0	2E-14	0	0	0
1.6	0	0	0	0	0	0	0
1.7	0	2E-14	0	-2E-14	0	0	0
1.8	0	0	-2E-14	2E-14	0	0	0
1.9	0	-2E-14	0	0	0	0	0
2	0	0	3.9E-14	-4E-14	0	0	0

Figure A.16: A_c and c

Alpha_coh

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	1.2E-14
1	-6E-16	0	0	0	0	0	-6E-16
1.333333333	-6E-16	0	0	0	0	0	6E-16
1.666666667	0	0	0	6E-16	0	0	0
2	0	0	0	6E-16	0	0	-7E-15
2.333333333	5.7E-16	0	0	0	0	0	6E-16
2.666666667	5.7E-16	0	0	-6E-16	0	0	0
3	-6E-16	0	0	0	0	0	-6E-16
3.333333333	0	0	0	6E-16	0	0	6.5E-15
3.666666667	5.7E-16	0	0	-6E-16	0	0	0
4	0	0	0	-6E-16	0	0	-7E-15
4.333333333	0	0	0	6E-16	0	0	6E-16
4.666666667	-6E-16	0	0	0	0	0	6E-16
5	-6E-16	0	0	0	0	0	-6E-16
5.333333333	5.7E-16	0	0	0	0	0	5.9E-15
5.666666667	0	0	0	-6E-16	0	0	0
6	-6E-16	0	0	0	0	0	-7E-15
6.333333333	5.7E-16	0	0	0	0	0	6.5E-15
6.666666667	1.1E-15	0	0	0	0	0	1.4E-14

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	0	0	0	-1E-16
1	-6E-17	0	0	0	0	0	0
1.333333333	-6E-17	0	0	0	5.9E-17	0	0
1.666666667	5.9E-17	0	0	0	0	-6E-17	0
2	5.9E-17	0	0	0	-6E-17	0	0
2.333333333	-6E-17	0	0	0	0	5.9E-17	0
2.666666667	0	0	0	0	0	0	5.9E-17
3	1.8E-16	0	0	0	0	0	0
3.333333333	-6E-17	0	0	0	0	0	0
3.666666667	-2E-16	0	0	0	0	0	0
4	0	0	0	0	0	-6E-17	0
4.333333333	5.9E-17	0	0	0	0	0	0
4.666666667	-6E-17	0	0	0	0	5.9E-17	0
5	-6E-17	-6E-17	0	0	0	-6E-17	0
5.333333333	5.9E-17	-6E-17	0	0	0	0	0
5.666666667	5.9E-17	5.9E-17	0	0	0	5.9E-17	0
6	0	5.9E-17	0	0	0	0	0
6.333333333	0	-6E-17	0	0	5.9E-17	-6E-17	0
6.666666667	1.2E-16	-1E-16	0	0	1.2E-16	-1E-16	0

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
1.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
2	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
2.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
2.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
3	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
3.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
3.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
4	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
4.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
4.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
5	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
5.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
5.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
6	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
6.333333333	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512
6.666666667	0.03614	0.0406	0.04377	0.04624	0.04792	0.04949	0.0512

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.666666667	0	0	0	1.2E-14	0	0	0
1	1.2E-14	0	0	5.9E-15	0	0	0
1.333333333	1.2E-14	0	0	0	0	0	0
1.666666667	-1E-14	0	0	-6E-15	0	0	0
2	-1E-14	0	0	0	0	0	0
2.333333333	1.2E-14	0	-4E-14	5.9E-15	0	0	0
2.666666667	0	0	0	0	0	0	0
3	5.9E-15	0	3.5E-14	-6E-15	0	0	0
3.333333333	1.2E-14	0	0	-6E-15	0	0	0
3.666666667	-6E-15	0	-2E-14	5.9E-15	0	0	0
4	0	0	0	0	0	0	0
4.333333333	-1E-14	0	-1E-14	0	0	0	0
4.666666667	0	0	0	0	0	0	0
5	1.2E-14	1.2E-14	0	0	0	0	0
5.333333333	0	5.9E-15	0	5.9E-15	0	0	0
5.666666667	-1E-14	-1E-14	3.5E-14	-6E-15	0	0	0
6	0	-6E-15	-4E-14	-6E-15	0	0	0
6.333333333	0	5.9E-15	0	5.9E-15	0	0	-6E-14
6.666666667	-2E-14	1.2E-14	7.1E-14	1.2E-14	0	0	-1E-13

Alpha_k

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.1595	-0.137	-0.1483	-0.162	-0.1701	-0.1703	-0.2322
0.5	-0.1219	-0.1043	-0.1132	-0.124	-0.1304	-0.1303	-0.1779
0.666666667	-0.0683	-0.0578	-0.0633	-0.0698	-0.0739	-0.0734	-0.1007
0.833333333	-0.0438	-0.037	-0.0408	-0.0454	-0.0482	-0.0478	-0.0659
1	-0.0303	-0.0257	-0.0287	-0.0321	-0.0342	-0.0339	-0.0469
1.166666667	-0.0221	-0.0188	-0.0212	-0.0239	-0.0257	-0.0255	-0.0353
1.333333333	-0.0166	-0.0143	-0.0164	-0.0186	-0.02	-0.0199	-0.0277
1.5	-0.0128	-0.0112	-0.013	-0.0149	-0.0161	-0.016	-0.0223
1.666666667	-0.0101	-0.0089	-0.0105	-0.0121	-0.0132	-0.0131	-0.0184
1.833333333	-0.008	-0.0072	-0.0087	-0.0101	-0.011	-0.011	-0.0155
2	-0.0064	-0.0059	-0.0072	-0.0085	-0.0094	-0.0094	-0.0132
2.166666667	-0.0052	-0.0049	-0.0061	-0.0073	-0.008	-0.0081	-0.0114
2.333333333	-0.0042	-0.0041	-0.0052	-0.0063	-0.007	-0.007	-0.01
2.5	-0.0034	-0.0034	-0.0045	-0.0055	-0.0061	-0.0062	-0.0088
2.666666667	-0.0027	-0.0029	-0.0039	-0.0048	-0.0054	-0.0055	-0.0078
2.833333333	-0.0022	-0.0024	-0.0034	-0.0042	-0.0048	-0.0049	-0.007
3	-0.0017	-0.0021	-0.0029	-0.0037	-0.0043	-0.0044	-0.0063
3.166666667	-0.0013	-0.0017	-0.0026	-0.0033	-0.0038	-0.0039	-0.0057
3.333333333	-0.0011	-0.0016	-0.0024	-0.0031	-0.0036	-0.0037	-0.0054

Times original value	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.0027	-0.0032	-0.0034	-0.0035	-0.0035	-0.0036	-0.0034
0.5	-0.002	-0.0024	-0.0026	-0.0027	-0.0027	-0.0028	-0.0026
0.666666667	-0.0008	-0.0012	-0.0014	-0.0015	-0.0016	-0.0016	-0.0014
0.833333333	-0.0003	-0.0007	-0.0009	-0.001	-0.001	-0.001	-0.0009
1	-3E-05	-0.0004	-0.0005	-0.0006	-0.0007	-0.0007	-0.0006
1.166666667	0.00015	-0.0002	-0.0003	-0.0004	-0.0005	-0.0005	-0.0004
1.333333333	0.00026	-6E-05	-0.0002	-0.0003	-0.0003	-0.0004	-0.0003
1.5	0.00034	2.4E-05	-0.0001	-0.0002	-0.0002	-0.0003	-0.0002
1.666666667	0.0004	8.8E-05	-5E-05	-0.0001	-0.0002	-0.0002	-0.0001
1.833333333	0.00044	0.00014	1.5E-06	-8E-05	-0.0001	-0.0001	-7E-05
2	0.00047	0.00017	4.1E-05	-4E-05	-8E-05	-1E-04	-3E-05
2.166666667	0.00049	0.0002	7.2E-05	-8E-06	-5E-05	-7E-05	5E-07
2.333333333	0.0005	0.00022	9.6E-05	1.8E-05	-2E-05	-4E-05	2.4E-05
2.5	0.00052	0.00024	0.00012	3.9E-05	4.3E-08	-2E-05	4.4E-05
2.666666667	0.00053	0.00025	0.00013	5.6E-05	1.8E-05	1E-06	5.9E-05
2.833333333	0.00053	0.00026	0.00015	7.1E-05	3.3E-05	1.6E-05	7.2E-05
3	0.00054	0.00027	0.00016	8.3E-05	4.5E-05	2.9E-05	8.3E-05
3.166666667	0.00054	0.00028	0.00017	9.3E-05	5.6E-05	4E-05	9.2E-05
3.333333333	0.00055	0.00029	0.00017	9.7E-05	6.1E-05	4.5E-05	9.6E-05

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-0.0149	-0.011	-0.0079	-0.0054	-0.0037	-0.002	-0.0003
0.5	-0.0133	-0.01	-0.0073	-0.0051	-0.0035	-0.0019	-0.0003
0.666666667	-0.0105	-0.0084	-0.0063	-0.0045	-0.0031	-0.0018	-0.0002
0.833333333	-0.0085	-0.0071	-0.0055	-0.004	-0.0028	-0.0016	-0.0002
1	-0.0071	-0.0062	-0.0049	-0.0036	-0.0026	-0.0015	-0.0002
1.166666667	-0.0061	-0.0054	-0.0044	-0.0033	-0.0024	-0.0014	-0.0002
1.333333333	-0.0053	-0.0048	-0.004	-0.0031	-0.0023	-0.0013	-0.0002
1.5	-0.0046	-0.0043	-0.0037	-0.0029	-0.0021	-0.0013	-0.0002
1.666666667	-0.0041	-0.0039	-0.0034	-0.0027	-0.002	-0.0012	-0.0002
1.833333333	-0.0036	-0.0036	-0.0031	-0.0025	-0.0019	-0.0012	-0.0002
2	-0.0033	-0.0033	-0.0029	-0.0023	-0.0018	-0.0011	-0.0002
2.166666667	-0.003	-0.003	-0.0027	-0.0022	-0.0017	-0.0011	-0.0002
2.333333333	-0.0027	-0.0028	-0.0025	-0.0021	-0.0016	-0.001	-0.0002
2.5	-0.0025	-0.0026	-0.0024	-0.002	-0.0015	-0.001	-0.0002
2.666666667	-0.0023	-0.0024	-0.0022	-0.0019	-0.0015	-0.0009	-0.0001
2.833333333	-0.0021	-0.0023	-0.0021	-0.0018	-0.0014	-0.0009	-0.0001
3	-0.0019	-0.0021	-0.002	-0.0017	-0.0014	-0.0009	-0.0001
3.166666667	-0.0018	-0.002	-0.0019	-0.0016	-0.0013	-0.0009	-0.0001
3.333333333	-0.0017	-0.0019	-0.0019	-0.0016	-0.0013	-0.0008	-0.0001

Times original value	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.333333333	-1.4588	-1.1007	-0.8005	-0.5525	-0.3776	-0.211	-0.0283
0.5	-1.2872	-0.9986	-0.74	-0.5179	-0.3571	-0.2013	-0.0272
0.666666667	-1.0023	-0.8238	-0.6342	-0.4565	-0.3207	-0.1838	-0.0253
0.833333333	-0.8097	-0.6967	-0.5538	-0.4084	-0.2917	-0.1698	-0.0238
1	-0.6719	-0.6004	-0.4906	-0.3696	-0.2679	-0.1581	-0.0225
1.166666667	-0.5693	-0.5251	-0.4395	-0.3375	-0.2479	-0.1482	-0.0214
1.333333333	-0.4902	-0.4648	-0.3973	-0.3104	-0.2309	-0.1397	-0.0204
1.5	-0.4278	-0.4154	-0.3619	-0.2873	-0.2161	-0.1322	-0.0196
1.666666667	-0.3775	-0.3744	-0.3317	-0.2672	-0.2032	-0.1256	-0.0188
1.833333333	-0.3362	-0.3398	-0.3057	-0.2496	-0.1917	-0.1197	-0.0182
2	-0.3018	-0.3102	-0.2832	-0.234	-0.1814	-0.1144	-0.0175
2.166666667	-0.2729	-0.2848	-0.2634	-0.2202	-0.1722	-0.1096	-0.017
2.333333333	-0.2482	-0.2627	-0.2459	-0.2078	-0.1639	-0.1052	-0.0165
2.5	-0.227	-0.2433	-0.2303	-0.1967	-0.1563	-0.1012	-0.016
2.666666667	-0.2085	-0.2262	-0.2164	-0.1865	-0.1494	-0.0974	-0.0156
2.833333333	-0.1924	-0.2111	-0.2038	-0.1773	-0.1431	-0.094	-0.0151
3	-0.1783	-0.1975	-0.1925	-0.1689	-0.1372	-0.0908	-0.0148
3.166666667	-0.1658	-0.1854	-0.1823	-0.1612	-0.1318	-0.0879	-0.0144
3.333333333	-0.1599	-0.1796	-0.1774	-0.1576	-0.1292	-0.0864	-0.0142

Figure A.17: $Anis_c$ and $Anis_k$

k_0

Times original value	%change FRP heave							%change FRP hor ero						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.07063	0.05233	0.05362	0.05671	0.0596	0.05936	0.08283	0.00469	0.00409	0.00375	0.00359	0.00347	0.00334	0.00325
0.4	0.07812	0.05815	0.05886	0.06159	0.06416	0.06358	0.08645	0.00498	0.00429	0.00393	0.00375	0.00361	0.00348	0.00337
0.7	0.09208	0.06935	0.06878	0.07068	0.07253	0.07134	0.09284	0.00552	0.0047	0.00427	0.00404	0.00389	0.00374	0.00359
1	0.10399	0.07954	0.07752	0.07841	0.0794	0.07776	0.0976	0.00603	0.00507	0.00457	0.00431	0.00413	0.00396	0.00378
1.3	0.11395	0.08859	0.08505	0.08482	0.08489	0.08293	0.10094	0.00649	0.00541	0.00485	0.00454	0.00434	0.00415	0.00394
1.6	0.12221	0.09649	0.09142	0.09003	0.08918	0.08699	0.1031	0.00692	0.00572	0.0051	0.00475	0.00452	0.00431	0.00406
1.9	0.12897	0.10326	0.09671	0.09418	0.09244	0.0901	0.10431	0.0073	0.006	0.00532	0.00493	0.00468	0.00445	0.00417
2.2	0.13444	0.10896	0.10102	0.09739	0.09485	0.09238	0.10474	0.00764	0.00626	0.00552	0.00509	0.00481	0.00456	0.00425
2.5	0.13876	0.11367	0.10446	0.09981	0.09654	0.09396	0.10455	0.00795	0.00649	0.0057	0.00523	0.00493	0.00466	0.00431
2.8	0.14208	0.11746	0.10713	0.10155	0.09764	0.09494	0.10386	0.00821	0.00669	0.00586	0.00535	0.00503	0.00474	0.00436
3.1	0.14452	0.12042	0.10912	0.10272	0.09825	0.09543	0.10279	0.00844	0.00687	0.00599	0.00545	0.00511	0.0048	0.0044
3.4	0.14616	0.12264	0.11053	0.1034	0.09845	0.0955	0.10141	0.00864	0.00703	0.00611	0.00555	0.00518	0.00485	0.00442
3.7	0.14712	0.12422	0.11143	0.10367	0.09832	0.09522	0.09979	0.00881	0.00717	0.00622	0.00562	0.00524	0.00489	0.00443
4	0.14746	0.12522	0.1119	0.10361	0.09792	0.09466	0.098	0.00895	0.00729	0.00631	0.00569	0.00529	0.00492	0.00443
4.3	0.14728	0.12572	0.112	0.10326	0.0973	0.09387	0.09608	0.00907	0.00739	0.00639	0.00575	0.00533	0.00494	0.00443
4.6	0.14663	0.12578	0.11178	0.10268	0.09649	0.09288	0.09407	0.00916	0.00748	0.00646	0.00579	0.00536	0.00496	0.00442
4.9	0.1462	0.12572	0.11161	0.10234	0.09604	0.09235	0.09304	0.00919	0.00752	0.00649	0.00581	0.00537	0.00496	0.00441

Times original value	%change FRP ver ero							%change FRP ver tra						
0	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	0.00029	-0.0003	-0.0004	-0.0004	-0.0003	-0.0001	-5E-07	-0.2728	-0.2599	-0.2056	-0.1543	-0.1082	-0.0594	-0.0067
0.4	0.00077	5.6E-05	-0.0001	-0.0002	-0.0001	-7E-05	1.2E-06	-0.1698	-0.1802	-0.15	-0.1159	-0.0829	-0.0465	-0.0054
0.7	0.00135	0.00055	0.00022	5.5E-05	4.8E-06	-5E-06	3.2E-06	-0.0324	-0.0715	-0.0729	-0.0622	-0.0473	-0.0282	-0.0036
1	0.00146	0.00068	0.00032	0.00013	5.1E-05	1.5E-05	3E-06	0.01615	-0.0293	-0.0412	-0.0395	-0.0319	-0.02	-0.0028
1.3	0.00144	0.0007	0.00034	0.00015	6.4E-05	2E-05	2.3E-06	0.03669	-0.0094	-0.0254	-0.028	-0.024	-0.0158	-0.0023
1.6	0.00137	0.00068	0.00034	0.00015	6.5E-05	1.9E-05	1.4E-06	0.04588	0.00109	-0.0166	-0.0214	-0.0194	-0.0133	-0.0021
1.9	0.00129	0.00065	0.00032	0.00014	6.1E-05	1.6E-05	5.1E-07	0.04977	0.00691	-0.0114	-0.0174	-0.0166	-0.0117	-0.0019
2.2	0.00121	0.00062	0.00031	0.00013	5.5E-05	1.3E-05	-3E-07	0.05098	0.01025	-0.0081	-0.0147	-0.0147	-0.0107	-0.0018
2.5	0.00113	0.00058	0.00029	0.00012	4.9E-05	9.2E-06	-1E-06	0.05077	0.01217	-0.0059	-0.0129	-0.0134	-0.01	-0.0017
2.8	0.00106	0.00055	0.00027	0.00011	4.3E-05	5.7E-06	-2E-06	0.04978	0.01324	-0.0044	-0.0117	-0.0124	-0.0095	-0.0016
3.1	0.001	0.00052	0.00025	0.00011	3.8E-05	2.4E-06	-2E-06	0.04839	0.01379	-0.0034	-0.0107	-0.0117	-0.0091	-0.0016
3.4	0.00094	0.00049	0.00024	9.7E-05	3.3E-05	-5E-07	-3E-06	0.04678	0.01401	-0.0027	-0.01	-0.0112	-0.0088	-0.0016
3.7	0.00088	0.00046	0.00023	9E-05	2.8E-05	-3E-06	-3E-06	0.04508	0.01401	-0.0022	-0.0094	-0.0107	-0.0085	-0.0015
4	0.00083	0.00044	0.00021	8.3E-05	2.4E-05	-5E-06	-3E-06	0.04336	0.01388	-0.0018	-0.009	-0.0104	-0.0084	-0.0015
4.3	0.00079	0.00042	0.0002	7.7E-05	2E-05	-8E-06	-4E-06	0.04166	0.01366	-0.0015	-0.0086	-0.0101	-0.0082	-0.0015
4.6	0.00074	0.0004	0.00019	7.2E-05	1.7E-05	-9E-06	-4E-06	0.04	0.01338	-0.0013	-0.0083	-0.0098	-0.0081	-0.0015
4.9	0.00072	0.00039	0.00019	6.9E-05	1.5E-05	-1E-05	-4E-06	0.03919	0.01323	-0.0012	-0.0081	-0.0097	-0.008	-0.0015

k_1

	%change FRP heave							%change FRP hor ero						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-1.0581	-0.987	-0.9671	-0.9572	-0.9474	-0.9384	-0.9693	-0.0551	-0.053	-0.0517	-0.0512	-0.0506	-0.0498	-0.0468
0.4	-0.6247	-0.5761	-0.5668	-0.5634	-0.5597	-0.5537	-0.581	-0.0317	-0.0304	-0.0298	-0.0295	-0.0291	-0.0287	-0.0271
0.7	-0.1341	-0.1148	-0.1161	-0.1187	-0.1208	-0.1188	-0.137	-0.0056	-0.0055	-0.0054	-0.0054	-0.0054	-0.0053	-0.0051
1	-0.0589	-0.0491	-0.0502	-0.052	-0.0535	-0.0527	-0.0631	-0.0023	-0.0023	-0.0023	-0.0023	-0.0023	-0.0023	-0.0022
1.3	-0.0329	-0.0271	-0.028	-0.0293	-0.0303	-0.03	-0.0367	-0.0012	-0.0013	-0.0013	-0.0013	-0.0013	-0.0013	-0.0012
1.6	-0.0208	-0.0171	-0.0178	-0.0188	-0.0196	-0.0194	-0.0241	-0.0006	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008
1.9	-0.0141	-0.0116	-0.0123	-0.0131	-0.0137	-0.0136	-0.0171	-0.0004	-0.0005	-0.0005	-0.0006	-0.0006	-0.0006	-0.0005
2.2	-0.0101	-0.0084	-0.0089	-0.0096	-0.0101	-0.0101	-0.0128	-0.0002	-0.0003	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004
2.5	-0.0075	-0.0062	-0.0068	-0.0073	-0.0078	-0.0078	-0.01	-8E-05	-0.0002	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
2.8	-0.0057	-0.0048	-0.0053	-0.0057	-0.0061	-0.0062	-0.008	-8E-06	-0.0001	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
3.1	-0.0043	-0.0037	-0.0042	-0.0046	-0.005	-0.0051	-0.0066	4.7E-05	-8E-05	-0.0001	-0.0002	-0.0002	-0.0002	-0.0002
3.4	-0.0034	-0.0029	-0.0034	-0.0038	-0.0041	-0.0042	-0.0055	8.8E-05	-4E-05	-9E-05	-0.0001	-0.0001	-0.0002	-0.0002
3.7	-0.0026	-0.0023	-0.0028	-0.0031	-0.0034	-0.0035	-0.0047	0.00012	-1E-05	-7E-05	-0.0001	-0.0001	-0.0001	-0.0001
4	-0.002	-0.0019	-0.0023	-0.0026	-0.0029	-0.003	-0.004	0.00014	1.3E-05	-4E-05	-8E-05	-1E-04	-0.0001	-0.0001
4.3	-0.0015	-0.0015	-0.0019	-0.0022	-0.0025	-0.0026	-0.0035	0.00016	3.2E-05	-2E-05	-6E-05	-8E-05	-9E-05	-9E-05
4.6	-0.0012	-0.0012	-0.0016	-0.0019	-0.0021	-0.0023	-0.0031	0.00018	4.7E-05	-9E-06	-4E-05	-6E-05	-8E-05	-8E-05
4.9	-0.001	0.06192	-0.0014	0.0429	-0.002	0.047	-0.0029	0.00019	0.00645	-2E-06	0.00446	-6E-05	0.00388	-7E-05

	%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0176	-0.0124	-0.0087	-0.0058	-0.0039	-0.0022	-0.0003	-1.5795	-1.1672	-0.8402	-0.5766	-0.3925	-0.2186	-0.0292
0.4	-0.0136	-0.01	-0.0071	-0.0049	-0.0033	-0.0019	-0.0002	-1.1726	-0.8974	-0.6619	-0.4624	-0.3187	-0.1796	-0.0243
0.7	-0.0082	-0.0066	-0.005	-0.0036	-0.0025	-0.0014	-0.0002	-0.6434	-0.5407	-0.4243	-0.3099	-0.2201	-0.1277	-0.018
1	-0.006	-0.0051	-0.0041	-0.003	-0.0021	-0.0012	-0.0002	-0.4611	-0.4096	-0.3345	-0.2517	-0.1826	-0.1082	-0.0156
1.3	-0.0048	-0.0043	-0.0035	-0.0026	-0.0019	-0.0011	-0.0002	-0.3651	-0.3379	-0.2846	-0.2194	-0.1619	-0.0976	-0.0144
1.6	-0.0041	-0.0037	-0.0031	-0.0024	-0.0018	-0.0011	-0.0002	-0.3046	-0.2914	-0.2518	-0.1982	-0.1486	-0.091	-0.0137
1.9	-0.0035	-0.0033	-0.0028	-0.0022	-0.0017	-0.001	-0.0002	-0.2623	-0.2581	-0.2281	-0.1829	-0.139	-0.0863	-0.0132
2.2	-0.0031	-0.003	-0.0026	-0.0021	-0.0016	-0.001	-0.0002	-0.2307	-0.2325	-0.2096	-0.171	-0.1316	-0.0827	-0.0129
2.5	-0.0028	-0.0027	-0.0024	-0.002	-0.0015	-0.001	-0.0001	-0.2059	-0.212	-0.1946	-0.1612	-0.1256	-0.0799	-0.0126
2.8	-0.0025	-0.0025	-0.0023	-0.0019	-0.0015	-0.0009	-0.0001	-0.1858	-0.195	-0.1819	-0.153	-0.1205	-0.0776	-0.0124
3.1	-0.0023	-0.0023	-0.0022	-0.0018	-0.0014	-0.0009	-0.0001	-0.1691	-0.1806	-0.171	-0.1458	-0.1161	-0.0756	-0.0123
3.4	-0.0021	-0.0022	-0.002	-0.0017	-0.0014	-0.0009	-0.0001	-0.1549	-0.1681	-0.1615	-0.1395	-0.1121	-0.0738	-0.0121
3.7	-0.0019	-0.002	-0.0019	-0.0017	-0.0013	-0.0009	-0.0001	-0.1427	-0.1572	-0.1529	-0.1337	-0.1086	-0.0722	-0.012
4	-0.0018	-0.0019	-0.0018	-0.0016	-0.0013	-0.0009	-0.0001	-0.132	-0.1474	-0.1452	-0.1285	-0.1053	-0.0707	-0.0119
4.3	-0.0016	-0.0018	-0.0017	-0.0015	-0.0012	-0.0008	-0.0001	-0.1227	-0.1387	-0.1383	-0.1237	-0.1023	-0.0694	-0.0119
4.6	-0.0015	-0.0017	-0.0017	-0.0015	-0.0012	-0.0008	-0.0001	-0.1144	-0.1309	-0.1319	-0.1193	-0.0995	-0.0681	-0.0118
4.9	-0.0015	0.02558	-0.0016	0.03452	-0.0012	0.04211	-0.0001	-0.1105	2.02593	-0.1288	2.81604	-0.0981	3.49174	-0.0118

Figure A.18: k_0 and k_1

k_2

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0405	-0.0344	-0.0357	-0.038	-0.0398	-0.0411	-0.0582	-0.0032	-0.0029	-0.0028	-0.0027	-0.0026	-0.0026	-0.0026
0.4	-0.0308	-0.0261	-0.0271	-0.0288	-0.0302	-0.0311	-0.0442	-0.0024	-0.0022	-0.0021	-0.002	-0.002	-0.002	-0.002
0.7	-0.0184	-0.0154	-0.016	-0.017	-0.0179	-0.0184	-0.0261	-0.0014	-0.0013	-0.0012	-0.0012	-0.0012	-0.0012	-0.0012
1	-0.014	-0.0117	-0.0122	-0.013	-0.0136	-0.014	-0.0199	-0.0011	-0.001	-0.0009	-0.0009	-0.0009	-0.0009	-0.0009
1.3	-0.0113	-0.0094	-0.0098	-0.0104	-0.0109	-0.0112	-0.016	-0.0009	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007
1.6	-0.0094	-0.0078	-0.0081	-0.0086	-0.0091	-0.0093	-0.0132	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006
1.9	-0.0079	-0.0065	-0.0068	-0.0073	-0.0076	-0.0078	-0.0111	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005
2.2	-0.0068	-0.0056	-0.0058	-0.0062	-0.0065	-0.0067	-0.0095	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004
2.5	-0.0059	-0.0048	-0.005	-0.0054	-0.0057	-0.0058	-0.0083	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004
2.8	-0.0052	-0.0042	-0.0044	-0.0047	-0.005	-0.0051	-0.0072	-0.0004	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
3.1	-0.0046	-0.0037	-0.0039	-0.0042	-0.0044	-0.0045	-0.0064	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
3.4	-0.0041	-0.0033	-0.0035	-0.0037	-0.0039	-0.004	-0.0057	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002
3.7	-0.0036	-0.003	-0.0031	-0.0033	-0.0035	-0.0036	-0.0051	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4	-0.0033	-0.0027	-0.0028	-0.003	-0.0031	-0.0032	-0.0046	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.3	-0.003	-0.0024	-0.0025	-0.0027	-0.0028	-0.0029	-0.0041	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.6	-0.0027	-0.0022	-0.0023	-0.0025	-0.0026	-0.0026	-0.0038	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.9	-0.0026	-0.0021	-0.0022	-0.0023	-0.0025	-0.0025	-0.0036	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0038	-0.003	-0.0023	-0.0016	-0.0012	-0.0007	-9E-05	-0.348	-0.2882	-0.2246	-0.1635	-0.1159	-0.067	-0.0094
0.4	-0.003	-0.0024	-0.0018	-0.0013	-0.0009	-0.0005	-7E-05	-0.2752	-0.2296	-0.1799	-0.1315	-0.0935	-0.0542	-0.0076
0.7	-0.002	-0.0016	-0.0013	-0.0009	-0.0006	-0.0004	-5E-05	-0.1817	-0.1545	-0.1227	-0.0907	-0.0649	-0.0379	-0.0054
1	-0.0016	-0.0014	-0.0011	-0.0008	-0.0005	-0.0003	-4E-05	-0.1481	-0.1275	-0.1022	-0.0761	-0.0548	-0.0322	-0.0046
1.3	-0.0014	-0.0012	-0.0009	-0.0007	-0.0005	-0.0003	-4E-05	-0.1255	-0.1092	-0.0883	-0.0661	-0.0478	-0.0282	-0.004
1.6	-0.0012	-0.001	-0.0008	-0.0006	-0.0004	-0.0002	-3E-05	-0.1085	-0.0953	-0.0775	-0.0584	-0.0424	-0.0251	-0.0036
1.9	-0.0011	-0.0009	-0.0007	-0.0005	-0.0004	-0.0002	-3E-05	-0.0949	-0.084	-0.0688	-0.052	-0.0379	-0.0225	-0.0032
2.2	-0.0009	-0.0008	-0.0006	-0.0005	-0.0003	-0.0002	-3E-05	-0.0839	-0.0748	-0.0615	-0.0468	-0.0342	-0.0203	-0.0029
2.5	-0.0008	-0.0007	-0.0006	-0.0004	-0.0003	-0.0002	-3E-05	-0.0747	-0.067	-0.0554	-0.0423	-0.031	-0.0185	-0.0027
2.8	-0.0008	-0.0007	-0.0005	-0.0004	-0.0003	-0.0002	-2E-05	-0.067	-0.0605	-0.0502	-0.0384	-0.0282	-0.0169	-0.0025
3.1	-0.0007	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-2E-05	-0.0605	-0.0548	-0.0457	-0.0351	-0.0258	-0.0155	-0.0023
3.4	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	-2E-05	-0.0549	-0.05	-0.0418	-0.0322	-0.0237	-0.0143	-0.0021
3.7	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	-2E-05	-0.05	-0.0457	-0.0384	-0.0296	-0.0219	-0.0132	-0.0019
4	-0.0005	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	-2E-05	-0.0458	-0.042	-0.0354	-0.0274	-0.0203	-0.0122	-0.0018
4.3	-0.0005	-0.0004	-0.0003	-0.0003	-0.0002	-0.0001	-2E-05	-0.0421	-0.0387	-0.0327	-0.0254	-0.0188	-0.0114	-0.0017
4.6	-0.0004	-0.0004	-0.0003	-0.0002	-0.0002	-0.0001	-2E-05	-0.0388	-0.0358	-0.0303	-0.0236	-0.0175	-0.0106	-0.0016
4.9	-0.0004	-0.0004	-0.0003	-0.0002	-0.0002	-0.0001	-1E-05	-0.0372	-0.0345	-0.0292	-0.0227	-0.0169	-0.0102	-0.0015

k_3

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0281	-0.0238	-0.0247	-0.0263	-0.0275	-0.0284	-0.0403	-0.0022	-0.002	-0.0019	-0.0019	-0.0018	-0.0018	-0.0018
0.4	-0.0244	-0.0206	-0.0214	-0.0227	-0.0238	-0.0246	-0.0349	-0.0019	-0.0017	-0.0016	-0.0016	-0.0016	-0.0015	-0.0016
0.7	-0.0183	-0.0153	-0.0159	-0.017	-0.0178	-0.0183	-0.026	-0.0014	-0.0013	-0.0012	-0.0012	-0.0012	-0.0011	-0.0012
1	-0.0142	-0.0119	-0.0124	-0.0132	-0.0138	-0.0142	-0.0202	-0.0011	-0.001	-0.0009	-0.0009	-0.0009	-0.0009	-0.0009
1.3	-0.0114	-0.0095	-0.0099	-0.0105	-0.011	-0.0113	-0.0161	-0.0009	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007
1.6	-0.0094	-0.0078	-0.0081	-0.0086	-0.009	-0.0093	-0.0132	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006
1.9	-0.0078	-0.0065	-0.0067	-0.0072	-0.0075	-0.0077	-0.011	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005
2.2	-0.0066	-0.0055	-0.0057	-0.0061	-0.0064	-0.0065	-0.0093	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004
2.5	-0.0057	-0.0047	-0.0049	-0.0052	-0.0055	-0.0056	-0.008	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0004
2.8	-0.005	-0.0041	-0.0042	-0.0045	-0.0047	-0.0049	-0.0069	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
3.1	-0.0043	-0.0036	-0.0037	-0.0039	-0.0042	-0.0042	-0.0061	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
3.4	-0.0038	-0.0031	-0.0033	-0.0035	-0.0037	-0.0037	-0.0053	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
3.7	-0.0034	-0.0028	-0.0029	-0.0031	-0.0033	-0.0033	-0.0048	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4	-0.0031	-0.0025	-0.0026	-0.0028	-0.0029	-0.003	-0.0043	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.3	-0.0028	-0.0022	-0.0023	-0.0025	-0.0026	-0.0027	-0.0038	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.6	-0.0025	-0.002	-0.0021	-0.0023	-0.0024	-0.0024	-0.0035	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
4.9	-0.0024	-0.0019	-0.002	-0.0022	-0.0023	-0.0023	-0.0033	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-0.0027	-0.0022	-0.0016	-0.0012	-0.0008	-0.0005	-7E-05	-0.2485	-0.2063	-0.1611	-0.1175	-0.0833	-0.0483	-0.0067
0.4	-0.0025	-0.002	-0.0015	-0.0011	-0.0008	-0.0004	-6E-05	-0.224	-0.1874	-0.1471	-0.1077	-0.0767	-0.0445	-0.0062
0.7	-0.002	-0.0016	-0.0013	-0.0009	-0.0006	-0.0004	-5E-05	-0.1818	-0.1545	-0.1227	-0.0906	-0.0649	-0.0379	-0.0054
1	-0.0017	-0.0014	-0.0011	-0.0008	-0.0006	-0.0003	-5E-05	-0.1508	-0.1298	-0.104	-0.0774	-0.0557	-0.0327	-0.0046
1.3	-0.0014	-0.0012	-0.0009	-0.0007	-0.0005	-0.0003	-4E-05	-0.1272	-0.1106	-0.0894	-0.067	-0.0484	-0.0285	-0.0041
1.6	-0.0012	-0.001	-0.0008	-0.0006	-0.0004	-0.0002	-3E-05	-0.1087	-0.0955	-0.0777	-0.0585	-0.0424	-0.0251	-0.0036
1.9	-0.0011	-0.0009	-0.0007	-0.0005	-0.0004	-0.0002	-3E-05	-0.094	-0.0832	-0.0681	-0.0515	-0.0375	-0.0223	-0.0032
2.2	-0.0009	-0.0008	-0.0006	-0.0005	-0.0003	-0.0002	-3E-05	-0.0821	-0.0732	-0.0602	-0.0458	-0.0334	-0.0199	-0.0029
2.5	-0.0008	-0.0007	-0.0006	-0.0004	-0.0003	-0.0002	-3E-05	-0.0724	-0.0649	-0.0537	-0.0409	-0.03	-0.0179	-0.0026
2.8	-0.0007	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-2E-05	-0.0643	-0.058	-0.0481	-0.0368	-0.027	-0.0162	-0.0023
3.1	-0.0006	-0.0006	-0.0005	-0.0003	-0.0002	-0.0001	-2E-05	-0.0575	-0.0521	-0.0434	-0.0333	-0.0245	-0.0147	-0.0021
3.4	-0.0006	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	-2E-05	-0.0517	-0.047	-0.0393	-0.0303	-0.0223	-0.0134	-0.002
3.7	-0.0005	-0.0005	-0.0004	-0.0003	-0.0002	-0.0001	-2E-05	-0.0467	-0.0427	-0.0358	-0.0276	-0.0204	-0.0123	-0.0018
4	-0.0005	-0.0004	-0.0003	-0.0003	-0.0002	-0.0001	-2E-05	-0.0425	-0.0389	-0.0327	-0.0253	-0.0187	-0.0113	-0.0017
4.3	-0.0004	-0.0004	-0.0003	-0.0002	-0.0002	-0.0001	-1E-05	-0.0388	-0.0356	-0.03	-0.0233	-0.0172	-0.0104	-0.0015
4.6	-0.0004	-0.0004	-0.0003	-0.0002	-0.0002	-1E-04	-1E-05	-0.0355	-0.0328	-0.0277	-0.0215	-0.0159	-0.0096	-0.0014
4.9	-0.0004	-0.0003	-0.0003	-0.0002	-0.0002	-9E-05	-1E-05	-0.034	-0.0314	-0.0266	-0.0206	-0.0153	-0.0093	-0.0014

D_0

Times original value	%change FRP heave ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP hor ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	-1.3225	-1.3543	-1.291	-1.2839	-1.0247	-0.815	-0.4053	0.00232	0.00155	0.001	0.00086	0.00075	0.00063	0.0004
0.833333333	-0.3761	-0.4449	-0.4524	-0.4739	-0.3656	-0.2883	-0.1395	0.00274	0.00179	0.00129	0.00101	0.00092	0.00078	0.00061
1.25	1.02961	0.94836	0.86052	0.81951	0.67278	0.53867	0.27968	0.00324	0.00214	0.00172	0.00136	0.00116	0.00102	0.00092
1.666666667	-0.2199	-0.1887	-0.1556	-0.1339	-0.1148	-0.0905	-0.0292	0.00338	0.0023	0.00171	0.00148	0.00125	0.00111	0.00105
2.083333333	-0.5329	-0.5014	-0.4554	-0.4348	-0.3538	-0.2781	-0.1221	0.00448	0.00304	0.00229	0.00187	0.00171	0.0015	0.00129
2.5	0.01145	0.01608	0.01441	0.01378	0.01401	0.01884	0.0304	0.00496	0.00343	0.00268	0.00217	0.0019	0.00175	0.00157
2.916666667	-0.4014	-0.3719	-0.3386	-0.3233	-0.2594	-0.1992	-0.0728	0.00527	0.00386	0.00304	0.00256	0.00224	0.00209	0.00194
3.333333333	0.03745	0.03091	0.02925	0.02682	0.02726	0.02762	0.04085	0.00613	0.00459	0.00374	0.00311	0.00285	0.00256	0.00223

Times original value	%change FRP ver ero ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	%change FRP ver tra ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.416666667	-0.0025	-0.0029	-0.0025	-0.0019	-0.0014	-0.0008	-0.0001	-0.223	-0.2655	-0.2413	-0.1884	-0.1384	-0.0823	-0.0115
0.833333333	-0.0008	-0.0015	-0.0015	-0.0013	-0.0009	-0.0006	-8E-05	-0.0886	-0.1508	-0.151	-0.1253	-0.0947	-0.0577	-0.0083
1.25	0.00126	0.00022	-0.0002	-0.0003	-0.0003	-0.0002	-3E-05	0.08392	0.00319	-0.0277	-0.0364	-0.0331	-0.0228	-0.0036
1.666666667	0.00195	0.00093	0.00041	0.00015	2.8E-05	-2E-05	-7E-06	0.14166	0.06536	0.02523	0.00529	-0.0029	-0.005	-0.0011
2.083333333	0.00228	0.00132	0.00076	0.00042	0.00024	0.00011	1.1E-05	0.17248	0.10249	0.05953	0.03255	0.01817	0.00778	0.00071
2.5	0.00242	0.00154	0.00099	0.00061	0.00038	0.0002	2.4E-05	0.18848	0.12515	0.08255	0.05194	0.03292	0.0171	0.0021
2.916666667	0.0025	0.00171	0.00117	0.00077	0.00051	0.00027	3.6E-05	0.19961	0.14244	0.1004	0.0678	0.04542	0.025	0.0033
3.333333333	0.00255	0.0018	0.00126	0.00085	0.00057	0.00031	4.2E-05	0.20587	0.15126	0.10959	0.07564	0.0519	0.02898	0.00391

Figure A.19: k_2, k_3 and D_0

D_1

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.178571429	0.03413	0.02821	0.0295	0.03157	0.03331	0.03409	0.04884	0.00254	0.00235	0.00224	0.00219	0.00216	0.00212	0.00215	0.00468	0.0041	0.00334	0.00251	0.00182	0.00108	0.00016	0.38348	0.35279	0.29635	0.22859	0.16868	0.1015	0.01489
0.357142857	0.02906	0.02404	0.0251	0.02684	0.0283	0.02897	0.04145	0.00218	0.00201	0.00191	0.00187	0.00184	0.00181	0.00183	0.00393	0.00341	0.00276	0.00206	0.00149	0.00088	0.00013	0.32801	0.29857	0.24897	0.191	0.1404	0.08416	0.01228
0.535714286	0.0219	0.01815	0.01891	0.02018	0.02124	0.02176	0.03106	0.00167	0.00152	0.00145	0.00142	0.00139	0.00136	0.00138	0.00285	0.00242	0.00193	0.00143	0.00102	0.0006	8.5E-05	0.24664	0.21959	0.18028	0.13667	0.09963	0.05923	0.00854
0.714285714	0.01842	0.01531	0.01593	0.01698	0.01785	0.01831	0.0261	0.00141	0.00129	0.00122	0.00119	0.00117	0.00115	0.00116	0.0023	0.00193	0.00152	0.00112	0.0008	0.00047	6.6E-05	0.20277	0.17803	0.14471	0.10887	0.07893	0.04667	0.00669
0.892857143	0.0162	0.0135	0.01404	0.01496	0.01572	0.01614	0.02298	0.00124	0.00114	0.00108	0.00105	0.00103	0.00101	0.00102	0.00195	0.00162	0.00127	0.00093	0.00066	0.00038	5.4E-05	0.17351	0.15074	0.1216	0.09094	0.06566	0.03867	0.00551
1.071428571	0.01555	0.01299	0.01351	0.01438	0.01511	0.01552	0.02209	0.0012	0.00109	0.00104	0.00101	0.00099	0.00097	0.00098	0.0018	0.00149	0.00116	0.00084	0.0006	0.00035	4.9E-05	0.16177	0.13928	0.11163	0.08307	0.05977	0.03508	0.00498
1.25	0.01545	0.01295	0.01345	0.01432	0.01503	0.01546	0.02198	0.00119	0.00109	0.00103	0.00101	0.00099	0.00097	0.00098	0.00173	0.00142	0.0011	0.0008	0.00056	0.00033	4.6E-05	0.15619	0.13343	0.10634	0.07878	0.05651	0.03307	0.00468
1.428571429	0.01518	0.01274	0.01323	0.01408	0.01478	0.0152	0.02161	0.00117	0.00107	0.00102	0.00099	0.00097	0.00095	0.00096	0.00167	0.00136	0.00105	0.00076	0.00054	0.00031	4.3E-05	0.15106	0.12854	0.10215	0.07551	0.05408	0.0316	0.00446

L_cutline

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.25	0	0	0	0	-6E-15	0	0	0.00627	0.00535	0.00473	0.00447	0.00418	0.00378	0.00302	0.00014	0.00011	8.7E-05	6.3E-05	4.4E-05	2.5E-05	3.4E-06	0	-2E-14	0	0	0	0	0
0.5	0	0	0	0	-3E-15	0	-8E-16	0.00606	0.00524	0.0047	0.00447	0.00424	0.00389	0.00304	0.00014	0.00012	8.8E-05	6.3E-05	4.4E-05	2.5E-05	3.4E-06	0	-8E-15	0	0	0	0	0
0.75	0	0	0	0	0	0	-8E-16	0.0054	0.00473	0.00434	0.00415	0.00399	0.00376	0.00313	0.00015	0.00012	8.9E-05	6.4E-05	4.5E-05	2.6E-05	3.4E-06	0	0	0	0	0	0	0
1	0	0	0	0	0	0	7.9E-16	0.00464	0.00411	0.00382	0.00367	0.00357	0.00343	0.00308	0.00015	0.00012	9E-05	6.4E-05	4.5E-05	2.6E-05	3.5E-06	0	0	0	7.9E-15	0	0	0
1.25	0	0	0	0	3.1E-15	0	0	0.00483	0.00425	0.00392	0.00374	0.00361	0.00344	0.00294	0.00015	0.00012	9.1E-05	6.5E-05	4.6E-05	2.6E-05	3.5E-06	-2E-14	0	0	7.9E-15	0	0	0
1.5	0	0	0	0	3.1E-15	0	-2E-15	0.00495	0.00431	0.00395	0.00375	0.00359	0.0034	0.00292	0.00015	0.00012	9.2E-05	6.6E-05	4.6E-05	2.6E-05	3.5E-06	0	0	0	-8E-15	0	0	0
1.75	0	0	0	0	-3E-15	0	-8E-16	0.00397	0.00352	0.00328	0.00316	0.00308	0.00301	0.00287	0.00015	0.00012	9.3E-05	6.7E-05	4.7E-05	2.7E-05	3.6E-06	0	0	0	0	0	0	0
2	0	0	0	0	-3E-15	0	7.9E-16	0.00374	0.00332	0.0031	0.00297	0.00289	0.00281	0.00263	0.00016	0.00012	9.4E-05	6.7E-05	4.7E-05	2.7E-05	3.6E-06	-2E-14	0	0	7.9E-15	0	0	0
2.25	0	0	0	0	0	0	7.9E-16	0.004	0.00352	0.00326	0.00309	0.00299	0.00286	0.00259	0.00016	0.00013	9.5E-05	6.8E-05	4.8E-05	2.7E-05	3.7E-06	7.9E-15	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0	0.00388	0.00342	0.00317	0.00302	0.00295	0.00287	0.00273	0.00016	0.00013	9.6E-05	6.8E-05	4.8E-05	2.7E-05	3.7E-06	0	0	0	0	0	0	0

d_rep

	%change FRP heave							%change FRP hor ero							%change FRP ver ero							%change FRP ver tra						
Times original value	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20	ng = 4	ng = 5	ng = 6	ng = 7	ng = 8	ng = 10	ng = 20
0.1	-5.8103	-5.707	-5.4021	-4.8527	-4.2922	-3.3052	-0.6244	-6.1564	-5.4098	-4.7892	-4.2463	-3.7737	-3.0007	-0.6713	-0.5748	-0.5684	-0.5507	-0.5273	-0.5128	-0.4666	-0.4094	0.22415	0.49478	1.18974	2.05433	2.57786	4.27814	7.58993
0.4	-3.3606	-3.1254	-2.823	-2.5261	-2.2153	-1.684	-0.2777	-3.1741	-2.7745	-2.4367	-2.1514	-1.9113	-1.5167	-0.3458	-0.3165	-0.3093	-0.2971	-0.2887	-0.2821	-0.2738	-0.2836	1.01036	1.46234	2.24378	2.79387	3.22769	3.77676	4.27873
0.7	-0.5147	-0.3288	-0.1526	-0.0906	-0.0552	-0.0244	0.06029	-0.1089	-0.0794	-0.049	-0.034	-0.0295	-0.0209	-0.0137	-0.038	-0.0334	-0.0367	-0.0439	-0.0481	-0.0681	-0.1143	2.00156	2.58106	2.75843	2.74098	2.768	2.19236	0.5662
1	-0.1009	-0.0483	-0.0176	0.02427	0.019	0.03267	0.05986	-0.0169	-0.013	-0.0096	-0.0083	-0.0074	-0.0066	-0.0054	-0.0164	-0.0176	-0.0266	-0.0332	-0.0385	-0.0453	-0.055	1.9003	2.18175	1.77665	1.43643	1.14252	0.73684	0.10706
1.3	-0.0329	0.01658	0.01311	0.01618	0.02474	0.03804	0.06634	-0.006	-0.0048	-0.0041	-0.0038	-0.0036	-0.0033	-0.003	-0.0129	-0.0177	-0.0217	-0.025	-0.0272	-0.0294	-0.0318	1.49922	1.21391	0.91262	0.63681	0.44714	0.25185	0.03456
1.6	0.01166	0.00696	0.01514	0.01924	0.02681	0.0363	0.06189	-0.0031	-0.0026	-0.0024	-0.0022	-0.0022	-0.0021	-0.0019	-0.0113	-0.0153	-0.0172	-0.0184	-0.0192	-0.0199	-0.0207	1.01597	0.60849	0.41062	0.2816	0.19443	0.10748	0.01461
1.9	0.01649	0.00832	0.0279	0.02552	0.02554	0.03659	0.05751	-0.0018	-0.0017	-0.0016	-0.0015	-0.0015	-0.0014	-0.0013	-0.0103	-0.0121	-0.0129	-0.0135	-0.0139	-0.0142	-0.0146	0.57335	0.34296	0.2388	0.16275	0.0976	0.05552	0.00709
2.2	0.00565	0.02154	0.01652	0.02199	0.02841	0.02578	0.04991	-0.0013	-0.0012	-0.0011	-0.0011	-0.0011	-0.001	-0.0009	-0.0085	-0.0095	-0.0101	-0.0103	-0.0105	-0.0107	-0.0108	0.37936	0.21999	0.12195	0.08504	0.05849	0.02898	0.00379
2.5	0.00118	0.02619	0.01853	0.02361	0.02687	0.03311	0.04841	-0.0009	-0.0009	-0.0008	-0.0008	-0.0008	-0.0008	-0.0007	-0.0072	-0.0076	-0.0079	-0.0081	-0.0082	-0.0083	-0.0083	0.20193	0.1408	0.07861	0.04791	0.03474	0.01817	0.00224
2.8	0.01959	0.01506	0.02709	0.02358	0.02692	0.032	0.05024	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0006	-0.0059	-0.0062	-0.0064	-0.0065	-0.0065	-0.0066	-0.0066	0.15767	0.08575	0.04897	0.03227	0.0219	0.0111	0.00143
3.1	0.02	0.01228	0.01833	0.02971	0.02536	0.03075	0.04824	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0049	-0.0052	-0.0053	-0.0053	-0.0053	-0.0054	-0.0054	0.11329	0.0533	0.03414	0.02265	0.01399	0.00741	0.00095
3.4	0.0101	0.02181	0.01813	0.02058	0.02522	0.03053	0.04607	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0042	-0.0043	-0.0044	-0.0044	-0.0045	-0.0045	-0.0045	0.07467	0.03734	0.02391	0.01443	0.00961	0.00523	0.00065
3.7	0.00887	0.0135	0.02502	0.01973	0.02413	0.02341	0.04099	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003	-0.0003	-0.0036	-0.0037	-0.0037	-0.0037	-0.0038	-0.0038	-0.0038	0.05642	0.02914	0.0163	0.01009	0.00729	0.00357	0.00047
4	0.01799	0.01488	0.01737	0.02069	0.02444	0.02824	0.04093	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0031	-0.0032	-0.0032	-0.0032	-0.0032	-0.0032	-0.0032	0.0366	0.02227	0.01267	0.00782	0.0053	0.00273	0.00034
4.3	0.01846	0.02167	0.01565	0.01959	0.02299	0.02745	0.04172	-0.0003	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0027	-0.0027	-0.0028	-0.0028	-0.0028	-0.0028	-0.0028	0.03103	0.01586	0.00872	0.00552	0.00395	0.00197	0.00026
4.6	0.01063	0.01371	0.01886	0.02403	0.02529	0.02685	0.04112	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0024	-0.0024	-0.0024	-0.0024	-0.0024	-0.0024	-0.0024	0.02387	0.01123	0.007	0.00445	0.00301	0.00153	0.0002
4.9	0.00361	0.00791	0.01514	0.02196	0.0251	0.03201	0.04207	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0022	-0.0023	-0.0023	-0.0023	-0.0023	-0.0023	-0.0023	0.01984	0.00949	0.0061	0.00411	0.00261	0.00142	0.00017

Figure A.20: D_1, $L_{Cut\ line}$ and d_{rep}

A.2. Local parameter tables

% change of each FRP as result of 10% increase in parameter value at Hedwigepolder value				
Parameter	% change heave	% change hor ero	% change ver ero	% change ver tra
Ac	0.00%	85.19%	59.10%	0.00%
c	0.00%	85.19%	59.10%	0.00%
Anis_c	0.00%	0.00%	-66.49%	0.00%
Anis_k	-3.06%	-3.33%	-12.66%	-11.93%
k0	3.74%	32.40%	17.21%	-13.34%
k1	-1.87%	-16.11%	-8.72%	-3.44%
k2	-0.98%	-8.53%	-4.57%	-1.86%
k3	-1.01%	-8.76%	-1.48%	-1.91%
D0	2.21%	18.71%	10.82%	4.96%
D1	1.07%	9.28%	5.00%	2.16%
Cutline	0.00%	45.29%	16.65%	0.00%
d_rep	5.14%	-94.81%	-69.70%	-363.75%

Figure A.21: Local influence of all parameters at $0/3 * D_0$

% change of each FRP as result of 10% increase in parameter value at Hedwigepolder value				
Parameter	% change heave	% change hor ero	% change ver ero	% change ver tra
Ac	0.00%	11.01%	4.26%	0.00%
c	0.00%	11.01%	4.26%	0.00%
Anis_c	0.00%	0.00%	-4.79%	0.00%
Anis_k	-0.06%	0.72%	-0.35%	-6.16%
k0	0.23%	6.55%	1.89%	-3.35%
k1	-0.12%	-2.54%	-0.77%	-2.04%
k2	-0.05%	-1.23%	-0.36%	-0.99%
k3	-0.06%	-1.26%	-0.11%	-1.02%
D0	9.38%	4.84%	1.35%	-1.91%
D1	0.06%	1.36%	0.40%	1.12%
Cutline	0.00%	5.15%	0.94%	0.00%
d_rep	-65.80%	-22.38%	-8.31%	19.77%

Figure A.22: Local influence of all parameters at $1/3 * D_0$

% change of each FRP as result of 10% increase in parameter value at Hedwigepolder value				
Parameter	% change heave	% change hor ero	% change ver ero	% change ver tra
Ac	0.00%	143.26%	53.08%	0.00%
c	0.00%	143.26%	53.08%	0.00%
Anis_c	0.00%	0.00%	-59.72%	0.00%
Anis_k	-2.18%	16.91%	-5.90%	-70.36%
k0	8.17%	114.86%	30.56%	-18.21%
k1	-3.09%	-42.25%	-12.12%	-31.38%
k2	-1.25%	-17.72%	-4.85%	-13.90%
k3	-1.27%	-18.08%	-1.33%	-14.21%
D0	-103.56%	80.08%	17.26%	-5.35%
D1	1.40%	19.89%	5.49%	15.87%
Cutline	0.00%	71.88%	9.71%	0.00%
d_rep	-8.01%	-398.58%	-96.76%	190.75%

Figure A.23: Local influence of all parameters at $2/3 * D_0$

% change of each FRP as result of 10% increase in parameter value at Hedwigepolder value				
Parameter	% change heave	% change hor ero	% change ver ero	% change ver tra
Ac	0.00%	0.72%	3.61%	0.00%
c	0.00%	0.72%	3.61%	0.00%
Anis_c	0.00%	0.00%	3.61%	0.00%
Anis_k	-3.03%	0.00%	-0.71%	-67.19%
k0	10.40%	0.60%	0.15%	1.62%
k1	-5.89%	-0.23%	-0.60%	-46.11%
k2	-1.40%	-0.11%	-0.16%	-14.81%
k3	-1.42%	-0.11%	3.61%	-15.08%
D0	-37.61%	0.27%	-0.08%	-8.86%
D1	1.62%	0.12%	0.19%	17.35%
Cutline	0.00%	0.46%	0.01%	0.00%
d_rep	-10.09%	-1.69%	-1.64%	190.03%

Figure A.24: Local influence of all parameters at $3/3 * D_0$

A.3. Influence parameter graphs

The red dotted line indicated the default (Hedwigepolder) value.

$0/3 * D_0$

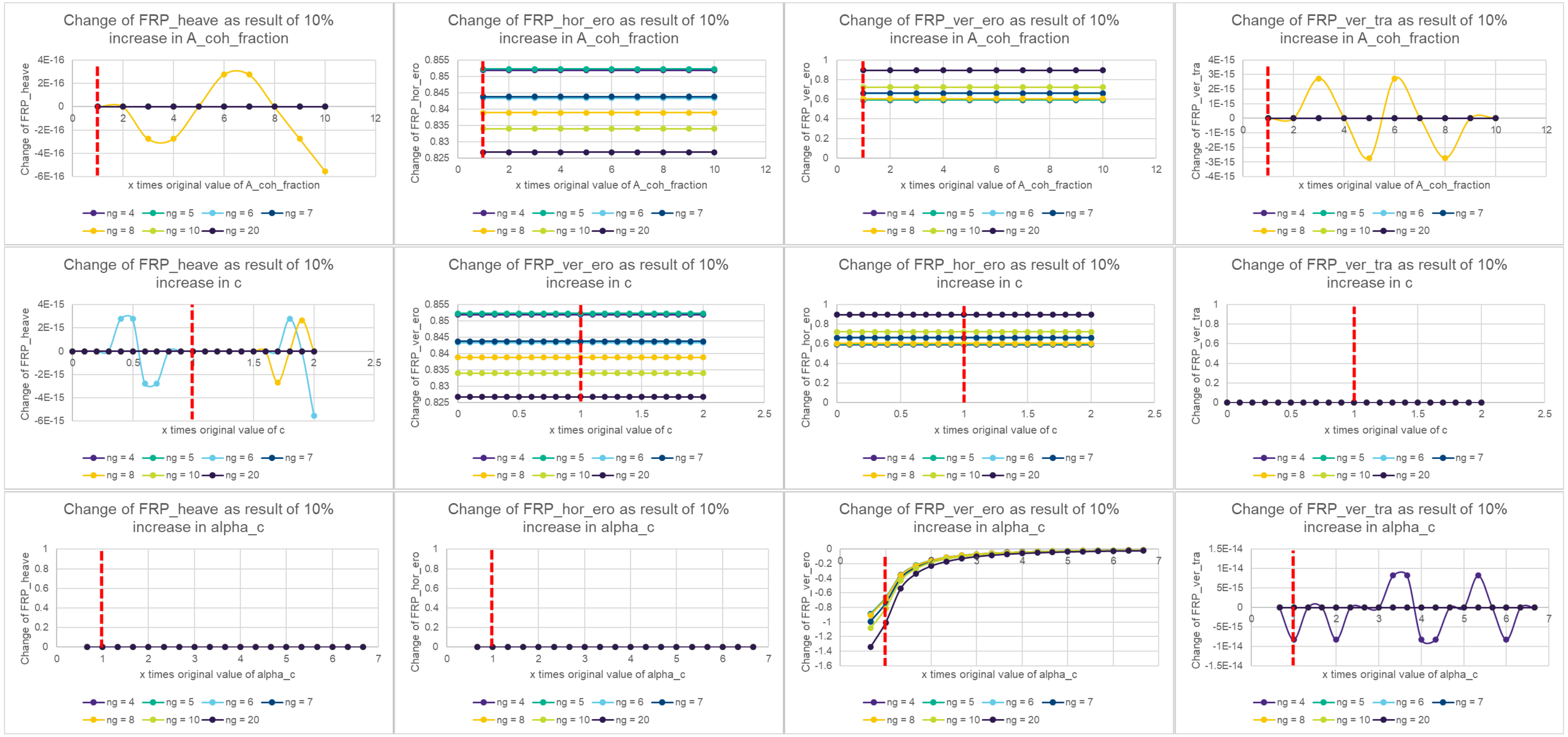

Figure A.25: A_c, c and $Anis_c$

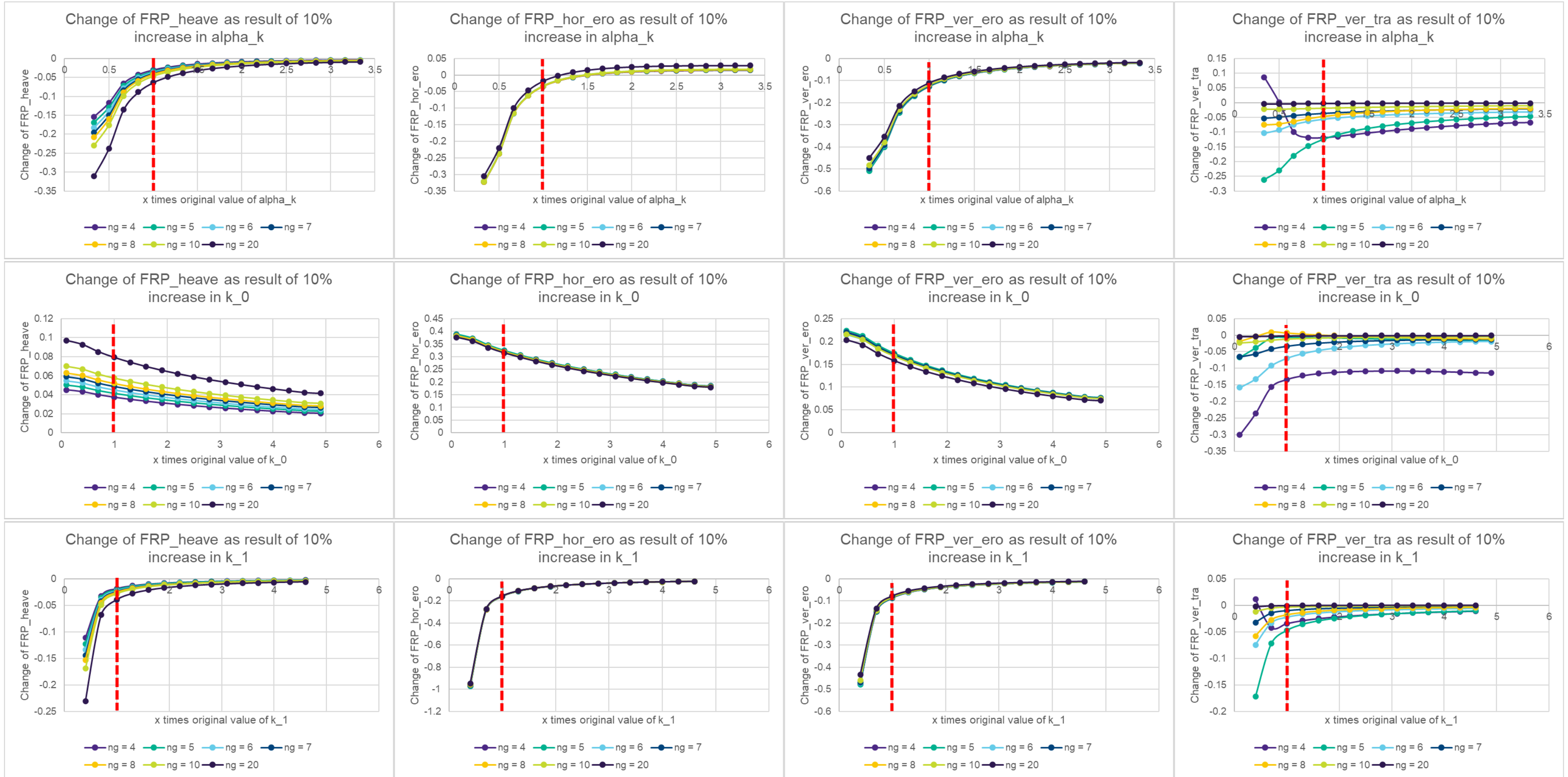

Figure A.26: $Anis_k$, k_0 and k_1

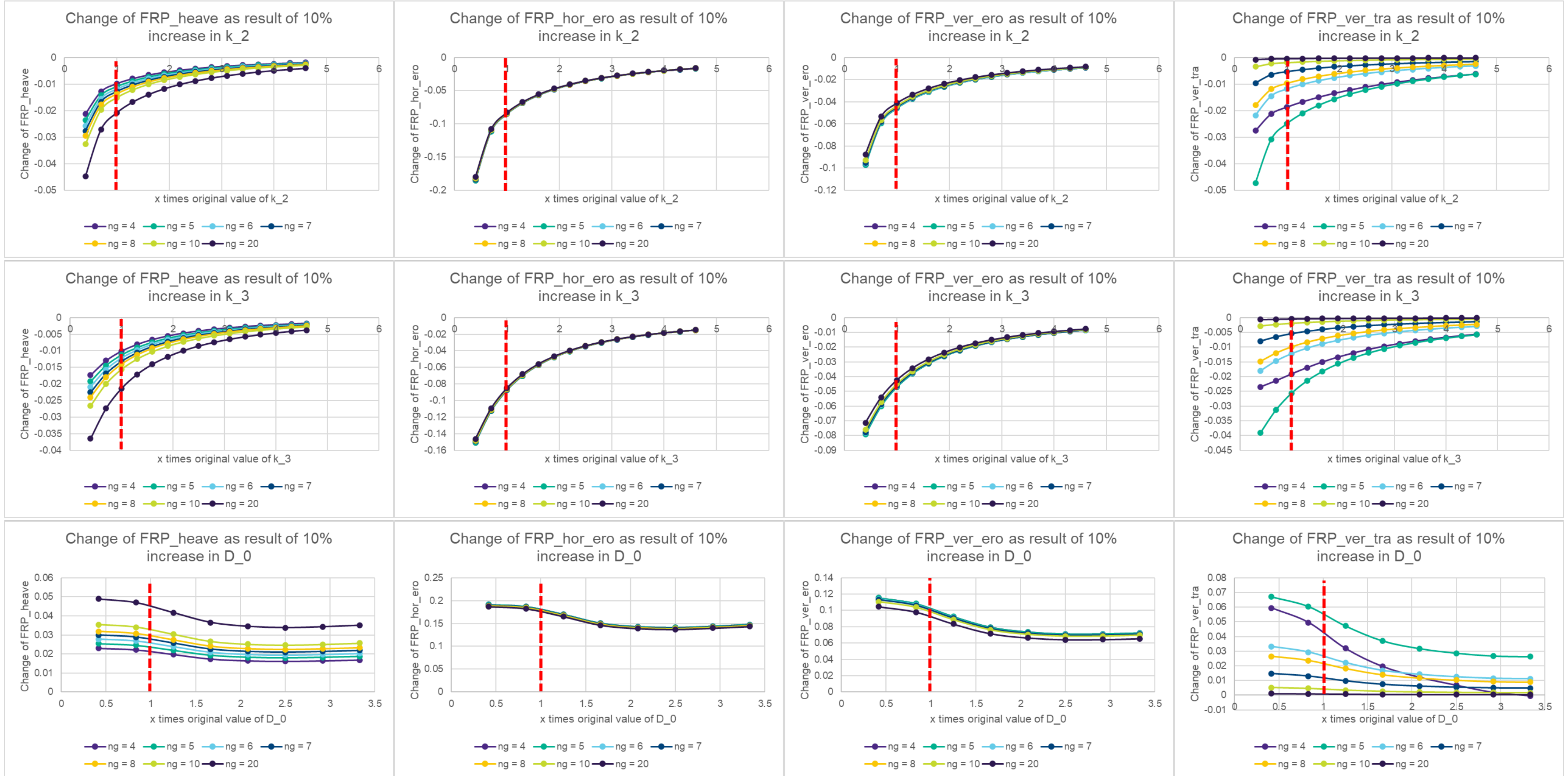

Figure A.27: k_2, k_3 and D_0

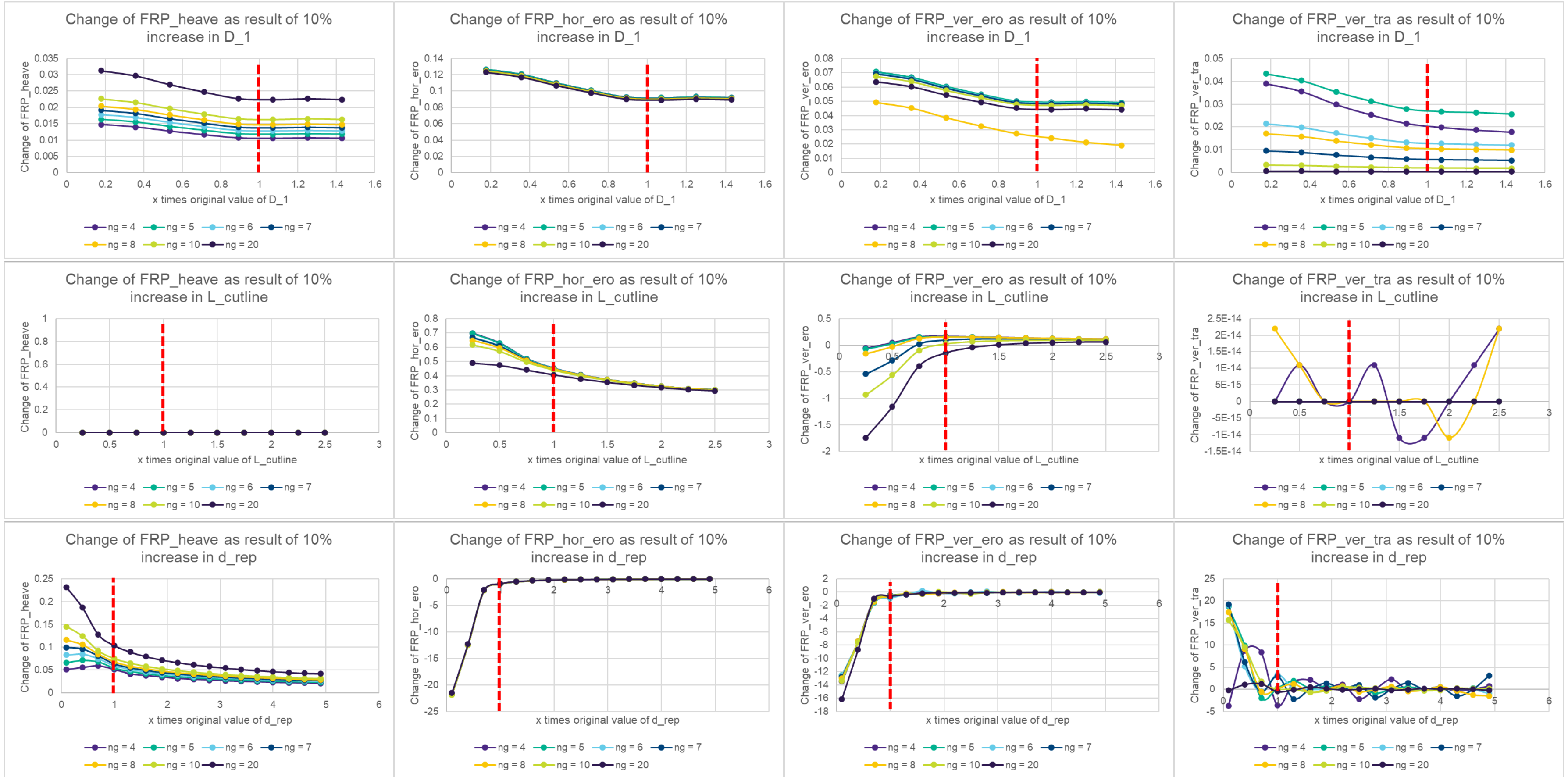

Figure A.28: D_1, $L_{Cut\ line}$ and d_{rep}

$1/3 * D_0$

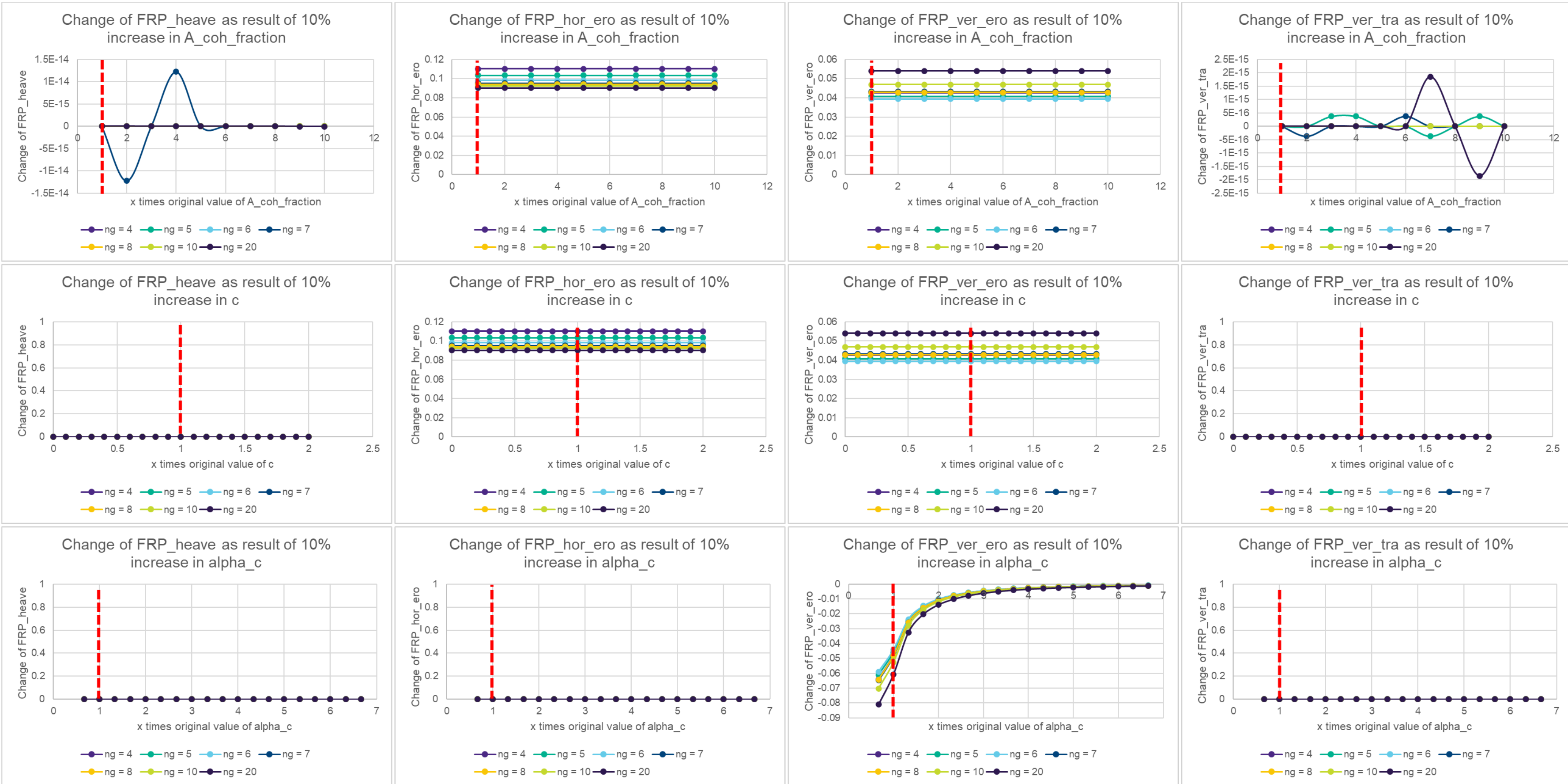

Figure A.29: A_c, c and $Anis_c$

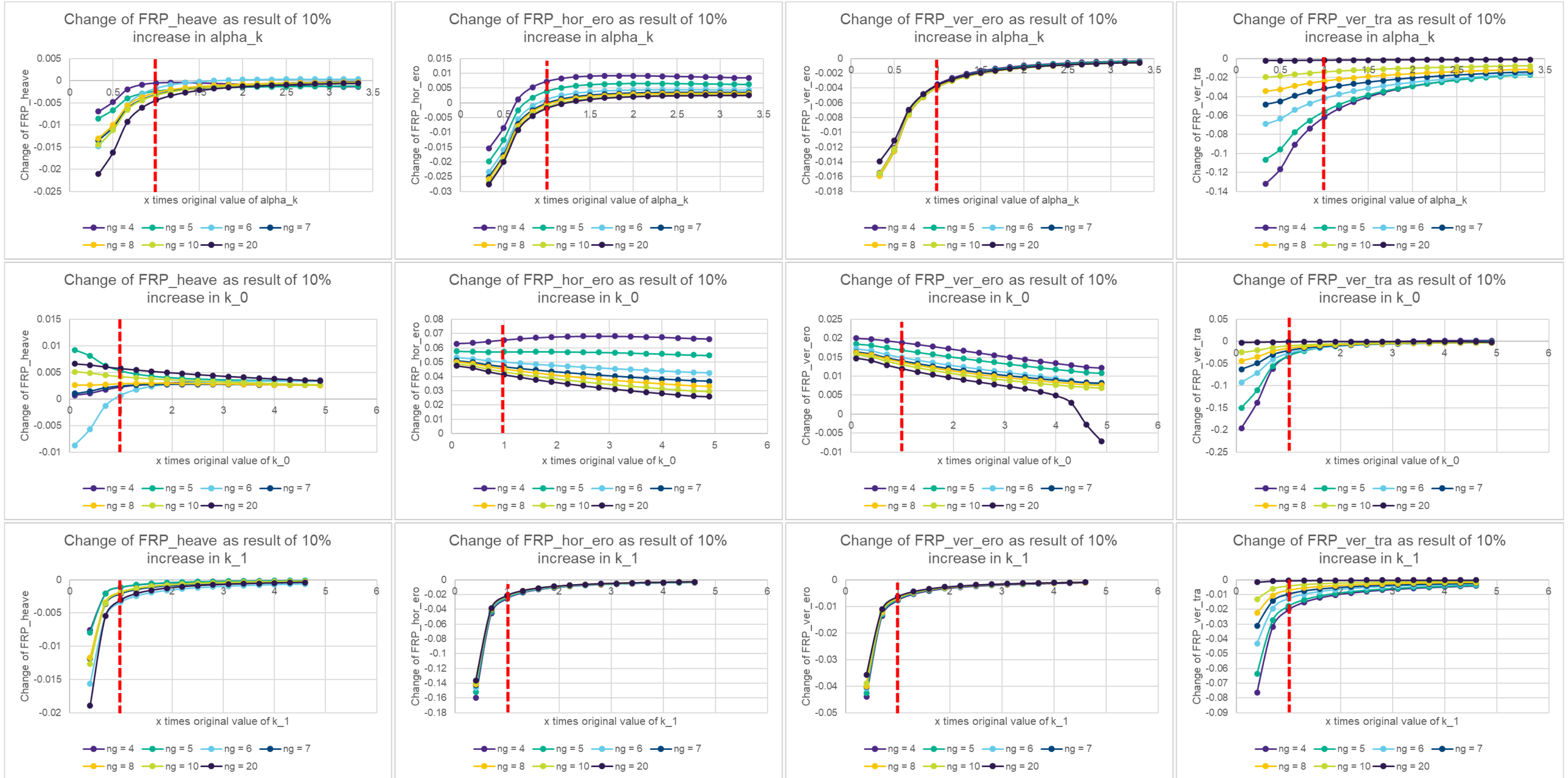

Figure A.30: $Anis_k$, k_0 and k_1

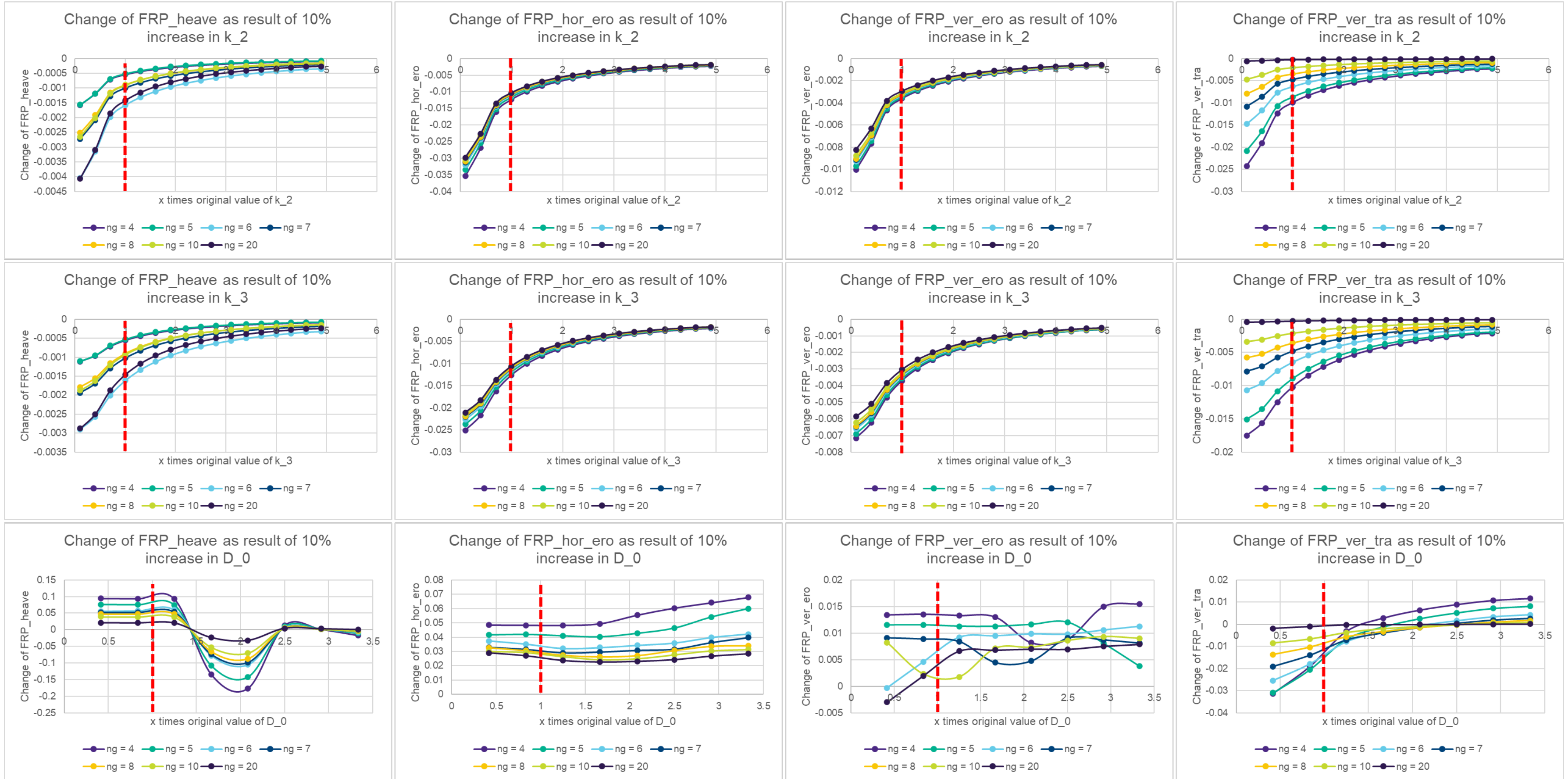

Figure A.31: k_2, k_3 and D_0

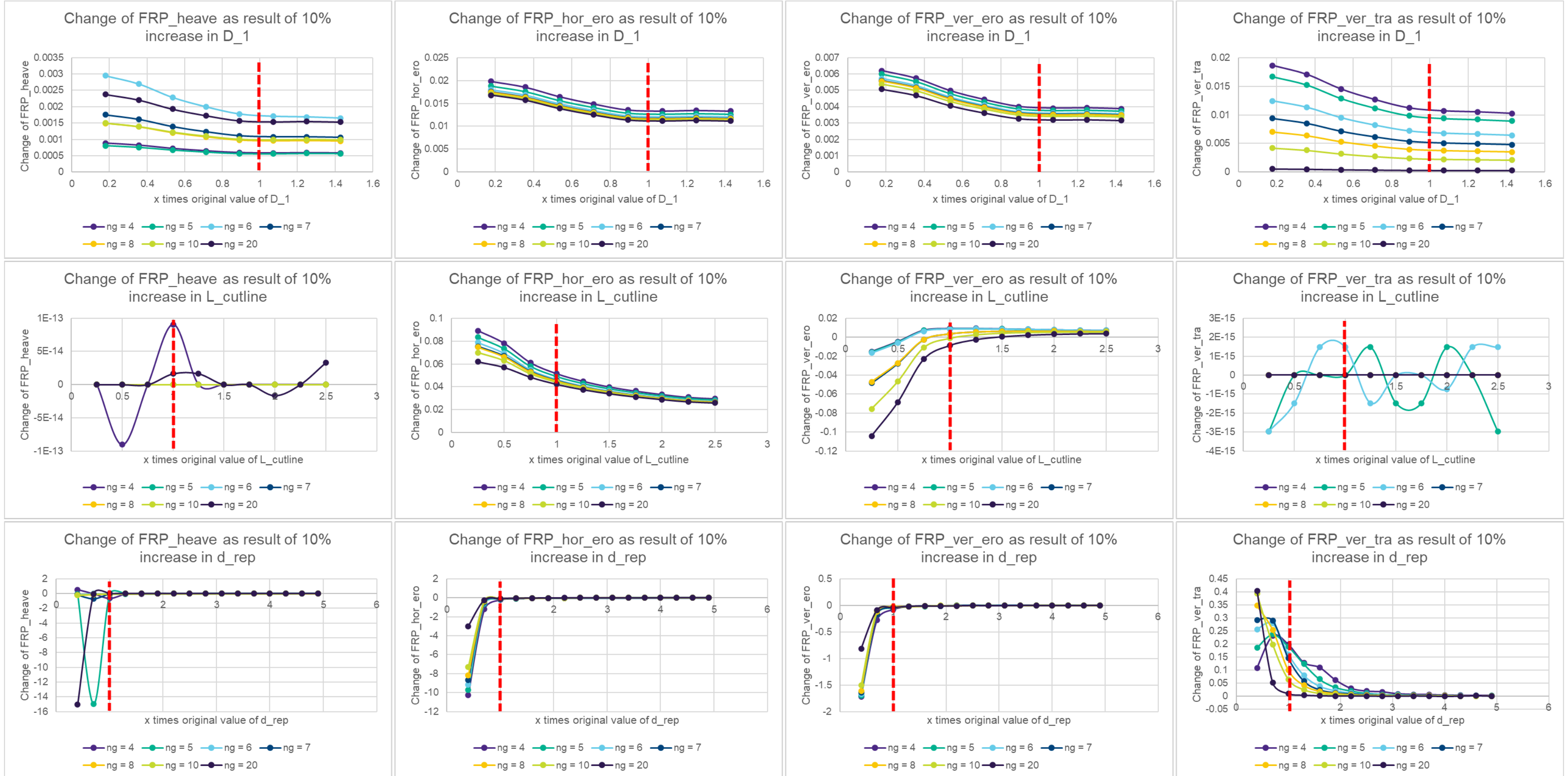

Figure A.32: D_1, $L_{Cut\ line}$ and d_{rep}

$2/3 * D_0$

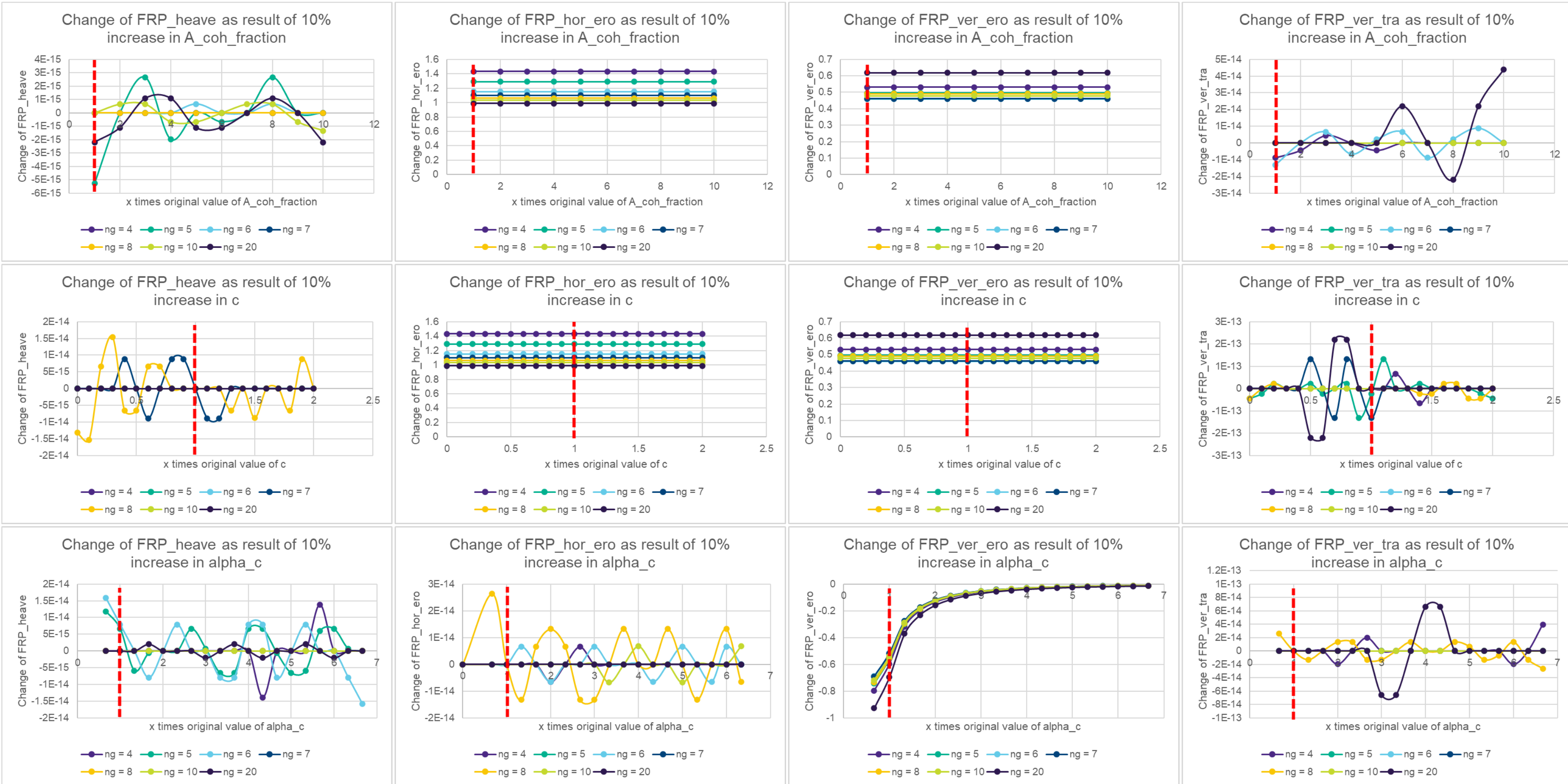

Figure A.33: A_c, c and $Anis_c$

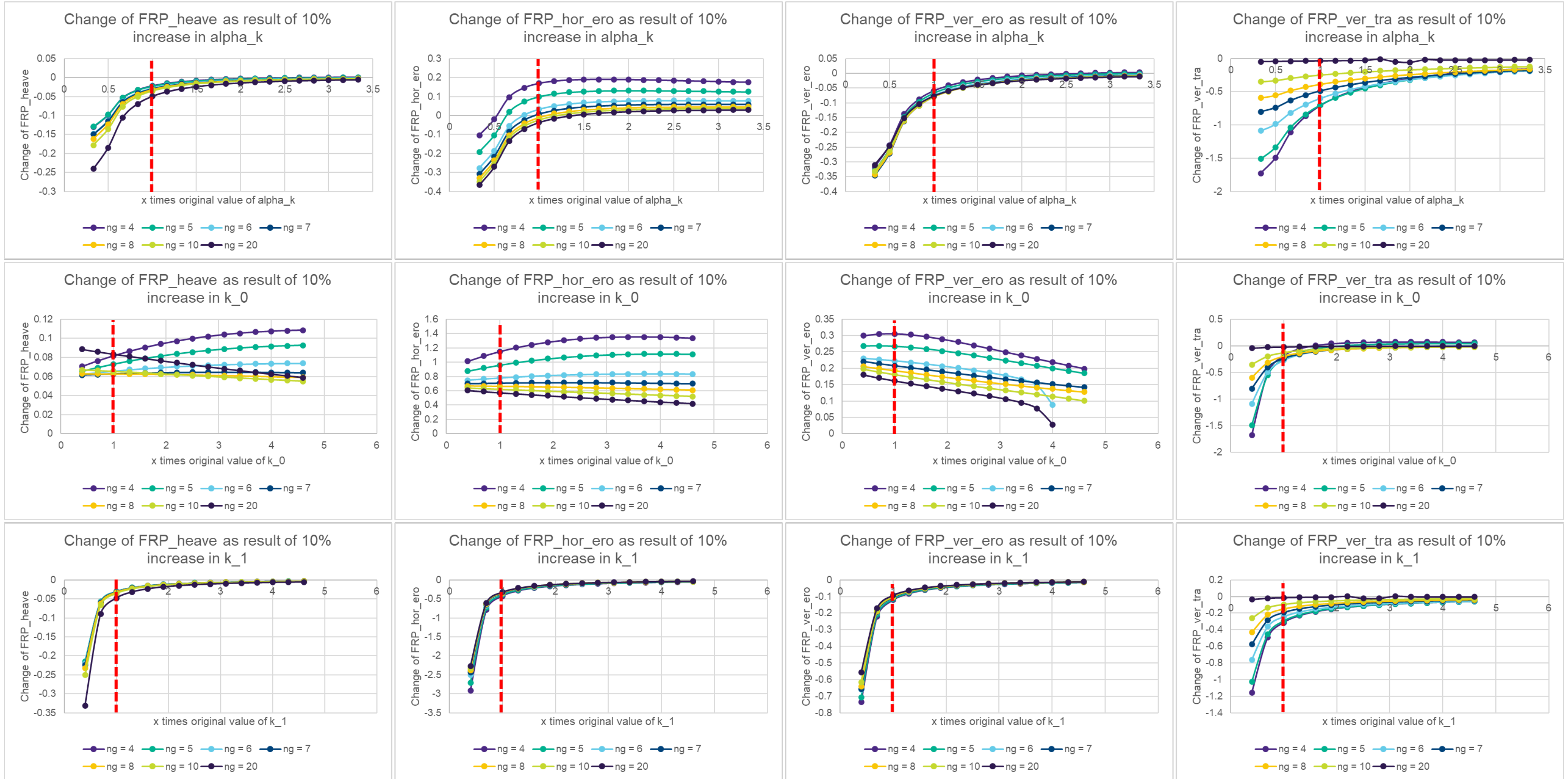

Figure A.34: $Anis_k$, k_0 and k_1

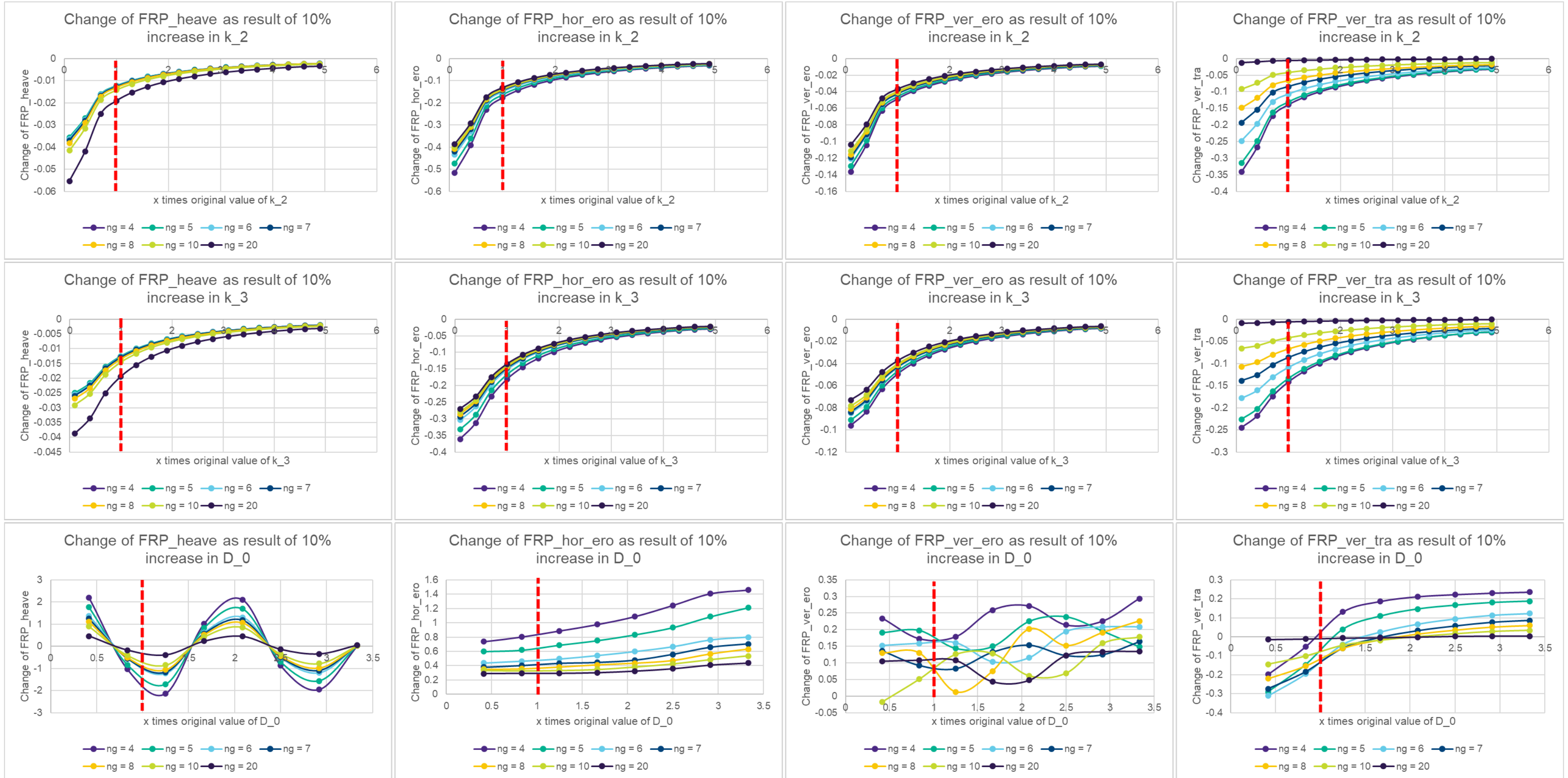

Figure A.35: k_2, k_3 and D_0

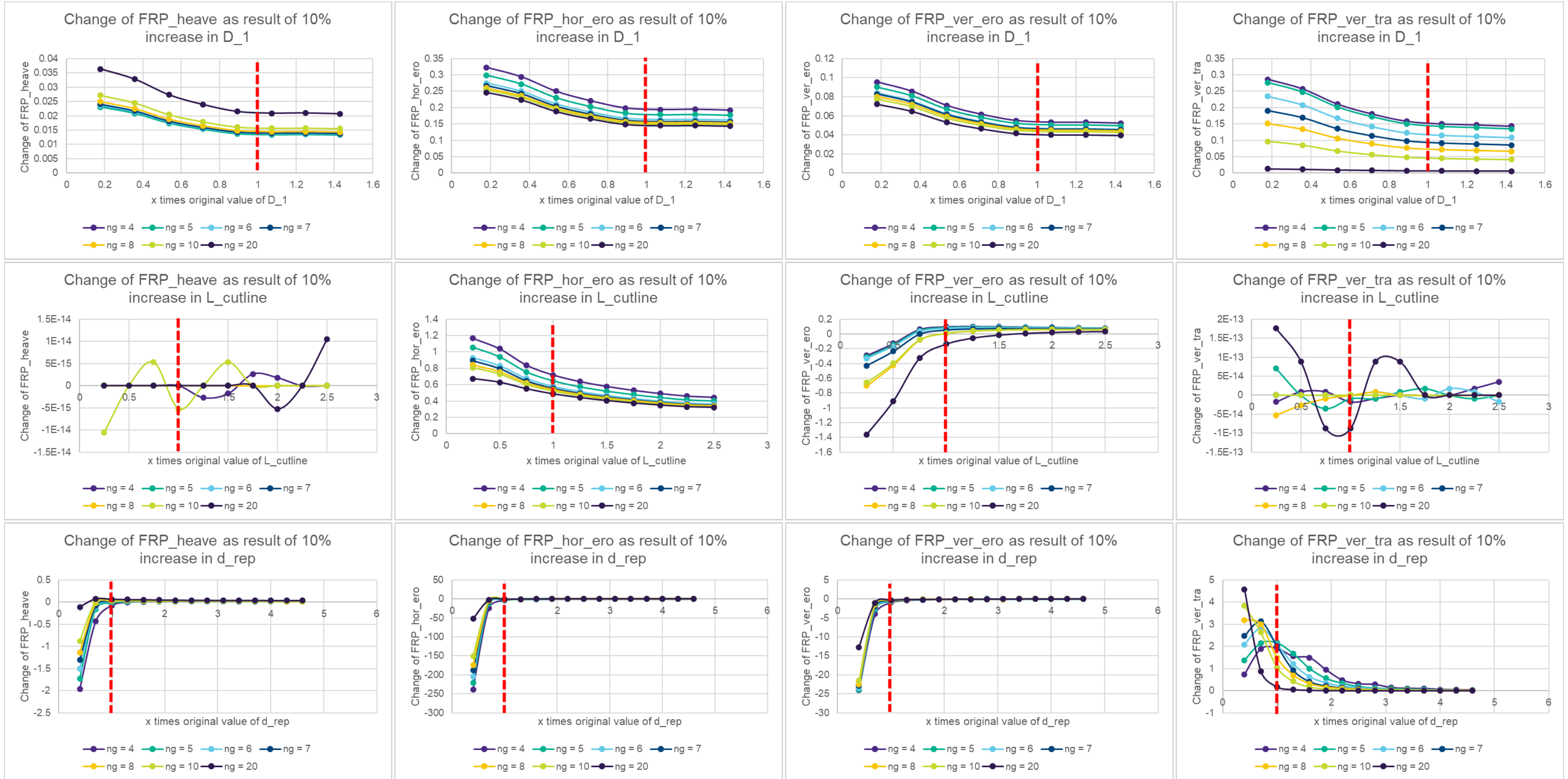

Figure A.36: D_1, $L_{Cut\ line}$ and d_{rep}

$3/3 * D_0$

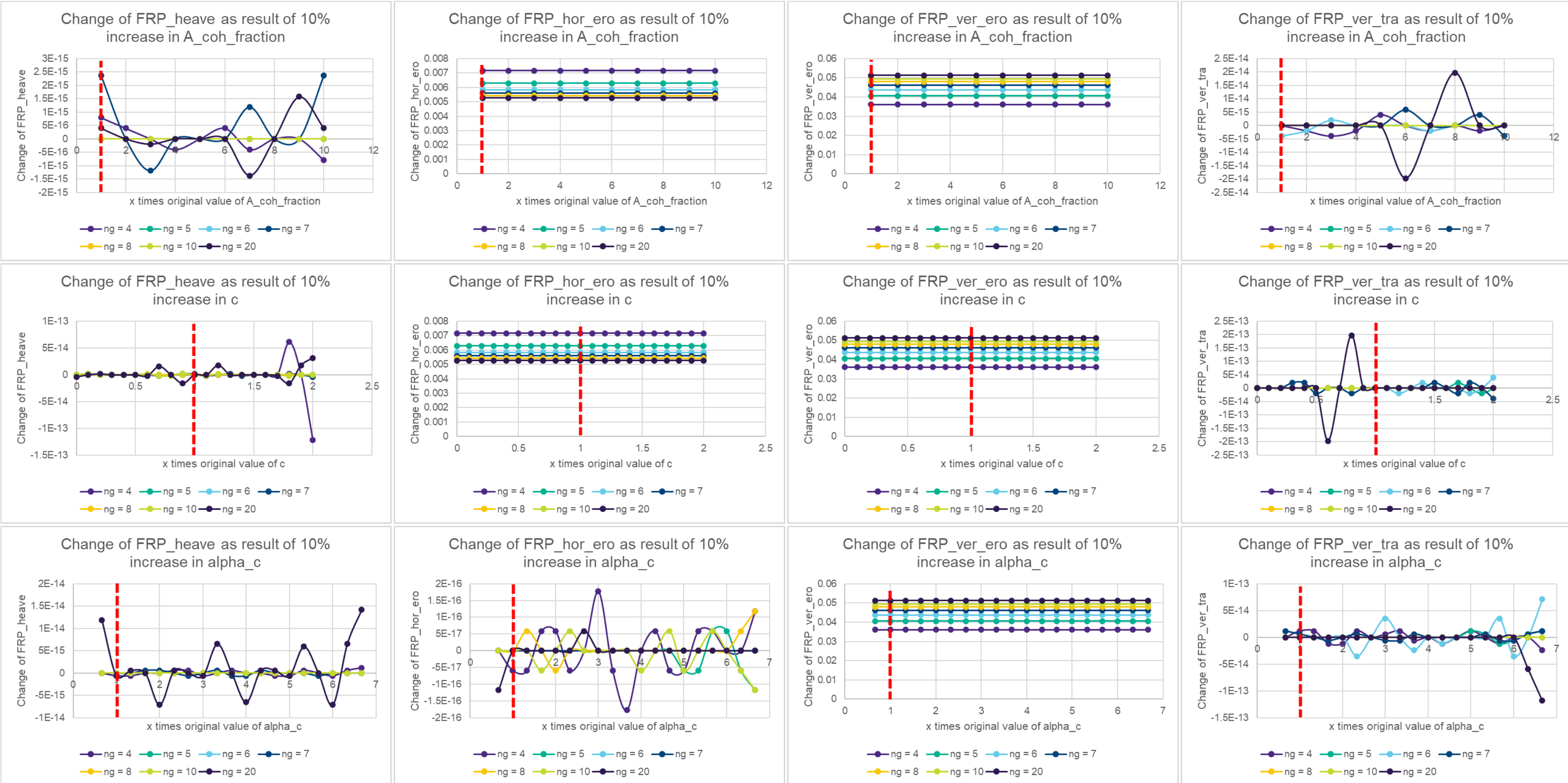

Figure A.37: A_c, c and $Anis_c$

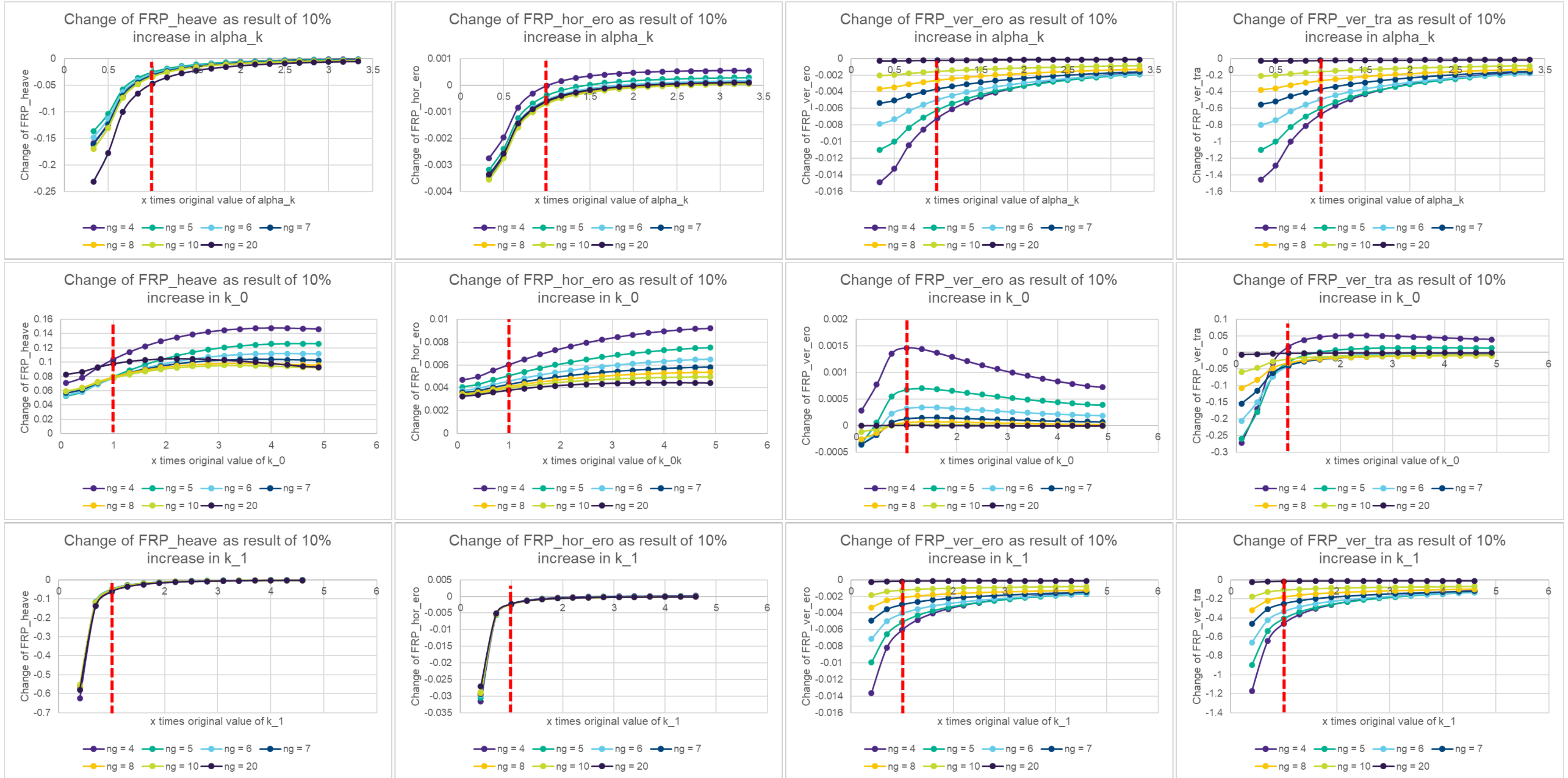

Figure A.38: $Anis_k$, k_0 and k_1

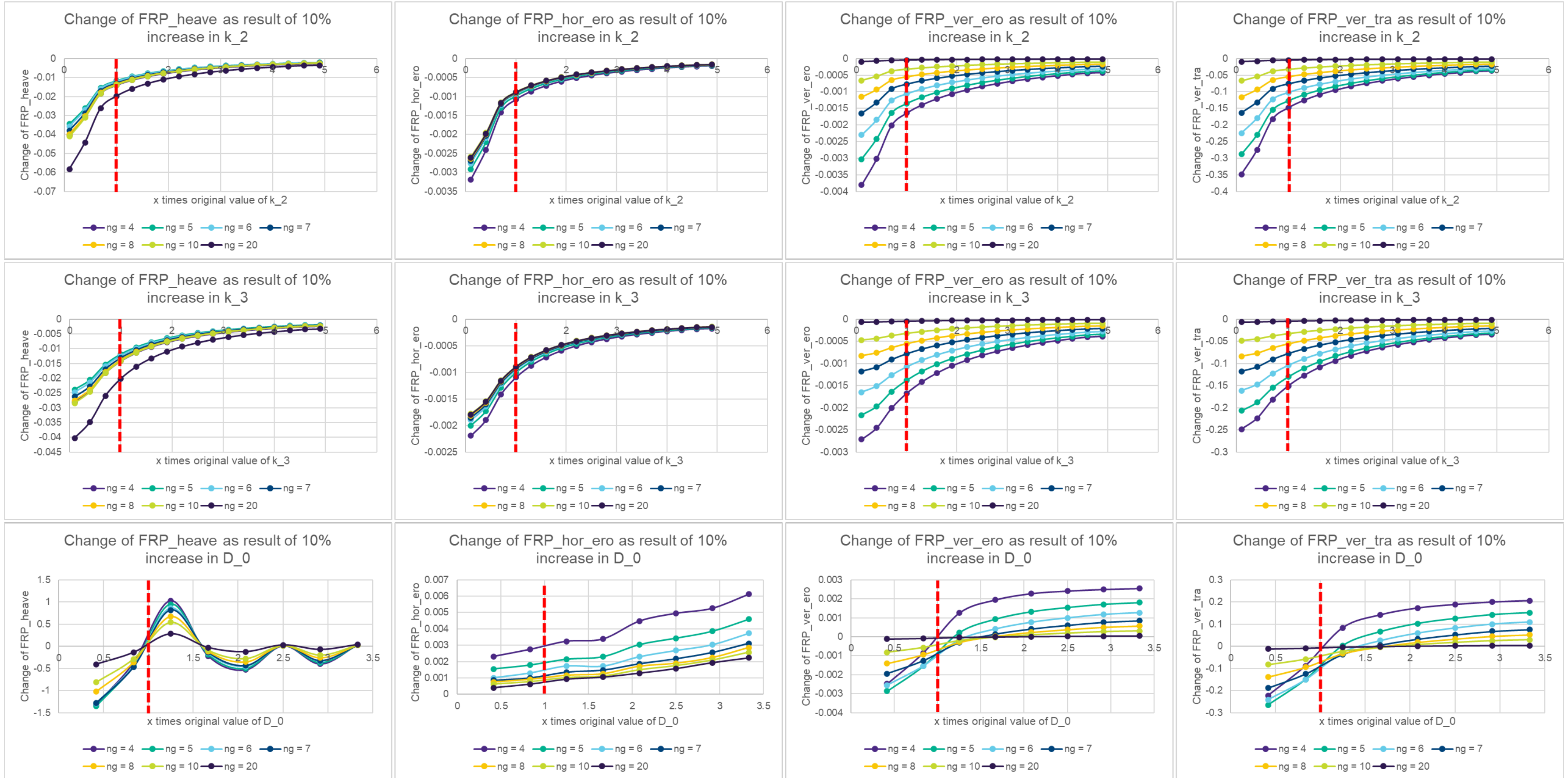

Figure A.39: k_2, k_3 and D_0

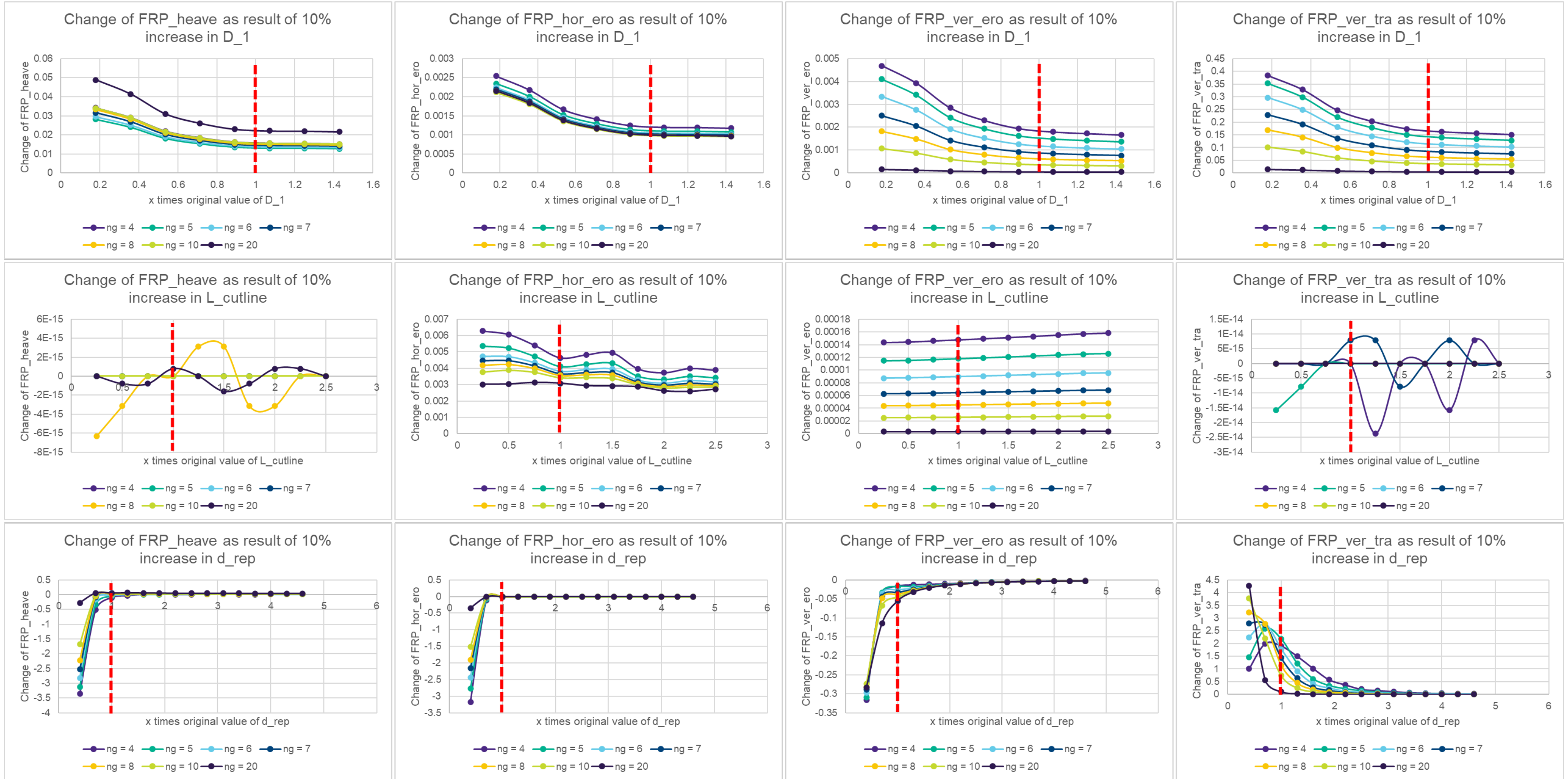

Figure A.40: D_1, $L_{Cut\ line}$ and d_{rep}