FAR-ZONE CONTRIBUTIONS TO THE GRAVITY FIELD QUANTITIES BY MEANS OF MOLODENSKY'S TRUNCATION COEFFICIENTS

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ABSTRACT

To reduce the numerical complexity of inverse solutions to large systems of discretised integral equations in gravimetric geoid/quasigeoid modelling, the surface domain of Green's integrals is subdivided into the near-zone and far-zone integration sub-domains. The inversion is performed for the near zone using regional detailed gravity data. The far-zone contributions to the gravity field quantities are estimated from an available global geopotential model using techniques for a spherical harmonic analysis of the gravity field. For computing the far-zone contributions by means of Green's integrals, truncation coefficients are applied. Different forms of truncation coefficients have been derived depending on a type of integrals in solving various geodetic boundary-value problems. In this study, we utilise Molodensky's truncation coefficients to Green's integrals for computing the far-zone contributions to the disturbing potential, the gravity disturbance, and the gravity anomaly. We also demonstrate that Molodensky's truncation coefficients can be uniformly applied to all types of Green's integrals used in solving the boundary-value problems. The numerical example of the far-zone contributions to the gravity field quantities is given over the area of study which comprises the Canadian Rocky Mountains. The coefficients of a global geopotential model and a detailed digital terrain model are used as input data.

Key words: far-zone contribution, geoid, gravity, Green's integrals, truncation coefficients

1. INTRODUCTION

For the gravimetric geoid/quasigeoid modelling from regional gravity data, the two-step approach is often used in practice. It involves the inverse of the Poisson integral equation (known as harmonic downward continuation) and consequently the

Stokes/Hotine integration. An alternative method to the two-step approach for computing the disturbing potential from regional gravity data by means of Green's integrals was formulated in *Novák (2003)*. The direct relation between the observed gravity disturbances/anomalies and the disturbing potential is defined in terms of a single integral equation. The inverse solution to the system of discretised integral equations (so-called the direct gravity inversion) directly provides the result in terms of the disturbing potential. To reduce the numerical complexity of the direct gravity inversion, the system of discretised integral equations is formed only for the near zone, while the far-zone contributions are treated separately. In *Tenzer and Novák (2008)*, the conditionality of inverse solutions to discretised Green's integral equations formed for the near zone was investigated. In this study, we derive the expressions for computing the corresponding far-zone contributions.

Molodensky's truncation coefficients (*Molodensky et al., 1960*) were reformulated for various deterministic and stochastic modifications of the Stokes kernel. Truncation coefficients for the Poisson kernel were implemented by *Witte (1967)*, *Martinec (1996)*, and *Huang (2002)*. In the above studies, the form of truncation coefficients depends on the type of integral kernel for which they are applied. A different concept for the definition of truncation coefficients is proposed in this study. The modified surface spherical harmonics for computing the far-zone contributions are introduced. The modified surface spherical harmonics are functionally related with the surface spherical harmonics by means of Molodensky's truncation coefficients. This relation allows to apply Molodensky's truncation coefficients uniformly to all types of Green's integrals used in solving the boundary-value problems. The optimalization of integral kernels by means of minimizing the far-zone contributions, and consequently the errors related to the evaluation of these quantities, are not discussed.

2. THEORY - SPHERICAL HARMONICS

Let us assume that the reference surface is approximated by the mean Earth's sphere of radius R. The disturbing potential T (i.e., difference between the true gravity potential and the normal gravity potential) on and outside this sphere is defined as (e.g., *Heiskanen and Moritz, 1967*, *Eq.2-152*)

$$T(r,\Omega) = \sum_{n=2}^{\infty} T_n(r,\Omega) = \sum_{n=2}^{\infty} \left(\frac{R}{r}\right)^{n+1} T_n(\Omega), \tag{1}$$

where $T_n(r,\Omega)$ are the solid spherical harmonics of T. The corresponding surface spherical harmonics $T_n(\Omega)$ in Eq.(1) read

$$T_n(\Omega) = \frac{2n+1}{4\pi} \iint_{\sigma} T(R,\Omega') P_n(\cos\psi)\, d\sigma = \sum_{m=-n}^{n} T_{n,m} Y_{n,m}(\Omega). \tag{2}$$

The 3-D position is defined in a frame of the geocentric spherical coordinates; where r is the geocentric radius, and Ω denotes the geocentric direction. In Eq.(2), σ is the unit sphere; P_n is the Legendre polynomial of degree n; ψ is the spherical angle between the external point $P(r, \Omega)$ and the running integration point $P'(R,\Omega')$; $Y_{n,m}(\Omega)$ and $T_{n,m}$ represent the surface spherical functions and their coefficients.

The gravity disturbance δg and the gravity anomaly Δg are defined in terms of spherical harmonics as follows (e.g., *Heiskanen and Moritz 1967, Eqs. 2-153 and 2-155*)

$$r\,\delta g(r,\Omega) = \sum_{n=2}^{\infty} (n+1)\left(\frac{R}{r}\right)^{n+1} T_n(\Omega)\,, \tag{3}$$

$$r\,\Delta g(r,\Omega) = \sum_{n=2}^{\infty} (n-1)\left(\frac{R}{r}\right)^{n+1} T_n(\Omega)\,. \tag{4}$$

3. FAR-ZONE CONTRIBUTIONS TO THE GRAVITY FIELD QUANTITIES

To derive the expression for computing the far-zone contribution to the gravity disturbance, we apply the binomial theorem to the radial component of the solid spherical harmonics in Eq.(3). For this, Eq.(3) is first rewritten as

$$r\,\delta g(r,\Omega) = \sum_{n=2}^{\infty} (n+1) T_n(\Omega)\left(1+\frac{r-R}{R}\right)^{-n-1}\,. \tag{5}$$

Application of the binomial theorem in Eq.(5) then yields

$$r\,\delta g(r,\Omega) = \sum_{n=2}^{\infty} (n+1) T_n(\Omega) \sum_{k=0}^{n} \binom{-n-1}{k}\left(\frac{r-R}{R}\right)^{k}\,. \tag{6}$$

From Eq.(6), we get

$$\begin{aligned} r\,\delta g(r,\Omega) = {} & \sum_{n=2}^{\infty} (n+1) T_n(\Omega) - \frac{r-R}{R}\sum_{n=2}^{\infty} (n+1)^2 T_n(\Omega) \\ & + \frac{1}{2}\left(\frac{r-R}{R}\right)^2 \sum_{n=2}^{\infty} (n+1)^2 (n+2) T_n(\Omega) \\ & + \sum_{n=2}^{\infty} (n+1) T_n(\Omega) \sum_{k=3}^{n} \binom{-n-1}{k}\left(\frac{r-R}{R}\right)^{k}\,. \end{aligned} \tag{7}$$

Combining Eqs.(2) and (7), the far-zone contribution to the gravity disturbance δg_{fz} is written as

$$r\,\delta g_{fz}(r,\Omega)=\frac{1}{4\pi}\sum_{n=2}^{\infty}(n+1)(2n+1)\int_{0}^{2\pi}\int_{\psi_0}^{\pi}T(R,\alpha,\psi)P_n(\cos\psi)\sin\psi\,\mathrm{d}\psi\mathrm{d}\alpha$$
$$-\frac{1}{4\pi}\frac{r-R}{R}\sum_{n=2}^{\infty}(n+1)^2(2n+1)\int_{0}^{2\pi}\int_{\psi_0}^{\pi}T(R,\alpha,\psi)P_n(\cos\psi)\sin\psi\,\mathrm{d}\psi\mathrm{d}\alpha \tag{8}$$
$$+\frac{1}{8\pi}\left(\frac{r-R}{R}\right)^2\sum_{n=2}^{\infty}(n+1)^2(n+2)(2n+1)\int_{0}^{2\pi}\int_{\psi_0}^{\pi}T(R,\alpha,\psi)P_n(\cos\psi)\sin\psi\,\mathrm{d}\psi\mathrm{d}\alpha,$$

where ψ_0 is the minimum spherical distance of the far zone. The higher than the second-degree terms ($k > 2$) in Eq.(8) are omitted causing a relative error of the order 1×10^{-8} or less. The surface integration domain in Eq.(8) is expressed in a frame of the polar spherical coordinates with the spherical azimuth α and the spherical angle ψ. Applying Molodensky's truncation coefficients $Q_n(\psi_0)$ obtained from orthogonality properties of the spherical harmonic functions and implemented into the expression for the modified surface spherical harmonics (cf. Appendix A, Eq.(A.6)), Eq.(8) becomes

$$r\,\delta g_{fz}(r,\Omega)=R\,\delta g_{fz}(R,\Omega)-\frac{1}{2}\frac{r-R}{R}\sum_{n=2}^{\infty}(n+1)^2(n-1)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega)$$
$$+\frac{1}{4}\left(\frac{r-R}{R}\right)^2\sum_{n=2}^{\infty}(n+1)^2(n-1)(n+2)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{m,m}(\Omega), \tag{9}$$

where

$$\delta g_{fz}(R,\Omega)=\frac{1}{2R}\sum_{n=2}^{\infty}(n+1)(n-1)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega)\,. \tag{10}$$

By analogy with Eq.(9), the far-zone contribution to the gravity anomaly Δg_{fz} reads

$$r\,\Delta g_{fz}(r,\Omega)=R\,\Delta g_{fz}(R,\Omega)-\frac{1}{2}\frac{r-R}{R}\sum_{n=2}^{\infty}(n-1)^2(n+1)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega)$$
$$+\frac{1}{2}\left(\frac{r-R}{R}\right)^2\sum_{n=2}^{\infty}(n-1)^2(n+1)(n+2)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{m,m}(\Omega), \tag{11}$$

where

$$\delta g_{fz}(R,\Omega)=\frac{1}{2R}\sum_{n=2}^{\infty}(n-1)^2Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega)\,. \tag{12}$$

Finally, the far-zone contribution to the disturbing potential T_{fz} is found to be

$$T_{fz}(r,\Omega) = T_{fz}(R,\Omega) - \frac{1}{2}\frac{r-R}{R}\sum_{n=2}^{\infty}(n-1)(n+1)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega) + \frac{1}{4}\left(\frac{r-R}{R}\right)^2\sum_{n=2}^{\infty}(n-1)(n+1)(n+2)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega), \tag{13}$$

where

$$T_{fz}(R,\Omega) = \frac{1}{2}\sum_{n=2}^{\infty}(n-1)Q_n(\psi_0)\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega). \tag{14}$$

Eq.(14) is the Stokes integral modified for computing the far-zone contribution (cf. Eq.A.3). The extended Stokes integral for computing the far-zone contribution is given in Eq.(13). Eqs.(9) and (11) define Green's integrals modified for computing the far-zone contributions to the gravity disturbance/anomaly by means of Molodensky's truncation coefficients. These Green integrals are somehow equivalent to Poisson's integral modified for computing the far-zone contribution to the gravity anomaly/disturbance. The main difference is in using Molodensky's surface truncation coefficients $Q_n(\psi_0)$ instead of the truncation coefficients for the Poisson kernel (cf. e.g., *Martinec, 1996*). Hotine's integral modified for computing the far-zone contribution by means of Molodensky's truncation coefficients is provided in Eq.(A.4). The relevant formulae for computing the far-zone contributions to the components of the deflection of the vertical can be found for instance in *Hirvonen and Moritz* (*1963*).

Molodensky's truncation coefficients $Q_n(\psi_0)$ can be evaluated either by a numerical integration through Eq.(A.1) or applying *Paul's (1973)* algorithm. Alternatively, they can be computed recurrently (cf., *Hagiwara, 1976*). We note that Green's integrals can be readily reformulated for the ellipsoidal approximation of the geoidal surface according to the approach described in *Vaníček et al. (1995)*.

4. NUMERICAL STUDY

The far-zone contributions to quantities of the Earth's gravity field were numerically investigated at the area of study bounded by the parallels of 42.5 and 67.5 arcdeg Northern latitude and the meridians of 210 and 270 arcdeg Eastern longitude. This area comprises a rough part of the Canadian Rocky Mountains, surrounding flat regions and a small part of the Pacific Ocean (see Fig. 1). Orthometric heights range from 0 to 5449 m, see Fig. 1. The computation was realized at the 5×5 arcmin equal-angular grid of points at the Earth's surface of which the orthometric heights were obtained from a detailed regional digital terrain model. The far zone was taken from 1 arcdeg of the spherical angle. The geopotential coefficients of the EGM-96 (*Lemoine et al., 1998*) complete to degree and order 180 were used for the computation, provided that the contribution of higher-degree terms is negligible when $\psi_0 \geq \pi/180$. The coefficients $T_{n,m}$

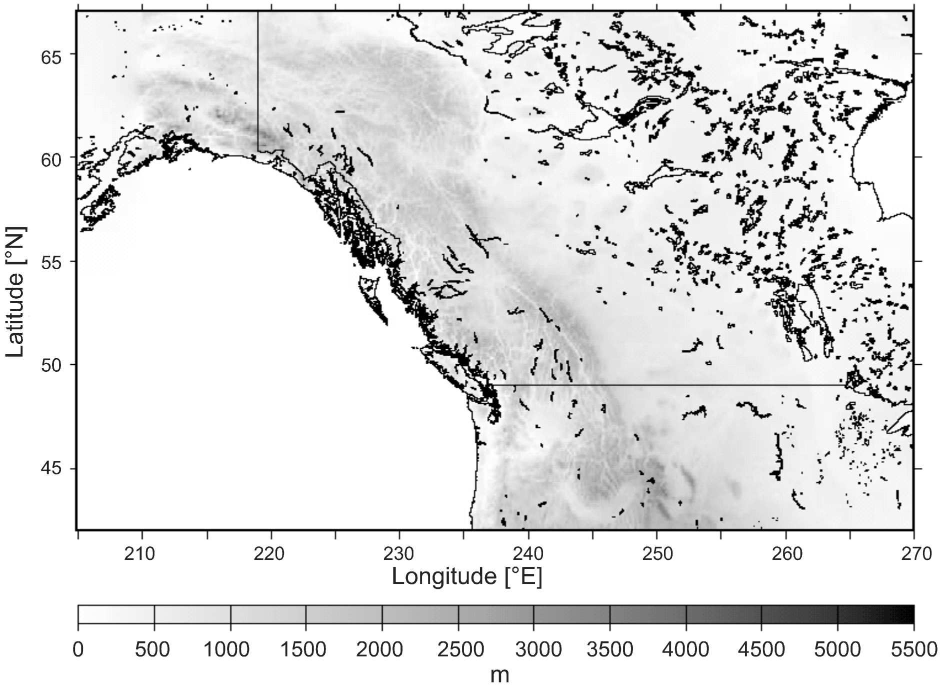

Fig. 1. Topography of the test area.

in Eqs.(9)–(14) are linked with the fully normalized coefficients $\bar{C}_{n,m}$ of the EGM-96 as follows

$$T_{n,m} Y_{n,m}(\Omega) = \frac{GM}{R} \left(\frac{a}{r} \right)^n \bar{C}_{n,m} \bar{Y}_{n,m}(\Omega) , \tag{15}$$

where $\bar{Y}_{n,m}(\Omega)$ are the fully normalized surface spherical functions, *GM* is the geocentric gravitational constant, and *a* is the major semi-axis of the geocentric reference ellipsoid (values *GM* and *a* are defined in *Lemoine et al., 1998, Table 3.3.1-1*). The value $R = 6371$ km was adopted.

The numerical results over the area of study revealed that the far-zone contribution to the gravity disturbances evaluated at the Earth's surface varies from −58.2 to 50.4 mGal (the mean value is −6.1 mGal); see Fig. 2. The far-zone contribution to the gravity anomalies varies from −45.2 to 46.4 mGal (the mean value is −1.0 mGal); see Fig. 3. The far-zone contribution to the disturbing potential varies from −415.0 to 126.6 m^2s^{-2} (the mean value is −161.7 m^2s^{-2}); see Fig. 4.

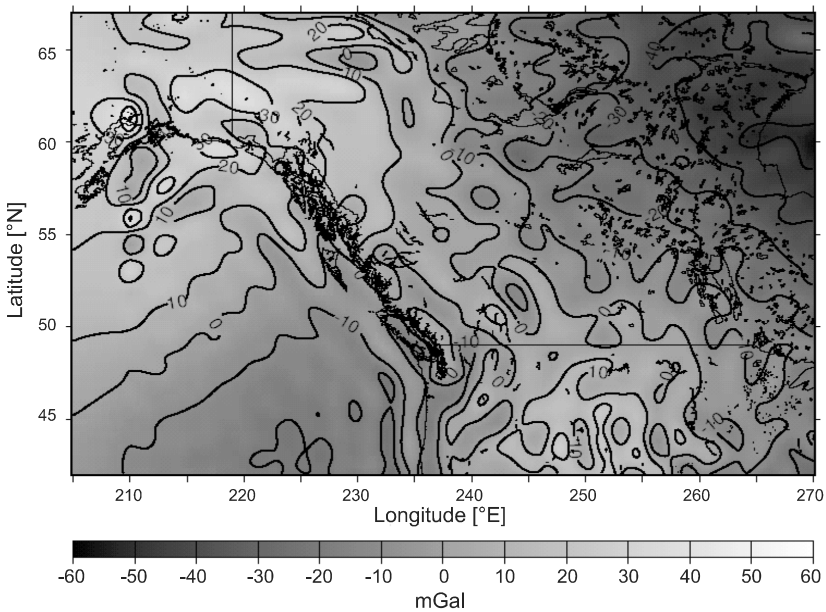

Fig. 2. Far-zone contribution to the gravity disturbances evaluated at the Earth's surface.

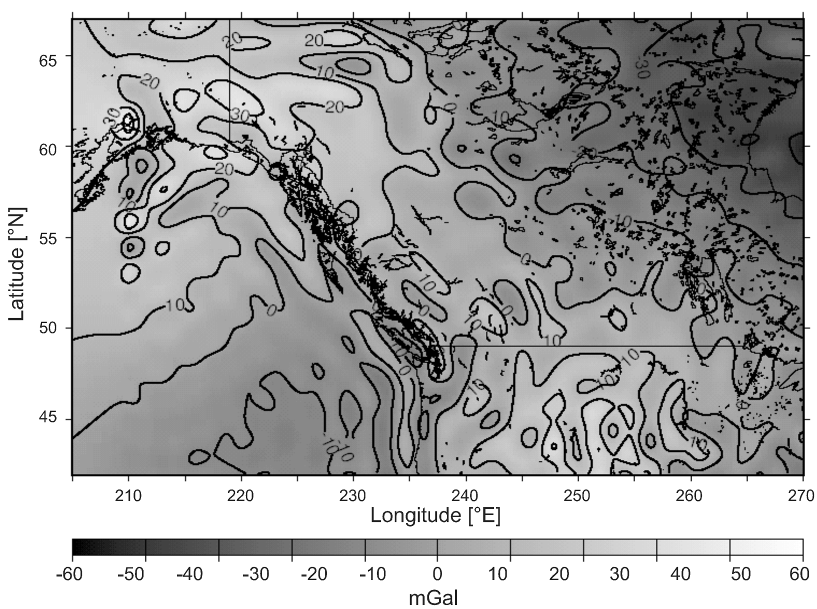

Fig. 3. Far-zone contribution to the gravity anomalies evaluated at the Earth's surface.

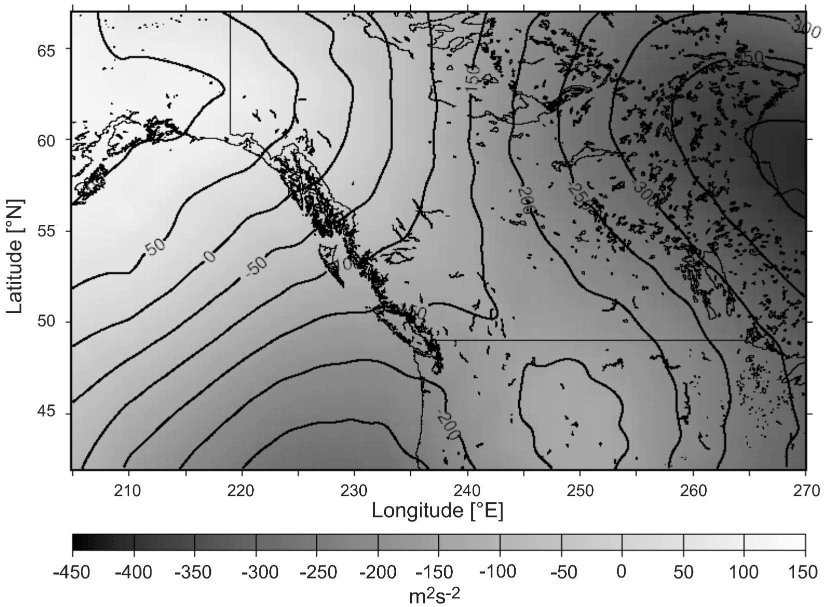

Fig. 4. Far-zone contribution to the disturbing gravity potential evaluated at the Earth's surface.

The far-zone contribution to the gravity anomalies/disturbances represents the quantity to be subtracted from the regional detailed gravity data in prior of solving the direct gravity inversion for the near zone. After the solution for the near zone is found in terms of the disturbing potential for the points at the Earth's surface, the far-zone contribution to the disturbing gravity is added. It provides the final result in terms of the gravimetric height anomalies when divided by the values of normal gravity at the telluroid. The treatment of topographical and atmospheric effects is out of the scope of this study.

The expressions for computing the far-zone contributions (Eqs.(9)–(14)) directly provide the reference gravity field when $\psi_0 = 0$. Due to the reasons summarized in *Vaníček and Sjöberg (1991)*, these expressions (for $\psi_0 = 0$) can be used to compute the low-degree reference gravity field from solely satellite-determined global geopotential model. The far-zone contributions are then estimated for the chosen value of ψ_0 from the geopotential coefficients above the upper degree of the reference gravity field.

5. CONCLUSIONS

A proper treatment of the far-zone contributions in regional gravimetric geoid/quasigeoid modelling is required when GPS-levelling data over the computation area are not available and consequently the systematic distortions of the gravimetric geoid/quasigeoid cannot be eliminated by applying either a low-degree polynomial corrector surface or an innovation function (see *Prutkin and Kless, 2008*).

In this study, we derived the expressions for computing the far-zone contributions to the disturbing potential, the gravity disturbance, and the gravity anomaly. Molodensky's truncation coefficients were utilized in these expressions. To do that, we first introduced the modified surface spherical harmonics which are functionally related with the surface spherical harmonics by means of Molodensky's truncation coefficients. Consequently, Green's integrals (of which the generic form is expressed in terms of the solid spherical harmonics) were expressed in terms of the surface spherical harmonics. This has been done through applying the binomial theorem to the radial component of the solid spherical harmonics. Combining the expressions for the modified surface spherical harmonics and for Green's integrals in terms of the surface spherical harmonics, the expressions for computing the far-zone contributions were obtained.

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APPENDIX A
MODIFIED SURFACE SPHERICAL HARMONICS BY MEANS OF MOLODENSKY'S TRUNCATION COEFFICIENTS

With reference to *Molodensky et al. (1960)*, the truncation coefficients $Q_n(\psi_0)$ are evaluated by the integral convolution of the Stokes function $S(\psi)$ and the Legendre polynomials $P_n(\cos\psi)$, whence

$$Q_n(\psi_0) = \int_{\psi_0}^{\pi} S(\psi) P_n(\cos\psi) \sin\psi \, \mathrm{d}\psi \, . \tag{A.1}$$

According to Molodensky's modification of the Stokes integral for the far-zone contribution, the disturbing potential T_{fz} reads (*Molodensky et al., 1960*)

$$\begin{aligned} T_{fz}(R,\Omega) &= R \sum_{n=2}^{\infty} \frac{Q_n(\psi_0)}{2} \frac{2n+1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \Delta g(R,\alpha,\psi) P_n(\cos\psi) \sin\psi \, \mathrm{d}\psi \mathrm{d}\alpha \\ &= R \sum_{n=2}^{\infty} \frac{Q_n(\psi_0)}{2} \Delta g_n(\Omega) \, . \end{aligned} \tag{A.2}$$

With reference to the relation between the surface spherical harmonics $T_n(\Omega)$ and $\Delta g_n(\Omega)$ for T and Δg respectively, i.e., $R\Delta g_n(\Omega) = (n-1) T_n(\Omega)$, see e.g., *Heiskanen and Moritz (1967, Chap. 2-17)*, Eq.(A.2) is further rewritten as

$$T_{fz}(R,\Omega)=\sum_{n=2}^{\infty}\frac{Q_n(\psi_0)}{2}(n-1)T_n(\Omega)\,. \tag{A.3}$$

Substitution for $T_n(\Omega)=R(n+1)^{-1}\delta g_n(\Omega)$ in Eq.(A.3) yields *Hotine's (1969)* integral modified for computing the far-zone contribution; $\delta g_n(\Omega)$ are the surface spherical harmonics for δg. It reads

$$T_{fz}(R,\Omega)=R\sum_{n=2}^{\infty}\frac{Q_n(\psi_0)}{2}\frac{n-1}{n+1}\delta g_n(\Omega)\,. \tag{A.4}$$

From the above equations, the modified surface spherical harmonics $\tilde{T}_n(\Omega,\psi_0)$ are introduced by

$$\begin{aligned}\tilde{T}_n(\Omega,\psi_0)&=\frac{2n+1}{4\pi}\int_0^{2\pi}\int_{\psi_0}^{\pi}T(R,\alpha,\psi)P_n(\cos\psi)\sin\psi\,\mathrm{d}\psi\mathrm{d}\alpha\\&=(n-1)\frac{Q_n(\psi_0)}{2}\frac{2n+1}{4\pi}\int_0^{2\pi}\int_{\psi_0}^{\pi}T(R,\alpha,\psi)P_n(\cos\psi)\sin\psi\,\mathrm{d}\psi\mathrm{d}\alpha\,.\end{aligned} \tag{A.5}$$

Combining Eqs.(2) and (A.5), we get

$$\tilde{T}_n(\Omega,\psi_0)=(n-1)\frac{Q_n(\psi_0)}{2}T_n(\Omega)=(n-1)\frac{Q_n(\psi_0)}{2}\sum_{m=-n}^{n}T_{n,m}Y_{n,m}(\Omega)\,. \tag{A.6}$$

$\tilde{T}_n(\Omega,\psi_0)$ can also be described in terms of the surface spherical harmonics $\delta g_n(\Omega)$ and $\Delta g_n(\Omega)$ as follows

$$\tilde{T}_n(\Omega,\psi_0)=\frac{n-1}{n+1}\frac{Q_n(\psi_0)}{2}R\delta g_n(\Omega)=\frac{Q_n(\psi_0)}{2}R\Delta g_n(\Omega)\,. \tag{A.7}$$

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