Optimization of an embedded rail structure using a numerical technique

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**This paper presents several steps of a procedure for design of a railway track aiming at the development of optimal track structures under various predefined service and environmental conditions.
The structural behavior of the track is analyzed using a finite element model in which the track and a moving train are incorporated. Parameters of the optimum track are determined by applying a numerical optimization technique. The optimization method employed here uses Mutipoint Approximations based on Response Surface fitting (MARS).
To demonstrate the robustness of the procedure, it is applied to a problem of optimal design of an innovative railway track for high-speed trains – a so-called Embedded Rail Structure. Requirements for the optimal design are related to the wear of the rails and wheels and the level of acoustic noise produced by a moving train. To obtain the optimal design, component dimensions and mechanical properties of the track are varied. Results of the optimization are presented and discussed.**

Key words: **Railway Engineering, Numerical Optimization, Dynamics**

1 Introduction

A classical railway track structure consists of a flat framework built up of two rails and sleepers connected to each other by fasteners, and a ballast bed as shown in Figure 1. The construction principle of the railway tracks has not been notably changed during the last decades. Yet some improvements of the design have recently been made such as introduction of concrete sleepers and new types of the fasteners. The main drawback of the classical railway structure is the high cost related to inspection and maintenance of the railway track. This cost is considerably increasing for high-speed train tracks. Because of strong *availability* requirements to modern railway tracks (i.e. they should always be available for trains), a reduction of the *maintenance effort* has become an important aspect in the design of new railway structures [2]. Other important requirements concern *bearing capacity* and *durability of the track, passenger's comfort* and level of the *acoustic noise* produced by a moving train.

HERON, Vol. 45, No. 1 (2000) ISSN 0046-7316

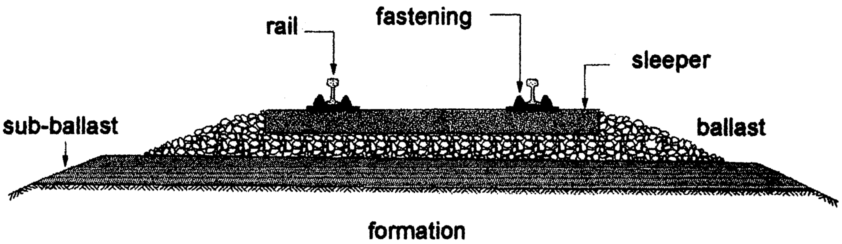

Fig. 1. Construction principle of classical track structure

During the last two decades, a considerable theoretical and practical effort has been made on the design of new railway structures satisfying the above mentioned requirements. One of such new non-conventional structures is a railway structure without ballast, a so-called Embedded Rail Structure (ERS). Around 1974 Nederlandse Spoorwegen (Dutch Railways) has started to use such a structure on concrete bridges. Since 1984 NS is using ERS also on steel bridges. Nowadays various types of Embedded Rail Structures (including tramway and metro) are actively used in many countries. It should be noted that only few types of railway structures without ballast have specifically been designed for high-speed trains.
Recently an experimental track of 3 km length with ERS on concrete slab (Figure 3) has been installed in the south of the Netherlands (near Best). This structure consists of a continuous reinforced concrete slab rested on a concrete stabilized roadbed, which in turn is placed on a sand base as shown in Figure 2. Two troughs in the slab at 1.5 m spacing serve to embed UIC54 rails, a visco-elastic compound, an elastic strip at the bottom of the troughs and some construction utensils (Figure 4).

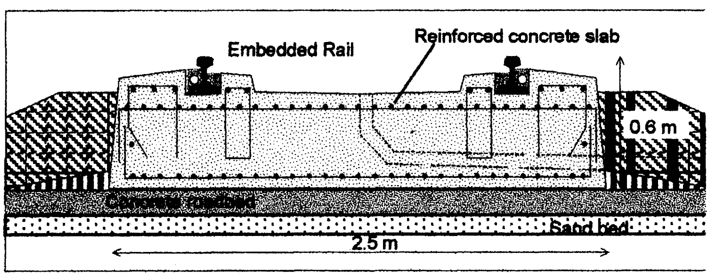

Fig. 2. Embedded Rail Structure

Fig. 3. Embedded Rail Structure installation near Best (the Netherlands)

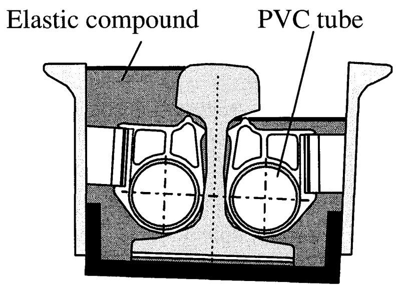

Fig. 4. Embedded rail

In the present paper, a procedure for optimal design of an ERS described above for high-speed trains using a numerical optimization technique is suggested. The dynamic behavior of the embedded rail structure is modeled using a finite element method. The track and a moving train are analyzed simultaneously as one mechanical system. A steady state dynamic analysis is performed in time domain. For the optimization, a specific type of an iterative mid-range approximation technique, namely Multipoint Approximations based on Response Surface fitting (MARS) has been chosen [6]. The approximations are obtained using the information about the original functions at several design points assigned by the optimizer, which are treated as a plan of experiments. The numerical procedure is only briefly presented here, for details we refer to [3,4]. More detailed information about the MARS method can be found in Reference [5].

2 Embedded Rail Structure

Numerical model

An ERC track with the train moving on it represents a complex mechanical system. The dynamic behavior of such a system depends on the geometric and mechanical characteristics of both the railway structures and the train. In this paper only vertical displacements are considered. Because of small magnitude of the displacements, the dynamic behavior of such a system can quite adequately be described by a 1-D finite element model.

In a simple case the train can be modeled by moving loads applied to the rails but to obtain more realistic results a more complex representation should be considered. In the mechanical model used here, the train is represented by the mass-spring system shown in Figure 5. Contact forces between the rails and wheels of the train are modeled using a Hertzian spring [3].

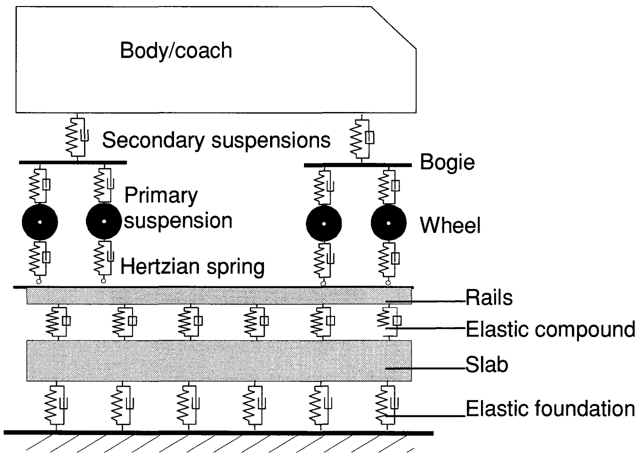

Fig. 5. Modelling of track and train in RAIL software (TU Delft)

The dynamic behavior of the embedded rail structure is defined by mechanical properties of its components (Figure 2). The rails and slab are modeled by elastic beam elements. The dynamic behavior of the elastic compound and foundation is described by the Winkler model (Figure 5). The irregularities of the rail surface are represented by a combination of sinus functions with various amplitudes and phase shifts. The amplitude of the irregularities can be considered as a measure of the rail roughness and therefore can be used for estimation of track maintenance effort. The above-described numerical model has been implemented in the computer program RAIL (TU Delft) [3].

Requirements to optimal design of ERS

Resonant frequencies of the track structure which can be obtained by analyzing the Frequency Response Function of the ERS should not coincide with vehicle resonant frequencies. Amplification of the structural response in the neighborhood of these frequencies should be restricted.

The acoustic characteristics of the track structure can be estimated by a specific response quantity, a so-called *attenuation rate* (distance damping). The attenuation rate characterizes the ability of a track structure to damp the vibrations on different distances from the source of the vibration.

The distance damping characterizes the ability of a mechanical system to reduce the acoustic noise. Acoustic noise radiating from the rails reduces as the attenuation rate of the structure increases.

Wheel-rail contact forces should be below prescribed values in order to reduce rail and wheel wear. Magnitude of these forces strongly depends on the rail surface geometry [2,6]. Usually, it is required that the standard deviation of the contact forces to be below 20% of the static wheel-rail contact load.

The above mentioned requirements will be used later for optimization of an embedded rail structure.

3 Optimization method

General optimization problem

To make a use of numerical optimization techniques the optimization problem should be stated in a general form that reads

Minimize

$$F_0(x) \rightarrow \min,\; x \in \boldsymbol{R}^{\mathrm{N}} \tag{1}$$

subject to

$$F_{\mathrm{j}}(x) \le 1,\; j = 1, \ldots, M \tag{2}$$

and

$$A_{\mathrm{i}} \le x_{\mathrm{i}} \le B_{\mathrm{i}},\; i = 1, \ldots, N. \tag{3}$$

Here

F_0 is *an objective function;*

$F_{\mathrm{j}}, j = 1, \ldots, M$ are *constraints;*

$x = (x_1, \ldots, x_{\mathrm{N}})^{\mathrm{T}}$ is a vector of *design* variables,

A_{i} and B_{i} are the so-called *side limits,* which define lower and upper bounds of the *i*-th design variable.

The components of the vector x represent various parameters of the mechanical system, such as geometry, material, stiffness and damping properties, which can be varied to improve the performance characteristics of the system. Depending on the problem under consideration the objective and constraint functions (1)–(2) can describe various structural and dynamic response quantities of the system such as weight, reaction forces, stresses, natural frequencies, displacements, velocities, accelerations, etc. Cost, maintenance and safety requirements can be used in the formulation of the optimization problem as well. The objective function provides a basis for improvement of the design whereas the constraints impose some limitations on the behavior characteristics of the system.

Solution of the optimization problem is an iterative process, which involves multiple evaluations of the objective and constraint functions (1)–(2). Typically, the values of the functions can be obtained using one of the numerical methods, e.g. the Finite Element Method.

Approximation concept

The optimization problem (1)–(3) can be solved using a conventional method of mathematical programming. However, for systems with many degrees of freedom the finite element analysis can be time consuming. As a result, the total computational effort of the optimization might become prohibitive. This difficulty has been mitigated in the mid-seventies by introducing approximation concepts [1].

According to the approximation concepts the original functions (1)–(2) are replaced with approximate ones which are computationally less time consuming. Instead of the original optimization problem (1)–(3) a succession of simpler approximated sub-problems similar to the original one formulated using the approximation functions is to be solved. Each simplified problem then has the following form:

Minimize

$$\tilde{F}_0^k(x) \to \min,\ x \in \boldsymbol{R}^N \tag{4}$$

subject to

$$\tilde{F}_j^k(x) \le 1,\ j = 1, \ldots, M \tag{5}$$

and

$$A_i^k \le x_i \le B_i^k,\ A_i^k \ge A_i,\ B_i^k \le B_i,\ i = 1, \ldots, N. \tag{6}$$

where the superscript k is the number of the iteration step, $\tilde{F}$ is the approximation of the original function F, A_i^k and B_i^k are *move limits* defining the range of applicability of the approximations. Since the functions (4)–(5) are chosen to be simple and computationally inexpensive, any conventional method of optimization [1] can be used to solve the problem (4)–(6). The solution of the problem x_*^k is then chosen as starting point for the (k+1)-th step and the optimization problem (4)–(6) reformulated with new approximation functions $\tilde{F}_j^{k+1}(x) \le 1$, ($j = 0, \ldots, M$) and move limits A_i^{k+1} and B_i^{k+1} is to be solved. The process is repeated until the convergence criteria are satisfied.

MARS optimization technique

The approximation is defined as a function of the design variables x and tuning parameters $\boldsymbol{a}$ (for brevity the indices k and j will be omitted). To determine the components of vector $\boldsymbol{a}$ the following weighted least-squares minimization problem is to be solved [5,6]:

Find vector $\boldsymbol{a}$ that minimizes

$$G(\boldsymbol{a}) = \sum_{p=1}^{P} \{ w_p^{(0)} [F(\boldsymbol{x}_p) - \tilde{F}(\boldsymbol{x}_p, \boldsymbol{a})]^2 \} \tag{7}$$

Here $F(\boldsymbol{x}_p)$ is the value of the original function from (1)–(2) evaluated at the point of the design parameters space $\boldsymbol{x}_p$, and P is the total number of such points; $w_p^{(0)}$ is a weight factor that characterises the relative contribution of the information about the original function at the point $\boldsymbol{x}_p$. For the numerical examples the multiplicative form of the approximating function has been chosen, which has the form

$$\tilde{F}(\boldsymbol{x}) = a_0 \prod_{i=1}^{P} (x_i)^{a_i}. \tag{8}$$

The optimization process is controlled by changing the move limits in each iteration step. The main rules of the strategy of changing of the move limits employed in the method are:

- if the approximating functions do not adequately represent the original ones in the current optimum point, what means that the search subregion is larger than the range of applicability of the current approximations, the move limits are changed to reduce the size of the search subregion;
- if the approximations are good and the solution of the optimization problem (4)–(6) is an internal point of the search subregion it could be considered as the solution of the original optimization problem (1)–(3), the search subregion is reduced;
- if the current optimum point belongs to the boundary of the search subregion (one of the move limits is active) whereas the approximations are good the size of the subregion is not changed on the next iteration.

The iteration process is terminated if the approximations are good, none of the move limits is active and the search subregion is small enough. More details about the weight coefficients assignment, the move limits strategy and the most recent developments of the method can be found in [5].

4 Optimization of ERS

Using the requirements to the Embedded Rail Structure formulated in the previous section, two optimization problems have been formulated. These problems have been solved for three different velocities of the moving train. For the model of the train the realistic data of the TGV train has been used. Both problems have been solved using the optimization method described above, whereas the multiplicative function (8) has been used to approximate the objective and constrain functions. The optimization problems and results are presented and discussed below.

Single criterion optimization of ersers

In this problem, the requirements related to the acoustic noise hinder and wheel-rail wear have been chosen for the optimum design of the ERS. To formulate the optimization problem, the first requirement has been taken as the objective function and the other requirement has been considered as a constraint. Thus, the following optimization problem is to be solved:

For a given train moving with a prescribed velocity along the ERS track with a given rail surface profile, minimize the inverse resonant frequency of the track structure f_0: $F_0(x) = \dfrac{1}{f_0} \rightarrow \min.$

The resonant frequency is obtained by applying an impulse load to ERS model and considering the Frequency Response Function (FRF) [4,6]. The constraint has been imposed on the standard deviation of the wheel-rail contact force $\sigma(P)$, which should not exceed 20% of its static value P_{st} that reads:

$$F_1(x) = \frac{\sigma(P)}{P_{st}} \leq 0.2\,.$$

The resonant frequency defines the level of the acoustic noise produced by a moving train, whereas the contact force describes the wheel-rail wear. The siffness (k) and damping (c) parameters of the visco-elastic compound have been chosen as design variables. Their lower and upper bounds are given in Table 1. The rail roughness parameter has been seen on 0.5.

Table 1. Design variables.

	Initial value	Lower bound	Upper bound	Unit
k	52	10	100	MN/m/m′
c	5.2	1	15	kNs/m/m′
s	0.5	0.1	1.5	-

The optimization problem has been solved for three typical train velocities. The numerical results of the optimization are given in Table 2, and the attenuation rates and contact forces are shown in Figure 7 and Figure 8 respectively. From Table 2 it can be seen that the optimum values of the stiffness and damping parameters of the compound decrease as the velocity of the train increases. Comparing the attenuation rate results it can be concluded that the optimum design of ERS for a low-speed train has better acoustic properties that the ERS for a high-speed train.

It should be noted that even though increasing the damping of the compound can reduce the wheel-rail contact forces the design variable c has not reach its upper bound. That happened because the value of contact forces had been treated in the optimization problem as a constraint and optimizer always tries to satisfy it, i.e. to make it equal to a prescribed value (in this case to $\sigma = 17.8$ kN). If further reduction of a response quantity is desired, the prescribed value should be adjusted. In addition, the response quantity can be included in the formulation of the objective function as it demonstrated in the following example.

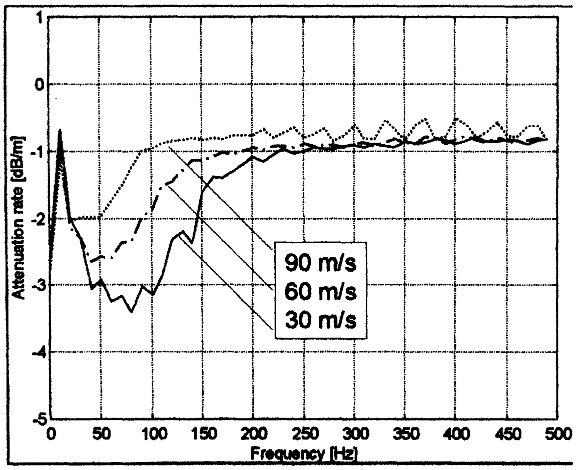

Fig. 6. Attenuation rate of optimum designs (two-criterion optimization).

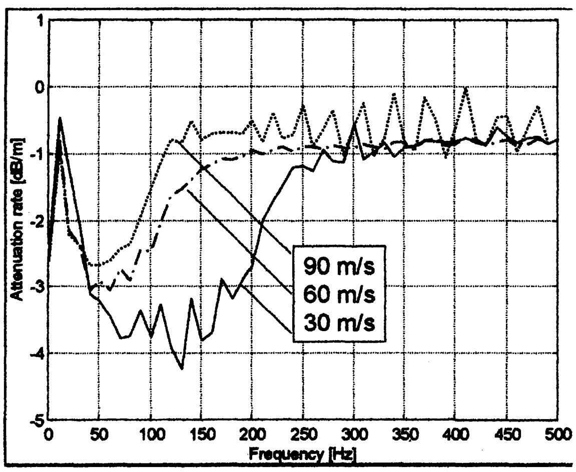

Fig. 7. Attenuation rate of optimum designs (single-criterion optimization).

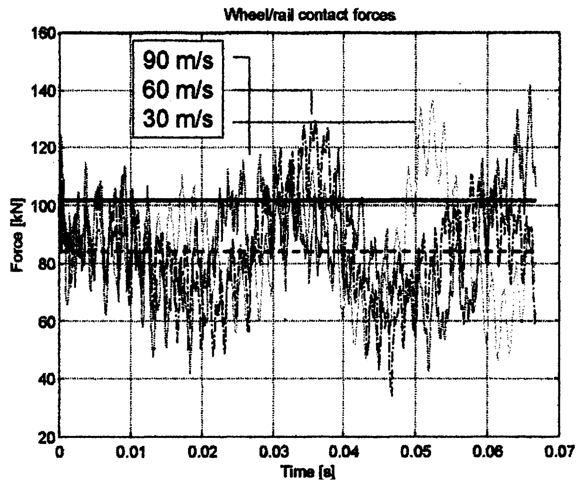

Fig. 8. Wheel-rail contact force of optimum designs (single criterion optimization).

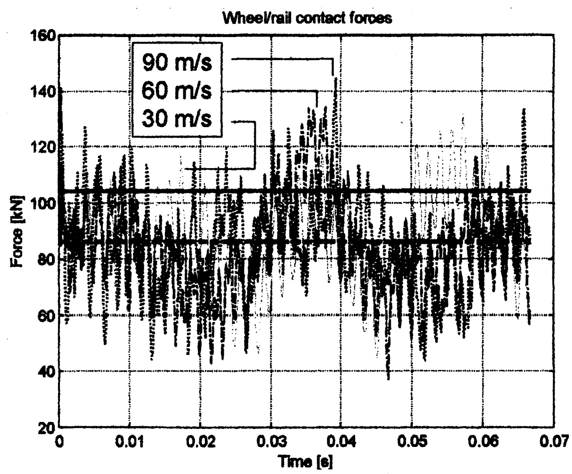

Fig. 9. Wheel-rail contact force of optimum designs (two-criterion optimization).

Table 2. Results of single criterion optimization of ERS (visco-elastic compound).

	30 m/s	60 m/s	90 m/s	Unit
Optimized stiffness k	88.8	28.4	20.9	MN/m/m′
Optimized damping c	12.5	13.8	6.46	kNs/m/m′
Resonant frequency	200	110	100	Hz
Attenuation rate (max.)	–4.2	–3.0	–2.7	dB/m
Scale of surface profile	0.5	0.5	0.5	–
St. dev. contact forces (σ)	17.8	17.8	17.8	kN
Static wheel load (μ)	85	85	85	kN
Ration (σ/μ)	20	20	20	%

Table 3. Results of two-criterion optimization of ERS (visco-elastic compound and rail surface geometry).

	30 m/s	60 m/s	90 m/s	Unit
Optimized stiffness k	39.5	21.1	10.0	MN/m/m′
Optimized damping c	15.0	15.0	10.8	kNs/m/m′
Resonant frequency	140	100	60	Hz
Attenuation rate (max.)	–3.4	–2.7	–2.8	dB/m
Optimized scale coef. (s)	1.11	0.61	0.71	–
St. dev. contact forces (σ)	17.8	17.8	17.8	kN
Static wheel load (μ)	85	85	85	kN
Ration (σ/μ)	20	20	20	%

Two-criterion optimization of ERS

In this problem, the optimization searches for an optimum design of ERS track that requires minimum maintenance effort and that has good acoustic properties. The maintenance effort can be estimated by the degree of roughness of the rail surface (parameters s) while the acoustic properties can be described by the resonant frequency of the ERS structure. The larger the value of the parameter s the less maintenance is required to keep the wheel-rail contact forces at a prescribed limit. To combine these two criterions, the following objective function has been used:

$$F_0(x) = \frac{f_{\max}}{f_0} + \frac{s_{\max}}{s} \rightarrow \min, \tag{9}$$

where s is the scaling coefficient defining the roughness of the rail surface f_{max} and s_{max} are the maximum values of f_0 and s respectively.
The optimum design should satisfy the wheel-rail wear requirement as well, i.e. the standard deviation of the contact force should be equal to the prescribed level (20% of static load). The vector of the design variables x comprises the stiffness and damping parameters of the compound, and the scaling coefficient s. The parameters of a train and velocity have not been changed during the optimization. Again, three optimization problems corresponding to the three different velocities of the train have been performed. The results are given in Table 3, Figure 6, and Figure 9. It should be noted that during the first two optimization runs the damping of the elastic compound has reached its upper bound value whereas at the end of the third optimization run the stiffness of the compound is equal to the lower limit. This explains the fact that the optimum value of the scaling coefficient is larger in the optimization for 90 m/s then that for 60 m/s. To prevent this, a critical damping indicating energy absorption of the structure during long time repetitive loading (fatigue test) should be taken into account, and additional constraint should be imposed on the value of the critical damping ratio.

Several criteria for the optimum design of an ERS have been proposed. Optimum mechanical properties of the ERS have been determined by performing single and two-criterion optimization. The applied criteria are related to the acoustic noise and vibration performance as well as track maintenance requirements. The problems have been solved for various train velocities.
The results indicated that the MARS method can effectively be used for the design of railway structures.

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