# ONSET OF REVERSAL AND CHAOS IN THERMALLY DRIVEN CAVITY FLOW

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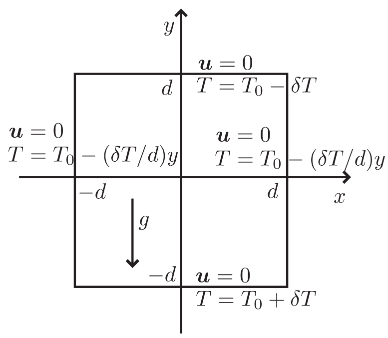
*Abstract* We investigate the onset of chaotic reversals of thermal convection cell in a two–dimensional square cavity using direct numerical simulation. To our knowledge, the chaotic reversal motion at the lowest Rayleigh number is related to the unicellular motion in this system. As Rayleigh number increases, the two antisymmetric attractors, which arise from the supercritical Hopf bifurcation, approach each other. However, because the basin boundaries of these attractors have multiple unstable directions, the behavior of the global changes seems to be complex for the small range of Rayleigh number around the onset of the chaotic reversal. On the other hand, period–doubling cascade from periodic reversal solutions leads to chaotic reversal as Rayleigh number decreases. At the conference we will report observation about the beginning of reversal and the quantities of the chaotic attractor.

## INTRODUCTION

There exists large-scale recirculating motion, often called the mean wind, in thermally driven turbulence in a finite container heated from below. The mean wind is observed widely to exhibit occasional and sudden reversals in a closed cylindrical container (Sreenivasan et al. 2002) and a thin (two-dimensional) rectangular vessel (Sugiyama et al. 2010). Similar reversals also occur in other configurations, such as the magnetic field of the earth and the earth's atmosphere. In this study the reversal mechanism of the mean wind is explored by numerically tracing the (global) bifurcation structure of thermal convection in a two-dimensional square container with respect to the Rayleigh number.

## SYSTEM

We consider a thermally stratified fluid under gravity in a two dimensional square domain. We adopt the Boussinesq approximation and assume the domain is surrounded by rigid and perfect thermal conductive frame heated from below. (See right figure.) This model is the exactly the same as the model in Mizushima & Adachi(1997). The non-dimensional governing equations can be written in usual non-primitive form as




$$\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}+\frac{\partial\psi}{\partial x}=\Delta\theta \ \text{ in } D\equiv(-1,1)\times(-1,1),$$

$$\frac{\partial\Delta\psi}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}=-RaPr\frac{\partial\theta}{\partial x}+Pr\Delta^2\psi \ \text{ in } D,$$

$$\theta=\psi=\frac{\partial\psi}{\partial n}=0 \ \text{ on } \partial D,$$

where $\psi$ and $\theta$ are stream function and temperature perturbation from hydrostatic equilibrium state (See Mizushima & Adachi(1997) for details). For spacial discretization, these two dependent variables are expanded by spectral Legendre–Galerkin method. The spectral coefficients are integrated using Crank–Nicolson and 2nd–order Adams–Bashforth method for the diffusion terms and remaining terms respectively. In the following, we set Prandtl number $Pr=7$ as in Mizushima & Adachi(1997) and investigate how the unicellular convection states depend on Rayleigh number $Ra$ especially around the global bifurcation point $(Ra_G)$ which leads to the chaotic reversal motion.

## ONSET OF CHAOTIC REVERSAL MOTION

The reflectional symmetric pair of unicellular solutions, which have opposite sign of angular momentum, arises from the hydrostatic equilibrium state by the supercritical pitchfork bifurcation.[3] (P1 in Figure 1) These solutions have the origin symmetry $(\psi(-\boldsymbol{x})=\psi(\boldsymbol{x}),\theta(-\boldsymbol{x})=-\theta(\boldsymbol{x}))$. Figure 1 shows the bifurcation diagram of the unicellular solutions. Only the branches which are related to the steady solution with positive angular momentum $(L=(1/2)\int_D\psi dD>0)$ are plotted by using averaged angular momentum $L$ (Local extreme maximum and minimum values are used for periodic and chaotic state.). All attractors in Figure 1 still have the origin symmetry. The periodic solution arises firstly from the supercritical Hopf bifurcation (H in Figure 1) from unstable steady solution. This branch becomes again stable above the pitchfork bifurcation point (P3 in Figure 1). Along the branch P3–G the distance between the orbit of this branch and the four–cellular steady solution, which exists on the basin boundary between the reflectional symmetric pair of the periodic solutions, becomes closer and closer.

Above the global bifurcation point $Ra_{\mathrm{G}} \simeq 50750$ (G in Figure 1), the orbit exhibits chaotic flow reversal. On the other hand, the periodic reversal solutions lead to the chaotic state by period–doubling cascade as $Ra$ decreases. Figure 2:left shows the period–doubling cascade from period–2 solution. Just above the critical point G, the chaotic orbit seems to include (infinity) many solutions which have the period with different prime numbers. (Figure 2: right–center)

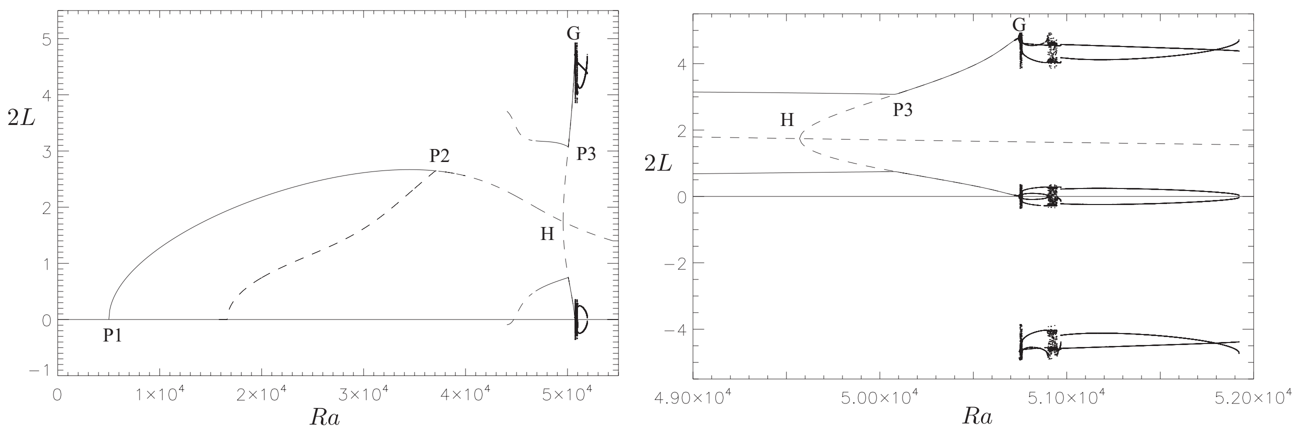



**Figure 1.** Bifurcation diagram of unicellular solutions. Solid lines and filled area are for attractors, and dashed lines for saddles. Local extreme values of averaged angular momentum $L \equiv (1/2)\int_D \psi dD$ are plotted. P1,P2 and P3 are pitchfork bifurcation points, H is Hopf bifurcation, and G means a global bifurcation point which leads to chaotic reversal motion. Below G only unicellular solutions with positive angular momentum are included.

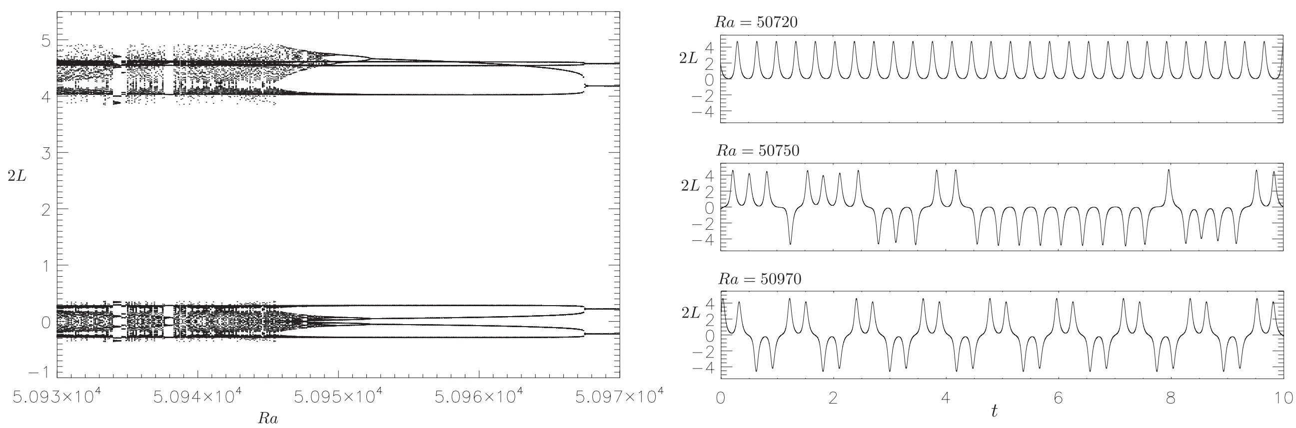



**Figure 2.** *left*:Period doubling cascade from a period-2 reversal solution. *right*: Time series of averaged angular momentum $L$.

## References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