

Experimental observations on long-term behavior of migrating alternate bars

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ABSTRACT: Migrating alternate bars form in alluvial channels as a result of morphodynamic instability. Extensive literature can be found on their origin and short-term development, but their long-term evolution has been poorly studied so far. In particular, it is not clear whether they eventually reach an equilibrium shape, since short-term experiments show that they may tend to elongate with time. We studied the long-term evolution of alternate bars by performing two independent long-duration laboratory experiments. In a straight flume, we carried out two tests, characterized by the same hydrodynamic conditions. In the first test (duration 3 weeks), a transverse plate created a permanent disturbance at the upstream boundary, forcing the formation of steady bars. In this case, both migrating and steady bars formed, but steady bars rapidly dominated the scene. In the second test (duration 10 weeks), the incoming flow was uniform, without any external disturbance. Migrating bars initially dominated the bed topography, but steady bars slowly developed from upstream, locally suppressing migrating bars. In both tests, migrating bars showed a periodic behavior, characterized by vanishing and reappearing at intervals of one or more weeks. Recurrent bar vanishing occurred also in another long-term experiment carried out in an annular flume, this time at intervals of 6-8 days. Migrating bar vanishing appears related to progressive steepening of the bar fronts. Without any external forcing, steady bar development seems to be caused by the presence of migrating bars, which may operate as forcing factors. Steady alternate bars are found to act similarly to point bars in sinuous channels, in their ability to locally suppress migrating bars. In conclusion, in the performed experiments migrating bars appear as a transitional phenomenon of alluvial channels having a cyclic character, whereas the bed topography is eventually dominated by steady bars.

1 INTRODUCTION

The linear stability analyses carried out in the 1970s showed that a flat alluvial bed may develop into a pattern of alternate or multiple bars, depending on the flow width-to-depth ratio and other

morphodynamic parameters (e.g. Hansen, 1967; Callander, 1969; Engelund, 1970; Engelund and Skovgaard, 1973; Parker, 1976; Fredsøe, 1978). Different bar wavelengths may be unstable, but according to the theory the selected wavelength corresponds to the bars with the largest temporal growth rate. This assumption, however, leads to systematic under-prediction of the bar wavelengths, typically by 30-40% (Nelson, 1990). This could be ascribed to the observed elongation of migrating bars with time (e.g. Fujita and Muramoto, 1985), since the wavelengths predicted by linear theories represent only the initial phases of the bar development. Using fully non-linear, physics-based, models, Nelson (1990) and Defina (2003) found that bar growth and migration rates decreased with increasing wavelength, which is in agreement with weakly non-linear theories (e.g. Tubino and Seminara, 1990). The question remained, therefore, whether migrating bars eventually reach a (dynamic) equilibrium configuration.

Here we investigate the long-term evolution of migrating alternate bars experimentally. We performed two independent long-duration laboratory experiments; the first one was carried out recently in a straight flume with sand bed and sediment recirculation. The other experiment was carried out in an annular flume with sand bed about thirty years ago, but the observed long-term bar evolution was never published (MSc thesis by Cornelisse, 1981). In the straight flume, we performed two tests (Crosato et al., 2010 and 2011): the first one was characterized by the presence of a transverse plate obstructing part of the flow at the upstream boundary. This plate created an external forcing leading to a typical “forced system” (Struiksmas & Crosato, 1989; Lanzoni, 2000). In the second test, the flow was not forced by any flow discontinuity as in a typical “free system”.

The bed topography of the forced system was very soon dominated by the presence of steady alternate bars. Migrating bars formed only in the second half of the channel. Instead, the bed topography of the free system was initially dominated by migrating bars. However, also in this case, steady bars, characterized by twice as large wave lengths and smaller amplitude, formed too, but became appreciable only after three weeks. We analyzed steady bar formation and its consequences in Crosato et al. (2011), whereas we focus here on the evolution of migrating bars.

In both tests, migrating bars initially increased in size, but after a few days the bars unexpectedly vanished to reform soon after with smaller wavelengths and higher celerity. Migrating bar vanishing involved several bars simultaneously, but in a few occasions it occurred all over the flume. Migrating bar vanishing repeated at intervals of one or more weeks. The much longer steady bars, instead, never vanished and once their amplitude had reached a value comparable to the amplitude of migrating bars, they locally suppressed migrating bars. We observed periodic bar vanishing also in the long-duration experiment carried out in an annular flume (Cornelisse, 1981), in this case bar vanishing occurred at regular intervals of 6-8 days.

2 LONG DURATION STRAIGHT FLUME EXPERIMENT

2.1 *Experimental setup*

The experiment consisted of recording the long-term water flow and channel bed evolution in a straight flume with movable bed, fixed banks, and constant discharge. The total length of the flume was 26 m, the channel width 60 cm. The bed was covered by an initially smooth layer of sand having a thickness of 25 cm. The median diameter of the sediment, D_{50} , was 0.238 mm. Water and sediment were re-circulated, but some water was added regularly, to compensate for small losses due to evaporation. The discharge was kept constant at 6.9 l/s.

A wire mesh was introduced at the upstream boundary to dissipate the excess energy of the incoming water and to distribute the flow uniformly. A honeycomb flow straightener was placed immediately downstream of it, followed by a floating sponge to further diminish large scale fluctuations and to suppress surface waves.

Due to the presence of relatively large dunes and ripples (Figure 1, left plate), the rough data had to be

filtered to clean out the bar signal. The filter used was based on the Matlab software ProcessV3 and optimized for bedforms having wavelength larger than 1 m (bars). Migrating bars characteristics were determined by plotting subsequent filtered bed level profiles, which allowed detecting their size and celerity. Non-migrating bars were identified by averaging the filtered bed level profiles over time, which smoothed out most unsteady signals. Turbulence and perturbation smoothing at the upstream boundary and the measuring and post-processing techniques adopted reduced the effective length of the channel to about 20 m.

Two experimental tests were carried out. In the first one, a transverse plate was placed at the upstream boundary to create a permanent external flow perturbation (*forced system test*). This test was meant to study the formation of steady and migrating bars in forced systems as in previous experiments (e.g. Struiksmma and Crosato, 1989; Lanzoni, 2000). In the second test, the transverse plate was removed and the initial bed carefully smoothed out to eliminate all perturbations to the flow (*free system test*). This test was meant to study the evolution of the bed topography in the same system, but without any external forcing.

2.2 Forced system test

The transverse plate, placed at the upstream cross-section of the channel near the right side wall, obstructed 2/3 of the channel width. This created a permanent flow non-uniformity at the inflow. At start, the bed was flattened and the flume was given a longitudinal slope equal to 3 ‰, but during the first two days the longitudinal slope gradually increased until it reached an equilibrium value of 3.54 ‰. The experimental conditions are summarized in Table 1.

Table 1 Experimental conditions (forced system test)

| channel length (m) | channel width (m) | longitudinal bed slope (-) | water discharge (m ³ /s) | median sed. diameter (mm) | mean water depth (m) | mean flow velocity (m/s) |
|--------------------|-------------------|----------------------------|-------------------------------------|---------------------------|----------------------|--------------------------|
| 26 | 0.6 | 3.54 ‰ | 6.9×10^{-3} | 0.238 | 0.051 | 0.225 |

During the experiment, the longitudinal profiles of bed and water levels were measured three times per day. The transverse velocity profile was measured across several sections, but with lower frequency.

Non-migrating alternate bars developed rapidly after experiment start, forced by the presence of the plate, starting from upstream. Their wavelength slowly increased from the initial value of 6.5 m to the final value of about 7.5 m (12.5 times the channel width), which was reached after about two weeks. After that, both wavelength and amplitude of the steady bars remained constant with time (Crosato et al., 2011). Migrating bars were present from the first day on, but only in the second half of the flume (Figure 1). They slowly migrated downstream, disappearing from the flume at the downstream boundary. Due to increasing dominance of steady bars the area in which migrating bars formed became shorter and shorter with time. Migrating bars did not form where steady bars had comparable or larger amplitude: this means that steady and migrating bars were present in the flume simultaneously, but at different locations (Figure 1). We can state that alternate steady bars acted as point bars inside sinuous channels, which are known to reduce or even suppress migrating bars (e.g. Kinoshita and Miwa, 1974; Tubino and Seminara, 1990). The averaged wavelength of migrating bars ranged between 2.6 and 4.1 m; their celerity between 22 and 39 cm/hour; their (filtered) amplitude between 5 and 18 mm. The ratio between steady and migrating bar wavelength ranged between 1.8 and 2.9, which is in agreement with previous studies (e.g. Olesen, 1984). Figure 2 shows the temporal evolution of wavelength and amplitude of migrating alternate bars (daily-averaged values of migrating bars in the second half of the flume). In Figure 2 it is possible to observe that migrating bars rapidly established themselves; their size however, decreased at day 4 to grow again in the next 4 days.

After 4 days, however, all bars unexpectedly vanished, to reform with smaller sizes the day after. Subsequently, the size of migrating bars increased again. A third drop in bar size was observed on day 14.

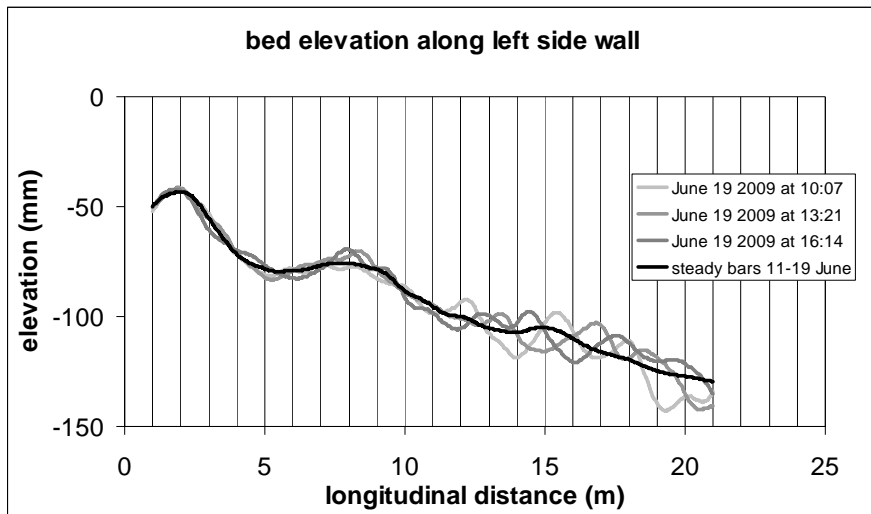


Figure 1 Successive measurements of bed level profile 5 cm from the left side wall (filtered data) showing the presence of steady and migrating alternate bars. Migrating bars are present only in the second half of the flume. Black line: time-averaged longitudinal profile June 11-19. Forced system test.

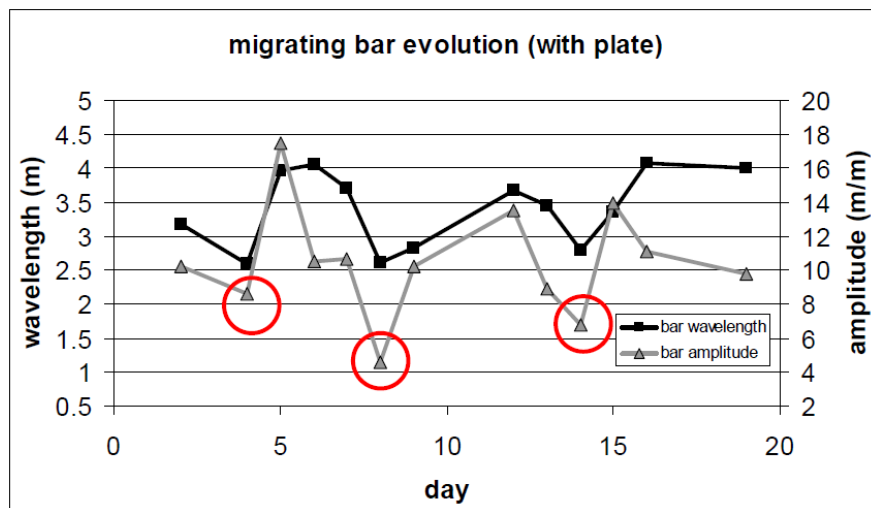


Figure 2 Temporal evolutions of spatially-averaged migrating bar wavelength and amplitude. Forced system test.

2.2 Free system test

This experimental test was characterized by uniform inflow and had the same constant discharge and sediment as the forced system test. The duration of this test was about 10 weeks, which means that it was much longer than any other published experiment of this type. The starting conditions were a smooth bed

with longitudinal slope of 3 ‰, just like in the previous experimental test. Subsequently, the bed slope gradually increased, reaching the equilibrium value of 3.74 ‰ after about two days. The experimental conditions are summarized in Table 2.

Table 2 Experimental conditions (experiment without transverse plate)

| channel length (m) | channel width (m) | long. bed slope (-) | water discharge (m ³ /s) | median sed. diameter (mm) | mean water depth (m) | mean flow velocity (m/s) |
|--------------------|-------------------|---------------------|-------------------------------------|---------------------------|----------------------|--------------------------|
| 26 | 0.6 | 3.74 ‰ | 6.9×10 ⁻³ | 0.238 | 0.049 | 0.235 |

The slightly higher equilibrium longitudinal slope and flow velocity and the slightly lower water depth with respect to the previous experiment can be attributed to the absence of the transverse plate, which increased energy dissipation in the forced system test. During the first week, the longitudinal profiles of bed and water levels were measured three times per day. The measurements were later carried out twice/day in the following 3 weeks, but only once every three days during the second month. In the last (10th) week, the measurements were carried out again with the initial frequency. The last measurement was taken 68 days after the start of the experiment.

A steady bar started to appear from the first week, but for the first three weeks this bar was weak and not always detectable. Its initial wavelength was about 7.0 m, but just like in the previous test, the wavelength gradually increased and reached the final value of about 7.5 m after about three weeks. Since then, the steady bar continued to grow in amplitude, although slowly, and two more bars started to appear about six weeks after experiment start (Crosato et al., 2011). Instead, migrating bars started to form immediately. As in the previous test, they could freely disappear from the flume at the downstream boundary while they re-formed upstream. Their averaged wavelength ranged between 2.5 and 4.9 m; their celerity between 23 and 40 cm/hour; their (filtered) amplitude between 5 and 16 mm. The ratio between steady and migrating bar wavelength ranged between 1.4 and 3.0. Initially, migrating bars were present along almost the entire flume length, but one month after the start of the experimental test they formed only in the second half of the flume, in a way similar to the forced system test and to prior experiments on both forced steady bars and migrating bars (Struiksmā and Crosato, 1989; Lanzoni, 2000). The area in which migrating bars formed reduced after the emerging of a steady bar in the upstream part of the flume (Figure 3). This means that, even without any upstream disturbance (demonstrated by measurements of transverse flow velocity), the channel bed topography gradually acquired the characteristics that could be expected with a permanent upstream forcing.

As in the forced system test, migrating bars had a cyclic behavior: they formed, vanished and re-formed again several times. We observed bar vanishing four times in two months. This is visible in Figure 4, which shows the temporal variations of daily-averaged bar wavelength and amplitude. We observed steepening of the bar front before vanishing. This observation, however, has some degree of uncertainty due to the smoothing that was applied to the rough data to clear up the presence of ripples.

The period between two successive vanishing events was 13-22 days, but it is possible that we missed one or more of these events, due to the relatively long interval between successive bed level profile measurements. Bar vanishing and re-forming occurred in less than one day, as shown in Figure 5.

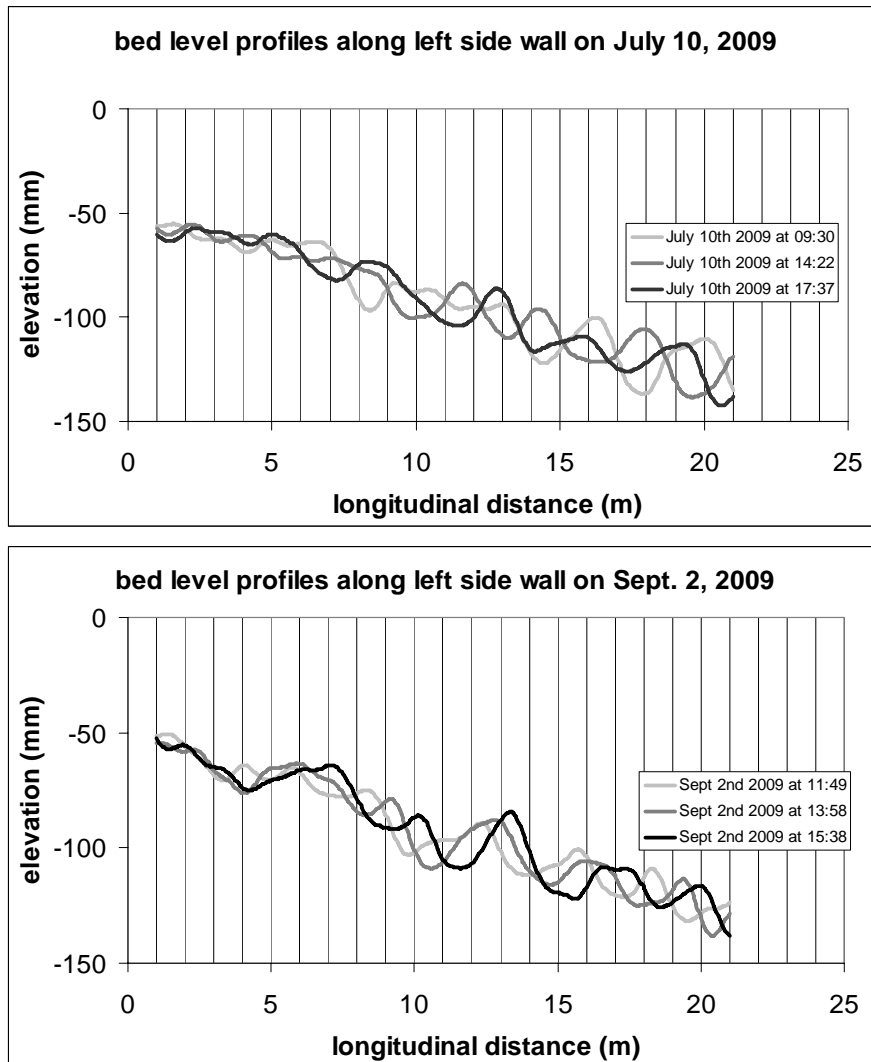


Figure 3 Successive measurements of bed level profile 5 cm from the left side wall (filtered data) showing the presence of both steady and migrating bars. It is possible to observe that the area in which migrating bars formed moved downstream and grew smaller with time. Free system test.

Contrary to migrating bars, steady bars never vanished. They tended towards an equilibrium configuration, although their growth rate was very small. This means that the free system tended to slowly transform into a forced system. As in the forced system, steady and migrating bars coexisted in the flume, but steady bars suppressed migrating bars locally, which means that steady and migrating bars were not found at the same location in the flume (Figure 3).

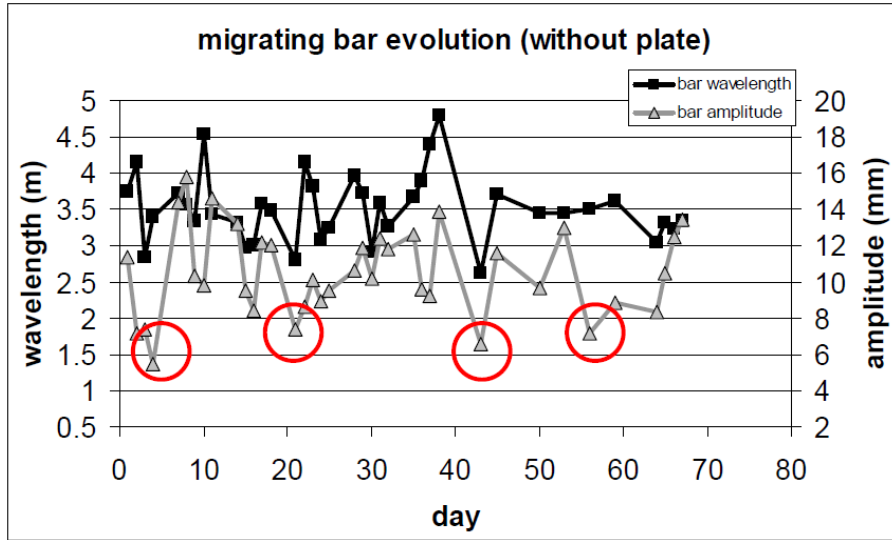


Figure 4 Temporal evolutions of spatially-averaged migrating bar wavelength and amplitude. Bar vanishing can be recognized by a drop in bar amplitude (red circles). Free system test.

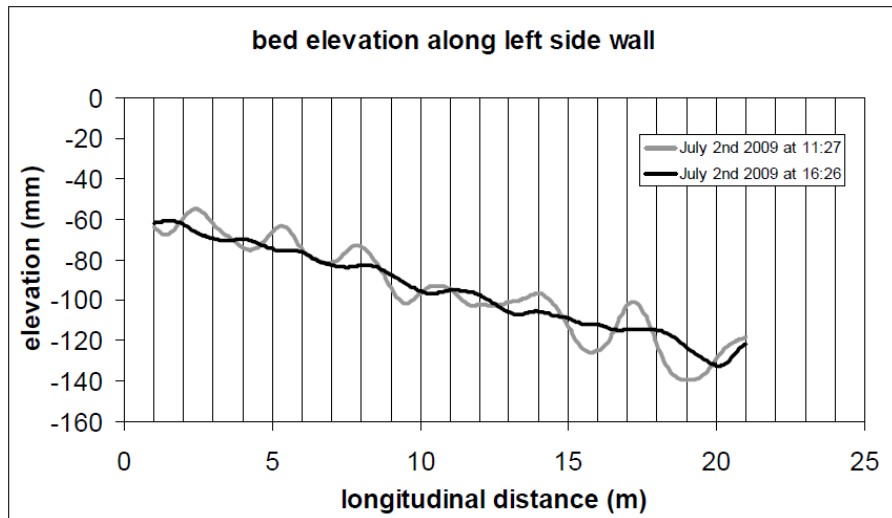


Figure 5 Evolution of bed topography 5 cm from the left side wall on July 2nd 2009. Migrating bars vanished in the second half of day 3. Free system test.

3 LONG DURATION ROTATING ANNULAR FLUME EXPERIMENT

Annular flumes are often used to study changes of channel bed topography in the laboratory, especially for cohesive sediment (Booij, 1994; Yang et al., 2000). In annular flumes there is no pump to re-circulate the sediment-laden water, because the flow is obtained by the rotation of the top lid. Counter-rotation of the channel is a way to minimize secondary circulation and lateral variability of shear stress, due to the differences in tangential velocity between the inner and the outer sides (Partheniades et al., 1966). The idea is to approach the idealized concept of an endless straight channel flow. However, complete

elimination of the secondary circulation is difficult, if not impossible, to achieve (Booij and Uijttewaai, 1999). For this, the flow in an annular flume finally resembles the flow in an infinitely-long mildly-curved channel, i.e. with a radius of curvature that is much larger than the radius of curvature of the flume. A 42 days long experiment was carried out at the Laboratory of Fluid Mechanics of Delft University of Technology, in 1981 (Cornelisse, 1981). The experiment was meant to study alternate bar formation and was similar to the one previously carried out by Engelund (1975). One long-duration test aimed at finding out whether an alluvial channel with migrating alternate bars would eventually develop into a dynamic equilibrium configuration, but the results were never published. The characteristics of the flume are described in Table 3, the flume lay out is shown in Figure 6.

Table 3 Experimental conditions (experiment in annular flume)

| inner wall diameter* (m) | outer wall diameter* (m) | channel width (m) | median sed. diameter (mm) | mean water depth (m) |
|-----------------------------|-----------------------------|----------------------|------------------------------|-------------------------|
| 1.94 | 2.34 | 0.2 | 0.200 | 0.06 |

* values relative to the inner side of the wall (inside the flume).

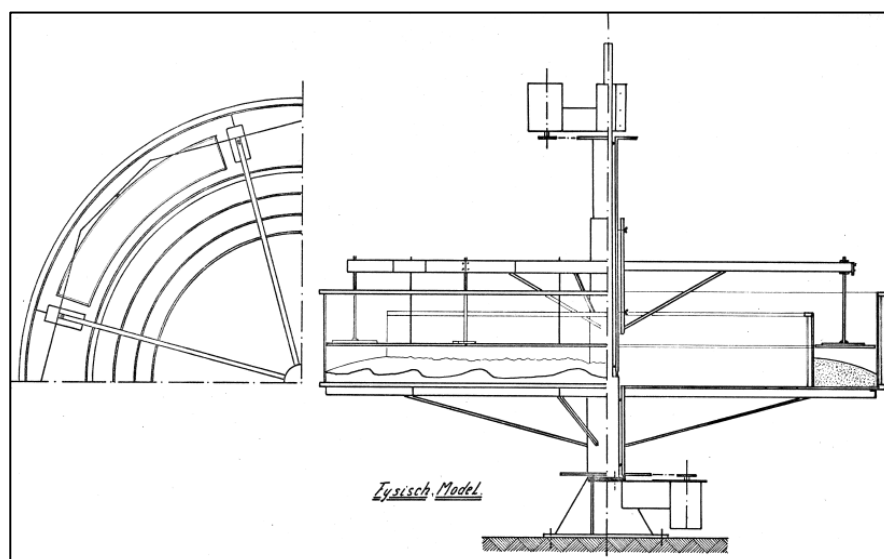


Figure 6 Annular flume lay out.

During the test there was no counter-rotation of the channel, the rotational velocity of the top lid, ω_t , being 0.9 rad./s. The rotational velocity of the channel, ω_b , was therefore equal to zero. This means that the experiment did not reproduce well the flow in a mildly-curved infinitely-long channel, but kept some asymmetry due to higher flow velocity near the outer wall. The experimental test started with a flat bed. Water was regularly added to compensate losses due to evaporation. The bed level was measured with a provo along 10 concentric circles across the channel width, i.e. every 2 cm starting 1 cm from the outer wall. Rough data were filtered to get rid of ripples.

The dominant alternate bar wavelength, defined as the one having the highest energy density, was determined by means of spectral Fourier analyses of the bed elevation measured along the largest concentric circle, i.e. the one having diameter equal to 2.32 m. It was not possible to correctly determine

the bar celerity, because it varied in radial direction, showing bars that were advancing slightly faster near the outer wall (probably due to having no counter-rotation of the channel). Unexpectedly, instead of tending towards an equilibrium configuration (amplitude, wavelength), alternate bars persistently showed a periodic behavior, characterized by repeating bar vanishing events. A dominant alternate bar having wavelength of 1.4 m and amplitude of the order of 5 cm was present during 90% of the time, but for the 10% of the time the channel bed was almost plane, without a clear dominant wavelength. Bars vanished at intervals of 6-8 days to re-appear a short time after (Figure 7, derived from digitalization of old drawings).

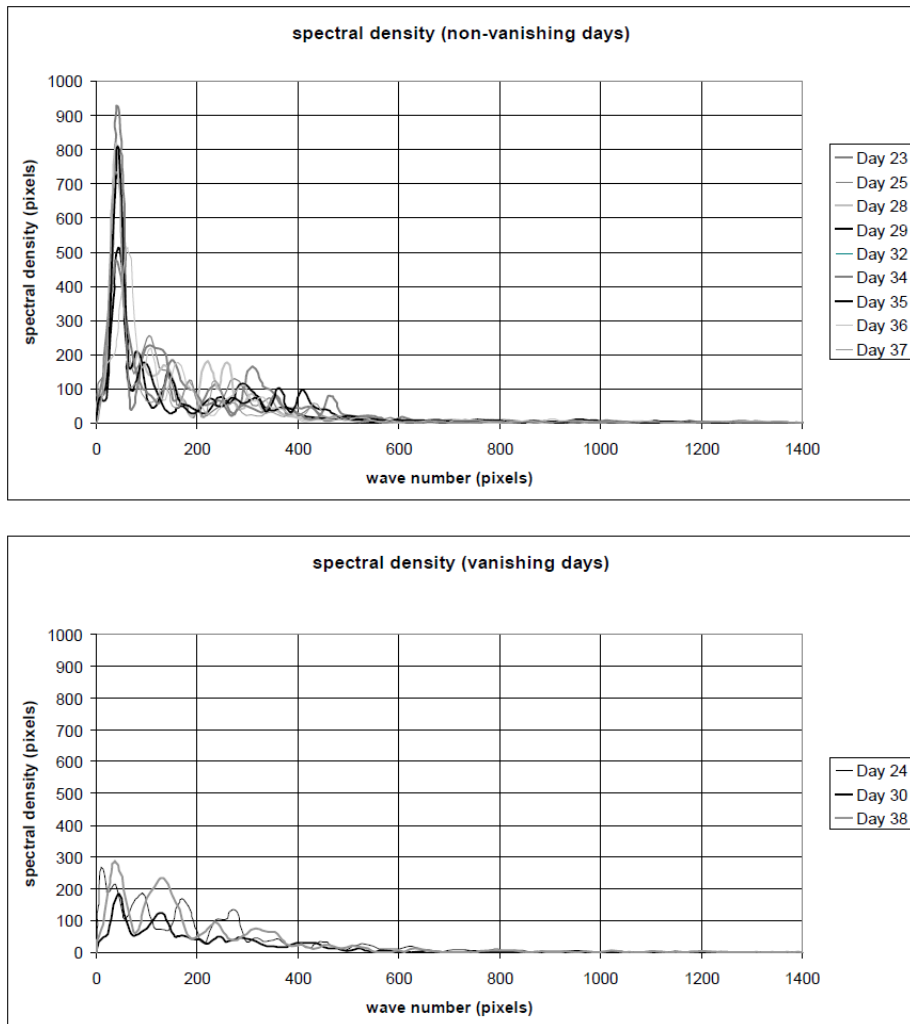


Figure 7 Spectral densities as a function of bar wave number (data corresponding to days 23-38). Above: days with well-developed bars. Below: days with vanishing bars: days 24, 30 and 38. Rotating annular flume experiment.

4 DISCUSSION AND CONCLUSIONS

We studied the long-term evolution of alternate bars by performing two independent long-duration laboratory experiments. Our experimental studies, carried out either in a straight flume or in a rotating annular flume, show that migrating alternate bars might never reach a stable (although dynamic due to their celerity) configuration. These features showed a periodic behavior characterized by vanishing and reforming. Moreover, in the straight flume tests, approximately twice as large steady bars developed too, even without any external disturbance. In both tests, steady bars gradually suppressed migrating bars. Contrary to migrating bars, steady bars never vanished, but tended towards an equilibrium configuration. This means that also the free system tended to slowly transform into a forced system. Steady bar development started immediately in the forced system, but the process was slow in the free system, where steady bars became clearly detectable only after three weeks (Crosato et al., 2011).

Apparently, previous experiments and numerical studies had been terminated before completion of the bed evolution process and therefore could not detect either migrating bar vanishing or steady bar development without external disturbances. A single bar vanishing event was reported by Takebayashi and Egashira (2001), who performed some laboratory tests on bar formation with graded sediment. They attributed bar vanishing to sediment non-uniformity. Their experiment, however, was not long enough to detect whether alternate bars vanished cyclically.

In our experiments, migrating bar vanishing seems related to progressive steepening of the bar fronts. Without any external forcing, steady bar development seems caused by the presence of migrating bars, which may operate as forcing factors, in a way similar to periodic discharge variations (Hall, 2004). Finally, steady alternate bars are found to act similarly to point bars in sinuous channels in their ability to reduce or suppress migrating bars (e.g. Kinoshita and Miwa, 1974; Tubino and Seminara, 1990).

In conclusion, in the performed experiments, migrating bars appear as a transitional phenomenon of alluvial channels having a cyclic character, whereas the bed topography is eventually dominated by the presence of steady bars. We believe, however, that bar vanishing and steady bar formation may only occur at certain morphodynamic conditions, whereas for other flow characteristics these processes might not take place. In order to have a full overview of formation and further developments of bars in alluvial channels, it is necessary to carry out a fully non-linear study of bar behavior in alluvial channels.

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