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Design of an Emergency Shelter

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Summary: This paper presents the design of a light and robust temporary emergency shelter with a triangulated polyhedral grid to transfer the lateral and vertical loads efficiently. To simplify the construction the variety of the elements is minimized, only two sizes of elements are applied. For the structural grid alternatives are designed using varying materials as cardboard, aluminum, steel and timber. The dimensions of the elements are validated. The alternatives are compared and ranked to minimize the environmental load due to the production, assemblage and transport.

Keywords: *Emergency shelter, grid shell, polyhedron, sustainability, transport, assemblage.*

1. INTRODUCTION

Every year many people become homeless due to storms, floods, earthquakes or other disasters. Generally national and international organizations, as for example the Red Cross, offer help by sending for example food, medicine and shelters. It takes time to repair and rebuilt homes, often the victims have to live for months in a temporary shelter. To be useful temporary shelters have to fulfill specifics and demands. The area of the plan has to be at least 9,29 m^2 [1]. The climate conditions are not specified before and can vary much, nevertheless the shelters must offer some protection to extreme temperatures, rain, snow and wind. Further the shelters must be assembled quickly on the site. The conditions of the site concerning the foundation can vary too, the soil can be of rock, sand or clay. The shelters must be shipped by planes and trucks over long distances to the site, so the weight and volume must be minimal. Fabric structures are the lightest structures of all [2] and can be transported easily. For light shelters the wind loads are often decisive for the design of the structure. To reduce the wind load acting on the structure the form and surface must be smooth and the height of the shelter must be rather small, so the wind can stream easily along and over it and does not lift the shelter from the foundation. Half spherical domes, with a height equal to the radius, do not turn over easily during a storm. Further a spherical dome is the most efficient way to enclose volume and to cover a surface [3]. The structure of a dome can be composed of small rigid bars. The most widely publicized are the geodesic domes associated with Buckminster Fuller [4]. A polyhedron with a triangulated grid is stiff, stable and strong, so these structures can resist horizontal and vertical loads quite well. Generally the grid of a geodesic is constructed by subdivision the edges of an regular polyhedron, mostly an icosahedron or an dodecahedron, in smaller parts and projecting the vertices on the circumscribing sphere. Unfortunately increasing the frequency of the partition increases the variation of the elements too. For an emergency shelter the variation of the elements has to be small, so the frequency of the partition has to be small too.

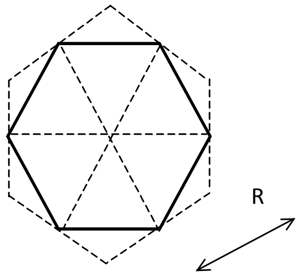

Fig. 1. View of the hexagonal roof of the shelter with a radius R.

For a half spherical dome the frequency of the partition has to be at least two if the grid is based on a icosahedron, dodecahedron, octahedron or any other polyhedron, so the edges are halved and the number of elements is doubled. To reduce the number and variety of the elements a polyhedral grid is developed with 18 and 6 identical edges. The shelter is composed of a hexagonal roof with six triangular faces, see figure 1, and a drum with twelve triangular faces. The vertices of the polyhedron touch the circumscribing half sphere, see figure 2. Using mathematics the vertex of the ring beam, between the roof and drum, are positioned nearly halfway the top and foundation, so that the edges of the roof, running from the top to the ring beam, and the edges of the drum, running from the ring beam to the foundation, are equal of length. Consequently the six congruent triangular faces of the roof are congruent to the six adjacent faces of the drum.

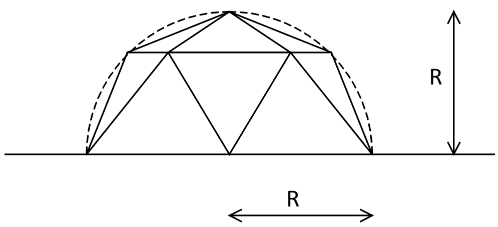

Fig 2. Façade of the shelter with the circumscribing sphere.

This paper concentrates on the design of a structural grid of a light and robust emergency shelter to be transported and constructed easily. The structure is designed with a triangulated polyhedral grid to transfer the lateral and vertical loads efficiently. Due to the spherical form the wind load is minimal. To simplify the construction the variety of the elements is minimized, only two sizes of elements, varying in length, are applied. For the structural grid alternatives are evaluated using varying materials, aluminum, steel, timber and even cardboard, just as for the paper dome [5]. The dimensions of elements are validated concerning strength and stiffness. The alternatives are compared and ranked to minimize the weight and environmental load due to the production, assemblage and transport.

2. DESIGN

A design can be considered as a response to the demands and wishes of the users and owners, so any process of design starts with an analysis of the demands.

2.1. Demands

For emergency shelters we can distinguish varying demands concerning use, comfort, cost, construction and transport.

- To accommodate the homeless during some months the area of the shelter must be at least 9,29 m^2 [1].
- The funds are provided by donators and limited, to help as many victims as possible, the cost of production, transport, construction and assemblage must be minimal.
- The materials are shipped to the site by planes and trucks, so the weight of the shelter must be minimal and the sizes of elements must be small enough to be transported easily.

- The shelters must be erected quickly using simple equipment. To simplify the construction the elements must be easy to handle. The weight of an element has to be smaller than 20 kg so one person can lift it. To simplify the assemblage the structure has to be composed of identical elements connected by preference with hinges above fixed joints.
- The disaster can happen everywhere, thus the climate conditions can varying much. Nevertheless the shelter must protect the inhabitants for rain, sun, wind, snow and extreme temperatures. On forehand the site is not specified, the ground can be of sand, clay, rock or another material. Still the foundation must be able to prevent the shelter to be lifted upward by an average wind load.
- According to the worldwide demand to reduce the emission of CO_2 the embodied energy and environmental load due to the production, transport and assembling of the elements must be minimal.

2.2. Loads

The structure has to resist varying loads for example the wind, snow and dead load. To facility transport and construction the dead weight of the structure has to be minimal. Probably in the tropics and subtropics the wind load will be decisive. Near the artic and in mountainous regions the snow loads can be quite large. For a region with extreme snowfall the structural resistance of the shelter has to be increased.

2.3. Materials

To reduce the dead weight and environmental load of the structure the shelter has to be composed of light materials so the elements can be transported well and constructed easily. For the enveloping skin we can use panels of paperboard, polypropylene or any other light and economic material. Nevertheless mostly a fabric is chosen to simplify the construction and minimize the weight. For the frame we can use elements of timber, aluminum, steel, cardboard or any other light, stiff and cost effective material [1]. The properties of cardboard vary much and depend on the direction of the fabrication, with: MD = machine direction, CD = cross machine direction [6]. Due to the variety of the properties the quality of cardboard elements must be put to extended test before use. Table 1 shows mechanical properties used for the design of the structures [6].

Table 1. Mass, stress and Young's modulus to compare steel, timber, cardboard and aluminum

	Mass $[kg/m^3]$	Tensile stress [MPa]	Compres s. stress [MPa]	Shear stress [MPa]	Young's modulus [MPa]
Steel	7800	235	235	134	210.10^3
Timber	500	11,5	16,8	1,89	$11,6.10^3$
Cardboard tubes CD	800	5 – 15	2,5 – 7,5		$1,34.10^3$
Aluminum	2700	90	90	54	70.10^3

3. FORM CONCEPT

To reduce the dead weight of a structure a form-active structure or a surface active structure seems quite effective. With cable structures large spans are made with a minimal dead weight. By preference temporary buildings are supported by a simple foundation. A simple foundation can resist upward forces if the weight of the building is larger than the uplift. The weight of a light shelter is often to small to resist the uplift caused by a storm, so most tents are pinned to the ground. The pins must be pushed or drilled in the ground, but probably heavy equipment, as used for pile foundations, is not available if the infrastructure is destroyed. By preference the structure of the shelter is designed in such a way that the foundation is not subjected by huge upward forces. Consequently arches are preferred above suspended structures with cables pulling at the foundation. Due to a vertical load the supports of an arch are subjected to a vertical force acting downward and a horizontal force, the thrust, acting outward. Often the thrust can be resisted by the foundation, but if the ground is to soft to resist the thrust then ties can be constructed between the supports. For structures with a circular plan the thrust can be taken with a circular ring. Nevertheless arches and domes must be able to resist upward wind loads. Every year many buildings are destroyed by tornados, so it will be very hard to protect a light shelter to extreme winds. To resist a huge wind load it can be necessary to increase the resistance against uplift with ballast, for example bags filled with earth and stones or any other material available at the side. The uplift acting at the shelter due to the wind load can be reduced substantially by designing the shelter in such a way that wind is mostly blowing over instead of against the shelter, so the form follows the loads. We can learn much from nomads who live in tents in harsh conditions. In the past the nomadic Indians, living on the prairies of the United States, built conical tents [2]. Due to the form, decreasing from the bottom to the top, the wind load is pretty small and acting just above the foundation so the over turning moment is small. In Mongolia nomads built yurts with a conical roof and a cylindrical base. The wind forces acting on a spherical dome are rather small too. For a spherical dome the surface starts perpendicular to the ground face so the usefulness of the interior is larger than for a conical shelter with an inclined surface.

3.1. Grid

Domes can be constructed with radial arches connected with a ring at the top and a ring the footing. Triangulating the surface gives a stiffer and stronger structure. To compose a spherical dome of triangular faces Walter Bauersfeld constructed the first geodesic dome in Jena in 1922. Later Buckminster Fuller got a patent for the principal of subdividing. Geodesic domes are made by projecting the nodes and edges of a polyhedron on the circumscribing sphere. Generally an icosahedron or a dodecahedron is chosen.

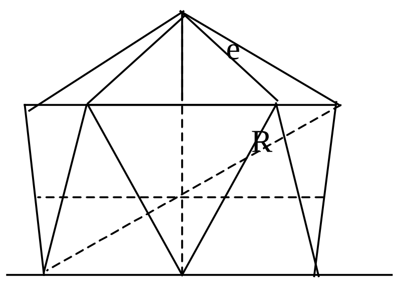

Fig. 3: Icosahedron truncated at the base, the height is about 1,45 times the radius.

Nevertheless geodesic domes can be created too by projecting the nodes and edges of a tetrahedron, octahedron or cube on the circumscribing sphere. For large geodesic domes the faces of the polyhedron will be subdivided into small triangles. The number of the subdivision of the edges is called the frequency. Increasing the number of the frequency increases the variation of the elements and increases the complexity as well. Especially for small emergency shelters it is advisable to decrease the number of elements as much as possible. The variation of the edges and vertices is minimal in case the edges are not subdivided in parts, so by preference the frequency is minimal. A tetrahedron gives a very simple grid, but the triangular ground plan is not very efficient. To create a useful interior the height of the tetrahedron must be quite large with respect to the plan. A cube composed of square faces is very efficient but not stable unless the faces are stiffened with diagonals. Also the dodecahedron needs diagonals for stability. The icosahedron is composed of triangular faces and stable. To create a useful plan the icosahedron must be truncated. Two possibilities arise: the structure is made of 15 faces or the structure is halved thorough the center. The height of a truncated icosahedron composed of 15 faces is rather large. Halving the icosahedron gives 10 halved faces. The height of the halved triangular faces is too small to make an opening for a door. To create a door the grid has to be adapted and this will increase the variation.

3.2. Hexagonal drum

To fulfill the demands a new polyhedron is designed with a pyramidal roof composed of six identical triangular faces, resting on a drum with twelve faces. Using identical elements simplifies the construction of the structure much. Symmetry is a sine qua non for the design. Using the symmetry of a sphere the vertices of the polyhedron are positioned on the half sphere with radius R. Further the vertices are positioned in such a way that six faces of the roof are congruent to six adjacent faces of the drum. Actually the drum is composed of twelve faces, six faces of the drum are identically to the faces of the roof, the other six faces of the drum are congruent too, but vary from the faces of the roof, see figure 4.

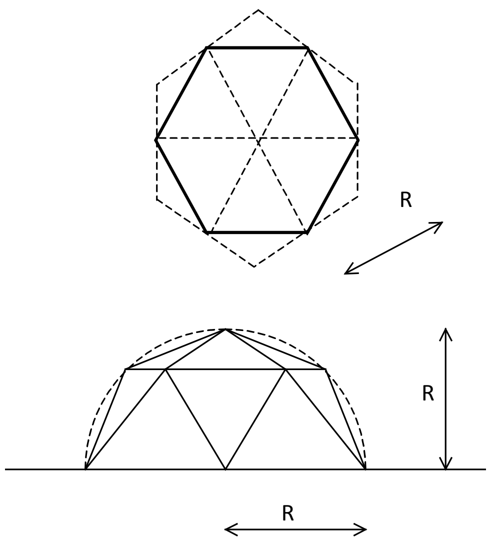

Fig. 4: Facetted dome with hexagonal plan and radius R

4. GEOMETRY

The polyhedron with a hexagonal plan is composed of a low rise pyramidal roof with six triangular faces, supported by a drum with twelve faces. The vertices of the polyhedron are situated on the circumscribing sphere with radius R. To decrease the variety of the elements the adjacent faces of the roof and drum are designed identically. The six faces of the drum and six adjacent faces of the roof have a common edge situated in a hexagonal face parallel to the ground face. The length of the edge in the ground face is larger than the edge in the ring between the six congruent faces of the roof and the identical adjacent faces of the drum, consequently the six faces of the drum standing on the ground face are larger than the other faces. The six triangles of the roof are congruent to the six adjacent triangles of the drum, thus the length of the edges of the faces of the roof must be equal to the length of the edges of the adjacent faces of drum. The length of the edges of the polyhedron is defined mathematically.

The two bisector lines of the identical triangles, named bs, are cords of a grand circle of the dome with radius R and cut the horizontal edge between the triangular face of the roof and the adjacent face of the drum, named rs, into two equal parts. The edges rs are situated on a horizontal hexagonal face with a radius r composed of six congruent faces, consequently the length of the edge rs is equal to r. This hexagonal face is positioned on a distance t of the top, the distance of the hexagon to the ground plan is equal to z = R – t.

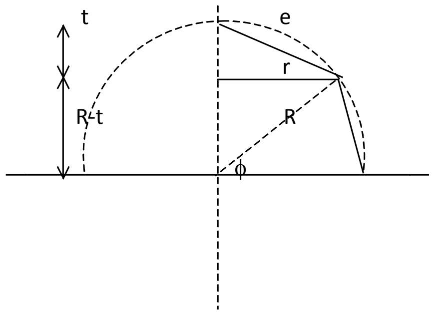

Fig. 5: Section over the grand circles of the polyhedron along edge e and the median of the chord of the hexagon on the ground face.

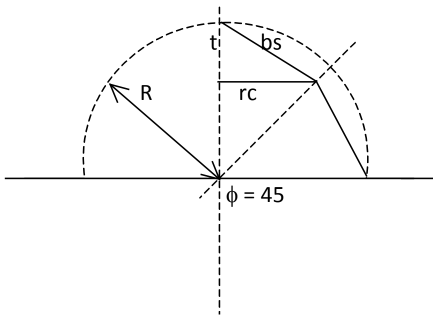

Fig. 6: Section over grand circles of the polyhedron along the bisector bs and the median rc.

Two vertical sections are drawn over the grand arches of the sphere. The first section is drawn over the edge e of the triangular face of the roof, see figure 5. The second section is made over the bisector bs of the face of the roof splitting rs into two equal parts, see figure 6.

From the section over the edge e, see figure 5, follows:

$$R^2 = (R-t)^2 + r^2 \;\rightarrow\; t = R - \sqrt{(R^2-r^2)} \qquad (1)$$

From the section over the bisector bs, see figure 6, follows:

$$R - t = r.\cos(\pi/6) \;\rightarrow\; t = R - r.\cos(\pi/6) \qquad (2)$$

Substituting t according to (2) into (1) to calculate r:

$$\sqrt{(R^2-r^2)} = r.\cos(\pi/6) \;\rightarrow\; r = R/\sqrt{[1+\cos^2(\pi/6)]} = 0{,}756.R \qquad (3)$$

Substitute r into (1) to calculate t:

$$t = R - r.\cos(\pi/6) = 0{,}345.R \qquad (4)$$

The edge e follows from:

$$e^2 = t^2 + r^2 \;\rightarrow\; e = 0{,}831.R \qquad (5)$$

The pairs of identical adjacent faces are joined with cords of the circle with radius r at a height of R - t. The length of the cords of the ring is equal to r. The angle α at the top of the triangular roof face follows from:

$$\sin\alpha = ½.\, rs/e = ½.0{,}756/0{,}831 = 0{,}455 \;\rightarrow\; \alpha = 27{,}05^o \qquad (6)$$

The height of the first ring is found for:

$$z = R - t = 0{,}655.R \qquad (7)$$

The length of the bisector of the face from the top T to the chord is equal too:

$$bs = e.\cos(\alpha) = 0{,}740.R \quad (8)$$

The area of a face of the roof is equal to:

$$A_1 = ½ * e * \cos(\alpha).\ rs = 0{,}288.\ R^2 \quad (9)$$

The distance from the center of gravity of the face to the edge e is equal to:

$$ce = {}^2/_3\ .\ bs.\sin(\alpha) = 0{,}2244.R \quad (10)$$

The angle ψ between bisector of the roof face and the horizontal face follows from:

$$\cos(\psi) = rc/bs = r.\cos(\pi/6) / bs \quad (11)$$

Substituting $bs = 0{,}740.R$ and $r = 0{,}756.R$ gives:

$$\psi = 27{,}8^o \quad (12)$$

Next the features of the other triangular faces standing on the ground plan are defined. These triangular faces have two sides equal to e, with: e = 0,831.R (5). The third side is positioned on the ground face, this face is a hexagon, so the length of the cord in the plan is equal to R. The angle β at the top of the triangular face of the drum follows from:

$$\sin(\beta) = ½.R/e = 0{,}602 \rightarrow \beta = 36{,}99^0 \quad (13)$$

The length of the bisector bs' running from the vertex of the ring to the chord of the ground face is equal too:

$$bs' = e.\cos(\beta) = 0{,}664.R \quad (14)$$

The area of the face on the ground is equal to:

$$A_2 = ½ * bs' * R = 0{,}332.R^2 \quad (15)$$

The angle γ between bisector of the face of the drum and the vertical Z-axis follows from:

$$\cos(\gamma) = z/bs' \quad (16)$$

Substituting $bs' = 0{,}664.R$ and $z = 0{,}655.R$ gives:

$$\gamma = 9{,}55^o \quad (17)$$

The results are checked with the following approach. Vertex 7 has to be at an equal distance from the top, vertex 13, and the vertex 1. The Cartesian coordinates of vertex 7 are:

$$x = R.\cos(\pi/6).\cos(\phi)\ ;\ y = R.\sin(\pi/6).\cos(\phi)\ ;\ z = R.\sin(\phi) \quad (18)$$

The length of the vector between vertices 7 and 13 is equal to:

$$[\{R.\cos(\pi/6).\cos(\phi)\}^2+\{R.\sin(\pi/6).\cos(\phi)\}^2+\{R.\sin(\phi) - R\}^2]^{0,5} \quad (19)$$

The length of the vector between vertices 1 and 7 is equal to:

$$[\{R.\cos(\pi/6).\cos(\phi) - R\}^2+\{R.\sin(\pi/6).\cos(\phi)\}^2+\{R.\sin(\phi)\}^2]^{0,5} \quad (20)$$

Both lengths are equal:

$$[\cos(\pi/6).\cos(\phi)]^2 + [\sin(\pi/6).\cos(\phi)]^2 + [\sin(\phi) - 1]^2 =$$

$$[\cos(\pi/6).\cos(\phi) - 1]^2 + [\sin(\pi/6).\cos(\phi)]^2 + [\sin(\phi)]^2 \quad (21)$$

With this equation the angle ϕ of the vector pointing to the vertices of the ring is calculated as follows:

$$2.\cos(\phi).\cos(\pi/6) = 2.\sin(\phi) \quad \rightarrow \quad \phi = 40{,}89^0 \quad (22)$$

The Cartesian coordinates of the vertices can be calculated easily using polar coordinates. With the radius R, the horizontal angle θ and the angle ϕ the Cartesian coordinates of the vertices are calculated with:

$$x = R.\cos(\theta).\cos(\phi);\ y = R.\sin(\theta).\cos(\phi);\ z = R.\sin(\phi) \quad (23)$$

Table 2 shows the Cartesian coordinates for a radius R = 1,0 m. The center of the coordinates X,Y,Z is positioned at the center of the plan of the dome, the Z-axis points upward. The angle θ is equal to $\theta = n.\pi/6$. For the ground plan n = 0, 2, 4 , 8 , 10 and 12 and for the ring n = 1, 3, 5, 7, 9 and 11. For the top to $\theta = 0$. For the vertices on the ring the angle ϕ is equal to: $\phi = 40{,}89^0$.

(11)

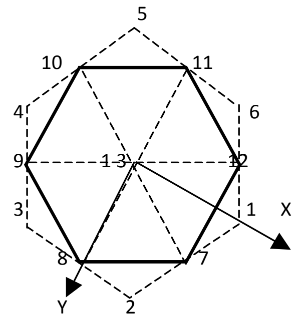

Fig. 7. Numbering of the vertices of the grid, X-axis and Y-axis on the plan, the Z-axis points upward.

Table 2: Cartesian coordinates for a dome with radius R = 1 m, the Z-axis is taken upward

	X	Y	Z
1	1	0	0
2	½	½ √3	0
3	-½ (14)	½ √3	0
4	-1	0	0
5	-½	-½ √3	0
6	½	-½ √3	0
7	0,655	0,378	0,6547
8	0	0,756	0,6547
9	-0,655	0,378	0,6547
10	-0,655	-0,378	0,6547
11	0	-0,756	0,6547
12	0,655	-0,378	0,6547
13	0	0	1

4.1. The radius of the dome

The area of the plan of the shelter has to be at least 10 m². the area of the ground plan is equal to:

$$A = 6 * 2 * ½ * R\sin(\pi/6).\ R\cos(\pi/6) = {}^3/_2.\ \sqrt{3}.R^2. \quad (24)$$

For R = 2,0 m the area is equal to:

$$A = 10{,}39 > 9{,}29\ m^2 \quad (25)$$

Next the length of the edges and chords are calculated by multiplying the lengths with the required radius R = 2,0 m.

4.2. The length of the edges and the Cartesian coordinates of the vertices

For a radius of R = 2,0 m the table 3 shows the length of the elements of the face of the roof and the identical face of the façade. Table 3 shows the length of the elements of the face of the façade standing on the ground face. Table 5 shows the area of the faces of the roof and facade. Table 6 shows the Cartesian coordinates of the vertices.

Table 3. Lengths of the face of the roof and identical face of the façade

	Length	[m]
edge e =	0,831.R =	1,662
Height of the first ring z = R-t	0,655.R =	1,309
Radius of the ring r =	0,756.R =	1,512
Cords of the ring rs = r =	0,756.R =	1,512
the bisector from the top to the ring bs = e.cos α =	0,740.R =	1,480

Table 4. Lengths of the face of the façade standing on the ground face

	Length	[m]
edge e =	0,831.R =	1,662
Cords of the ground plan c = r	1,0 .R =	2,0
Bisector from the top to the chord radius of the ring bs =	0,664.R =	1,328

Table 5.The area of the roof and facade

		Area	[m^2]
Roof face	A_{f1} =	0,27976.R^2 =	1,119
Identical face of the façade	A_1 =	0,2796.R^2 =	1,119
Face standing on the ground face	A_2 =	0,3319.R^2 =	1,328

Table 6: Cartesian coordinates for R = 2,0 m, the Z axis points upward according to the FEM program

Node	X	Y	Z
1	2	0	0
2	1	1,732	0
3	-1	1,732	0
4	-2	0	0
5	-1	-1,732	0
6	1	-1,732	0
7	1,309	0,756	-1,309
8	0	1,512	-1,309
9	-1,309	0,756	-1,309
10	-1,309	-0,756	-1,309
11	-0	-1,512	-1,309
12	1,309	-0,756	-1,309
13	0	0	-2

5. LOADS

The shelter is subjected to the permanent load and the live loads.

5.1.1. Permanent load

The envelope can be made of a fabric or of panels of cardboard. The grid can be made of cardboard, aluminum or steel. The self weight of the structure is rather small. The load of the fabric is equal to p = 0,1 kN/m^2. The vertices are subjected to a force F = p.ΣA_i/3 with A_i according to table 7.

Table 7. Vertical forces due to the permanent load, p = 0,1 kN/m^2

Vertex		F_i [kN]
F_{13} =	p.6. A_1/3 =	0,224
$F_7 = F_8 = F_9 = F_{10} = F_{11} = F_{12}$ =	p. (4.A_1 + A_2)/3=	0,193

5.1.2. Snow load

The snow load acting on the ground face according to the Euro code NEN 1991-1-3, 2005 [7] follows from:

$$s_e = u_1.s_k \tag{26}$$

For a dome u_1 = 0,8. As reference the snow load is calculated for the Netherlands with s_k = 0,7 kN/m^2. For an arctic area or a height of more than 1000 m above sea level, the load has to be increased. The snow load acting on a face of a inclined roof with an angle ϕ the distributed load acting on the roof face follows from:

$$p_s = u_1.s_k \cdot \cos\phi = 0{,}495 \text{ kN/m}^2 \tag{27}$$

The vertices are subjected to a force F = p_s.ΣA_i/3 with A_i according to table 8.

Table 8. Vertical forces due to the permanent load, p = 0,495 kN/m^2

Vertices		F_i [kN]
F_{13} =	p_s.6. A_1/3 =	1,11
$F_7=F_8=F_9=F_{10}=F_{11}=F_{12}$ =	p_s. 2.A_1 /3=	0,37

5.1.3. Wind Load

According to the Euro code NEN 1991-1-4, 2005 [8] the wind load is calculated with:

$$w_e = c_{pe}.q(z) \tag{28}$$

With: c_{pe} is coefficient depending on the form of the structure and direction of the wind and q(z) is the wind load depending on the height and the region.

The wind load depends on the location. By preference the shelters will be built at a side protected for severe storms. To limit the cost the shelter is designed to resist a reasonable load. As reference is chosen the maximum wind load acting on the coast of The Netherlands with a velocity equal to 28,5 m/s. Of course the shelters will not be able to resist the forces of a twister or hurricane. It will be better to replace the shelters then to design the shelters hurricane proof. The height of the structure is equal to R = 2,0 m. For a height of z = 2,0 m, the extreme pressure is respectively: q(z=2) = 1,11 kN/m^2.

5.1.4. Coefficients internal over and under pressure:

The coefficient for an internal over pressure acting to the exterior is equal to c_{pe} = + 0,2.The coefficient for an internal under pressure acting inward is equal to c_{pe} = - 0,3. The sign is positive for a pressure acting on the face and negative for a pressure acting in the direction from the face (sucking).

5.1.5. Coefficients pressure and sucking

For domes the pressure coefficients depend on the position of the face to the direction of the wind, the diameter of the dome d and the height f. For a half spherical dome the diameter d is equal to 2* R and the height f is equal to the radius f = R. The wind load is acting perpendicular to the faces. For a dome the wind loads are acting radial. The coefficients are given into table 9. The wind load acting at a face is assumed to be constant.

Table 9: Coefficients for a spherical dome, with height R and diameter d = 2.R

Area	Coefficient c
A Pressure windward side	0,8
C Sucking leeward side	- 0,4
B Sucking, perpendicular to the wind direction	- 1,2

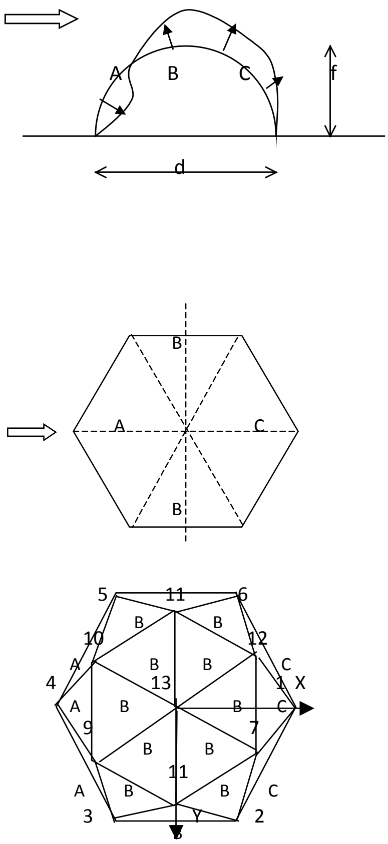

Figure 8: The faces of the facades subjected to wind load and the zones with a constant pressure coefficient A,B and C.

The wind load, acting perpendicular to the faces, is calculated with:

$$F = Area * p \qquad (29)$$

The area of a regular face is equal to $A_1 = 1{,}119\ m^2$. The area of a face at the ground plan is equal to $A_2 = 1{,}328\ m^2$. The vertex at the top is connected to 6 faces. For the triangular faces the load acting at the surface is distributed over the vertices with $F = p.A/3$.

The angle between the normal of a roof face and the Z axis is equal to $\psi = 27{,}8^0$, see (12). The angle between the normal of a congruent face of the drum and the X –axis is also equal to $\psi = 27{,}8^0$ degrees. For the face on the ground plan the angle between the normal and the X-axis is equal to $\gamma = 9{,}55^0$. The loads acting on the faces are solved into components parallel to the axis X,Y and Z, see table 10.

Table 10. Components of the wind load acting at the vertices

Faces	F_x	F_x	F_z
Roof face:	$p.A_1.\sin\psi.\cos\theta$	$p.A_1.\sin\psi.\sin\theta$	$p.A_1.\cos\psi$
Adjacent face	$p.A_1.\cos\psi.\cos\theta$	$p.A_1.\cos\psi.\sin\theta$;	$p.A_1.\sin\psi$
Face on ground plan	$p.A_2.\cos\gamma.\cos\theta$	$p.A_2.\cos\gamma.\sin\theta$	$p.A_2.\sin\gamma$

The load acting on the faces is taken by the vertices, Every vertex of the face takes $^1/_3$ of the load. A vertex of the ring at height R-t is connected with four faces. The vertex of the top is connected with 6 faces. The force acting on a vertex is calculated by summarizing the forces acting on the jointed faces: $F = \Sigma\ ^1/_3\ F$. The following tables show the wind loads calculated with: $p = c.q(z)$, with $q(z) = 1{,}11\ kN/m^2$. The coefficient c depends on the wind direction For zone A c = 0,8, for zone B c = 1,2 and for zone C c = 0,4. The following tables show the forces acting at the vertices due to the overpressure, under pressure and a wind load acting parallel to the X-axis.

Table 11: Loads acting at the vertices due to the overpressure c = 0,2

Node	X	Y	Z
7	0,252	0,145	-0,24
8	0	0,291	-0,24
9	-0,252	0,145	-0,24
10	-0,252	-0,145	-0,24
11	0	-0,291	-0,24
12	0,252	-0,145	-0,24
13	0	0	-0,769

Table 12: Loads acting at the vertices due to the under pressure c = - 0,3

Node	X	Y	Z
7	-0,378	-0,218	0,36
8	0	-0,436	0,36
9	0,378	-0,218	0,36
10	0,378	0,218	0,36
11	0	0,436	0,36
12	-0,378	0,218	0,36
13	0	0	1,15

Table 13: Loads acting at the vertices due to the wind load parallel to the X-axis

Node	X	Y	Z
7	0,882	0,678	-1,22
8	0	1,744	-1,44
9	-0,061	0,007	-0,89
10	-0,061	-0,007	-0,89
11	0	-1,744	-1,22
12	0,882	-0,678	-1,22
13	0	0	-4,61

6. OUTPUT MATRIX FRAME

With a fine element program the normal forces acting at the members are calculated for the varying loads.

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Table 14: Forces acting at the members due to the permanent load p = 0,1 kN/m² ↓ and the snow load p = 0,495 kN/m²↓

	Member	N_g [kN]	N_{sn} [kN]
Roof	S_{7-13},S_{8-13},S_{9-13},S_{10-13},S_{11-13},S_{12-13}	-0,09	-0,45
Ring	S_{7-8},S_{8-9},S_{9-10},S_{10-11},S_{11-12},S_{12-7}	-0,02	-0,31
Façade	S_{1-7},S_{2-7},S_{2-8},S_{3-8},S_{3-9},S_{4-9},S_{4-10},S_{5-10},S_{5-11},S_{6-11},S_{6-12},S_{1-12}	-0,23	-0,35

Table 15: Forces acting at the members due to overpressure p = 0,2 *1,11 kN/m²↑ and under pressure p = 0,3 * 1,11 kN/m²↓

	Member	Now [kN]	N_{wu} [kN]
Roof	S_{7-13},S_{8-13},S_{9-13},S_{10-13},S_{11-13},S_{12-13}	0,31	-0,46
Ring	S_{7-8},S_{8-9},S_{9-10},S_{10-11},S_{11-12},S_{12-7}	0,07	-0,11
Façade	S_{1-7},S_{2-7},S_{2-8},S_{3-8},S_{3-9},S_{4-9},S_{4-10},S_{5-10},S_{5-11},S_{6-11},S_{6-12},S_{1-12}	0,24	-0,35

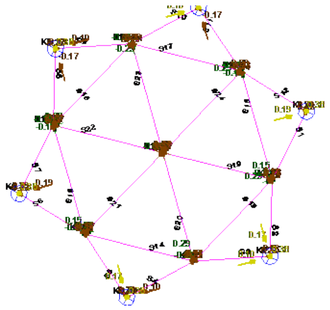

Fig. 9. View from above

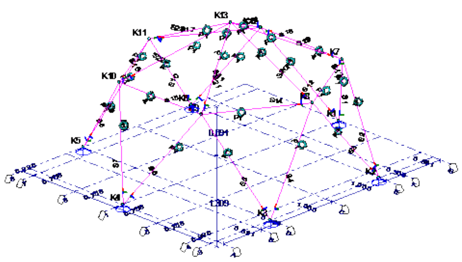

Fig. 10. The frame of the shelter

Table 16 Normal forces due to wind acting parallel X-axis p = c * 1,11 kN/m²

	Member	N_w [kN]
Roof	S_{7-13},S_{8-13},S_{9-13},S_{10-13},S_{11-13},S_{12-13}	+2,79
Ring	S_{7-8} , S_{11-12}	-0,07
Ring	S_{8-9} , S_{10-11}	-0,84
Ring	S_{9-10}	-1,45
Ring	S_{7-12}	+0,19
Façade	S_{1-7},S_{1-12},	+1,07
Façade	S_{2-7},S_{6-12},	+1,20
Façade	S_{2-8},S_{6-11},	+1,10
Façade	S_{3-8},S_{5-11},	+2,20
Façade	S_{3-9},S_{5-10},	+0,46
Façade	S_{4-9},S_{4-10},	+1,40

7. BENDING MOMENTS

The faces of the dome are subjected to normal forces and bending moments. The faces are subjected to an equally distributed load p. Due to this load the edges supporting the face are subjected to triangular distributed load $q = c_e.p$. Due to this load the bending moment acting on an edge with length e is equal to:

$$M = q.e^2/12 \qquad (30)$$

For a triangular face of the roof and the identical face of the façade the distance from the centre of the face to the edge is equal to:

$$ce = {}^2/_3 * 1{,}48 * \sin 27{,}065 = 0{,}45 \text{ m} \qquad (31)$$

For a triangular face of the facade standing on the ground face the distance from the centre of the face to the edge is equal to:

$$ce = {}^2/_3 * 1{,}33 * \sin 36{,}99 = 0{,}53 \text{ m} \qquad (32)$$

Due to the permanent load the edges of the roof are subjected to a load equal to p = 0,1 kN/m². With (28) the bending moment is equal to:

$$M = 2* 0{,}45 * 0{,}1 * 1{,}662^2/12 = 0{,}021 \text{ kNm} \qquad (33)$$

Due to the snow load the edges of the roof are subjected to a load equal to p = 0,495 kN/m². With (28) the bending moment is equal to:

$$M = 2* 0{,}45 * 0{,}495 * 1{,}662^2/12 = 0{,}103 \text{ kNm} \qquad (34)$$

The surface is made of a fabric connected at the vertices of the grid. An edge is subjected to a maximal bending moment in case the wind presses on the face. The maximal load due to wind pressure and an internal under pressure is equal to:

$$p_{we} = c_{pe}.q(z) = (0{,}8 + 0{,}3) * 1{,}11 = 1{,}21 \text{ kN/m}^2 \qquad (35)$$

Due to this load the bending moment acting on an edge of the façade with length e is equal to:

$$M = (0{,}45 + 0{,}53) * 1{,}21.1{,}662^2/12= 0{,}273 \text{ kNm} \qquad (36)$$

8. STIFFNESS

The structure must be stiff, strong and stable. The stiffness of the elements must fulfill some demands so the structure does not fail due to buckling or deforms to much. For the design of the structure the buckling ratio has to be by preference larger than 5.

$$n = \pi^2.EI/(N_d.l_c^2) \ > 5 \qquad (37)$$

With this equation the minimal second moment of the area can be defined for elements of steel, timber, cardboard or aluminum. For the ring, with an edge equal to 1,51 [m], the maximal normal force due to the permanent load, internal under pressure and wind load parallel X-axis is equal to:

$$N_d = \gamma_g.N + \gamma_e.N \qquad (38)$$

For the permanent load the load factor is equal to γ_g = 1,2 (not favorable). For the live load the load factor is equal to γ = 1,5. For the elements of the ring, length 1,51 m, the maximal normal force due to the permanent load, under pressure and wind load is equal to:

$$N_d = 1{,}2 * 0{,}02 + 1{,}5 * (0{,}11 + 1{,}36) = 2{,}23 \text{ kN} \qquad (39)$$

Substituting this force into (37) gives the requited stiffness of the elements:

$$EI > 5.N_d.l_c^2/\pi^2 = 5 * 2{,}23.10^3*1510^2/\pi^2 = 2{,}58.10^9 \text{ Nmm}^2 \qquad (40)$$

The deformation of an edge subjected by a distributed load increasing to the middle of the span is equal to:

$$w = \ q.l^{\,4}/(120.EI) \qquad (41)$$

The deformation of an edge is by preference smaller than 0,004.*l*. Substituting this demand into (370 gives an equation for the stiffness:

$$EI > q.l^3/0{,}48 \qquad (42)$$

Due to the wind pressure and the under pressure the maximal load acting on an edge is equal to q = (0,45 + 0,53) * 1,21 kN/m. Substituting the load and length into (42) gives the minimal needed stiffness EI:

$$EI > (0,45+0,53)*1,21*1662^3/0,48 = 11,34.10^9 \text{ Nmm}^2 \quad (43)$$

The stiffness required to limit the deformation is larger than the stiffness needed to prevent buckling. With the calculated stiffness the minimal magnitude of second moment of the area of the elements can be defined for the varying materials, see table 17.

Table 17 minimal second moment of the area

	Young's modulus [MPa]	Second moment of the area [mm⁴]
Steel	210.10^3	$0,054.10^6$
Timber	$11,6.10^3$	$0,98.10^6$
Cardboard	$1,34.10^3$	$8,46.10^6$
Aluminum	70.10^3	$0,16.10^6$

9. STRESSES

According to the Theory of Linear Elasticity the stresses are calculated for the preliminary design of the structure with:

$$\sigma = \gamma_g. N/A +/- \gamma_e. M/W < f_{max} \quad (44)$$

For the permanent load the load factor is equal to $\gamma_g = 0,9$ (favorable) or 1,2 (not favorable). For the live load the load factor is equal to $\gamma_e = 1,5$.

The bending moment is maximal for the load combination wind pressure parallel X-axis and internal under pressure, see (32), M = 0,273 kNm. The ultimate bending moment is equal to:

$$M_d = \gamma_e.M_d = 1,5 * 0,273 = 0,41 \text{ kNm} \quad (45)$$

The normal force acting in this element due to the permanent load is equal to N = -0,23 kN. The normal force due to the internal under pressure is equal to N = -0,35 kN. The normal load due to the wind load acting parallel the X-axis is equal to +0,46 kN. Thus the maximal normal load is equal to:

$$N_d = -1,2 * 0,23 – 1,5 * 0,35 + 1,5 * 0,46 = -0,11 \text{ kN} \quad (46)$$

Next the stresses are validated with equation (40) for the following elements of steel, timber, cardboard and aluminum.

Table 18: Area and second moment of the area

		Area [mm²]	Second moment of the area [mm⁴]
		$\pi.[D^2-d^2]/4$	$\pi.[D^4-d^4]/64$
Steel	Ǿ40-4	452	$0,074.10^6$
Timber	Ǿ65	3318	$0,88.10^6$
Cardboard	Ǿ130-20	6912	$10,8.10^6$
Aluminum	Ǿ50-4	578	$0,154.10^6$

Table 19: Normal and bending stress

	Section	Normal stress [MPa]	Bending stress [MPa]	Maximal stress [MPa]	Ultimate stress f_{max} [MPa]
		$-\gamma_g.N/A$	$\pm\gamma_e.M/W$		
Steel	∅40-4	-0,24	±110,8	-111	235
Timber	∅65	-0,03	± 15,14	- 15,2	16,0
Cardboard	∅130-20	-0,02	± 2,47	- 2,5	2,5
Aluminum	∅50-4	-0,19	± 66,6	- 67	90

10. JOINTS

For space frames of steel, timber and aluminium several types of joints are designed [9]. The architect Shiguru Ban designed with the Dutch firm Octatube, Delft, a geodetic dome, the Paper Dome, composed of cardboard tubes ∅200-20 [5]. This dome was built in Amsterdam, 2003, rebuilt in Utrecht, 2004, and deconstructed last year, 2013. The cardboard tubes were post tensioned with steel bars to transfer tensile forces from the joints to the tubes. Recently Octatube developed a system to joint the cardboard tubes with spherical joint for a space frame supporting the roof of a sport accommodation in Delft. Probably it is possible to glue wooden tabs into the tubes to transfer the tensile loads to the joints. These joints with glued tabs must be researched further.

Fig.12. Removing the fabric from the grid of the Paper Dome, 2012.

11. EVALUATION

Table 20 shows the volume and weight of the elements. The volume of the steel and aluminum tubes is much smaller than the volume of the timber and card board tubes. The weight of the timber and aluminum tubes is much smaller than the weight of the steel and cardboard tubes. The total length of the edges of this shelter is about 29 m, The weight of the structure exclusive joints and fabric varies from 45 kg to 160 kg. Reduction of the weight is quite important in case the shelter is shipped by plane, then light aluminum tubes are favorable. Concerning the sustainability the timber elements are favorable. Table 21 shows the embodied energy and CO_2 emission described by Ashby [10]. Wood stores CO_2 so the emission is negative, of course later the stored CO_2 will be released back into the environment. For cardboard Ashby does not give any data. Generally cardboard is made of waste materials. The fibers are glued and the production of the glue will use some energy. The embodied energy to produce cardboard is about 20% more than timber [11]. Table 22 shows the embodied energy calculated for the given sections. The embodied energy is for timber minimal.

Table 20: Mass, volume and weight of the elements

	Section	Mass	Volume	Weight
		[kg/m³]	[m³/m]	[kg/m]
Steel	∅40-4	7800	$0,45.10^{-3}$	3,53
Timber	∅65	500	$3,32.10^{-3}$	1,66
Cardboard	∅130-20	800	$6,91.10^{-3}$	5,53
Aluminum	∅50-4	2700	$0,58.10^{-3}$	1,56

Table 21: Embodied energy and CO_2 emission [10]

	Embodied energy [MJ/kg]	CO_2 emission [kg/kg]
Steel	22,4 - 22,8	1,9 - 2,1
Timber	14,4 –15,9	-1,2 – -1,0
Aluminum	184 – 203	11,6 - 12,8

Table 22: Embodied energy for the elements with a length of 1 m

	Section	Weight [kg/m]	Average embodied energy [MJ/kg]	Embodied energy [MJ]
Steel	∅40-4	3,53	22,6	79
Timber	∅65	1,66	14,6	24
Cardboard	∅130-20	5,53	17,5	97
Aluminum	∅50-4	1,56	190	296

12. CONCLUSIONS

The emergency shelter is quite robust, due to the spherical form the structure can resist heavy loads. The variety of the faces, edges and vertices is minimal, the structure is composed of two types of faces and two types of edges varying in length only.

For temporary structures cardboard is efficient and sustainable, but for emergency shelters which have to be shipped fast by planes to the site the weight of the elements has to be reduced further. The strength of the elements varies much. Controlling the production, increasing the strength and stiffness, reducing the deviation of the features will decrease the weight of the elements.

At the present a half spherical dome with a structure composed of timber elements fulfills the demands well. The timber poles are light and the volume, embodied energy and CO_2 emission is minimal. This frame is light, strong, stiff, sustainable, transportable and easy to erect. Further research is needed to develop and optimize the structure, to create a low cost, robust and sustainable emergency shelter.

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