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MEASUREMENT OF THE DERIVATIVE z_w FOR OSCILLATING SWEEPBACK WINGS

by

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C R A N F I E L D

Measurement of the Derivative z_w for Oscillating
Sweptback Wings

-by-

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SUMMARY

Measurements have been made of the derivative z_w for rigid sweptback wings mounted at zero incidence and oscillated with simple harmonic motion. The Reynolds number was in the range 1.2×10^5 to 4.1×10^5 .

The wings were of trapezoidal planform, chosen to indicate the effects of sweepback, aspect ratio and taper ratio. In each case the variation of z_w with frequency parameter was determined, and the effect of z_w amplitude of oscillation checked, and found to be fairly small.

It was found that the effects of the planform parameters on the derivative were, in general, similar to those on lift curve slope. The curves obtained, however, suggested higher values of $(-z_w)$ than given by theory for zero frequency parameter, in all cases.

This Report was submitted in 1952 as a part requirement for the award of the Diploma of the College of Aeronautics.

MEP

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1. Notation

A	Aspect ratio
a	Lift curve slope
c	Mean aerodynamic chord
C_D	Drag coefficient
C_L	Lift coefficient
f_N	Natural frequency of oscillation
f_R	Resonant frequency of oscillation
\bar{l}	Amplitude of forcing displacement
S	Wing area
V	Windspeed (ft./sec.)
\bar{z}	Amplitude of oscillation of wing
\bar{z}_R	' ' ' ' ' at resonance
Z_W	Damping derivative ($= \partial Z / \partial w$)
Z_W	$Z_W / \rho S V$
α	Angle of incidence
Λ	Angle of sweepback

Notation (Contd.)

λ	Taper ratio
ρ	Air density
μ	Viscous damping coefficient
$\sigma_{1,2}$	Spring stiffnesses
ω	$= 2\pi f_R c/V$ Frequency parameter.

2. Introduction

A knowledge of the aerodynamic derivative z_w for a wing in unsteady motion is required in making flutter calculations. In the absence of readily applicable information it has been common practice to evaluate z_w from the expression

$$- \frac{1}{2} \left(\frac{\partial C_L}{\partial \alpha} + C_D \right), \text{ corresponding to steady flow. Dimensional}$$

analysis, however, indicates that it may depend on frequency parameter ω and amplitude parameter z/c , as well as Reynolds number and Mach number, in unsteady flow.

The object of the present series of tests was to determine the effects of the frequency and amplitude parameters on z_w for a series of trapezoidal sweptback wings oscillating with simple harmonic motion. The planforms were chosen to demonstrate dependence on sweepback angle, aspect ratio and taper ratio. All the wings tested were of symmetrical section (NACA 0018 and NACA 0020), having rather a large thickness/chord ratio by present standards for sweptback wings.

The Reynolds number of the tests was somewhat low for reliable extrapolations of the results to full scale to be made. A further point is that results are obtained at low Mach number, whereas the main interest in flutter is at speeds where compressibility effects are important. It may be possible, however, using Glauert's correction, or some other means, to modify incompressible data, as was done in Ref. 1.

3. Previous Work using the same Test Rig

It is useful, at this stage, to summarise the main points arising from earlier work using the same rig, insofar as they have a bearing on the present tests.

The first systematic programme was carried out in 1950, and is described in Ref. 2. The equation of motion for the rig, with wing attached, was obtained in the latter paper, based on the assumptions that:-

- (1) only viscous damping was present,
- (2) the inertia of the springs was negligible,
- (3) the motion was simple harmonic and of constant amplitude, and
- (4) the transient effects (free motion) had decayed,

an expression for z_w was developed from the solution for steady state forced motion, in terms of measurable physical quantities. It was found that the effect of the amplitude parameter \bar{z}/c on the value of z_w was negligible for $\bar{z}/c \leq 0.15$, and hence efforts were turned to measuring the effect of frequency parameter ω .

In order to compare the results obtained with those derived theoretically by W.P. Jones, rectangular wings of varying aspect ratio were used. Tests were also made on two 45° sweptback wings of different aspect ratio. When the amplitude ratio at resonance, \bar{z}_R/\bar{l} , was plotted against the reciprocal of the wind speed, the points were found to lie closely on a straight line, for a particular pair of springs. Consequently points taken off the straight lines were used to calculate z_w . When the latter was plotted against frequency parameter, ω , good agreement was found between measurements taken using different sets of springs. Scatter was greatest in the case of the sweptback wings.

Curves were extrapolated to $\omega = 0$ and $\omega = 0.5$, and it was assumed that the effect of wind tunnel constraint was as developed for steady flow at $\omega = 0$, falling to zero at $\omega = 0.5$. From the results corrected in this way it appeared that z_w increased with increasing aspect ratio, and decreased with sweepback. The form of the curves was very similar to that suggested by theory, though actual magnitudes differed somewhat. The latter was accounted for by the finite thickness of the wings tested experimentally.

In the second test programme (1951) attention was turned exclusively to sweptback wings. The aim was to determine the effects of sweepback, aspect and taper ratios on z_w , and the approach was basically the same as before. Measurements appear to have been taken at uniform intervals of V ,

rather than $1/V$, and only three sets of springs were used for each aerofoil. Values of \bar{z}_R/\bar{l} were picked off the best straight lines, and the $z_w - \omega$ graphs derived as in Ref. 2.

The resulting curves (Ref. 3) were much less uniform in character than those previously obtained, and considerable scatter was apparent in the results from different springs. As a consequence it was not possible to apply tunnel corrections, and only the broadest trends were apparent from the results. Since the aspect ratio of the tapered wings was not constant, it was rather difficult to separate these two effects.

4. Preliminary Tests

In view of the rather inconclusive nature of the tests on sweptback wings described in Ref. 3, it was decided to repeat the programme. Since it was thought possible that partial spring closure had taken place during the previous tests, a new pair of springs was made, differing from the original ones in being open-wound. In order to give a positive connection with these springs, 'eye'-type attachments were fitted in place of the hook type.

Prior to the commencement of tests the calibrations, including that of windspeed in the working section, were checked, using the methods described in Ref. 2.

As a preliminary to the main programme, some tests were made on the unswept wing of aspect ratio 3 to check earlier measurements. Since the condition of the weaker springs previously used had deteriorated, only the stiffest pair (E springs) were used, together with the new open-wound F springs. The technique used was very similar to that described in Ref. 2, care being taken to avoid spring closure by suitable pretensioning. In view of the previous findings that the amplitude parameter had no effect on results over the normal operating range, a suitable exciting amplitude was set, and retained while taking measurements throughout the speed range. This reduced the time needed to make a complete run since it obviated the need to stop after each reading to reset the eccentric.

Results were plotted initially in the form \bar{z}_R/\bar{l} against $1/V$, and found to lie closely on a straight line. On making further runs with different exciting amplitudes, different lines were obtained, roughly parallel to the first but corresponding to decreasing \bar{z}_R/\bar{l} with increasing \bar{l} at a particular speed. This effect was apparent with both pairs of springs. Consequently runs were made varying the exciting amplitude so as to keep the resonant amplitude of the wing approximately constant.

The derivative z_w was obtained from the faired straight lines in the usual w way for all these runs. For the runs made with constant exciting amplitude there was a general tendency for z_w to decrease with decreasing frequency parameter. The results corresponding to constant resonant amplitude, however, showed the opposite effect, and agreed with those of Ref. 2 in this respect. The actual values of $(-z_w)$ were higher than those previously measured. In the case of the E springs the values of \bar{z}_R/\bar{l} were higher than in Ref. 2, but this was more than offset by increased spring stiffness.

Just before the completion of the above tests a failure of one of the E springs occurred. A new pair of springs was made as a replacement and designated G springs. These G springs were of similar form to the F springs but less stiff (thinner wire).

Later tests on the same wing, using F and G springs and the constant resonant amplitude technique, gave values of $(-z_w)$ in good agreement between themselves but higher than found in Ref. 2 (see Fig. 18).

5. Main Test Programme

Data on the wings tested is given in Table I and the planforms shown in Fig. 1. As previously explained, the main object of the tests was to determine the effects of sweepback angle, aspect ratio and taper ratio on z_w for sweptback wings. It was anticipated that the effect of taper ratio would be relatively small, and therefore it was considered desirable to vary it while keeping the other parameters constant. The existing models did not enable this to be done, and so two new ones were made, having a quarter-chord sweepback of 30° and an aspect ratio of 4.63 (as for one of the existing untapered wings) and taper ratios of 1:2 and 2:3 respectively.

The open coiled F and G springs were used throughout, and, following the experience gained in the preliminary tests on the rectangular wing, it was decided to use the constant resonant amplitude technique. Measurements were taken with each pair of springs at resonant amplitudes of approximately 0.3in. and 0.4in., i.e. a total of four complete runs per wing. The spring pretension (based on extension) was maintained constant.

Readings were taken at windspeeds giving roughly constant increments of $1/V$. The method used followed closely that described under 'Final Procedure' in Ref. 2, except that there were normally two operators.

The difficulties encountered were similar to those described in Ref. 2. Close control of the exciting motor

speed was not possible, even with the aid of the 'fine-control' rheostat. Fluctuations of exciting speed were particularly marked when other plant was running in the laboratory. As a result the rig was run through the resonance speed several times at each windspeed and the maximum value of \bar{z} noted. Early attempts to take measurements at 50 f.p.s. proved unsatisfactory due to the sharpness of the response curve, and much care was required to obtain resonance at 60 ft./sec.

At the higher windspeeds considerable fluctuations were apparent in the readings of windspeed as measured on the Prandtl manometer. The technique used was to set the eyepiece at the position corresponding to the nominal windspeed, and to keep the mean height of the fluid column as close to it as possible, using the fine speed control. It was not considered advisable to use the manometer damping system, since the latter prevented changes in the mean value becoming rapidly apparent. Another feature, also noted previously, was the random variation in the centre of oscillation of the rig at high speed (about 200 f.p.s.). Since resonance is not critically dependent on exciting speed at the higher windspeeds, however, it was still possible to measure the resonant amplitude fairly consistently.

Results were plotted in the form (\bar{z}_R/l) against $1/V$ for all four runs with a particular wing. When it appeared desirable, points were checked at the end of each run. As an additional check, at the end of the programme, one of the wings was retested to indicate any effects arising in fitting the wing and springs. The previous measurements were repeated closely.

6. Results

The expressions used to derive z_w and ω are those given in Ref. 2,

$$z_w = - \frac{1}{\rho V S} \left[\frac{\sigma_2 l}{2\pi f_N \bar{z}_R} - \mu \right]$$

$$\omega = \frac{2\pi f_R c}{V}$$

Both pairs of springs were calibrated before commencing the main test programme, and again after its completion. They are quoted in lb./ft.

	F		G	
	σ_1	σ_2	σ_1	σ_2
Before Tests	258	236	131	130
After tests	246	240	130	136

A linear variation in σ_2 during the tests was assumed in calculating results. Following previous practice the natural frequency, f_N , was taken equal to the resonant frequency, f_R , it having been shown in Ref. 2 that the difference between them was negligible.

The curves of \bar{z}_R/\sqrt{l} against $1/V$ are given in Figs. 2-9. The previous practice was to draw the best straight lines through the experimental points. From the results of the present tests, however, considerable deviations from a linear relation are apparent, and in the circumstances it has been thought best to use measured values of \bar{z}_R/\sqrt{l} in the above expression for z_W , rather than those taken from smooth curves. Values of the rig damping coefficient, μ , were taken from the measurements quoted in Appendix I.

For clarity the results obtained with the different wings are plotted separately in Figs. 10-17. Since the effect of amplitude is, in general, fairly small a mean curve is drawn for each pair of springs.

Tunnel Corrections

It may be seen from Figs. 10-17 that, in addition to amplitude effects, there is considerable variation between results obtained using the two pairs of springs. In order to use the method of correcting for tunnel constraint given in Ref. 2, extrapolation to $\omega = 0$ and $\omega = 0.5$ is necessary, and this can not be done accurately in the present case. In these circumstances no corrections have been applied to the results.

Final results

Final curves, based on the mean values for the two sets of springs, are plotted in Figs. 19-24 in such a way as to separate aspect ratio and sweepback effects, and show the effect of taper.

If it is assumed that $\omega = 0$ corresponds to steady flow, the theoretical value of z_W can be found at that point from $z_W = -\frac{1}{2} \partial C_L / \partial a$. Values of $\partial C_L / \partial a$ were obtained from Ref. 4, and the Weissinger correction for sweepback

$\left(\frac{a}{a_{\Delta=0}} = 1 - \frac{\Delta^2}{2} \cdot \frac{A}{A+2} \right)$ was applied. The corresponding values of z_w are shown in Figs. 19-24.

7. Discussion

The results obtained for the unswept wing, and shown in Fig. 18, indicate good agreement between the two sets of springs used. In addition, extrapolation to $\omega = 0$ gives a value of $(-z_w)$ only slightly greater than the theoretical value corresponding to this aspect ratio and trailing-edge angle. On the other hand, the most striking feature of the results for sweptback wings (Figs. 10-17) is the rather large discrepancy between the results obtained with the two sets of springs.

In general the value of $(-z_w)$ increases as the amplitude of oscillation increases, but this effect is secondary to that caused by different springs. As mentioned previously, experimental points have been plotted direct, (i.e. not using faired curves of \bar{z}_R/\bar{l} against $1/V$) and in view of possible errors in measuring physical quantities (see Appendix II) a certain amount of scatter is to be expected. Nevertheless fairly closely defined curves are obtained for each spring in most cases.

The F springs always give higher values of $(-z_w)$, but the magnitude of the discrepancy varies from wing to wing. Since the windspeed, at a given value of the frequency parameter, does not differ greatly for the two sets of springs, it seems unlikely that the effect is associated with unsteady flow in the tunnel. The good agreement in the case of the rectangular wing, however, would seem to indicate that no important factors have been omitted from the analysis. In the circumstances it is difficult to offer any adequate explanation of the effect, though one method of checking the results would be to repeat some tests with a third set of springs.

The purpose of the mean curves, plotted in Figs. 19-24, is to indicate the broad effects of the planform parameters on z_w , and to give a comparison with the theoretical values for steady flow. Those showing aspect ratio effect (Figs. 19-21) indicate an increase in $(-z_w)$ with increasing A for sweepbacks of 30° and 45° . At 60° sweepback, however, this trend is reversed. Sweepback effect at roughly constant aspect ratio is shown in Figs. 22 and 23. In both cases $(-z_w)$ is found to decrease with increasing sweepback, the effect being most marked at the higher aspect ratio in the case of 60° sweepback. In all cases the curves suggest higher values than the theoretical at $\omega = 0$.

The effect of taper on a 30° swept wing was found to be small (Fig. 24), though again values are considerably higher than theory.

The general effects described are in fairly good agreement with the findings of Ref. 3, but the magnitudes of $(-z_w)$ are greater throughout.

8. Conclusions

(i) In general the results obtained for $(-z_w)$ show a similar dependence on sweepback, aspect ratio and taper ratio as do lift curve slopes. However, the curves suggest consistently higher values than the theoretical as the frequency parameter tends to zero.

(ii) The effect of varying the amplitude parameter is not negligible for a particular pair of springs. It is small in relation to the discrepancies arising between different springs.

(iii) The form of the mean curves of $(-z_w)$ against frequency parameter varies somewhat from wing to wing. This is attributed to the neglect of tunnel interference effects.

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TABLE I
MODEL DETAILS

Wing	Aerofoil Section	Root Chord	Aspect Ratio	Sweepback ($\frac{1}{4}$ chord)	Taper Ratio
I	NACA 0018	3.80"	2.63	30°	1:1
II	'	'	4.74	'	'
III	'	'	'	'	2:3
IV	'	'	'	'	1:2
V	NACA 0020	3.75	3	45°	1:1
VI	'	'	5	'	'
VII	NACA 0018	3.80	2.63	60°	'
VIII	'	'	4.74	'	'

APPENDIX I

The measurement of the viscous damping coefficient μ

The method of measuring the viscous damping coefficient, μ , by the free oscillation method was described in Ref. 2. It was pointed out that practical difficulties arose in using the forced oscillation technique for this purpose, but nevertheless a further attempt was made to do so. With the smallest obtainable exciting amplitude (about 0.1mm.), however, it was not found possible to obtain readings of resonant amplitude within the limits set by the scale and spring closure.

Earlier measurements of μ were made wind-off. It was noted in Ref. 3 that spring pretension and amplitude range had a marked effect. Although μ had previously been found to be small relative to z_w , it was decided to make a rather fuller investigation of the subject. This was done, using the E springs, at the time when the tests on the unswept wing were in progress.

The moving parts of the rig were weighed and the equivalent mass calculated. The time for a free oscillation to decay to half amplitude was taken over the three amplitude ranges 0.4in. to 0.2in., 0.3in. to 0.15in., and 0.2in. to 0.1in., wind off and through the speed range up to 200 f.p.s. In each case the mean of about 5 readings was taken. As an additional check the tests were repeated with three different pairs of weights attached to the rig.

μ was found to be greatest at the higher amplitude range, and showed erratic, though relatively small, variation with windspeed. The weights used had very little effect.

A more limited series of tests was made with the F springs, and later with the G springs, using the medium weights. With the G springs particularly, the time for the oscillation to decay at high windspeeds was rather irregular, depending quite critically on the actual windspeed.

When the results obtained for the unswept wing were found to give higher values of $(-z_w)$ than previous tests, some further consideration was given to the rig damping. In particular non-viscous damping arising from spring hysteresis was thought likely to have some effect. It is shown in Ref. 5 that non-viscous damping can be dealt with for the resonance condition, provided it is not large enough to affect seriously the simple harmonic nature of the motion. The concept used is that known as equivalent viscous damping, and is based on considerations of energy dissipation. Since the natural frequency, at which the rig damping coefficient, μ , is measured, is very close to the resonant frequency, the hysteresis damping is already taken account of in μ .

Although the values obtained for μ were relatively small, those corresponding to the particular amplitude and windspeed have been used in calculating z_w . All measurements were made with the same spring pretension as used for obtaining \bar{z}_R/l .

Windspeed ft./sec.		60	70	85	100	120	150	200
F	$\bar{z}_R=0.3\text{in.}$	0.0070	0.0068	0.0065	0.0063	0.0063	0.0063	0.0060
	$\bar{z}_R=0.4\text{in.}$	0.0088	0.0086	0.0084	0.0082	0.0081	0.0081	0.0078
G	$\bar{z}_R=0.3\text{in.}$	0.0035	0.0035	0.0035	0.0035	0.0037	0.0039	0.0029
	$\bar{z}_R=0.4\text{in.}$	0.0045	0.0045	0.0044	0.0044	0.0045	0.0045	0.0039

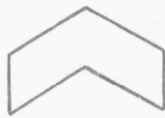
Values of μ

APPENDIX II

EFFECT ON z_w OF ERRORS IN MEASUREMENT

The percentage error in z_w due to small errors in measuring physical quantities is found in a typical case (Wing VII).

<u>Type and Magnitude of measuring error</u>	<u>Corresponding error in z_w</u>	
	<u>60 ft./sec.</u>	<u>200 ft./sec.</u>
$2\bar{z}$ (Full excursion of exciter) ± 0.005 mm.	± 1.5	± 0.5
$2\bar{z}_R$ (Full excursion of wing at resonance) ± 0.01 in.	± 1.7	± 1.5
Resonant frequency $\pm \frac{1}{2}\%$	± 0.5	± 0.5
Spring stiffness $\pm \frac{1}{2}\%$	± 0.5	± 0.5
Rig Damping Coefficient, $\mu \pm 5\%$	± 0.5	± 0.1
Windspeed	± 1.0	± 1.5



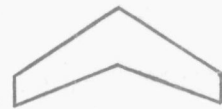
I



II



III



IV



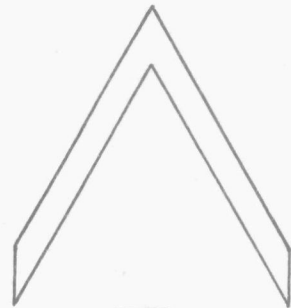
V



VI

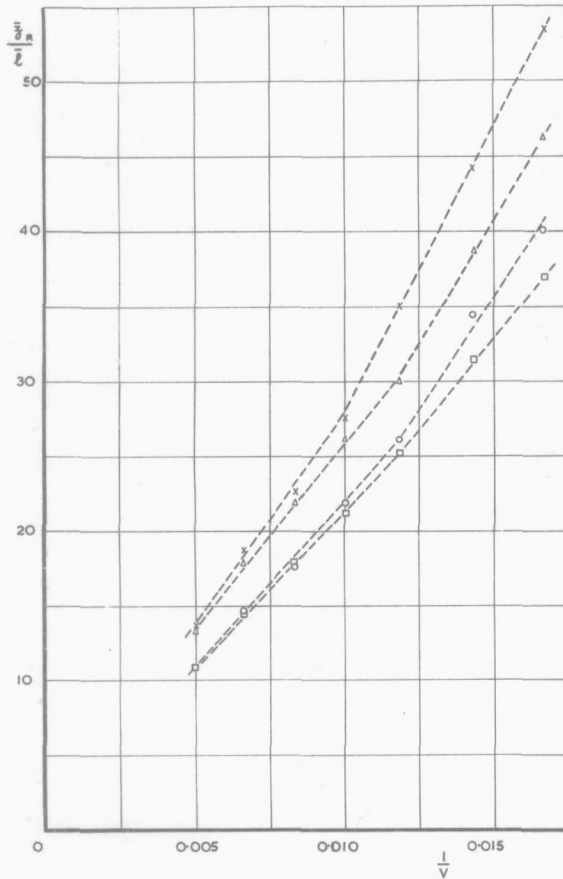


VII

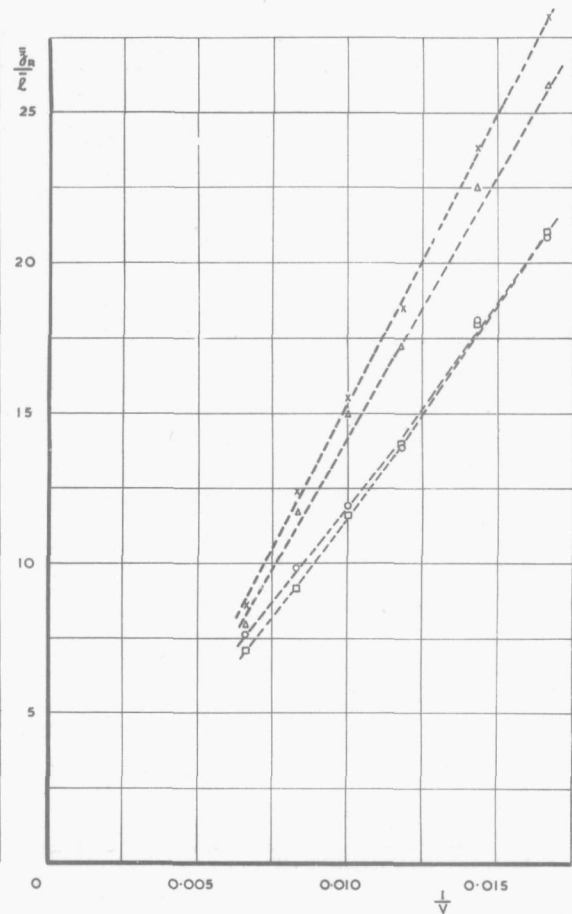


VIII

FIG. I.



WING. 5.
 $A = 3, \Delta \alpha_A = 45^\circ, \lambda = 1:1$

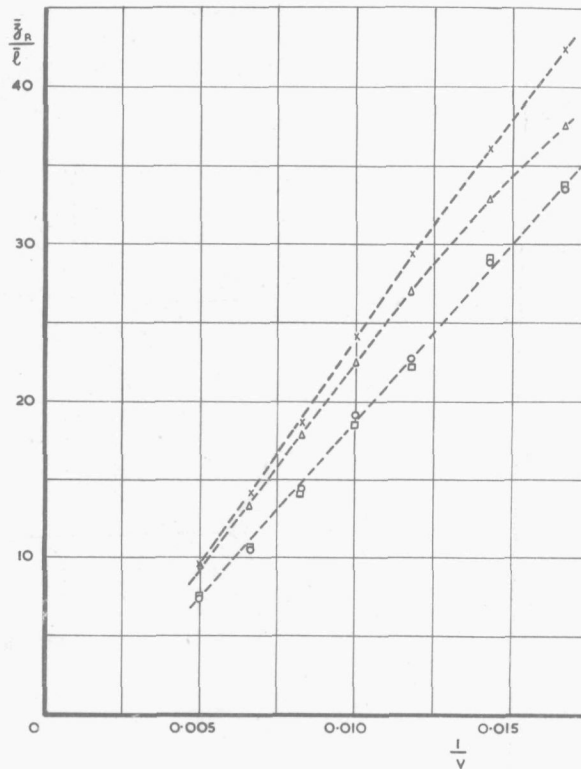
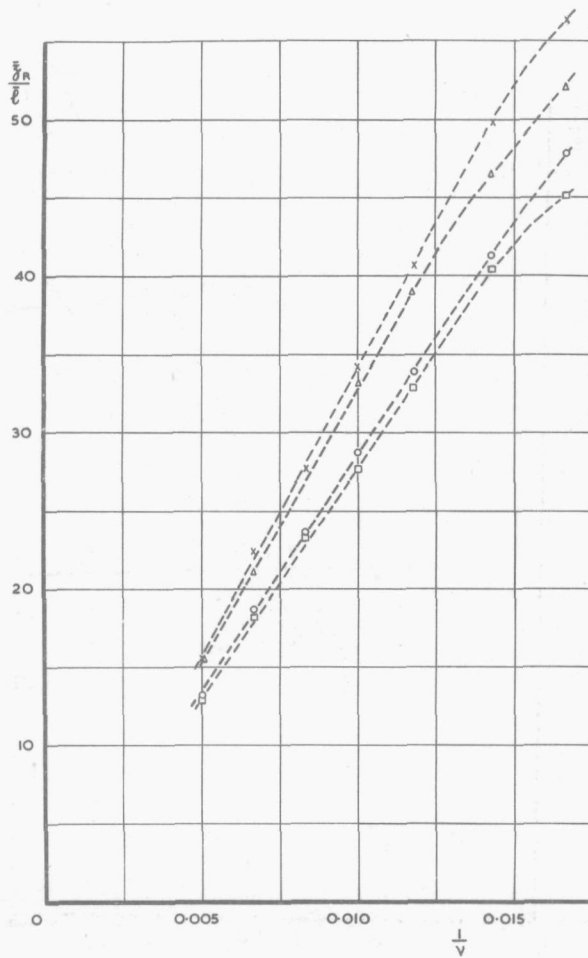


WING. 6.
 $A = 5, \Delta \alpha_A = 45^\circ, \lambda = 1:1$

SPRING F $\begin{cases} \frac{V^2}{L^2} = 0.3'' \text{---} x \text{---} \\ \frac{V^2}{L^2} = 0.4'' \text{---} \Delta \text{---} \end{cases}$
SPRING G $\begin{cases} \frac{V^2}{L^2} = 0.3'' \text{---} o \text{---} \\ \frac{V^2}{L^2} = 0.4'' \text{---} \square \text{---} \end{cases}$

FIG. 6.

FIG. 7.

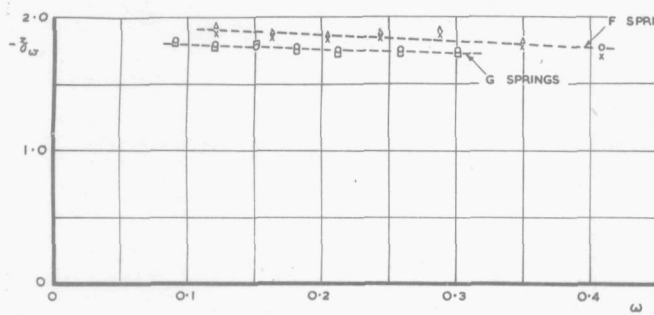


SPRINGS F $\begin{cases} \frac{Q_{Rf}}{Q_{Rf}} = 0.3'' & \text{---X---} \\ \frac{Q_{Rf}}{Q_{Rf}} = 0.4'' & \text{---}\Delta\text{---} \end{cases}$

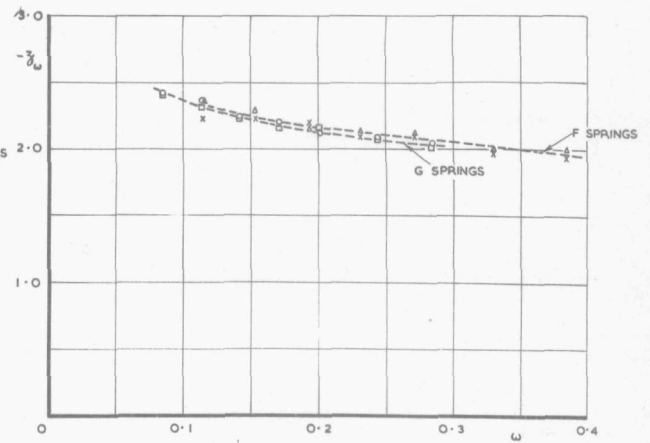
SPRINGS G $\begin{cases} \frac{Q_{Rf}}{Q_{Rf}} = 0.3'' & \text{---}\circ\text{---} \\ \frac{Q_{Rf}}{Q_{Rf}} = 0.4'' & \text{---}\square\text{---} \end{cases}$

FIG. 8.

FIG. 9.



WING 1.
 $A = 2.63$, $\Lambda c/A = 30^\circ$, $\lambda = 1:1$

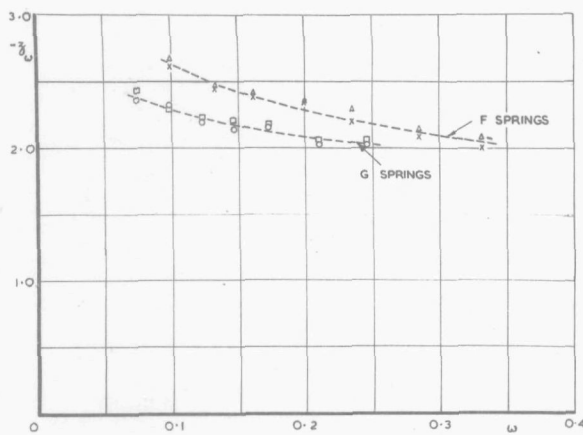


WING 2.
 $A = 4.74$, $\Lambda c/A = 30^\circ$, $\lambda = 1:1$

FIG. 10.

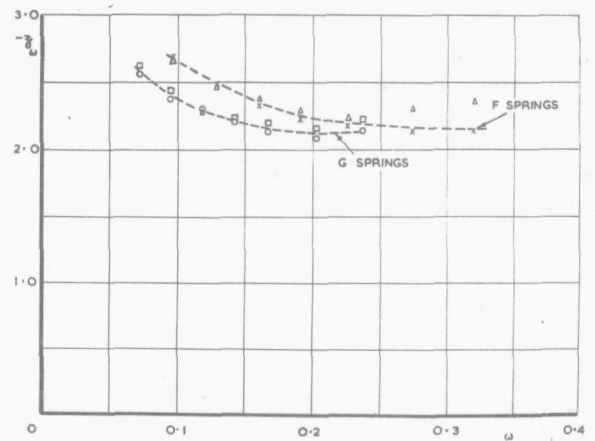
FIG. 11.

SPRINGS $\frac{\pi}{\delta_R} = 0.3$ $\frac{\pi}{\delta_R} = 0.4$
 F x Δ
 G o \square



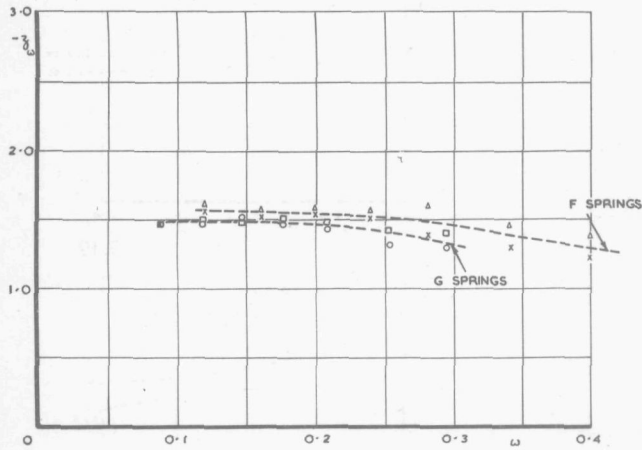
WING 3.
 $A = 4.74$, $\Lambda c/A = 30^\circ$, $\lambda = 2:3$

FIG. 12.



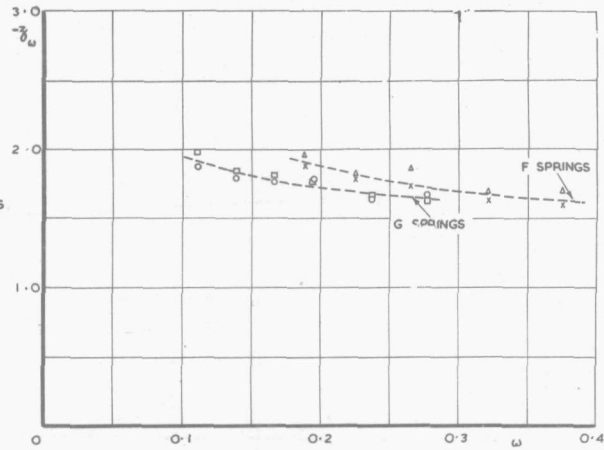
WING 4.
 $A = 4.74$, $\Lambda c/A = 30^\circ$, $\lambda = 1:2$

FIG. 13.



WING. 5.
 $A = 3, \Delta c_A = 45^\circ, \lambda = 1:1$

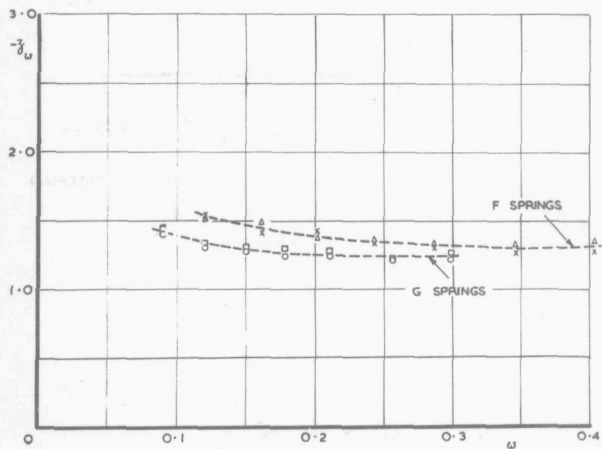
FIG. 14.



WING. 6.
 $A = 5, \Delta c_A = 45^\circ, \lambda = 1:1$

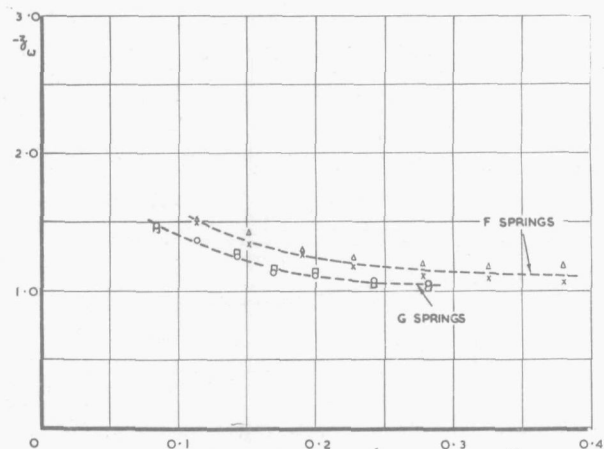
FIG. 15.

SPRINGS $\frac{Z}{\delta_R} = 0.3''$ $\frac{Z}{\delta_R} = 0.4''$
 F X Δ
 G O \square



WING. 7.
 $A = 2.63, \Delta c_A = 60^\circ, \lambda = 1:1$

FIG. 16.



WING. 8.
 $A = 4.74, \Delta c_A = 60^\circ, \lambda = 1:1$

FIG. 17.

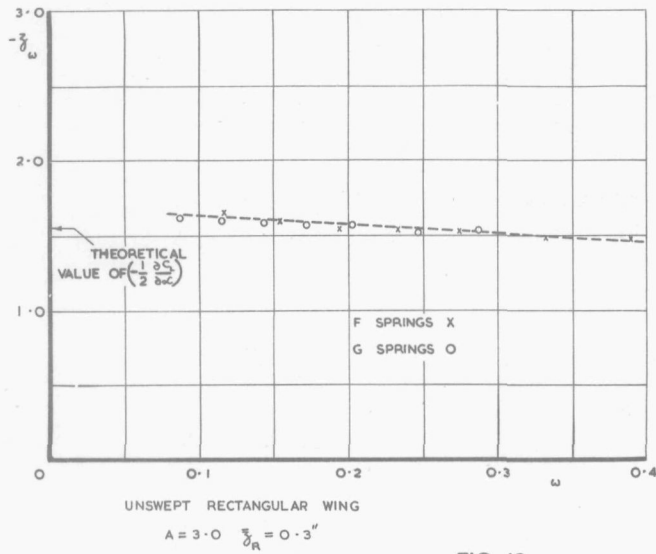


FIG. 18.

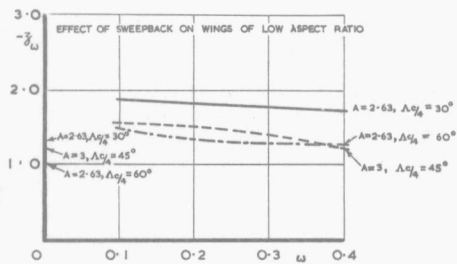


FIG. 22.

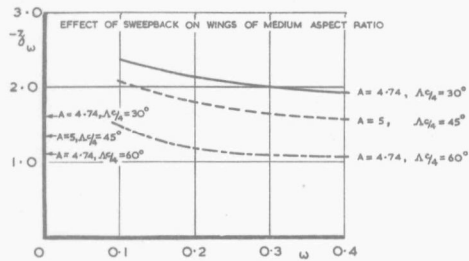


FIG. 23.

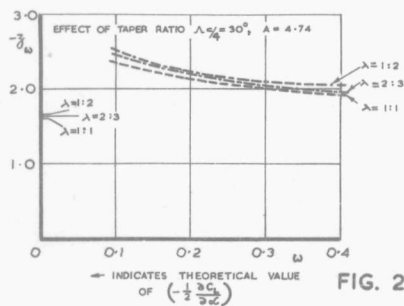


FIG. 24.

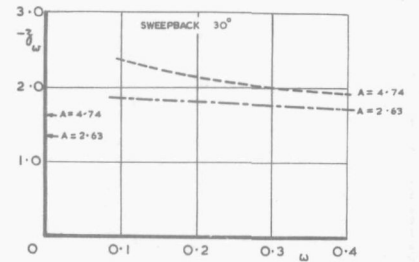


FIG. 19.

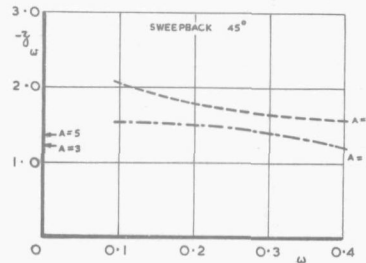


FIG. 20.

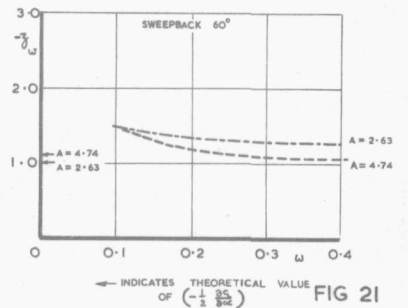


FIG. 21.

EFFECT OF ASPECT RATIO AT CONSTANT SWEEPBACK

VARIATION OF $\frac{1}{2} \frac{dC_p}{d\alpha}$

WITH FREQUENCY PARAMETER ω