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IDENTIFICATION OF MULTIPLE LOCALIZED FORCES ON A FOOTBRIDGE

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ABSTRACT

An existing joint input-state estimation algorithm is extended for applications in structural dynamics. The estimation of the input and the system states is performed in a minimum-variance unbiased way, based on a limited number of response measurements and a system model. An additional method is proposed to identify the noise statistics, which are needed for the joint input-state estimation procedure and which can be used to quantify the uncertainty on the estimated forces and system states. The proposed methodology is illustrated using data from an in situ experiment on a footbridge.

Keywords: *force identification, state estimation, noise statistics, uncertainty interval*

1. INTRODUCTION

For civil engineering structures, knowledge of the dynamic loads is crucial to design purposes. Very often these dynamic loads are not well known or cannot be measured directly, e.g. wind loads or footfall excitation. Inverse identification techniques may then be used to identify the unknown forces. The joint input-state estimation algorithm proposed in this work is an extension of an existing algorithm proposed by Gillijns and De Moor [1], which was used for applications in structural dynamics by Lourens et al. in [2]. The algorithm has the structure of a Kalman filter [3], except that the true value of the input is replaced by a minimum-variance unbiased estimate. As opposed to many existing state estimation algorithms, e.g. [4, 5], no assumptions about the dynamic evolution of the forces are needed. The application of the algorithm is restricted to the linear domain.

The noise statistics are essential when using the proposed joint input-state estimation algorithm, especially when quantification of uncertainty on the estimation is aimed. Several methods have been proposed in the literature to identify the noise statistics, both offline [6] and online [7, 8, 9]. Very often, in structural dynamics applications, operational loads (e.g. wind loads) are modeled as

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stochastic noise processes. This can be directly taken into account for the noise identification procedure, which then boils down to a force identification problem.
This paper consists of a theoretical part, followed by a practical illustration. Firstly, a brief overview of system models as commonly used in structural dynamics is given. Secondly, the joint input-state estimation algorithm is presented. Thirdly, a method for the identification of noise statistics corresponding to the operational loads is proposed. The methodology is finally illustrated for the practical case of a footbridge, where multiple forces are identified.

2. MATHEMATICAL FORMULATION

2.1. System model

Consider the continuous-time governing equations of motion for a linear structure discretized in space:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) = \mathbf{S}_{\mathrm{p}}(t)\mathbf{p}(t) \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^{n_{\mathrm{DOF}}}$ is the vector of displacements, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{DOF}}}$ denote the mass, damping and stiffness matrix, respectively, and $\mathbf{f}(t) \in \mathbb{R}^{n_{\mathrm{DOF}}}$ is the excitation vector. The excitation is factorized into an input force influence matrix $\mathbf{S}_{\mathrm{p}}(t) \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{p}}}$ and the vector $\mathbf{p}(t) \in \mathbb{R}^{n_{\mathrm{p}}}$ representing the n_{p} force time histories.

After performing a modal decomposition, and assuming proportional damping, the decoupled governing equations of motion in modal coordinates become

$$\ddot{\mathbf{z}}(t) + \mathbf{\Gamma}\,\dot{\mathbf{z}}(t) + \mathbf{\Omega}^2\mathbf{z}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathrm{p}}(t)\mathbf{p}(t) \tag{2}$$

where $\mathbf{z}(t) \in \mathbb{R}^{n_{\mathrm{DOF}}}$ is the vector of modal coordinates. $\mathbf{\Gamma} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{DOF}}}$ is a diagonal matrix containing the terms $2\,\xi_j\omega_j$ on its diagonal, where ω_j and ξ_j are the natural frequency and modal damping ratio according to mode j, respectively. $\mathbf{\Omega} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{DOF}}}$ is a diagonal matrix, containing the natural frequencies ω_j on its diagonal. $\mathbf{\Phi} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{DOF}}}$ is a matrix with the eigenvectors of the structure $\boldsymbol{\phi}_j$ as columns. In many cases, modally reduced order models are applied, i.e. the matrices $\mathbf{\Gamma}$, $\mathbf{\Omega}$, and $\mathbf{\Phi}$ only contain the contribution of a limited number of structural modes, denoted by n_{m}. Note that the modes can be either determined from a first principle model or directly identified from measurements on a structure.

The decoupled equations can be written into state-space form, which after time discretization yields:

$$\mathbf{x}_{[k+1]} = \mathbf{A}\,\mathbf{x}_{[k]} + \mathbf{B}\,\mathbf{p}_{[k]} \tag{3}$$

where $\mathbf{x}_{[k]} = \mathbf{x}(k\Delta t)$ and $\mathbf{p}_{[k]} = \mathbf{p}(k\Delta t)$ $(k = 1, \dots, N)$, Δt is the sampling time step, and N is the total number of samples. The state vector $\mathbf{x}_{[k]}$ consists of the modal displacements and velocities:

$$\mathbf{x}_{[k]} = \begin{bmatrix} \mathbf{z}_{[k]} \\ \dot{\mathbf{z}}_{[k]} \end{bmatrix} \tag{4}$$

When using a zero-order hold assumption on the force, the expressions for **A** and **B** are given by:

$$\mathbf{A} = \exp\left(\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -\mathbf{\Gamma} \end{bmatrix} \Delta \mathrm{t}\right), \quad \mathbf{B} = [\mathbf{A} - \mathbf{I}] \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -\mathbf{\Gamma} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathrm{p}} \end{bmatrix} \tag{5}$$

The output vector is generally written as

$$\mathbf{d}(t) = \mathbf{S}_{\mathrm{a}}\mathbf{\Phi}\,\ddot{\mathbf{z}}(t) + \mathbf{S}_{\mathrm{v}}\mathbf{\Phi}\,\dot{\mathbf{z}}(t) + \mathbf{S}_{\mathrm{d}}\mathbf{\Phi}\,\mathbf{z}(t) \tag{6}$$

Where $\mathbf{S}_\mathrm{a}$, $\mathbf{S}_\mathrm{v}$, and $\mathbf{S}_\mathrm{d} \in \mathbb{R}^{n_\mathrm{d} \times n_\mathrm{DOF}}$ are selection matrices indicating the degrees of freedom corresponding to the acceleration, velocity, and displacement measurements, respectively. Eq. (6) is transformed into its state-space form, using Eq. (2):

$$\mathbf{d}_{[k]} = \mathbf{G}\,\mathbf{x}_{[k]} + \mathbf{J}\,\mathbf{p}_{[k]} \tag{7}$$

The expressions for the state-output matrix $\mathbf{G}$ and the direct transmission matrix $\mathbf{J}$ are given by

$$\mathbf{G} = [\mathbf{S}_\mathrm{d}\boldsymbol{\Phi} - \mathbf{S}_\mathrm{a}\boldsymbol{\Phi}\,\boldsymbol{\Omega}^2 \quad \mathbf{S}_\mathrm{v}\boldsymbol{\Phi} - \mathbf{S}_\mathrm{a}\boldsymbol{\Phi}\,\boldsymbol{\Gamma}]\,,\;\; \mathbf{J} = \left[\mathbf{S}_\mathrm{a}\boldsymbol{\Phi}\boldsymbol{\Phi}^\mathrm{T}\mathbf{S}_\mathrm{p}\right] \tag{8}$$

When process noise and measurement noise are added to Eq. (3) and Eq. (7), respectively, a discrete-time combined deterministic-stochastic state space description of the system is obtained:

$$\mathbf{x}_{[k+1]} = \mathbf{A}\,\mathbf{x}_{[k]} + \mathbf{B}\,\mathbf{p}_{[k]} + \mathbf{w}_{[k]} \tag{9}$$
$$\mathbf{d}_{[k]} = \mathbf{G}\,\mathbf{x}_{[k]} + \mathbf{J}\,\mathbf{p}_{[k]} + \mathbf{v}_{[k]} \tag{10}$$

2.2. Joint input-state estimation

An existing joint input-state estimation algorithm for linear systems with direct feedthrough [1] is extended in order to include the correlation between the process noise $\mathbf{w}_{[k]}$ and the measurement noise $\mathbf{v}_{[k]}$, which is often important for structural dynamics applications. The system under consideration is described by Eq. (9) and Eq. (10). The noise processes $\mathbf{w}_{[k]} \in \mathbb{R}^{n_\mathrm{m}}$ and $\mathbf{v}_{[k]} \in \mathbb{R}^{n_\mathrm{d}}$ are assumed to be zero mean and white, with known covariance matrices $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{S}$:

$$\mathbb{E}\left[\begin{pmatrix}\mathbf{w}_{[k]}\\ \mathbf{v}_{[k]}\end{pmatrix}\begin{pmatrix}\mathbf{w}_{[k]}^\mathrm{T} & \mathbf{v}_{[k]}^\mathrm{T}\end{pmatrix}\right] = \begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^\mathrm{T} & \mathbf{R}\end{bmatrix}\delta_{[k-l]} \tag{11}$$

with $\mathbf{Q} \geq 0$, $\mathbf{R} > 0$ and $\delta_{[k]} = 1$ for $k = 0$ and 0 otherwise.

Joint input-state estimation consists of estimating the forces $\mathbf{p}_{[k]}$ applied to the system and the corresponding system states $\mathbf{x}_{[k]}$, from a set of response measurements $\mathbf{d}_{[k]}$. A state estimate $\hat{\mathbf{x}}_{[k|l]}$ is defined as an estimate of $\mathbf{x}_{[k]}$, given the output sequence $\{\mathbf{d}_{[n]}\}_{n=0}^{l}$. The corresponding error covariance matrix, denoted as $\mathbf{P}_{[k|l]}$, is defined as $\mathbb{E}\left[\left(\mathbf{x}_{[k]} - \hat{\mathbf{x}}_{[k|l]}\right)\left(\mathbf{x}_{[k]} - \hat{\mathbf{x}}_{[k|l]}\right)^\mathrm{T}\right]$. An input estimate $\hat{\mathbf{p}}_{[k|l]}$ and its error covariance matrix $\mathbf{P}_{\mathrm{p}[k|l]}$ are defined in a similar way. For the estimation procedure, an initial unbiased estimate $\hat{\mathbf{x}}_{[0|-1]}$ is assumed to be known, as well as its error covariance matrix $\mathbf{P}_{[0|-1]}$. The estimate $\hat{\mathbf{x}}_{[0|-1]}$ is assumed independent of $\mathbf{w}_{[k]}$ and $\mathbf{v}_{[k]}$ for all k. Finally, it is also assumed that the rank of the direct transmission matrix $\mathbf{J}$ equals the number of applied forces n_p.

The filtering algorithm is initialized using the initial state estimate $\hat{\mathbf{x}}_{[0|-1]}$ and its error covariance matrix $\mathbf{P}_{[0|-1]}$. Hereafter, it propagates by computing the force and state estimates recursively in three steps, i.e. the input estimation step, the measurement update and the time update:

Input estimation

$$\tilde{\mathbf{R}}_{[k]} = \mathbf{G}\,\mathbf{P}_{[k|k-1]}\mathbf{G}^\mathrm{T} + \mathbf{R} \tag{12}$$
$$\boldsymbol{\mathcal{M}}_{[k]} = \left(\mathbf{J}^\mathrm{T}\tilde{\mathbf{R}}_{[k]}^{-1}\mathbf{J}\right)^{-1}\mathbf{J}^\mathrm{T}\tilde{\mathbf{R}}_{[k]}^{-1} \tag{13}$$
$$\hat{\mathbf{p}}_{[k|k]} = \boldsymbol{\mathcal{M}}_{[k]}(\mathbf{d}_{[k]} - \mathbf{G}\,\hat{\mathbf{x}}_{[k|k-1]}) \tag{14}$$
$$\mathbf{P}_{\mathrm{p}[k|k]} = \left(\mathbf{J}^\mathrm{T}\tilde{\mathbf{R}}_{[k]}^{-1}\mathbf{J}\right)^{-1} \tag{15}$$

Measurement update

$$\mathbf{L}_{[k]} = \mathbf{P}_{[k|k-1]}\mathbf{G}^\mathrm{T}\tilde{\mathbf{R}}_{[k]}^{-1} \tag{16}$$
$$\hat{\mathbf{x}}_{[k|k]} = \hat{\mathbf{x}}_{[k|k-1]} + \mathbf{L}_{[k]}(\mathbf{d}_{[k]} - \mathbf{G}\,\hat{\mathbf{x}}_{[k|k-1]} - \mathbf{J}\,\hat{\mathbf{p}}_{[k|k]}) \tag{17}$$
$$\mathbf{P}_{[k|k]} = \mathbf{P}_{[k|k-1]} - \mathbf{L}_{[k]}\left(\tilde{\mathbf{R}}_{[k]} - \mathbf{J}\mathbf{P}_{\mathrm{p}[k|k]}\mathbf{J}^\mathrm{T}\right)\mathbf{L}_{[k]}^\mathrm{T} \tag{18}$$
$$\mathbf{P}_{\mathrm{xp}[k|k]} = \mathbf{P}_{\mathrm{px}[k|k]}^\mathrm{T} = -\mathbf{L}_{[k]}\mathbf{J}\mathbf{P}_{\mathrm{p}[k|k]} \tag{19}$$

Time update

$$\hat{\mathbf{x}}_{[k|k-1]} = \mathbf{A}\,\hat{\mathbf{x}}_{[k|k]} + \mathbf{B}\,\hat{\mathbf{p}}_{[k|k]} \tag{20}$$

$$\boldsymbol{\mathcal{L}}_{[k]} = \mathbf{L}_{[k]}\big(\mathbf{I} - \mathbf{J}\,\boldsymbol{\mathcal{M}}_{[k]}\big) \tag{21}$$

$$\mathbf{P}_{[k+1|k]} = [\mathbf{A} \quad \mathbf{B}]\begin{bmatrix}\mathbf{P}_{[k|k]} & \mathbf{P}_{\mathrm{xp}[k|k]}\\ \mathbf{P}_{\mathrm{px}[k|k]} & \mathbf{P}_{\mathrm{p}[k|k]}\end{bmatrix}\begin{bmatrix}\mathbf{A}^{\mathrm{T}}\\ \mathbf{B}^{\mathrm{T}}\end{bmatrix} + \mathbf{Q} - \big(\mathbf{A}\boldsymbol{\mathcal{L}}_{[k]} + \mathbf{B}\boldsymbol{\mathcal{M}}_{[k]}\big)\mathbf{S}^{\mathrm{T}} - \mathbf{S}\,\big(\mathbf{A}\boldsymbol{\mathcal{L}}_{[k]} + \mathbf{B}\boldsymbol{\mathcal{M}}_{[k]}\big)^{\mathrm{T}} \tag{22}$$

In the equations above, the system is assumed to be time-invariant. The algorithm can, however, also be applied to time-variant systems by indexing the system matrices, i.e. $\mathbf{A}_{[k]}$, $\mathbf{B}_{[k]}$, $\mathbf{G}_{[k]}$, and $\mathbf{J}_{[k]}$.

2.3. Estimation noise statistics

In structural dynamics applications, very often operational loads, e.g. wind loads, are modeled as stochastic noise processes. The corresponding noise terms are then far more important than sensor noise, and the noise statistics $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{S}$, under the assumption of white noise processes, can be estimated from a preliminary vibration experiment where only noise sources are present ($\mathbf{p}_{[k]} = 0$).

For the noise identification procedure, the noise sources are modeled as a set of stochastic forces $\mathbf{p}_{\mathrm{s}[k]}$. This implies that the ambient forces are assumed to be concentrated in a limited number of structural nodes. The power spectral density (PSD) of the forces $\mathbf{S}_{\mathrm{p_s p_s}}(\omega) \in \mathbb{C}^{n_{\mathrm{p_s}} \times n_{\mathrm{p_s}}}$ at frequency ω is obtained from the PSD of the measured structural response $\mathbf{S}_{\mathrm{dd}}(\omega) \in \mathbb{C}^{n_{\mathrm{d}} \times n_d}$ and the frequency response function (FRF) matrix $\mathbf{H}(\omega) \in \mathbb{C}^{n_{\mathrm{d}} \times n_{\mathrm{p}_s}}$, as follows:

$$\mathbf{S}_{\mathrm{p_s p_s}}(\omega) = \mathbf{H}^{+}(\omega)\mathbf{S}_{\mathrm{dd}}(\omega)\mathbf{H}^{*+}(\omega) \tag{23}$$

where $\blacksquare^{+}$ denotes the Moore-Penrose pseudo-inverse of a matrix and $\blacksquare^{*}$ denotes the Hermitian transpose of a matrix. Defining the PSD of a sampled time series as

$$\mathbf{S}_{\mathrm{pq}}(\,\omega) := \sum_{k=0}^{N-1} \mathbf{R}_{\mathrm{pq}[k]} \exp(-i\omega k \Delta t)\,, \tag{24}$$

where

$$\mathbf{R}_{\mathrm{pq}[k]} := \mathbb{E}\{\mathbf{p}_{[l+k]}\mathbf{q}_{[l]}^{\mathrm{T}}\} \tag{25}$$

The force PSD $\mathbf{S}_{\mathrm{p_s p_s}}(\omega)$, obtained from Eq. (23), equals the force covariance matrix $\mathrm{Cov}\big(\mathbf{p}_{\mathrm{s}[k]}\big)$ $\big(:= \mathbb{E}[\mathbf{p}_{\mathrm{s}[k]}\mathbf{p}_{\mathrm{s}[k]}^{\mathrm{T}}]\big)$, under the assumption of a stationary discrete-time white noise process. This holds for each frequency. The noise covariance matrices are calculated as:

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^{\mathrm{T}} & \mathbf{R}\end{bmatrix} = \begin{bmatrix}\mathbf{B}\\ \mathbf{J}\end{bmatrix}\mathrm{Cov}(\mathbf{p}_{\mathrm{s}[k]})[\mathbf{B}^{\mathrm{T}} \quad \mathbf{J}^{\mathrm{T}}] \tag{26}$$

In the literature, several methods have been proposed to estimate the cross-PSD of two sampled time series $\mathbf{p}_{[k]}$ and $\mathbf{q}_{[k]}$ [10, 11]. For the remainder of this paper, the correlogram approach [12] is used to calculate the output PSD $\mathbf{S}_{\mathrm{dd}}(\omega)$ from a set of output measurements. The FRF matrix can be obtained from an updated finite element model of the structure or can be directly obtained from system identification techniques.

For a frequency ω, the number of modes significantly contributing to the response can become less than the number of stochastic forces to be estimated ($= n_{\mathrm{p_s}}$). The problem of estimating the force PSD from a set of output measurements then becomes ill-posed and rank deficient. As a consequence, modeling errors and measurement errors will result in an erroneous estimate of the force PSD. The stochastic force covariance matrix is estimated hereafter by averaging the force PSD over a number of frequencies where the ill-posedness of the problem is minimal. The accuracy and effectiveness of the algorithm at a frequency ω is therefore assessed based on an algorithm proposed by Fabunmi [13].

3. IN SITU EXPERIMENT ON A FOOTBRIDGE

In this section, the effectiveness of the force identification procedure is investigated by means of an in situ experiment on a footbridge, located in Ninove (Belgium). It is a two-span cable-stayed steel bridge (Figure 1) with a main and secondary span of 36 m and 22.5 m, respectively.

Figure 1 The footbridge in Ninove, Belgium.

The system matrices are constructed from an updated FE model of the bridge. The FE model is updated using a set of experimental modal parameters and considering the stiffness of the neoprene bearings, the bridge deck, the pylons, and the effective stiffness of the cables as updating variables. The experimental modal parameters are obtained through an output-only system identification, based on ambient vibrations due to, amongst others, wind. The output-only data, collected at 56 measurement locations along the bridge deck, have been processed using the reference-based data-driven stochastic subspace identification algorithm (SSI-data/ref) [14]. Table 1 presents a comparison between the experimentally identified modal characteristics and those calculated from the updated FE model. The experimental modal damping ratios ξ_{id} and the MAC values [15] between the measured mode shapes and the ones obtained from the updated FE model are shown as well.

Table 1 Comparison between the experimentally identified modal characteristics and the modal characteristics of the updated FE model ($\boldsymbol{f}_{\mathbf{id}}$: identified natural frequency, $\boldsymbol{\xi}_{\mathbf{id}}$: identified modal damping ratio, $\boldsymbol{f}_{\mathbf{fem}}$: undamped natural frequency updated FE model, $\boldsymbol{\epsilon}$: error $\boldsymbol{f}_{\mathbf{fem}}$ w.r.t. $\boldsymbol{f}_{\mathbf{id}}$, **MAC**: MAC-value).

Mode No.	f_{id} [Hz]	ξ_{id} [%]	f_{fem} [Hz]	ϵ [%]	MAC [−]
1	2.97	1.18	2.90	-2.39	0.98
2	3.06	1.92	3.09	1.03	0.94
3	3.79	0.78	3.71	-2.02	0.92
4	-	-	5.02	-	-
5	6.00	0.68	5.84	-2.75	0.99
6	-	-	7.01	-	-
7	6.93	0.59	7.15	3.14	0.92
8	7.99	0.78	7.68	-3.90	0.99
9	-	-	8.61	-	-
10	-	-	9.60	-	-
11	9.73	1.12	9.99	2.63	0.81
12	10.86	1.21	10.59	-2.46	0.91
13	-	-	11.74	-	-
14	12.55	1.96	12.81	2.05	0.93
15	13.57	0.66	13.13	-3.23	0.91
16	14.71	0.58	13.95	-5.20	0.91
17	13.86	0.58	14.15	2.07	0.91
18	-	-	15.65	-	-
19	-	-	17.00	-	-
20	18.58	0.44	18.02	-3.02	0.84
21	17.15	1.06	18.49	7.81	0.91

A reduced-order discrete-time state-space model is constructed from the updated finite element model of the footbridge, applying a zero order hold assumption on the force. The model includes the first 21 eigenmodes of the FE model, listed in Table 1. For each of the 21 modes, the mass normalized mode shape is assumed to be known from the FE model. For the 14 modes identified using the SSI procedure, the natural frequency as well as the modal damping ratio are taken as the identified values. For each of the remaining, unidentified modes, the natural frequency is determined using an H1 transfer function estimation procedure [12], whereas the modal damping ratio is obtained by performing a manual updating procedure.

A set of vibration experiments was performed on the footbridge, considering both exogenous and ambient excitation. For the first set of experiments, with exogenous excitation, the bridge was excited by hammer excitation and/or harmonic excitation by two pneumatic artificial muscles. Originally sampled at 200 Hz, all data used in the inverse calculations are resampled at a lower sampling rate in order to include only frequencies within the range of the modes included in the model. Using a decimation factor of 5, the data are lowpass filtered using a Chebyshev type I filter at 16 Hz and subsequently resampled at 40 Hz. Since the working range of the accelerometers has its lower bound at 0.5 Hz, all measured signals are additionally high-pass filtered using an eighth order Chebyshev type I filter with a cutoff frequency of 0.5 Hz.

3.1. Estimation noise statistics

The data used in the following analysis contain the response of the footbridge to ambient excitation. The measurement setup is shown in Figure 2. The response data consist of the vertical (z-direction) and lateral (y-direction) acceleration measurements for each of the five sensor locations, collected using wired accelerometers. A time period of 500 s is considered. As an example, the vertical acceleration at node 27 is shown in Figure 3.

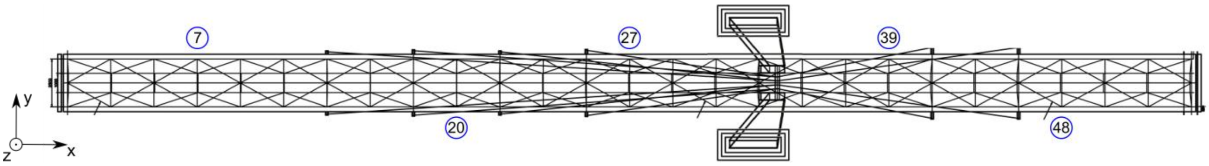

Figure 2 Overview measurement setup ambient excitation. Sensor positions are indicated in blue.

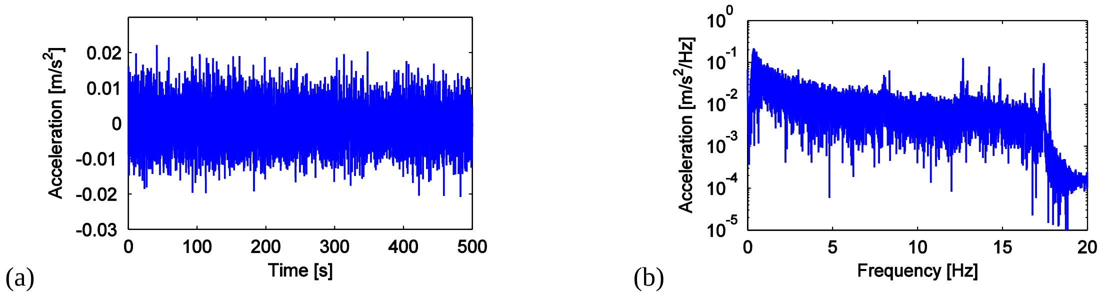

Figure 3 (a) Time history and (b) frequency content up to 20 Hz, of the vertical accelerations at node 27 (accelerations due to ambient excitation).

The response data and the system model are now used to estimate the ambient force covariance matrix, hereby applying the method as proposed in section 2.3. For each acceleration signal a stochastic force is assumed, acting at the same node and along the same direction. In this way, a set of 10 stochastic forces is estimated from 10 acceleration measurements. The averaged force PSD values, which are a measure for the force covariance matrix, are shown in Figure 4. Since the force covariance matrix has real elements, the averaging is performed for the modulus of the PSD values over the frequency range where the ill-posedness of the problem is minimal (see section 2.3).

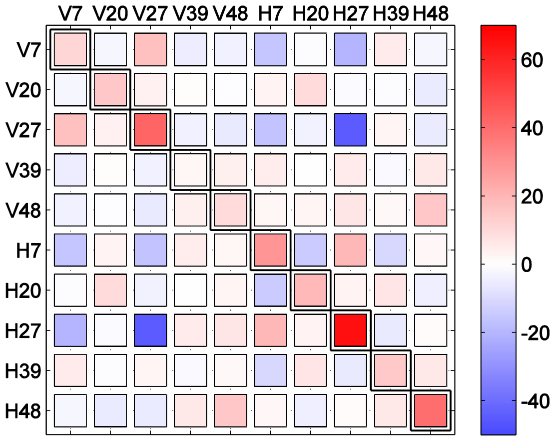

Figure 4 Estimated stochastic forces covariance matrix in [N^2] (Vi: vertical force and Hi: lateral force node i).

In general, the lateral force components Hi are characterized by a larger variance than the vertical force components Vi. This is expected, since the ambient loads mainly consist of wind loads. The largest force variance occurs at node 27, both for the vertical and the lateral force component and, in addition, both force components are strongly correlated. The covariance matrix obtained seems to be reasonable and can now be used to calculate the noise covariance matrices.

3.2. Force identification

The data used in the following analysis are obtained during the excitation of the footbridge by two vertical hammer forces at the bridge deck, one at node 27 and one at node 48. The measurement setup is shown in Figure 5. A time period of 25 seconds is considered, containing one impact at both nodes. The forces time histories and the corresponding frequency content are shown in Figure 6.

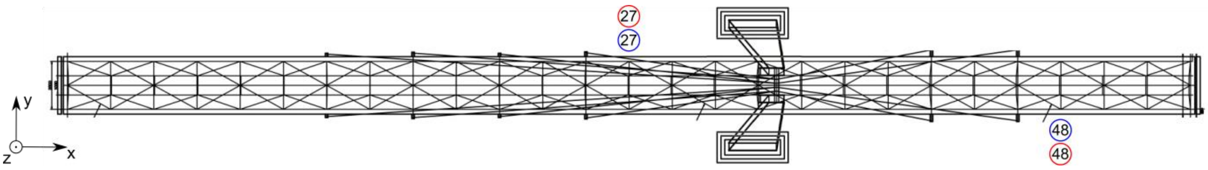

Figure 5 Overview measurement setup force identification. Sensor positions are indicated in blue, hammer force locations are indicated in red.

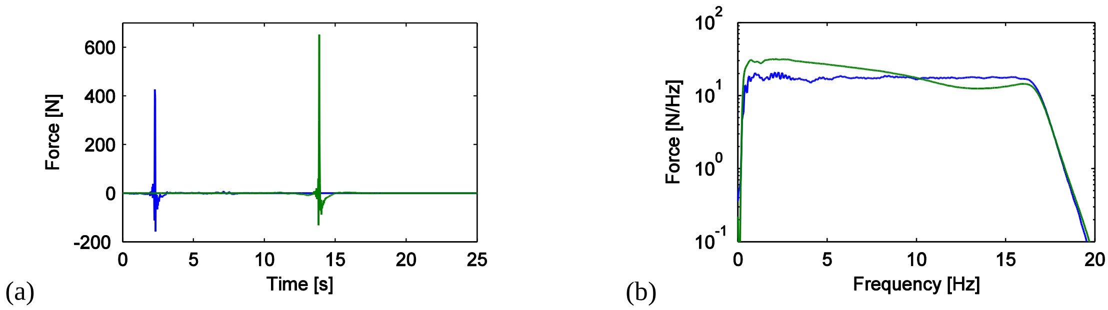

Figure 6 (a) Time history and (b) frequency content up to 20 Hz, of the hammer forces applied vertically to the bridge deck (blue: node 27, green: node 48).

The force identification is performed using the joint input-state estimation algorithm, proposed in section 2.2. The acceleration measurements taken into account are the vertical accelerations at nodes 27 and 48, i.e. the two driving point accelerations. The noise covariance matrices **Q**, **R**, and **S** are calculated from the estimated stochastic force covariance matrix, obtained from section 3.1. The initial state estimate $\hat{\mathbf{x}}_{[0|-1]}$ is assumed zero and its error covariance matrix $\mathbf{P}_{[0|-1]}$ is assigned a diagonal matrix with values of 10^{-1} on its diagonal. This large value indicates the high level of uncertainty regarding the initial state estimate. For the analysis, the force locations and directions are assumed known.

The reconstructed forces are characterized by a low frequency drift. This is due to the fact that only acceleration measurements are taken into account, but also to the large influence of ambient excitation at low frequencies. For frequencies up to 2.6 Hz, the ambient excitation becomes more important than the hammer force, in this way giving rise to a large error on the identified force signals. The low frequency drift is removed by applying a fourth order Butterworth high-pass filter with a cutoff frequency of 2.6 Hz to the identified force signals. The measured force signal is filtered using the same filter.

The results of the force identification are shown in Figure 7 and Figure 8, for the hammer forces at node 27 and node 48, respectively.

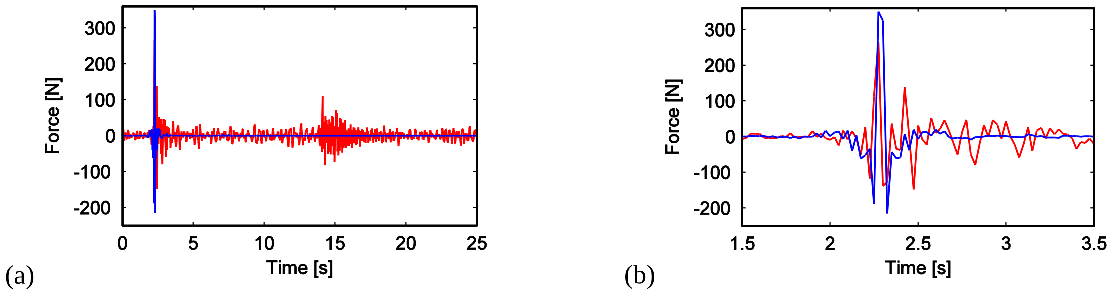

Figure 7 (a) Complete time history and (b) detail of the time history of the identified hammer force at node 27 (blue: measured, red: identified).

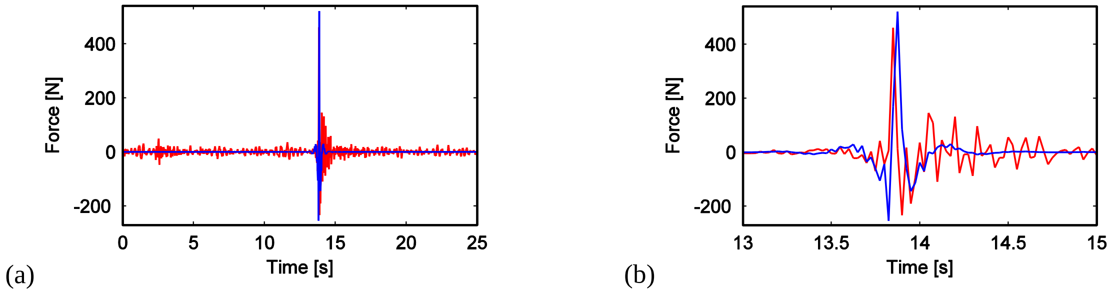

Figure 8 (a) Complete time history and (b) detail of the time history of the identified hammer force at node 48 (blue: measured, red: identified).

Both for the force applied at node 27 and at node 48 a very good correspondence between the measured and the identified force signals is obtained.

The uncertainty on the identified force signals can be quantified by means of the force error covariance matrix $\mathbf{P}_{\mathrm{p}[k|k]}$ (see Eq. (15)). The diagonal elements of this matrix are a measure for the variance of the estimation error and are used to define an uncertainty bound on the results obtained. Since only acceleration measurements are taken into account, the low frequency content of the force cannot be retrieved from the data. As a consequence, the error covariance matrix $\mathbf{P}_{\mathrm{p}[k|k]}$ grows unbounded. By adding displacement or strain measurements, the error covariance matrix does no longer grow unbounded and this approach yields an uncertainty bound on the estimation. This is, however, not illustrated for the case of the Ninove footbridge, since no displacement or strain measurements are available.

4. CONCLUSIONS

An existing joint input-state estimation algorithm was extended for applications in structural dynamics. The algorithm can be used to identify forces applied to a structure when their positions are known. In addition, a method was proposed to identify the process noise and measurement noise characteristics, which are needed for the joint input-state estimation procedure. The methodology was illustrated for a set of data collected from an in situ experiment on a footbridge. Multiple hammer excitation forces have been identified from a limited set of acceleration data. The identified force signals are a very good estimate of the true applied forces.

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