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A new detection method for noisy channels with time-varying offset

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***Abstract*—We consider noisy communications and storage systems that are hampered by varying offset of unknown magnitude such as low-frequency signals of unknown amplitude added to the sent signal. We study and analyze a new detection method whose error performance is independent of both unknown base offset and offset's slew rate. The new method requires, for a codeword length $n \geq 12$, less than 1.5 dB more noise margin than Euclidean distance detection. The relationship with constrained codes based on mass-centered codewords and the new detection method is discussed.**

***Keywords*–** channel mismatch, constrained code, Pearson code, Pearson distance, slew rate, varying offset.

I. Introduction

Euclidean-distance-based detection of transmitted or stored encoded data signals is optimal in the presence of white Gaussian noise, but its error performance is vulnerable in the presence of channel mismatch, such as offset of unknown magnitude. Unknown offset magnitude variations may be caused by a variety of interference sources. For example, in optical disc recording, scratches and finger prints on the disc [1] cause low-frequency varying offset in the read-out signal.

In nonvolatile memories (NVMs) data are represented by stored charge [2, 3, 4]. The stored charge can leak away from the floating gate through the gate oxide or through the dielectric. The amount of leakage, called *drift*, depends on various physical parameters, such as, for example, the device temperature and the time elapsed between writing and reading [5].

In [6, 7], the authors assume that the unknown offset mismatch can be approximated by a zeroth-order, constant, term for all symbols in a codeword. In this model, the offset term may vary from word to word, but is fixed within a codeword. The authors advocate detection based on the *Pearson distance*, which is resilient to unknown, but constant within a codeword, offset and gain (scaling) of the received signal [6, 7, 8]. The zeroth-order model can be overly simplistic in specific communications channels, where the offset or low-frequency interference may vary so rapidly that the basic premise that the offset is constant within a codeword is false.

A low-frequency varying offset can be segmented into an approximately piecewise linear function of time, where the 'pieces' have a length equal to the codeword length. As discussed in [9], memory cells of nonvolatile data storage products that are closer to warmer spots lose their data charge more rapidly than memory cells closer to colder spots, so that offset loss is not constant within a codeword [4]. Evidently, the (varying) offset cannot be considered to be equal for all symbols in a codeword, and alternative detection methods have been sought for.

It is assumed in this paper that the unknown time-varying offset can be approximated by a word-wise first-order term that varies linearly over the codeword symbols, where both the *base* offset and offset's *slew rate* (the offset's first-order rate of change) are unknown. Both unknown terms, the base offset and offset's slew rate, may vary from codeword to codeword, but are fixed within a codeword. The quest for advanced detection techniques that are immune to unknown, first-order, offset variation is not new. Skachek and Immink [9] introduced *mass centered* codewords whose detection is independent of both unknown base offset and offset's slew rate. They concluded that the redundancy of their scheme is prohibitively large for many applications. Bu and Weber [10] also addressed a channel model where the offset varies within a codeword. They introduced Pearson-distance-based detection in conjunction with a difference operator and a pair-constrained code. Their adopted code has significantly less redundancy than the previously proposed mass-centered codes [9]. However, it requires a 3 dB higher noise margin, which makes it less suitable for noise-dominant channels.

Alternative solutions are wanted that are less costly in terms of noise figure or redundancy. To that end, we propose and analyze a detection method based on a new distance measure, whose error performance is independent of both unknown base offset and offset's slew rate. The rate of the requisite binary constrained code is very high as only one codeword must be barred from the repertoire of 2^n possible codewords. It requires less than 1.5 dB more noise margin than Euclidean distance detection, or less than 1 dB with respect to Pearson distance detection, both for $n = 12$.

The paper is organized as follows. In Section II, we start with preliminaries, a description of the adopted channel model, and a description of the properties of prior art minimum Pearson distance detection. In Section III, we propose a novel detection method that improves the detector's resilience in case the received signal is distorted by changing offset. We analyze the error performance of the new detection method, and offer results of simulations. Receiver complexity is discussed in Section IV. Section V furnishes the conclusions of our paper.

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II. Preliminaries, channel model, prior art

Let $\mathcal{S}$ be a codebook of selected binary codewords $\boldsymbol{x} = (x_1, \ldots, x_n)$, where the integer n denotes the codeword length, and $x_i \in \{0,1\}$. The received signal is denoted by the vector $\boldsymbol{r}$ having real entries r_i, which is defined by

$$r_i = x_i + \nu_i + I_i, \;\; i = 1, 2, \ldots, n. \tag{1}$$

The (real) variables ν_i denote zero-mean white additive Gaussian noise samples with variance σ^2, for example caused by thermal noise. The (real) variable I_i denote time-varying interfering offset. It is assumed that the interfering offset, I_i, can be approximated by a word-wise linear function waveform (ramp), which is denoted by

$$I_i = b_0 + b_1 i, \;\; i = 1, 2, \ldots, n, \tag{2}$$

where the (real) coefficients b_0 and b_1 denote the unknown *base* offset and the unknown offset's *slew rate*, respectively. Both offset and offset's slew rate are assumed to be constant within a codeword; they may vary from codeword to codeword.

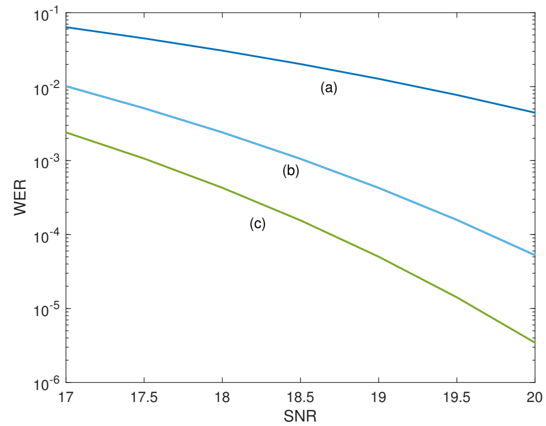

Fig. 1. Word error rate, WER versus signal-to-noise ratio, SNR = $-20\log(\sigma)$ (dB). Curve (a) shows Euclidean detection with channel mismatch, $b_0 = 0.1$, $b_1 = 0.01$, Curve (b) shows the new detection method, which is detailed in Section III, and Curve (c) shows Euclidean detection for the ideal AWGN channel, no mismatch, $b_0 = b_1 = 0$. All curves for $n = 12$.

A. Motivating example

The error performance of Euclidean detection is seriously deteriorated in the face of relatively small mismatch. In order to demonstrate this, Figure 1 shows, for $n = 12$, the word error rate (WER) versus signal-to-noise ratio, SNR = $-20\log(\sigma)$ (dB), of in Curve (c) Euclidean detection, ideal noisy channel without mismatch, i.e. $b_0 = b_1 = 0$, in Curve (a) noisy channel with offset mismatch, $b_0 = 0.1$ and $b_1 = 0.01$. The diagram clearly shows that the error performance of Euclidean detection is seriously degraded by a small offset. Curve (b) is obtained using a novel detection scheme, whose error performance is independent of both channel mismatch terms b_0 and b_1; the new method is described in Section III. We may observe that in the matched case, the error performance of the new method is inferior to that of Euclidean detection, but in case of mismatch the situation changes, and the new method has a superior error performance. The development and analysis of the new method is the main topic of our paper, and described in Section III and further.

B. Prior art, constant offset

In order to make the error performance independent of unknown (base) offset mismatch, Immink and Weber [6] introduced the (modified) Pearson distance between two n-vectors. Let $\boldsymbol{x}, \hat{\boldsymbol{x}} \in \mathcal{S}$, be two n-vectors, where $\mathcal{S}$ is the set of chosen codewords. For the base offset mismatch case, $I_i = b_0$, they proposed the distance measure between the received vector $\boldsymbol{r}$ and $\hat{\boldsymbol{x}}$

$$\delta'(\boldsymbol{r}, \hat{\boldsymbol{x}}) = \sum_{i=1}^{n} \left(r_i - \hat{x}_i + \overline{\hat{x}}\right)^2, \tag{3}$$

where $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ denotes the average of the entries of an n-vector $\boldsymbol{y}$. Since $\delta'(\boldsymbol{r}, \boldsymbol{0}) = \delta'(\boldsymbol{r}, \boldsymbol{1})$, the receiver cannot distinguish between the all-0 word and the all-1 word, denoted by $\boldsymbol{0}$ and $\boldsymbol{1}$, respectively. The ambiguity can be remedied by arbitrarily excluding one of them, say the word $\boldsymbol{0}$, so that $\mathcal{S} = \{0,1\}^n \setminus \{\boldsymbol{0}\}$.

A minimum Pearson distance detector outputs the codeword

$$\boldsymbol{x}_{\rm o} = \arg\min_{\hat{\boldsymbol{x}} \in \mathcal{S}} \delta'(\boldsymbol{r}, \hat{\boldsymbol{x}}). \tag{4}$$

By substituting $r_i = x_i + \nu_i + b_0$ into (3), we can easily verify that the outcome of (4) is independent of b_0 for codewords in $\mathcal{S}$. Note that offset may vary from codeword to codeword, but not within a codeword. In many practical situations of interest, however, the offset within a codeword is not constant, but slowly varying, i.e, in our model $b_1 \neq 0$. In the next section, we develop a new detection method that can cope with varying offset $I_i = b_0 + b_1 i$, $b_1 \neq 0$.

III. New distance measure

In the vein of (3), we establish a distance measure that is independent of the varying offset term $I_i = b_0 + b_1 i$.

A. Definition of distance measure

Define

$$\delta(\boldsymbol{r}, \hat{\boldsymbol{x}}) = \sum_{i=1}^{n} \left(r_i - \hat{x}_i + \varphi_i(\hat{\boldsymbol{x}})\right)^2, \tag{5}$$

where the proposed $\hat{\boldsymbol{x}}$-dependent term is

$$\varphi_i(\hat{\boldsymbol{x}}) = \beta_0(\hat{\boldsymbol{x}}) + \beta_1(\hat{\boldsymbol{x}}) i. \tag{6}$$

The coefficients $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$ are to be determined to ensure that the outcome of the detection is independent of the unknown offset parameters b_0 and b_1. The decoded codeword, $\boldsymbol{x}_{\rm o}$, is found, as in (4), by the minimization process

$$\boldsymbol{x}_{\rm o} = \arg\min_{\hat{\boldsymbol{x}} \in \mathcal{S}} \delta(\boldsymbol{r}, \hat{\boldsymbol{x}}). \tag{7}$$

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After substituting (1) into (5), we have

$$\begin{aligned}\delta(\boldsymbol{r},\hat{\boldsymbol{x}}) &= \sum_{i=1}^{n}(x_i+\nu_i-\hat{x}_i+\varphi_i(\hat{\boldsymbol{x}}))^2 \\ &+ 2\sum_{i=1}^{n}(-\hat{x}_i+\varphi_i(\hat{\boldsymbol{x}}))\,I_i \\ &+ \sum_{i=1}^{n}[I_i^2+2I_i(x_i+\nu_i)]. \end{aligned} \tag{8}$$

The first term is independent of the offset term I_i. The third term is independent of $\hat{\boldsymbol{x}}$, and therefore irrelevant in view of the minimization process (7). The second term

$$2\sum_{i=1}^{n}(-\hat{x}_i+\varphi_i(\hat{\boldsymbol{x}}))\,(b_0+b_1 i), \tag{9}$$

is independent of the unknown variables b_0 and b_1 if we choose the coefficients $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$ in such a way that, see (6),

$$\sum_{i=1}^{n}(-\hat{x}_i+\beta_0(\hat{\boldsymbol{x}})+\beta_1(\hat{\boldsymbol{x}})i)=0 \tag{10}$$

and

$$\sum_{i=1}^{n}i(-\hat{x}_i+\beta_0(\hat{\boldsymbol{x}})+\beta_1(\hat{\boldsymbol{x}})i)=0. \tag{11}$$

After substituting the well-known expressions for $\sum i^k$, $k=1,2$, we obtain two equations for the unknown coefficients, $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$:

$$\left\{\begin{array}{rlll} & \beta_0(\hat{\boldsymbol{x}})+\frac{n+1}{2} & \beta_1(\hat{\boldsymbol{x}}) & =\zeta_0(\hat{\boldsymbol{x}}) \\ \frac{n+1}{2} & \beta_0(\hat{\boldsymbol{x}})+\frac{(n+1)(2n+1)}{6} & \beta_1(\hat{\boldsymbol{x}}) & =\zeta_1(\hat{\boldsymbol{x}}), \end{array}\right. \tag{12}$$

where the zeroth and first moment of the codeword $\hat{\boldsymbol{x}}$ are defined by

$$\zeta_0(\hat{\boldsymbol{x}})=\frac{1}{n}\sum_{i=1}^{n}\hat{x}_i \text{ and } \zeta_1(\hat{\boldsymbol{x}})=\frac{1}{n}\sum_{i=1}^{n}i\hat{x}_i. \tag{13}$$

Solving the linear system (12) in the unknown coefficients, $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$, yields

$$\beta_0(\hat{\boldsymbol{x}})=2\frac{(2n+1)\zeta_0(\hat{\boldsymbol{x}})-3\zeta_1(\hat{\boldsymbol{x}})}{n-1} \tag{14}$$

and

$$\beta_1(\hat{\boldsymbol{x}})=6\frac{-(n+1)\zeta_0(\hat{\boldsymbol{x}})+2\zeta_1(\hat{\boldsymbol{x}})}{n^2-1}, \tag{15}$$

which establishes with (5) and (6) the new detector algorithm. In the next subsection, we analyze the error performance of the new detection method based on (5).

B. Analysis of the error performance

We adopt here the same set of codewords, $\mathcal{S}=\{0,1\}^n\backslash\{\mathbf{0}\}$, which is used in conjunction with the prior art modified Pearson distance detector [6]. Let $\boldsymbol{x}\in\mathcal{S}$ be the sent codeword, and let $\hat{\boldsymbol{x}}\in\mathcal{S}$, $\hat{\boldsymbol{x}}\neq\boldsymbol{x}$. In view of (7), $\delta(\boldsymbol{r},\hat{\boldsymbol{x}})$ can be rewritten as an equivalent expression, which is convenient for the computation of the error performance. The detector's performance is independent of a term $c_1+c_2 i$, (c_1 and c_2 arbitrary constants), thus we may delete $I_i=b_0+b_1 i$ or subtract $\varphi_i(\boldsymbol{x})=\beta_0(\boldsymbol{x})+\beta_1(\boldsymbol{x})i$ without effect on the outcome of (7). Then, exploiting the linearity of the expressions (13), (14), (15) in φ_i, we derive from (8) after deleting irrelevant terms:

$$\begin{aligned}\delta(\boldsymbol{r},\hat{\boldsymbol{x}}) &\equiv \sum_{i=1}^{n}(x_i-\hat{x}_i+\varphi_i(\hat{\boldsymbol{x}})+\nu_i)^2 \\ &\equiv \sum_{i=1}^{n}(x_i-\hat{x}_i+\varphi_i(\hat{\boldsymbol{x}})-\varphi_i(\boldsymbol{x})+\nu_i)^2 \\ &= \sum_{i=1}^{n}(x_i-\hat{x}_i-\varphi_i(\boldsymbol{x}-\hat{\boldsymbol{x}})+\nu_i)^2. \end{aligned} \tag{16}$$

where the equivalence symbol $\equiv$ denotes that the expressions on both sides of $\equiv$ yield $\boldsymbol{x}_{\text{o}}$ after the minimization (7). Let $\boldsymbol{e}=\boldsymbol{x}-\hat{\boldsymbol{x}}$, then we obtain

$$\delta(\boldsymbol{r},\hat{\boldsymbol{x}})\equiv\sum_{i=1}^{n}(e_i-\varphi_i(\boldsymbol{e})+\nu_i)^2. \tag{17}$$

The detector errs, if it restores $\hat{\boldsymbol{x}}$ instead of the sent $\boldsymbol{x}$, that is, if

$$\delta(\boldsymbol{r},\hat{\boldsymbol{x}})<\delta(\boldsymbol{r},\boldsymbol{x}), \tag{18}$$

or, after using (17),

$$2\sum_{i=1}^{n}(e_i-\varphi_i(\boldsymbol{e}))\nu_i+\sum_{i=1}^{n}(e_i-\varphi_i(\boldsymbol{e}))^2<0. \tag{19}$$

The noise samples, ν_i, are assumed to be white and drawn from $N(0,\sigma^2)$, so that the *pairwise error probability* $Pr(\boldsymbol{x}\rightarrow\hat{\boldsymbol{x}})$ equals

$$\begin{aligned} Pr(\boldsymbol{x}\rightarrow\hat{\boldsymbol{x}}) &= Pr(\delta(\boldsymbol{r},\hat{\boldsymbol{x}})<\delta(\boldsymbol{r},\boldsymbol{x})) \\ &= Q\left(\frac{d(\boldsymbol{x},\hat{\boldsymbol{x}})}{2\sigma}\right), \end{aligned} \tag{20}$$

where

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}du, \tag{21}$$

and the squared noise distance $d^2(\boldsymbol{x},\hat{\boldsymbol{x}})$ between $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$ is

$$d^2(\boldsymbol{x},\hat{\boldsymbol{x}})=\sum_{i=1}^{n}(e_i-\varphi_i(\boldsymbol{e}))^2=\delta(\mathbf{0},\boldsymbol{e}). \tag{22}$$

The *union bound* offers a useful tool to approximate the average word error rate (WER). The WER is upperbounded by [6]

$$\text{WER}\leq\sum_d K_d Q\left(\frac{d(\boldsymbol{x},\hat{\boldsymbol{x}})}{2\sigma}\right), \tag{23}$$

where K_d is the average number of neighbors at distance $d=d(\boldsymbol{x},\hat{\boldsymbol{x}})$. The minimum noise distance between any pair of distinct codewords, denoted by $d_{\min}$, is defined by

$$d_{\min}=\min_{\boldsymbol{x},\hat{\boldsymbol{x}}\in\mathcal{S},\boldsymbol{x}\neq\hat{\boldsymbol{x}}}d(\boldsymbol{x},\hat{\boldsymbol{x}}). \tag{24}$$

The *union bound estimate* of the word error rate is

$$\text{WER}\approx N_{d_{\min}}Q\left(\frac{d_{\min}}{2\sigma}\right),\ \sigma\ll 1, \tag{25}$$

where the average number of neighbors at minimum noise distance $d_{\min}$ is denoted by $N_{d_{\min}}$.

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1) Analysis of minimum noise distance $d_{\min}$: For relatively small values of n we can, using exhaustive search (24), find the worst case error vector $\boldsymbol{e}$. Substituting the found $\boldsymbol{e}$ into (22) yields the expression (26) for various values of n, $n < 30$ (the maximum word length, n, of our search), so that

$$d^2_{\min} = \begin{cases} \frac{n^2-1}{16n}, & n = 3,5,7,9, \\ \frac{n(n^2-4)}{16(n^2-1)}, & n = 2,4,6,8,10, \\ \frac{(n-1)(n-2)}{n(n+1)}, & 11 \leq n < 30. \end{cases} \quad (26)$$

Figure 2 displays the minimum noise distance, $20\log(d_{\min})$ (dB) versus codeword length n, where the results are obtained using (26). As a reference we plotted the minimum noise distance of the prior art method that offers constant offset immunity, based on the distance measure (3). We notice for small values of n a significant loss in the receiver's noise margin with respect to conventional Euclidean distance detection due to the decrease in $d_{\min}$. For $n \geq 12$, the loss is less than 1.5 dB, (for $n \geq 18$ the loss is less than 1 dB). Note that the method advocated by Bu *et al.* [10] that aims to solve the same problem has a 3 dB noise penalty, irrespective of the codeword length.

For large n, the minimum distance computation is amenable for analysis. We approximate $\beta_0(\boldsymbol{e})$ and $\beta_1(\boldsymbol{e})$ by, see (14) and (15),

$$\beta_0(\boldsymbol{e}) \approx \frac{4nw - 6s}{n^2} \text{ and } \beta_1(\boldsymbol{e}) \approx \frac{12s - 6nw}{n^3},$$

where we use the short-hand notation $w = n\zeta_0(\boldsymbol{e})$ and $s = n\zeta_1(\boldsymbol{e})$. Then, after working out (22), we obtain

$$\delta(\boldsymbol{0}, \boldsymbol{e}) \approx d_{\rm H} + 4\frac{3nws - n^2w^2 - 3s^2}{n^3}, \quad (27)$$

where $d_{\rm H} = \sum e_i^2$ denotes the Hamming distance between $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$. For $1 \leq d_H \leq n-1$, we find that minimizing (27) over w and s gives

$$\min_{s,w} \delta(\boldsymbol{0}, \boldsymbol{e}) \approx d_{\rm H} - \frac{4n^2d^2_{\rm H} - 6nd^3_{\rm H} + 3d^4_{\rm H}}{n^3}, \; n \gg 1, \quad (28)$$

where the minimum is achieved at the maximum values for w and s, i.e., $w = d_H$ and $s = n(n+1)/2 - (n-d_H)(n-d_H+1)/2$. The expression in (28) is at a minimum for $d_H = 1$ and $d_H = n-1$, which shows that a large Hamming distance does not necessarily lead to a large noise distance $\delta(\boldsymbol{0}, \boldsymbol{e})$. Observe that this minimum is approximately $1-4/n$. For the remaining case $d_H = n$, we find from (27) that $\min_{s,w} \delta(\boldsymbol{0}, \boldsymbol{e}) \approx 4$, achieved when w and s are maximum, i.e., $w = n-2$, and $s = n(n+1)/2 - 2$. Note that the choice $w = d_H = n$ and $s = n(n+1)/2$, corresponding to $\boldsymbol{e} = \boldsymbol{1}$, would lead to $\delta(\boldsymbol{0}, \boldsymbol{e}) = 0$, but this undesirable case is avoided by excluding the all-zero vector from the code $\mathcal{S}$. In conclusion, we obtain

$$d^2_{\min} \approx 1 - \frac{4}{n}, \; n \gg 1. \quad (29)$$

For detection based on (3), which refers to a constant offset, the minimum squared noise distance equals

$$d^2_{\min} = 1 - \frac{1}{n}. \quad (30)$$

The minimum distance of the new method is smaller than that of the prior art, which accounts for the immunity against varying offset that we created.

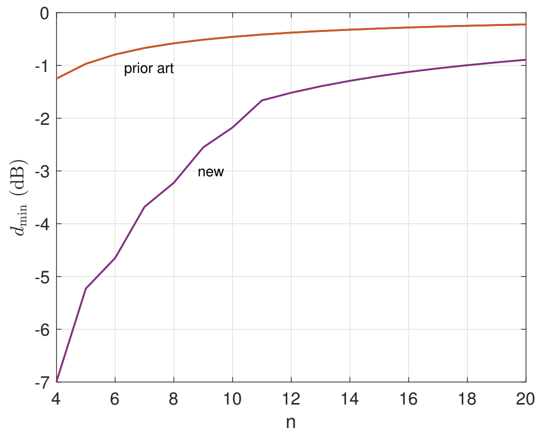

Fig. 2. Minimum noise distance, $20\log(d_{\min})$ (dB), versus codeword length n for the new scheme and the prior art using (3) as a reference.

2) Analysis of $N_{d_{\min}}$: Let $\boldsymbol{1}$ be the all-1 word, and $\boldsymbol{x}_1 = (0,\ldots,0,1)$, and $\boldsymbol{x}_2 = (1,0,\ldots,0)$. For $n > 11$, each codeword $\boldsymbol{x} \in \mathcal{S} \setminus \{\boldsymbol{1}, \boldsymbol{x}_1, \boldsymbol{x}_2\}$, has two words $\hat{\boldsymbol{x}}$ at minimum noise distance $d(\boldsymbol{x}, \hat{\boldsymbol{x}}) = d_{\min}$ that differ at the first or last position. The words $\boldsymbol{x}_1 = (0,\ldots,0,1)$ and $\boldsymbol{x}_2 = (1,0,\ldots,0)$ also have two neighbors at minimum noise distance, namely $\hat{\boldsymbol{x}} = \boldsymbol{1}$ and $\hat{\boldsymbol{x}} = (1,0,\ldots,0,1)$. The all-1 word, $\boldsymbol{x} = \boldsymbol{1}$, has four neighbors at minimum noise distance, namely $(1,0,\ldots,0)$, $(0,1,\ldots,1)$, $(0,\ldots,0,1)$, and $(1,\ldots,1,0)$, so that the average number of neighbors at minimum noise distance is $N_{d_{\min}} = 2 + 2/|\mathcal{S}| \approx 2$.

C. Results

We have conducted simulations and computations to evaluate the error performance of the new detection method. We started by comparing the word error rate of the new detection scheme as computed by two methods and by simulations. Results are shown in Figure 3, which displays the WER versus SNR = $-20\log(\sigma)$ (dB) for the union bound (23), union bound estimate (25), and computer simulations. The error performance is independent of mismatch terms b_0 and b_1. We note that the simulations agree favorably with the union bound (23); the difference between union bound and union bound estimate is large in the range of small SNR.

We also appraised the error performance of various detection schemes. Figure 1 shows results of computations (union bound), where we compare the word error rate of various scenarios of offset and detection schemes.

Figure 4 displays the word error rate versus SNR for three detection methods, namely Curve 1) Euclidean distance detection, Curve 2) modified Pearson distance detection using distance measure (3), and Curve 3) the new detection method. Results are shown for the ideal noisy channel and the mismatched channel, $b_0 = 0, b_1 = 0.025$. Without offset mismatch, both Euclidean and modified Pearson distance detection perform better than the new method. The situation changes when there is a varying offset. Then, both Euclidean and modified Pearson distance detection perform less than the new detection method. The error performance of the new

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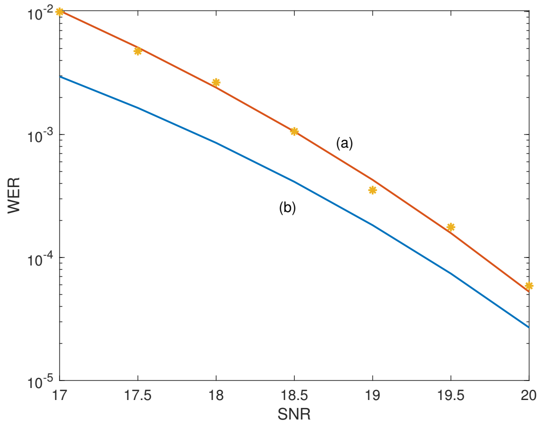

Fig. 3. Word error rate, WER versus SNR = $-20\log(\sigma)$ (dB) of the new detection method computed using (a) union bound (23), (b) union bound estimate (25), and by simulations. The points marked with '*' result from simulations. Note that Curve (a) is the same as Curve (b) in Figure 1. For all curves, $n = 12$.

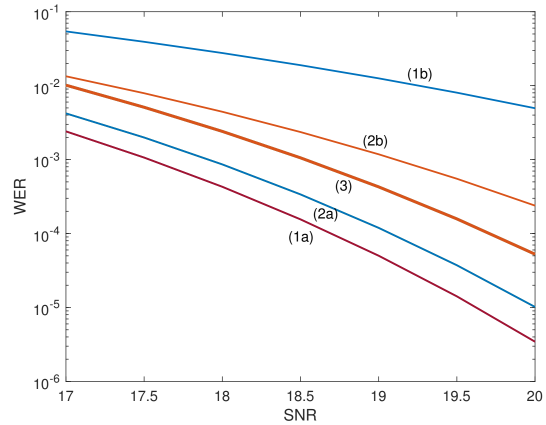

Fig. 4. Word error rate, WER versus SNR = $-20\log(\sigma)$ (dB) of (1a) Euclidean detection, no offset; (1b) Euclidean detection, $b_0 = 0, b_1 = 0.025$; (2a) Pearson detection, no offset; (2b) Pearson detection, $b_0 = 0, b_1 = 0.025$; (3) new detection method, with and without offset. For all curves, $n = 12$.

detection method is independent of the offset terms b_0 and b_1. If the channel is ideal $b_0 = b_1 = 0$, the new detection method loses error performance with respect to the prior art detection methods. If, however, the channel is mismatched $b_0, b_1 \neq 0$, we notice, see Curve (3), that the error performance is unaffected, while the alternative schemes lose performance. For all curves, we have $n = 12$. The performance curves for the ideal channel, $b_0 = b_1 = 0$, have been computed for Curve (2a) (Pearson distance detection) using, see [6, eqn. (28)],

$$\mathrm{WER_p} \approx nQ\left(\frac{1}{2\sigma}\sqrt{1-\frac{1}{n}}\right), \tag{31}$$

and for Curve (1a) (Euclidean distance detection) by

$$\mathrm{WER_e} \approx nQ\left(\frac{1}{2\sigma}\right), \tag{32}$$

where $\mathrm{WER_p}$ and $\mathrm{WER_e}$ denote the word error rate of Pearson and Euclidean distance detection, respectively. We used computer simulations for the mismatched case shown by Curves (1b) and (2b). Curve (3) showing the new method's error performance was computed using union bound (23) and confirmed by simulations.

D. Relationship with mass-centered codewords

In [9], an alternative approach has been disclosed for obtaining immunity against varying offset by using a constrained code. The constrained set, $\mathcal{S}_\mathrm{m}$, of mass-centered codewords is defined by [9]

$$\mathcal{S}_\mathrm{m} = \{\boldsymbol{x} \in \{0,1\}^n : \Omega(\boldsymbol{x}) = 0\}, \tag{33}$$

where

$$\Omega(\boldsymbol{x}) = 2\sum_{i=1}^{n}\left(i - \frac{n+1}{2}\right)x_i. \tag{34}$$

We simply rewrite (34) as, see (13) and (15),

$$\begin{aligned}\Omega(\boldsymbol{x}) &= 2\sum_{i=1}^{n} ix_i - (n+1)\sum_{i=1}^{n} x_i \\ &= n\left(2\zeta_1(\boldsymbol{x}) - (n+1)\zeta_0(\boldsymbol{x})\right) \\ &= \frac{n(n^2-1)}{6}\beta_1(\boldsymbol{x}).\end{aligned} \tag{35}$$

For mass-centered codewords, where $\Omega(\boldsymbol{x}) = 0$, we may write

$$\zeta_1(\boldsymbol{x}) = \frac{n+1}{2}\zeta_0(\boldsymbol{x}),$$

so that, using (14) and (15),

$$\beta_0(\boldsymbol{x}) = \zeta_0(\boldsymbol{x}) = \overline{x} \text{ and } \beta_1(\boldsymbol{x}) = 0. \tag{36}$$

Clearly, by selecting a set of mass-centered codewords, we are able to significantly simplify the detection routine as the term $\beta_1(\boldsymbol{x})i$ is absent in (5), at the cost of extra code redundancy. We also note that as $\beta_0(\boldsymbol{x}) = \overline{x}$, the distance measure $\delta(\boldsymbol{r}, \hat{\boldsymbol{x}})$ changes into the prior art $\delta'(\boldsymbol{r}, \hat{\boldsymbol{x}})$, see (3). The rate of a mass-centered code is not attractive for many applications, see, for example, Table 1 of [9], which shows the size of $\mathcal{S}_\mathrm{m}$ versus n.

IV. Receiver implementation complexity

The complexity of the encoder and detector/decoder can be partitioned into three major blocks, namely a) encoding arbitrary user data into a codeword in the Pearson code $\mathcal{S}$ and *vice versa*, b) computation or storage of the coefficients $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$, and c) evaluation of the distance measure (7) for all codewords in $\mathcal{S}$.

A. Encoder/decoder complexity

Systematic methods for designing Pearson codes that efficiently translate (arbitrary) source data into n-bit codewords in the codebook $\mathcal{S} = \{0,1\}^n \setminus \{\mathbf{0}\}$ and *vice versa* have been presented in [11, 12, 13]. The rate of the encoder,

$R = 1 - \frac{1}{2^{n-1}}$, presented in [13] is close to the maximum possible, and the complexity of the encoder and decoder scales linearly with n.

B. Coefficient storage

The receiver requires the coefficients $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$ for computing (5). The coefficients $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$ can be calculated 'on the fly' for each codeword $\hat{\boldsymbol{x}} \in \mathcal{S}$, but in order to save computation time, they are preferably pre-calculated and stored in a memory. The coefficients storage requires at first sight $2^n - 1$ memory cells. We may, however, save on the coefficients storage as illustrated by the following observation.

We partition the codebook set $\mathcal{S}$ in distinct subsets of codewords that have equal zeroth- and first-order moments. Let m_0 and m_1 be two positive integers. The codeword subset $\mathcal{S}_{m_0,m_1}$ with prescribed moments m_0 and m_1 is defined by

$$\mathcal{S}_{m_0,m_1} = \left\{ \boldsymbol{x} \in \mathcal{S} : \sum_{i=1}^{n} x_i = m_0 \wedge \sum_{i=1}^{n} i x_i = m_1 \right\}. \quad (37)$$

Let $N(n)$ denote the number of distinct nonempty subsets $\mathcal{S}_{m_0,m_1}$.

Theorem 1: The number of distinct nonempty subsets $\mathcal{S}_{m_0,m_1}$ is upper bounded by

$$N(n) \leq \frac{n(n^2+5)}{6}. \quad (38)$$

Proof: Let $q = \sum_{i=1}^{n} x_i = m_0$ denote the number of 1's in $\boldsymbol{x}$ (weight), then

$$m_1 = \sum_{i=1}^{n} i x_i \in \{q_1, q_1+1, q_1+2, \ldots, q_2\},$$

where $q_1 = q(q+1)/2$ (all q 1's at the beginning of $\boldsymbol{x}$) and $q_2 = nq - q(q-1)/2$ (all q 1's at the end of $\boldsymbol{x}$). Then the number of distinct values of m_1 for a given m_0 equals $q_2 - q_1 + 1 = nq - q^2 + 1$. The number of distinct pairs (m_0, m_1) is

$$\sum_{q=1}^{n} (nq - q^2 + 1) = \frac{n(n^2+5)}{6},$$

which proves the theorem. ■

We conclude that the number of distinct coefficient pairs $\beta_0(\hat{\boldsymbol{x}})$ and $\beta_1(\hat{\boldsymbol{x}})$ equals $N(n)$, so that the decoder storage complexity grows polynomially, $\propto n^3$, with the codeword length n. If the size of the subset $\mathcal{S}_{m_0,m_1}$ is small, this does not provide great solace, but for larger subset sizes it may offer an attractive saving in coefficient storage.

C. Time complexity

For evaluating (7), the decoder requires $|\mathcal{S}| = 2^n - 1$ computations of $\delta(\boldsymbol{r}, \hat{\boldsymbol{x}})$ plus comparisons, which makes the new method unattractive for very large n. It is shown in [6] that the (time) complexity of the prior art method based on (3) can be reduced to n computations and comparisons using Slepian's method [14]. This significant reduction in time-complexity is possible as $\mathcal{S}$ consists of n permutation codes. We cannot apply Slepian's method here as the subset $\mathcal{S}_{m_0,m_1}$ is not a simple permutation code. Although the storage requirements of the precalculated coefficients can be reduced as shown above, the evaluation of (7) requires $2^n - 1$ computations and comparisons per decoded codeword.

V. Conclusions

We have presented a new detection method for noisy channels with an offset of monotonically increasing time-varying magnitude. The error performance of the new detection method is independent of both unknown base offset and offset's slew rate. The rate of the requisite constrained code is very high as only one codeword has to be barred. Computer simulations have been conducted to appraise the error performance of the new detection method. The simulations compare favorably with theory based on the union bound. For large signal-to-noise ratios, the new method requires, for a codeword length $n \geq 12$, less than 1.5 dB more noise margin than Euclidean distance detection or less than 1 dB loss with respect to Pearson detection. The relationship with constrained codes based on mass-centered codewords and the new detection method has been discussed.

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