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Review

A Review of Mechanical Design Approaches for Balanced Robotic Manipulation

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Abstract

Robot manipulators are suitable for many industrial tasks, such as assembly and pick-and-place operations. However, high-acceleration motions result in shaking forces and moments to the base, which can cause vibration of the manipulator and instability in the case of a mobile base. Furthermore, gravity compensation of the manipulator links requires additional motor torque, which can increase energy consumption. Balanced manipulators address these problems by employing a mechanical design that results in the balancing of gravity and other static forces, or the removal of shaking forces and/or moments. This review paper provides an overview of mechanical design approaches for balanced robotic manipulation, with an emphasis on experimentally prototyped designs. We first define the types of balancing according to the literature. We then provide an overview of different approaches to the mechanical design of balanced manipulators, along with simple examples of their implementation. Experimental prototypes in this field are then comprehensively presented and summarized to allow readers to compare their development maturity. At the end of the paper, we outline challenges and future directions of research.

Keywords: static balancing; dynamic balancing; force balancing; robot manipulators; mechanical design

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1. Introduction

Robotic manipulation has been an active field of research for many decades. These manipulators are used in a variety of industrial tasks, such as inspection, assembly, pick and place, etc. They are attached to rigid bases mounted on the ground or to mobile robots [1]. Ground-fixed manipulators are common in industrial automation applications such as manufacturing and warehousing [2]. Mobile manipulators, consisting of a moving base and a manipulator, operate in a variety of environments [3], such as ground terrain (wheeled robots), underwater (autonomous underwater vehicles) [4], air (unmanned aerial vehicles) [5–7], or space (satellites) [8]. The manipulator provides the robot with the ability to interact with the environment through a sensor or actuator, called a payload, mounted at the end of the manipulator's kinematic chain, called the end-effector. Examples of payloads include a camera for inspection or a gripper for pick and place and assembly tasks.

In gravitational environments, the manipulator exerts static forces and moments on the base due to the gravitational force acting on its links [9]. Furthermore, during motion, the manipulator exerts additional inertial forces and moments on the base, termed shaking forces and shaking moments [10]. For fast accelerations on a ground-fixed base, these can induce vibrations [11]. For mobile manipulators, the base undergoes unwanted disturbances that move the base and have to be compensated for through control of the base while executing the manipulation task. These result in a lack of precision, low dexterity, or even instability of the base.

A solution to this problem is to design balanced robotic manipulators for which the static forces and moments and/or shaking forces and moments are mitigated or negligible. The mechanical design of these manipulators facilitates different levels of balancing behaviors, from compensating the gravity force to being fully reactionless for any motion. By reducing the shaking forces and moments, we reduce the disturbances and vibrations on the base, thereby increasing precision and reducing fatigue.

A related problem is to design statically balanced manipulators, where the static forces, particularly gravity, are balanced through mechanical design. A statically balanced robot manipulator has lower gravity compensation torques, which increases the maximum allowable payload at the end-effector under static conditions [12]. Approaches that address shaking force and shaking moment balancing often result in static balance as well.

In the 19th century, Otto Fischer studied the shaking force balancing of Crank-Slider mechanisms and developed the method of principal vectors for calculating the inverse dynamics of a human in motion, which was also relevant for balancing [13]. The method was mentioned by Berkhof and Lowen in 1968 [14], who then began a detailed study on various approaches for static and shaking force/moment balancing [15,16].

In the period from 1970 to now, research on balanced mechanisms has been further developed by Agrawal [17,18], Arakelian [19], Gosselin [9], Kochev [20], van der Wijk [21], and others. The focus during this period has been on developing theoretical tools to derive the mathematical relationships that result in balance, which are known as the balance conditions. These tools were then employed to analyze novel balanced mechanism design approaches. In the past decade, some of these concepts have also been validated through experimental prototypes applied in real-world scenarios.

Whereas the theoretical developments in balanced mechanisms began in the 19th century, the practical applications in robot manipulation have only seen adoption in the last three decades after the advent of industrial robotics, with a notable uptake in the last 15 years. Previous review papers provide a high-level overview [22], an in-depth examination [10,19,20], or a comprehensive analysis [23] of theoretical approaches with numerical examples. However, a review of practical prototypes built and tested experimentally in this field, along with the developments since the last review by Wei and Zhang in 2020 [22], is missing. The goal of this review paper is to provide an overview of gravity, static, shaking-force, and shaking-moment balancing approaches relevant to robotic manipulation, and provide a concise yet comprehensive review of real-world prototypes and their experimental results.

In addition to extensive theoretical and experimental work on creating balanced rigid body manipulators, some preliminary research has also been conducted on the balance of flexible bodies [24–27]. In particular, the reduction in vibration modes through balancing holds many advantages for high payload-to-weight ratio tasks.

Force and moment balancing for robot manipulation can also be achieved using control and motion planning-based approaches, applied in space manipulation applications [28,29] and legged robot locomotion [30]. A hybrid approach applying control-based balancing for soft robotics links for legged robotic links was also investigated [31,32], which is outside the scope of robot manipulation. These approaches constitute a separate field of research employing fundamentally different mathematical tools and optimization goals than those used in mechanical design. For that reason, we will exclude control-based approaches from this review.

The structure of the paper is as follows: Section 2 outlines the types of balancing with their definitions. The approaches used to achieve various balancing types are summarized with relevant literature in Section 3. Experimental prototypes are categorized and elaborated in detail in Section 4. Based on our analysis of the development of the field so far, we provide an assessment of challenges and future directions for this field in Section "Challenges and Future Work". The conclusion (Section 5) summarizes our review findings.

2. Types of Balancing

We define a robot manipulator as a mechanism consisting of rigid links connected by joints that allow motion, attached to a ground-fixed or mobile base with one or more fixed joints for attachment. The combination of spatial geometry and joint type determines the degrees of freedom (DOF) of their motion. The most common joints are revolute (1-DOF rotational motion) and prismatic (1-DOF translational). To facilitate spatial (3D) motion, spherical and universal passive joints that allow motion with multiple DOF are also used. The manipulator carries out the intended task by moving the payload at the end-effector (a sensor or actuated mechanism such as a gripper) attached at the end of a kinematic chain. The manipulator can be serial, consisting of a single open-chain of links, or parallel, consisting of one or more closed chains. Simple 2D examples of both types of robotic manipulator platforms are shown in Figure 1.

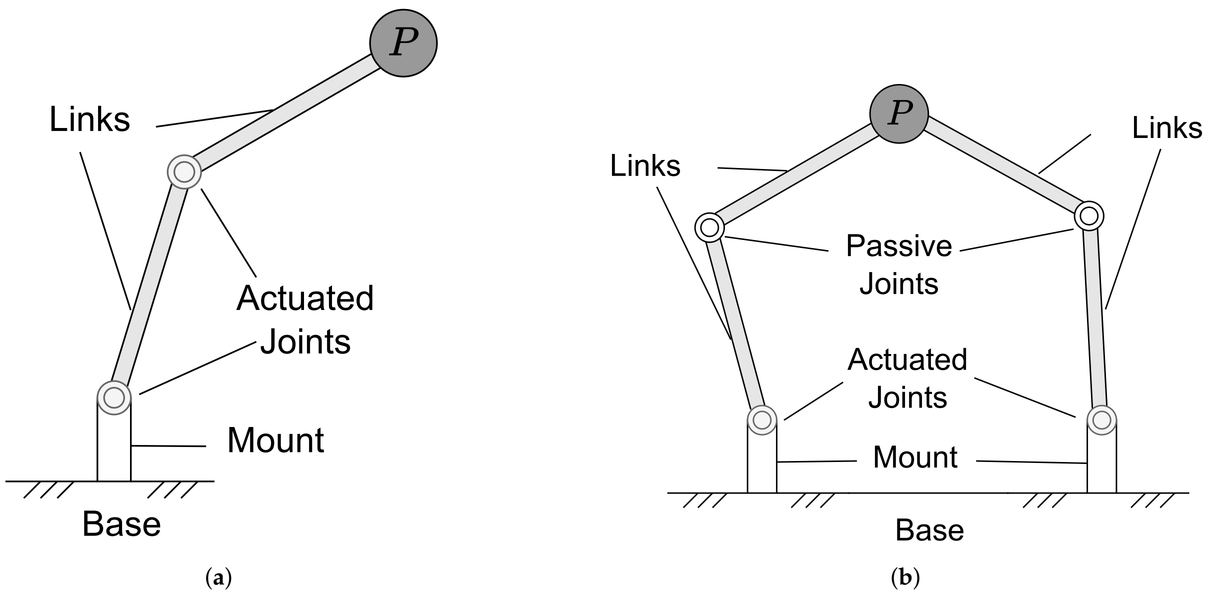


Figure 1. Two-DOF planar robotic manipulator examples: (**a**) serial, (**b**) parallel. These consist of links, connected by joints that allow motion, attached to a base (ground-fixed or mobile). P is the payload at the end-effector that is manipulated in order to fulfill the manipulation task. The robot moves the actuated joints and manipulates the payload through the mechanism geometry and joint kinematics.

As a manipulator moves, it exerts forces and moments on the base. These originate from (1) gravity, (2) motion of the links (acceleration, centrifugal, and Coriolis components), and (3) external forces/moments, such as actuators, contact forces/moments, loads, and support constraint forces.

As an example, these are illustrated for a serial planar manipulator in Figure 2. There are two reaction forces and one reaction moment for two DOF motion, which counteract the forces and moments generated by the manipulator. For a spatial manipulator, there are three reaction forces and three reaction moments, corresponding to six DOF motion.

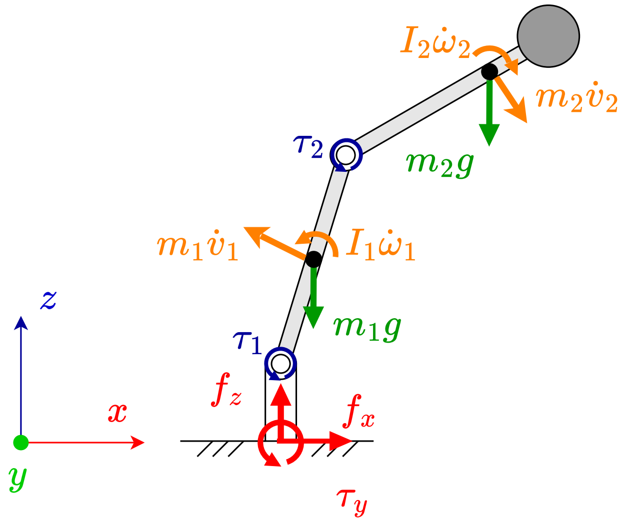


Figure 2. Reaction forces and moments exerted by a 2-DOF serial manipulator. The world frame axes are shown on the left. Gravity acts in the negative z direction, and acts at the center of mass of the links as f_{g_1} and f_{g_2}. Reaction forces applied on the base are f_x and f_z, and the reaction moment is τ_y. The manipulator has gravity forces (in green), actuated joint torques τ_1 and τ_2, and inertial forces/moments (in yellow).

We first define the kinematics of the robotic manipulator using the Denavit–Hartenberg convention. It consists of n moving bodies (links) attached to joints, both indexed as i. Frame O_B is the base-fixed reference frame of the manipulator, and each frame O_i is fixed to link i with its origin at the parent joint. Each body has mass $m_i \in \mathbb{R}$, inertia $\boldsymbol{I}_i \in \mathbb{R}^{3\times3}$, and center of mass at $\boldsymbol{r}_i \in \mathbb{R}^3$ with respect to O_B. The system is described in terms of generalized coordinates $\boldsymbol{\theta} = \begin{bmatrix}\theta_1 & \theta_2 & \dots & \theta_n\end{bmatrix}^\mathsf{T}$, which are determined by the joint DOF. We also denote as $\boldsymbol{r}_C \in \mathbb{R}^3$ the position of the center of mass of the system. A simple illustrative example of a 2-DOF open-chain planar robotic manipulator composed of revolute joints is shown in Figure 3.

For a closed-chain manipulator, the motion of its links is constrained, and additional closed-loop kinematic equations are needed to fully define the end-effector motion. Using this approach can often remove redundant generalized coordinates.

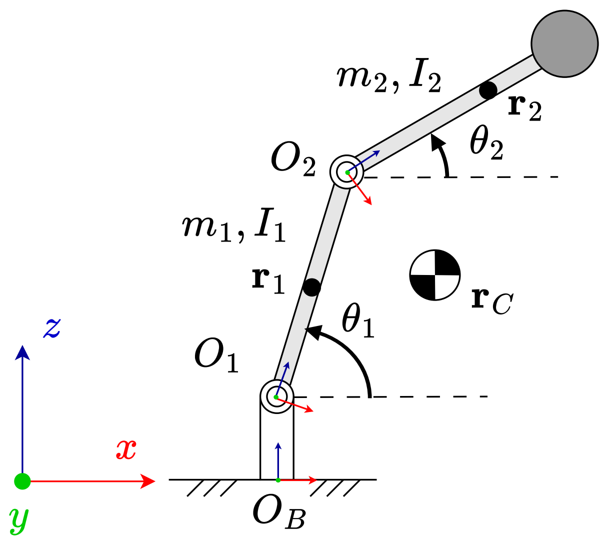


Figure 3. The kinematics of a simple 2-DOF open chain, fully actuated manipulator. The world frame axes is shown as colored on left, and the pose of frames O_1 and O_2 are also shown with the same colors. The center of mass of the manipulator is the checkered circle and its position vector is $\boldsymbol{r}_C$.

According to [9], we identify the following types of balancing: (1) gravity, (2) static, (3) shaking force (or force), (4) shaking moment (or moment), and (5) dynamic (both shaking force and shaking moment). A moment-balanced mechanism also has force balance properties, as shown in Section 2.3. We provide mathematical definitions from the literature for these types of balancing in the following sections.

2.1. Static and Gravity Balancing

A mechanism is defined as statically balanced if the static forces (e.g., the gravity force from the weight of the links and forces exerted by energy-conserving components) are balanced so that the actuators do not need to apply torque to maintain the manipulator in its equilibrium configuration [9,33]. The utility of such balancing can be observed in Figure 2, where an unbalanced manipulator requires actuation torques to maintain the manipulator in this configuration.

A definition is provided from [17] in terms of the potential energy of the system, which remains constant $\bar{T}$ for all possible configurations $\boldsymbol{\theta}$, where $T_g(\boldsymbol{\theta})$ is the total gravitational potential energy and $T_p(\boldsymbol{\theta})$ is the potential energy stored by other mechanical elements (e.g., springs):

$$\bar{T} = T_p(\boldsymbol{\theta}) + T_g(\boldsymbol{\theta}) \tag{1}$$

Static balancing purely for the gravity force is called gravity balancing or gravity compensation. In (2), where f_g is the total gravity force acting on all the system bodies, f_p is the total gravity-aligned (z-axis) force from potential conserving mechanical elements (e.g., springs or coupled counter masses), and f_z is the reaction force applied by the base:

$$f_z + f_g + f_p = 0. \tag{2}$$

2.2. Force Balancing

A mechanism is defined as shaking-force balanced if the net reaction force exerted on the base is constant for any motion of the links [9], which occurs when the net forces about the center of mass of the manipulator are zero.

The linear momentum $\boldsymbol{L} \in \mathbb{R}^3$ of the system is the summation of linear momenta of n bodies with index i, each with mass m_i and velocity vector $\boldsymbol{v}_i = \dot{\boldsymbol{r}}_i$, with respect to the balance position (manipulator center of mass). In a balanced mechanism, it remains constant for any velocity within the range of admissible motions of the manipulator:

$$\boldsymbol{L} = \sum_{i=1}^{n} m_i \boldsymbol{v}_i = \bar{\boldsymbol{L}}. \tag{3}$$

The force balance condition (3) can be interpreted in two other ways:

1. As the change in linear momentum is caused by a net force on the manipulator $\boldsymbol{f} \in \mathbb{R}^3$, these are zero about the center of mass for any configuration:

$$\frac{d}{dt}\boldsymbol{L} = \boldsymbol{f} = 0. \tag{4}$$

2. The linear momentum of the system can also be expressed in terms of the velocity of the manipulator center of mass, where m_t is the total mass:

$$\boldsymbol{L} = \sum_{i=1}^{n} m_i \boldsymbol{v}_i = m_t \dot{\boldsymbol{r}}_C \tag{5}$$

As m_t is constant, this means that in a force-balanced system $\dot{\boldsymbol{r}}_C = 0$, hence the center of mass $\boldsymbol{r}_C$ remains static at fixed position $\bar{\boldsymbol{r}}_C$:

$$\boldsymbol{r} = \frac{1}{M}\sum_{i=1}^{n} m_i \boldsymbol{r}_i = \bar{\boldsymbol{r}}_C. \tag{6}$$

Note that for solving the force balance conditions, it is more useful to express $\boldsymbol{r}_i$ and $\boldsymbol{r}_C$ as functions of the generalized coordinates $\boldsymbol{\theta}$.

2.3. Moment Balancing

For a system to be shaking moment balanced, the angular momentum $\boldsymbol{K} \in \mathbb{R}^3$ of the manipulator about the balance point (the center of mass) must remain constant for the range of admissible motions:

$$\boldsymbol{K} = \sum_{i=1}^{n} \boldsymbol{I}_i \boldsymbol{\omega}_i = \bar{\boldsymbol{K}}, \tag{7}$$

where $\boldsymbol{I}_i \in \mathbb{R}^{3\times 3}$ and $\boldsymbol{\omega}_i \in \mathbb{R}^3$ are the inertia and angular velocity of body i with respect to the center of mass.

This also implies that the change in angular momentum, which is caused by net moments on the manipulator $\boldsymbol{\tau} \in \mathbb{R}^3$, with respect to the balance point, is zero:

$$\frac{d}{dt}\boldsymbol{K} = \boldsymbol{\tau} = 0 \tag{8}$$

A body experiencing linear motion perpendicular to a rotational axis also has angular momentum with respect to that axis. The relationship between the linear momentum of a body $\boldsymbol{L}_i$ and its corresponding angular momentum component $\boldsymbol{K}_i$ is expressed as the following, where $\boldsymbol{r}_i$ is the position vector in the fixed-frame coordinates:

$$\boldsymbol{K}_i = \boldsymbol{r}_i \times \boldsymbol{L}_i = m_i(\boldsymbol{r}_i \times \boldsymbol{v}_i) \tag{9}$$

Furthermore, moments are also caused by static forces acting perpendicular to the rotational axis, which contribute to the reaction moments. Therefore, a moment-balanced system is also force-balanced, as zero net force about the center of mass is a necessary condition for zero net moments.

3. Approaches to Design Balanced Manipulators

Different types of balancing can be achieved by modifying the design of a manipulator by adding components that introduce balancing in the system's dynamic behavior. These components are added at selected places in the manipulator in order to balance the system. The required parameters of these components are determined through resolving the balance conditions of the dynamic equations with the added components. In choosing the type and position of these components, there is often a tradeoff between mass/inertial addition and space.

We categorize approaches to designing balanced manipulators into three broad categories: *Passive Component Addition*, *Active Component Addition*, and *Inherently Balanced Design*. In addition to this, the *Synthesis* approach combines the aforementioned and/or control-based approaches. The definition of each approach and the sub-approaches within these categories are explained in the following subsections, accompanied by simple illustrative examples.

3.1. Passive Component Addition

In passive component addition, non-actuated elements are added to the mechanism, resulting in static, force, or moment balancing. These elements do not require any input forces or torques, and their motions are coupled with the kinematics of the original unbalanced mechanism. Therefore, the total energy and momentum of the system remain constant, depending on the type of balance desired. Several types of balancing components are used to achieve different types of balance, which are summarized below.

3.1.1. Springs

Springs can be used to achieve gravity and static balancing, as they store potential energy to keep the mechanism in an equilibrium configuration [33]. The total potential energy in the system remains constant; however, the springs only balance the static forces, such as gravity and other springs. Joint torques, as well as the dynamic forces and moments from linear and rotational acceleration, still result in shaking forces and moments; thus, a spring-balanced manipulator cannot be force- or moment-balanced.

Balancing the gravity force with springs also reduces the joint torques through gravity compensation at the equilibrium position, which increases the maximum allowable payload and reduces energy consumption. For this reason, springs have been extensively demonstrated to be an effective balancing method for manipulators, particularly where the goal is to minimize energy consumption with minimal mass additions.

A simple example of a 1-DOF manipulator consisting of a link balanced by a single linear spring is shown in Figure 4. We assume that deflections in the x-axis are negligible, such that the spring force only acts in the z-axis, in order to simplify the balance equations.

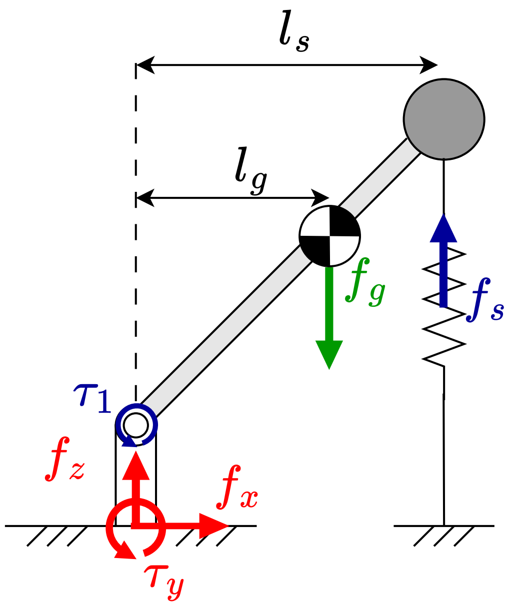


Figure 4. A 1-DOF manipulator with spring attached to the base. The spring is compressed along the z-axis (with horizontal component negligible) at equilibrium and its force counteracts the gravity of the link.

If we look at the moment balance about the joint origin, we can see that there is an equilibrium for which $\tau_1 = 0$ and $\theta_1 \neq 0$, which is the first equation for static balance, where f_g is the gravity force and f_s is the spring force:

$$f_s l_s - f_g l_g = 0. \tag{10}$$

The static balance equation in the gravity axis for the example shown is:

$$f_z + f_s - f_g = f_z - kd - f_g = 0, \tag{11}$$

where we expand the spring force $f_s = -kd$ according to the relationship between the spring stiffness k and the spring extension or compression along the positive z-axis d.

At equilibrium, the spring is compressed ($d < 0$); hence $-kd > 0$, and the gravity force is balanced by both the spring and reaction forces. A torque τ_1 is required to rotate the link by providing rotational acceleration as well as counteracting the spring or gravity force away from the equilibrium configuration, which is why the spring-balanced system only results in static, and not force or moment balance.

Spring additions for static and gravity balancing of manipulators can reach significant levels of complexity. Static balancing using springs has been investigated for multiple DOF serial [34], parallel planar [35], and spatial mechanisms [36,37]. A zero free length spring can also be created through a cable and pulley arrangement, and is often used for static balancing [38,39].

A variety of configurations—such as spring-linkage combinations, as well as pulley, cross-mechanism, and hydro-pneumatic springs for torque compensation—were investigated for the static balancing of articulated robots in [40]. Torsional springs are also an effective and space-efficient method of balancing serial manipulators [41,42]. Gear-spring modules were designed in [43] for static balancing of a Delta-robot, where the theoretical peak torques are reduced between 38.4 and 55.4% within 86% of the original kinematic workspace. A patent [44] also describes an internal spring-based weight-balancing device for a robotic arm link.

Methods for calculating optimal spring configurations have also been widely investigated, for example, by analyzing the stiffness matrix [45,46], genetic optimization [34], and eigenvalue analysis of the joint-space potential energy function [47]. Several experimental prototypes employing springs for gravity and static balancing are summarized in Section 4.

3.1.2. Counter Mass (CM)/Counter Inertia (CI)

Masses can be added to modify the center of mass motion. By balancing individual or groups of links with counter masses, the center of mass of the mechanism can be fixed to a static position or constrained to motion along an axis; for example, along the gravity axis in the case of gravity balancing, or fixed at a point for all motions, thereby achieving force balance.

Whereas balancing links with revolute joints using counter masses is simple due to the coupled motion of the link and counter mass (CM) [19], more complex solutions, such as a negatively geared rack and pinion or Scott–Russell mechanisms, are required for the passive balancing of prismatic joints [48].

A simple 1-DOF manipulator consisting of a revolute joint, force-balanced with a single CM, is illustrated in Figure 5. The original link has mass m, and its center of mass is at length l along the joint-frame x axis (shown by the red arrow). A CM is positioned opposite, with the link extended in the opposite direction (counter-link). The mass of the counter link extension with counter mass is m^*, and its center of mass is at $-l^*$. For the mechanism to force balance about the joint origin, the relationship between the link and counter-link kinematic parameters is [49]:

$$m^* l^* = ml. \tag{12}$$

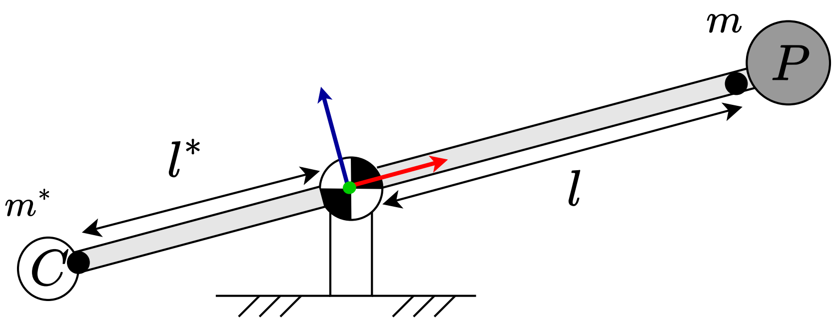


Figure 5. A 1-DOF manipulator where the link and payload (P) are balanced with a counter-mass. The link is extended behind and the counter mass (C) is attached, so that the center of mass (checkered circle) is positioned directly at the joint origin (colored axes).

There is a linear tradeoff between the proportions of $m^* : m$ and $l^* : l$, i.e., to achieve minimal mass increase, the length of the counter mass must be increased. For example, in (12), we can select $m^* = m/2$ and $l^* = 2l$, which achieves force balance with a 50% increase in total mass.

The most common application of CM is to force balance low DOF serial [50] or parallel mechanisms [35]. CMs can also be applied to spatial parallel manipulators to achieve force balancing, with effective exploitation of symmetries [51]. In [52,53], counter masses are added to force balance the arms of a spatial three DOF robot manipulator (see Section 4.3 for more details).

3.1.3. Counter Rotating Inertia (CR)/Counter Rotating Counter Mass (CRCM)

Counter Rotating Inertias or simple Counter Rotations (CR) add moments to the system in order to counteract the rotational imbalance [54]. They are compact; however, they can also result in significant mass addition. An additional inertial element is negatively geared to the manipulator joint and cancels the moments caused by the rotational motion of the links [55]. For serial and many types of closed-chain manipulators, moment balancing cannot be achieved through CM/CI alone, and CR is required [19,56,57]. However, the implementation of CR can be complex in practice, as mechanical transmissions, such as gear trains, are required to couple their motion to the manipulator.

An extension to the earlier 1-DOF planar CM example (Figure 5) is shown in Figure 6, where an additional CR is added to moment balance the manipulator. The link with the counter mass has combined planar inertia $I \in \mathbb{R}$ about the center of mass (also the joint origin). A CR, geared to the revolute joint with a constant transmission ratio k, rotates in the opposite direction with angular velocity $\dot{\theta}^* = k\dot{\theta}$ and has planar inertia $I^* \in \mathbb{R}$ about its centroid. Using the conservation of angular momentum, the relationship between the CR and link motions for the balanced manipulator is as follows [55]:

$$I\dot{\theta} - I^* k\dot{\theta} = (I - kI^*)\dot{\theta} = 0. \qquad (13)$$

Counter rotations can also be integrated with counter masses to form a single unit called a Counter Rotating Counter Mass (CRCM), which is more compact, requires a lower mass/inertia addition, and can achieve both force and moment balance together [58]. A simple 1-DOF manipulator balanced with a CRCM is illustrated in Figure 7. The planar inertia of the CRCM $I^* \in \mathbb{R}$ (measured about the axis of the joint connecting the CRCM to the main link) includes both a mass component, which assists in the force balance of the link, as well as its own inertial contribution from the material that is considered in the moment balance Equation (13).

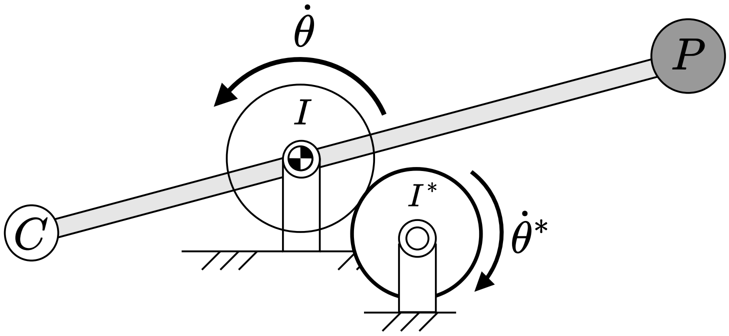


Figure 6. A 1-DOF manipulator balanced with a CM (to balance the CoM about the joint) and separate CR (to counter the joint actuation torques). The CoM of the link and CM is at the joint origin (checkered circle).

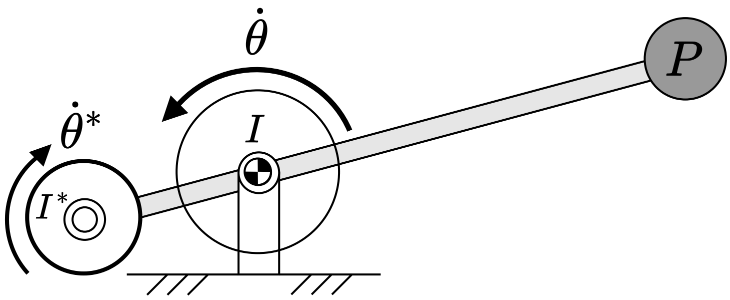


Figure 7. A 1-DOF manipulator balanced with a CRCM. It is attached to the link and the inertial element, negatively coupled to θ with a constant transmission ratio. Combined with the counter mass, it has inertia I^* about the joint origin (also center of mass), indicated as the checkered circle. The CRCM rotates with velocity $\dot{\theta}^*$ relative to the manipulator velocity $\dot{\theta}$.

3.1.4. Additional Linkage Mechanism (ALM)

Mechanisms consisting of links and moving masses are added to keep the center of mass of the system at a fixed point during motion. Examples include pantographs [35], idler loops, duplicate mechanisms [55], or counter mechanisms consisting of linkages and CR elements [59].

An example of a duplicate mechanism for the dynamic balancing of a 1-DOF manipulator is shown in Figure 8. The mechanism is mirrored both vertically and horizontally. The center of mass is in the center of the manipulator, and I is the inertia of each component about it. The total mass and inertia of the system are increased to four times the original.

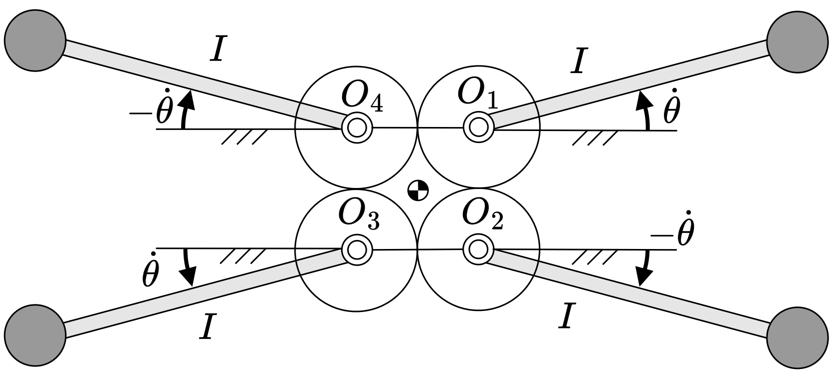


Figure 8. A 1-DOF manipulator balanced using duplicate mechanisms. Four copies are needed for dynamic balancing, geared to the original joint O_1. O_2, O_3, O_4 are the origins of the duplicate links. The center of mass (checkered circle) is in the center of all 4 mechanisms.

Van der Wijk et al. compare the various configurations of CR, CRCM, and ALMs in terms of the tradeoff between mass and space addition to achieve dynamically balance for 1 and [54] 2 DOF [55] planar serial manipulators. Theoretical derivations and simulated results show that ALM-based approaches result in the lowest mass addition; however, this comes with a quadrupling of the space required. CRCM achieves dynamic balance for a larger mass addition, but without a significant increase in space. With optimized transmission and inertial shape, ALM can be matched or improved.

Laliberte and Gosselin [59] built and tested prototypes of several ALM arrangements for a 2-DOF manipulator and demonstrated the reduction in reaction moments compared to their unbalanced equivalents (see Section 4.4).

3.2. Active Component Addition

This approach for balancing requires adding an actuated element to the manipulator that actively generates forces and/or torques to cancel out the inertial forces and moments generated by the mechanism during motion.

3.2.1. Active CM/CR/CRCM

This is similar to a passive CM or CRCM, but driven by an actuation force and/or torque. They allow for the decoupling of joint and CM/CR motions, which facilitates more compact sizing and convenient positioning. Active CM has been proposed for both static and force balancing [60], depending on the control method and maximum extension of the component. Active CRCM can be used to consolidate multiple Counter Rotations into one, in order to balance an entire mechanism without requiring complex gear mechanisms [61].

In most designs, a moving mass is actuated with an actuated prismatic joint (motor and slider) to keep the center of mass offset to a minimum along a single axis, which is a form of gravity or static balancing. A patent was filed in [62] describing an intelligent adaptive active balancing CM for gravity balancing. The authors of [6,63] use the drone battery as an actively controlled CM for statically balancing an aerial manipulator. Other experimental prototypes that move the battery or other counter masses for the force balance of serial manipulators are presented in [64–66]. These are described in more detail in Section 4.3.

A bio-inspired tail design that uses aerodynamic drag to balance a robot mechanism was developed in [67], and was shown to be more effective than reaction wheels (a form of Active CR) in balancing a robotic mechanism.

3.2.2. Active Dynamic Balancing Unit (ADBU)

These integrate Active CM and CR in all axes to provide full six-DOF balancing in the form of a single actively driven unit. They are highly compact and particularly effective in multiple DOF parallel or kinematically redundant spatial manipulators, where it is infeasible to achieve complete moment balance through the addition of passive elements alone. In [68], a mechatronic design of an ADBU is presented, which was proposed earlier in [69] as a method to dynamic balance a Delta robot that is already statically balanced with counter masses.

Active balancing in the form of a redundant drive was developed and experimented with as a method of improving the dynamic balance of a flexible four-bar linkage [27].

3.3. Inherent Balancing

For many types of manipulator linkage arrangements, the whole manipulator can be dynamically balanced in the design stages through the derivation of the balancing conditions. These are the required relationships between link geometries, mass, and inertia that result in the desired type of balance. The required mass and inertial parameters of each link in the balanced manipulator are achieved through modifying the link shape [70,71] or mass/inertial additions.

Typically, the balance conditions are found through momentum balance equations. We can express (3) as a function of the Mass Matrix $\boldsymbol{M}(\boldsymbol{\theta})$ and generalized joint velocities $\dot{\boldsymbol{\theta}}$:

$$\boldsymbol{L} = \boldsymbol{M}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}. \tag{14}$$

From (3), we can see that $\boldsymbol{L} = 0$ when $\boldsymbol{M}(\boldsymbol{\theta}) = \boldsymbol{0}$ for all admissible $\dot{\boldsymbol{\theta}}$. This is typically done by setting all the coefficients of $\boldsymbol{\theta}$ variables in $\boldsymbol{M}(\boldsymbol{\theta})$ to 0. The derived equations are known as the force balance conditions. Alternative methods of finding force balance conditions are to solve for the center of mass motion (6) or zero net force (4).

Similarly, we can express the angular momentum (7) as a function of $\dot{\boldsymbol{\theta}}$ and mass-inertial matrix $\boldsymbol{I}(\boldsymbol{\theta})$:

$$\boldsymbol{K} = \boldsymbol{I}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}. \tag{15}$$

From (7), we can see that setting $\boldsymbol{I}(\boldsymbol{\theta}) = \boldsymbol{0}$ for all admissible $\dot{\boldsymbol{\theta}}$ results in $\boldsymbol{K} = 0$. The common approach is the same as for the force balance conditions, which is to set all the coefficients of $\boldsymbol{\theta}$ in $\boldsymbol{I}(\boldsymbol{\theta})$ to 0 [15].

Note that for inherently balanced designs, the moment balance conditions also fulfill the force balance conditions, as explained in Section 2.3. Hence, these are typically force or dynamically balanced.

Typically, (14) and (15) are formulated through the calculation of open chains for all the linkages, followed by the substitution of closed-loop constraint equations to remove redundant joint coordinates [21]. Alternative methods to using momentum equations include algebraic formulations based on complex numbers [72] or screw theory [73].

Some approaches based on balancing established kinematic designs are summarized in the following sub-sections.

3.3.1. Parallel Mechanisms

Mechanisms consisting of four-, five-, and six-bar closed-chain linkages have been extensively explored in the literature. The force balance conditions for a four-bar linkage were first explored by Berkof and Lowen [15] and subsequently expanded to moment balance in [16]. These conditions were further analyzed to determine feasible designs in [74–76]. A theoretical example of a 1-DOF force-balanced four-bar linkage, as presented by [75], is shown in Figure 9. The detailed design of the linkage profile itself, using topology optimization to achieve dynamic balancing, is presented in [77,78].

The force and moment balance conditions of 2–4 closed chains of three successive serial revolute joints (RRR) were derived by van der Wijk [21]. A method of determining a partially dynamically balanced five-bar linkage was also analyzed using optimization methods for zero reaction torques [79]. An analysis of the counter mass design considerations for force and moment balancing was presented by Arakelian [80].

Most of the mechanisms outlined above have only been balanced in the case of planar systems, and building spatial systems requires synthesizing planar mechanisms into a spatial arrangement, which is further elaborated on in Section 3.4. An approach that designs dynamically balanced mechanisms in 3D using Parallel-Pipeds and finding spatial balance conditions has been explored by Gosselin and Wu [81,82]. These designs typically require very large mass additions and are only feasible for ground-based manipulators.

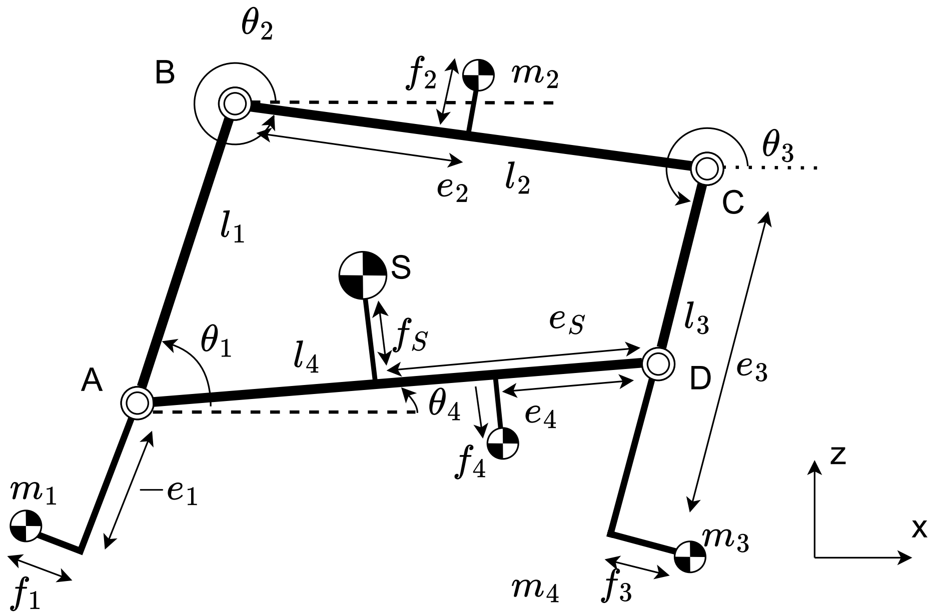


Figure 9. A force-balanced 1-DOF four-bar linkage, where the centers of masses of the links are indicated with e_i and f_i. The center of mass S is static with respect to the fixed bar AD. The linear momentum is zero for all admissible configurations and joint velocities.

The dynamic balance of flexible four-bar linkages is a topic of interest, as four-bar linkages are a common closed-chain mechanism used for industrial automation purposes. In [83], the dynamic equations are modeled using cubic spline functions for beam discretization. The authors found that the counter masses/inertias attached to inherently balance such a mechanism are the dominant factor in vibrations.

Kalas [84] investigated the dynamic balance of a five-bar linkage with a single compliant flexible link by modeling the links as pseudo-rigid. Meijaard and van der Wijk [26] presented two approaches to the dynamic balance of flexible link mechanisms: (1) the similarity of a mechanism and a balancing mechanism, and (2) modal balancing, where the lowest-order linear vibration modes suppress the shaking forces and moments, according to the material properties of the link structure. These can be classified under inherent balancing, as the combined dynamic behavior of the system is optimized for dynamic balance, which now includes the material properties of the links. According to their analysis, for flexible mechanisms, there is a trade-off between force balance and moment balance.

3.3.2. Principal Vector Method

A simplification of the equations of motion for a complex mechanism consisting of multiple links aligned with a set of principal axes was derived by Otto Fischer for biomechanics [13], called the Principal Vector method. The Fischer method reformulates the motion of the centers of mass of linkages along coordinate bases aligned with their principal axes. This was later generalized by van der Wijk to multiple closed loops [85,86]. It has been extensively applied to the design of force and moment balanced mechanisms [21]. An example of a 4-DOF mechanism developed from the proposed designs is shown in Figure 10.

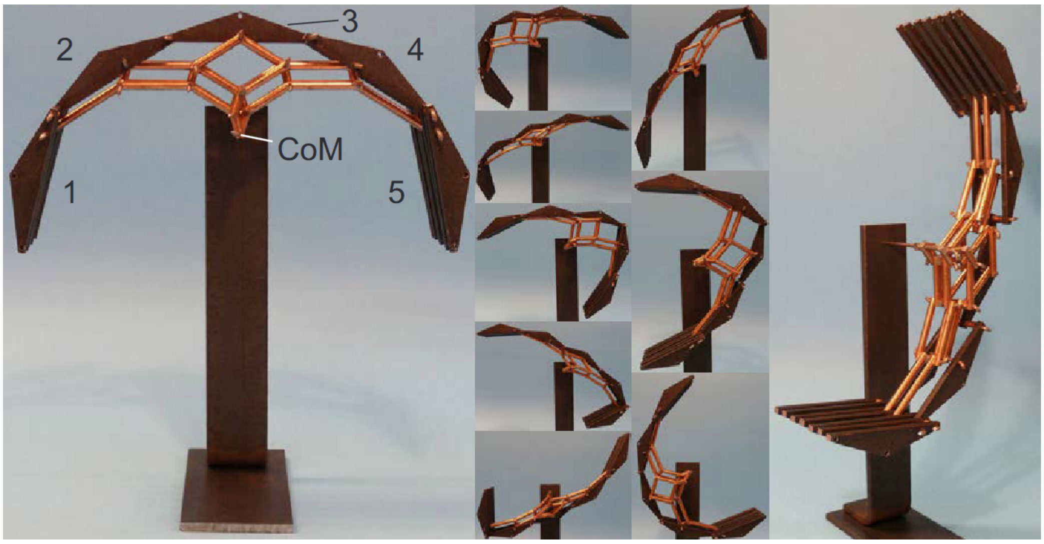


Figure 10. A physical prototype of a principal vector 5-link balanced mechanism, developed by van der Wijk [21].

The most common implementation of the Principal Vector approach is to balance parallelogram [87] and pantograph-based manipulators that require lower mass additions compared to other parallel mechanisms [10,52,53,69]. Pantograph designs are also used as a form of balancing ALM for mechanisms [19,51]. While the majority of balanced pantograph designs are planar configurations, van der Wijk [88] has extended them to spatial mechanisms.

Often, multiple DOF mechanisms resulting from principal vector designs can be complex to implement in practice. Girgenti and van der Wijk [89] present methods of substituting complex kinematic designs with passive components in order to simplify the design and conserve space.

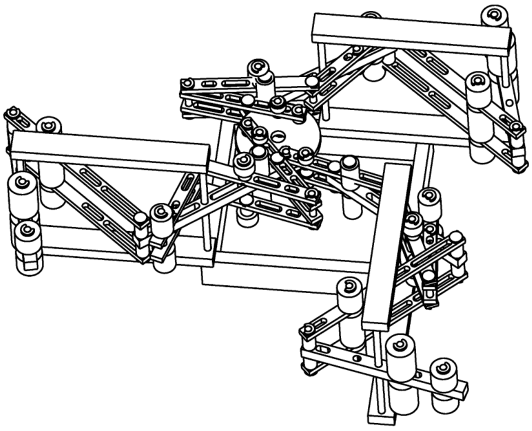
3.4. Synthesis

The inherent balancing approach for designing dynamically balanced mechanisms (Section 3.3) provides a powerful tool for designing balanced one or more degrees of freedom (DOF) planar mechanisms. Combining lower-DOF balanced mechanisms into kinematic arrangements of higher DOF and more complex balanced manipulators, both planar and spatial, is a popular and mature approach explored extensively in this field.

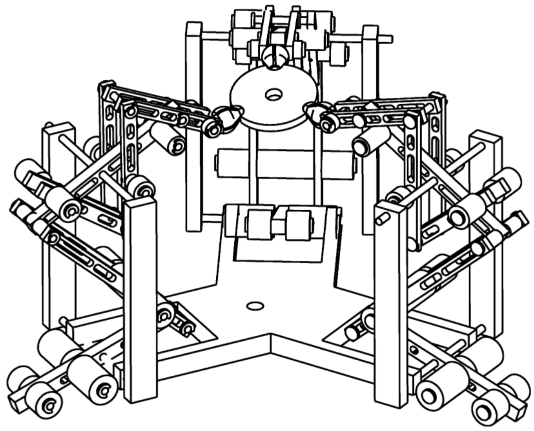
While synthesized mechanisms allow for easier design of high DOF manipulators, these designs are often not optimal in terms of mass and inertia addition. In that case, closed-loop constraints are used to find further balance conditions in order to reach the optimal balanced design [87].

Synthesis has been applied extensively for parallel mechanisms [90], pantographs [91], and designs with both inherent balance properties and added passive components [59]. Jean and Gosselin [92] presented an approach for synthesizing 1–3 DOF planar manipulators from statically balanced four-bar linkages and further applied an inherent balancing approach to derive balance conditions [92].

Gosselin then extended this approach to synthesize force and moment balanced four-bar linkage mechanisms into 2–6 DOF dynamically balanced planar and spatial mechanisms [75,76,93]. The four-bar linkages either jointly balance a load at an end-effector or are stacked on top of each other with recursive balancing of four-bar linkages through a chain, as shown in Figure 11.



(a)



(b)

Figure 11. Three-DOF manipulators synthesized from one-DOF dynamically balanced four-bar linkages [75]: (**a**) planar, (**b**) spatial.

More than one balancing approach can also be synthesized to design a balanced mechanism. Auxiliary parallelograms and springs have been employed in [17,94] and studied at a conceptual level in [95] to achieve gravity balancing for a spatial manipulator. A patent [96] describes a balance mechanism synthesizing a parallelogram ALM, a spring-loaded cam assembly, and a constant force spring for the static balancing of robot auxiliary equipment.

The force and moment balance conditions of serial three revolute joint (RRR) planar manipulators were derived by van der Wijk [21], which were then synthesized to design a variety of planar 2–4 DOF force-balanced mechanisms [97]. In this case, the inherent dynamic balancing conditions of the synthesized mechanisms were derived using the closed-loop conditions, thereby further reducing the mass and inertial addition.

In some cases, moment balance conditions are too restrictive to implement in practice for designing a manipulator. In that case, a hybrid approach of mechanical design and control is adopted. The set of dynamic motion conditions for moment-balanced trajectories are derived, and trajectories are selected to be along or close to these reactionless trajectories or paths. Papadopoulos and Abu-Abed [98] designed a nine-bar finger mechanism for space manipulation that is dynamically balanced through a combination of dynamic parameter changes and finding reactionless paths. Ouyang et al. [99] propose the Adjusted Kinematic Parameter (AKP) method, which force-balances a five-bar mechanism in conjunction with real-time Proportional-Derivative control. A hybrid mechanical-design and control approach for balancing a 3-RRR manipulator was presented in [100]. van der Wijk et al. [97] designed and experimentally validated a 4-RRR parallel mechanism that uses inherent balancing and optimal trajectories along reactionless paths for dynamic balancing.

Synthesized mechanisms are the predominant method by which experimental manipulator prototypes have been built and tested for force balance (e.g., PAMINSA [101]) and dynamic balance (e.g., DUAL-V [97]). A summary, along with a detailed elaboration of these mechanisms, is given in Section 4.

3.5. Summary and Comparison of Approaches

Previous subsections (Sections 3.1–3.4) showed all the possible approaches to design balanced manipulators. These approaches are not mutually exclusive, and many proposed designs combine a variety of these approaches. The methods described, along with their advantages, disadvantages, and relevant papers, are summarized and compared in Table 1.

Table 1. A summary of all the methods for dynamic balancing as categorized in this paper.

Category	Approach	Advantages	Disadvantages	Relevant Papers
Passive Component Addition	Springs	- Lightweight - Compact - Low Mass Addition	- Only achieves static balance - Expends energy during motion from equilibrium - Constrains the motion to be within the linear elastic region of deflection	[17,35–37,40,41,43,51,79,89,102]
	Counter Mass (CM) Counter Inertia (CI)	- Simple modifications to an existing mechanism - Kinematic equations do not increase in complexity - Can be implemented compactly	- CM Only achieves force balance - Increases overall mass and inertia - Tradeoff between mass/inertia and space increase - Can increase moments/joint torques at high accelerations - CI requires precise modifications of link inertia	[12,49,52,54,56,59,60,63,64,69,103–106]
	Counter Rotating Counter Mass (CRCM)	- Simultaneously low mass and low space addition	- Complex kinematic design - Mechanically complex to implement in practice	[49,55,56,58,105,107]
	Additional Linkage mechanism (ALM)	- Low mass addition - Increases stiffness and rigidity	- Significant increase in space - Can restrict workspace due to additional complexity in kinematics	[17,48,51,55,59,87,105,108]
Active Component Addition	Active CRCM/ADBU	- Highly Compact - Low Mass/Inertia addition - Can be placed in a variety of locations - Can balance all forces and torques simultaneously	- Requires actuation - Mechanical limits to forces and torques that can be balanced - Complex to design and implement	[61,67–69,109]

Table 1. *Cont.*

Category	Approach	Advantages	Disadvantages	Relevant Papers
Inherent Dynamic Balancing	Parallel Mechanisms	- Can synthesize to form higher DOF planar or spatial mechanisms. - Lower mass increase than CM/CI applied to serial mechanisms - Multiple kinematic designs for any set of balancing conditions	- Designs can restrict workspace due to interference or singularities - Complex kinematics and mechanical design	[11,16,57,71,74,75,77,78,93,98,101,110–118]
	Principal Vector Designs	- Low mass increase - Well-established mathematical methods of solving for balance	- Very large mass and inertia - Complex mechanical design (particularly joint design)	[85,88,91]
	Parallel-Piped	- Design optimizes in 3D for balance conditions	- Large mass increase - Complex mechanical design (particularly joint design)	[81,82]
Synthesized mechanisms		- Can combine the advantages of different methods (from the above) - Build complex mechanisms from simpler balanced components which are easier to solve for balance	- Complex combinations of designs which can be difficult to implement practically - Redundant components (masses, inertias) are added which could be reduced if the entire mechanism is optimized	[35,53,76,82,90–94,97–99,116]

4. Experimental Prototypes

The overwhelming majority of research in the field of dynamic balancing is theoretical, and even application papers are mostly validated in simulation. We identified 23 papers in the literature that present physically built and validated balanced manipulator prototypes with experimental results. These are elaborated on in the following subsections and summarized in Table 2.

Table 2. A summary of all papers with a built prototype, categorized according to type of balancing, dimension (planar or spatial), balanced DOF, and motion DOF. For each paper, the balancing method as well as experimental metrics for results are provided.

Type of Balance	Dimension	Balanced DOF	Motion DOF	Paper	Method	Experimental metrics
Gravity	Planar	1	2	[119]	Spring	Joint torques
		1	2	[120]	Spring	Joint angles
		1	3	[87]	ALM, Active CM	Joint angles
		1	3	[42]	Spring	Joint torques
		1	3	[121]	Spring	Joint torques
	Spatial	1	3	[38]	Active ALM	Joint torques, end-effector position
Static	Planar	1	2	[66]	Active CM	Reaction moments, base position
		1	3	[64]	Active CM	Base rotation
		1	5	[6]	Active CM	Joint torques, base orientation
		3	3	[35]	CM/CR, CRCM, ALM	Joint torques (simulation)
	Spatial	3	4	[122]	Spring, ALM	Joint torques
		3	6	[108]	Active ALM	Base displacement
Static and Moment	Planar	2	3	[65]	Active CM, Inherent Balancing	Base orientation
Force	Planar	2	2	[103]	CM	Joint Torques
	Spatial	3	3	[53]	Synthesized Mechanism	Reaction forces
		3	4	[101]	Synthesized Mechanism	Actuator torques
		3	6	[52]	CM	Reaction forces and moments
Dynamic	Planar	3	1	[118]	Inherent Balancing	Angular acceleration, joint torque
		3	2	[123]	Inherent Balancing	Reaction forces and moment
		3	2	[59]	ALM, CM/CR, CRCM	Joint torques
		3	3	[124]	Synthesized Mechanism	End-effector position
		3	4	[97]	Synthesized Mechanism	End-effector position, reaction forces/moments
	Spatial	3	4	[109]	ADBU	End-effector position

4.1. Gravity Balanced

The earliest example of a balanced manipulator prototype is by Ulrich and Kumar [119], who gravity balanced a 1- and 2-DOF link manipulator through a spring connected with a pulley system.

Agrawal et al. [87] then developed the next instance of an experimental prototype, a gravity-balanced 3-DOF serial manipulator. The center of mass was identified by balancing the manipulator using auxiliary parallelograms, which was then balanced using a single actively driven counter mass. The actual center of mass closely tracked the desired as extrapolated from the dynamic model and joint encoder angles.

Spring-based gravity compensation methods have been prototyped by several different researchers. Cho et al. [120] presented a 2-DOF spring-based gravity balancing mechanism for a link with roll and pitch motion, operating in a hemispheric workspace. A spring-based balancing mechanism was employed for the gravity balancing of a 3-DOF Prismatic-Revolute-Revolute manipulator by [42]. Using a different number of coil loops for the wire that connects the spring successively to all the joints, the mechanism ensured static balance and joint torque reduction rates of 61–90%.

Most recently, a spring cam mechanism was developed to balance a serial 3-DOF planar manipulator [121]. The mechanism attaches to the base and connects to the links via pulley mechanisms with calculated force factor reductions between the links, so that the manipulator is balanced with a single spring.

For gravity-balancing spatial manipulators, Baradat et al. [38] developed a method of gravity balancing a 3-DOF Delta robot using a pantograph mechanism that was actuated with a control law to provide a balancing force at the end-effector. This method is preferable to mass additions or springs, as it can also adjust for different loads applied at the end-effector. The mechanism also reduces precision errors caused by deformations of the manipulator that lead to misalignment of the end-effector due to changes in altitude and inclination.

They tested their concept on an experimental prototype built by modifying the *SurgiScope*(*R*), a robotized navigation tool holder. The results showed a reduction in the root mean square value of the input torque by 99.5% and a significant reduction in the position errors, from 86.8% to 97.5%.

4.2. Statically Balanced Manipulators

The most basic application of static balancing is 1-DOF displacement of a battery to offset the horizontal (i.e., gravity orthogonal) center of mass displacement of an aerial manipulator by Ruggiero et al. [6]. Al Akhras et al. [64] improve this to a lightweight arm powered by pulleys, concentrating the mass near the mount and optimizing the link profiles, which require smaller battery displacements. Luxton et al. [66] statically balanced a 2-DOF manipulator with a separate active counter mass attached to the manipulator. It has a mass of 312 g, of which 90 g is the counter mass for a payload of 30 g.

In all of these cases, the aerial manipulator exhibited lower base displacements with static balancing, which also improved the base stability. An experimental prototype of this simple static balancing concept is shown in Figure 12.

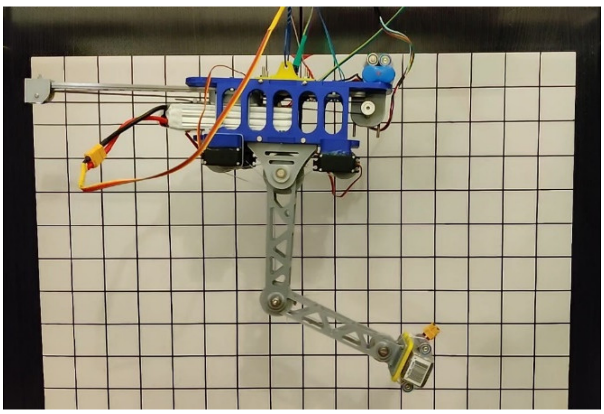


Figure 12. A lightweight 2-DOF manipulator with active CM-based balancing in the horizontal axis, by Al Akhras et al. [64].

A statically balanced 4-DOF robot arm mimicking a human shoulder and elbow using a parallelogram structure and springs was presented by Vermeulen and Wisse [122]. It was tested in a pick-and-place operation with a 2 kg mass over 0.8 m in 1.5 s, using a small 1.5 W DC motor assisted by balanced behavior.

A statically balanced aerial manipulator was developed by Imanberdiyev et al. [108] using 2 lightweight 6-DOF arms, which can be operated in multiple modes for different tasks. In one of the modes, one of the arms conducted the manipulation task while the other balanced the center of mass of the overall system using control-based methods. This can be categorized as an ALM approach, as it is a duplicate mechanism. The balancing behavior for the balanced manipulation was validated in an experiment by measuring base motions.

An aerial manipulator that, in addition to static balancing, also attempted moment balancing was presented by Ohnishi et al. [65]. By kinematically constraining a joint, the angular momentum of the 2-DOF manipulator remains the same for all motions of the first joint. The authors tested their design in a live aerial manipulation scenario and found that the attitude disturbances were reduced when the sliding battery balancing mechanism was activated.

A statically balanced manipulator consisting of synthesizing legs of a pantograph and spring-pulley system was designed by Ebert-Uphoff and Johnson [39]. Laliberte et al. [35] designed a prototype of a 3-DOF parallel manipulator using a parallelogram that is statically balanced with counterweights and springs. In both of these papers, they display a prototype, but the results provided are purely from simulation.

4.3. Force-Balanced Manipulators

Counter mass-based force balancing of an industrial PUMA-760 serial manipulator was attempted early in the development of this field [103]. The mass increase is significant, with over 100 kg added to a link to balance a 5 kg payload.

A force-balanced Delta robot, designed by modifying each arm into a pantograph and balancing it with counter masses, was formulated by van der Wijk and Herder [69] and prototyped by Clark et al. [52] for aerial manipulation applications. The results showed a reduction in shaking forces and moments; however, the improvement in precision was not significant.

Similar to [52], a force-balanced pantograph was synthesized as a 3-DOF spatial manipulator by Suryavanshi et al. [53,125], with its kinematic configuration similar to that of a Delta robot. It allows operation in two modes similar to the *DYMO* parallel robot [126]: a translation mode and rotation mode, which is based on the different joint angle combinations within domains bounded by singular configurations. Force sensors arranged parallel and perpendicular to gravity on the base validate the force balance by measuring reaction forces, which are of a small magnitude of up to 0.2 N. The manipulator is shown in Figure 13.

An advancement in the techniques of synthesizing force-balanced mechanisms was developed as the PAMINSA manipulator family (Figure 14). Briot et al. [101] presented a methodology to design force-balanced pantograph legs from various combinations of revolute (R) and prismatic (P) joint kinematic chains. These pantograph legs are then synthesized to form 3 to 6 DOF spatial manipulators based on different kinematic configurations. The force-balanced counter masses as well as reduced "partially balanced" counter mass which result in minimal reaction torques for desired high-acceleration trajectories were calculated. The built prototype of a 4-DOF manipulator synthesized from 3 Revolute-Prismatic-Revolute (RPR) chains was tested with and without a 200 N load. The partially balanced manipulator has 79% of the mass of the when fully counter mass balanced, yet has the same reduced joint torques while manipulating a 200 N load compared to an unbalanced manipulator. A further simulation study [48] investigated dynamic balancing using rotation of the wrist to counteract reaction torques.

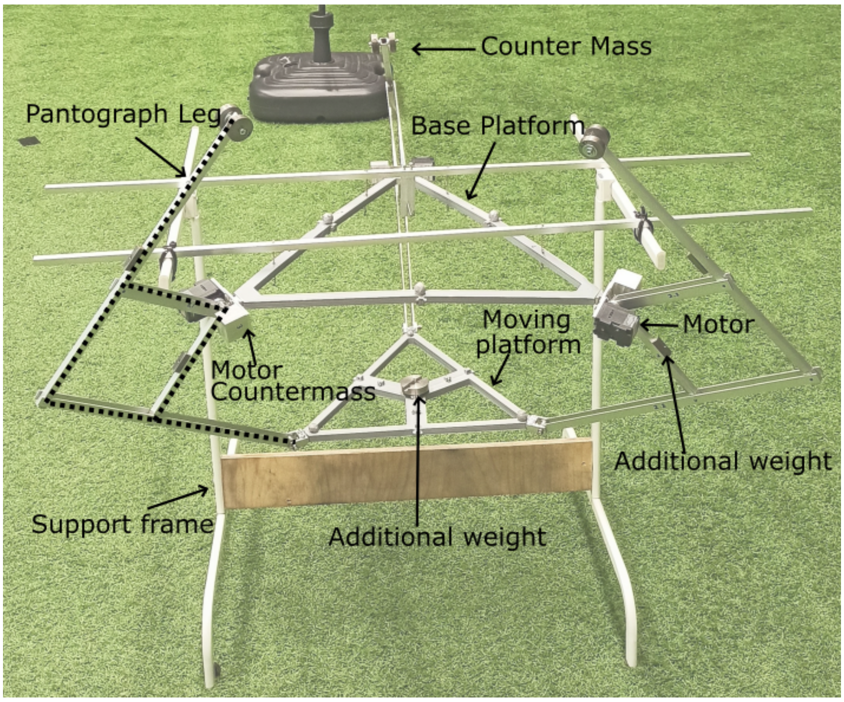


Figure 13. The ADAPT manipulator presented in [125], from 3 synthesized force-balanced pantograph legs.

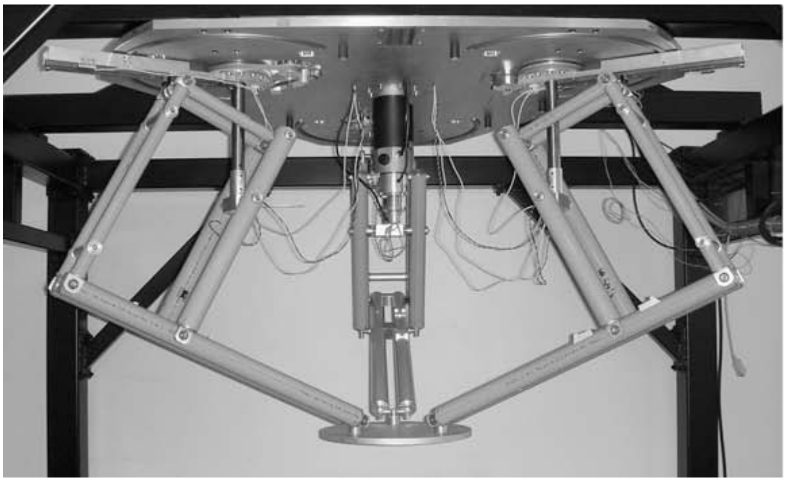


Figure 14. The PAMINSA manipulator [101].

4.4. Dynamically Balanced Manipulators

A dynamically balanced four-bar linkage manipulator designed with an inherent balancing approach was built by Zomerdijk et al. [118] and achieved up to 21G tip accelerations, 99.3% reduction in reaction forces, and 97.8% reduction in reaction moment. A structural analysis for the finite element model was also performed to address in-plane and out-of-plane deformation for the first three modes, addressing issues from flexible body behavior which degrades the precision. The manipulator is shown in Figure 15.

This was extended to a 2-DOF planar mechanism, where a parallelogram manipulator is dynamically balanced with a counter parallelogram [123]. The manipulator parallelogram is extended using inverted four-bar linkages that add stiffness and rigidity, and contain the actuating joints. The reaction force was reduced by 93.6% and reaction moment by 88.9%, reaching up to 3G acceleration in experiments. A finite element analysis was also conducted to analyze the degradation of performance due to flexible body deformations. However, the balancing modifications required a mass 9.9 times and inertia roughly 4 times that of the unbalanced manipulator. This manipulator, called Super-B, is shown in Figure 16, and has been patented [127].

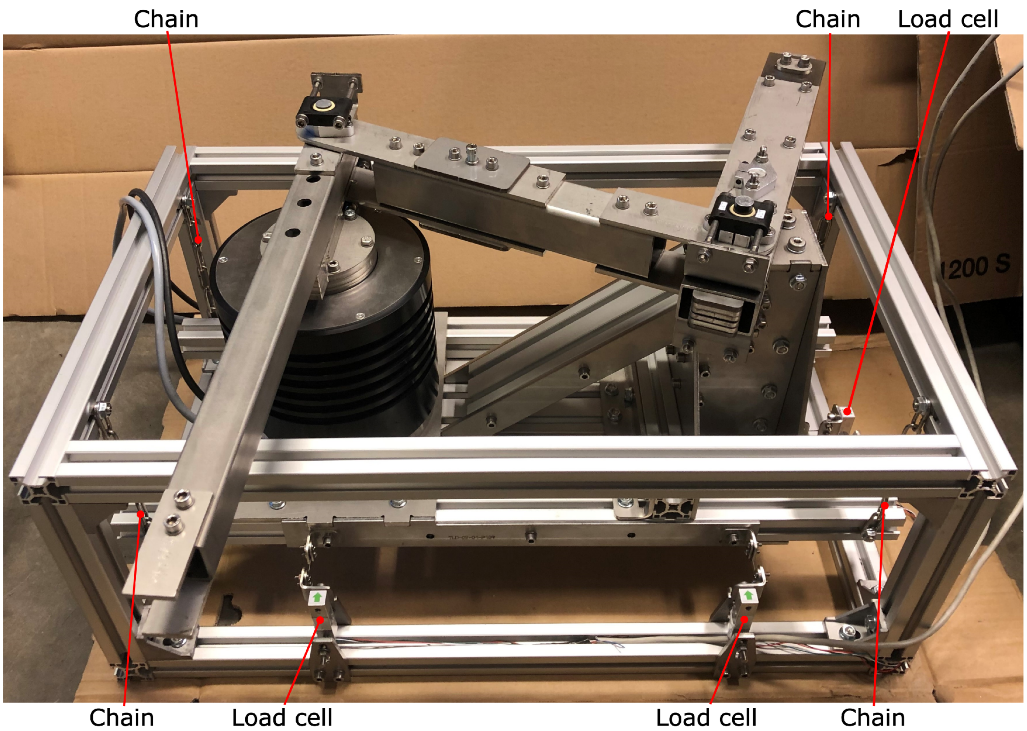


Figure 15. The prototype of a 1-DOF dynamically balanced manipulator developed in [118].

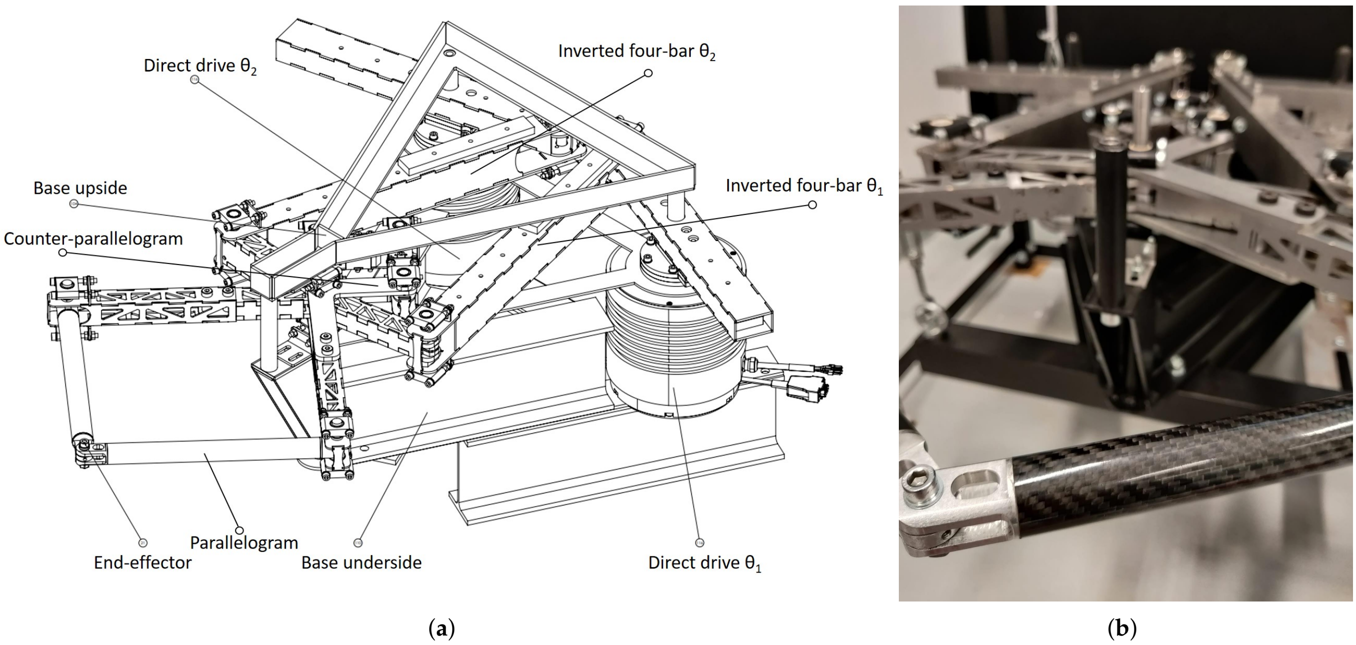


Figure 16. The Super-B manipulator developed by Boere et al. [123]: (**a**) design, (**b**) prototype.

Bom [128] attempted to design and prototype a 2-DOF manipulator synthesized with 2 four-bar linkages suitable for aerial manipulation; however, further work is required to fully validate moment balancing behavior.

Two parallelogram mechanisms were synthesized to create a 3-DOF planar manipulator that is force-balanced and moment balanced for certain trajectories [129]. Dynamic balancing was verified using tracking the displacement of a point on the base, which was below 1 mm for the balanced mechanism and up to 12 mm for the unbalanced.

A 4-DOF planar dynamically balanced manipulator, designed from inherently balancing 4-RRR 2-DOF serial chains (DUAL-V), was prototyped, tested in [21,97], and patented [130]. The manipulator is force-balanced for all translational motions of the platform, and moment balanced for motion along the reactionless paths along central axes. The manipulator achieved 93–97% reduction in shaking forces and up to 96% reduction in shaking moments for reactionless trajectories, with a maximum acceleration of 10.3G. Due to the increase in inertia, for high-acceleration applications the actuator joint torques are increased 47% higher than those for the unbalanced manipulator. The manipulator is shown in Figure 17.

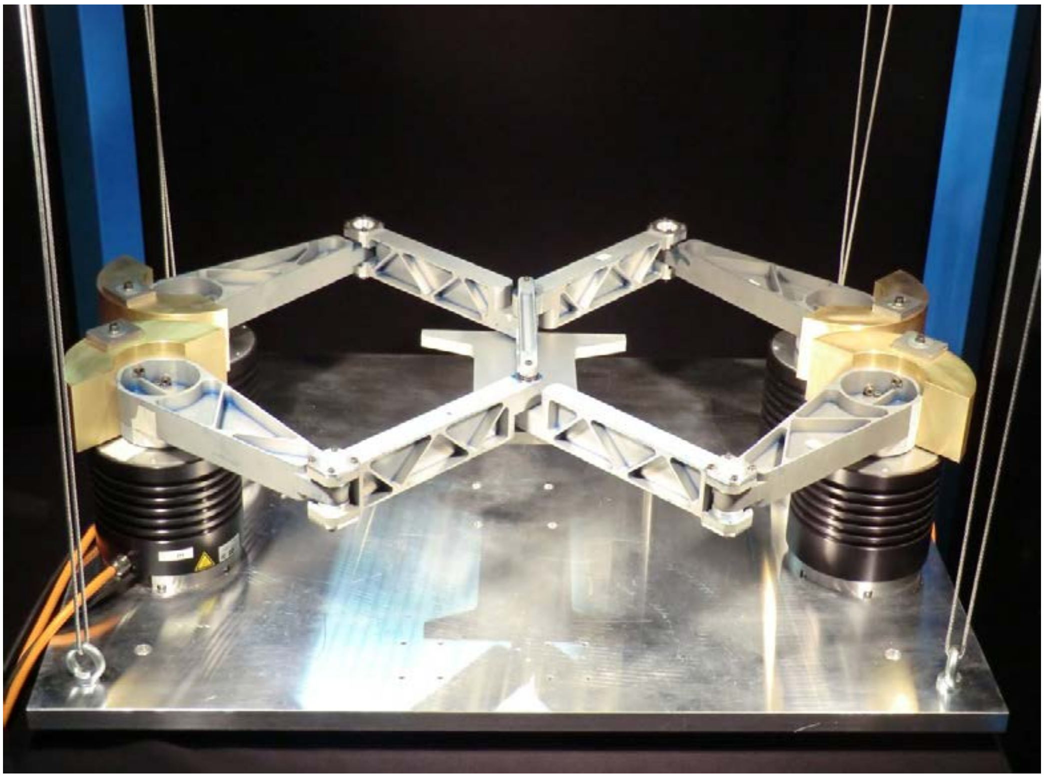


Figure 17. DUAL-V manipulator built and tested by van der Wijk et al. [97].

Laliberte and Gosselin [59] prototype 3 different types of passive component additions that dynamically balance a 2-DOF parallel mechanism: a CR to result in a constant inertia, proximal CM/CI, and an ALM balancing pantograph. The comparison showed that balancing greatly reduced reaction torques and forces, as well as joint torques. However, there is a up to 5-fold mass/inertia addition involved with dynamic balancing, and a trade-off between decreasing mass and increasing inertia relating to the space occupied by components.

Finally, a Delta arm was balanced using an ADBU by Mirz et al. [109]. The balancing unit had two translational and one rotational DOF, and controlled for shaking forces and moments using (1) open-loop control based on the manipulator dynamic model and (2) closed-loop control using independent modal space control. The results found that the maximal vibration amplitude in the principal direction of motion can be reduced up to 86%.

The experimental prototypes discussed in this section are summarized in Table 2. Only research papers which demonstrate real-world experimental results validated through metrics are included. We categorize in the following order: type of balancing, dimension (planar or spatial), number of spatially balanced DOF, and end-effector motion DOF. We differentiate the latter 2 columns as often manipulators have higher motion DOF, of which a subset are balanced, or vice versa, where the dimensionality of balanced DOF is greater than the DOF determining the motion of the end effector. For example, a dynamically balanced 2-DOF manipulator is force-balanced in 2 Cartesian dimensions and moment balanced orthogonal to the plane, while only having 2-DOF motion that can uniquely determine the Cartesian position of the end effector. A summary of the method, experimental metrics, and the citation for every paper is also included.

5. Conclusions

In the past few decades, balanced manipulators have seen many advancements, starting from theoretical contributions that are now being implemented as real-world prototypes. They offer many advantages for both ground-fixed and mobile robot manipulation, where disturbances pose a problem for precise and robust task execution.

The scope of theoretical designs is wide due to the variety of methods available; however, the feasibility of implementing these in practice reduces that scope. The various design approaches can be selected based on the task and environment for the manipulator, which will determine the requirements and tradeoffs in area, mass/inertia, and maneuverability.

At a fundamental level, approaches to designing balancing manipulators involve adding passive or active components, or inherent balancing based on adjusting the geometric, kinematic, and dynamic parameters. Low DOF balanced manipulators designed using these approaches can also be synthesized into complex higher DOF manipulators.

While theoretical contributions to this field are prolific, real-world applications remains in their infancy. We identified 23 experimental prototypes of real-world balanced systems that have been built and tested in experimental scenarios. The validated results show that balancing robotic manipulators results in smaller base displacements, higher maximum accelerations, and lower maximum joint torques during the execution of manipulation tasks. However, this often results in a significant tradeoff in terms of mass/inertia or area increase.

Challenges and Future Work

Balanced manipulator designs often result in infeasibly high masses, mass/inertia values that are difficult to achieve in practice, or a large increase in workspace that is not feasible in a constrained industrial automation factory floor or on a mobile robot. Perhaps for this reason, we could not find any attempts to build an inherently designed experimental spatial manipulator prototype with dynamic balance.

Designing for highly accurate mass and inertia of links restricts the scope of designs and also requires very precise fabrication of parts. The balance conditions for many inherently balanced designs are often highly sensitive to parameter errors. While the mass of links can be easily adjusted using adjustable counter masses, achieving precise inertias for dynamic balance requires high-quality machining.

The increase in mass/inertia of manipulators from force/moment balancing modifications also increases the inertial moments; hence, the joint torques also increase for higher accelerations, overcoming any improvements from the reduction in gravity compensation. In such cases, a tradeoff analysis for improvement in precision vs. increase in joint torques [101] or a hybrid mechanical design and optimal trajectory control approach [28,97,100] is more suitable.

Control-based approaches or ADBU units can also reduce shaking forces and moments without significant mass or inertial addition, which makes them a preferred option over mechanical design approaches. For these reasons, force balance remains the more feasible and, hence, preferred type of balancing for prototyped manipulators, spanning a variety of planar and spatial designs.

There are several challenges to overcome and promising directions that offer ample research opportunities, as well as commercial applications:

1. Continued research in designing, building, and experimentally validating physical prototypes of abstract design concepts for real world applications. These will provide more examples for researchers on the practical issues involved in the design and manufacturing of balanced manipulators.
2. Analysis and comparison of the practical mechanical implementation of abstract components (such as CRCM, ADBU, etc.) that comply with the design constraints inherent in real-world manipulator designs.
3. More research into the design of low-mass and low-inertia components from a mechanical perspective, as mass and inertia increase poses a major challenge to the practical implementation of balanced manipulators in mobile platforms.
4. Synthesis of mechanical design and control approaches for dynamic balancing. This idea was proposed in [28] and explored in some other studies. Control approaches have been extensively explored in the field of space manipulators [18]. However, real-world experimental results remain sparse in the literature. Designing dynamic balanced manipulators can be very difficult in practice; however, a synthesis of force balancing and control along reactionless trajectories can achieve this objective without requiring complex and precise designs.

We anticipate that in the years to come, research on balanced robotic manipulators will continue to develop. These advances, particularly when accompanied with experimental prototypes, will show the improved benefits of balanced manipulators for many manipulation tasks and pave the way for increased commercial application.

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Abbreviations

The following abbreviations are used in this manuscript:

ADBU	Active Dynamic Balancing Unit
ALM	Additional Linkage Mechanism
DC	Direct Current
DOF	Degree of Freedom
CI	Counter Inertia
CM	Counter Mass
CR	Counter Rotation
CRCM	Counter Rotating Counter Mass
G	Gravitational-Force equivalent
PID	Proportional-Integral-Derivative (Control)
RRR	Revolute-Revolute-Revolute

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