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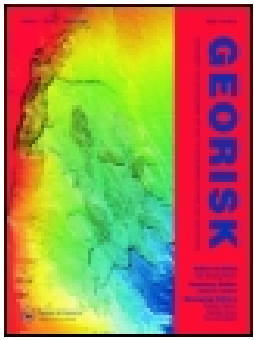
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Uncertainty in spatial average undrained shear strength with a site-specific transformation model

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ABSTRACT

Transformation models are used to infer geotechnical properties from indirect measurements. A site-specific transformation model can be calibrated with direct and indirect measurements from a site. When such a model is used, then spatial variability, measurement errors and statistical uncertainty propagate into the uncertainty of the spatial average, which is the variable of interest in most geotechnical analyses. This research shows how all components enter the total uncertainty of a transformation model for undrained shear strength from cone resistance. A method is proposed to estimate the uncertainty in the spatial average undrained shear strength, particularly focusing on the role of averaging of all spatially variable error components. The main finding is that if a considerable share of the measurement and transformation errors is random or spatially variable, the uncertainty estimates can be considerably lower compared to methods proposed earlier, and hence, characteristic values can be considerably higher.

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1. Introduction

Soil properties are variable in space, because of various processes during the formation process (e.g. Lumb 1966). This spatial variability can be described by random fields with the statistical properties mean, variance and scale of fluctuation (Vanmarcke 1977, 1983). The impact of spatial variability on geotechnical engineering problems is widely recognised (see, e.g. Fenton and Griffiths 2002; Griffiths, Huang, and Fenton 2009; Cho 2007; Hicks and Samy 2002; Ahmed and Soubra 2012). Besides spatial variability, geotechnical parameters are also uncertain because of measurement uncertainty, statistical uncertainty and transformation uncertainty, see Figure 1 (Phoon and Kulhawy 1999a, 1999b; Baecher and Christian 2003; Uzielli 2008; Cao, Wang, and Li 2017).

According to, among others, Phoon and Kulhawy (1999a, 1999b), the total uncertainty is a combination of the uncertainties from the various sources. Since geotechnical failure modes usually involve a volume of soil, we are in reliability-based design interested in the (total) uncertainty in the spatial average of the geotechnical parameter (e.g. shear strength), involving both spatial variability and epistemic uncertainties. Phoon and Kulhawy (1999b) propose to apply variance reduction as proposed by Vanmarcke (1977) to the variance related to spatial

variability ($\sigma_{spatial}^2$), and add the remaining variances to determine the total uncertainty:

$$\sigma^2 = \Gamma^2 \cdot \sigma_{spatial}^2 + \sigma_{measurement}^2 + \sigma_{statistical}^2 + \sigma_{transformation}^2 \quad (1)$$

Essentially, determining the total variance as in Equation (1) implies the assumption that measurement uncertainty, statistical uncertainty and transformation uncertainty relate to systematic errors, which are not subject to (spatial) averaging. While this assumption is certainly conservative in the sense that it will lead to a high uncertainty estimate in engineering applications, the question is whether we should consider these error terms as entirely systematic and not subject to spatial averaging and what difference it would make in terms of probability distributions and characteristic values, if we followed a more differentiated approach. In this paper, we will investigate these questions for the example of site-specific transformation models.

To estimate geotechnical parameters from indirect measurements (e.g. CPTs) we often use transformation models. Instead of using a generic transformation model from literature, we can calibrate a site-specific transformation model using direct measurements of a soil property

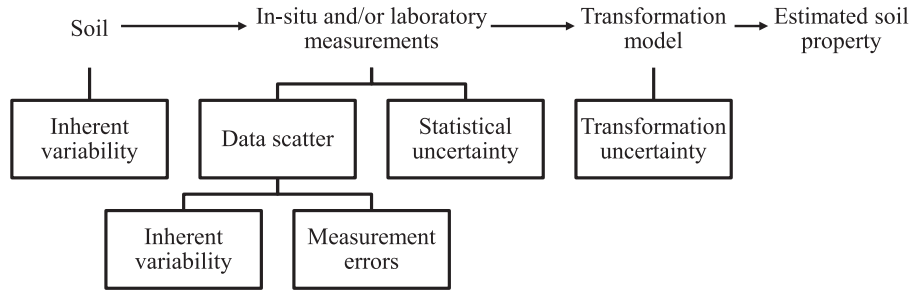


Figure 1. Overview of geotechnical uncertainties (Cao, Wang, and Li 2017; adapted from Phoon and Kulhawy 1999a).

(e.g. laboratory results) and indirect measurements (e.g. cone resistance) at the same location, across the site of interest. Such a site-specific transformation model can then be used to estimate the geotechnical parameter of interest using less costly indirect measurements. In Dutch dike design, for example, a site-specific transformation model is often used to estimate the depth-average undrained shear strength (ratio) from normalised cone resistance. Because of all errors mentioned above, the prediction by such an empirical transformation model is uncertain. In practice, it remains challenging to properly quantify this uncertainty, in particular the distinction between spatially variable components and random errors which are subject to spatial averaging and systematic errors which are not.

In Sections 2 and 3, we analyse how the different error terms propagate into the uncertainty in the estimated (detrended) spatial average using a site-specific transformation model for undrained shear strength from cone resistance, for a synthetic random field example. We particularly focus on the distinction between random and systematic errors, and how this affects the uncertainty in the spatial average of the undrained shear strength. Ultimately, we propose a method to estimate the uncertainty in the spatial average at locations of interest with a CPT in Section 4, accounting for the averaging of random errors and accounting for systematic errors. We also propose a method to estimate the spatial average and the uncertainty in the spatial average at other locations in the same statistically homogeneous deposit, based on the observed variability of multiple CPTs.

2. Characterisation of the spatial average parameter using CPTs

2.1. Introduction

In line infrastructure projects, we often want to estimate the depth-average of the geotechnical parameter in a statistically homogeneous layer, since failure mechanisms typically involve a vertical zone of influence which is much larger than the vertical scale of fluctuation, for

instance, shear planes crossing a soil layer by several metres vertically. Usually, this concerns a geological deposit that is present over a site of about 1–10 km, at least for long linear infrastructures such as dikes, roads or railways. We use CPTs to estimate the undrained shear strength at locations where a CPT is available. The estimated depth-average shear strength at a cross-section is then representative for a section shorter than the horizontal scale of fluctuation. At cross-sections without a CPT, the depth-average can be estimated based on the average of the entire site and the variability of the spatial averages across the site.

In both occasions, we need to estimate the uncertainty in the estimated spatial average. For the remainder of this article, the term *spatial average* refers to the *depth-average*. In this section, we propose how to estimate the uncertainty in the spatial average at cross-sections with a CPT and cross-sections without a CPT, based on known or estimated error statistics, particularly focussing on the difference between random and systematic errors. The observed variability is compared with the estimated uncertainty in the spatial average, using a synthetic random field with known values and an assumed perfect transformation model.

2.2. Uncertainty in the spatial average

The undrained shear strength at a cross-section where a CPT is available, can be estimated using a transformation model, see Equation (2). Here, s_u^I is the indirectly measured undrained shear strength, q_{net} the normalised cone resistance¹ and N_{kt} the transformation model parameter. The depth-average (denoted by a bar) parameter $\bar{s}_u^I$ can be estimated by the numerical mean of all CPT measurements N (typically every 2 cm), see Equation (3) and Figure 2.

$$s_u^I = \frac{q_{net}}{N_{kt}} \quad (2)$$

$$\bar{s}_u^I = \frac{1}{N} \sum_{i=1}^{i=N} s_{u,i}^I \quad (3)$$

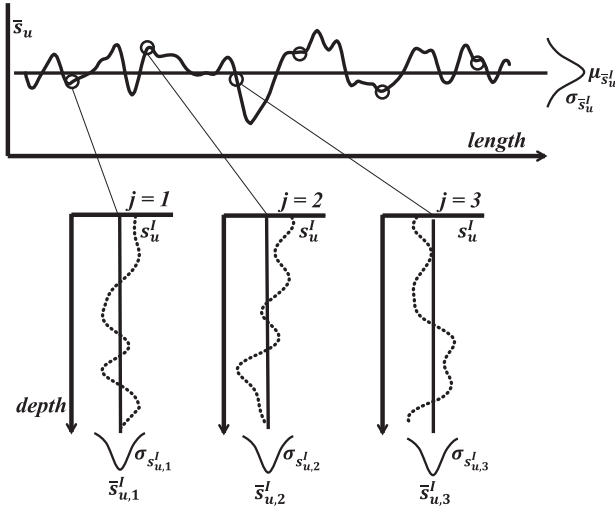


Figure 2. Schematic overview of a 2D site (length and depth).

We can estimate the spatial average at the cross-section accurately, because a CPT measurement is almost continuous. The observed data scatter around the spatial average stems from small-scale vertical spatial variability and random errors (white noise). The spatial variability around the spatial average can be estimated from the total point variance if we assume that the total point variance of the site (σ^2) is the linear sum of vertical fluctuation (σ_f^2) and fluctuation of the spatial average (σ_{av}^2). When we substitute the definition of the variance reduction function (Vanmarcke 1977) ($\Gamma = \sigma_{av}/\sigma$) into this, we obtain an expression for the expected variance around the spatial average in a CPT: $\sigma_f^2 = (1 - \Gamma^2) \cdot \sigma^2$. The observed data scatter stems also from random measurement error ($\varepsilon_{q_{net}}$), which is an independent error source and typically modelled as multiplicative error. Hence, we write the total observed scatter in terms of coefficient of variation (CoV), see Equation (4).

$$\sigma_{s_{u,j}}^I = \bar{s}_u^I \cdot \sqrt{\left((1 - \Gamma^2) \cdot CoV_{spat}^2 + CoV_{\varepsilon_{q_{net}}}^2 \right)} \quad (4)$$

In line with Phoon and Kulhawy (1999b), the total uncertainty in the spatial average is the linear sum of spatial variability, statistical uncertainty, systematic measurement error (bias) and transformation error, where the error terms are assumed to be independent, see Equation (5). The statistical uncertainty in the spatial average is dependent on the number of measurements N_j and the observed variance in CPT j ($\sigma_{s_{u,j}}^I$), from Equation (4). For the remainder of this article, we will use the term random error for white noise in the individual measurements and systematic error for a bias in a CPT or for the entire field (i.e. site).

$$\sigma_{\bar{s}_u}^2 = \frac{1}{N_j} \cdot \sigma_{s_{u,j}}^I + \sigma_{meas,sys}^2 + \sigma_{trans}^2 \quad (5)$$

In a statistically homogeneous layer, our best estimate for the spatial average at a cross-section without a CPT, is the mean value of the spatial averages from M CPTs across the site, see Equation (6) and Figure 2. We can estimate the uncertainty in the spatial average at a cross-section without a CPT from the variance of the spatial averages from multiple CPTs, accounting for statistical uncertainty, see Equation (7).

The goal of this paper is to investigate the effect of random errors in contrast with the assumption of only systematic errors. To have a clear comparison, we base ourselves on the same assumptions as Phoon and Kulhawy (1999b): a linear combination of spatial variability and error terms, see Equation (8).

$$\mu_{\bar{s}_u} = \frac{1}{M} \sum_{j=1}^M \bar{s}_{u,j}^I \quad (6)$$

$$\sigma_{\bar{s}_u}^2 = \left(1 + \frac{1}{M} \right) \cdot \frac{1}{M-1} \sum_{j=1}^M (\bar{s}_{u,j}^I - \mu_{\bar{s}_u})^2 \quad (7)$$

$$\sigma_{\bar{s}_u}^2 = \Gamma^2 \cdot \sigma_{spat}^2 + \sigma_{meas,sys}^2 + \sigma_{trans}^2 + \sigma_{stat}^2 \quad (8)$$

2.3. Example

We evaluate the uncertainty estimation according to Equation (5) with a synthetic random field example with known values for the undrained shear strength s_u and errors. For the sake of a good comparison, we use synthetic data for which we generate the true but subsequently unknown values. To this end, we generate a stationary Gaussian random field of 200×2000 (v:h) cells, representing a site with a 2 m thick soil layer over 2 km length, with Circulant Embedding (Kroese and Botev 2015). The field size and resolution are expected to be large enough to sample enough independent samples and small enough to be computationally efficient. The soil property in the synthetic true field is normally distributed with mean value $\mu_{s_u} = 20$ kPa and standard deviation $\sigma_{s_u} = 4$ kPa. In this example, it is ignored that most geotechnical parameters are non-negative and therefore other probability distributions might be more suitable. The assumption of a statistically homogeneous random field with constant mean is justified if the field data is de-trended. A squared exponential correlation function is applied with a horizontal and vertical correlation length of respectively 25 and 0.25 m (equivalent with $\delta_h = 44$ m and $\delta_v = 0.44$ m), consistent with literature (among others, Phoon and Kulhawy 1999a). From the true known field s_u , the measured field for the indirect measurement q_{net} is generated,

according to Equation (9).

$$q_{net} = s_u \cdot N_{kt} \cdot \epsilon_{q_{net}} \quad (9)$$

This example considers a perfect transformation model with a deterministic value of $N_{kt} = 20$. The multiplicative random measurement error $\epsilon_{q_{net}} \sim N(1, CoV_{\epsilon_{q_{net}}})$ is varied. We estimate the depth-average shear strength $\bar{s}_{u,j}^I$ at a location with a CPT, according to Equation (2) and estimate the uncertainty in the spatial average at this CPT, based on the observed data scatter, using Equation (5). The analysis is done for a layer thickness of 0.2, 1.0 and 2.0 m and a measurement interval of 2 cm, such that $N_j = [10, 50, 100]$. We compare the estimated uncertainty (according to Equation (5)) with the modelled uncertainty in this synthetic example: the difference between the estimated and the true known spatial average $\sigma_{\bar{s}_{u,j}^I} = \bar{s}_{u,j}^I - \bar{s}_{u,j}$. When there is only a random measurement error, the estimated uncertainty coincides quite well with modelled uncertainty, see Figure 3. As the random error increases, it dominates over the statistical uncertainty which is causing the difference between the true and estimated uncertainty (only 1000 simulations are done).

3. Uncertainty in the site-specific transformation model

3.1. Introduction

We have shown how to estimate the spatial average and the uncertainty in the local spatial average based on a CPT or based on the variability of multiple CPTs. In the example in Section 2.3, we assumed a perfect

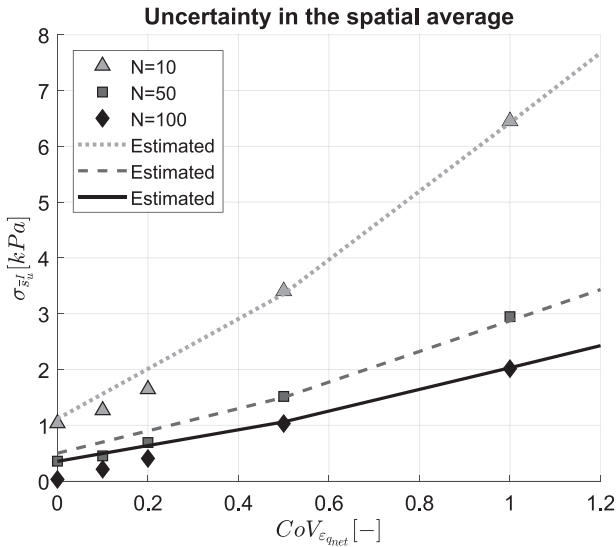


Figure 3. Modelled uncertainty versus estimated uncertainty in the indirectly estimated spatial average from a CPT.

transformation model, which is unrealistic but served the purpose of clarification. When we use a “generic” transformation model from literature it is likely that the empirical model is biased for the entire site (Ching, Phoon, and Wu 2016). This systematic transformation uncertainty should be accounted for in the estimated spatial average parameter, see Equation (5). For transformation models calibrated and used at a specific site, we do not expect a systematic bias for the entire site. However, locally the transformation model parameter may deviate from the site-average, since the transformation model error is most certainly spatially variable, because it is, at least to some degree, due to missing factors that are spatially variable, such as over consolidation ratio, water content and plasticity index. Since the stress state (e.g. loading history) is constant in a vertical profile, we assume that the transformation error is largely systematic per CPT, but independent from one location to the other, if the distance between two locations is larger than the scale of fluctuation. Ching, Phoon, and Wu (2016) showed that the vertical scale of fluctuation of the transformation error is relatively large, compared to the layer thickness. Hence we can justify the assumption that the transformation error is fully correlated in depth, and independent per CPT, at least for practical engineering purposes.

Fundamentally, transformation uncertainty is a model uncertainty. In principle, model error is meant to cover the model prediction errors for perfectly known model inputs. In practical terms it is, however, impossible to determine model uncertainty in a clean fashion, nor transformation uncertainty for that matter, because such perfect conditions are not available. It is, for instance, practically impossible to calibrate a site-specific transformation model where two paired measurements are at exactly the same location. Therefore, spatial variability causes additional error in the transformation uncertainty estimate. Moreover, there is measurement error in both CPT and laboratory measurements, which will have random and systematic components. This section analyses the propagation of these extraneous errors into the uncertainty in the transformation model parameter, as the question to be answered is to what degree transformation error is ultimately random or systematic. Subsequently, we will use this information to assess which components eventually matter for the uncertainty in the spatial average.

3.2. Calibration of the transformation model

The empirical transformation model used to estimate the undrained shear strength s_u^I from the normalised cone tip resistance has been presented in Equation (2). After pairing measured cone resistance with direct

(laboratory) measurements from (nearly) the same location ($q_{net,i} ; s_{u,i}^D$), we perform a linear regression analysis on n pairs from different locations within the same site (and deposit) to obtain an estimate for the transformation model parameter $\hat{N}_{kt}$ and the variability in s_u^I , represented by the residuals. Note that because of soil mechanical considerations, the regression line is forced through the origin.

Two regression methods are compared: minimising the standard deviation (SD) and minimising the coefficient of variation (CoV), see Equations (10) and (11), respectively. If the variability around the regression line is constant, minimising the standard deviation is the expected correct regression method; if the scatter around the regression line increases with the mean, minimising the coefficient of variation is expected to be the better option.

$$\sigma_{s_u^I} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (s_{u,i}^D - q_{net,i}/\hat{N}_{kt})^2} \quad (10)$$

$$CoV_{s_u^I} = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n \left(\frac{s_{u,i}^D - q_{net,i}/\hat{N}_{kt}}{q_{net,i}/\hat{N}_{kt}} \right)^2} \quad (11)$$

3.3. Example

The calibration of the site-specific transformation model is demonstrated for the synthetic random field example

as in Section 2.3. From the random field with true known values for the undrained shear strength s_u we sample fields for the direct measurement s_u^D and the indirect measurement q_{net} , by adding random measurement errors (e.g. sample disturbance) and model errors to the samples, according to Equations (12) and (13). Then we select n locations where we couple a measured shear strength to a measured cone resistance and perform a regression analysis to obtain the estimate for the transformation model parameter, see Figure 4.

$$s_u^D = s_u \cdot \varepsilon_{s_u^D} \quad (12)$$

$$q_{net} = (s_u \cdot (N_{kt} \cdot \varepsilon_t)) \cdot \varepsilon_{q_{net}} \quad (13)$$

First, we consider an ideal transformation model with a deterministic value of $N_{kt} = 20$ and perfect measurements ($CoV_{\varepsilon_{q_{net}}} = CoV_{\varepsilon_{s_u^D}} = CoV_{\varepsilon_t} = 0$). The transformation model for the site is calibrated with 25 CPTs (minimum spacing 50 m) and 25 laboratory test at arbitrary depths (see Figure 5). We estimate the transformation model parameter from the slope of the regression (Figure 6): $\hat{N}_{kt} = 1/0.05 = 20$. The scatter in the regression is zero due to the absence of measurement and transformation errors.

3.4. Error propagation into the transformation model parameter

In this section, we will analyse how spatial variability, measurement errors and transformation errors

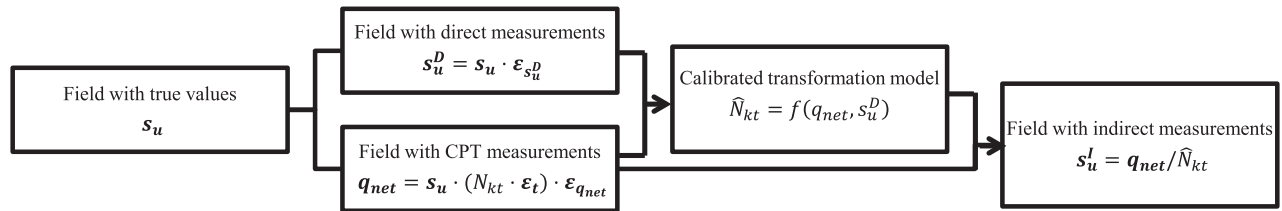


Figure 4. Schematic representation of the simulated random fields and transformations.

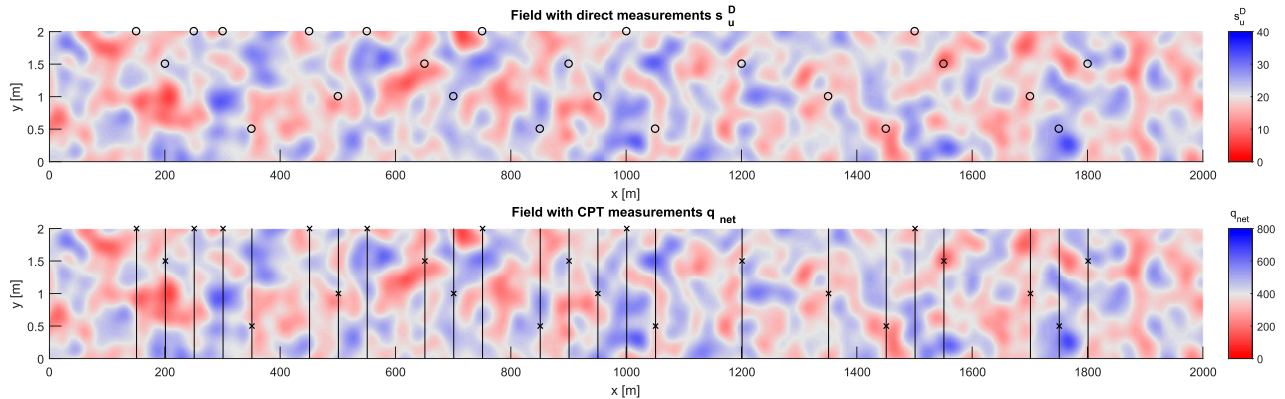


Figure 5. Direct and indirect measurements (circles and crosses, respectively) from synthetic random fields.

propagate into the transformation model. From the regression analysis, we obtain the variability in the indirectly measured undrained shear strength $CoV_{s_u^D}$. If the CoV is constant, then total uncertainty in the transformation model parameter $CoV_{N_{kt}} = CoV_{s_u^D}$. We assume the errors to be independent and therefore the uncertainty in the transformation model parameter N_{kt} can be written as

$$CoV_{N_{kt}}^2 = CoV_{spat}^2 + CoV_{\varepsilon_{su}^D}^2 + CoV_{\varepsilon_{qnet}}^2 + CoV_{\varepsilon_t}^2 \quad (14)$$

There will always be a non-zero distance between direct and indirect measurements. Therefore, the underlying true values of undrained shear strength will not exactly be the same, but strongly correlated if close together. We can use the semi-variogram to estimate the contribution of (true) spatial variability between the measured values to the uncertainty in the transformation model parameter:

$$CoV_{spat} = \sqrt{2 \cdot CoV_{s_u}^2 \cdot (1 - \rho(\Delta x, \Delta y))} \quad (15)$$

For the measurement and transformation errors, we expect that only random errors and spatially variable errors lead to variability of the indirectly measured undrained shear strength. Therefore, $CoV_{\varepsilon_{su}^D}$, $CoV_{\varepsilon_{qnet}}^2$ and $CoV_{\varepsilon_t}^2$ in Equation (14) relate to the random and spatially variable errors. For errors in the independent variable (cone resistance), we also expect a bias due to the nature of the regression analysis, see, e.g. Greene (2002).

Systematic measurement errors in the CPT measurements do not add to the variability in the transformation model but will lead to a higher or lower value of the

transformation model parameter. However, the measurements are still correlated to the correct direct measurements and therefore, systematic measurement errors in the CPT measurements cancel out if we use equally biased measurements with a biased transformation model. Systematic measurement errors in the direct measurement, however, are problematic, because those lead to a non-quantifiable bias in the transformation model.

3.5. Results

We use the example from Section 3.3 to investigate the effect of the above-mentioned errors on the calibrated transformation model parameter $\hat{N}_{kt}$ and the uncertainty. To that end, we assume a horizontal and vertical distance between direct and indirect measurements of respectively 1.0 m and 0.10 m and the following (model) error assumptions:

- random measurement error direct measurement $CoV_{\varepsilon_{su}^D}=10\%$; such that $\varepsilon_{su}^D \sim N(1, 0.1)$
- random measurement error indirect measurement $CoV_{\varepsilon_{qnet}}=10\%$; such that $\varepsilon_{qnet} \sim N(1, 0.1)$
- spatially variable transformation error $CoV_{\varepsilon_t}=10\%$; such that $\varepsilon_t \sim N(1, 0.1)$

The results of a simulation are shown in Figure 7. The uncertainty in the transformation model parameter (and variability in the indirectly measured undrained shear strength) is estimated to be: $CoV_{N_{kt}} = \sqrt{0.11^2 + 0.1^2 + 0.1^2 + 0.1^2} = 0.21$.

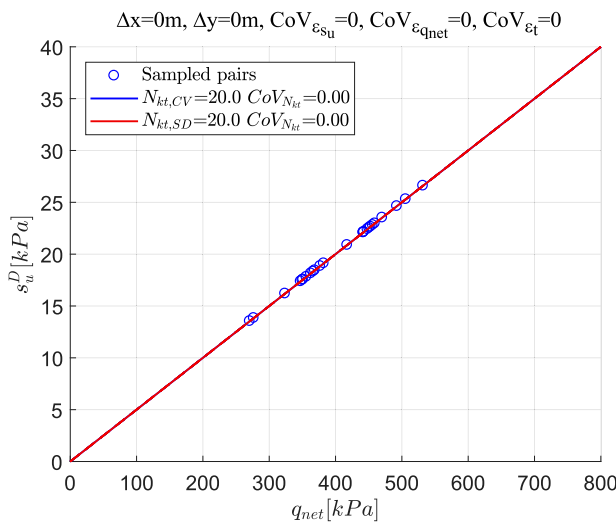


Figure 6. Calibration of the transformation model parameter without uncertainty.

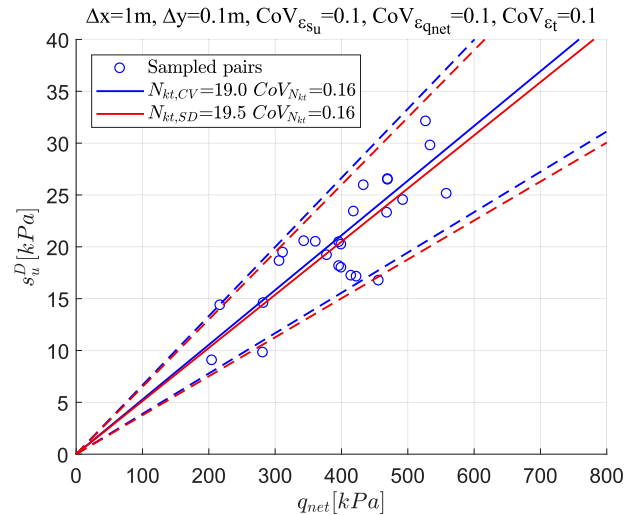


Figure 7. Calibration of the transformation model parameter with extraneous uncertainty. The dashed lines indicate the 90% confidence bounds.

Note, that the total scatter is relatively large, compared to the spatial variability in s_u^I itself, which is quite common in geotechnical engineering. Figure 8 shows the results for $\hat{N}_{kt}$ and $CoV_{N_{kt}}$ for 1000 random fields. On average, the transformation model parameter is biased for both regression methods. This bias is caused by the scatter due to spatial variability, random measurement error in the cone resistance and transformation error. Random measurement errors in the shear strength from laboratory tests lead to scatter, but do not contribute to the bias. Compared to the statistical uncertainty, regression with minimising SD is virtually unbiased for a Gaussian (normally distributed values) field, see Figure 8. The results for $\hat{N}_{kt}$ and CoV with different values of the CoV of the error terms are shown in Table 1.

It is found that the variability in indirectly measured undrained shear strength (or uncertainty in the transformation model parameter) obtained by minimising SD, is on average slightly higher than what was expected based on the modelled errors.

The difference increases with increasing variability and can likely be attributed to additional model error due to the regression method.

The statistical uncertainty in $\hat{N}_{kt}$ is a systematic error and depends only on the number of independent measurement pairs. For this example, the uncertainty is $CoV_{trans,stat} = CoV_{N_{kt}}/\sqrt{n}$. Note, that multiple measurement pairs in one CPT can be not fully independent, because they can have a correlated error.

If the soil property in the random field is assumed to be lognormally distributed, it is found that both methods are equally biased. In this case, there is no preference for one of the two regression methods.

4. Uncertainty in the spatial average

4.1. Introduction

We showed that a virtually unbiased transformation model parameter can be obtained for a site with the

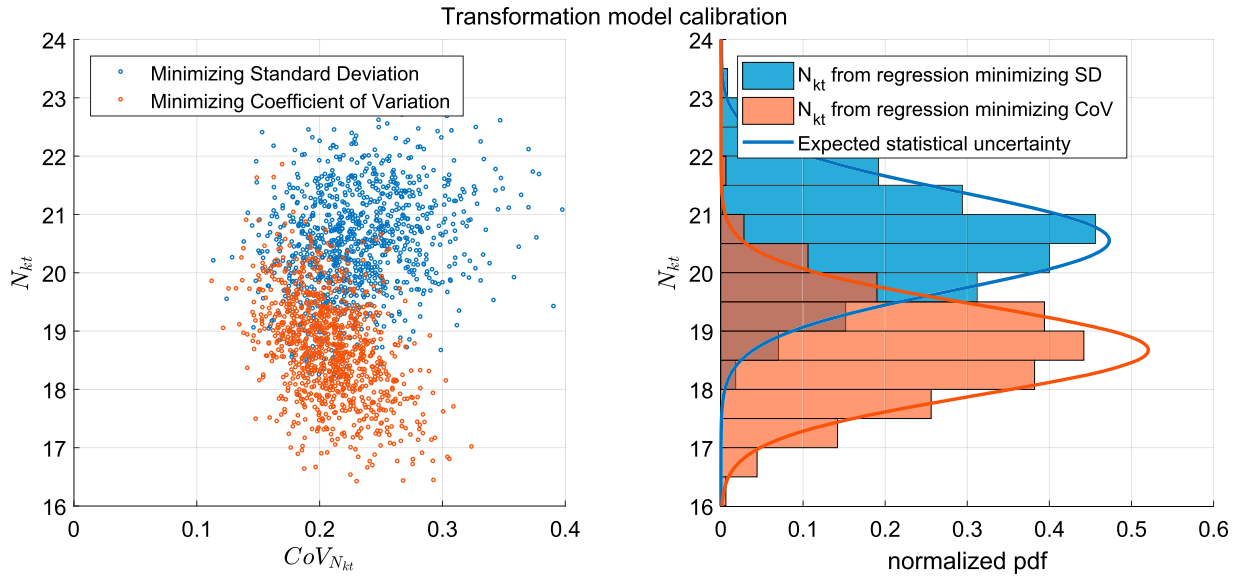


Figure 8. Results of the 1000 times repeated calibration of the transformation model parameter, using two different regression methods.

Table 1. Average results for 1000 times repeated calibration of the transformation model parameter and uncertainty, for different combinations of errors and using two different regression methods.

	Modelled extraneous errors				Regression method				Expected uncertainty
	Error term CoV			$\Delta x = 1\text{m}$ $\Delta y \text{ [m]}$	Min. CoV		Min. SD		
	$\varepsilon_{s^0_t}$ [-]	$\varepsilon_{q_{net}}$ [-]	ε_t [-]		N_{kt}	$CoV_{N_{kt}}$	N_{kt}	$CoV_{N_{kt}}$	
Case 1	0	0	0	0	20.0	0.01	20.0	0.01	0.01
Case 2	0.05	0.05	0.05	0.05	19.6	0.11	20.1	0.11	0.10
Case 3	0.10	0.10	0.10	0.10	18.7	0.21	20.6	0.23	0.21
Case 4	0.15	0.15	0.15	0.15	17.1	0.31	21.2	0.39	0.30
Case 5	0.20	0.20	0.20	0.20	15.3	0.40	22.1	0.59	0.40

described method minimising the SD, except for the statistical uncertainty. We have seen that, besides transformation error, also extraneous sources of uncertainty propagate into the uncertainty of the transformation model parameter obtained by regression analysis.

The considerations so far have been about estimating point values, whereas for most geotechnical problems we are interested in spatial averages, for example, along a shear plane. The approaches proposed in literature in this respect consider only the averaging of the true spatial variability (Vanmarcke 1977; Phoon and Kulhawy 1999a, 1999b), whereas also other random (i.e. non-systematic) errors are subject to averaging, at least when multiple measurements are available. Applying Equation (1) straightforwardly would therefore lead to an overestimation of the total uncertainty. Therefore, the systematic error component of the transformation uncertainty, arguably, is the uncertainty we are actually faced with when executing a geotechnical analysis based on indirect measurements.

We propose two methods to estimate this systematic uncertainty in the transformation model parameter. We investigate the appropriateness of these uncertainty estimates by analysing the difference between the indirectly estimated spatial average and the true known value, while paying special attention to the difference between random, systematic and spatially variable errors. We differentiate between cross-sections where a CPT is present and cross-sections without.

4.2. Proposed method

As argued above, only the systematic part of the transformation model parameter constitutes the uncertainty in the spatial average. To estimate this systematic part, we assume that the total variance in the transformation model parameter consists of a random and a systematic part: $CoV_{N_{kt}}^2 = CoV_{N_{kt},sys}^2 + CoV_{N_{kt},rand}^2$. We introduce the ratio of random variability and total point variability: $r = CoV_{N_{kt},rand}^2 / CoV_{N_{kt}}^2$, such that the systematic component in the transformation uncertainty can be estimated as follows: $CoV_{N_{kt},sys}^2 = (1 - r) \cdot CoV_{N_{kt}}^2$. The random and total part in the variability follow from Equation (14):

$$r = \frac{CoV_{spat}^2 + CoV_{e_{su}}^2 + CoV_{e_{qnet}}^2}{CoV_{N_{kt}}^2} \quad (16)$$

The systematic part of the transformation uncertainty in the indirectly estimated spatial average can be estimated using Equation (17), while accounting for statistical

uncertainty, as concluded in Section 3.5.

$$\begin{aligned} \sigma_{trans} &= \mu_{s_u} \cdot \sqrt{CoV_{N_{kt},sys}^2 + CoV_{N_{kt},stat}^2} \\ &= \mu_{s_u} \cdot \sqrt{\frac{1}{n} + (1 - r) \cdot CoV_{N_{kt}}} \end{aligned} \quad (17)$$

However, in practice, we often do not have this quantitative information on the random (and systematic) error. Therefore, we propose an approach that is based on the observed variability of the indirectly measured undrained shear strength. We can write the total point variability of the indirectly measured undrained shear strength of the entire site as the summation of random and a systematic variance, i.e. fluctuations around the spatial average in CPT j and fluctuations of the spatial average: $\sigma_{s_u}^2 = \sigma_{s_{u,j}}^2 + \sigma_{\bar{s}_u}^2$, see Figure 2. Then, the ratio r can then be rewritten as follows:

$$r \approx \frac{\sigma_{s_{u,j}}^2}{\sigma_{\bar{s}_u}^2} \quad (18)$$

This estimation for the share of random and systematic uncertainty in the transformation model parameter is dependent on the spatial variability, through the total uncertainty (Equations (4) and (5)) and hence, the estimated value according to Equation (18) may deviate from the definition in Equation (16). However, when epistemic uncertainty is dominant in the total uncertainty (which is often the case in geotechnical engineering, see, e.g. Nadim (2015)), then the expected difference is small.

Including the estimated systematic transformation uncertainty in the uncertainty estimate of the spatial average for a cross-section with a CPT (Equation (5)) leads to:

$$\begin{aligned} \sigma_{\bar{s}_u}^2 &= \frac{1}{N_j} \cdot \sigma_{s_{u,j}}^2 + \sigma_{meas,sys}^2 \\ &+ \left(\mu_{s_u} \cdot \sqrt{\frac{1}{n} + (1 - r) \cdot CoV_{N_{kt}}} \right)^2 \end{aligned} \quad (19)$$

The uncertainty in the spatial average at cross-sections without a CPT is estimated by Equation (7). Since this uncertainty is based on the spatial average (of indirect measurements), it includes the averaging of random errors and true spatial variability already.

4.3. Example

The appropriateness of the proposed approach is shown for the synthetic random field in Section 3.5. We use the calibrated transformation model factor from the 1000 random fields to estimate the spatial average undrained

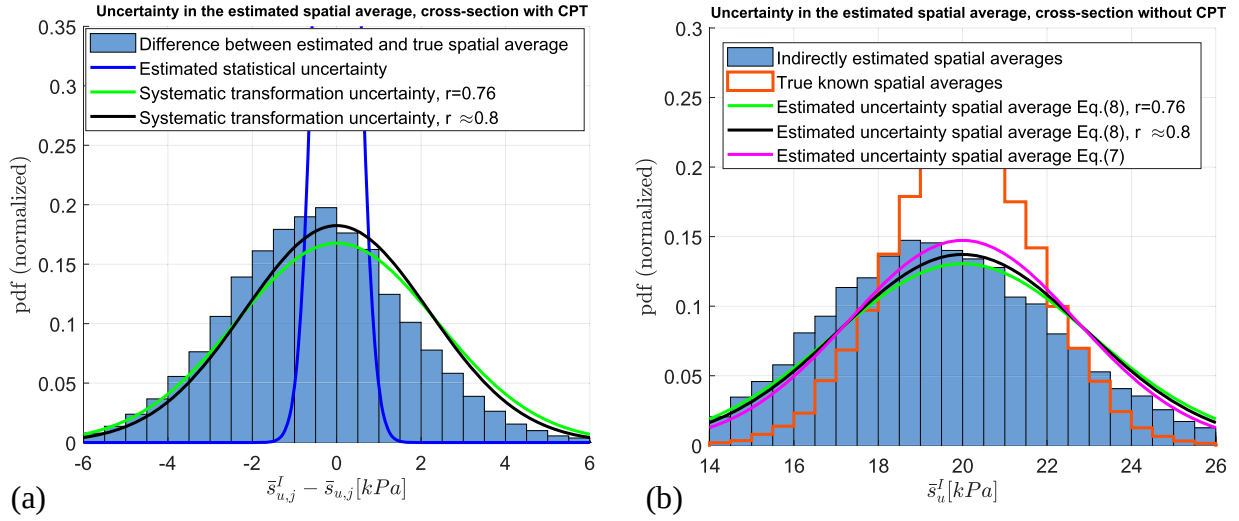


Figure 9. Uncertainty (a) and variability (b) of the spatial average estimated using indirect measurements.

shear strength at cross-sections with and without a CPT. The histogram of the uncertainty in the spatial average from a CPT ($\bar{s}_{u,j}^I - \bar{s}_{u,j}$) is shown in Figure 9(a).

The total uncertainty is, indeed, bigger than only statistical uncertainty in the transformation model (blue solid line), because of the systematic part in the transformation uncertainty. In this example, we determine the systematic part of the transformation uncertainty based on the imposed random errors (values in Table 1) using Equation (14): $r = (0.11^2 + 0.1^2 + 0.1^2) / (0.11^2 + 0.1^2 + 0.1^2 + 0.1^2) = 0.76$. The factor r based on the variability of the indirect measurements: $r \approx \sigma_{s_{u,j}}^2 / \sigma_{s_u}^2 = 0.80$. The estimated value is higher than the value obtained by Equation (16), because the ratio of local point variance and total point variance comprises, besides averaging of random errors, also averaging of true spatial variability. Therefore, the systematic part in the transformation uncertainty is in this case underestimated. In this numerical example where epistemic uncertainty is dominant, the effect on the uncertainty is negligible, demonstrated by the green and black solid lines in Figure 9(a). Both lines match quite well, except a small shift that represents the bias in the transformation model parameter caused by the regression method.

Figure 9(b) shows the variability of the indirectly estimated spatial averages for 1000 random fields. It includes both variability of the spatial average and the systematic part of the epistemic uncertainties as described in Equation (8), depicted by the green and black line. The magenta line shows the estimated uncertainty according to Equation (7) based on the observed variability, which substantiates the appropriateness of the proposed approach.

5. Practical implications

The presented method appropriately accounts for averaging of both spatial variability and random errors in the uncertainty estimate of the spatial average from indirect CPT measurements. The present example has contemplated one end of the spectrum in the sense that measurement errors have been assumed entirely random (i.e. white noise). On the other hand, assuming these errors entirely systematic as done in Phoon and Kulhawy (1999b) is quite conservative. The comparison in Figure 10, in terms of 5%-quantile characteristic values, $s_{u, kar}$, shows that there is considerable margin between these two assumptions. Even with half of the measurement error being systematic and half random, it is still likely that a substantial part of the

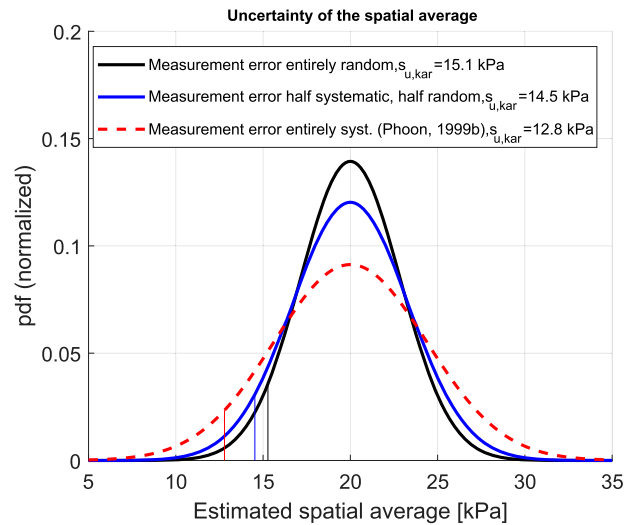


Figure 10. Comparison presented method with established method.

transformation uncertainty is random, because of spatial variability in the transformation model.

6. Conclusions and recommendations

We showed that we can use direct measurements (e.g. laboratory tests) and indirect measurements (e.g. CPTs) from a site to calibrate a site-specific transformation model. This site-specific transformation model can be used to estimate the spatial average of the soil parameter of interest using indirect measurements, which are often cheaper and provide better spatial coverage. On average a virtually unbiased transformation model for a site can be obtained by linear regression with proper choices, contrary to generic transformation models, which can be biased for the entire site.

We demonstrated that the uncertainty in the transformation model parameter contains random errors, which are subject to averaging in the estimation of the spatial average. Therefore, we should not only account for spatial averaging of the actual soil heterogeneity, but also for averaging of random measurement errors. The remaining component in the uncertainty in an indirectly estimated spatial average is, hence, the (locally unknown) systematic bias in the transformation model.

This systematic component of the uncertainty in the site-specific transformation model can be estimated using information on estimates of the random and systematic errors involved, or based on the ratio local versus total (site) point variance. Therefore, we should not only focus our site investigation on estimating the heterogeneity of the subsoil, but also on differentiating between systematic and random errors, e.g. by repetitive laboratory measurements or analysing the spatial variability of the transformation error.

The considerations and results in this article imply that there are several possibilities to reduce the uncertainty in the indirect estimate of the undrained shear strength (or any other parameters obtained in a similar manner). One option is to minimise the distance between a direct and indirect measurements, as spatial variability propagates into transformation uncertainty. Because the transformation model error is expected to be at least to some degree systematic in a vertical, it is recommended to add direct measurements at different CPTs/boreholes, rather than at different depths in the same vertical. The notion that reducing measurement error helps, too, is trivial, yet we have shown that bias in direct measurements is to be avoided particularly.

The advantage of the method proposed in this article is that it is suitable for practical application, since only basic knowledge of statistics is required. The method can be used to determine the soil property and

uncertainty at cross-sections with a CPT and cross-sections without a CPT. In addition, we could also use the method as a starting point for more sophisticated analysis, such as Kriging, or to establish priors for Bayesian analysis, see for example the proposed method by Yang, Xu, and Wang (2017). Such methods require expert knowledge of Bayesian theory or random fields, which is not always mastered by practicing engineers.

Note

1. CPT cone tip resistance (q_c) corrected for pore water pressures (u_2) through the cone factor (a) and normalized for the in situ vertical stress (σ_{v0}): $q_{net} = q_c + u_2 \cdot (1 - a) - \sigma_{v0}$.

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