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Design of a Folded Leaf Spring with high support stiffness at large displacements using the Inverse Finite Element Method

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Abstract. Compliant (flexure) elements provide highly precise motion guiding because they do not suffer from friction or backlash. However, their support stiffness drops dramatically when they are actuated from their home position. In this paper, we show that the existing Inverse Finite Element (IFE) method can be used to efficiently design flexure elements such that they have a high support stiffness in their actuated state. A folded leaf spring element was redesigned using an IFE code written in Matlab^{TM}. The design was validated using the commercial Finite Element software package Ansys^{TM}, showing the desired high support stiffness in the actuated state. The proposed method could aid in the design of more compact flexure mechanisms with a larger useful range of motion.

Keywords: Compliant mechanisms, Flexures, Inverse Finite Elements, Precision, Support stiffness

1 Introduction

Conventional mechanisms based on sliding or rolling contacts (for example ball bearings) typically have limited precision due to the inherent friction and play. *Compliant mechanisms* provide motion differently, by deflection of slender segments called *flexures* [1, 2]. In essence, these mechanisms can be regarded as highly deformable structures and therefore do not suffer from friction or play. This results in a highly repeatable behavior which is essential in high precision applications. However, the support stiffness of a flexure element tends to drop dramatically when the mechanism has undergone a displacement in the actuated direction [3–6]. This limits the useful range of motion and results in bulky designs.

Efforts to increase support stiffness in the actuated state include the addition of torsion reinforcement structures [6, 7], using pre-curved flexures [8, 6, 9], addition of elements in parallel [10, 11] and thickening the middle part of a flexure [2, 12, 13]. These methods all aim at designing the flexure mechanism in its relaxed state, in which it will be fabricated. However, the loss of support stiffness occurs in the actuated (loaded) state, on which the methods do not have direct control.

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In this paper, we show that a flexure element can be designed in its actuated state, such that it will provide a high support stiffness in that position. To enable this, we will use the *Inverse Finite Element (IFE) Method*, originally proposed by Givindjee and Mihalic [14] and recently introduced to the compliant mechanisms community by Albanesi et al. [15]. Albanesi et al. have used the IFE method before to design compliant mechanisms. However, the authors use the method to control contact forces (for example in a compliant gripper). Instead, we use the IFE method to alter the support stiffness of the compliant mechanism itself. In other previous work [8, 6, 9], flexures have been pre-curved in order to obtain a high support stiffness in the actuated state. However, the authors use an optimization method, whereas we will use the IFE method which is computationally more efficient. Furthermore, this paper focuses on a different flexure element which avoids the high actuation forces mentioned in [8, 9].

First, the essence of the IFE method will be outlined. Second, the method used to increase support stiffness will be demonstrated by redesigning an existing flexure element. We will reflect on the work in the Results and Discussion section and the main constributions to literature will be summarized in the Conclusions section.

2 Method

In this section, first the essence of the IFE method from [15, 14] will be outlined. Second, a flexure element commonly used in industry will be introduced. Using the proposed method, this element will be redesigned in order to obtain a high support stiffness in its actuated state. Lastly, an application example will be given.

2.1 The Inverse Finite Element Method

The Inverse Finite Element (IFE) Method can be used to retrieve the relaxed (stress-free) shape of a structure when its loaded (stressed) shape and its external forces are specified [15, 14]. This includes analyses with large nonlinear deflections. Note that a compliant mechanism can be regarded as a structure undergoing large deflections.

First consider a regular (that is, forward) nonlinear Finite Element analysis in which the shape of a structure in its relaxed state is specified. The goal is to compute the unknown loaded shape as a result of specified external forces. Consider these forces to be independent of the shape (no follower-forces). The goal then is to solve

$$\mathbf{R}(\mathbf{U}) = \mathbf{0}, \tag{1}$$

which is the residual vector containing the imbalance of internal and external forces and moments in the loaded state. $\mathbf{U}$ is a vector containing the nodal displacements and rotations which need to be found. Note that for large deflections, $\mathbf{R}$ can have a nonlinear dependence on $\mathbf{U}$. We could also write

$$\mathbf{U} = \mathbf{X} - \mathbf{X_0}, \tag{2}$$

where $\mathbf{X_0}$ and $\mathbf{X}$ are the nodal coordinates in the relaxed and loaded states, respectively. Now we can write

$$\mathbf{R}(\mathbf{X_0}, \mathbf{X}). \tag{3}$$

In the forward FE method, $\mathbf{X_0}$ is specified and remains unchanged in the analysis. After applying boundary conditions, the residual equation is generally solved using some iterative method (for instance Newton-Rhapson) often relying on the gradient with respect to the unknowns $\mathbf{X}$:

$$\mathbf{K}_{fwd} = \frac{\partial \mathbf{R}(\mathbf{X_0}, \mathbf{X})}{\partial \mathbf{X}} \tag{4}$$

which is the stiffness matrix of the structure.

In the IFE method, the loaded shape $\mathbf{X}$ is specified as an input in $\mathbf{R(X_0}, \mathbf{X})$ which is then solved for the unknown relaxed shape $\mathbf{X_0}$ using the gradient with respect to these unknowns:

$$\mathbf{K}_{inv} = \frac{\partial \mathbf{R}(\mathbf{X_0}, \mathbf{X})}{\partial \mathbf{X_0}} \tag{5}$$

Note that mathematically, the IFE method is very similar to the regular forward FE method. Because of this similarity, their computational cost is also similar. Furthermore, note that the internal forces of the body are not needed as an input in the IFE method, since these are already determined once the loaded and unloaded shapes are known. Only the loaded shape, boundary conditions and external forces have to be known. The IFE routine can be modified such that instead of an external force, a displacement can be imposed. This modification is the same as in the case of the forward FE method (see for example [16]).

We have written an IFE code in Matlab using the 2D Euler-Bernoulli beam formulation. Large, nonlinear deflections are included using a co-rotational formulation as in [17]. The material is assumed to behave linearly elastic.

2.2 The folded leaf spring

Figure 1 shows a linear guide commonly used in industry but less prevalent in literature [2, 18, 19]. Its central body is assumed rigid and is guided along the indicated Y direction by the six Folded Leaf Springs (FLS), which allow this motion by bending deformations. The blocks at the extremities of the leaf springs are attached to the fixed world. The design challenge typically is to provide a low stiffness along the actuated direction (Y in this case), while maintaining a high support stiffness in all other five spatial degrees of freedom of the central body.

Figure 2a shows one FLS element isolated from the mechanism. In its initial, relaxed state, the element provides a high support stiffness to the middle body in the indicated Z direction. This is because the complete mechanism in its turn constrains (stiffens) rotations around the X and Y axes of the FLS at this point

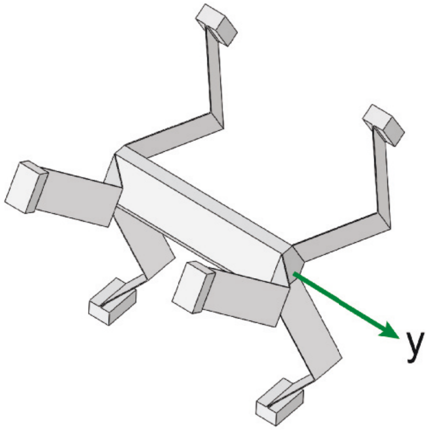
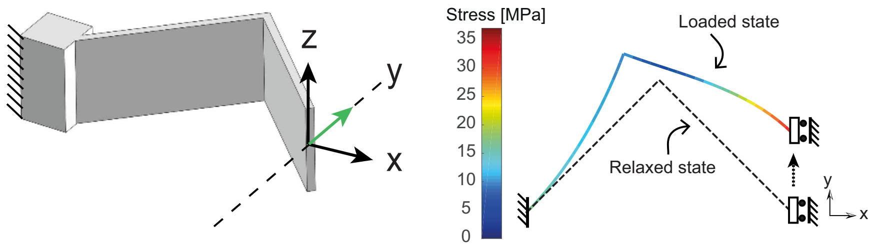


Fig. 1. A compliant linear guide commonly used in industry, existing of six Folded Leaf Springs (FLS) [2, 18, 19]. These slender elements allow for movement in the Y direction by elastic bending deformations.

(refer to [2] for a more detailed explanation). Figure 2b shows a 2D representation of the same FLS in the XY-plane. The left side is attached to the fixed world. The right side is constrained in rotation and in the X-direction to simulate the connection to the central rigid body. The dashed line represents the relaxed state of the FLS, while the colored solid line shows the FLS in actuated (loaded) state. The colors indicate the stresses. In this loaded state the FLS inevitably becomes curved and thereby loses a significant part of its support stiffness in Z direction [3–6]. This issue will be addressed in the next section.



(a) Isolated FLS. (b) FE simulation of the FLS

Fig. 2. Figure a) shows a single FLS isolated from the linear guide from Fig. 1. Figure b) shows the curvature occurring in its loaded state, causing the dramatic decrease in support stiffness in Z direction.

2.3 Redesign of the folded leaf spring using the IFE method

In this section, we will redesign the FLS element such that it provides a high support stiffness in its actuated (loaded) state, instead of in its relaxed home position. The main idea is to design the FLS such that it has an initial curvature in its relaxed state. This curvature should be such that the FLS becomes straight in its loaded state, thereby increasing its support stiffness. Figure 3 shows this FLS element. Using the IFE method, the FLS can be designed in its actuated state at some specified displacement. The IFE method will output the relaxed, pre-curved shape of the FLS. Note that only the loaded shape and displacement have to be specified to fully determine the relaxed shape shown in Fig. 3. The displacement is chosen as 21.4 mm. The straight beams both have a length of 111.8 mm. The choice of these numbers will be explained in the next section.

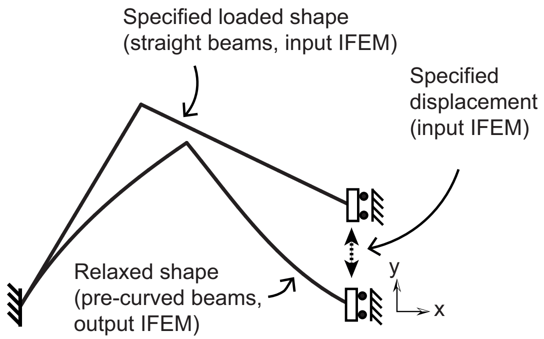
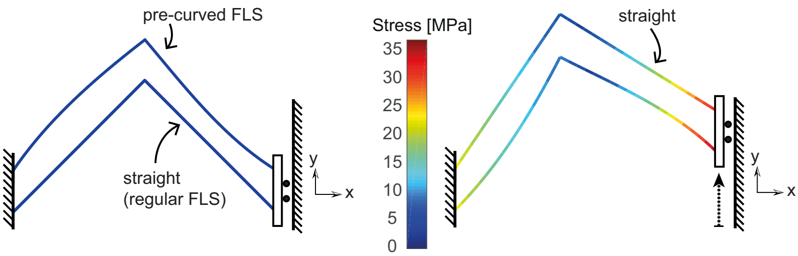


Fig. 3. This FLS is initially curved such that it will attain a straight shape after actuation in Y direction, providing a high support stiffness in this position. The pre-curved relaxed shape can be computed efficiently using the IFE method.

2.4 Design and validation of the combination element

The pre-curved FLS designed in the previous section will provide a high support stiffness around its actuated position, but a low support stiffness around its home position. To realize a high support stiffness around both positions, we can add a pre-curved FLS to a regular FLS to form a *combination element* as in Fig. 4. Figure 4a shows this combination element in the relaxed state, where the mechanism is in its relaxed home position. Here, the regular, straight flexure provides support stiffness in the Z direction. In the actuated state shown in Fig. 4b, the pre-curved FLS has become straight and now provides support stiffness in its turn.



(a) Home position (relaxed) (b) Actuated state (loaded)

Fig. 4. Combination element in which a pre-curved FLS is combined with a regular FLS. In a), both flexures are relaxed and the straight flexure provides support stiffness in the Z direction. In its actuated state shown in b), the pre-curved flexure has become straight and provides support stiffness in its turn.

The combination element is designed as follows. First some properties and dimensions of the regular FLS are chosen as summarized in Table 1. Using the commercially available FE package Ansys^{TM}, the support stiffness of this FLS is computed while it is actuated along its motion range Y as in Fig. 2b. Beam188 elements are used, with the option for nonlinear geometry activated. Note that this data cannot be computed using the code written in Matlab, because that code only considers beams in 2D. Using the obtained support stiffness data, we can now decide at which point in the displacement range the pre-curved FLS should become straight in order to compensate for the stiffness loss of the initially straight FLS. The shape of the relaxed pre-curved FLS will be computed using the IFE code written in Matlab. Finally we will validate the support stiffness of the combination element over its full range using Ansys. For a fair comparison with the state-of-the-art, the combination element will be compared to an element consisting of two regular, initially straight FLS elements.

Table 1. Properties of the folded leaf spring as shown in Fig. 2.

Variable	Value
Size X direction	100 mm
Size Y direction	50 mm
Size Z direction	10 mm
Flexure thickness	1 mm
E-modulus (Polyactic Acid, PLA)	4 GPa

3 Results and Discussion

Figure 5 shows the support stiffness of the combination element shown in Fig. 4 along its displacement range (red solid line). This stiffness is compared to that of an element with two regular, initially straight FLS elements shown by the black solid line. As anticipated, the combination element shows a lower support stiffness for small displacements because only one of the FLS elements provides support stiffness at this point. However, at larger displacements, the combination element outperforms the double regular FLS design because the pre-curved flexure has become straight. Furthermore, the support stiffness of the combination element is more constant along the displacement range. This results in a more straight trajectory of the mechanism under influence of gravity forces.

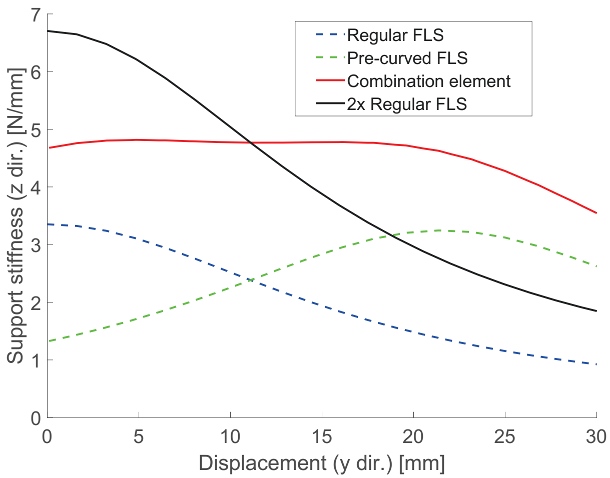


Fig. 5. Support stiffness of the combination element shown in Fig. 4 (red), compared to the case with two regular, straight flexures (black), showing an extension of the range with high support stiffness. The dashed lines in green and blue show the contributions of the two flexures forming the combination element.

More pre-curved FLS elements could be added to cover a larger region of high support stiffness. These can be designed such that they cover the region in the negative Y direction, using the same proposed method. Adding more elements will increase the stiffness in the actuation direction y, which is generally not desired. However, this could be dealt with by *static balancing techniques* [20].

The green dashed line in Fig. 5 shows the support stiffness of the pre-curved redesigned FLS. The stiffness peak is shifted to the actuated state, but its shape is similar to the stiffness peak of the regular FLS shown in dashed blue. This result validates the use of the IFE code in Matlab to shift support stiffness from the home position to the actuated state. The shifting distance is specified as 21.4 mm, in order to provide an optimal stiffness overlap resulting in a constant support stiffness of the combination element.

Computing the shape of the pre-curved FLS shown in Fig. 3 was done efficiently using the IFE code written in Matlab. Such a computation takes around half a second using the code on a regular laptop. Using IFE routines to shift support stiffness will be more advantageous when considering flexure mechanisms exhibiting complex spatial behavior. When shell elements need to be used instead of beam elements, the analysis can become computationally too expensive for the use of optimization methods. The IFE method could be used in these cases to provide a solution.

The validation of the combination element was done only theoretically. Flexures are often fabricated out of high-strength steels using Wire-Electrical Discharge Machining. This technique readily allows for fabrication of initially curved flexures. However, it is anticipated that considerable attention should be paid to tolerances in thickness of the flexures, since the bending stiffness has a cubic relation to this property. Furthermore, the combination element in Fig. 4 is partly *overconstrained* (see [2] for a detailed explanation). Overconstraints could result in unpredictable behavior under the influence of temperature changes or manufacturing errors. However, this overconstraint is only present in the region of the displacement range where the two FLS elements provide comparable support stiffness. In the other configurations, only one of the FLS elements effectively provides support stiffness and so the system can be considered to not be overconstrained.

4 Conclusion

In this paper, we showed that the existing Inverse Finite Element (IFE) Method can be used to efficiently design flexure elements such that they have a high support stiffness in their actuated state. This is beneficial because most existing flexure elements only provide support stiffness at their relaxed home position. The method is efficient because it does not need design iterations, as is the case in optimization methods.

As an example, a Folded Leaf Spring (FLS) element with a high support stiffness in its actuated state was designed using an IFE code written in Matlab. The resulting *pre-curved* design was analyzed using the commercial Finite Element software package Ansys$^{\mathrm{TM}}$. The results show that the element indeed has a large support stiffness in its actuated state instead of in its home position.

We showed an example implementation of this pre-curved FLS, in which it is combined with a regular FLS element such that it will provide a high support stiffness in both its home position and its actuated state. Using Ansys, this combination element was compared to a benchmark design consisting of two regular FLS elements. As expected, the benchmark design has a higher support stiffness at the home position, but the combination element outperforms it at the actuated positions and shows a more constant support stiffness along its range of motion.

The proposed method could aid in improving support stiffness of flexure mechanisms in their actuated state, resulting in more compact designs with larger range of motion.

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