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LQR Optimal Control of Four-steering Vehicle Based on Particle Swarm Optimization Algorithm

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ABSTRACT

This paper proposes a linear quadratic controller based on particle swarm algorithm for the rear wheel control of four-wheel steering vehicle. Particle swarm optimization with fitness functions is used to optimize the coefficients of the weight matrix offline. The fuzzy rules following the controller is used if the road condition is terrible. The simulation results show that the LQR control model based on particle swarm optimization makes the trajectory tracking of the vehicle better and the side slip angle of the vehicle lower. It can be proved that the controller has positive effect on handling stability of the vehicle and safety of drivers.

CCS CONCEPTS

• **General and reference** → Document types; Surveys and overviews.

KEYWORDS

LQR, particle swarm optimization, side slip angle, fuzzy controller

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1 INTRODUCTION

With the rapid development of automobile technology, people have put forward higher requirements for automobile speed, handling stability and safety. The steering stability of the vehicle when turning is very important. At present, the steering technology of automobiles is mainly divided into front-wheel steering (FWS) and four-wheel steering. Although the rear wheel angle of the four-wheel steering vehicle is relatively small, it significantly improves the steering stability of the vehicle. The difficulty of four-wheel steering(4WS) technology lies in the control strategy.

Many scholars have done a lot of research on the control strategies of the rear wheel angle. In 1986, Sano proposed a 4WS controller with a fixed ratio between front and rear wheels' steering angle [1]. The disadvantage of the above control method is that the vehicle's yaw rate gain will vary with the front wheel angle, which would be detrimental to the driver's manipulation. Various control algorithms have been applied to 4WS vehicle including sliding mode control [2-3], fuzzy control [4-5]. Many experts research four-wheel steering systems with robust characteristics and optimal control characteristics in recent years. In 2008, Canale et al. proposed a robust controller based on 4WS vehicles, taking into account the understeering property and stability requirements of the vehicle [6]. Du Feng and others used a linear quadratic regulator (LQR) for the rear wheel angle [7]. Luo Yutao and others proposed a linear quadratic controller based on genetic algorithm for the coordinated control of torque of 4WS vehicle [8]. Liu Qijia used the weight function method to perform the tire cornering stiffness based on the linear optimal quadratic control strategy [9]. Xie Xianyi et al. proposed a linear quadratic control method with variable weight coefficients in view of different road friction coefficient conditions [10]. Xinbo Chen et al. proposes a linear quadratic regulator combined with feedback control which shows good robustness [11].

This paper presents a control method for the rear wheel angle of a 4WS vehicle. The rear wheel angle works on the optimization of

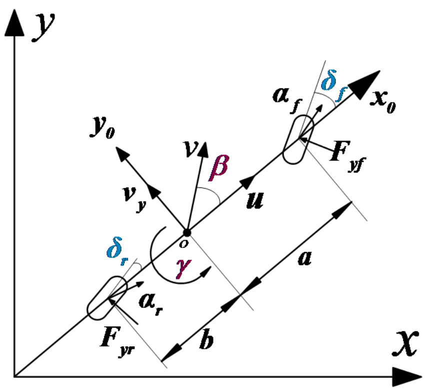


Figure 1: Two-DOF Reference Model

the handling stability of the vehicle. Controlling the vehicle's yaw rate and side slip angle could increase the margin of tire lateral force and improve handling stability. In this paper, an optimal control method is used to control the rear wheel angle which is thesum of the feedforward of front wheel angle, and the feedback of yaw rate and side slip angle. At the same time, the particle swarm algorithm is used to control the weight matrix coefficient of LQR in a reasonable range. The optimal search is carried out in the internal, so that the objective function of the error is minimized, and the best control effect is achieved.

2 VEHICLE MODEL

This article uses a LQR based on particle swarm optimization (PSO) to control the rear wheel angle of a 4WS vehicle for better handling and stability of the vehicle. First, the dynamics of the vehicle need to be modeled to characterize the kinematics and dynamics of the vehicle. In this paper, a two-degree-of-freedom (2-DOF) dynamic model of a 4WS vehicle as shown in Figure. 1 is used to express vehicle handling stability. According to the equilibrium relationship between the lateral force and torque of the vehicle, the simplified kinematics equation of the vehicle can be obtained as following:

$$\begin{cases} \dot{\beta} = \frac{(k_f+k_r)}{mu}\beta + (\frac{ak_f-bk_r}{mu^2} - 1)\gamma - \frac{k_f}{mu}\delta_f - \frac{k_r}{mu}\delta_r \\ \dot{\gamma} = \frac{ak_f-bk_r}{I_z}\beta + \frac{a^2k_f+b_rk_r}{I_z u}\gamma - \frac{ak_f}{I_z}\delta_f + \frac{bk_r}{I_z}\delta_r \end{cases} \tag{1}$$

As shown in Figure 1, F_{yf}, F_{yr} represents the lateral force of the ground against the two front wheels and the two rear wheels, respectively. δ_f, δ_r represents the steering angle of the front and rear wheels of the vehicle, a,b represents the distance from the vehicle centroid to the front and rear axles, v represents the longitudinal velocity of the vehicle and u is the lateral velocity, β is the side slip angle, and γ is the yaw rate.

Since the tire side slip angle of both the front and rear wheels is very small, the linear expression of the lateral force and angle could be approximated as follows:

$$\begin{cases} F_{yf} = k_f\alpha_f \\ F_{yr} = k_r\alpha_r \end{cases} \tag{2}$$

$$\begin{cases} \alpha_f = \frac{v+a\gamma}{u} - \delta_f \\ \alpha_r = \frac{v-b\gamma}{u} - \delta_r \end{cases} \tag{3}$$

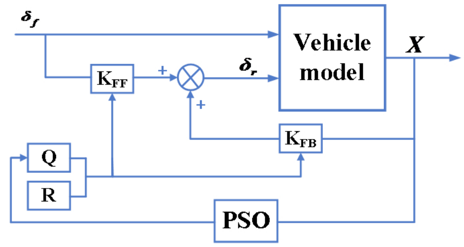


Figure 2: Structure of PSO-LQR

Where k_f,k_r represent the tire cornering stiffness of the front and rear wheels, and α_f, α_r represent tire side slip angle of the front and rear wheels respectively. Since the turning angles of the front and rear wheels of the vehicle are small, it can be approximated as $\beta \approx v/u$,$\cos\delta_f = 1$, $\cos\delta_r = 1$. Substitute Eq. 2) and Eq. 3) into Eq. 1), the 2-DOF vehicle dynamics model can be expressed as follows:

$$\begin{cases} \dot{\beta} = \frac{(k_f+k_r)}{mu}\beta + (\frac{ak_f-bk_r}{mu^2} - 1)\gamma - \frac{k_f}{mu}\delta_f - \frac{k_r}{mu}\delta_r \\ \dot{\gamma} = \frac{ak_f-bk_r}{I_z}\beta + \frac{a^2k_f+b_rk_r}{I_z u}\gamma - \frac{ak_f}{I_z}\delta_f + \frac{bk_r}{I_z}\delta_r \end{cases} \tag{4}$$

the form of the equation of state can be expressed as follows:

$$\dot{X} = AX + BU + CW \tag{5}$$

The matrices in above equation are expressed as follows:

$$A = \begin{bmatrix} \frac{(k_f+k_r)}{mu} & \frac{ak_f-bk_r}{mu^2} - 1 \\ \frac{ak_f-bk_r}{I_z} & \frac{a^2k_f+b_rk_r}{I_z u} \end{bmatrix}, B = \begin{bmatrix} -\frac{k_r}{mu} \\ \frac{bk_r}{I_z} \end{bmatrix}, C = \begin{bmatrix} -\frac{k_f}{mu} \\ -\frac{ak_f}{I_z} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix}, X = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, U = [\delta_r], W = [\delta_f].$$

3 CONTROLLER DESIGN

3.1 A Linear Quadratic Regulator

In this part, a linear quadratic controller based on particle swarm algorithm is designed.

As shown in Figure 2, K_{FB} is the feedback gain matrix of the state variable, K_{FF} is the feedforward gain matrix of the front wheel. Q and R are both weighting matrices. The feedback gain of state varaible and the feed forward gain of the front wheel are used to control the rear wheel. The optimal control function of the rear wheel is expressed as following:

$$U = \delta_r = K_{FB}X + K_{FF}\delta_f \tag{6}$$

According to the difference value between the system state and the ideal state and the size of the control quantity, an appropriate weight coefficient is designed for the optimal control strategy of the rear wheel angle.

When the vehicle enters a steady state, the yaw rate remains unchanged. Therefore, the system state could be expressed as:

$$\dot{\gamma} = \dot{\beta} = 0 \tag{7}$$

Then the equation is expressed as follows:

$$\begin{cases} 0 = \frac{(k_f+k_r)}{mu}\beta + (\frac{k_f a-k_r b}{mu^2} - 1)\gamma - \frac{k_f}{mu}\delta_f \\ 0 = \frac{ak_f-bk_r}{I_z}\beta + \frac{a^2k_f+b_rk_r}{I_z u}\gamma - \frac{ak_f}{I_z u}\delta_f \end{cases} \tag{8}$$

The ideal yaw rate of 4WS vehicle can be expressed as follows:

$$\gamma_d = \frac{u \cdot k_f k_r (a+b)\delta_f}{k_f k_r (a+b)^2 + mu^2 (k_f a - k_r b)} \tag{9}$$

The ideal side slip angle of a car which is steering is 0. The ideal model of the control system can be expressed as follows:

$$X_d = \begin{bmatrix} \beta_d \\ \gamma_d \end{bmatrix} = A_d \delta_f = A_d W \tag{10}$$

Where $A_d = [\begin{smallmatrix} 0 \\ \frac{u \cdot k_f k_r (a+b)\delta_f}{k_f k_r (a+b)^2 + mu^2 (k_f a - k_r b)} \end{smallmatrix}]$.

In order to find the optimal control strategy, the difference value should be minimized. The fitness function is designed as follows:

$$J = \frac{1}{2}\int_0^\infty [(X - X_d)^T Q (X - X_d) + U^T R U] dt \tag{11}$$

The first term in the formula is the error between the system state and the ideal state, and the second term is the value of the control variable. The weighting matrix Q and R is used to define a target midway between the two to achieve the optimal control effect.

Using the variational method to solve the quadratic optimal problem, and the following Hamiltonian function with three equations are constructed as follows:

$$H = \frac{1}{2}(X - X_d)^T Q (X - X_d) + \frac{1}{2} U^T R U + \lambda(t)(AX + BU + CW) \tag{12}$$

The governing equation is as following:

$$\frac{\partial H}{\partial U} = RU + B^T \lambda(t) = 0 \tag{13}$$

Where the control variable $U(t) = -R^{-1} B^T \lambda(t)$.

The regular equation is as following:

$$\dot{X} = \frac{\partial H}{\partial \lambda} = AX - BR^{-1} B^T \lambda(t) + CW \tag{14}$$

The adjoint equation is as following:

$$-\frac{\partial H}{\partial X} = \lambda(t) = -Q(X - X_d) - A^T \lambda(t) \tag{15}$$

In Eq. (16) $P(t)$ is expressed as following:

$$\begin{cases} \lambda(t) = P(t)X - \varepsilon(t) \\ \dot{\lambda}(t) = P(t)X + P(t)X - \dot{\varepsilon}(t) \end{cases} \tag{16}$$

$\varepsilon(t)$ are intermediate variables.

The matrix A, B, Q is assumed as constant matrices, $t \longrightarrow \infty$,P(t) = 0, ε(t) = 0.Substitute these formulas into Eq. (16):

The Riccati equation is as following:

$$PA + A^T P - PBR^{-1} B^T P + Q = 0 \tag{17}$$

Solving Eq. (17), the control variable is as following:

$$U(t) = -R^{-1} B^T P X + R^{-1} B^T (PBR^{-1} B^T - A^T)^{-1} (QA_d - PC) W \tag{18}$$

Where

$$\begin{cases} K_{FB} = -R^{-1} B^T P \\ K_{FF} = R^{-1} B^T (PBR^{-1} B^T - A^T)^{-1} (QA_d - PC) \end{cases} \tag{19}$$

Table 1: Setting of Optimization Parameters of Particle Swarm Optimization

Parameters	Value
Size of population	10
Size of generation	10
The range of q_v	[1, 3]
The range of q_r	[0.008,0.012]
The range of v_x	[-0.5,0.5]
The range of v_y	[-0.003,0.003]
Learning factor C_1 and C_2	0.5,0.5
Inertia weight	0.8

3.2 PSO-LQR

The coordinated control effect of LQR on the side slip angle and yaw rate of vehicle depends on the selection of the weight matrix parameters. When the road friction coefficient is low, the side slip angle of the vehicle should to be smaller, and when the road friction coefficient is high, it pays more attention to tracking the ideal yaw rate. Therefore, a LQR controller is designed based on particle swarm algorithm, and the fitness function is defined for the minutest cumulative tracking error. The fitness function is as following:

$$\min J = \int_0^T t \left| e_\beta(t) \right| dt + \int_0^T t \left| e_\gamma(t) \right| dt \tag{20}$$

Where $e_\beta = \beta - \beta_d$, $e_\gamma = \gamma - \gamma_d$.The optimization objective of the LQR controller based on the particle swarm algorithm is the Q matrix, which is denoted as:

$$Q = \begin{bmatrix} q_v & 0 \\ 0 & q_r \end{bmatrix} \tag{21}$$

Set the optimization parameters as shown in Table 1. The particle swarm algorithm takes a long time to calculate, and it cannot meet the requirements of real-time vehicle motion. Therefore, the simulation is performed offline to obtain the optimal weight matrix under a specific road.

Taking the working condition with a road friction coefficient of 0.85 as an example, the optimization process of the fitness function of the particle swarm algorithm is shown in Figure 3. It can be seen from the figure that PSO is searching for better parameters and the populations' fitness value is decreasing.

3.3 Fuzzy Controller

When the vehicle speed is high and the road friction coefficient is small, sild slip angle is way worth more than yaw rate for handling stability of the vehicle. This paper designs a fuzzy controller when the road friction coefficient is low. The particle swarm algorithm obtains the optimal weight matrix for the maximum and minimum road friction coefficients at first. Then the fuzzy controller is used to adjust and allocate the weight coefficients dynamically. The established fuzzy rules are as shown in Table 2 [8], and the surfaces of the fuzzy controller are shown in Figure 4.

Table 2: Fuzzy Control Rules

μ	q_v	q_r
NB	PB	NB
NM	PS	NS
NS	PS	NS
ZE	ZE	ZE
PS	NS	PS
PM	NS	PS
PB	NB	PB

Table 3: Vehicle Parameters

Symbol	Unit	Value	Symbol	Unit	Value
m	kg	1270	I_z	kg·m^2	1536.7
a	m	1.015	C_f	N/rad	-66450.8
b	m	1.895	C_r	N/rad	-47730.9

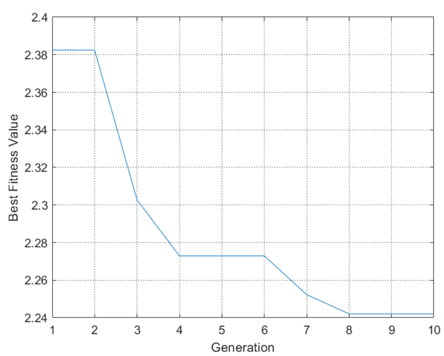


Figure 3: The Iterative Process

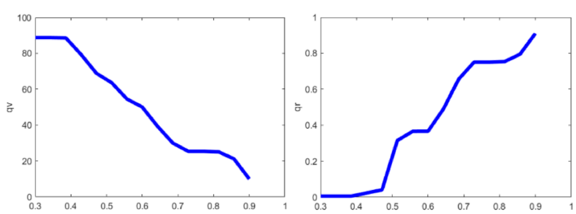


Figure 4: Fuzzy Surfaces

4 SIMULATION VALIDATION

Carsim is a vehicle model simulation software which provides high-precision vehicle dynamics simulation. The 2-DOF vehicle dynamics model is built in Carsim and the PSO-LQR is expressed in Matlab/Simulink. The controller parameters of the vehicle are shown in Table 3

4.1 Simulation A

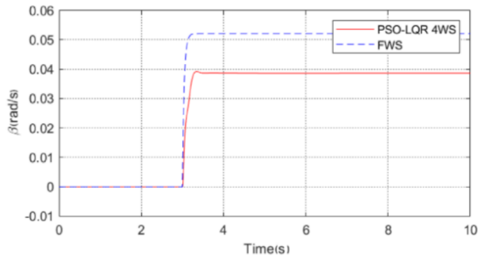
Input 0.1 radians of angle for the front wheel, the parameters of matrix Q is given by PSO. The simulation results are shown in Figure 5.

It can be seen from Figure 5 (a) that, when the front wheel angle is input by 0.1 radians at the third second, the 4WS vehicle based on PSO-LQR quickly responds within 0.15 seconds and reaches a steady state. Compared with the FWS vehicle in the Figure 6 (b), the 4WS vehicle has an optimization degree of 25.9% for the sideslip angle. It can be seen from Figure 5 (b) that the yaw rate of 4WS vehicle based on PSO-LQR reaches the steady state within 0.08 seconds. At this time, the steady-state yaw rate of the vehicle is slightly larger, which ensures a smaller turning radius of the vehicle. It can be seen from Figure 5 (c) that the controller controls the rear wheel angle of the vehicle within a reasonable range to ensure the stability of the vehicle.

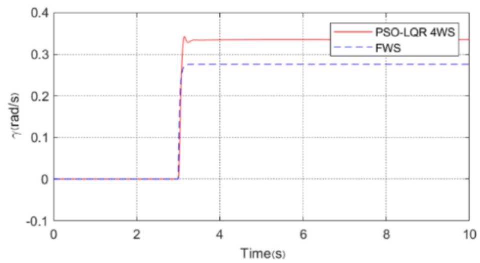
4.2 Simulation B

In order to verify the control effect of the controller when the vehicle is on double lane change, the weight matrix is optimized offline. According to ISO3888-1:1999, the double-line shifting experiment was carried out, and the road friction coefficient was set to 0.85. The speed of the vehicle is 60km/h. The simulation results are shown in Figure 6.

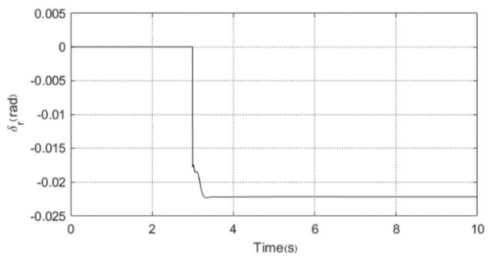
As can be seen from Figure 6 (a), The 4WS vehicle obtains a smaller side slip angle than the FWS vehicle with the driver model in Carsim under the double lane change test, and the peak value is reduced by 96.1%. Compared with the Front-Feedback 4WS vehicle, the side slip angle of PSO-LQR is rather lower. It is obvious that the side slip angle almost vanished. It can be seen from Figure 6 (b) that the difference of yaw rate between the ideal yaw rate and the 4WS vehicle controlled by PSO-LQR is the smallest of the three. This will reduce the fatigue of drivers.



(a) The vehicle's side slip angle



(b) The vehicle's yaw rate



(c) The vehicle's rear wheel angle

Figure 5: Simulation results on $v_x = 60km/h$, $\mu = 0.85$

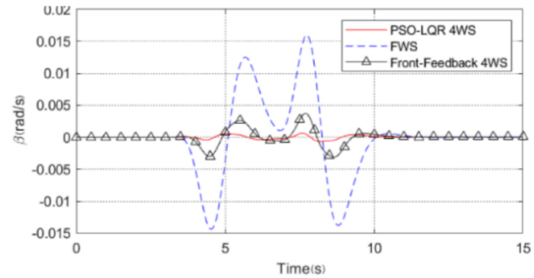
4.3 Simulation C

This simulation is used to verify the effect of the controller when the vehicle is at a high speed and the road friction coefficient is low. The speed of the vehicle is 100km/h and the road friction coefficient is 0.4. The results of the simulation are shown in Figure 7.

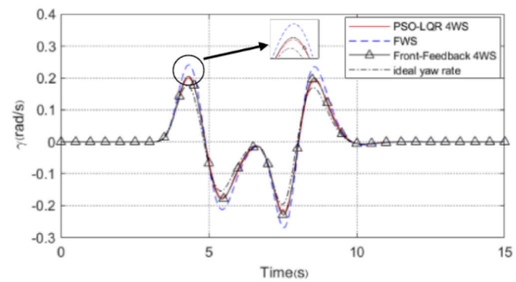
It is visible from the Figure 7 (a) that, compared with the traditional LQR controller, PSO-LQR ensures that the error between the vehicle's trajectory and the standard double lane change is smaller. In Figure 7 (b), the side slip angle of PSO-LQR is rather lower.

5 CONCLUSION

In this paper, the particle swarm algorithm is used to adjust the parameters of the linear quadratic regulator. The effectiveness of the control strategy is verified in the Simulink-Carsim environment. It is visible that the PSO-LQR could reduce the side slip angle widely and track the ideal yaw rate nicely. The controller still has a lower side slip angle and great effect on trajectory tracking when the road is ice-snow or rainy and the speed of the vehicle is high. The controller ensures better handling stability of the vehicle and safety of drivers.

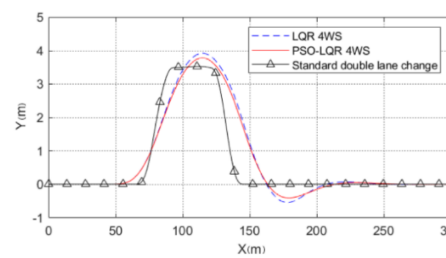


(a) The vehicle's side slip angle

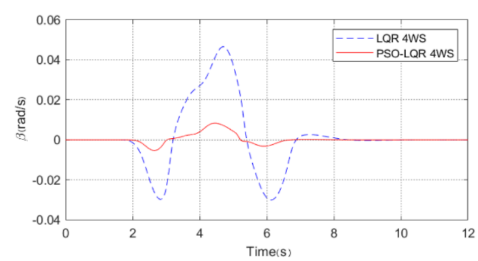


(b) The vehicle's yaw rate

Figure 6: Simulation results on $v_x = 60km/h$, $\mu = 0.85$, double lane change



(a) The vehicle's trajectory



(b) The vehicle's side slip angle

Figure 7: Simulation results on $v_x = 100km/h$, $\mu = 0.4$

At present, the research on four-wheel steering technology is still in developing. This article believes that the 4WS control strategy should focus on the following aspects in the future. First, the estimation of state parameters is important. The accurate state estimation should be carried out for state parameters that are not easily available, such as the side slip angle of the vehicle. This will ensure the accuracy of control strategy. Second, more consideration should be given to the influence of the non-linear characteristics of tires. At last, the mature design of the control system is very important, otherwise an inaccurate output of rear wheel angle may bring serious safety hazards to the car.

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