Integrating ride-hailing services with public transport
a stochastic user equilibrium model for multimodal transport systems

Liu, Bing; Ji, Yuxiong; Cats, Oded

DOI
10.1080/23249935.2023.2236240

Publication date
2023

Document Version
Final published version

Published in
Transportmetrica A: Transport Science

Integrating ride-hailing services with public transport: a stochastic user equilibrium model for multimodal transport systems

Bing Liu, Yuxiong Ji & Oded Cats

TRANSPORTMETRICA A: TRANSPORT SCIENCE
https://doi.org/10.1080/23249935.2023.2236240

Taylor & Francis
Taylor & Francis Group

OPEN ACCESS Check for updates

Integrating ride-hailing services with public transport: a stochastic user equilibrium model for multimodal transport systems

Bing Liu [a,b], Yuxiong Ji[a] and Oded Cats[b]

[a]Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, Shanghai, People's Republic of China; [b]Department of Transport & Planning, Delft University of Technology, Delft, The Netherlands

ABSTRACT

Public transport (PT) agencies are increasingly keen on integrating ride-hailing (RH) services with PT to improve overall mobility. Understanding the traffic flow distribution in the integrated system is vital for the policy decision-making and services design of such a system. We propose a stochastic user equilibrium (SUE) model for multimodal transport systems consisting of private car, PT and RH. The travel costs in the SUE model are investigated using a multimodal graph representation to capture the relationship of different travel modes in the integrated system. We apply the proposed model to a toy case and a real-world case. A RH subsidy strategy is compared with the benchmark to demonstrate travellers' route and mode shifts in the integrated system. Our findings offer insights on subsidising RH services through the proposed model, and provide valuable knowledge on the planning and design of the integrated system.

ARTICLE HISTORY
Received 22 September 2022
Accepted 8 July 2023

KEYWORDS
Stochastic user equilibrium; ride-hailing; public transport; multimodal network

1. Introduction

Ride-hailing (RH) services provided by transportation network companies (TNCs) such as Didi and Uber are changing the mobility landscape in many cities worldwide. RH services improve the mobility experience of travellers by offering a flexible door-to-door travel option. From the perspective of government and public transport (PT) agencies, there is a debate on the benefit of RH services given the ambiguous interplay between RH and other travel modes such as PT and private cars. RH services may supplement the gap of PT in low-demand areas and encourage a car-independent lifestyle, while they may also absorb passenger flow from PT and induce congestion in road networks (Tirachini 2020).

There has been a fervent debate on the relationship between RH and PT in previous studies. Some empirical studies drew the conclusion that RH services are competing with PT and reducing the PT passenger flow (Graehler, Mucci, and Erhardt 2019; Hall, Palsson, and Price 2018). However, some researchers pointed out that the relationship between PT and RH is more nuanced than a simple dichotomy. For example, Cats et al. (2022) investigated the Uber trip data in six cities in the United States and Europe. The results showed

CONTACT Yuxiong Ji yxji@tongji.edu.cn

that between 20%–40% RH trips have no attractive PT alternative and the mode share of RH trips compared to the PT alternative is related to their travel time competitiveness. The competitive and complementary relationship between RH and PT further affects road traffic. The evidence of RH services increasing the vehicle kilometres travelled (VKT) was found in empirical studies (Erhardt et al. 2019) and simulation results (Tirachini and Gomez-Lobo 2020). However, several studies argue that RH services could encourage car users to shift their travel mode to PT by offering better first/last-mile connections to PT stations (Clewlow and Mishra 2017; Erhardt et al. 2019).

Some new concepts of RH services and PT integration have been proposed in recent years to enhance the complementary effect between the two modes and then stimulate the mode shift from private car to PT (Reck and Axhausen 2020; Yan, Levine, and Zhao 2019b; Zhang and Khani 2021). One possible integration strategy is to subsidise the first/last-mile RH trips of PT. The RH services are provided by transport network companies such as Uber and Didi and PT agencies offer the subsidy directly to travellers. In this way, RH becomes an affordable and flexible alternative for riders to reach the PT system. In addition, the PT agencies adjust the bus routes in the RH subsidised areas to avoid capacity underutilisation and reduce operational costs. The subsidising strategy has been piloted in more than 10 cities in the USA (Curtis et al. 2019). For example, CapMetro started a pilot project from June 2018 in Austin, TX, which fully subsidises the rides connecting bus stations with the Exposition Innovation Zone (Curtis et al. 2019). SacRT collaborates with Lyft and Uber in Sacramento, CA, providing a fixed amount subsidy (5$ per trip) for the first/last mile rides connecting six light rail stations (Curtis et al. 2019). Innisfil Transit was launched in May 2017 in Innisfil, Ontario, Canada, in partnership with Uber, which subsidised rides connecting the town with key destinations as an alternative to costly fixed PT services (Cane 2017). Benaroya, Sweet, and Mitra (2023) conducted an empirical case study on the Innisfil Transit project and the results show that the residents benefitted from cheaper RH costs.

While the integration of RH and PT services is conceptually appealing, service providers need to understand how the travellers react to integration strategies in a multimodal system incorporating PT, RH and private car. Some studies investigated integration strategies by analytical models. Siddiq, Tang, and Zhang (2022) presented a game-theoretic model to evaluate incentive mechanisms in the transport system incorporating car and mixed-mode (e.i. combining PT with RH service). Zhu et al. (2021) proposed a bi-level game-theoretic approach to model the cooperative and competitive relationship between the TNCs and the government with a first/last mile RH subsidy.

The abovementioned studies analysed the equilibrium of travel mode choice and transport markets at an aggregated level while ignoring the underlying network configuration. However, the potential mode shift between RH and PT is related to several network-related features, including the spatial relationship between RH trips and PT lines, travel distance and the quality of PT alternatives (Babar and Burtch 2020; Cats et al. 2022; Wang and Ross 2019; Young, Allen, and Farber 2020). Moreover, the integration of RH and PT may also induce a path shift (Geržinič et al. 2021) since RH services make it possible for travellers to access further PT stations and then choose a PT route with shorter travel time and less transfer.

The aim of this study is to help planners to understand how traffic flows are distributed across the network in integrated transport systems and how integration strategies affect system performance. We propose a stochastic user equilibrium (SUE) multimodal transport network model with integrated PT and RH trips. The travel costs of different travel modes are analysed in detail for the underlying transport networks. A toy case and a real-world case are deployed to demonstrate and test the proposed model. A subsidy strategy is applied in the case study and the impact of the strategy on the mode share and traffic flow distribution as well as overall system performance is demonstrated. A series of sensitivity analysis are carried out to explore the impact of the subsidy under different scenarios.

The main contributions of this paper are threefold. Firstly, this study models the multimodal transport system with combined PT and RH trips at a network level. We consider the specific stations and segments in the network instead of an aggregated transport corridor. This enables the model to capture the impact of network characteristics on mode and path choices. Moreover, it enables the model to investigate the traffic flow caused by multiple OD pairs on specific segments, which may support the development of the integrated system in tactical planning aspects such as pricing strategies, bus service redesign and infrastructure planning. Secondly, the SUE model proposed in this study extends the classic SUE model to a multiclass multimodal transport system incorporating car, RH, PT and combined trips of RH and PT. In addition, flow-related costs including RH waiting time and bus crowding are considered in the proposed SUE model, which is vital for traveller choice behaviour in RH and PT systems. Thirdly, a possible integration strategy of RH and PT — RH subsidy — is tested under different circumstances and its impacts across the network are assessed. The results provide some insights into the application conditions and pricing strategy of RH subsidy.

The remainder of this paper is structured as follows. In section 2, we review the literature on multimodal transport assignment. In section 3, we introduce the multimodal transport system and analyse the generalised cost of travellers. Section 4 formulates the equilibria of the system and provides a solving algorithm. A toy case and a real-world case study are given in Section 5. Section 6 concludes the paper and suggests directions for future research.

2. Literature review

The traffic assignment problem in multimodal transport networks has been studied extensively in the past few decades. Some studies assume that travellers can choose different travel modes in a multimodal transport system but cannot transfer between modes. Florian (1977) proposed an equilibrium model for transport systems with private car and one or more PT modes. In the past decades, extensive research has been devoted to extending the bi-modal model to include more realistic features in the model representation, such as multiple user classes (Boyce and Bar-Gera 2004; De Cea et al. 2005; Lam and Huang 1992), non-separable/asymmetric travel costs(Cantarella, Cartenì, and de Luca 2015; De Cea et al. 2005) and elastic demand(Cantarella 1997; Cantarella, Cartenì, and de Luca 2015). Kitthamkesorn et al. (2013) incorporated bicycles in an equilibrium model.

Recently, some studies extended the multimodal traffic assignment model to emerging means of travel. Ridesharing is an innovative transport mode that allows drivers to pick up riders with similar travel needs. In some studies, ridesharing services are assumed to be provided by existing drivers who also have an itinerary. Xu, Ordóñez, and Dessouky (2015a) introduced ridesharing in the traffic assignment model under the assumption that drivers only pick up riders that have the same OD pair. The traffic assignment model is combined with a market pricing model to facilitate the analysis of the impact of ridesharing regulatory interventions. Xu et al. (2015b) relaxed the non-detour assumption and considered the ridesharing between drivers and riders from different OD pairs. Some policy measures, including HOT lanes (Di et al. 2017; Di et al. 2018; Li et al. 2019), compensation (Yan et al. 2019a) and entry restriction (Sun and Szeto 2021), are incorporated in ridesharing traffic assignment models to support policy decision-making.

Different from ridesharing, RH is a service provided by hired personal drivers. For ridesharing services, the detour caused by sharing is the main determinant of service quality. However, for RH services, the key factors are the waiting time and a relatively high price charged for a private ride. Ban et al. (2019) developed a general equilibrium model for transport systems composed of solo drivers and RH services. The choices made by travellers are affected by factors including trip price, RH waiting time, convenience and safety. Wei, Vaze, and Jacquillat (2022) considered RH services as a competitive travel mode for PT and developed a framework to optimise PT schedules under the equilibrium of a transport system consisting of PT, RH and an opt-out option.

Instead of assuming that travellers cannot transfer between different travel modes, few studies explicitly considered mixed trips in multimodal transport systems, especially in the context of park-and-ride (Kitthamkesorn et al. 2013; Liu et al. 2018; Fan et al. 2022) and bike access to metro (Fan et al. 2022). For example, García and Marín (2005) proposed a network equilibrium model for a multimodal transport system with two main modes: car and PT, in which the PT can be accessed by walking, bicycle or auto (park-and-ride). However, research considering RH services as an access mode of PT is scarce. Zhang and Khani (2021) modelled the equilibrium of integrated PT and RH systems as a fixed-point problem. In the proposed model, travellers can either access a main PT line by RH services or walk to a bus stop near the origin and then take a bus.

Our proposed model differs from the stochastic user equilibrium model proposed by Zhang and Khani (2021) in three critical aspects. First, apart from PT, private car and door-to-door RH services are also included in this study. Thus, the proposed model can not only estimate how traffic flows are distributed in PT networks but also obtain the mode shares among different main travel modes. Second, we consider multi-class users in the proposed model to capture the mode choice behaviour variation of car owners and non-car owners. Third, bus crowding is considered in this study, which is vital for the planning of bus services such as frequency setting. This is especially relevant in the case of RH subsidy for PT access/egress trips, since this integration policy may induce more bus demand and thereby lead to severe bus crowding.

3. A multimodal transport system

In this section, we first introduce a multimodal transport system considering the access and egress legs of PT. Then, we analyse the generalised cost of each travel mode.

3.1. Setting and assumptions

As shown in Figure 1, we consider a multimodal transport system with regular travel demand such as commuting. The system consists of three main travel modes: PT, RH and

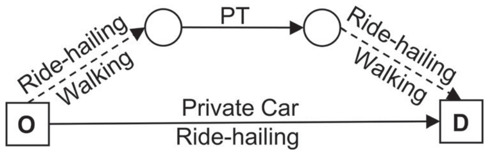

Figure 1. Trips of three main modes in multimodal transport systems.

private car. The PT services and RH services are provided by PT agencies and transport network companies, respectively.

Travellers can travel from their origins to destinations directly by either private car or RH, whereas for travellers opting for PT their journey consists of three legs: access, egress and PT. PT travellers can choose between RH services and walking as their access and egress modes. Two user classes are considered in this study, car owner and non-car owner. All the main travel modes are available for car owners, whereas non-car owners cannot use private cars.

Travellers make travel decisions according to a probabilistic mode choice model considering the generalised travel cost of travel modes and routes. For each OD pair, the travellers are first divided into car, RH and PT users. Car and RH users travel on the same road network but encounter different costs. PT users travel in a transport network consisting of the PT network and walking and RH arcs for access and egress trips. Then, travellers are assigned to specific routes in the road and transport network.

The proposed model involves the following assumptions:

PT service

- We capture the congestion on road networks but assume that the PT travel time is not affected by road traffic. It is reasonable for PT services travelling on exclusive lanes, including bus rapid transit, tram, metro and train. For other bus services, the proposed model may be considered an approximation.
- We assume that all the PT users are able to board the first arriving vehicle.

RH service

- The model considers simplified RH operations. We assume that the RH fleet size for a certain area is fixed, whereas in reality, it depends on the collective outcome of individual driver choices.
- The waiting time of RH services is assumed to be a piecewise linear function of RH travel demand in the area. In reality, the waiting time is affected by multiple factors, including driver behaviour and passenger-vehicle matching and routing algorithms.
- The impact of access/egress RH trips on traffic congestion is considered negligible. This is considered to be a reasonable assumption in the context of this study because the access/egress trips always take place in low-density areas where congestion is often mild. For high-demand areas with advanced PT networks, PT stations are within walkable distance and the market share of RH as access/egress mode is likely to be small. Thus, the traffic flow caused by RH services as an access/egress mode can be disregarded.

- The impact of empty RH cruising on congestion is not included in the model. We considered the supply of RH services as exogenous and the RH demand does not affect the empty kilometres of RH. Note that, the increase of RH demand may either increase or decrease the empty kilometres.

Choice behaviour

- The route decision is made by travellers before departure according to the information available, including fare, travel time and crowding level. En-route decision processes such as optimal strategies in PT assignment (Spiess and Florian 1989) are not considered in this study.
- The route choice behaviour is assumed to follow a random utility model. The disutility is represented by the generalised travel cost and the random errors follow the Gumbel distribution, in line with the assumptions underlying the logit model.

3.2. Notations

The notations employed in this section are listed in Table 1.

Table 1. Notations.

Notations	Description
A_c, A_d	Set of connectors and road arcs in the road network
$A_w, A_r, A_t, A_b^+, A_b^-, A_b$	Set of walking arcs, RH arcs, transfer arcs, alighting arcs, boarding arcs and in-vehicle arcs in the transport network
N_c	Set of zone centroids
N_d	Set of road nodes
N_a	Set of access nodes
N_p	Set of platform nodes
R_c	Set of paths in the road network
R_t	Set of paths in the transport network
W	Set of OD pair
R_w^c	Set of paths in the road network between OD pair w
R_w^t	Set of paths in the transport network between OD pair w
x_a	Aggregated flow on arc a
q_{wk}	Flows on path k between OD pair w
q_w^c, q_w^r, q_w^t	Demand of car user, RH user and PT user between OD pair w
δ_{ak}	1 if arc a is on path k; 0 otherwise
λ_1, λ_2	Value of time for travelling and waiting
μ_c, μ_r, μ_b	Coefficient of distance-based monetary cost for car users, RH services and PT services
τ_0^r, τ_0^b	Fixed fare of RH and PT services
p_t	Transfer penalty
u_0, v_1, v_2, b_1, b_2	Parameters in RH waiting time function
$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5,$	Standard deviation of perceived error in multinomial logit models
α_1, β_1	Parameters in BPR function for road network
α_2, β_2	Parameters in BPR function for PT service

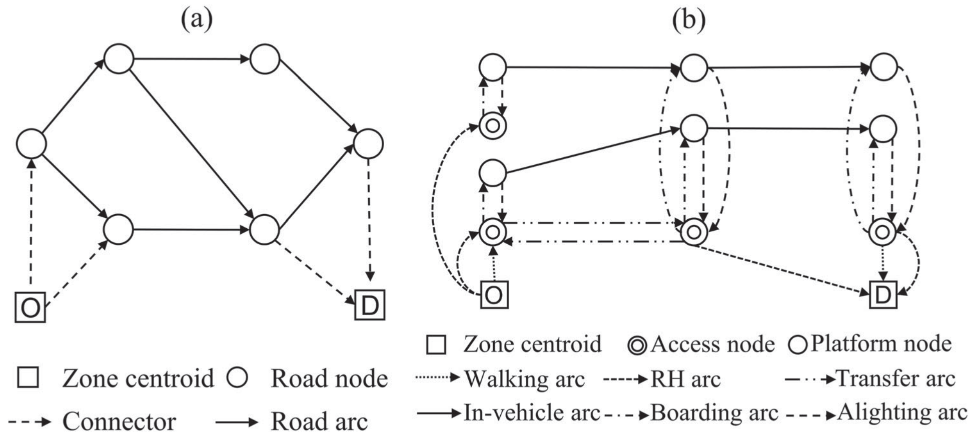

Figure 2. (a) Road network and (b) transport network.

3.3. Network representation

Two sub-networks are introduced in this study to model the movement of three types of users: (i) a transport network for PT users and (ii) a road network for car and RH users. As shown in Figure 2a, the road network consists of two types of nodes, zone centroids and road nodes. Zone centroids represent the origins and destinations of travellers and the road nodes refer to the intersections in the road network. The arcs connecting the zone centroids and road nodes are connectors and the arcs between two road nodes are the road arcs corresponding to the road segments in the real network.

In the transport network, PT users can access the PT system by walking or RH, thereby three travel modes are considered in the transport network: walking, RH, and PT. Figure 2b illustrates the nodes and arcs in the transport network. The zone centroids are as defined above. Access nodes and platform nodes correspond to the PT stations in the transport network. Access nodes represent the physical position of stations. For each PT line passing at the same station, there is a corresponding platform node. These platform nodes are connected to the access node representing the station, thereby allowing for interchanging between lines.

Walking and RH arcs connect zone centroids with access nodes, representing the access and egress legs of PT journeys. Only the walking arcs within walkable distance are included in the network. The PT stations connected by RH arcs are also in a specific distance range from zone centroids. The walkable distance and the distance range of RH arcs can be determined by the PT density of the application area. Similar to Nguyen and Pallottino (1988), three types of arcs are introduced to the proposed model to depict the movement of passengers in PT networks. In-vehicle arcs between two consecutive platform nodes represent the PT route segments. Boarding arcs connect access nodes to platform nodes, representing the passenger waiting and boarding process at PT stations. Alighting arcs connect platform nodes to access nodes, depicting the alighting process. Thus, for each stop of each PT line, there is one boarding arc and one alighting arc, correspondingly. In addition, we introduce transfer arcs as part of the transport network, which connect access nodes which are within walkable distance, to allow travellers to transfer between different PT stations.

3.4. Generalised travel cost analysis

The travel cost of different modes has various components, including travel time, waiting time, monetary cost and transfer penalty. All the travel costs other than RH waiting time are assigned to arcs in the proposed network and are assumed to be additive. In this section, we introduce the generalised cost for car, RH and PT users, respectively.

3.4.1. Car users

The cost encountered by car users consists of travel time cost and monetary cost. The travel time of connector a is assumed to be a fixed value t_a. The classic US Bureau of Public Roads (BPR) function is adopted in this study to express the congestion effect on the road network. The travel time attached to road arc a can be expressed as:

$$t_a = t_a^0\left(1+\alpha_1\left(\frac{x_a}{m_a^d}\right)^{\beta_1}\right), \quad a \in A_d \tag{1}$$

where x_a is the aggregated flow on arc a. t_a^0 and m_a^d refer to the free-flow travel time and road capacity of arc a, respectively. α_1, β_1 are parameters related to the congestion effect. In addition to the travel time, the car users endure a distance-based fee for fuel and maintenance. The monetary cost on arc a is given by:

$$\tau_a^c = \mu_c d_a, \quad a \in A_c \cup A_d \tag{2}$$

where μ_c is the monetary cost per distance unit and d_a denotes the distance of arc a.

We assume the travel time and monetary cost of car users to be additive in the road network, thereby the total travel time and monetary cost for the car users on path k are given by:

$$t_k^c = \sum_{a\in A_c\cup A_d} \delta_{ak} t_a, \quad k \in R_c \tag{3}$$

$$\tau_k^c = \sum_{a\in A_c\cup A_d} \delta_{ak} \tau_a^c, \quad k \in R_c \tag{4}$$

Thus, the generalised travel cost of path k is formulated as:

$$c_k^c = \lambda_1 t_k^c + \tau_k^c, \quad k \in R_c \tag{5}$$

where λ_1 is the value of time for travelling.

3.4.2. Ride-hailing users

The travel cost of RH users is comprised of travel time, waiting time and monetary cost. The RH users and car users share the same road network. Thereby, the travel time of RH users takes the same form as car users.

The waiting time is imposed on the connectors starting from zone centroids, which is a function of the RH vehicle utilisation rate in the zone. The RH vehicle utilisation rate of zone centroid i is the ratio of RH passenger volume departing from the zone represented by centroid i to the available RH fleet size m_{ri} in zone i. The RH volume is composed of both the RH users and the PT users who take RH services as their access/egress mode. Thus, the vehicle utilisation rate of zone centroid i is formulated as:

$$v_i = \frac{\sum_{a\in\{a|a^-\in S_i, a\in A_r\}} x_a + \sum_{k\in\{k^-\in S_i, k\in R_c\}} q_k^r}{m_i^r}, \quad i \in N_c \tag{6}$$

where a^- and x_a refer to the head node and aggregated flow of arc a, respectively. k^- is the origin node of path k and A_c, q_k^r is the RH flow on path k. S_i is the set of nodes in zone i. Following Pinto et al. (2020), the RH waiting time is assumed to be a linear function of utilisation rate v_i, which is given by:

$$u_i^r = \begin{cases} u_0 & v_i < v_1 \\ u_0 + b_1(v_i - v_1) & v_1 \le v_i < v_2 \\ u_0 + b_1(v_2 - v_1) + b_2(v_i - v_2) & v_i \ge v_2 \end{cases}, \quad i \in N_c \tag{7}$$

where u_0, v_1, v_2, b_1, b_2 are the parameters of the piecewise linear function.

The monetary cost of RH services has two components: fixed cost and distance-based cost. The distance-based cost of RH users takes the same form as car users but at a higher cost coefficient μ_r. Thereby, the total monetary cost for RH users on path k is:

$$\tau_k^r = \tau_0^r + \mu_r \sum_{a\in A_c\cup A_d} \delta_{ak} d_a, \quad k \in R_c \tag{8}$$

where τ_0^r refers to the fixed cost of RH service.

The generalised travel cost for RH users on path k is then given by:

$$c_k^r = \lambda_1 t_k^c + \lambda_2 u_{k^-}^r + \tau_k^r, \quad k \in R_c \tag{9}$$

where k^- refers to the origin node of path k. λ_1 and λ_2 are the values of time for travelling and waiting, respectively.

3.4.3. Public transport users

PT users endure the access and egress cost of walking and RH, the cost of PT and the penalty of transfer between RH and PT or different PT lines. For walking, we consider the walking time t_a on walking arc a as a fixed cost, thereby the walking time of path k is given by:

$$t_k^w = \sum_{a\in A_w} \delta_{ak} t_a, \quad k \in R_t \tag{10}$$

For the travellers taking RH as an access and/or egress mode, this trip leg is represented by a single RH arc. The travel time t_a and monetary cost τ_a attached to the RH arc a is the cost of the whole trip, which is calculated in advance. Thus, the travel time and monetary cost of RH on path k are:

$$t_k^r = \sum_{a\in A_r} \delta_{ak} t_a, \quad k \in R_t \tag{11}$$

$$\tau_k^r = \sum_{a\in A_r} \delta_{ak} \tau_a, \quad k \in R_t \tag{12}$$

The waiting time for RH arc a in the transport network is the same as the time endured by RH users on the road network. The total RH waiting time for path k is the sum of all the

RH arcs on path k:

$$u_k^r = \sum_{a \in A_r} \delta_{ak} u_{a^-}^r, \quad k \in R_t \tag{13}$$

The cost of PT includes perceived travel time, waiting time, monetary cost and transfer cost. For an in-vehicle arc a, there is a fixed free-flow travel time t_a^0. The impact of crowding on perceived travel time is explicitly accounted for in this study. Thus, the perceived travel time for in-vehicle arc a is formulated as a classic BPR function (US Bureau of Public Roads 1964):

$$t_a = t_a^0 \left(1 + \alpha_2 \left(\frac{h_a x_a}{m_a^t}\right)^{\beta_2}\right), \quad a \in A_b \tag{14}$$

where x_a is the passenger flow on in-vehicle arc a. m_a^t represents the vehicle standing area of the PT line represented by in-vehicle arc a, and h_a is the departure headway of this line. α_2, β_2 are the parameters related to the PT crowding effect, which can be calibrated by empirical data (Shao et al. 2022). Thus, the perceived travel time of PT on path k is formulated as:

$$t_k^b = \sum_{a \in A_b} \delta_{ak} t_a, \quad k \in R_t \tag{15}$$

The PT waiting time is determined by the headway of PT services. The waiting time attached to boarding arc a can be defined as a piecewise function (Luethi, Weidmann, and Nash 2007):

$$u_a = \begin{cases} \frac{h_a}{2} & 0 < h_a \leq 5min \\ 3.19 \cdot \log_{10}(h_a) & h_a > 5min \end{cases}, \quad a \in A_b^- \tag{16}$$

Thus, the waiting time for PT on path k is:

$$u_k^b = \sum_{a \in A_{b-}} \delta_{ak} u_a, \quad k \in R_t \tag{17}$$

The monetary cost of PT includes fixed and distance-based costs. The travellers pay the fixed cost τ_0^b once for each line they take in the whole trip and the distance-based cost is determined by the distance of in-vehicle arcs. The monetary cost of PT on path k is formulated as:

$$\tau_k^b = \tau_0^b \sum_{a \in A_{b-}} \delta_{ak} + \mu_b \sum_{a \in A_b} \delta_{ak} d_a, \quad k \in R_t \tag{18}$$

where μ_b is the distance cost coefficient for PT.

Two types of transfer costs are considered in this study, the walking cost for transfer between different stations and the additional discomfort caused by the unreliability and inconvenience induced by transferring. Similar to the cost for access/egress walking, we consider the walking time t_a on transfer arc a as the cost for transfer walking. We impose a fixed transfer penalty p_t for each boarding arc and RH arc in the transport network to represent the discomfort of transfer. The total transfer cost for path k can be expressed as:

$$\kappa_k^t = \lambda_1 \sum_{a \in A_t} \delta_{ak} t_a + p_t \left(\sum_{a \in A_b^- \cup A_r} \delta_{ak} - 1 \right), \quad k \in R_t \tag{19}$$

The generalised cost of PT users on path k can be formulated as:

$$c_k^t = \lambda_1 (t_k^w + t_k^r + t_k^b) + \lambda_2 (u_k^r + u_k^b) + \tau_k^r + \tau_k^b + \kappa_k^t, \quad k \in R_t \tag{20}$$

4. Stochastic user equilibrium in the multimodal transport system

4.1. Equilibria of stochastic user equilibrium problem

A multinomial logit model is applied in the route choice process. At equilibria, the flow on path k between OD pair w is given by:

$$q_{wk} = \begin{cases} q_w^c \frac{\exp(-\theta_1 c_k^c)}{\sum_{n \in R_w^c} \exp(-\theta_1 c_n^c)} + q_w^r \frac{\exp(-\theta_2 c_k^r)}{\sum_{n \in R_w^c} \exp(-\theta_2 c_n^r)} & k \in R_w^c \\ q_w^t \frac{\exp(-\theta_3 c_k^t)}{\sum_{n \in R_w^t} \exp(-\theta_3 c_n^t)} & k \in R_w^t \end{cases} \quad w \in W \tag{21}$$

where q_w^c, q_w^r and q_w^t are the total demand for car, RH and PT between OD pair w, respectively. θ_1, θ_2 and θ_3 refer to the standard deviation of perceived error when car users, RH users and PT users choosing among paths in the road network and the transport network. The larger the θ, the higher the probability that a traveller chooses the shortest route (Sheffi 1985).

A multinomial logit model is built to model the modal split between car, RH and PT. Two user classes are considered in this study: car owners and non-car owners. For car owners, all three main travel modes are available. At an equilibrium state, for each OD pair w, the travel demands of car owners by three travel modes are given by:

$$q_w^{nt} = q_w^n \frac{\exp(-\theta_4 c_w^t)}{\exp(-\theta_4 c_w^t) + \exp(-\theta_4 c_w^r) + \exp(-\theta_4 c_w^c)}, \quad w \in W \tag{22}$$

$$q_w^{nr} = q_w^n \frac{\exp(-\theta_4 c_w^r)}{\exp(-\theta_4 c_w^t) + \exp(-\theta_4 c_w^r) + \exp(-\theta_4 c_w^c)}, \quad w \in W \tag{23}$$

$$q_w^{nc} = q_w^n - q_w^{nr} - q_w^{nt}, \quad w \in W \tag{24}$$

where q_w^{nt}, q_w^{nr} and q_w^{nc} denote the travel demand of car owners taking PT, RH service and private car, respectively. qn w is the total demand of car owners between OD pair w. c_w^c, c_w^r and c_w^t refer to the expected disutility for choosing private car, RH and PT, respectively, which are given by (Williams 1977):

$$c_w^c = -\frac{1}{\theta_1} \ln \sum_{k \in R_w^c} \exp(-\theta_1 c_k^c), \quad \forall w \in W \tag{25}$$

$$c_w^r = -\frac{1}{\theta_2} \ln \sum_{k \in R_w^c} \exp(-\theta_2 c_k^r), \quad \forall w \in W \tag{26}$$

$$c_w^t = -\frac{1}{\theta_3} \ln \sum_{k \in R_w^t} \exp(-\theta_3 c_k^t), \quad \forall w \in W \tag{27}$$

The non-car owners have two options for their main mode: RH and PT. For each OD pair w, the travel demand of non-car owners choosing for each travel mode is given by:

$$q_w^{mt} = q_w^m \frac{\exp(-\theta_5 c_w^t)}{\exp(-\theta_5 c_w^t) + \exp(-\theta_5 c_w^r)}, \quad \forall w \in W \tag{28}$$

$$q_w^{mr} = q_w^m - q_w^{mt}, \quad \forall w \in W \tag{29}$$

where q_w^{mt} and q_w^{mr} denote the travel demand of non-car owners choosing for the PT and RH, respectively. $qm\ w$ is the total demand of non-car owners between OD pair w.

Thus, the total demands of car users, RH users and PT users between OD pair w are given by:

$$q_w^c = q_w^{nc}, \quad \forall w \in W \tag{30}$$

$$q_w^r = q_w^{nr} + q_w^{mr}, \quad \forall w \in W \tag{31}$$

$$q_w^t = q_w^{nt} + q_w^{mt}, \quad \forall w \in W \tag{32}$$

4.2. Solving algorithm

Due to the non-additive costs (e.g. RH waiting time) in the generalised travel costs we mentioned above, it is difficult to obtain the derivative information for the flow-cost mapping function in the proposed SUE problem. Thus, we develop an iterative algorithm based on the method of successive average (MSA) to solve the equilibrium problem. The proposed algorithm is presented in Algorithm 1. In each iteration, the auxiliary mode demands and path flows are calculated by multinomial logit models. Then, the mode demands and path flows are updated by MSA. The algorithm stops when the gap satisfies the convergence criterion or it reaches the maximum iteration number. The gap evaluates the similarity of the path and mode flows in the last iteration and the auxiliary flows, which is given by:

$$gap = \frac{\sum_{w \in W}(|q_w^c - h_w^c| + |q_w^r - h_w^r| + |q_w^t - h_w^t|)}{\sum_{w \in W}(q_w^n + q_w^m)} + \frac{\sum_{w \in W}\sum_{k \in R_w^c \cup R_w^t}|q_{wk} - h_{wk}|}{\sum_{w \in W}(q_w^n + q_w^m)} \tag{33}$$

where h_w^c, h_w^r and h_w^r are the auxiliary mode flow of car, RH and PT between OD pair w, respectively. h_{wk} is the auxiliary flow of path k between OD pair w. q_w^c, q_w^r, q_w^r and q_{wk} are the mode and path flow in the last iteration. Thus, the first and second terms are the similarity of the mode flows and path flows, respectively.

5. Case study

In this section, the proposed model is tested in a toy network and a real-world network. We first conduct a series of experiments in a toy network to explore how the RH subsidy affects the distribution of traffic flow in a multimodal transport network and analyse the impact of demand level, RH fleet size and subsidy amount on the system performance. Then, the influence of subsidy strategy on large-scale networks is tested by the transport network in Jiading District, Shanghai.

For both cases, the convergence precision ϵ is set to be 0.001 and the maximal iteration number N is 1000. Other key parameters for a typical weekday during the morning peak are listed in Table 2 according to the field data in Shanghai, China.

Algorithm 1. MSA-based algorithm for SUE in multimodal transport systems.

Input: Road network ($N_c \cup N_d$, $A_c \cup A_d$)
Transport network ($N_c \cup N_a \cup N_p$, $A_w \cup A_r \cup A_t \cup A_b \cup A_b^+ \cup A_b^-$)
Path set R_w^c, R_w^t, $w \in W$
Parameter δ_{ak}, $a \in A_w \cup A_r \cup A_t \cup A_b \cup A_b^+ \cup A_b^- \cup A_d \cup A_c$, $k \in R_w^c \cup R_w^t$, $w \in W$
Parameters involved in generalised travel cost functions
Demand q_w^c, q_w^m, $w \in W$
Convergence precision ϵ and maximum iteration number N

Output: Path flow q_{wk}, $k \in R_w^c \cup R_w^t$, $w \in W$
Mode flow q_w^c, q_w^t, q_w^r, $w \in W$

Step 0: Initialization
  for $w \in W$:
    $q_w^c = q_w^t = q_w^r = 0$;
    for $r \in R_w^c \cup R_w^t$: $q_{wr} = 0$;
  for $a \in A_b \cup A_d$: calculate perceived travel time t_a by (1), (14)
  for $i \in N_c$: calculate RH waiting time v_i by (6), (7)
  $n = 1$;
Step 1: Update generalised path costs
  for $w \in W$:
    for $k \in R_w^c$: calculate c_k^c, c_k^r by (2)–(5), (8), (9);
    for $k \in R_w^t$: calculate c_k^t by (10)–(13), (15),(17)–(20);
Step 2: Update expected disutility for each main mode
  for $w \in W$: calculate c_w^c, c_w^r, c_w^t by (25)–(27);
Step 3: Calculate auxiliary flows
  for $w \in W$:
    calculate h_w^c, h_w^t, h_w^r by (22)–(24), (28)–(32)
    for $k \in R_w^c \cup R_w^t$: calculate h_{wk} by (21);
Step 4: Check convergence
  $gap = \frac{\sum_{w \in W}(|q_w^c - h_w^c| + |q_w^r - h_w^r| + |q_w^t - h_w^t|)}{\sum_{w \in W}(q_w^n + q_w^m)} + \frac{\sum_{w \in W}\sum_{k \in R_w^c \cup R_w^t}|q_{wk} - h_{wk}|}{\sum_{w \in W}(q_w^n + q_w^m)}$
  If $n > N$ or $gap < \varepsilon$: **break**;
Step 5: Update mode flows and path flows by MSA
  for $w \in W$:
    $q_w^c = q_w^c + \frac{(h_w^c - q_w^c)}{n}$, $q_w^r = q_w^r + \frac{(h_w^r - q_w^r)}{n}$, $q_w^t = q_w^t + \frac{(h_w^t - q_w^t)}{n}$
    for $k \in R_w^c \cup R_w^t$: $q_{wk} = q_{wk} + \frac{(h_{wk} - q_{wk})}{n}$
Step 6: Update link and node costs
  for $a \in A_b \cup A_d$: calculate perceived travel time t_a by (1), (14)
  for $i \in N_c$: calculate RH waiting time v_i by (6), (7)
  $n = n + 1$;
  go to **Step 1**;

Table 2. Parameters.

Notations	Value
λ_1, λ_2	23.77RMB/h, 38.51 RMB/h
μ_c, μ_r, μ_b	1.5 RMB /km, 3 RMB /km, 0 RMB /km
τ_0^r, τ_0^b	12RMB, 2RMB
p_t	2RMB
u_0, v_1, v_2, b_1, b_2	3, 20, 50, 0.5, 0.8
$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5,$	2
α_1, β_1	0.15, 4
α_2, β_2	0.0021, 2.85

5.1. Toy case

5.1.1. Case setting

We modify the network proposed by Spiess and Florian (1989), which is commonly used for in the PT network assignment literature, to include the road network and access/egress

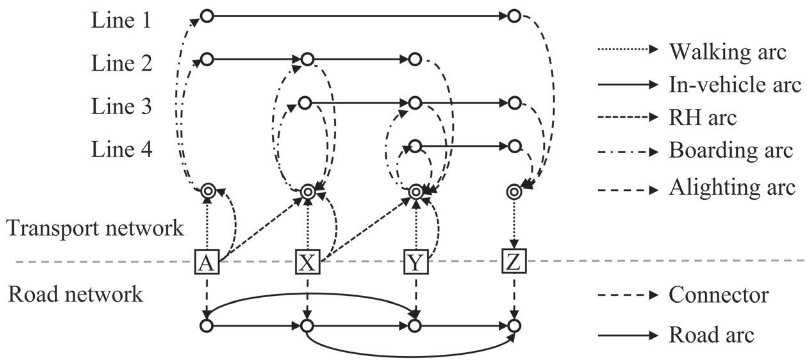

Figure 3. Toy network.

Table 3. Scenario design.

Scenario	PT level	Integration strategy
US^-	Urban area	Unsubsidised
US^+	Urban area	Fully subsidised
RS^-	Rural area	Unsubsidised
RS^+	Rural area	Fully subsidised

Table 4. RH subsidy investment and benefits.

	Investment (RMB)	Time saving (h)	VKT decrease (km)
US	43187.9	558.8	6867.1
RS	69361.9	907.8	7560.8

arcs of the PT network to allow the application for of the proposed model, see Figure 3. The complete network and demand data are detailed in Appendix A.

Four scenarios with different levels of PT service provision and integration strategies are designed, as summarised in Table 3. The access time from each origin to the nearest access node is set in scenarios US^- and US^+ to 5 and 1 min by walking and RH, respectively, representing areas with dense PT services (urban areas). Scenarios RS^- and RS^+ are devised to represent areas with sparse PT service (rural areas), the access time of which is set to 17 and 3 min by walking and RH, respectively. The access RH mode is available in all of the scenarios, but in the US^+ and RS^+ scenarios, the RH service as an access mode of PT is fully subsidised, i.e. the RH service is offered for free.

5.1.2. Results

The investment in RH subsidy and its benefits are shown in Table 4. Two benefits are evaluated here: the time saving of travellers and the VKT decrease in the road network. In both scenarios, RH subsidy benefits travellers and reduce the VKT in the system. The investment under the RS scenario is higher than under the US one and subsequently the benefits of RH subsidy in RS are larger than US correspondingly. We fully subsidise all access RH trips in the case study. In practice, planners can design the subsidy amount of RH considering the trade-off between the investment in RH subsidy and the abovementioned benefits given the locally prevalent planning objectives.

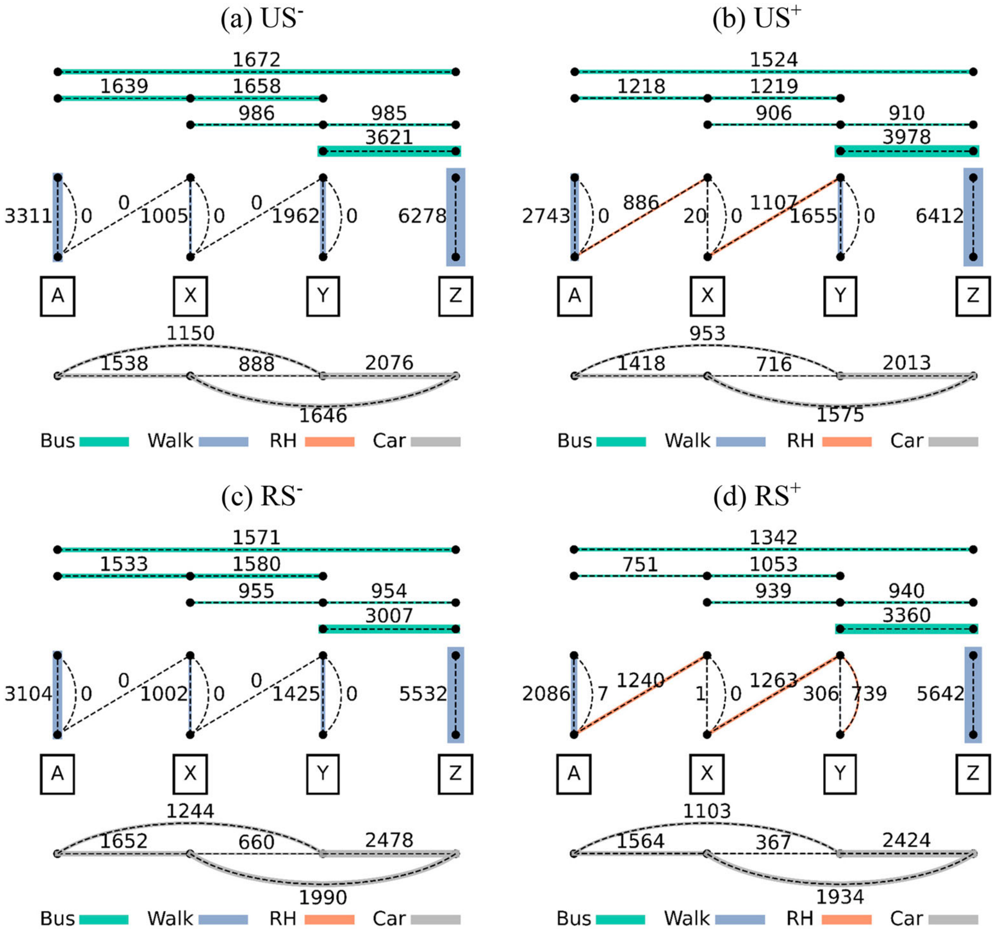

Figure 4. Traffic flow distributions on the toy network.

The traffic flow distributions on the network are illustrated in Figure 4. Car users and RH users are directly assigned to the road network. PT users choose first an access mode and are then distributed across the PT network. For example, there are 3629 PT users departing from origin A in Figure 4b, 2743 of which walk to the bus stop located within area A and 886 of which take a RH service to the bus stop located at area X. For the 2743 PT users who take the bus departing from the bus stop within area A, 1524 of them take bus line 1 and 1218 of them take bus line 2.

Comparing the subsidised scenarios (US$^+$ and RS$^+$) with unsubsidised scenarios (US$^-$ and RS$^-$), a RH subsidy strategy decreases the traffic flow on all the segments in the road network in subsidised, as shown in Figure 4. In contrast, in the absence of a subsidy, no PT user chooses RH as their access/egress mode due to the costly service fee. In the subsidised scenarios, some travellers not only shift their access mode from walking to RH but also change their access nodes to avoid transfers. Due to the long access distance for PT lines in the rural area scenario, RH is more attractive in the rural area scenario than in the urban area scenario when it is subsidised. In addition, the subsidy strategy reduces the passenger flow on Lines 1-3. It can be explained by the route shift of travellers departing from A

Table 5. Generalised travel costs and mode shares.

			GTC (RMB)						Mode share		
Scenario	O	D	PT		Car		RH		PT	Car	RH
US$^-$	A	Z	25.5		24.4		50.6		55.2%	44.8%	0.0%
	X	Z	19.3		16.3		37.9		50.2%	49.7%	0.1%
	Y	Z	9.8		11.5		29.2		98.1%	1.8%	0.1%
US$^+$	A	Z	23.5	(−7.9%)	22.8	(−6.5%)	49.0	(−3.1%)	60.5%	39.5%	0.0%
	X	Z	16.4	(−15.0%)	15.4	(−5.7%)	37.8	(−0.5%)	56.4%	43.6%	0.0%
	Y	Z	10.4	(+5.8%)	10.7	(−6.7%)	28.4	(−2.6%)	82.8%	17.2%	0.0%
RS$^-$	A	Z	33.6		31.9		58.2		51.7%	48.2%	0.1%
	X	Z	28.0		23.0		44.7		50.1%	49.9%	0.0%
	Y	Z	18.6		18.4		36.2		71.3%	28.7%	0.0%
RS$^+$	A	Z	31.0	(−7.8%)	29.9	(−6.2%)	58.0	(−0.3%)	55.6%	44.4%	0.0%
	X	Z	22.2	(−20.9%)	21.7	(−5.9%)	45.3	(+1.2%)	63.2%	36.8%	0.0%
	Y	Z	18.8	(+1.4%)	17.3	(−6.3%)	35.0	(−3.2%)	52.3%	47.7%	0.0%

and X. As shown in Figure 4a, all the travellers from A take Line 1 or 2. In Figure 4b, some travellers shift their routes to Line 3 because the RH subsidy is provided between A and X, which leads to a decrease in the flow of Lines 1 and 2 and increases the flow of Line 3. Similarly, the route shift of travellers from X results in the flow decrease of Line 3 and increase of Line 4. The result implies that there is potential for PT agencies to cut down operational costs by redesigning the bus frequency or vehicle capacity of bus services. Notably, although the RH trips are fully subsidised, the passenger flow on the RH arcs connecting the origins to the nearest stations in Figure 4b is zero, implying that the subsidy cannot affect the access mode choice of travellers when the station is close to the origin.

The generalised travel costs (GTCs) and mode shares for different travel modes are reported in Table 5. In all scenarios, the GTC of private RH service is higher than PT and car for all OD pairs due to the costly RH monetary cost. Thereby, the mode share of RH is close to zero.

The RH subsidy reduces the GTC of all car users. However, the GTC of PT users for OD pair Y-Z increases in the subsidised scenarios (US$^+$ and RS$^+$). This is caused by the increase in passenger flow of Line 4 (Figure 4**)**. Thanks to the affordable RH service, some travellers departing from node X shift their route from Line 3 to Line 4, and travellers from node A shift their route from Line 1 to Line 2 plus Line 4.

Compared with the urban area scenarios, the PT mode shares are lower in the rural area scenarios due to the low accessibility of PT services for all OD pairs without RH subsidy. Comparing the unsubsidised scenarios with the subsidised scenarios, the RH subsidy stimulates some car users departing from A or X to shift their travel mode to PT. While the RH subsidy reduces the GTC experienced by both car users and PT users, it does make PT relatively more competitive than car. However, for OD pair Y-Z, the PT mode share decreases due to the increase in GTC of PT and the decrease in GTC of car as explained above.

5.1.3. Sensitivity analysis

In this section, we carry out a series of experiments to explore how the performance of the RH subsidy varies from different demand levels and RH fleet sizes. Furthermore, we analyse the impact of subsidy amount on the multimodal transport system.

Figure 5 shows the GTC and PT mode share under different demand levels. The increase in demand yields an increasing GTC and PT mode share for all scenarios where demand

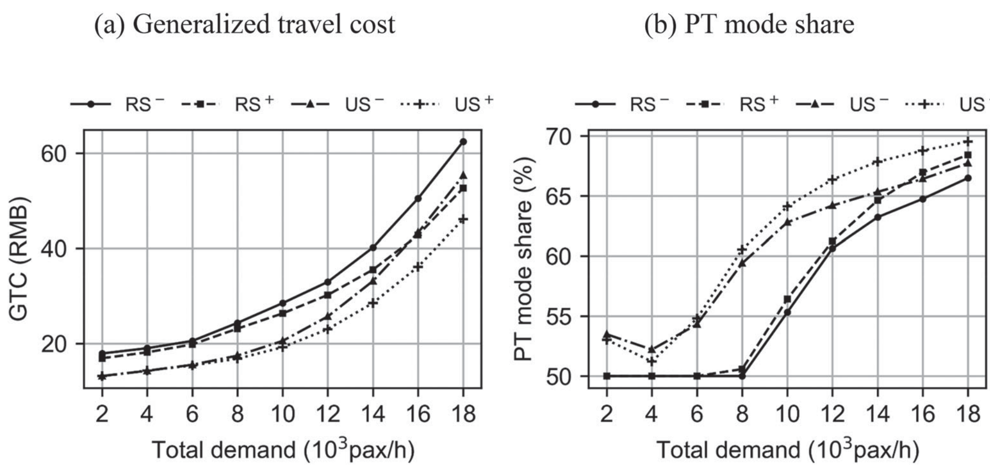

Figure 5. (a) GTC and (b) PT mode share for different demand levels.

density is larger than 4000 pax/h. However, in the US^- and US^+ scenario, the PT mode share decreases when the demand increases from 2000 pax/h to 4000pax/h. This is because the travel cost in PT network is more sensitive than the travel cost in road network to the increase of demand at this demand level. The tendency is different in RS and US scenarios because the sensitivity depends on the underlying network configuration. This finding suggests that the detailed network configuration plays a vital role in mode split.

There is no significant difference in GTC and PT mode share between unsubsidised and subsidised scenarios when the demand density is low ($<$ 4000pax/h). With the increase in demand, a sharper rise in GTC occurs under unsubsidised scenarios than under subsidised scenarios. It is because the RH subsidy offers PT travellers more affordable travel route options (access to more PT lines), which enables passengers to avoid crowded PT routes to a larger extent than in the absence of subsidies. Correspondingly, the PT mode share increases more significantly in the subsidised scenarios than in unsubsidised ones.

The impact of the RH fleet size on GTC and PT mode share is presented in Figure 6. Although the increase of RH fleet size reduces the RH waiting time, it does not compensate for the monetary cost of RH services in the unsubsidised scenarios. Thus, no traveller is using the relatively expensive RH services, even when the RH fleet is large and waiting times are minimal. Consequently, an increase in the RH fleet size has no impact on modal split and overall performance in the unsubsidised scenarios. For the subsidised scenarios, the GTC and PT mode share show a downtrend and uptrend with the rise of RH fleet size, respectively. When the fleet size is small (200 veh/h), the impacts of subsidy are limited due to the sharp increase of waiting time of RH service. With the increase in RH fleet size, the RH waiting time decreases, which reduces the GTC of travellers and absorbs more travellers to PT. Moreover, beyond a certain fleet size (2800 veh/h for US^+ and 3400 veh/h for RS^+) the fleet size does not affect the GTC and PT mode share any more since there are sufficient RH vehicles so that the RH waiting time reaches the minimal value.

Hitherto, the subsidy strategy covered all RH-related costs. However, the subsidy may only partially cover user costs. In the following analysis, the impact of the level of subsidy on the GTC and PT mode share is analysed by comparing it to the scenario without subsidy in both urban and rural settings, see Figure 7. When the subsidy is lower than 12 RMB

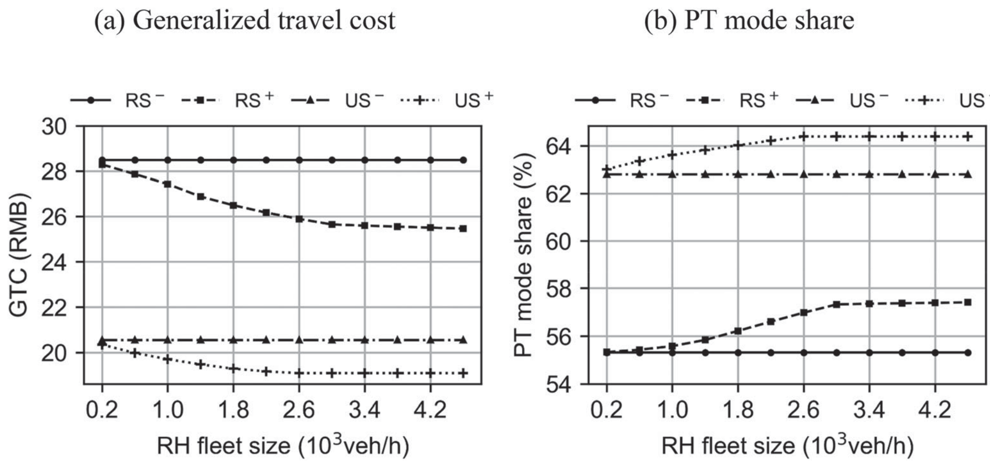

Figure 6. The impact of RH fleet size on (a) GTC and (b) PT mode share.

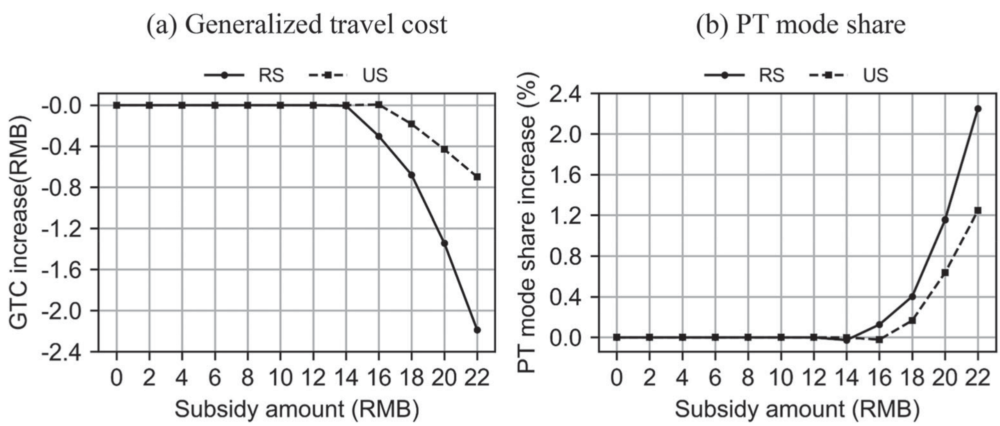

Figure 7. The impact of subsidy amount on (a) GTC and (b) PT mode share.

per trip, neither the GTC nor the PT mode share are impacted. With the increase in subsidy level, the GTC and PT mode share of US show a steady decrease and increase trends, respectively. However, there is a slight decrease in PT mode share in RS when the subsidy rises from 12 RMB to 14 RMB. This is because the subsidy does not only reduce the GTC of subsidised OD pairs but may also affect travellers travelling between other OD pairs who share the same corridor. The increase in subsidy reduces the GTC of PT users departing from X and then increases the PT mode share of OD pair X-Z. However, some travellers departing from X shift their route from Line 3 to Line 4, which makes Line 4 more crowded. Due to the crowding level, some travellers departing from Y shift their travel mode from PT to car, which leads to a drop in the PT mode share. The PT mode share increase for OD pair X-Z is more than cancelled-out by the decrease for Y-Z, resulting in a decrease in PT mode share when calculated for the entire system.

To explore how the subsidy affects the passenger flow across the PT network, we plot the maximal section flow of each bus line with different subsidy levels in Figure 8. With

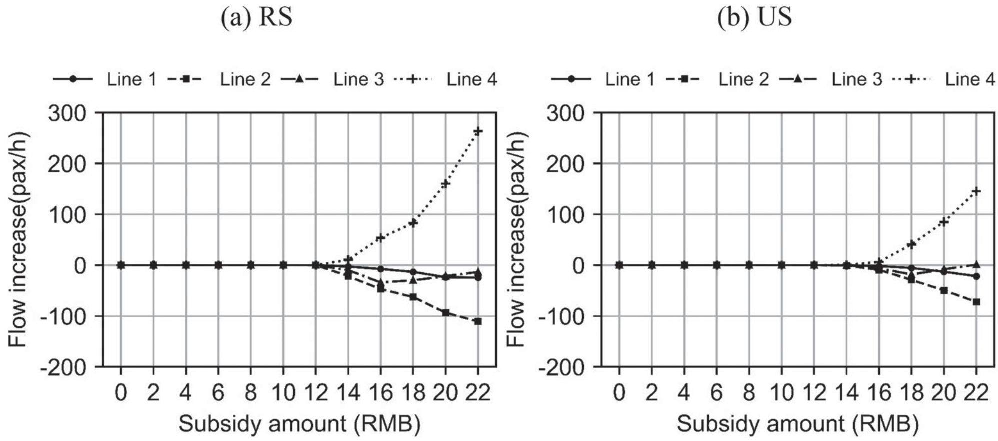

Figure 8. The impact of subsidy amount on bus passenger flow for (a) rural scenario and (b) urban scenario.

the increase in subsidy level, we observe a sharp increase in passenger flow for Line 4 in both scenarios. It implies that the RH subsidy enhances the supplementary effect of RH service and Line 4 by providing an affordable RH access service to Line 4. The increase in RH subsidy leads to a drop in passenger flow for Lines 1–3, indicating that the RH service competes with these lines. However, there is a slight increase in the flow for Line 3 both in RS and US when the subsidy amount reaches 16 RMB. The main source of passenger demand for Line 3 consists of travellers travelling between the OD pair X-Z. With the increase in subsidy level, more and more passengers take the RH service to access Line 4, which departs more frequently than Line 3. Thus, results in a passenger flow reduction for Line 3 when the subsidy amount is less than 16RMB. However, the greater subsidy level encourages a part of passengers from A taking RH service to access Line 3 to avoid the crowding on-board Line 1 and Line 2. When this part of route shift becomes dominant, the passenger flow for Line 3 rises.

5.2. Real-world case

5.2.1. Case setting

The proposed model is tested in Jiading District, Shanghai, China. It is a 464.2 km^2 mixed-use residential, industrial and commercial area located in northwestern Shanghai. There are 1.8 million inhabitants living in Jiading District. The study area is divided into 392 Traffic Analysis Zones (TAZs). There are 81 bus lines and one metro line serving the case study area. The road network and PT network are illustrated in Figure 9.

The OD matrix is extracted from taxi data for all weekday morning peaks (7:00–9:00) during April 2018. The OD pairs are sorted by travel flow and the top 2171 OD pairs amounting to 80% of the travel demand in this case study are selected for further analysis. The origin and destination distributions are presented in Figure 10. The OD matrix is scaled up to 100000pax/h in total, and the proportion of car owners is set to be 50%.

To show the impact of integration strategies on the multimodal transport system, two scenarios are tested in this case study: (i) benchmark without any integration strategies and

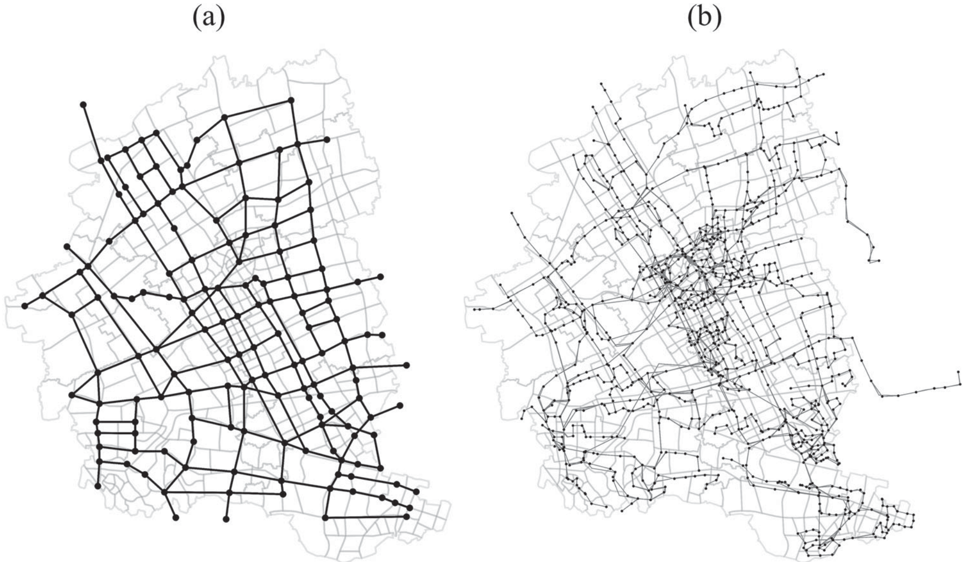

Figure 9. (a)Road network and (b) PT network in the case study area.

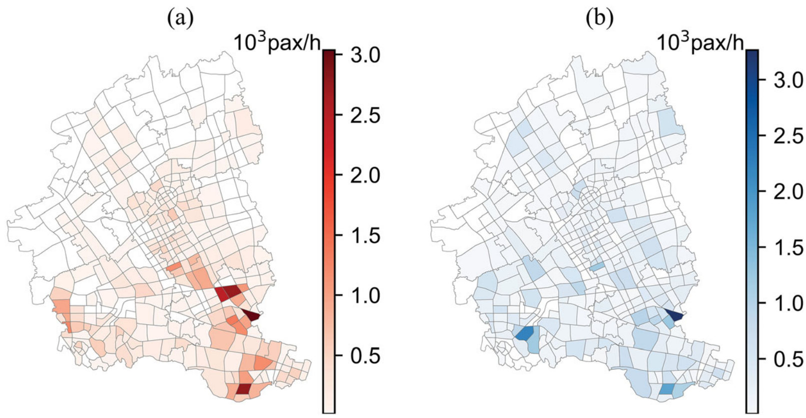

Figure 10. The (a) origin and (b) destination distribution in the case study area.

(ii) subsidised scenario in which we subsidise the fixed cost of RH service for the RH trips connecting the areas with low accessibility of PT services. We use the weighted cost of all access and egress arcs — the walking arcs and RH arcs connecting the area — to evaluate the access/egress cost of area i, which can be measured as:

$$l_i = \frac{\sum_{a\in\{a|a^-\in N_i, a\in A_w\}} \lambda_1 x_a t_a + \sum_{a\in\{a|a^-\in N_i, a\in A_r\}} x_a(\lambda_1 t_a + \lambda_2 u_a + \tau_a)}{\sum_{a\in\{a|a^-\in N_i, a\in A_w\cup A_r\}} x_a} \tag{34}$$

where l_i is the average travel cost access/egress cost of area i. The denominator is the sum of traveller flow departing or arriving in the area. The first term of the numerator is the sum of travel costs on all walking arcs connecting area i with PT stations. The second term is the total travel cost of access/egress RH services in area i.

Table 6. GTC and mode share in the benchmark scenario.

		Mode share		
User class	GTC (RMB)	PT	Car	RH
Car owner	20.3	49.4%	50.6%	0.0%
Non-car owner	25.7	93.2%	0.0%	6.8%
Total	23.0	71.3%	25.3%	3.4%

Table 7. Access and egress mode share of PT users in the benchmark scenario.

	Walking	RH
Access	99.7%	0.3%
Egress	99.8%	0.2%
Total	99.8%	0.2%

5.2.2. Results

5.2.2.1. Benchmark. The traffic volumes in the PT network are shown in Figure 11a. In the PT network, it can be observed that the metro line absorbs a heavy travel demand due to its high-speed and high-frequency characteristics. To further observe the traffic distribution in the bus network, we zoom in the figure and only present the bus volumes in Figure 11b. There is a high-demand area of bus service in the south-eastern and the south-western of the study area, which resonates with the demand distribution shown in Figure 10. The traffic flows in the road network are shown in Figure 11c and the traffic volumes on high-demand corridors are marked on the arcs. There is a high commuting demand connecting the case study area and the centre of Shanghai during the morning peak. Two high-volume corridors connecting the case study area to the city centre of Shanghai are observed in the road network. We also zoom in Figure 11d to show the distribution of traffic flow on the network for all parts other than the main corridors (note the difference in scale).

The average GTC and mode share in the benchmark are presented in Table 6. The GTC of car owners is lower than non-car owners because of the extra private car mode option for car owners. More than two-thirds of travellers choose PT as their main travel mode in total. For car owners, half of the travellers use PT but no one selects direct RH as their main mode. It can be explained by the high monetary cost of RH services, which is always higher than driving by themselves. The majority of non-car owners take PT as their main travel mode. Although the monetary cost of RH service is high, there are still some non-car users choosing RH service because of the inconvenience (e.g. long-walking distance, transfer) of PT between certain OD pairs.

Table 7 further presents the mode share of access and egress leg for PT users. Due to the high monetary cost, only a few PT users use RH services as their access or egress mode in the benchmark.

Figure 12 illustrates the access/egress cost of each area in the case study area. We select 25 areas with high access/egress cost ($>$ 10 RMB) to be subject to subsidies in the subsidised scenario. These areas are mainly distributed in the periphery of the case study area, where the density of PT stations and lines is low (Figure 9b).

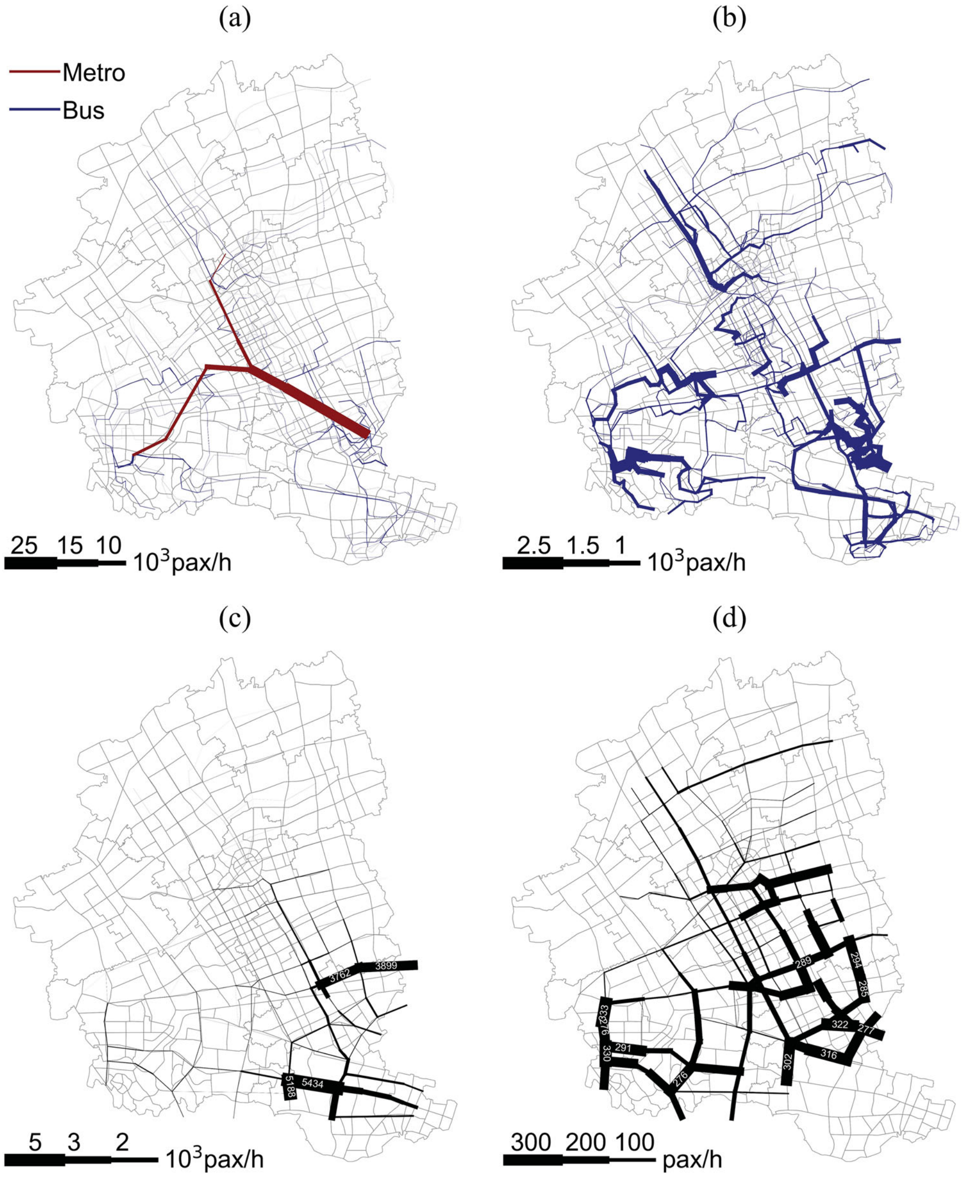

Figure 11. Peak period traffic volumes in the (a) PT network, (b) bus network, (c) road network, and (d) road network without main corridors.

5.2.2.2. Subsidised scenario. As mentioned in previous sections, the case study area is divided into 392 TAZs and 25 of them are subsidised for RH access and egress services. There are 108 OD pairs with origin or destination in the subsidised areas. The travel cost of subsidised travellers is affected directly by the subsidy, and other travellers can also be further influenced by the change of traffic flow caused by subsidised travellers. In this section, we focus on the impact of subsidy strategy on the subsidised travellers.

The subsidy strategy can reduce the travel cost of travellers and absorb more travellers to use PT. The GTC and mode share of subsidised travellers in benchmark and subsidised

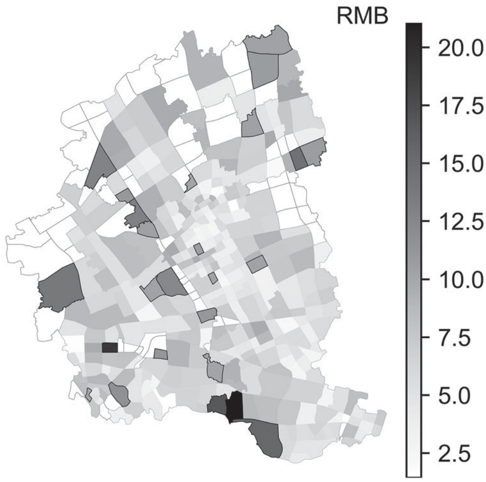

Figure 12. Weighted access/egress cost in the case study area.

Table 8. GTC and mode share of travellers, subsidised OD pairs only.

Scenario	User class	GTC (RMB)	Mode share		
			PT	Car	RH
Benchmark	Car owner	23.2	34.9%	65.1%	0%
	Non-car owner	29.9	94.2%	–	5.8%
	Total	26.6	64.6%	32.5%	2.9%
Subsidised	Car owner	22.2 (−4.3%)	40.0%	60.0%	0%
	Non-car owner	27.8 (−7.0%)	97.2%	–	2.8%
	Total	25.0 (−6.0%)	68.6%	30.0%	1.4%

scenario are presented in Table 8. The RH subsidy reduces the GTC by 6.0% in total. The improvement of GTC for non-car owners is larger than car owners due to the difference in PT mode share between the two user classes. The subsidy is offered to PT users, thereby the benefit of the subsidy strategy is larger if there are more PT users. The decrease in PT cost also stimulates some car users and RH users to shift their main mode to PT. There is a 5.1% and 3.0% mode shift from car and RH to PT for car owners and non-car owners, respectively.

The access and egress mode shares of PT users are shown in Table 9. There is a significant increase of the share of RH use in the subsidised scenario. The amount of subsidy for each OD pair is the same, but the impact of subsidy on GTC and mode shift varies for different OD pairs. There are 108 OD pairs which are subsidised, the mode shift in Table 9 is mainly attributed to 20 of them. For the other 88 OD pairs, the RH subsidy barely affects the mode choice of travellers. For any given OD pair, the impact of RH subsidy on access and egress mode shift is related to the position of origin and destination and the PT lines connecting the origin and destination.

The impact of the subsidy is not limited to a change in the access and egress mode of travellers, but may also lead to a shift of access and egress station and PT route in some cases. Figure 13 illustrates the path shift of an OD pair in the case study. The path shown in Figure 13 is the lowest-cost PT path for the relevant cases. In the subsidised scenario, the travellers chose to take a PT route which connects origin and destination directly but has

Table 9. Access and egress mode share of PT users, subsidised OD pairs only.

	Benchmark		Subsidised	
	Walking	RH	Walking	RH
Access	99.3%	0.7%	92.8%	7.2%
Egress	95.3%	4.7%	74.4%	25.6%
Total	97.3%	2.7%	83.6%	16.4%

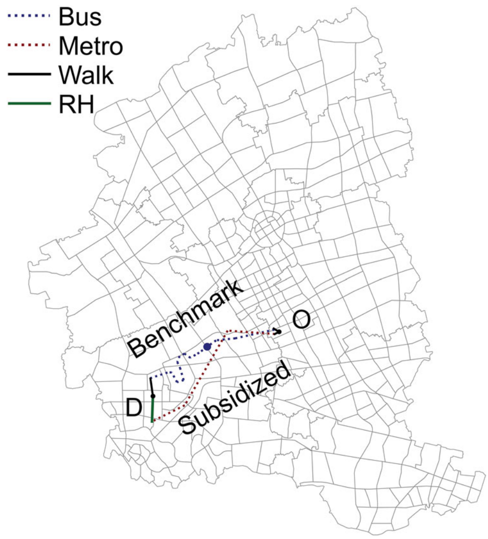

Figure 13. Station and route shift in the subsidised scenario.

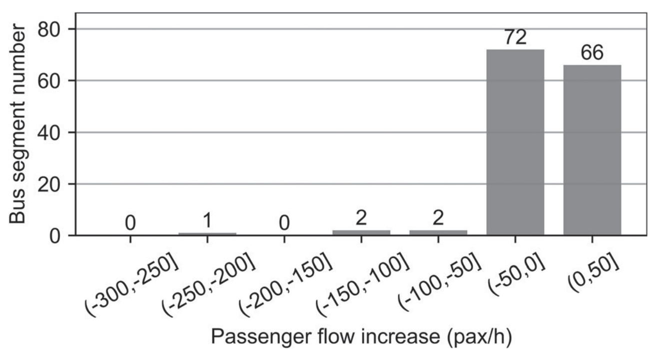

Figure 14. Distribution of passenger flow increase in the PT network.

a further egress station which can be reached by subsidised RH services. Thereby, the travellers can avoid long-distance walking and transfer between PT routes. At the same time, the PT leg of travellers shifts from bus to metro, which is faster and has a shorter waiting time than the bus.

The subsidy strategy results in different impacts on different segments of the PT network. In the subsidised scenario, both increased and decreased segment flow are observed in the

PT network. Figure 14 shows the distribution of bus segment traffic flow increase. Most of the segments see a slight increase or decrease ($<$ 50pax/h) in passenger flow, which may not influence the operation of bus services. Some bus segments encounter a dramatic passenger flow decrease, which may lower bus occupancy and cause energy waste.

6. Conclusion and future work

We propose an SUE model for multimodal transport systems consisting of private car, RH and PT, where the RH services are considered both as an independent travel mode as well as an access/egress mode of PT. The costs of travellers are analysed at a network level to capture the impact of network-related features on mode and path choices. An MSA-based algorithm is adopted for solving the proposed problem. The model is tested on a toy network and a series of sensitivity analysis is conducted to evaluate the impact of demand level, RH fleet size and RH subsidy level on the multimodal transport system. The model is also applied for a real-world case study using data from Jiading District, Shanghai, China. A scenario where the RH trips in low accessibility areas are subsidised is tested to demonstrate how it stimulates traveller mode and path shifts. The proposed model can support decision-makers in assessing the implications of alternative interventions in a multimodal transport system where the demand for RH impacts both car and PT performance.

Our findings suggest that capturing the relation between PT and RH and their underlying transport network is necessary in analysing a multimodal system. The results show that providing subsidies to access/egress RH trips reduces the GTC and makes some travellers shift their mode from private car or door-to-door RH to PT. However, the impact of subsidy varies among different network configurations and subsidy amounts. The results of the toy case show that the RH subsidy has a more significant effect when the PT service is sparse (in rural areas). With the increase in demand level, the RH subsidy hampers the increase in GTC by providing more affordable options of PT. However, the performance of the RH subsidy is limited when the RH fleet size is small. The amount of subsidy offered is also a key factor. If the amount of RH subsidy is not sufficiently high to stimulate travellers to change their travel mode, then RH subsidy may not affect the system performance or may even have a negative effect on PT mode share. The real-world case also shows that the impact of subsidy varies for different ODs. Modal shift is not observed for some subsidised ODs in the subsidised scenario, which can be explained by low competitiveness compared to other travel alternatives. Path shift of PT users is usually observed for ODs which are associated with a path characterised by a longer PT access/egress distance but a shorter travel time and fewer transfers than the path selected in the unsubsidised scenario.

Interestingly, the impact of subsidies is not limited to direct effects for those benefiting from a subsidised trip but also extends to secondary effects for those who travel in the same corridor, for example, due to increased crowding levels caused by path shifts. Thus, policy makers are advised to evaluate policy performance on the whole transport system rather than only focusing on subsidised travellers in order to adequately assess its impact.

There are some limitations that remain to be solved for more general scenarios. First, we simplify the operation of RH services in our study to develop a tractable model. In reality, the waiting time of RH services is influenced by multiple factors including fleet size, driver behaviour and pricing. In this study, we assume the waiting time is fully determined by the fleet size and the fleet size is fixed. This assumption is considered reasonable when the strategy of drivers and operators of RH services does not change during the analysis period. Second, we consider RH and walking as the access/egress mode of PT but some other travel modes such as private car and bike can also become the access/egress options in areas with park-and-ride facilities or bike-sharing services. The proposed model can be extended to a general model including more travel modes by combining our work with some existing studies. Third, two user classes are considered in this study: car owners and non-car owners. However, the heterogeneity in these two user classes is not discussed in this study, such as the heterogeneity of value of time. In reality, in areas with a high Gini coefficient, travellers with different values of time are expected to respond differently to the subsidy. Those heterogeneous features of travellers can be captured in the proposed model by introducing more user classes into the model.

For future research, there are some promising directions related to the integration of PT and RH services. First, important research opportunities lie in the two-sided mobility systems considering the choice of supply-side RH drivers (de Ruijter et al. 2022). Second, the model can be extended to a multi-class SUE model to account for travellers' heterogeneity. Third, more access modes of PT (e.g. private car and bicycle) can be included in the proposed model to allow for comparing the impact of strategies for different travel modes (e.g. park-and-ride and bike-sharing) on multimodal transport systems. Fourth, the specification of the subsidy strategy can be optimised in relation to policy objectives, possibly in conjunction with the adjustment of related bus operations (e.g. frequency adjustments).

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by National Natural Science Foundation of China: [Grant Number 52272335]; European Research Council and Amsterdam Institute for Advanced Metropolitan Solutions: [Grant Number 804469]; China Scholarship Council: [Grant Number 202106260129].

ORCID

Bing Liu http://orcid.org/0000-0003-2052-9924

References

Babar, Yash, and Gordon Burtch. 2020. "Examining the Heterogeneous Impact of Ride-Hailing Services on Public Transit use." *Information Systems Research* 31 (3): 820–834. https://doi.org/10.1287/isre.2019.0917

Ban, Xuegang, Maged Dessouky, Jong-Shi Pang, and Rong Fan. 2019. "A General Equilibrium Model for Transportation Systems with e-Hailing Services and Flow Congestion." *Transportation Research Part B: Methodological* 129: 273–304. https://doi.org/10.1016/j.trb.2019.08.012.

Benaroya, Anne, Matthias Sweet, and Raktim Mitra. 2023. "On-demand Ride Hailing as Publicly Subsidized Mobility: An Empirical Case Study of Innisfil Transit." *Case Studies on Transport Policy* 11), https://doi.org/10.1016/j.cstp.2022.100944.

Boyce, David, and Hillel Bar-Gera. 2004. "Multiclass Combined Models for Urban Travel Forecasting." *Networks and Spatial Economics* 4: 115–124. https://doi.org/10.1023/B:NETS.0000015659.39216.83

Cane, T. 2017. "Demand-Responsive Transit Implementation—Stage 1." In.: Publication DSR-042–17. Innisfil, ON: Town of Innisfil Report.

Cantarella, Giulio Erberto. 1997. "A General Fixed-Point Approach to Multimode Multi-User Equilibrium Assignment with Elastic Demand." *Transportation Science* 31 (2): 107–128. https://doi.org/10.1287/trsc.31.2.107.

Cantarella, Giulio Erberto, Armando Cartenì, and Stefano de Luca. 2015. "Stochastic Equilibrium Assignment with Variable Demand: Theoretical and Implementation Issues." *European Journal of Operational Research* 241 (2): 330–347. https://doi.org/10.1016/j.ejor.2014.09.010.

Cats, Oded, Rafal Kucharski, Santosh Rao Danda, and Menno Yap. 2022. "Beyond the Dichotomy: How Ride-Hailing Competes with and Complements Public Transport." *Plos One* 17 (1): e0262496. https://doi.org/10.1371/journal.pone.0262496.

Clewlow, Regina R., and Gouri S. Mishra. 2017. "Disruptive Transportation: The Adoption, Utilization, and Impacts of Ride-Hailing in the United States." In.

Curtis, Terra, Meg Merritt, Carmen Chen, David Perlmutter, Dan Berez, and Buffy Ellis. 2019. "Partnerships between Transit Agencies and Transportation Network Companies (TNCs)." In.

De Cea, Joaquin, J. Enrique Fernández, Valerie Dekock, and Alexandra Soto. 2005. "Solving Network Equilibrium Problems on Multimodal Urban Transportation Networks with Multiple User Classes." *Transport Reviews* 25 (3): 293–317. https://doi.org/10.1080/0144164042000335805

de Ruijter, Arjan, Oded Cats, Rafal Kucharski, and Hans van Lint. 2022. "Evolution of Labour Supply in Ridesourcing." *Transportmetrica B: Transport Dynamics* 10 (1): 599–626. https://doi.org/10.1080/21680566.2021.2024917

Di, Xuan, Henry X. Liu, Xuegang Ban, and Hai Yang. 2017. "Ridesharing User Equilibrium and Its Implications for High-Occupancy Toll Lane Pricing." *Transportation Research Record: Journal of the Transportation Research Board* 2667 (1): 39–50. https://doi.org/10.3141/2667-05.

Di, Xuan, Rui Ma, Henry X. Liu, and Xuegang Ban. 2018. "A Link-Node Reformulation of Ridesharing User Equilibrium with Network Design." *Transportation Research Part B: Methodological* 112: 230–255. https://doi.org/10.1016/j.trb.2018.04.006.

Erhardt, Gregory D, Sneha Roy, Drew Cooper, Bhargava Sana, Mei Chen, and Joe Castiglione. 2019. "Do Transportation Network Companies Decrease or Increase Congestion?" *Science Advances* 5 (5): eaau2670. https://doi.org/10.1126/sciadv.aau2670

Fan, Yinchao, Jianxun Ding, Haoxiang Liu, Yu Wang, and Jiancheng Long. 2022. "Large-Scale Multimodal Transportation Network Models and Algorithms-Part I: The Combined Mode Split and Traffic Assignment Problem." *Transportation Research Part E: Logistics and Transportation Review* 164: 102832. https://doi.org/10.1016/j.tre.2022.102832

Florian, Michael. 1977. "A Traffic Equilibrium Model of Travel by Car and Public Transit Modes." *Transportation Science* 11 (2): 166–179. https://doi.org/10.1287/trsc.11.2.166.

García, Ricardo, and Angel Marín. 2005. "Network Equilibrium with Combined Modes: Models and Solution Algorithms." *Transportation Research Part B: Methodological* 39 (3): 223–254. https://doi.org/10.1016/j.trb.2003.05.002

Geržinič, Nejc, Oded Cats, Niels van Oort, Sascha Hoogendoorn-Lanser, and Serge Hoogendoorn. 2021. "What is the market potential for on-demand services as a train station access mode?" *arXiv preprint arXiv:2107.00980*.

Graehler, Michael, Richard Alexander Mucci, and Gregory D. Erhardt. 2019. Understanding the recent transit ridership decline in major US cities: Service cuts or emerging modes. Paper presented at the 98th Annual Meeting of the Transportation Research Board, Washington, DC.

Hall, Jonathan D., Craig Palsson, and Joseph Price. 2018. "Is Uber a Substitute or Complement for Public Transit?" *Journal of Urban Economics* 108: 36–50. https://doi.org/10.1016/j.jue.2018.09.003

Kitthamkesorn, Songyot, Anthony Chen, Xiangdong Xu, and Seungkyu Ryu. 2013. "Modeling Mode and Route Similarities in Network Equilibrium Problem with Go-Green Modes." *Networks and Spatial Economics* 16 (1): 33–60. https://doi.org/10.1007/s11067-013-9201-y.

Lam, William HK, and Hai-Jun Huang. 1992. "A Combined Trip Distribution and Assignment Model for Multiple User Classes." *Transportation Research Part B: Methodological* 26 (4): 275–287. https://doi.org/10.1016/0191-2615(92)90038-X

Li, Meng, Xuan Di, Henry X. Liu, and Hai-Jun Huang. 2019. "A Restricted Path-Based Ridesharing User Equilibrium." *Journal of Intelligent Transportation Systems* 24 (4): 383–403. https://doi.org/10.1080/15472450.2019.1658525.

Liu, Zhiyuan, Xinyuan Chen, Qiang Meng, and Inhi Kim. 2018. "Remote Park-And-Ride Network Equilibrium Model and Its Applications." *Transportation Research Part B: Methodological* 117: 37–62. https://doi.org/10.1016/j.trb.2018.08.004

Luethi, Marco, Ulrich Weidmann, and Andrew Nash. 2007. "Passenger Arrival Rates at Public Transport Stations." Paper presented at the TRB 86th Annual Meeting Compendium of Papers.

Nguyen, Sang, and Stefano Pallottino. 1988. "Equilibrium Traffic Assignment for Large Scale Transit Networks." *European Journal of Operational Research* 37 (2): 176–186. https://doi.org/10.1016/0377-2217(88)90327-X

Pinto, Helen KRF, Michael F Hyland, Hani S Mahmassani, and I. Ömer Verbas. 2020. "Joint Design of Multimodal Transit Networks and Shared Autonomous Mobility Fleets." *Transportation Research Part C: Emerging Technologies* 113: 2–20. https://doi.org/10.1016/j.trc.2019.06.010

Reck, Daniel Jan, and Kay W. Axhausen. 2020. "Subsidized Ridesourcing for the First/Last Mile: How Valuable for Whom?" *European Journal of Transport and Infrastructure Research* 20 (4): 59–77. https://doi.org/10.3929/ethz-b-000444035.

Shao, Minhua, Congcong Xie, Tianye Li, and Lijun Sun. 2022. "Influence of in-Vehicle Crowding on Passenger Travel Time Value: Insights from Bus Transit in Shanghai, China." *International Journal of Transportation Science and Technology* 11 (4): 665–677. https://doi.org/10.1016/j.ijtst.2021.09.001.

Sheffi, Yosef. 1985. *Urban Transportation Networks. Vol. 6*. Englewood Cliffs, NJ: Prentice-Hall.

Siddiq, Auyon, Christopher S Tang, and Jingwei Zhang. 2022. "Partnerships in Urban Mobility: Incentive Mechanisms for Improving Public Transit Adoption." *Manufacturing & Service Operations Management* 24 (2): 956–971. https://doi.org/10.1287/msom.2021.0987

Spiess, Heinz, and Michael Florian. 1989. "Optimal Strategies: A new Assignment Model for Transit Networks." *Transportation Research Part B: Methodological* 23 (2): 83–102. https://doi.org/10.1016/0191-2615(89)90034-9

Sun, S., and W. Y. Szeto. 2021. "Multi-class Stochastic User Equilibrium Assignment Model with Ridesharing: Formulation and Policy Implications." *Transportation Research Part A: Policy and Practice* 145: 203–227. https://doi.org/10.1016/j.tra.2020.12.011.

Tirachini, Alejandro. 2020. "Ride-hailing, Travel Behaviour and Sustainable Mobility: An International Review." *Transportation* 47 (4): 2011–2047. https://doi.org/10.1007/s11116-019-10070-2

Tirachini, Alejandro, and Andres Gomez-Lobo. 2020. "Does Ride-Hailing Increase or Decrease Vehicle Kilometers Traveled (VKT)? A Simulation Approach for Santiago de Chile." *International Journal of Sustainable Transportation* 14 (3): 187–204. https://doi.org/10.1080/15568318.2018.1539146

US Bureau of Public Roads. 1964. *Traffic Assignment Manual for Application with a Large, High Speed Computer*. Washington: US Department of Commerce. Office of Planning. Urban Planning Division.

Wang, Fangru, and Catherine L. Ross. 2019. "New Potential for Multimodal Connection: Exploring the Relationship Between Taxi and Transit in New York City (NYC)." *Transportation* 46 (3): 1051–1072. https://doi.org/10.1007/s11116-017-9787-x

Wei, Keji, Vikrant Vaze, and Alexandre Jacquillat. 2022. "Transit Planning Optimization Under Ride-Hailing Competition and Traffic Congestion." *Transportation Science* 56 (3): 725–749. https://doi.org/10.1287/trsc.2021.1068.

Williams, Huw CWL. 1977. "On the Formation of Travel Demand Models and Economic Evaluation Measures of User Benefit." *Environment and Planning A* 9 (3): 285–344. https://doi.org/10.1068/a090285

Xu, Huayu, Fernando Ordóñez, and Maged Dessouky. 2015a. "A Traffic Assignment Model for a Ridesharing Transportation Market." *Journal of Advanced Transportation* 49 (7): 793–816. https://doi.org/10.1002/atr.1300.

Xu, Huayu, Jong-Shi Pang, Fernando Ordóñez, and Maged Dessouky. 2015b. "Complementarity Models for Traffic Equilibrium with Ridesharing." *Transportation Research Part B: Methodological* 81: 161–182. https://doi.org/10.1016/j.trb.2015.08.013.

Yan, Chen-Yang, Mao-Bin Hu, Rui Jiang, Jiancheng Long, Jin-Yong Chen, and Hao-Xiang Liu. 2019a. "Stochastic Ridesharing User Equilibrium in Transport Networks." *Networks and Spatial Economics* 19 (4): 1007–1030. https://doi.org/10.1007/s11067-019-9442-5.

Yan, Xiang, Jonathan Levine, and Xilei Zhao. 2019b. "Integrating Ridesourcing Services with Public Transit: An Evaluation of Traveler Responses Combining Revealed and Stated Preference Data."

Transportation Research Part C: Emerging Technologies 105: 683–696. https://doi.org/10.1016/j.trc.2018.07.029
Young, Mischa, Jeff Allen, and Steven Farber. 2020. "Measuring When Uber Behaves as a Substitute or Supplement to Transit: An Examination of Travel-Time Differences in Toronto." *Journal of Transport Geography* 82: 102629. https://doi.org/10.1016/j.jtrangeo.2019.102629
Zhang, Yufeng, and Alireza Khani. 2021. "Integrating Transit Systems with Ride-Sourcing Services: A Study on the System Users' Stochastic Equilibrium Problem." *Transportation Research Part A: Policy and Practice* 150: 95–123. https://doi.org/10.1016/j.tra.2021.05.008
Zhu, Zheng, Ailing Xu, Qiao-Chu He, and Hai Yang. 2021. "Competition between the Transportation Network Company and the Government with Subsidies to Public Transit Riders." *Transportation Research Part E: Logistics and Transportation Review* 152: 102426. https://doi.org/10.1016/j.tre.2021.102426

Appendix A. Demand and network data in toy case

Appendix A1. OD Matrix.

Origin	Destination	Car owner (%)	Non-car owner (%)
A	Z	16.7	16.7
X	Z	20.0	20.0
Y	Z	13.3	13.3

Appendix A2. Public transport data.

Line	Segment	Running time (min)	Distance (km)	Headway (min)	Area (m^2)
Line 1	(A, Z)	25	10	6	20
Line 2	(A, X)	7	3.5	6	20
Line 2	(X, Y)	6	3	6	20
Line 3	(X, Y)	4	3	15	20
Line 3	(Y, Z)	4	3	15	20
Line 4	(Y, Z)	10	3	3	20

Appendix A3. Road network data.

Segment	Capacity	Travel time (min)	Distance (km)
(A, X)	800	5	3.5
(X, Y)	800	5	3
(A, Y)	800	10	6.5
(Y, Z)	800	5	3
(X, Z)	800	9	6