# Experimental evaluation of the viscous contribution to mean drift forces on vertical cylinders

A.K. Dev and J.A. Pinkster

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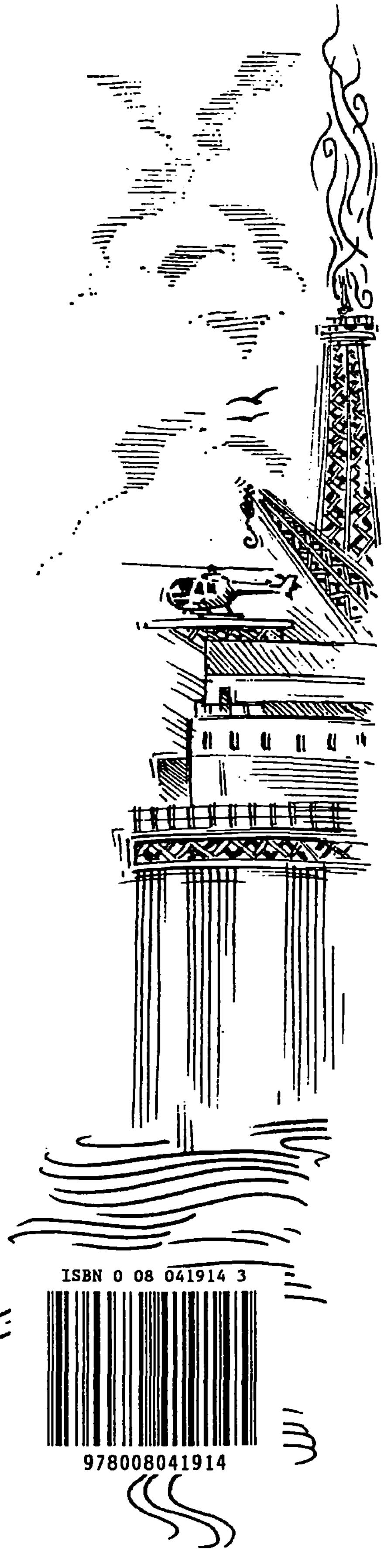

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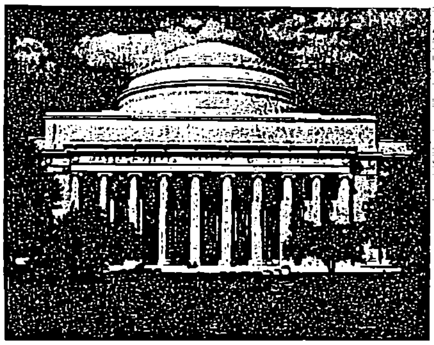

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# EXPERIMENTAL EVALUATION OF THE VISCOUS CONTRIBUTION TO MEAN DRIFT FORCES ON VERTICAL CYLINDERS

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## ABSTRACT

Discrepancies, according to some literature, are observed between the measured and the theoretical horizontal mean drift forces in regular and irregular waves on moored offshore structures like semi-submersibles and tension leg platforms. Such divergence is dominant in the low frequency range where diffraction effects are less for slender body type structures and is thus considered to be caused by the viscous effects. The theory to evaluate such viscous contributions has been developed based on the viscous drag force term of the Morison equation via the linear (Airy) wave theory over the splash zone (wave stretching zone) for a wavy flow field and a wave-current co-existing flow field. To substantiate the theory and its range of applicability and its dependency on different hydrodynamic parameters, experiments have been conducted with fixed and truncated vertical surface piercing cylinders of different diameters and of segmented construction in regular waves at zero and with forward velocities simulating the effect of currents. Test results in regular waves of varying amplitudes with and without forward speed show that the viscous effects are indeed significant. Such contributions are further influenced by presence of currents. The theory based on a relative horizontal velocity and a relative surface elevation and using experimentally obtained values of the mean drag coefficients, when applied to a floating semi-submersible in frequency domain for regular waves, improves the theoretical predictions for the horizontal mean forces.

## KEYWORDS

Potential; viscous; mean drift forces; vertical cylinders; in waves and in waves and currents; model testing; hydrodynamic parameters; mean drag coefficients

## INTRODUCTION

It is now well known that floating structures in an irregular seaway exhibit a wave frequency response, a mean response and a low frequency (slowly varying) response at the natural frequency of the moored floating structure. The mean response in regular waves is caused by the mean drift force in the horizontal mode. While the first order force with the wave frequency is linear with the wave height, the mean force being non-linear is quadratic with the wave height. These quadratic wave forces are believed to be due to potential effects and as such are treated by the linear potential theory using the pressure integration method (near field approach) (Pinkster, 1980) or conservation of momentum principle (far field approach) (Maruo, 1960; Newman, 1967). Such methods prove to be quite satisfactory when viscous effects are less prominent. For floating structures such as semi-submersibles and tension leg platforms, viscous effects are equally important because of their inherent structural geometry, i.e. having surface piercing column structures. These viscous contributions are further pronounced when currents co-exist with waves due to their non-linear interactions.

Several authors have treated viscous effects in the mean drift force on floating structures such as semi-submersibles and tension leg platforms. Pijfers and Brink (1977) considered the viscous drift force due to waves and currents in their analysis of two semi-submersibles' drift forces. Denise and Heaf (1979) considered the drag force using empirical drag and friction coefficients while analyzing the response of a tension leg platform. Ferretti and Berta (1981) applied the Morison equation (Morison *et al.*, 1950) to calculate the mean drift force on a vertical cylinder due to potential effects. The influence of wave height on the splash zone was shown to cause the viscous drift force. Finally the wave-current interaction effects were demonstrated at the mean water level (mwl). Lundgren *et al.* (1982) discussed the different contributions for the potential and viscous drift force on a fixed cylinder providing approximate analytical expressions. The horizontal relative velocity model in the Morison equation has been applied by (Burns, 1983) on a tension leg platform while comparing the extreme horizontal excursion in both regular and irregular waves in frequency and time domain. Chakrabarti (1984) presented closed form analytical solutions for both potential and viscous drift forces on a fixed vertical cylinder to find their relative importance such as where the viscous or potential drift force predominates. Kobayashi *et al.* (1985), while investigating the response of a tension leg platform in regular and irregular waves, considered viscous contributions to the wave drift forces by using the horizontal relative velocity in the drag term of the Morison equation. Standing *et al.* (1991) gave an expression for the mean drag force on a single column of a semi-submersible. Both relative horizontal velocity and relative surface elevation were accounted for. Comparison (Pinkster, 1993) of measured and computed mean drift forces on two types of semi-submersibles in both regular and irregular waves shows consistent divergence between 3-D predictions and results of experiments. Chitrapu (1993) presented a method to compute the wave and current induced viscous drift forces and moments on a tension leg platform in regular and irregular waves.

It can be concluded from the above review that most of the approaches share a common view of treating a single surface piercing vertical cylinder representing the column of a semi-submersible or a tension leg platform. Furthermore, wave elevation up to the instantaneous sea level has been the cause of the viscous mean drift force due to waves only. Such forces are calculated by exploiting the drag force term of the Morison equation. Wave-current interaction effects have been shown for the structure up to the mean water level only. Not much attention has been paid regarding the values of the mean drag coefficients for different flow fields. In most of the cases, only horizontal relative velocity has been considered because the numerical models were mostly for tension leg platforms.

In this study, a theoretical evaluation (Dev, 1992a) has been carried out for finding the viscous contributions to the horizontal mean drift force. The cylinder is considered divided into two parts namely the splash zone (from the mwl up to the actual sea level) and the submerged zone (from the mwl down to the bottom of the cylinder). In theory, the value of $C_{D0}$ is suppressed by taking its value as unity. Model tests have been carried out to deal with such evaluation experimentally for fixed cylinders in order to validate the theory. Finally, calculations including viscous contributions for the mean drift force in regular waves, for a complete semi-submersible have been compared with the available model test results for verifying the predictions.

## HORIZONTAL MEAN DRIFT FORCES ON A FIXED CYLINDER

Regular waves having the following wave kinematics for deep water condition are used.

$$\phi = \frac{\zeta_a g}{\omega} e^{kz} \sin(kx - \omega t) \tag{1}$$

$$\zeta = \zeta_a \cos(kx - \omega t); \qquad u = \zeta_a e^{kz} \omega \cos(kx - \omega t) = u_m \cos(kx - \omega t) \tag{2}$$

Use of the linear (Airy) wave theory can be questionable because of its validity up to mwl with finite wave amplitudes. On the other hand, modification via wave stretching proposed by (Wheeler, 1970; Chakrabarti, 1984, 1990) leads to zero viscous mean drift forces in deep water conditions (Chakrabarti, 1984) which is not true as would be seen later from the experimental results. Under these circumstances, either constant velocity in the wave crest or extension of the linear (Airy) wave theory can be exercised. The former one is applied in the following estimates.

### Viscous Mean Drift Forces in Waves Only

The viscous drag force for a unit length cylindrical section is, according to the Morison equation, as follows:

$$\begin{aligned} F_D &= 1/2 \ \rho \ C_D \ D \ (u_m \cos \omega t) \ | u_m \cos \omega t | \\ &= 1/2 \ \rho \ C_D \ D \ \zeta_a^2 \ \omega^2 \ 8/(3 \pi) \ \cos \omega t \end{aligned} \tag{3}$$

The mean drift force originating from viscous effects in the splash zone is as follows:

$$\begin{aligned} \bar{F}_D &= 4/(3\pi) \ \rho \ C_{D0} \ D \ \zeta_a^2 \ \omega^2 \ 1/T \int_0^T \int_0^\zeta \cos \omega t \ dz \ dt \\ &= 2/(3\pi) \ \rho \ g \ k \ C_{D0} \ D \ \zeta_a^3 \end{aligned} \tag{4}$$

The mean drift force on the splash zone is thus found to vary with cube of the wave height and for a particular wave height, it would increase linearly with wave frequency squared.

### Viscous Mean Drift Forces in Waves and Currents

Splash Zone. The viscous drag force in presence of currents now becomes:

$$F_D = 1/2 \ \rho \ C_D \ D \ (u_m \cos \omega t + U) \ | u_m \cos \omega t + U | \tag{5}$$

The application of eq. (5) depends on the magnitude of U with respect to that of $u_m$.

$$\bar{F}_D = 1/2 \ \rho \ C_{D0} \ D \ 1/T \int_0^T \int_0^\zeta (U^2 + 2 U u_m \cos \omega t + u_m^2 \cos^2 \omega t) \ dz \ dt \tag{6}$$

$$\bar{F}_D = 1/2\ \rho\ C_{D0}\ D\ \zeta_a\ U\ u_m \qquad \text{For } |U| \geq u_m \tag{7}$$

$$\bar{F}_D = 1/\pi\ \rho\ C_{D0}\ D\ \zeta_a\ u_m^2 \{(\gamma^2 \sin\Theta) + 1/12\ (\sin 3\Theta + 9 \sin\Theta) + \gamma/2\ (\sin 2\Theta - \pi + 2\Theta)\} \qquad \text{For } |U| < u_m \tag{8}$$

Submerged Zone. The mean drift force due to wave current interaction effects at mwl ($z=0$) is given by the following equations. For the complete submerged zone, the wave particle velocity $u_m$ is to be replaced by $\zeta_a\ \omega\ e^{kz}$ and computations are to be repeated for a number of segments.

$$\bar{F}_D = 1/2\ \rho\ C_{D0}\ D\ 1/T \int_0^T (U^2 + 2\,U\,u_m \cos\omega t + u_m^2 \cos^2\omega t)\ dt \tag{9}$$

$$\bar{F}_D = 1/2\ \rho\ C_{D0}\ D\ u_m^2\ (\gamma^2 + 1/2) \qquad \text{For } |U| \geq u_m \tag{10}$$

$$\bar{F}_D = 1/(2\pi)\ \rho C_{D0} D u_m^2 \{\gamma^2(2\Theta - \pi) + 1/2\ (2\Theta - \pi + \sin 2\Theta) + 4\gamma \sin\Theta\} \qquad \text{For } |U| < u_m \tag{11}$$

The value of $\Theta$ is $\cos^{-1} = (-U/u_m)$ and for negative U, the value of $\Theta$ is $\cos^{-1} = (U/u_m)$. The above phenomena of positive and negative currents including their magnitude and thus their relative effects on wave-current coexisting field are illustrated in Fig. 1 and Fig. 2.

## Computations

For a vertical cylinder of 10 m diameter and 20 m draft which is similar to a vertical column of a semi-submersible or tension leg platform, computations have been done based on the above outlined theory. For $D/\lambda \leq 0.20$, the drag forces cannot be disregarded and their effects on the total wave drift force can be equally or more important. Some results are presented here.

Figure 3 shows the mean drift force due to the viscous drag force term in the Morison equation. It is apparent that with the increase of the wave height, the mean drift force increases with the cubic power of it. The total drift force calculation after viscous effects are added are shown in Fig. 4. While considering the mean drift force including viscous effects, it is no longer independent of the wave height and such trends are clearly shown in Fig. 4. On the other hand, the mean drift force due to potential effects, when non-dimensionalized, is independent of the wave height. The mean drift force due to potential effects has been calculated by DELFRAC, a 3-D diffraction program developed by Delft University of Technology.

Figure 5 clearly suggests that the wave-current interaction is much more pronounced at or immediately below the mean water level. It increases with the increase of wave frequencies for a wave height. As the draft increases, the interaction becomes weaker. The decrement is rather drastic in the sense that the higher the frequency, the higher the rate of the decrement. Such phenomena are governed by the exponential term which depends on both draft and frequency. The above characteristic can further be seen in Fig. 6 where the effects of wave heights are also shown. It is worth mentioning here that assuming constant velocity for the submerged zone as mentioned in (Lundgren *et al.*, 1982) would produce a much higher force.

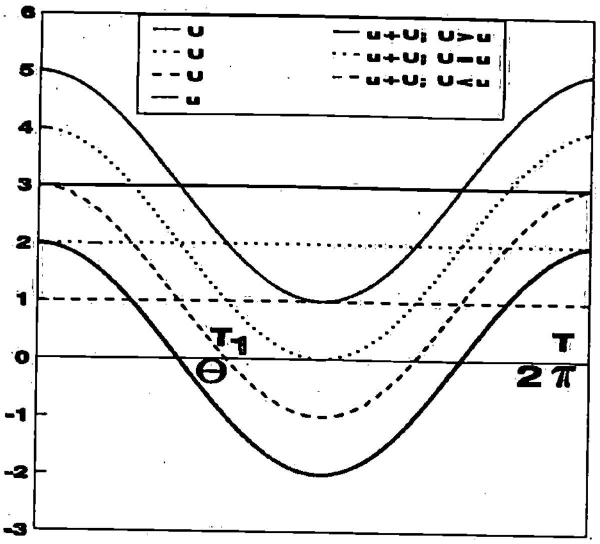

Fig. 1 Effects of positive currents on horizontal water particle velocity

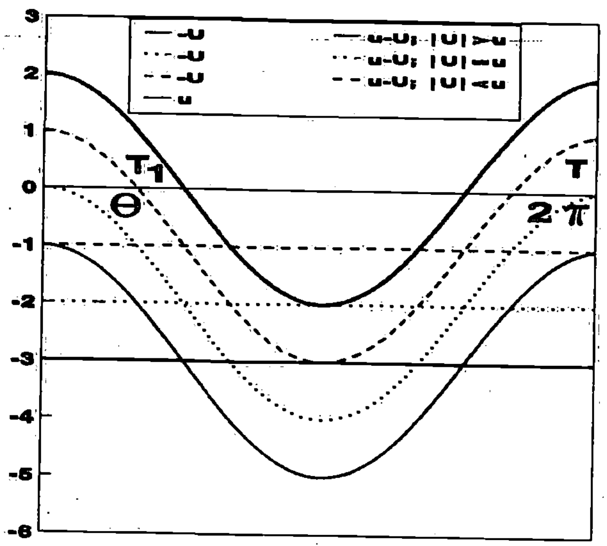

Fig. 2 Effects of negative currents on horizontal water particle velocity

## HORIZONTAL MEAN DRIFT FORCES ON A FLOATING CYLINDER

### Relative Horizontal Velocity and Relative Surface Elevation

In case of a floating cylinder, only translatory motions, i.e. only horizontal and vertical modes of motions are considered. Thus the cylinder is subject to a relative horizontal velocity and a relative surface elevation.

For a floating cylinder, the relative surface elevation $\zeta_r$ is replaced by $\zeta_{r\,m} \cos(\omega t + \epsilon_\zeta)$.

$$\zeta_{r\,m} = \zeta_a \sqrt{\{1 + (RAO)_z^{\,2} - 2\,(RAO)_z \cos\epsilon_z\}};\quad \epsilon_\zeta = \arctan\{z_a \sin\epsilon_z / (\zeta_a - z_a \cos\epsilon_z)\} \qquad (12)$$

Similarly, the horizontal relative velocity $u_r$ is replaced by $u_{r\,m} \cos(\omega t + \epsilon_u)$.

$$u_{r\,m} = \zeta_a\, \omega \sqrt{\{1 + (RAO)_x^{\,2} - 2\,(RAO)_x \sin\epsilon_x\}};\quad \epsilon_u = \arctan\{-\dot{x}_m \cos\epsilon_x / (u_m - \dot{x}_m \sin\epsilon_x)\} \qquad (13)$$

### Viscous Mean Drift Forces in Waves Only

Using the viscous drag term of the Morison equation and replacing u by $u_r$ the mean drift force due to viscous effects on a floating cylinder is as follows:

$$\begin{aligned}\overline{F}_D &= 1/2\ \rho\, C_{D0}\, D\ 1/T \int_0^T \int_0^{\zeta_r} u_{r\,m}^{\,2}\ |\cos(\omega t + \epsilon_u)|\ \{\cos(\omega t + \epsilon_u)\}\ dz\ dt \\ &= 2/(3\,\pi)\ \rho\, C_{D0}\, D\, u_{r\,m}^{\,2}\, \zeta_{r\,m} \cos(\epsilon_u - \epsilon_\zeta)\end{aligned} \qquad (14)$$

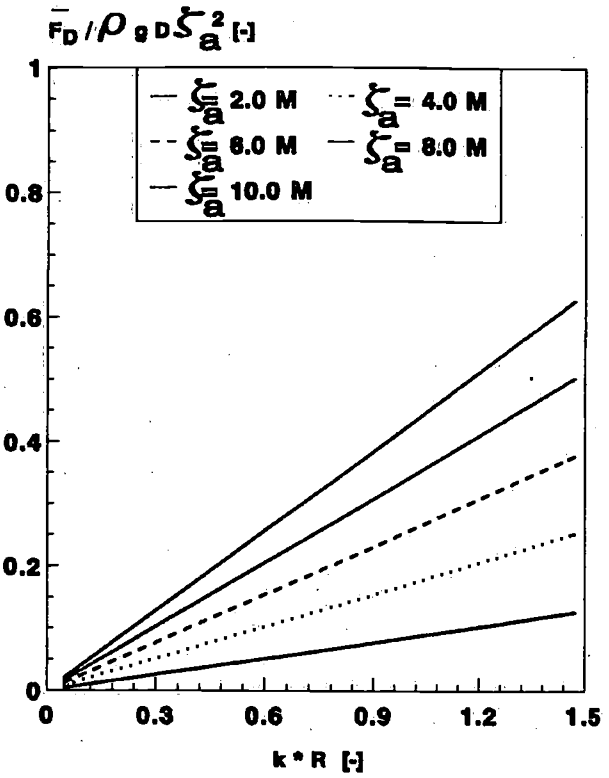

Fig. 3. $\bar{F}_D$ versus k*R in waves only for different wave amplitudes

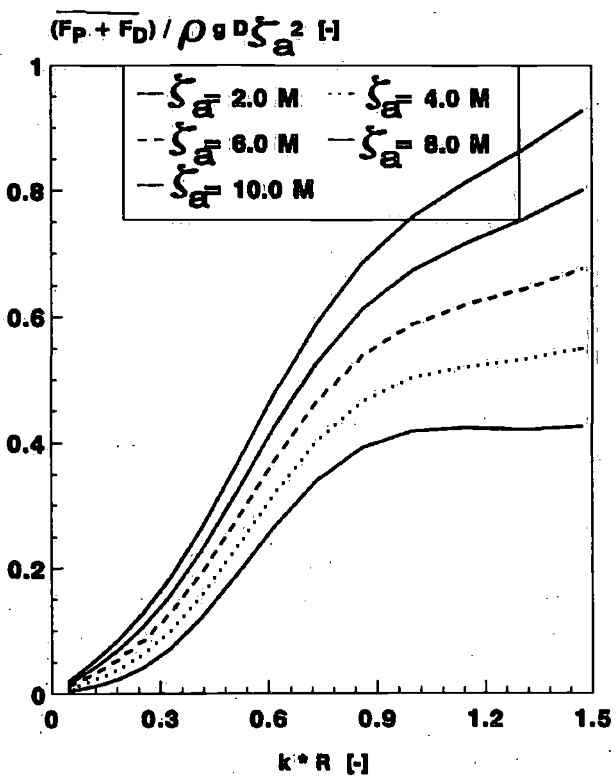

Fig. 4. $\bar{F}_T$ versus k*R in waves only for different wave amplitudes

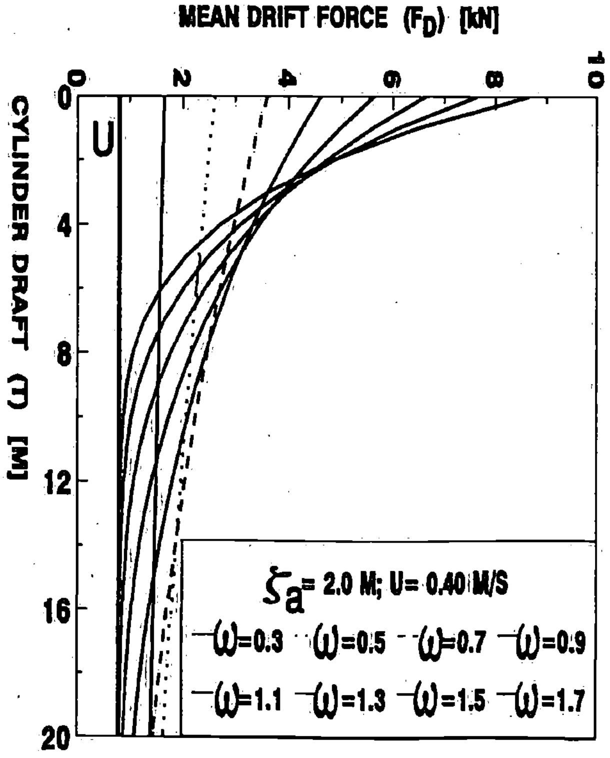

Fig. 5. $\bar{F}_D$ over T in waves and currents for different wave frequencies

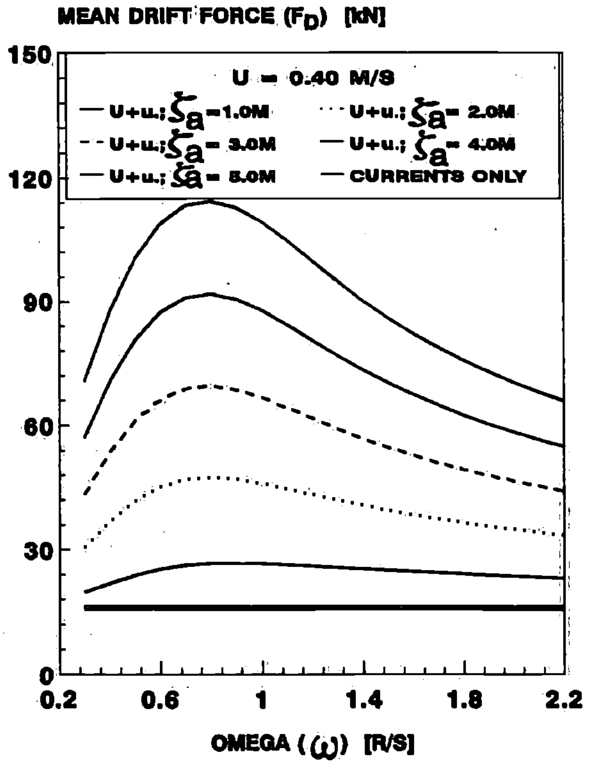

Fig. 6. $\bar{F}_D$ versus wave frequency for different wave amplitudes

Viscous Mean Drift Forces in Waves and Currents

In presence of positive currents, the viscous drag force is as follows:

$$F_D = 1/2 \ \rho \ C_D \ D \ (u_r + U) \ \ | u_r + U | \qquad (15)$$

Similar to the fixed cylinder, the application of eq. (15) is to be performed depending on the magnitude of the current velocity U with respect to that of the relative velocity $u_{r\,m}$.

Splash Zone. The mean drift force due to wave-current interaction effects becomes:

$$\bar{F}_D = 1/2 \ \rho \, C_{D0} D \ 1/T \int_0^T \int_0^{\zeta_r} \{U^2 + 2\,U\,u_{r\,m} \cos(\omega t + \epsilon_u) + u_{r\,m}{}^2 \cos^2(\omega t + \epsilon_u)\} \ dz\,dt \qquad (16)$$

$$\bar{F}_D = 1/2 \ \rho \ C_{D0} \ D \ U \ u_{r\,m} \ \zeta_{r\,m} \ \cos(\epsilon_u - \epsilon_\zeta) \qquad \text{For } |U| \ge u_{r\,m} \qquad (17)$$

$$\begin{aligned} \bar{F}_D = 1/\pi \ \rho \ C_{D0} \ D \ \zeta_{r\,m} \ u_{r\,m}{}^2 [\{(\gamma^2 \sin\Theta \cos\epsilon_\zeta)\} + \gamma/2 \ \{(2\Theta - \pi) \ \cos(\epsilon_u - \epsilon_\zeta) \\ + \ \sin 2\Theta \cos(\epsilon_u + \epsilon_\zeta)\} + (1/3 \ \sin\Theta) \{\cos\epsilon_\zeta (1/2 \ \cos 2\Theta (\cos 2\epsilon_u + 1)) \\ + \ \cos\epsilon_u (2 \cos(\epsilon_u - \epsilon_\zeta) + (\cos 2\Theta + 1) \sin\epsilon_u \sin\epsilon_\zeta)\}] \qquad \text{For } |U| < u_{r\,m} \end{aligned} \qquad (18)$$

Submerged Zone. The mean drift force at mwl is given by the following equations:

$$\bar{F}_D = 1/2 \ \rho \ C_{D0} \ D \ 1/T \int_0^T \{U^2 + 2\,U\,u_{r\,m} \cos(\omega t + \epsilon_u) + u_{r\,m}{}^2 \cos^2(\omega t + \epsilon_u)\} \ dt \qquad (19)$$

$$\bar{F}_D = 1/2 \ \rho \ C_{D0} \ D \ u_{r\,m}{}^2 \ (\gamma^2 + 1/2) \qquad \text{For } |U| \ge u_{r\,m} \qquad (20)$$

$$\begin{aligned} \bar{F}_D = 1/(2\pi) \ \rho \ C_{D0} \ D \ u_{r\,m}{}^2 \{\gamma^2 (2\,\Theta - \pi) + 4\,\gamma \sin\Theta \cos\epsilon_u \\ + \ 1/2 \ (2\Theta - \pi + \sin 2\Theta \cos 2\epsilon_u)\} \qquad \text{For } |U| < u_{r\,m} \end{aligned} \qquad (21)$$

Computational Results

For the same cylinder particulars, computations have also been done for the floating condition as well. Some results are presented here.

In Fig. 7, comparison between the mean drift force due to viscous effects for a fixed and floating cylinder is shown for unit wave amplitude. The difference is appreciable. But with the increase of wave height, the viscous effects would increase in a similar manner as for the fixed cylinder. The mean drift force for a floating cylinder shows a blunt peak at the heave natural frequency.

In Fig. 8, wave-current interaction effects are shown for the fixed and the floating cylinder in their splash zone. For a fixed wave amplitude, the interaction effects increase with the increase of the current velocity. This is the same for a fixed as well as for a floating cylinder.

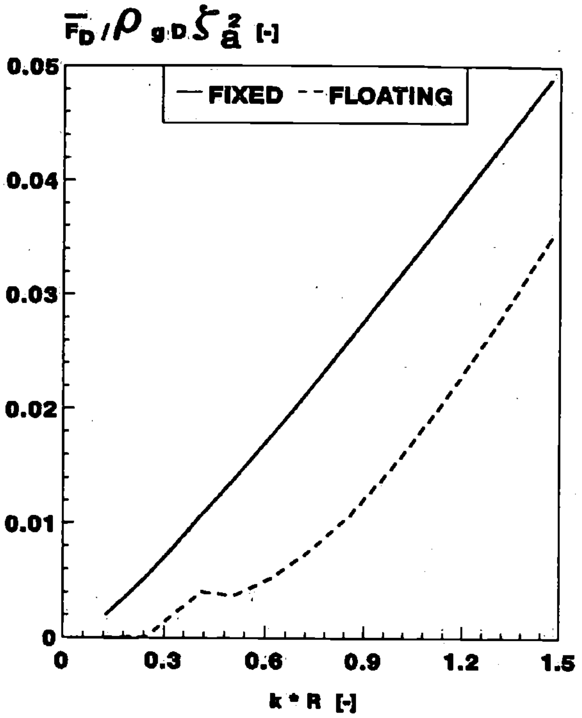

Fig. 7. $\bar{F}_D$ versus k*R in waves only for a fixed and floating cylinder

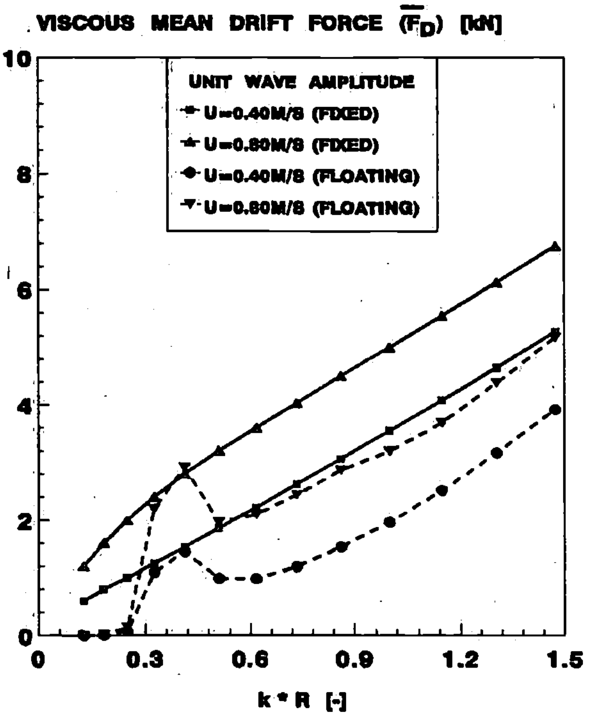

Fig. 8. $\bar{F}_D$ versus k*R in waves and currents for a fixed and floating cylinder

## CONTROLLING HYDRODYNAMIC PARAMETERS

The Reynolds number, $N_{Re}$, is usually used for expressing the hydrodynamic force coefficients in a uniform flow field. The Keulegan-Carpenter number, $N_{K-C}$, is used similarly in a waves only field whether the structure is oscillating in an in-line direction in still water or fixed in a harmonically oscillating flow. For a waves only field, both $N_{Re}$ and $N_{K-C}$ again lead to a new hydrodynamic parameter known as 'frequency parameter' (a ratio of $N_{Re}/N_{K-C}$) as introduced by (Sarpkaya, 1981).

It has been revealed through different existing studies that only a few have handled properly the mean drag coefficients for viscous mean forces. Pijfers and Brink (1977) considered the drag coefficient for the mean drift force as an average value based on $N_{Re}$ and $N_{K-C}$. Moe and Verley (1980) produced results for the mean drag coefficients as a function of the Reduced Velocity (hereinafter referred as the Moe-Verley number) $N_{M-V}$ and $N_{K-C}$. Koterayama (1984) used the same hydrodynamic parameters for a fixed vertical cylinder in waves and currents to express the viscous mean drag coefficients. Chakrabarti (1984) has mentioned a single value of $C_{D0}$ in a wave-current interaction field without mentioning any dominant hydrodynamic parameter.

The individual hydrodynamic parameter for individual flow fields and also in interaction flow fields are given below. For either uniform flow or harmonically oscillating flow (bodies), $N_{Re}$ can be expressed as the followings:

$$N_{Re} = \rho\, U\, D / \mu \text{ (for uniform flow field)}; \quad N_{Re} = \rho\, u_m\, D / \mu \text{ (for wavy flow field)} \qquad (22)$$

For harmonically oscillating flow (bodies), $N_{K\text{-}C}$ is expressed as follows:

$$N_{K\text{-}C} = u_m T / D = 2 \pi \zeta_a / D = u_m / (n D) \tag{23}$$

For harmonically oscillating flow (bodies) in uniform flow, $U_{red}$ or $N_{M\text{-}V}$ is expressed as follows:

$$U_{red} = N_{M\text{-}V} = U / (n D) \tag{24}$$

Sarpkaya (1981) proposed a modified $N_{K\text{-}C}{}^{+}$ in a wave-current co-existing flow field and that is defined as follows:

$$N_{K\text{-}C}{}^{+} = (u_m + U) T / D = u_m T / D + U / (n D) \tag{25}$$

The modified $N_{K\text{-}C}{}^{+}$ is a combination of $N_{K\text{-}C}$ and $N_{M\text{-}V}$. In practice, U should be replaced by $|U|$ as the uniform flow (steady current) can be either positive or negative. So, use of the modified $N_{K\text{-}C}{}^{+}$ obviates the necessity of introducing another hydrodynamic parameter $N_{M\text{-}V}$. Similar to the above, $N_{Re}$ can also be modified by replacing the velocity term by the combined velocity of waves and currents. $\beta$ is not capable of showing any distinctive nature for wave-current interaction because it is a ratio and so remains as an identical value both for a waves only field as well as for a co-existing flow field. So, the only predominant hydrodynamic parameter left would be the Keulegan-Carpenter number. Now the remaining thing that needs further treatment is how the $N_{K\text{-}C}{}^{+}$ behaves in a co-existing field rather than as defined earlier. The controlling factors to be chosen are now the magnitude as well as the direction of U with respect to those of $u_m$ and also the crest and trough phases of waves in a co-existing flow field.

Such newly proposed Keulegan-Carpenter number, $N_{K\text{-}C}{}^{*}$, (Iwagaki *et al.*, 1983) would have an advantage of defining a flow field under a single parameter rather than introducing additional parameters. In addition, flow fields defined by $N_{K\text{-}C}{}^{*}$ for a co-existing flow field can easily be compared to the similar one under a waves only field (Sarpkaya, 1981). The basic definition of $N_{K\text{-}C}$ has been used for the newly proposed number, i.e. the physical meaning of $N_{K\text{-}C}$ is rather geometric and can be considered as the ratio of the moving distance of a water particle in one side direction of the cylinder, S, to the cylinder diameter, D.

If U is either positive or negative and $|U| \geq u_m$ :

$$N_{K\text{-}C}{}^{*} = (\pi/D) \; \{2 \int_0^{T/2} (u_m \cos\omega t \pm U)\, dt\} = (\pi/D)\, (\pm U)\, T = \pi\, |U|\, T/D \tag{26}$$

If U is positive and $U < u_m$ :

$$N_{K\text{-}C}{}^{*} = (\pi/D) \; \{2 \int_0^{T_1} (u_m \cos\omega t + U)\, dt\} = T/D \; u_m\, (\sin\Theta - \Theta \cos\Theta) \tag{27}$$

where $\Theta = \cos^{-1}(-U/u_m)$ ; $\pi/2 \leq \Theta \leq \pi$

If U is negative and $|U| < u_m$ :

$$N_{K\text{-}C}{}^{*} = (\pi/D) \; \{2 \int_{T_1}^{T/2} (\,|u_m \cos\omega t - U|\,)\, dt\} = T/D \; u_m\, \{\sin\Theta + (\pi - \Theta) \cos\Theta\} \tag{28}$$

where $\Theta = \cos^{-1}(U/u_m)$ ; $0 \leq \Theta \leq \pi/2$

So, $N_{K\text{-}C}$ for a waves only field is different from that in a wave-current co-existing field not only by the additional magnitude of the combined velocity but also by the influence of the crest and trough phases of the interacting waves. Furthermore, $N_{K\text{-}C}^*$ in the co-existing field is governed by the magnitude as well as by the direction of U compared to those of $u_m$.

The ratio of the second term to the first term in eq. (25) turns up also as an important non-dimensional factor of $U/u_m$. Like 'beta parameter', which is a ratio of $N_{Re}$ to $N_{K\text{-}C}$, the velocity ratio of $U/u_m$ can be expressed as the following:

$$\gamma = U/u_m = N_{M\text{-}V} / N_{K\text{-}C} \tag{29}$$

## MODEL TESTING

A detailed experimental study (Dev, 1992b, c, 1993) was carried out in order to evaluate the presence and extent of the viscous mean drift forces on fixed cylinders of different diameters in waves only as well as in waves and currents. Both positive and negative currents were used, i.e. by towing the carriage into the waves and out of the waves during the tests. Furthermore, the model cylinder was constructed in a unique way to represent the two separate hydrodynamic zones like the splash zone to represent the effects of the wave elevation during the crest and trough phases and the submerged zone which is always immediately under the trough phase of the wave. To the authors' knowledge, a few experiments were done to investigate the viscous mean force and all such experiments were done on the complete submerged test sections of either vertical or horizontal cylinder thus without giving any definite result on the splash zone that practically exists in case of surface piercing vertical columns of a semi-submersible or a tension leg platform in addition to its constantly submerged zone.

The main objectives of the experimental investigations were (a) to assess the magnitude of the viscous mean force in a wavy flow field as well as in a wave-current co-existing flow field including their effects on the two separate important hydrodynamic zones of the cylinder, (b) to obtain the time averaged values of the viscous mean drag coefficients over a dominant range of the wave frequencies in the said flow fields and finally (c) to find the suitability of the single controlling hydrodynamic parameter to express the viscous mean drag coefficients due to wave-current interactions so that the co-existing flow field can be made analogous to the flow field due to a wavy flow field.

### Experimental Apparatus and Procedure

Two cylinders of different diameters were used in the model tests. The diameter of one model cylinder was 75 mm (scale factor of 100) and that of the other was 315 mm (scale factor of 35). The basic construction is same for both of them. The model cylinder was made up of four segments - the lowest and the topmost being the dummy ones and the intermediate ones as the test sections. Both test sections contained their individual load cell to measure the forces. To eliminate the end effects of the circular cylinder, the lower dummy cylinder was used. The slits between the cylindrical sections of the whole construction were covered with thin rubber.

The setup accessories (see Fig. 9 and Fig. 10) along with the model cylinder were mounted on the towing carriage over the water surface with the model cylinder immersed under water. Arrangements were also made to slide the cylinder up and down vertically to equal the splash zone to the incoming wave amplitude.

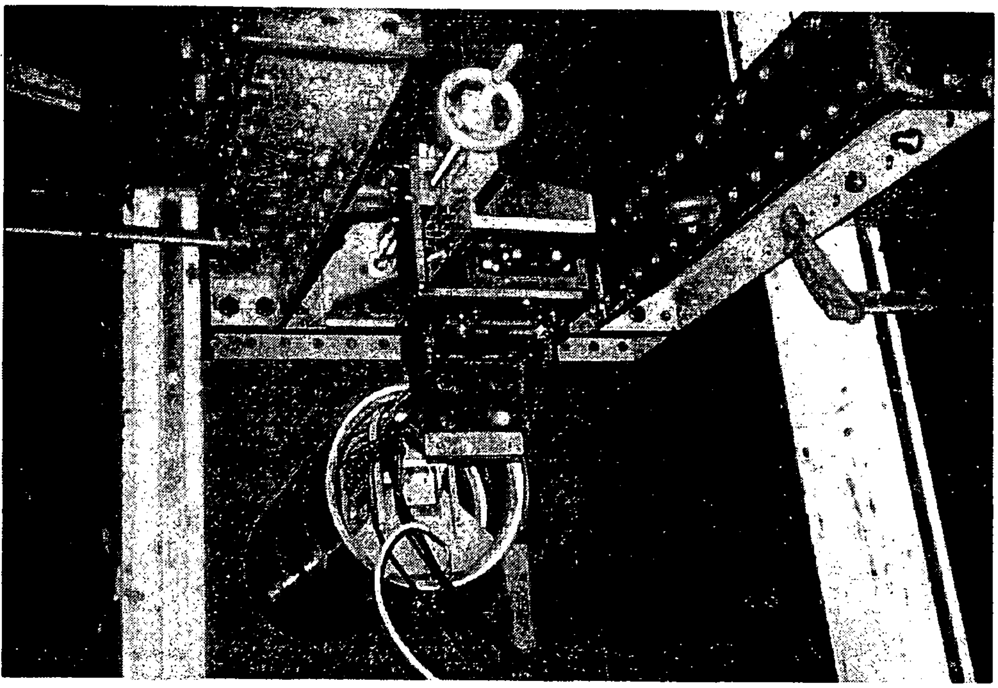

Fig. 9. Larger (315 mm) diameter cylinder at Seakeeping Basin of MARIN

Fig. 10. Larger (315 mm) diameter cylinder at Towing Tank No. 1 of TUD

Tests with the smaller diameter cylinder were conducted at the Towing Tank No. 2 of Ship Hydromechanics Laboratory of Delft University of Technology. The dimensions of the Tank are length: 85 m x breadth: 2.75 m x depth: 1.25 m having a flap-type wave maker and a towing carriage. The tests were conducted for a frequency range of 3.0 - 10.0 r/s. For each frequency, three wave amplitudes (highest, intermediate and lowest) were used. At every intermediate wave amplitude for each frequency, tests were also conducted for both positive and negative currents in presence of waves. Two uniform velocities 0.15 m/s and 0.30 m/s were used.

Tests with the larger diameter cylinder were conducted at the Towing Tank No. 1 of Ship Hydromechanics Laboratory of Delft University of Technology. The dimensions of the Tank are length: 142 m x breadth: 4.22 m x depth: 2.50 m having a flap type (electro hydraulic) wave maker and a towing carriage. Tests were conducted for a frequency range of 3.25 - 6.50 r/s. For each frequency, three wave amplitudes were utilized. For each wave frequency at each wave amplitude, tests were conducted for both positive and negative currents in presence of waves. Three carriage speeds were used and they are 0.173 m/sec, 0.261 m/sec and 0.348 m/sec.

It has been mentioned earlier that the particulars and the basic construction of the model cylinder have been kept the same. But in order to observe the relative wave elevation around the cylinder, two additional wave probes were fitted to the fore (facing the wave maker) and the aft end (facing the beach). The gap between the probes and the cylinder wall was less than 10.0 millimeters. Tests were conducted at the sea-keeping basin of MARIN (Maritime Research Institute Netherlands). The dimensions of the basin are 100 m x 24.5 m having a water depth of 2.50 m with a maximum carriage speed of 4.50 m/s. Tests were conducted for a frequency range of 2.66 - 5.32 r/s. For each wave frequency, two to three different wave amplitudes were used as before. For each wave frequency, at each wave amplitude, tests were conducted for both positive and negative currents in presence of waves. One carriage speed of 0.261 m/s was used.

For each test, the vertical position of the cylinder was adjusted in such a way that the trough of the passing wave always remains at the separation line between the splash zone and the submerged zone. Thus, the submerged zone was always maintained as fully submerged throughout all the tests.

The ranges of the different hydrodynamic parameters of the experiments are shown in Table 1.

## Data Analysis of Measurements

The measured forces in waves only or in waves and currents expanded in a Fourier Series up to third order are as follows:

$$F = \bar{F}_0 + \sum_{n=1}^{3} (F_{I_n} \sin n\omega t + F_{D_n} \cos n\omega t) \tag{30}$$

One of the main objectives is to find the viscous contribution towards the measured mean force, i.e. to express the measured mean force as the following $\bar{F}_0 = \bar{F}_P + \bar{F}_D$.

The potential mean drift force on the splash zone is due to the contribution of the relative elevation and the second order pressure (velocity squared term of Bernoulli's equation) and is calculated by the Program DELFRAC. So, the viscous mean drift force is calculated out as follows $\bar{F}_D = \bar{F}_0 - \bar{F}_P$.

From the above, the time averaged mean drag coefficients on the splash zone can be obtained by using the expressions as outlined earlier for the theoretical viscous mean drift forces on a fixed cylinder. For the submerged zone, based on the application of the Morison equation, the contribution to the mean force is purely of potential origin and can thus be calculated using the second order pressure as mentioned before.

For the cylinder in waves and currents, similar treatment can be applied except the theoretical viscous mean drift forces need to be considered for the conditions when U is greater or equal to $u_m$ or less than $u_m$ for the splash zone as well as for the submerged zone. Potential contributions are to be considered properly with forward speed effects using the equations proposed by (Clark *et al.*, 1993). The values of the mean drag coefficients can then be obtained from the measured mean forces and the theoretical ones.

Table 1. Non-dimensional hydrodynamic parameters

| | Small Cylinder | Larger Cylinder | Larger Cylinder |
|---|---|---|---|
| | | For Wavy Flow Field | |
| $N_{K\text{-}C}$ | 0.670 - 7.460 | 0.798 - 2.690 | 1.020 - 2.690 |
| $N_{Re}$ | $0.491\text{x}10^4$ - $0.243\text{x}10^5$ | $0.745\text{x}10^5$ - $0.126\text{x}10^6$ | $0.781\text{x}10^5$ - $0.160\text{x}10^6$ |
| $\beta$ | $0.732\text{x}10^4$ - $0.325\text{x}10^4$ | $0.933\text{x}10^5$ - $0.466\text{x}10^5$ | $0.763\text{x}10^5$ - $0.514\text{x}10^5$ |
| | | For Wave-Current Co-existing Flow Field | |
| $N_{K\text{-}C}{}^{+}$ | 2.010 - 13.700 | 0.930 - 4.830 | 2.000 - 3.950 |
| $N_{Re}{}^{+}$ | $0.164\text{x}10^5$ - $0.333\text{x}10^5$ | $0.868\text{x}10^5$ - $0.225\text{x}10^6$ | $1.530\text{x}10^5$ - $2.350\text{x}10^6$ |
| $N_{M\text{-}V}$ | 1.260 - 8.380 | 0.530 - 2.100 | 0.970 - 1.200 |
| $N_{K\text{-}C}{}^{*}$ | 3.950 - 26.30 | 1.820 - 6.950 | 3.090 - 4.970 |

## EXPERIMENTAL RESULTS

In Fig. 11 through Fig. 16, the results for the mean drift forces are shown for the three sets of tests carried out in regular waves at zero forward speed. The mean horizontal forces on the splash zone and those on the submerged zone are compared with the theoretical calculations of the relevant contributions to such forces based on the 3-D potential theory (the near field method). The results clearly indicate that the deviations between the theory and the experiment occur mainly in the splash zone whereas for the submerged zone differences are not that consistent in sense and tend to show the potential calculations' trends.

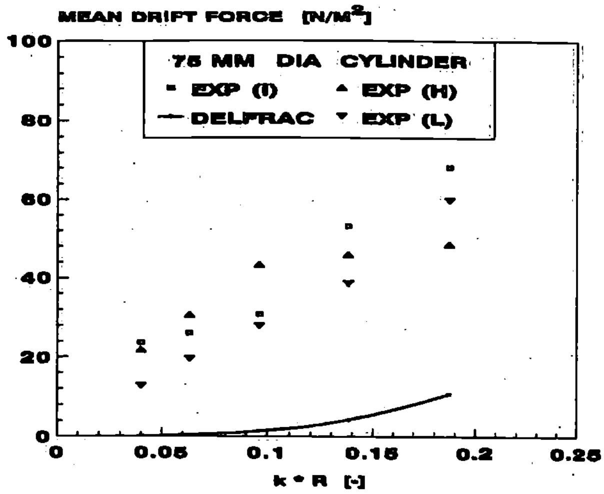

Fig. 11. $\bar{F}$ versus k*R in waves only for Spl. Z.

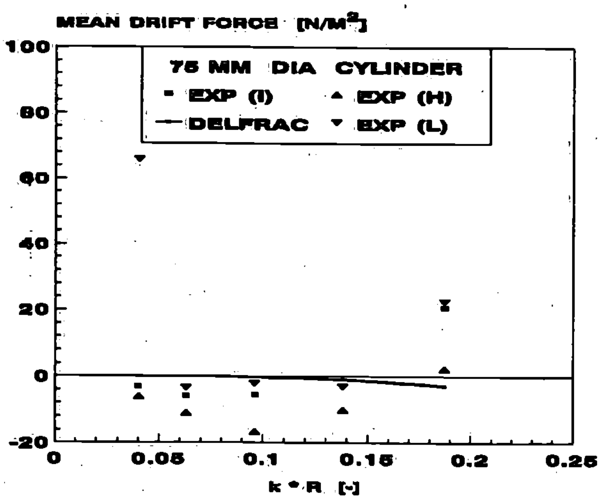

Fig. 12. $\bar{F}$ versus k*R in waves only for Sub.Z.

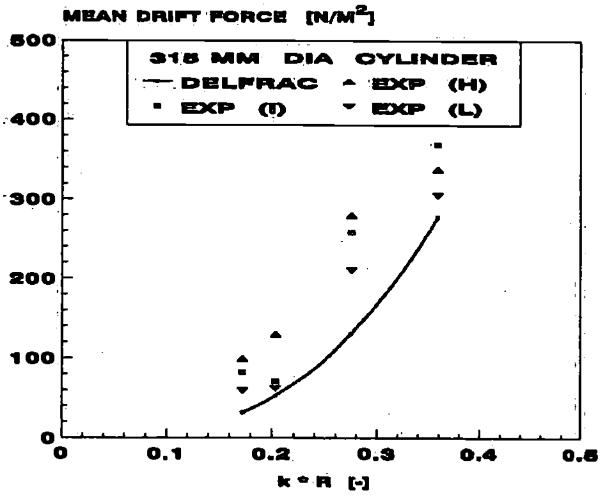

Fig. 13. $\bar{F}$ versus k*R in waves only for Spl. Z.

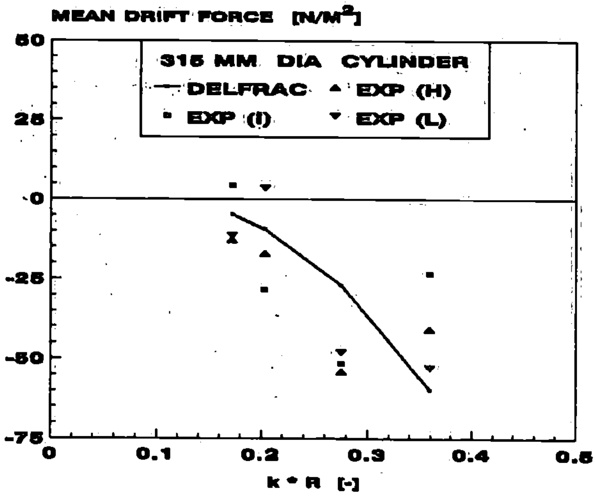

Fig. 14. $\bar{F}$ versus k*R in waves only for Sub.Z.

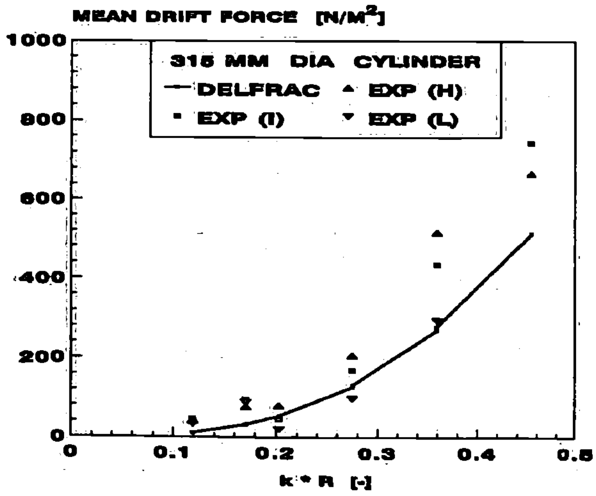

Fig. 15. $\bar{F}$ versus k*R in waves only for Spl. Z.

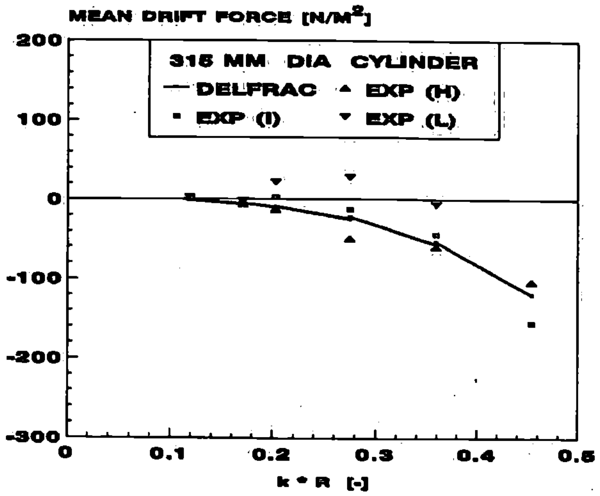

Fig. 16. $\bar{F}$ versus k*R in waves only for Sub.Z.

Based on the experimental data obtained from the experiments, Fig. 17 is produced where the three curves $R_{ft}$ (theoretical ratio of viscous to potential mean drift force) equal to 5, 1 and 1/5 representing 80% viscous, viscous equal to potential and 80% potential respectively. At H/D > 1 and at very low values of k*D, the force is dominated mainly by viscous effects which is also indicated by the experimental results which were obtained from the smaller diameter cylinder. Around the line, $R_{ft} = 1$, both viscous and potential forces are equally important. Experimental results from the smaller as well as from the larger diameter cylinder show the trends with a few disparity. Figure 18 establishes the values of the mean drag coefficients in a waves only field as functions of $N_{K-C}$ (from 3 to 8) where the average value is about 1.50. Force regimes where both potential and viscous effects are important, the values of the mean drag coefficients, $C_{D0}$, are thus expected to be better expressed as functions of wave steepness k*H = (H/D)*(k*D).

For the submerged zone in waves and currents, the values of the mean drag coefficients are plotted as functions of $N_{M-V}$ for different values of $N_{K-C}$. From Fig. 19, it is clearly seen that around a value of 4-6 of $N_{M-V}$, the experimental values of $C_{D0}$ become quite large for certain values of $N_{K-C}$. Similar results were also obtained by (Koterayama, 1984). In Fig. 20, the experimental values of $C_{D0}$ are all close to and around 0.5 which is similar to the results shown in Fig. 19 for the same range of the values of $N_{M-V}$.

For the splash zone as well as for the submerged zone, plots are made in Fig. 21 and Fig. 22 for the experimental values of $C_{D0}$ in waves and currents as a function of single variable $N_{K-C}^*$ based on the analysis of the three sets of model tests carried out so far. Using these experimental values of the mean drag coefficients in a wave-current coexisting flow field, the theoretical computations have been done and thus compared with the experimental results. They are shown in Fig. 23 and Fig. 24 and the calculation is predicted better than using a unit value of the mean drag coefficient, $C_{D0}$.

During the experiment with the larger diameter cylinder at MARIN, the relative wave elevation at the fore and aft of the cylinder at zero forward speed was measured for different wave amplitudes. Comparison with the 3-D potential theory calculations is shown in Fig. 25 and Fig. 26 revealing the fact that the theoretical calculations of the mean drift force due to potential effects are consistent with the experimental analysis.

Computations have been done for a complete semi-submersible for regular waves in frequency domain. The particulars of the model (SEDCO 700) are shown in Fig. 27. The potential mean drift force including first order horizontal and vertical motions including their phases are obtained from the 3-dimensional potential theory calculations. The theory outlined before regarding the viscous effects on a single floating cylinder has been extended based on the trigonometric relations on the columns of the floating semi-submersible. The values of the mean drag coefficients used in the calculations are based on the experimentally obtained values as functions of appropriate hydrodynamic parameters. In case of $N_{K-C}$, $u_m$ is replaced by $u_{r\ m}$ for the floating semi-submersible. Results of model tests for SEDCO 700 are available in (Pinkster, 1993). Measurements of the mean horizontal wave drift force on the model in regular waves were carried out by using a soft-spring mooring system.

Comparison in Fig. 28 and Fig. 29 show that in the lower wave frequency range, the measured mean drift forces are consistently higher than those calculated by the 3-D potential theory. After the viscous contributions have been added to the potential mean drift force, the newly predicted mean horizontal forces for both head sea and beam sea conditions are notably recuperated especially in the low frequency range ($\omega < 1$) which are referred as extreme seas under severe storm conditions.

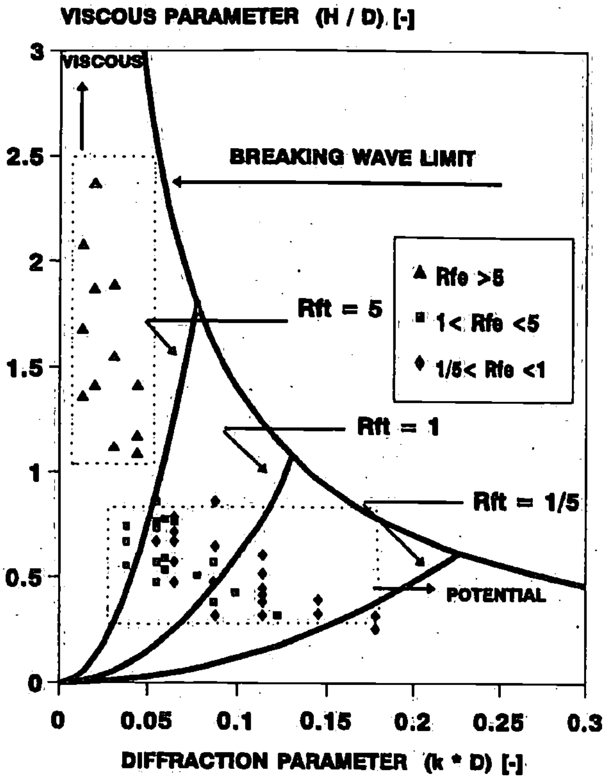

Fig. 17. Mean drift force regimes in waves only for the splash zone

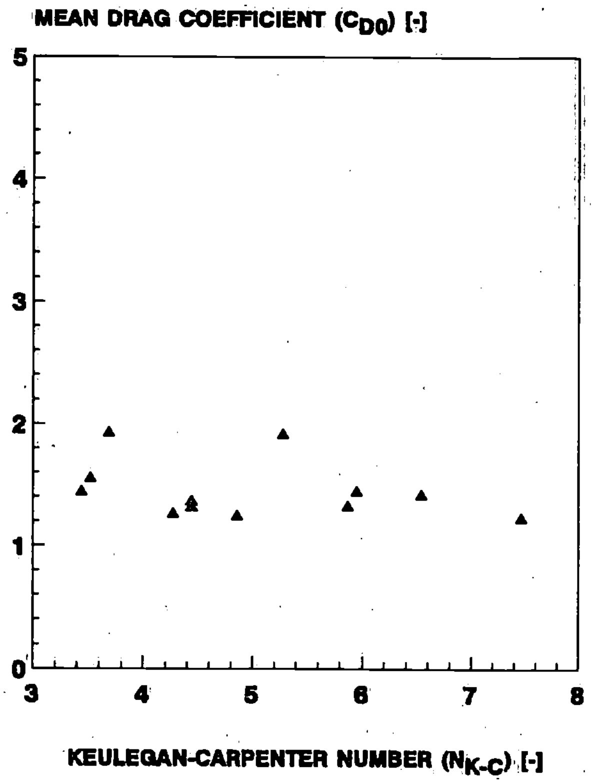

Fig. 18. $C_{D0}$ versus $N_{K-C}$ in waves only for the splash zone

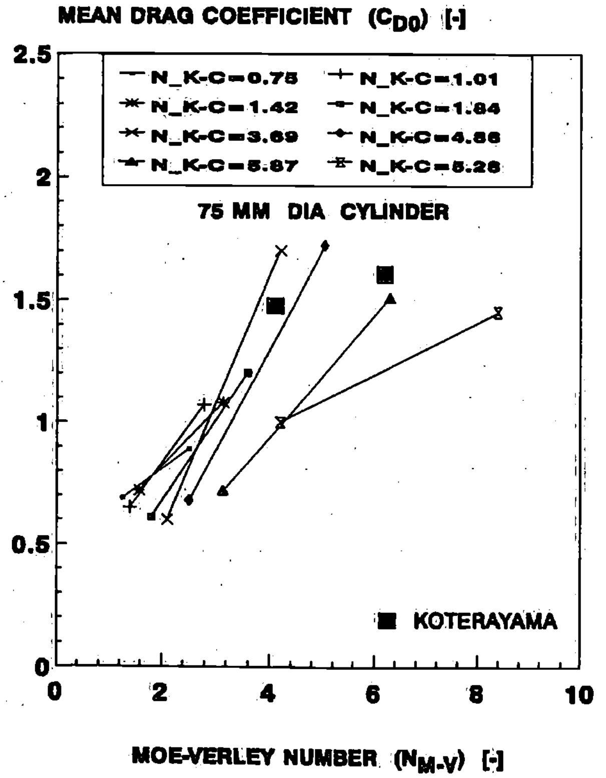

Fig. 19. $C_{D0}$ versus $N_{M-V}$ in waves and currents for differenr $N_{K-C}$

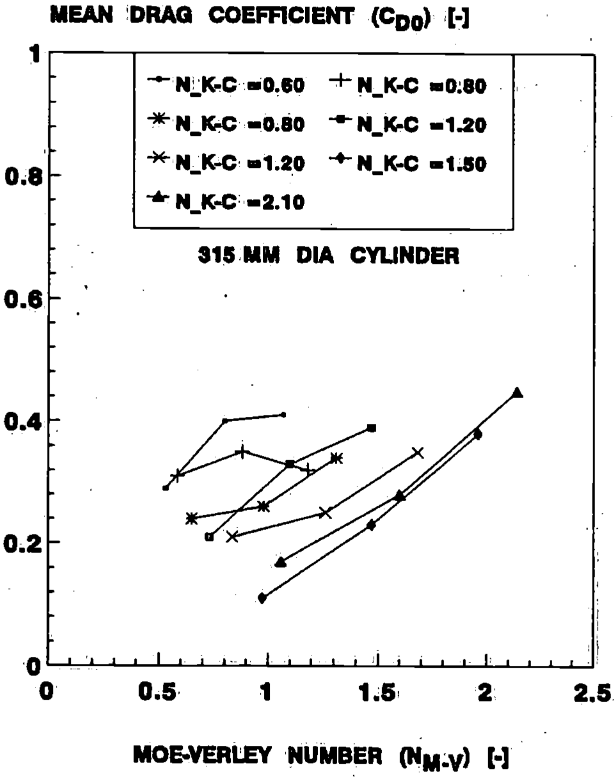

Fig. 20. $C_{D0}$ versus $N_{M-V}$ in waves and currents for different $N_{K-C}$

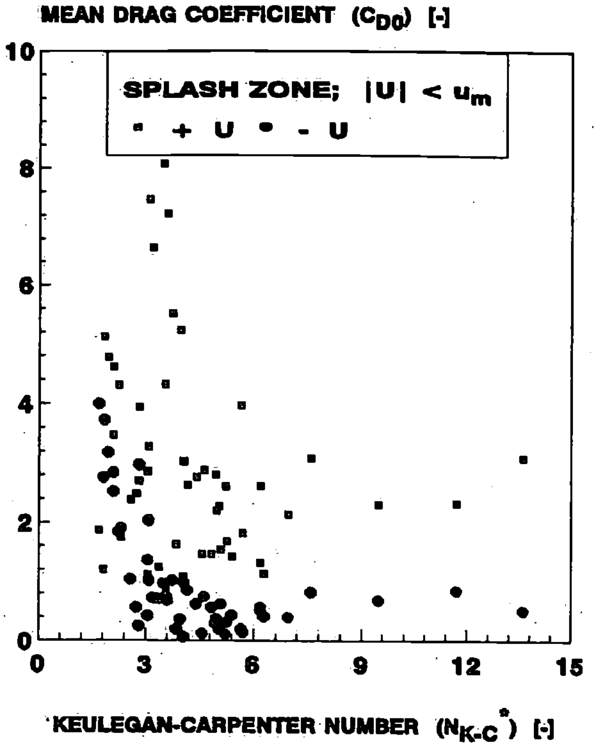

Fig. 21. $C_{D0}$ vs. $N_{K\_C}^*$ in waves and currents

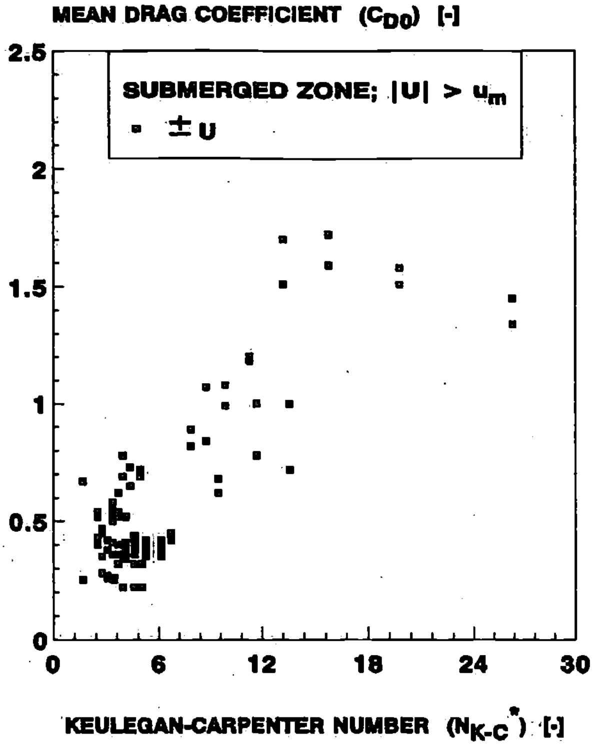

Fig. 22. $C_{D0}$ vs. $N_{K-C}^*$ in waves and currents

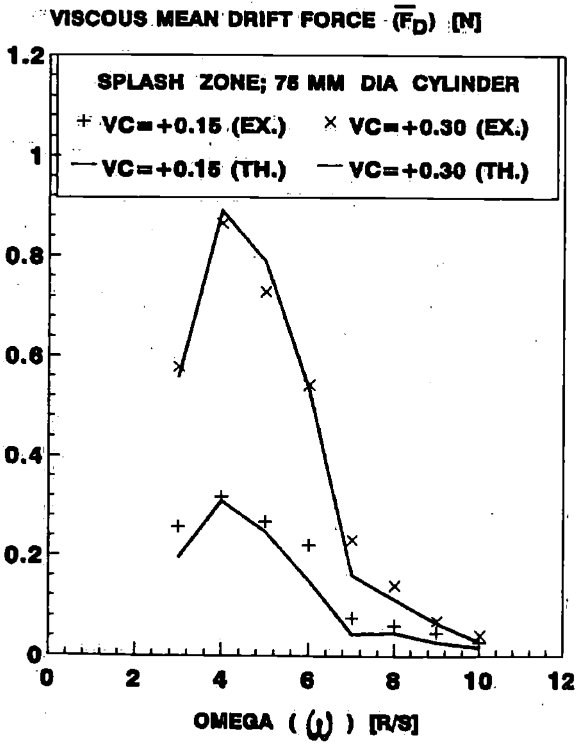

Fig. 23. $\bar{F}_D$ versus ω in waves and currents

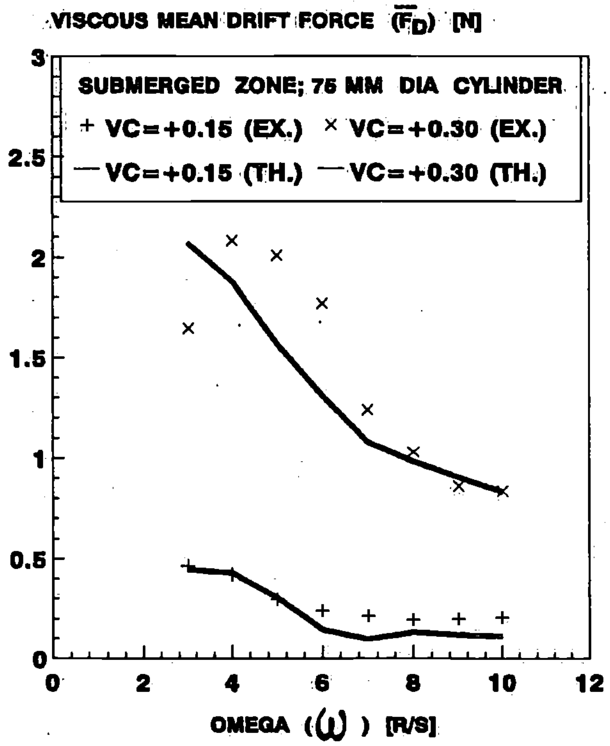

Fig. 24. $\bar{F}_D$ versus ω in waves and currents

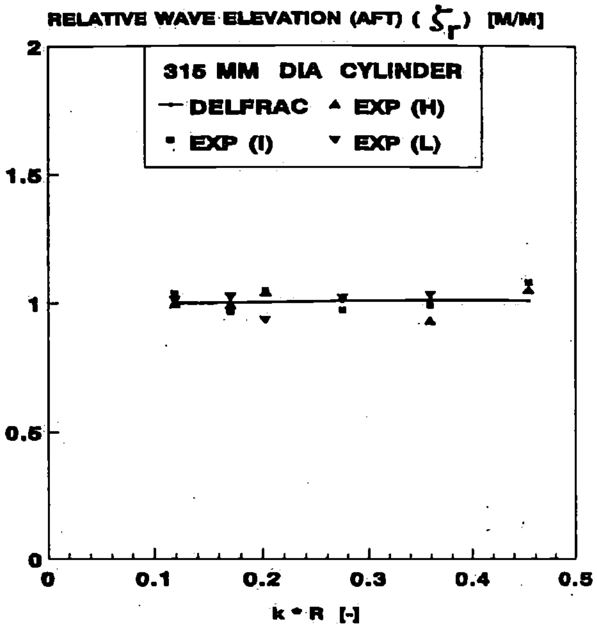

Fig. 25. $\zeta_r$ versus k*R in waves only

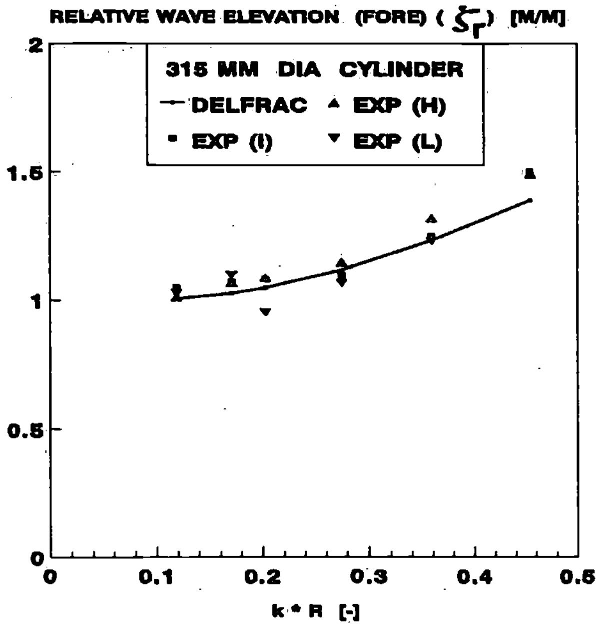

Fig. 26. $\zeta_r$ versus k*R in waves only

## CONCLUDING REMARKS

The horizontal mean drift forces on fixed and truncated surface piercing vertical cylinders of different diameters in regular waves with or without currents were investigated. It was thus possible to get an insight into different force regimes like potential vs. viscous and the values of the mean drag coefficients in different flow fields as functions of fitting hydrodynamic parameters.

Model tests clearly signify the splash zone of the vertical columns as the major source of the viscous contributions in waves even without currents. Accordingly, the viscous mean drift forces in irregular waves should not be treated as quadratic transfer functions (viscous mean drift forces are cubic transfer functions) by subtracting the steady force due to currents from that due to waves and currents. The values of the mean drag coefficients ($C_{D0}$) can be well represented as functions of the Keulegan-Carpenter number ($N_{K\text{-}C}$) at very low diffraction parameter (k*D) whereas those in the equal force regimes need to be evaluated as functions of the wave steepness (k*H), a combination of both the viscous parameter (H/D) and the diffraction parameter (k*D).

Similar to a waves only field, the viscous mean drift forces in a wave-current co-existing flow field should better be dealt with two separately hydrodynamic zones - the splash zone and the submerged zone respectively. Even presence of a small amount of current can cause the large mean drift force due to non-linear wave-current interaction effects which again largely depend on the values of current velocities and wave heights . The mean drag coefficients while expressed as functions of current velocity, wave particle velocity, crest phase, trough phase, etc. would simplify presenting them against a single suitable hydrodynamic parameter $N_{K\text{-}C}^{*}$.

Theoretically for floating cylinders, the viscous effects are expected to behave in a similar pattern as have been observed during the tests for fixed cylinders in regular waves with and without currents.

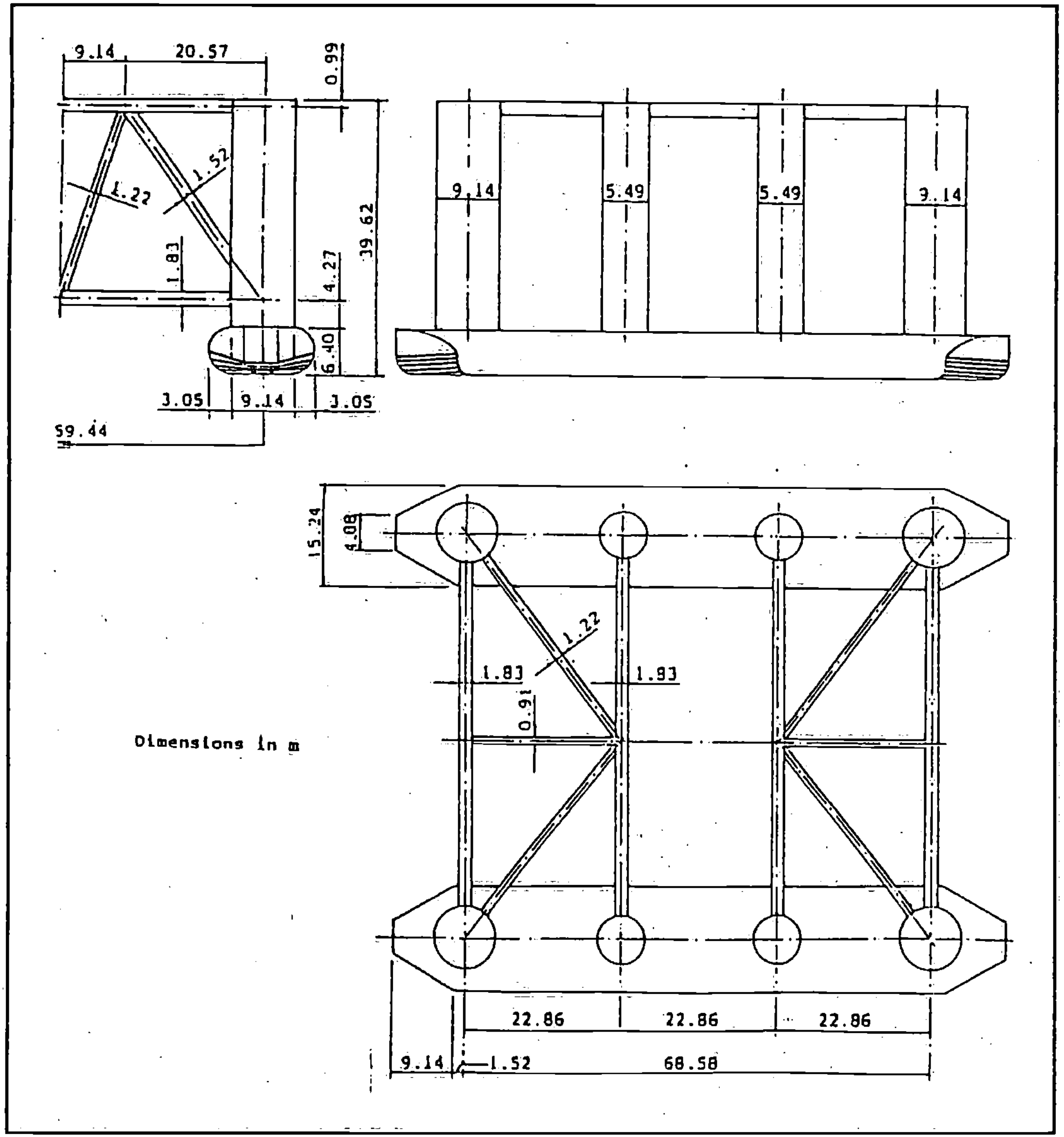

Fig. 27. General arrangement of semi-submersible SEDCO 700

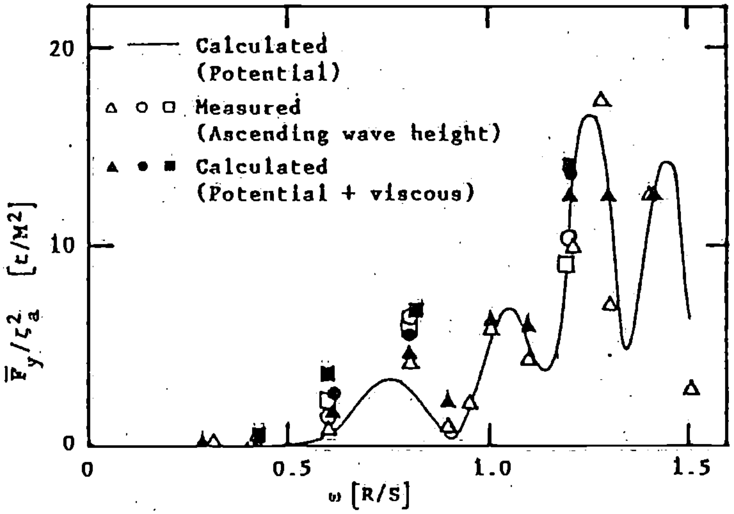

Fig. 28. Mean drift force in regular beam waves

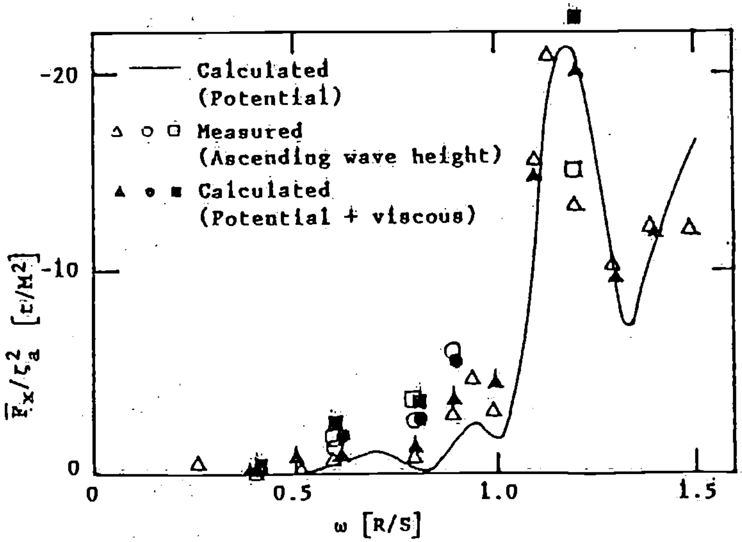

Fig. 29. Mean drift force in regular head waves

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