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HOW TO MEASURE PATH LENGTH ?

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ABSTRACT

Path length measuring is a relevant engineering problem. Leonardo Da Vinci designed for the military appropriate equipment, the podometer, to do so. Modern equipment such as step meters and map meters are quite similar to Da Vinci's design, despite geometrical statistical - stereological - methods based on theorems of Cauchy and Buffon that were potentially available for a long period of time for doing a better job. The theorems have moreover been applied earlier for the engineering purposes indicated in this paper. Even Saltikov's re-introduction for quantitative image analysis purposes in 1945 was ignored. Gradually, the last half of a century, stereological methods became more popular in concrete technology. Nevertheless, the stereology-based global averaging operation required to make the step from materials technology to engineering properties is inherent to making errors, as the literature demonstrates.

The methodological framework has been described in earlier papers by this author as to path length measurements in sections of cracked concrete and on X-ray images of steel fibre reinforced concrete (SFRC). By laying side by side in this paper the direct engineering approach (from Da Vinci to map meter) and the science (stereology)-based approaches, the profit in economy and reliability can nevertheless be stressed. Particularly the element of geometric averaging based on simple mathematical-statistical notions is highlighted, because the literature reflects still major violations to these scientific principles. Three-dimensional information in the indicated fields of materials engineering is also readily obtained, provided proper sampling is guaranteed.

Keywords
Concrete, fibres, cracks, podometer, map meter, Cauchy, Buffon, Saltikov, image analysis, geometric averaging.

INTRODUCTION

A quite common problem engineers are confronted with is how to measure *path length,* either in a plane (2D) or in space (3D). This problem could be expanded to surface area in 3D. This author has paid earlier attention to old roots of stereology, the proper science for solving such problems. Discussed were theorems of Cavalieri, Buffon and Cauchy and their impact was demonstrated on (also path) measuring methodology in concrete technology [1].

The exposition "Da Vinci, the Genius" (Rotterdam, 2013) displayed (a model of) Da Vinci's podometer (Fig. 1), which convincingly proved path measuring equipment (in this case path length covered by the military in the field) to be even much older. Two *modern* developments relate to the podometer, *i.e.* the step meter, in the USA denoted a hike-o-meter, and the map meter, map measurer, opisometer, mile-o-graph or curvi(o)meter. The first measures the number of steps and the path length covered by its user. The second measures route (path) length on a map. The similarity with the half a millennium earlier developed podometer is obvious (Fig. 2).

Fig. 1. Podometer on the exposition: “Da Vinci, the Genius”, Rotterdam 2013

Fig. 2. Two types of map meters of which the similitude to that of Da Vinci is quite striking.

Therefore, nowadays one could (and should) have been doing much better with Cauchy's methods of 1.5 century ago [2]. Basically, all is required is making normal projections of the route (path) on a number of randomly oriented *straight* lines, which thereupon renders possible easy *measuring* of these total projections. Averaging leads straightforwardly to an accurate estimate of path length, superior to that of the map meter. The step to measuring the length of a crack trace in a section is an obvious one. It can be shown that this approach can be replaced by *counting* intersections between the traces and randomly oriented grids of straight lines. This allows analysing in a very efficient way a large number of cracks in a section plane as to average or total crack length. The total length of steel fibre projections in an X-ray image of SFRC is solved in an analogous way. By using intersection counting instead of total projection measuring for SFRC, *de facto* we also conform to the even one century older Buffon set up [1].

All approaches could be extended to the most relevant 3D engineering world, provided sampling is properly arranged. Most important is here to recognize that optimum path measuring methods were potentially available for a very long time. Unfortunately, the afore-mentioned present-day experimental methodology did not exploit such potentials, instead harked back to practical solutions, like that of Leonardo Da Vinci.

SO, LET'S DO BETTER

The question now to be raised is whether the length of a curved line on a plane cannot be assessed in a more economic and accurate way. The wide variety of map meters that are for sale suggest that a negative answer should be given. However, here mathematical-statistical tools should be employed. Not new ones, because the method to be referred to here is due to Cauchy [2] and is more than one century old. Of historic relevance is that Steinhaus [3] was reported having used Cauchy's method already before the nineteen thirties for the present purpose.

The idea is not to concentrate directly on the route delineated on the map, but on its total projection on a line, which can easily be measured. This is repeated for a number of random or systematic directions, whereupon the total projected length is simply averaged. When L is the length of the route, its total projection in a direction β can be written as $L'(\beta)$. Hence, the result of averaging will be

$$L = \frac{\pi}{2}\overline{L'(\beta)} \tag{1}$$

allowing an *easy* assessment of the route's length.

How many times should this operation be accomplished for a satisfying result? Kendall and Moran [4] give the following lower and upper bounds for 2n repetitions

$$\frac{\pi\cos\frac{\pi}{2n}}{2n\sin\frac{\pi}{2n}} \le \frac{L}{L_{2n}} \le \frac{\pi}{2n\sin\frac{\pi}{2n}} \tag{2}$$

whereby L is the true length and L_{2n} its estimate for 2n repetitions. For $\pi/2n = x$, eq. (2) transforms into the well-known form

$$\frac{x\cos x}{\sin x} \le \frac{L_{2n}}{L} \le \frac{x}{\sin x} \tag{3}$$

Both bounds have limiting values of 1 since $x/\sin x \to 1$ for $x \to 0$. Let us take n=3 for getting an idea about the *economy* of the approach. This leads approximately to $0.91<L_{2n}/L<1.05$, which should be quite acceptable for normal applications.

CRACKS IN CONCRETE

In research one is frequently confronted with patterns of curved lines in a plane. An example is the pattern of micro-cracks in a section of a (non-)loaded concrete specimen (Fig. 4). A quite complicated and laborious task would be to measure total crack length by a map meter. This was a problem frequently encountered in our research whereby the effects of loading type and level on damage characteristics (*e.g.* total crack length and orientation distribution) of different types of concrete were emphasized [5,6].

Basically, the framed crack pattern (with *A* as frame area) can be treated similarly as before. For that purpose the contrast in the image plane should be improved. In the case of Fig. 4, the section surface had been sprayed by a fluorescent delineating the cracks, so that they can be photographed under illumination by UV light. Mostly, such images are digitized and measure-ment procedures automated. However, this leads to serious biases in obtained results, as again demonstrated in a recent publication [7]. Of course, this can easily be understood by using our imagination, seeing the differences resulting from conventional digitization illustrated in Fig. 3. In our research we have therefore copied the crack patterns manually, providing for an analogue image that can be subjected to the so called *method of random (or directed) secants,* the 1945-re-invention by Saltikov of Cauchy's theorems [8,9].

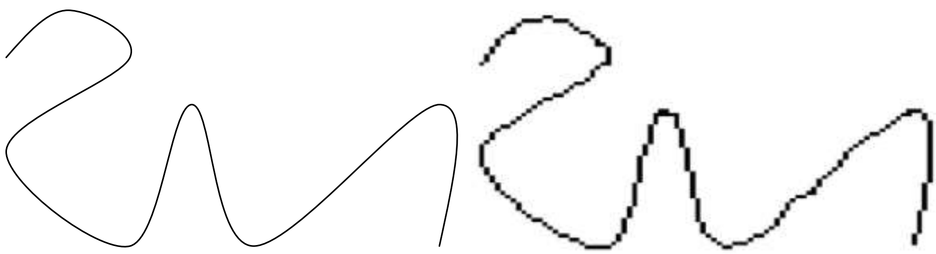


Fig. 3. Analogue representation of smoothly curved crack trace in a section (left) is replaced by an orthogonal set of straight line segments due to a conventional procedure of digitization (right).

Crack length in images such as Fig. 5 could be analysed (and indeed has been so) by application of Cauchy's formula. Hence, a set of randomly or systematically oriented lines is projected onto the framed image. However, a far more economical approach involves using a grid of such lines covering the image and counting the number of interesections, *P*, between the test lines ("secants") and the cracks. This is depicted in Fig. 5 by a superimposed *orthogonal* set of a limited number of grids lines (the so called "Stroeven-concept").

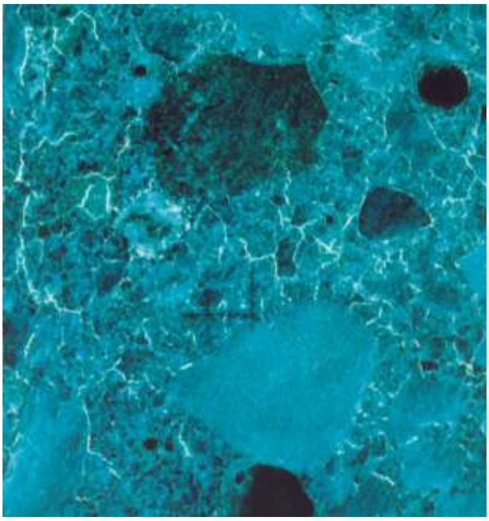


Fig. 4. Part of a cross-section of a pre-loaded concrete specimen reveals myriads of small cracks that are visualized by the fluorescent spray technique [8].

The formula relating the number of points P per unit of total test line length and averaged over a number of systematically oriented grids is given by

$$L_A = \frac{\pi}{2}\overline{P_L(\beta)} \tag{4}$$

in which L_A is the total crack length per unit of frame area and $\overline{P_L(\beta)}$ is the number of intersection points P per unit of the total length of grid lines in a particular direction β averaged over a systematic set of grid orientations. Eq. (4) directly reflects Cauchy's relationship in eq. (1); it offers a very economic and reliable approach to analysing multiple curved lines in a plane. This application does not have to be restricted to line patters on microlevel. Landscape images produced by airplane or helicopter photography can and have been analysed the same way. Only in the latter case one could imagine Da Vinci's podometer to be emoployed for checking the results!

Note that the discussion so far was restricted to 2D space. The world around us is 3D, however. Also for this situation Cauchy presented a formula relating the surface area, S, of an object in space to its average total projected area, A', on a random or systematic set of planes

$$S = 2\overline{A}' \tag{5}$$

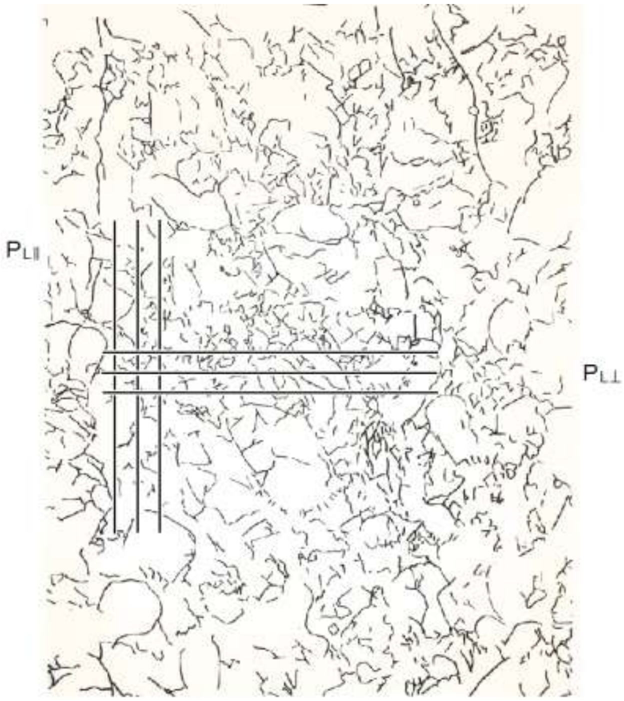


Fig. 5. Manually copied crack pattern of a full section image of a conventionally-sized prismatic concrete specimen (note the aggregate grain sections). Orthogonal line grids are superimposed for intersection counting, yielding measurements in horizontal and vertical directions, respectively (as indicated).

This has been demonstrated allowing to interprete the results of the directed secants approach to 2D crack patterns into crack surface area in the real 3D concrete world [5,6]. The appropriate formula for a system of cracks dispersed in 3D can be demonstrated equal to

$$S_V = 2\overline{P}_L \qquad (6)$$

in which S_V represents the crack surface area per unit of volume. The averaging of the point intersection density P_L has to be accomplished over a set of uniformly at random oriented images; a complicated action in concrete technology! However, in the so called "Stroeven-concept" (inspired by Saltikov [6,9]), secants scanning can be restricted to orthogonal directions in axial sections (Fig. 5). This is possible in concrete subjected to prevailing tensile or compressive stresses leading to an axis of symmetry in the internal crack structure. For a complete methodological treatment, the interested reader is referred to a series of relatively old publications, such as [5].

STEEL FIBRES IN CONCRETE

As depicted in Fig. 6, the similitude with the cracks in a section is obvious. The X-ray image can be subjected to a grid of parallel lines whereupon the number of intersections is counted. Hence, eq. (4) holds also for this case, whereby the projected fibre length is denoted by *L*'.

$$L_A' = \frac{\pi}{2}\overline{P_L(\beta)} \tag{7}$$

When expanding the information to 3D, the only tricky element is the derivation of the relationship between total fibre length in 3D, *L*, and its projected value*, L*'. In fact, we deal with the theorem of Cauchy that constitutes the basis for *global averaging*. Since this is a common operation required for relating information on material structure to global engineering properties, we will later on present the simple proof of the theorem. Since it combines geometry and statistics, it is part of the disciplines of geometrical statistics, geometric probability theory and stereology.

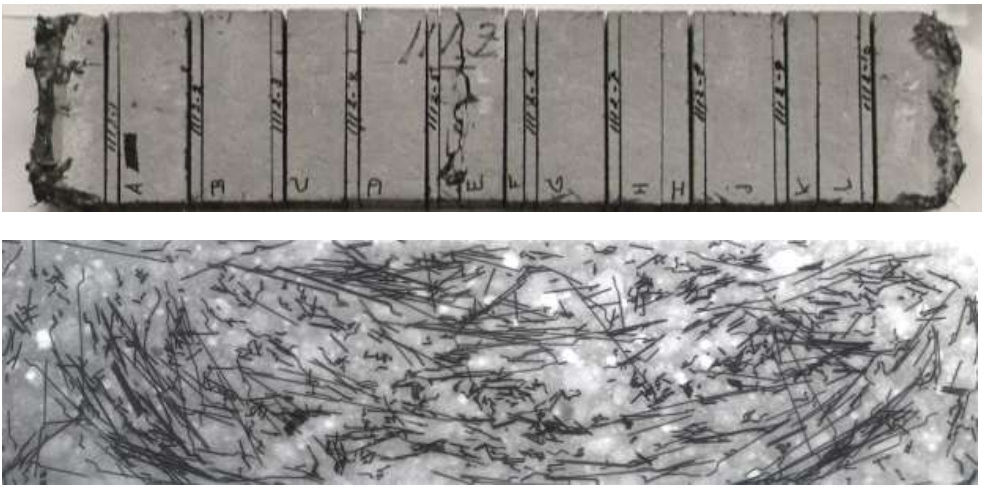


Fig. 6. SFRC specimen of 70x200x1000 mm (top) is sliced at regular distances to produce 10 tiles. An X-ray radiograph of such a "vertical slice" is shown at the bottom); gravitation during compaction was in vertical direction (perpendicular to top surface) [10].

Generalizing the approach, path length in 3D can be considered consisting of a sequence of straight 'unit' length elements. For the proof of Cauchy fibres can be considered composed of a sequence of straight unit length elements, so fibres can be of arbitrary length and curved. Fig. 7 presents a detail of projected mono-size steel fibres (in a tile of SFRC) and the unit sphere composed of the unit fibre elements collected and united with one end in the origin of the sphere. The length of the narrow strip on the sphere's surface for fibres enclosing an angle θ with vertical direction *z* represents the relative frequency of such fibres, being sin θ. Consequently, when distributed isotropic uniformly at random (IUR) in bulk, frequency will be a function of $\sin\theta$. Or in other words, the number density of fibers enclosing the same angle θ with the *z*-axis in Fig. 7 is a function of $\sin\theta$. So, the frequency distribution of IUR fibers in 3D can be expressed by

$$f(\theta)_{3D} = \frac{\pi}{2}\sin\theta \tag{8}$$

Note that this sine-weighed orientation distribution is not produced in 'compucrete' set ups whereby fibre orientation is defined by connecting "randomly" generated nodal points, leading to biases in applications in which this 'compucrete' is employed.

The projection image obtained by X-raying is 2D random when obtained from an IUR fiber dispersion in 3D; the image displays projections of pieces of fibres included in a tile with thickness t sawn from larger SFRC units (Fig. 6, at the top). Projection direction is perpendicular to the tile's surface.

It can be shown that an analysis by the sweeping test line system allows bridging the dimensional gap between intersection density of line grid and fiber projections, P_L , and fibre length per unit of volume, L_V . Hence, [11,12]

$$L_{V3} = \frac{2}{t}\bar{P}_L \tag{9}$$

which is basically similar to eq. (2) covering the case of dispersed cracks in concrete. It also directly reflects Cauchy's expression in eq. (5). Eq. (8) will yield an unbiased estimate of L_{V3} .

An estimate for fibre length per unit of volume, L_V can also be based on counting fibre intersections in a section plane. For an IUR fibre dispersion, a single image will suffice (for getting a reliable estimate, we need more section images, of course). For the more realistic case of non-IUR fibres, a random set of section images would be required, a too complicated set up in concrete technology. The estimate is given by the Cauchy expression

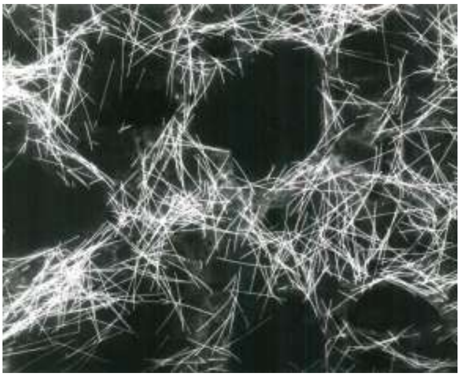
$$L_{V3} = 2\bar{N}_A \tag{10}$$

Herein, N_A stands for the number of fibers per unit of the section area.

Still, the section plane misses 50% of the fibers in its near neighborhood [8]. This is due to the probability of intersection of section and fibre that is proportional to the tangent height of the fibre perpendicular to the section plane. This equals cosθ in the coordinate system of Fig. 7. Hence, spatial fibre orientation distribution is given by

$$f_{N_A}(\theta)_{3D} = \sin 2\theta \tag{11}$$

Obvioulsly, Fig. 6 presents a non-isometric case that more closely represents the situation in engineering situations. Also in this case (like with the cracks) application of the 'Stroeven concept' is possible because an axis of symmetry is generally due to the impact of compaction by vibration. Formulas have been published in numerous articles to which the interested reader is referred [11,12,13]. Formulas directly reflect the theorems of Cauchy and Buffon, (and application for similar cases by Steinhaus in 1930), however the methodology had to be re-introduced by Saltikov in 1945.



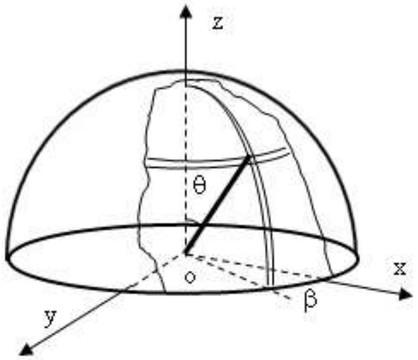


Fig. 7. Visualization by sphere model of orientation distribution of mono-size fibre elements (visualized for illustrative purposes in X-ray projection image (left)) in SFRC specimen. Fibre elements are considered translated from bulk to join in one of their ends in point O. As a consequence, other ends will UR cover the surface of the sphere with unit radius (right).

As promised, here are the global 2D and 3D averaging operations underlying Cauchy's formulas

$$\overline{L'} = \frac{\int_0^{\pi/2} \cos\theta \mathrm{d}\theta}{\int_0^{\pi/2} \mathrm{d}\theta} L = \frac{2}{\pi} L \tag{12}$$

$$\overline{A'} = S \frac{\int_0^{\pi/2}\int_0^{\pi/2} \sin\theta\cos\theta \mathrm{d}\theta \mathrm{d}\beta}{\int_0^{\pi/2}\int_0^{\pi/2} \sin\theta \mathrm{d}\theta \mathrm{d}\beta} = \frac{1}{2} S \tag{13}$$

CONCLUSIONS

Da Vinci's podometer was a practical concept for measuring path length. Engineering modernized the concept into a range designs for step and map meters. Modernization should have been based however on already for six or seven generations available geometrical statistical methodology. Moreover, the theoretical concepts were also *used* for the purpose of path length measurements by Steinhaus, but generally overlooked.

In concrete technology, the practical impact of the theorems of Cauchy and Buffon was not understood for a long time. Even Saltikov's implementation of such theorems into a practical methodology for path measuring (*i.e.*, directed and random secants) was overlooked for quite some time. Gradually, the last half of a century this economic and reliable approaches to analyzing crack and fibre dispersions in concrete became more widespread [14,15].

In addition to providing modern methodology for path measuring, the theorems provide the basic principles for geometric averaging. In that respect, the concrete literature is still full of misunderstandings and misinterpretations. The spacing concept for fibres is just a single example. Therefore, the aim of this paper was laying side-by-side the engineering approach from Da Vinci, the modern step and map meters, and the materials science-based approaches making profitable use of the appropriate geometrical-statistical principles underlying optimized path measuring methodology in 2D as well as 3D.

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