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# EVALUATING LINK AND PATH INCENTIVES: WHICH IS THE MOST EFFECTIVE STRATEGY FOR MITIGATING TRAFFIC CONGESTION?

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**ABSTRACT**
This study investigates the potential of link- and path-based incentives to mitigate congestion in urban transportation networks. Both incentive schemes are formulated as non-linear optimisation problems with complementarity constraints. Mathematically, it is demonstrated that the feasible region of the link-based model is a subset of the feasible region of the path-based model. Consequently, path-based incentives exhibit greater potential for shifting the user equilibrium flow pattern toward system optimum compared to link incentives. A column generation-based iterative solution technique, which generates new paths at each iteration, is devised to efficiently solve both optimisation problems. Numerical experiments conducted for various transport networks also highlight the superiority of path-based incentives in reducing total travel time in urban transportation networks.

*Keywords*: Incentive scheme, System optimum, Traffic assignment, Traffic management.

# INTRODUCTION

## Motivation

Providing drivers with sensible route advice is considered a successful traffic management tool, with the potential to reduce congestion (*1*–*4*), thereby improving network efficiency and sustainability (*5, 6*), although it may increase individual travel cost (distance and/or time) for some users (*7*). This implies that some drivers may need to follow routes longer than their individual optimal paths for the benefit of the community. This situation in which the total social benefit reaches the highest level is called system optimum (SO) and is in contrast to user equilibrium (UE), which aims at achieving the highest individual benefits (*8*). Studies estimated a wide range (5% - 25%) of benefits, in terms of reduced total travel time (TTT), in typical road networks, when SO traffic flow is achieved (*9*–*12*).

Since SO is an ideal situation where a central authority dictates routes for all users, leading to increased (individual) travel times, a stimulus is needed to encourage such changes in drivers' behaviour. Road pricing (*13*–*16*) has been traditionally considered to shift the UE flow pattern toward SO. However, incentivising schemes with voluntary participation (*17*–*20*) have recently gained more popularity due to public dissatisfaction (*21*) and inequitable welfare distribution across the population (*22, 23*) resulted from road pricing. Due to limited resources, an efficient allocation of incentives within a limited budget is crucial. Yet, optimally assigning incentives to achieve the highest network efficiency in a complex real traffic network can be challenging due to the optimisation problem being computationally intensive. This is particularly more challenging than assigning first-best tolls due to a budget limit. Moreover, even without a budget limit, the incentive problem could differ from the first-best pricing in that it does not necessarily match the marginal cost of a link with a positive sign.

## Objectives and contributions

Similar to tolls, incentives can be easily assigned to links. However, advancements in mobile apps and navigation systems have made path-based incentives feasible. Despite the rich body of literature on incentivising drivers, a key research gap concerns the rationale for selecting either link- or path-based incentives to manage urban traffic. With the emergence of technologies that enable us to track travellers through their journeys and the widespread usage of navigation apps, path-based pricing/incentivising has become technically feasible. To the best of the authors' knowledge, little attention has been devoted to assessing the efficiency of link- and path-based incentives and no study has yet shed light on the potential superiority of one incentive type over the other. In this work, we bridge this fundamental gap as follows:

1. We introduce two distinct optimisation problems aimed at minimising TTT under both link- and path-based incentive schemes with various participation rates of travellers in the incentivising program within the constraints of a limited budget;
2. We propose an innovative solution algorithm capable of efficiently solving both link- and path-based incentive optimisation problems in real-size transportation networks;
3. We conduct a thorough comparison between link- and path-based incentives, offering valuable insights into their respective performances.

Together, these contributions advance our understanding of incentive-based approaches in traffic management and pave the way for improved urban transportation strategies.

## LITERATURE REVIEW

Achieving an SO traffic flow, (*24*) showed that the drivers who comply with the routing advice need to take routes slightly longer than the shortest path. Still, a strong stimulus, e.g., an incentive is required to push drivers to take a route that might be significantly worse than their preferred (e.g., faster) route. (*25*), (*26*), and (*27*) showed that drivers would accept longer routes under incentive strategy, compared to other stimuli for contributing to a more liveable, safer, and less polluted city, while (*28*) showed incentives' positive impact using real-world data. These findings imply that employing incentives can play a vital role in the success of a routing advice system aiming at steering flow toward SO.

One of the first studies investigating the application of link-based incentives to achieve SO has been conducted in two small toy networks with 2 and 4 links (*29*). The study showed that the difference in total travel times between UE and SO was flattened when the demand increased beyond a certain threshold, indicating that the application of incentives may not have economic significance beyond that threshold. This happens because when the entire network becomes congested, redistributing the traffic only places an additional burden on other parts of the network.

Considering path-based incentives, (*30*) investigated the impact of applying them to designated safe routes on traffic network performance. A logit model was employed to assign traffic to routes based on their generalised costs consisting of travel time, fuel cost, and safety measures minus incentive. Their numerical results indicated that depending on the incentive program setup, the incentive scheme can be beneficial or not. (*31*) designed a personalised incentive framework generated by processing travel information through a decision tree and evolutionary game theory to adjust the mode and route choices of travellers while taking into account a balance between multiple goals. (*32*) employed personalised monetary incentives to adjust the departure time and route choice of travellers to minimise energy consumption. They observed that by offering an incentive 27% of travellers would change their routes while 20% would change their departure time, and the system can achieve 8.7% energy saving. Finally, (*33*) proposed a path-based personalised incentive chosen from a predetermined set to minimise TTT under various budget limits and user participation levels of the incentive scheme. They showed that the value of saved time was usually larger than the cost of offering incentives, however, for large budget limits the value of saved time might be smaller than the amount spent on incentives.

Recently, (*34*) conducted a comparison between link- and path-based incentives to analyse their potential to reduce TTT. They formulated single-level optimisation problems to compare the two types of incentives under budget limits and various participation levels of drivers. Their numerical examples in two transportation networks showed that in most cases path-based incentives outperformed link-based incentives, while for a low participation level of drivers, the link incentive reduced TTT more than path incentives. We adopt a similar specification of link and path incentive optimisation problems to compare the performance of these two types of incentives. Nevertheless, our research differs in numerous aspects. First, our proposed solution algorithm computes the shortest paths in each iteration, generating at least 10 paths for each origin-destination (OD) pair, while (*34*) enumerated only 3 paths for each OD pair a-priori, resulting in the flow-independent shortest path. Second, we introduce a column generation approach that solves the optimisation problem at each iteration of the algorithm using a solver, while (*34*) utilised a customised branch-and-bound algorithm to solve the optimisation problem once. Third, even though the shortest path problem is solved at least 10 times[1] in this research compared to only once in (*34*) and a more complex network is employed, our proposed approach significantly outperforms it in terms of computation time. Finally, we use theory to prove that link incentives cannot outperform path incentives and, conversely, that link-based incentives work at best as well as path-based incentives.

## METHODOLOGY

In this section, we formulate the path- and link-based problems as two single-level optimisation problems called P1 and P2, respectively, to determine the optimal incentive schemes under budget limitations.

### Path-based and link-based incentive optimisation problems

We represent a transportation network by a graph $G(V,A)$, where $V$ is the set of nodes and $A \subset V \times V$ is the set of links. Let $W \subset V \times V$ be the set of OD pairs, and let the travel demand, $q_w$, be described by the fixed number of vehicles travelling between the OD pair $w \in W$.[2] Table 1 defines all the parameters and variables used in the formulated optimisation problems.

The single-level optimisation problem for path-based incentives called **P1**, with budget limit $B$, under participation rate $r$ of the cooperative travellers who voluntarily participate in the incentive scheme, where $\hat{u}_w$ and $\tilde{u}_w$ denote the minimum travel cost for cooperative and non-cooperative travellers between origin-destination pair $w$, respectively, is formulated as follows.

$$Z1 = \min_{\tilde{\boldsymbol{f}},\hat{\boldsymbol{f}},\bar{\boldsymbol{y}}} \Sigma_{a\in A}(x_a t_a) \tag{1}$$

s.t.

$$\Sigma_{p\in P_w} \hat{f}_w^p = q_w \cdot r \qquad \forall w \in W \tag{2}$$

$$\Sigma_{p\in P_w} \tilde{f}_w^p = q_w \cdot (1-r) \qquad \forall w \in W \tag{3}$$

$$\Sigma_{a\in A} \delta_a^p\, t_a - \bar{y}^p - \hat{u}_w \geq 0 \qquad \forall p \in P_w,\ \ w \in W \tag{4}$$

$$(\Sigma_{a\in A} \delta_a^p\, t_a - \bar{y}^p - \hat{u}_w)\hat{f}_w^p = 0 \qquad \forall p \in P_w,\ \ w \in W \tag{5}$$

$$\Sigma_{a\in A} \delta_a^p\, t_a - \tilde{u}_w \geq 0 \qquad \forall p \in P_w,\ \ w \in W \tag{6}$$

$$(\Sigma_{a\in A} \delta_a^p\, t_a - \tilde{u}_w)\tilde{f}_w^p = 0 \qquad \forall p \in P_w,\ \ w \in W \tag{7}$$

$$\Sigma_{w\in W}\Sigma_{p\in P_w} \hat{f}_w^p\, \bar{y}^p \leq B \tag{8}$$

$$x_a = \Sigma_{w\in W}\Sigma_{p\in P_w} \delta_a^p(\hat{f}_w^p + \tilde{f}_w^p) \qquad \forall a \in A \tag{9}$$

$$t_a = t_a(x_a) \qquad \forall a \in A \tag{10}$$

$$\tilde{\boldsymbol{f}}, \hat{\boldsymbol{f}}, \bar{\boldsymbol{y}}, \boldsymbol{u}, \tilde{\boldsymbol{u}} \geq 0 \tag{11}$$

The objective function, $Z1$, minimises the network total travel time with respect to path flows and incentives, $\mathbf{f}$ and $\bar{\mathbf{y}}$. Constraints (2) and (3) guarantee the flow conservation of vehicles for cooperative and non-cooperative travellers, respectively. Constraints (4) and (5) are the complementarity constraints ensuring Wardrop's first principle with generalised travel times, de-

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[1] This is to ensure solution stability, as shown in Table 2 of Section 4.

[2] We acknowledge the potential risk of induced car demand associated with incentive schemes. In our proposed method, we do not offer high incentives that could generate revenue for drivers, i.e., negative generalised travel cost. This restraint is guaranteed by Constraint (4). By refraining from assigning high incentives, we can assume that the attraction of travellers from other modes to car trips is prevented, leading to inelastic demand, $q_w$.

fined as travel time minus incentives for the cooperative travellers. Similarly, complementarity Constraints (6) and (7) ensure a UE flow pattern for non-cooperative travellers. Note that these constraints are defined solely based on travel times, as non-cooperative travellers do not receive incentives. Constraint (8) imposes the budget limitation, Constraint (9) maps path flows to link flows, Constraint (10) defines link travel times as a function of link flows, and Constraint (11) ensures non-negativity for all variables. Note that function $Z1$, accompanied by Constraints (2), (3), and (9) - (11), represents the SO problem in a transportation network under adequate regularity assumptions (*35*).

We can similarly formulate the budget-constrained link-based incentive problem **P2**, with budget limit, $B$, under participation rate, $r$, as follows.

$$Z2 = \min_{\tilde{\boldsymbol{f}},\hat{\boldsymbol{f}},\boldsymbol{y}} \Sigma_{a\in A}(x_a t_a) \tag{12}$$

s.t.

$$\Sigma_{p\in P_w} \hat{f}^p_w = q_w \cdot r \qquad \forall w \in W \tag{13}$$

$$\Sigma_{p\in P_w} \tilde{f}^p_w = q_w \cdot (1-r) \qquad \forall w \in W \tag{14}$$

$$\Sigma_{a\in A} \delta^p_a (t_a - y_a) - \hat{u}_w \geq 0 \qquad \forall p \in P_w,\ w \in W \tag{15}$$

$$(\Sigma_{a\in A} \delta^p_a (t_a - y_a) - \hat{u}_w)\hat{f}^p_w = 0 \qquad \forall p \in P_w,\ w \in W \tag{16}$$

$$\Sigma_{a\in A} \delta^p_a\, t_a - \tilde{u}_w \geq 0 \qquad \forall p \in P_w,\ w \in W \tag{17}$$

$$(\Sigma_{a\in A} \delta^p_a\, t_a - \tilde{u}_w)\tilde{f}^p_w = 0 \qquad \forall p \in P_w,\ w \in W \tag{18}$$

$$\hat{x}_a = \Sigma_{w\in W}\Sigma_{p\in P_w} \delta^p_a\ \hat{f}^p_w \qquad \forall a \in A \tag{19}$$

$$\Sigma_{a\in A} \hat{x}_a y_a \leq B \tag{20}$$

$$x_a = \Sigma_{w\in W}\Sigma_{p\in P_w} \delta^p_a (\hat{f}^p_w + \tilde{f}^p_w) \qquad \forall a \in A \tag{21}$$

$$t_a = t_a(x_a) \qquad \forall a \in A \tag{22}$$

$$\tilde{\boldsymbol{f}}, \hat{\boldsymbol{f}}, \boldsymbol{y}, \boldsymbol{u}, \tilde{\boldsymbol{u}} \geq 0 \tag{23}$$

Similar to problem **P1**, the objective function, $Z2$, minimises the total travel time in the whole network with the path flow, **f**, under link-based incentive, **y**, with Constraints (13)–(23) follow the same structure as those of P1.

**Differences between path-based and link-based incentive problems**

**Theorem**: Total travel time obtained by optimally solving **P1** is never higher than the total travel time obtained from optimally solving **P2** under the same budget limit $B$.

**Proof**: Assume that the pair ($\boldsymbol{f}$, $\boldsymbol{y}$) satisfies Constraints (13)–(23), i.e., it is a feasible pair for **P2**. We can show that there is a pair ($\boldsymbol{f}$, $\bar{\boldsymbol{y}}$) with the exact same path flows that satisfies Constraints (2)–(11), i.e., that the feasible solution set of **P1** encompasses the feasible solution set of **P2**. Since the two optimisation problems have identical objective functions, **P1** always results in flow patterns with total travel times at most as low as those of **P2**.

Assume incentive of $y_a$ is assigned to link $a$. All paths (and cooperative users) that traverse link $a$ will receive this incentive. Therefore, travellers on path $p$ will receive a link-additive path incentive as $\bar{y}^p = \Sigma_{a\in A}\delta^p_a y_a$. We can then rewrite Constraint (15) as follows:

$$\Sigma_{a\in A}\delta^p_a(t_a - y_a) - \hat{u}_w = \Sigma_{a\in A}\delta^p_a t_a - \Sigma_{a\in A}\delta^p_a y_a - \hat{u}_w = \Sigma_{a\in A}\delta^p_a t_a - \bar{y}^p - \hat{u}_w, \tag{I}$$

which results in Constraint (4).

**TABLE 1**: Notation for variables and parameters

| Symbol | Definition |
|---|---|
| $A$ | Set of links |
| $V$ | Set of nodes |
| $W$ | Set of all OD pairs |
| $q_w$ | Travel demand between OD pair $w \in W$ |
| $P_w$ | Set of all paths between OD pair $w \in W$ |
| $t_a$ | Travel time on link $a \in A$ |
| $x_a$ | Vehicle flow on link $a \in A$ |
| $\hat{x}_a$ | Cooperative vehicle flow on link $a \in A$ |
| $f_w^p$ | Vehicle flows on path $p \in P_w$ between OD pair $w \in W$ |
| $\hat{f}_w^p$ | Cooperative vehicle flow on path $p \in P_w$ between OD pair $w \in W$ |
| $\tilde{f}_w^p$ | Non-cooperative vehicle flow on path $p \in P_w$ between OD pair $w \in W$ |
| $y_a$ | Incentive on link $a \in A$ |
| $\bar{y}^p$ | Incentive on path $p \in P_w$ between OD pair $w \in W$ |
| $B$ | Total budget available for the incentive scheme |
| $\hat{u}_w$ | Minimum travel time for cooperative travellers between OD pair $w \in W$ |
| $\tilde{u}_w$ | Minimum travel time for non-cooperative travellers between OD pair $w \in W$ |
| $r$ | Participation rate of cooperative drivers |
| $\delta_a^p$ | Link-path incident matrix |

With similar substitutions, we can show that Constraint (16) can be rearranged to result in Constraint (5). Now, we can rewrite Constraint (20) by substituting $\hat{x}_a$ with its definition, i.e., $\hat{x}_a = \Sigma_{w \in W} \Sigma_{p \in P_w} \delta_a^p \hat{f}_w^p$, as follows:

$$\Sigma_{a \in A} \hat{x}_a y_a = \Sigma_{a \in A} y_a \Sigma_{w \in W} \Sigma_{p \in P_w} \delta_a^p \hat{f}_w^p = \Sigma_{w \in W} \Sigma_{p \in P_w} \Sigma_{a \in A} y_a \delta_a^p \hat{f}_w^p = \Sigma_{w \in W} \Sigma_{p \in P_w} \bar{y}^p \hat{f}_w^p, \quad \text{(II)}$$

which is equal to the path incentive budget constraint, i.e., Constraint (8).

Considering the two statements (I) and (II) shows that for any pair of $(\boldsymbol{f}, \boldsymbol{y})$ that satisfies Constraints (13)–(23), there is a pair $(\boldsymbol{f}, \bar{\boldsymbol{y}} =< \Sigma_{a \in A} \delta_a^p y_a >)$ that satisfies constraints (2)–(11). Therefore, the feasible region of the optimisation problem **P1** encompasses the feasible region of the optimisation problem **P2**. Since the incentives collected by drivers do not change, the link flows and budget spent by incentive schemes remain identical for the two pairs.□

## SOLUTION ALGORITHM

The incentivised UE problem with a budget limit presented in problems **P1** and **P2** can be directly solved in simple networks by enumerating all paths where the number of paths is small. In the case of a large-scale transportation network, such an approach is not efficient since it is computationally expensive to enumerate all the paths of the network. Therefore, a column generation-based approach is developed that is able to generate new paths as needed as the algorithm proceeds. Column generation-based approaches, in principle, can lead to the optimal solution if they iterate long enough to enumerate all the paths in the network (*36*). However, it can be stopped when the improvement in two consecutive iterations falls below a certain threshold resulting in a balance between computation time and solution quality.

To solve problems **P1** and **P2** in a real-size network, we propose the following column generation-based method that considers each path as a column, adds new columns at each iteration, and stops when the minimum iteration number, $N$, is reached and the relative difference of total travel times in two consecutive iterations falls below a predefined value, $\varepsilon$. The steps of the proposed column generation algorithm are as follows.

1. **Initialisation**:
   (a) Define values for $N$ and $\varepsilon$.
   (b) For each OD pair $w \in W$, set $P_w = \emptyset$.
   (c) Set $n = 1$, $\boldsymbol{y} = 0$, $\boldsymbol{f} = 0$, $\boldsymbol{x}^0 = \boldsymbol{x}(\boldsymbol{f})$, and $\boldsymbol{t}^0 = \boldsymbol{t}(\mathbf{x}^0)$.
2. **Shortest path**: for each OD pair $w \in W$,
   (a) Find the shortest path $p$ such that $p \notin P_w$, and
   (b) set $P_w = P_w \cup p$.
3. **User equilibrium**: solve the optimisation problem, and find traffic flows, $\boldsymbol{f}$, and corresponding incentive values $\boldsymbol{y}$.
4. **Updating**: set $\boldsymbol{x}^n = \boldsymbol{x}(\boldsymbol{f})$ and $\boldsymbol{t}^n = \boldsymbol{t}(\boldsymbol{x}^n)$.
5. **Stopping criteria**:
   (a) Calculate $\bar{\varepsilon} = \frac{|\Sigma_{a\in A}(x_a^n t_a^n) - \Sigma_{a\in A}(x_a^{n-1} t_a^{n-1})|}{\Sigma_{a\in A}(x_a^n t_a^n)}$.
   (b) If $\bar{\varepsilon} \leq \varepsilon$ and $n \geq N$ stop, otherwise set $n = n+1$ and go to step 2.

Note that we generate a new path that does not belong to the current active path set at each iteration of the column generation process to prevent getting trapped around a local optimum. However, such a path cannot be found by solving a standard shortest path problem. Therefore, we propose to solve the integer problem shown in equations (24)–(28) to find the shortest path between each OD pair while imposing a high penalty for choosing a path between OD pair $w \in W$ that already exists in the current active path set.

$$\lambda_w = \min_{\boldsymbol{\beta}} \Sigma_{a\in A}(\beta_a t_a) + \Sigma_{p\in P_w}(M\rho^p) \tag{24}$$

s.t.

$$\Sigma_{a\in A} \mid \delta_a^p - \beta_a \mid \geq 1 - \rho^p \qquad \forall p \in P_w \tag{25}$$

$$\Sigma_{a\in A:\text{start}(a)=v}\beta_a - \Sigma_{a'\in A:\text{end}(a')=v}\beta_{a'} = \begin{cases} +1 & \text{if } v \text{ is the origin node} \\ -1 & \text{if } v \text{ is the destination node} \\ 0 & \text{otherwise} \end{cases} \qquad \forall v \in V \tag{26}$$

$$\beta_a \in \{0,1\} \qquad \forall a \in A \tag{27}$$

$$\rho^p \in \{0,1\} \qquad \forall p \in P_w \tag{28}$$

where $\beta_a$ is a binary variable that equals 1 if link $a$ is on the shortest path and 0 otherwise; $\rho^p$ is a binary variable that takes the value of 1 if path $p$ that already belongs to the current active path set $P_w$ is selected as the shortest path and 0 otherwise. The objective function (24) is designed to select a set of links with the least travel time while avoiding to choose one of the current active paths with a big penalty coefficient $M$. Constraint (25) ensures that the variable $\rho^p$ takes the value of 1 if and only if all constituting links of path $p \in P_w$ exist in the shortest path found by the optimisation problem. Constraint (26) makes sure that the solution of the model is a path. Note that start($a$) and end($a$) indicate the starting and end nodes of link $a$.

Constraint (25) is nonlinear due to the presence of an absolute value function. A linearised version can be used to reduce its complexity. We linearise absolute value functions by introducing non-negative variables $(\eta_a^p)^+$ and $(\eta_a^p)^-$ and binary variable $\phi_a^p$. We first set the argument of each absolute value function equal to the subtraction of the two non-negative variables as shown in the following equation.

$$\delta_a^p - \beta_a = (\eta_a^p)^+ - (\eta_a^p)^- \quad \forall p \in P_w,\ a \in A \tag{29}$$

Then, the following constraints ensure that one of the non-negative variables, $(\eta_a^p)^+$ and $(\eta_a^p)^-$, take the value of zero using the big-M method.

$$(\eta_a^p)^+ \leq M\phi_a^p \qquad \forall p \in P_w,\ a \in A \tag{30}$$

$$(\eta_a^p)^- \leq M(1-\phi_a^p) \qquad \forall p \in P_w,\ a \in A \tag{31}$$

Therefore, the output of the absolute value function becomes equal to the summation of the two non-negative variables.

Constraint (25) is finally linearised by replacing the absolute value functions with the summation of their associated non-negative values as follows.

$$\Sigma_{a\in A}\left((\eta_a^p)^+ + (\eta_a^p)^-\right) \geq 1-\rho^p \quad \forall p \in P_w \tag{32}$$

Therefore, the linearised shortest path problem optimises the cost function (24), subject to constraints (29)–(32) and (26)–(28).

## NUMERICAL EXPERIMENTS

This section presents three transportation networks to test the proposed problem formulation and solution technique. The first network is a toy network that is used to show how the shortest path model, (24) - (28), works. The second network, Nguyen-Dupuis (ND), is used to test the correctness of the proposed solution technique and compare the link and path incentive-based optimisation problems. The third network, Sioux Falls (SF), is used to show the ability of the proposed solution technique to solve bigger networks. The link volume-delay function in all networks follows the BPR function (*37*)

$$t_a(x_a) = t_a^0\left(1+0.15\left(\frac{x_a}{C_a}\right)^4\right), \tag{33}$$

where $t_a^0$ is the free flow travel time in link $a$ and $c_a$ its capacity in vehicles per time unit.

### Shortest path example

A sample transportation network with three nodes and three links is shown in Figure 1. There are two paths between the OD pair (1,3). Path 1 consists of links 1 and 2, and path 2 consists of links 1 and 3. Assume that at an iteration of the solution algorithm, path 1 has been generated and added to the path set $P_{(1,3)} = \{1\}$, which means that $\delta_1^1 = \delta_2^1 = 1$ and $\delta_3^1 = 0$. In addition, assume that link travel times at the current iteration are $t_1 = t_2 = 1$ and $t_3 = 2$. The optimisation problem (24)–(28) can only choose one of the two paths because any other combinations of links would make constraint (26) infeasible. For instance, if we write constraint (26) for node 2, where $\beta_a$ takes the value of one only for link 1, we will have $0-1=0$ that is infeasible. Now, we need to find the objective value for the two feasible solutions for the shortest path model and select the one with the lowest objective value as follows:

1. Path 1: in this case, $\beta_1 = \beta_2 = 1$ and $\beta_3 = 0$. Note that since path 1 is already in the current path set, we have $\delta_1^1 = \delta_2^1 = 1$ and $\delta_3^1 = 0$. Therefore, constraint (25) for path 1 can be written as follows:
   $|\delta_1^1 - \beta_1| + |\delta_2^1 - \beta_2| + |\delta_3^1 - \beta_3| = |1-1| + |1-1| + |0-0| = 0 \geq 1-\rho^1$
   that forces $\rho^1$ to take a value of one. Therefore, the objective value will take a value of $\lambda_{(1,3)} =$

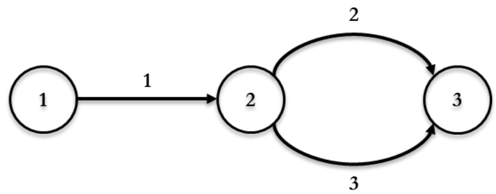


**FIGURE 1**: Sample transportation network

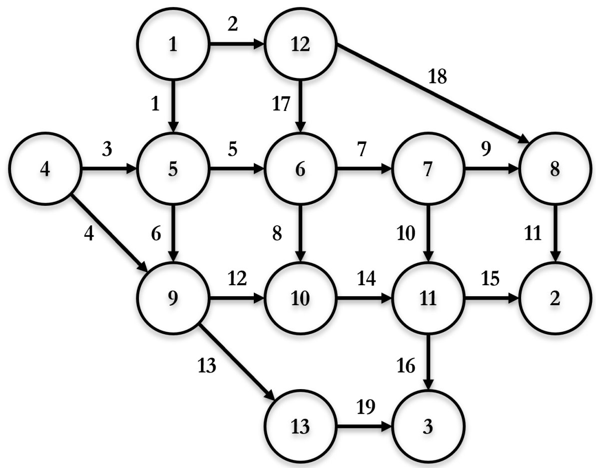


**FIGURE 2**: Nguyen-Dupuis network

$\beta_1 t_1 + \beta_2 t_2 + \beta_3 t_3 + M\rho^1 = 2 + M.$

2. Path 2: in this case, $\beta_1 = \beta_3 = 1$ and $\beta_2 = 0$. Constraint (25) for path 1 in this scenario can be written as follows:
   $|\delta_1^1 - \beta_1| + |\delta_2^1 - \beta_2| + |\delta_3^1 - \beta_3| = |1-1| + |0-1| + |1-0| = 2 \geq 1 - \rho^1$
   which lets $\rho^1$ take either a value of one or zero. However, the objective function will fix the value of $\rho^1$ at zero to keep the objective value as low as possible. Hence, the objective value will take a value of $\lambda_{(1,3)} = \beta_1 t_1 + \beta_2 t_2 + \beta_3 t_3 + M\rho^1 = 3$. Note that constraint (25) is defined over the current path set, therefore, there is no need to write it for path 2.

The objective function will select path 2 over path 1 despite its longer travel time because it is the shortest path that is not included in the current path set $P_{(1,3)}$.

**Nguyen-Dupuis network**

As shown in Figure 2, the Nguyen-Dupuis network contains 13 nodes, 19 links, and four OD pairs. This network is used to show the correctness of the proposed solution algorithm by comparing its results with those obtained from solving **P1** and **P2** directly when all the paths are generated. This network contains 25 paths that are summarised in Table 2. Given the small number of paths and the ease with which all paths between each OD pair can be generated, we validate the accuracy of the proposed algorithm. Additionally, we examine the impact of path and link incentives in this network. The parameters of the BPR function, equation (33), and the demand between each OD pair in the ND network are sourced from (*34*).

**TABLE 2**: All paths between OD pairs of Nguyen-Dupius network.

| OD | Path | Constitutive links |
|---|---|---|
| (1,2) | 1 | 2,11,18 |
| | 2 | 2,7,9,11,17 |
| | 3 | 2,7,10,15,17 |
| | 4 | 2,8,14,15,17 |
| | 5 | 1,5,7,9,11 |
| | 6 | 1,5,7,10,15 |
| | 7 | 1,5,8,14,15 |
| | 8 | 1,6,12,14,15 |
| (1,3) | 9 | 1,6,13,19 |
| | 10 | 1,6,12,14,16 |
| | 11 | 1,5,8,14,16 |
| | 12 | 1,5,7,10,16 |
| | 13 | 2,7,10,16,17 |
| | 14 | 2,8,14,16,17 |
| (4,2) | 15 | 3,5,7,9,11 |
| | 16 | 3,5,7,10,15 |
| | 17 | 3,5,8,14,15 |
| | 18 | 3,6,12,14,15 |
| | 19 | 4,12,14,15 |
| (4,3) | 20 | 3,5,7,10,16 |
| | 21 | 3,5,8,14,16 |
| | 22 | 3,6,12,14,16 |
| | 23 | 4,13,19 |
| | 24 | 3,6,13,19 |
| | 25 | 4,12,14,16 |

*Benchmark analysis*

To directly solve the optimisation problems **P1** and **P2**, they are modeled in GAMS software (*38*) and its NLP and MIP solvers are used. A case with a budget limit of 3000 time unit and 50% participation rate is selected for the analysis. The results obtained from direct and column generation-based approaches for both link and path incentive-based optimisation problems are summarised in Table 3. As can be seen, the proposed column generation-based solution technique successfully solves link and path incentive-based models with optimality gaps of 0.00019% and 0.00004%, respectively. Moreover, the difference in link flows between direct and column generation-based approaches is always less than 0.0075%.

*Link and path incentives under full participation*

Figure 3 shows the total travel times obtained from solving both the link and path incentive-based optimisation problems with different budget limits and 100% participation rate. As expected, path incentives result in smaller total travel times under all budget limits. Additionally, path incentives enable the optimisation problem to achieve the SO flow pattern with a lower budget limit.

**TABLE 3**: Link flow and TTT comparison between column generation-based and direct approaches

| Link | Path incentives | | | Link incentives | | |
|---|---|---|---|---|---|---|
| | **Direct** | **CG** | **Diff. (%)** | **Direct** | **CG** | **Diff. (%)** |
| 1 | 733.8933 | 733.8900 | -0.00045 | 732.6700 | 732.6699 | -0.00001 |
| 2 | 466.1067 | 466.1100 | 0.00071 | 467.3300 | 467.3301 | 0.00002 |
| 3 | 407.8727 | 407.8700 | -0.00065 | 409.0500 | 409.0512 | 0.00030 |
| 4 | 392.1273 | 392.1300 | 0.00068 | 390.9500 | 390.9488 | -0.00031 |
| 5 | 654.8634 | 654.8600 | -0.00053 | 672.5200 | 672.5319 | 0.00177 |
| 6 | 486.9025 | 486.9000 | -0.00052 | 469.2000 | 469.1892 | -0.00230 |
| 7 | 685.7051 | 685.7100 | 0.00072 | 703.4500 | 703.4551 | 0.00072 |
| 8 | 68.9466 | 68.9500 | 0.00489 | 67.3300 | 67.3301 | 0.00015 |
| 9 | 307.7062 | 307.7100 | 0.00123 | 303.4700 | 303.4554 | -0.00481 |
| 10 | 377.9989 | 378.0000 | 0.00030 | 399.9700 | 399.9997 | 0.00743 |
| 11 | 674.0246 | 674.0200 | -0.00069 | 672.5500 | 672.5322 | -0.00265 |
| 12 | 523.0781 | 523.0800 | 0.00037 | 496.4000 | 496.3926 | -0.00149 |
| 13 | 355.9518 | 355.9500 | -0.00050 | 363.7500 | 363.7454 | -0.00128 |
| 14 | 592.0247 | 592.0300 | 0.00090 | 563.7300 | 563.7227 | -0.00129 |
| 15 | 325.9754 | 325.9800 | 0.00142 | 327.4500 | 327.4678 | 0.00543 |
| 16 | 644.0482 | 644.0500 | 0.00028 | 636.2500 | 636.2546 | 0.00073 |
| 17 | 99.7883 | 99.7900 | 0.00173 | 98.2600 | 98.2533 | -0.00684 |
| 18 | 366.3184 | 366.3200 | 0.00043 | 369.0700 | 369.0768 | 0.00185 |
| 19 | 355.9518 | 355.9500 | -0.00050 | 363.7500 | 363.7454 | -0.00128 |
| TTT | 179671.60 | 179671.68 | 0.00004 | 179992.66 | 179993.00 | 0.00019 |

*Link and path incentive under partial participation*

To compare the performance of the two incentive schemes under various levels of participation rate, we solve the two optimisation problems with different participation rates ranging from 25% to 100% at 25% increments. Let us define $\gamma_B$ as the difference in total travel times obtained by solving the two optimisation problems relative to the total possible improvements, i.e., the difference between the UE and SO total travel times, under the budget limit $B$. The $\gamma_B$ value is calculated using equation (34), where $T^{P1}$, $T^{P2}$, $T^{UE}$, and $T^{SO}$ represent the total travel times under path incentives, link incentives, user equilibrium, and system optimal flow patterns, respectively. Note that a negative $\gamma_B$ value indicates the higher performance of the path incentive compared to the link incentive under the budget limit of $B$.

$$\gamma_B = \frac{T^{P1} - T^{P2}}{T^{UE} - T^{SO}} \tag{34}$$

Table 4 shows the $\gamma$ values under different budget limits in the ND network. As shown, link incentives never outperform path incentives in terms of total travel time. However, they result in identical total travel times with a low participation rate and high budget limit. In addition, the difference tends to decrease as the budget increases. This demonstrates the effectiveness of path incentives in alleviating congestion using limited resources.

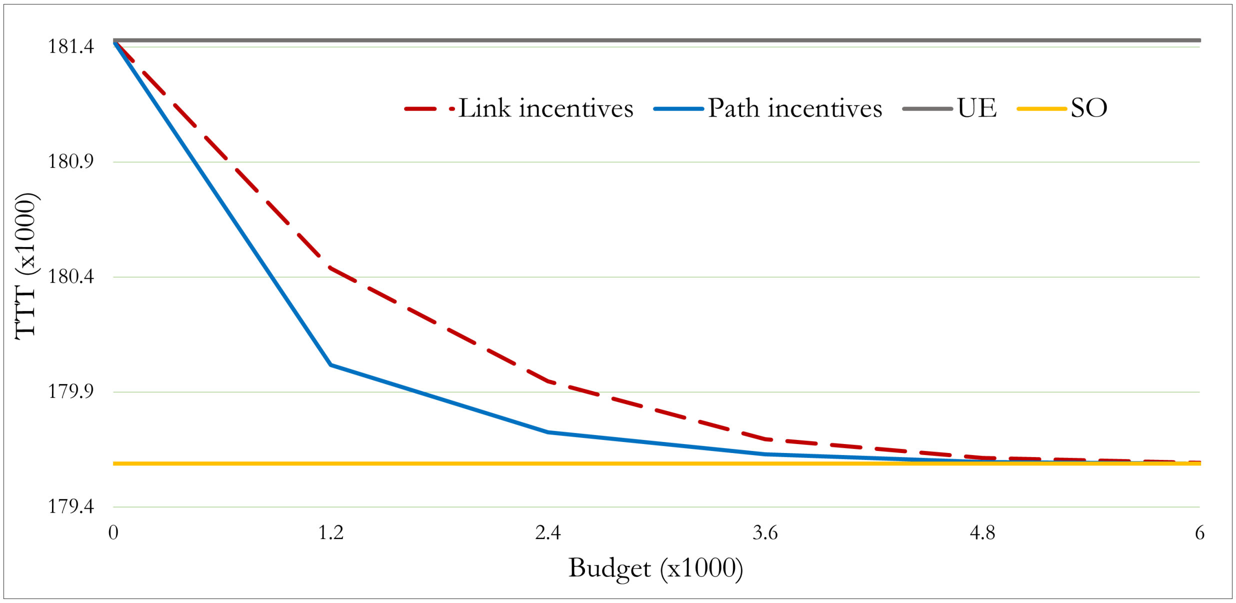


**FIGURE 3**: Total travel time of link- and path-based incentive schemes with different budget limits and 100% participation rate in ND network

**TABLE 4**: Comparing link- and path-based incentive schemes with partial participation in ND network using the $\gamma_B$ indicator

| Budget | Participation rate | | | |
|---|---|---|---|---|
| | 25% | 50% | 75% | 100% |
| 1000 | -26.19 | -33.80 | -27.52 | -29.64 |
| 2000 | -8.39 | -22.77 | -22.35 | -16.23 |
| 3000 | -2.34 | -28.02 | -7.06 | -7.06 |
| 4000 | 0.00 | -6.78 | -1.93 | -1.98 |
| 5000 | 0.00 | -0.05 | -0.07 | -0.77 |

**Sioux Falls network**

In this section, we aim at solving the incentive problems **P1** and **P2** for a larger network to compare the performance of link- and path-based incentives in a realistic traffic network. The SF network consists of 24 nodes and 76 links, as shown in Figure 4. The link volume-delay function in this network follows the BPR function (33), and the details of the links and OD matrix are sourced from (*39*). This network includes 552 OD pairs with a huge number of paths. Thus, the direct approach is not efficient for solving this network. Consequently, we employ only the proposed column generation method, and we then investigate its convergence rate under various incentive strategies and budget limits.

*Link and path incentives under full participation*

As can be seen in Figure 5, for all budget considerations, path incentives consistently outperformed their link-based counterparts. This finding highlights the superiority of the path-based approach in achieving more favorable outcomes.

In order to compare the benefits associated with each incentive scheme, we employ a cost-

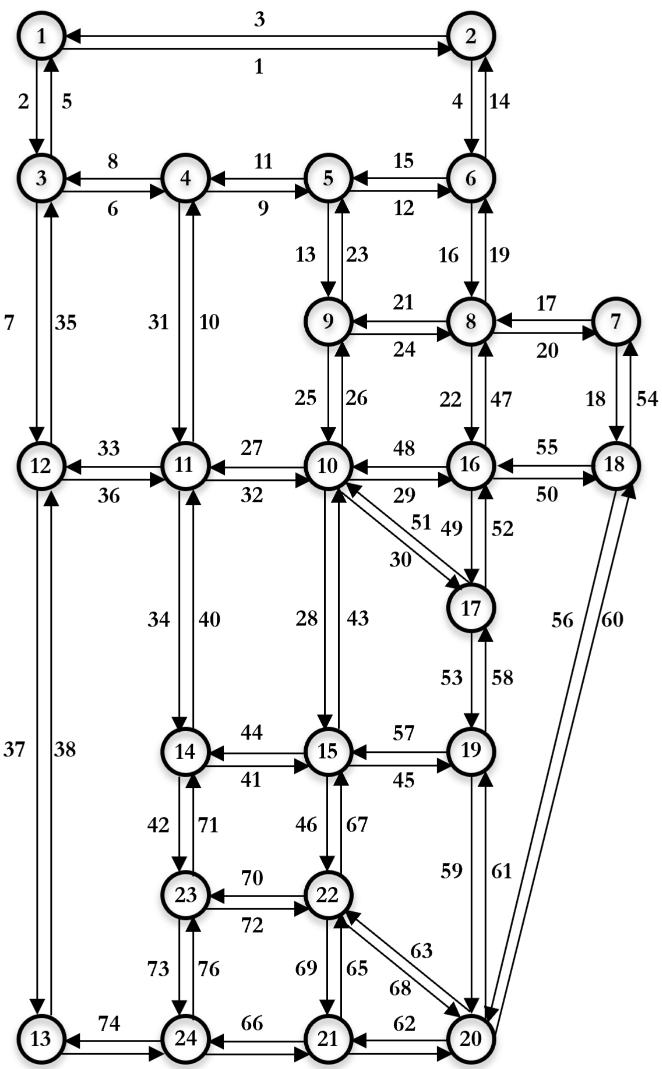


**FIGURE 4**: Sioux Falls network

benefit analysis that can reveal the benefits we gain, i.e., the difference between TTT in a specific case and a benchmark TTT, from the cost that we pay, i.e., the budget allocated to an incentive scheme compared to a benchmark situation, which we define as a B/C index. Figure 6 represents the B/C index in the SF network under both link- and path-based incentives where the benchmark is considered to be the previous budget limit, so the benefit and cost of each incentive scheme are compared to the previous one with a lower budget, i.e., incremental B/C values. According to this figure, when the budget is small, the benefit of investing in the incentive scheme outweighs its associated extra cost as the B/C values are higher than 1. For budgets over 18,000 in path incentives and 10,000 in link incentives, the benefits gained from a specific incentive scheme, relative to the previous budget limit, are insufficient to justify the costs incurred. This case study indicates that the marginal benefits associated with increasing the incentive budget are getting gradually smaller (until negative).

Looking into the convergence of the employed column generation algorithm, we can show

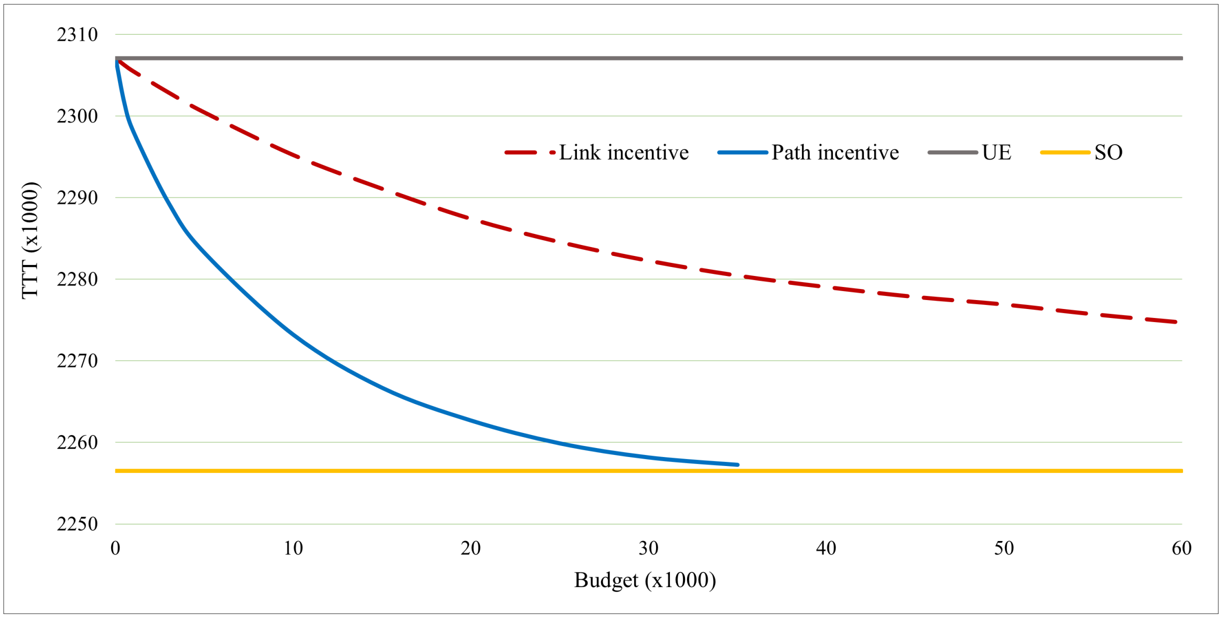


**FIGURE 5**: Total travel time of link- and path-based incentive schemes with various budget limits in Sioux Falls network

the efficiency of the proposed solution method is acceptable. As can be seen in Table 5, under a wide range of budget limits, for both link and path incentives, the algorithm converges after a few iterations with a maximum computation time of 7800 seconds. Still, the proposed algorithm works better for the link incentive problem compared to the path incentive, especially under a small budget limit that converges via 3 steps. However, we enforce the algorithm to iterate 10 times to ensure that the algorithm is not trapped around a local optimum and its solution is stable and consistent across multiple iterations.

*Link and path incentives under partial participation*

Similar to the ND network, the differences in total travel times achieved under link and path incentives are investigated and summarised in Table 6. Link incentives never yielded a smaller total travel time than path incentives. Moreover, the difference increases as the participation rate rises under all budget limits. This is mainly due to the complex layout of the network and the high number of OD pairs, which require more delicate treatment that path incentives can provide.

## CONCLUSION

In this paper, path- and link-based incentives are used to push the user equilibrium flow pattern toward the system optimum in order to minimise total travel time in the network with partial participation of travellers in the incentivising prgram. Wardrop's first principle is applied to the generalised travel time, which is defined as travel time minus link or path incentives. Both incentivising schemes are formulated as non-linear optimisation problems with complementarity constraints and the objective of minimising total travel time. The mathematical properties of the two models reveal that the feasible region of the path-based optimisation problem encompasses that of the link-based problem. Therefore, link incentives cannot yield a smaller total travel time than path incentives. Since generating all paths for a real-sized network is computationally ex-

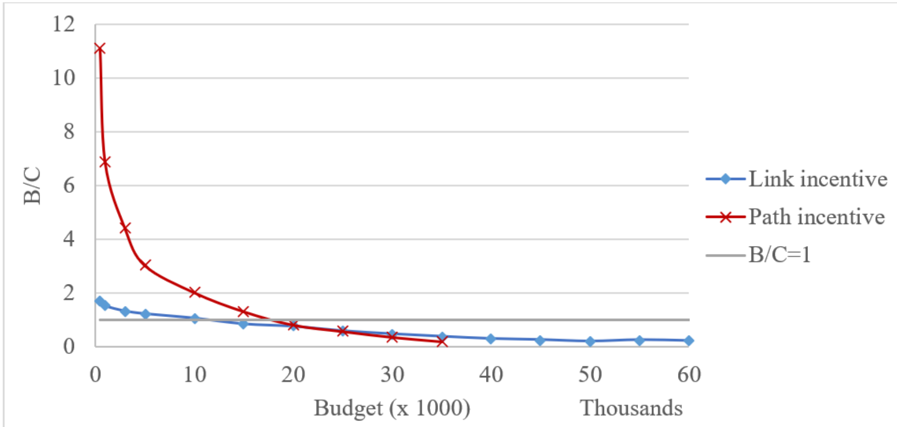


**FIGURE 6**: Cost-benefit analyses for the link- and path-based incentive schemes in Sioux Falls network

**TABLE 5**: The error term, $\bar{\varepsilon}$, computed in each iteration of the proposed column generation algorithm

| Budget (x 1000) | 5 | | 30 | |
|---|---|---|---|---|
| Iteration | Link incentive | Path incentive | Link incentive | Path incentive |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0.742 | 0.756 | 0.762 | 0.783 |
| 3 | 0.006 | 0.005 | 0.008 | 0.006 |
| 4 | 0 | 1.12E-14 | 3.32E-04 | 1.60E-04 |
| 5 | 0 | 0 | 0 | 3.57E-14 |
| 6 | 0 | 0 | 0 | 6.00E-05 |
| 7 | 0 | 0 | 0 | 6.69E-15 |
| 8 | 0 | 0 | 0 | 7.52E-15 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 6.27E-16 |

pensive, an iterative column-generation-based solution technique is proposed that generates a new path between each origin-destination pair at each iteration.

A benchmark analysis is conducted in the Nguyen-Dupuis network to evaluate the accuracy of the column generation-based approach in comparison to a direct approach, where all paths in the network are enumerated prior to solving the optimisation models. The results indicate that the column generation-based approach can successfully provide high-quality solutions without the need to enumerate all paths beforehand. The results of the two optimisation problems in both Nguyen-Dupuis and Sioux Falls networks demonstrate their effectiveness in reducing total travel time across the network under different budget limits and participation rates. Notably, the reduction rate under the path-based incentive is higher than that under its counterpart link-based scheme. Thus, it is advisable to use path incentives, especially when facing tight budget constraints.

This study assumes that all drivers will accept the provided incentivised routes. Additionally, it simplifies the optimisation problems by overlooking the elastic nature of travel demand

**TABLE 6**: Comparing link- and path-based incentive schemes with partial participation in SF network using the $\gamma_B$ indicator

| Budget | Participation rate | | | |
|---|---|---|---|---|
| | 25% | 50% | 75% | 100% |
| 10000 | -13.98 | -20.17 | -37.48 | -43.58 |
| 20000 | -15.34 | -24.10 | -39.15 | -48.92 |
| 30000 | -4.10 | -24.10 | -44.46 | -47.72 |

and the impacts of new technologies, such as connected and automated vehicles. Relaxing these assumptions in future studies would yield more realistic results.

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## AUTHOR CONTRIBUTIONS

The authors confirm their contribution to the paper as follows: Study conception and design: RN, SV, CR, MR; Modeling: RN; analysis and interpretation of results: RN, SV, CR, MR, RC; manuscript preparation: RN, SV, CR, MR, RC;

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