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Proof Load Testing Method by AASHTO and Suggestions for Improvement

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ABSTRACT: Due to the aging of infrastructure, methods are explored by which the reliability of existing bridges and viaducts can be assessed. In case limited information of the structure is available or its condition is of concern, proof load testing may be used to demonstrate sufficient load-carrying capacity. Proof load tests in the USA are typically performed using the Manual for Bridge Evaluation (MBE) published by AASHTO. The proof load is expressed by the regular live-load model magnified by the target proof load factor. The level of reliability obtained using the target proof load factor is not explicitly stated in the MBE, but is of particular interest. In this article relevant background documents are investigated to uncover the underlying calculations, assumptions and input data. Current challenges in proof load testing are described in which the consideration of time-dependence, stop criteria, available information and system-level assessment are highlighted. Subsequently, improvements to the MBE proof load testing background are suggested. An example calculation using traffic data from the Netherlands shows that the HL93 load model and Eurocode LM1 provide a reasonably constant proof load factor with span length for bending and shear. However, the HS20 load model does not scale well with increasing span length. It is found that the magnitude of the target load as specified through the proof load factor is directly related to the desired level of reliability. Although the MBE proof load testing method is practical, several challenges remain.

1. INTRODUCTION

Normally, a structure that has just been completed fulfills functional and safety requirements as specified by the prevailing design standards. But after years of use, the environment or societal demands may have changed (for instance larger traffic intensity or more stringent safety requirements). In addition, the structure may have suffered from degradation. To evaluate if the structure fulfils the requirements, an assessment needs to be carried out (Lantsoght et al. 2017). Proof load testing is one of the methods available for assessment, competing with desk studies that often making use of finite element models.

Recent advances concern the usage of load test information to update finite element models and structural reliability estimations (Alampalli et al. 2021), and are collected in the TRB Circular Primer on Bridge Load Testing (Alampalli et al. 2019). Proof load testing as a means to assess the structural reliability found its way into the literature in the 80s (Grigoriu and Hall 1984; Lin and Nowak 1984; Rackwitz and Schrupp 1985). The probabilistic treatment of proof load testing can result in appropriate target loads depending on the load rating, dead/live load ratios, degradation, bridge age, reference period and prior service loads (Faber et al. 2000). In Europe proof load factors were developed as part of the large scale ARCHES (Assessment and Rehabilitation of Central European Highway Structures) project (Casas and Gómez 2013). Recently, efficient strategies for bridge reclassification based on probabilistic decision analysis have gained attention (Medha et al. 2019; Schmidt et al. 2020; Zhang et al. 2021).

In a proof load test a relatively large load is applied to a bridge or viaduct to demonstrate sufficient load-carrying capacity. If the structure is able to withstand the large load without showing signs of distress, the test is a success. The load can be applied by using one or more heavy trucks (as is common in the USA), a loading frame with ballast, or a specialized load testing vehicle (Figure 1).



Figure 1: The German BELFA load testing vehicle on viaduct Vlijmen-Oost in The Netherlands. Reprinted with permission from (Lantsoght et al. 2017).

The magnitude of the load to be applied in the proof load test is commonly referred to as the *target load*. If the target load could not be reached during the proof load test, because signs of distress were detected, then load posting (load restrictions) may be applied, or the bridge needs to be renovated/replaced. Such decisions depend on the load level reached during the test and the nature of the observed distress.

In case a proof load test is performed in the USA, the Manual for Bridge Evaluation (MBE) (AASHTO 2018) is used as a guideline. This article focusses on the reliability background of the MBE described in the Lichtenstein (1993) report which was also included in the 1998 Manual for Bridge Rating Through Load Testing (NCHRP 1998). Suggestions for improvement of the MBE are provided and current challenges in proof load testing are highlighted – all within the context of structural reliability.

2. RELIABILITY BACKGROUND OF THE MBE PROOF LOAD TESTING METHOD

For a full description of the proof load testing method, the reader is referred to Section 8.8.3 of the MBE (AASHTO 2018). This article is mainly concerned with the calculation of the target load and its relation to structural reliability. Therefore, only the relevant parts of the method are described here.

2.1. Target proof load

Bridge-specific circumstances may be included via the adjusted proof load factor (X_{pA}). The proof load factor is increased or decreased by an associated percentage (see Table 1, which is Table 8.8.3.3.1-1 of the MBE). In case multiple considerations apply, the adjustment percentages are summed. The value of the adjusted proof load factor is calculated via $X_{pA} = X_p(1 + \Sigma\% / 100)$.

The target proof load (L_T) is expressed in terms of the load model and is magnified by an (adjusted) proof load factor, leading to the following expression:

$$L_T = X_{pA} L_R (1 + IM) \tag{1}$$

where L_R is the comparable unfactored live load due to the rating vehicle for the lanes loaded and IM is the dynamic load allowance (or impact). The background to the target proof load factor (X_p) may be found in the 1998 Manual for Bridge Rating Through Load Testing (NCHRP 1998), that references and attaches a report by Lichtenstein (1993). In Chapter 3 of that technical report the default value $X_p = 1.4$ is derived from a probabilistic analysis.

Table 1: Adjustment to target proof load (AASHTO 2018).

Consideration	Adjustment	
One-lane load controls	+15%	
Nonredundant structure	+10%	
Fracture-critical details present	+10%	
Bridges in poor condition	+10%	
In-depth inspection performed	-5%	
Ratable, existing $RF \ge 1$	-5%	
<i>ADTT</i> ≤ <i>1000</i>	-10%	
<i>ADTT</i> ≤ <i>100</i>	-15%	

2.2. Probabilistic analysis

In Lichtenstein (1993) a simply supported bridge with span of l = 60 ft (18.3 m) is considered as a base case. The limit state function adopted for the probabilistic calculation is:

$$Z = R - (D + L + I) \tag{2}$$

where R is the resistance of the structure, D is the dead load (permanent load), L is the live load, and I is the impact load (dynamic load effect).

The resistance and dead load are regarded as deterministic values in the calculation. It is assumed their values are known after the proof load test. The value of the dead load effect is taken equal to the AASHTO HS20 live load effect, i.e. $D = L_A = 807$ kip·ft per lane. The effect of different dead load contributions in relation to the live load is studied in Lichtenstein (1993) as well. The live load (L) and impact load (I) are incorporated in the model as normally distributed random variables.

The mean value of the 75-year maximum traffic load is equal to 1.79 times the HS20 load effect (L_A) as determined by extrapolation of a traffic survey (Nowak 1993). For a reference

period of two years, it is 1.65 times the HS20 load effect. If two lanes are considered, a reduction factor of 0.85 applies due to expected redistribution of the load between lanes. The live load has a coefficient of variation of 0.18. The dynamic load allowance is estimated to be about 0.1 of the live load, with a coefficient of variation of 0.8.

If the proof load test was successful, the resistance of the structure is at least equal to the sum of the target proof load (L_T) and the dead load. In the probabilistic analysis the mean resistance of the structure is calculated as:

$$R = 1.12(L_T + D)$$

$$= 1.12 \left[X_p L_A (1 + C_{I,A}) + D \right]$$
(3)

where the factor 1.12 is thought to account for higher mean strengths in respect to the nominal (or 'design') strengths used in code regular calculations. The value of the AASHTO impact coefficient for 60 ft (18.2 m) is $C_{I,A} = 50 / (60 + 125) = 0.27$.

The reliability index (β) that results from the probabilistic calculation for various values of X_p is provided in Table 2. The values in the column *Recalculated* have been obtained by the author and are in correspondence with the original numbers. The value $X_p = 1.4$ was selected because a reliability index of $\beta = 2.3$ was found to be in line with the operating level according to the AASHTO Load and Resistance Factor Design (LRFD) studies (AASHTO 2020). Additional calculations in relation to varying span lengths provided similar results (Lichtenstein 1993).

Table 2: Calculated reliability with increasing proof load factor.

Dwoofload	Reliability index for 2 years (β)		
Proof load factor (X_p)	Lichtenstein	Recalculated	
	(1993)		
1.2	1.26	1.26	
1.3	1.89	1.89	
1.4	2.57	2.52	
1.5	3.15	3.15	
1.6	3.78	3.77	

3. SUGGESTED IMPROVEMENTS TO THE MBE PROOF LOAD TESTING BACKGROUND

In this section various improvements are suggested to be incorporated in the background of the MBE proof load testing method. It does however not result in a ready-to-be adopted new format. Instead, the most important facets are highlighted and improvements are suggested.

3.1. Probabilistic model

In Lichtenstein (1993) the dead load is treated as a deterministic value equal to the live load. The value of the dead load (effect) is not exactly known, nor does it need to be. When including the dead load as a random variable, it can also be eliminated from the limit state function – as shown in Eq. (4).

An additional factor of 1.12 is used in Eq. (3) to convert from nominal to mean strength. Such a factor is appropriate when R is a design or nominal strength. However, here R is a random variable. After a successful proof load test, it is known that the resistance must be equal to or larger than the load effect following from self-weight and the target load ($R \ge D + L_T$). Assigning the resistance with a value that is 12% higher than obtained from the test is speculative.

With the suggested alterations the limit state function may be rewritten such that only the liveload and the dynamic amplification remain as random variables. In essence the probability of failure of the structural part or cross-section is directly reformulated into the probability that a future live-load effect (including dynamic amplification) exceeds the load effect produced during the proof load test:

$$Z = R - (D + L + I)$$

$$= (D + L_T) - (D + L + I)$$

$$= L_T - (L + I)$$
(4)

Here L_T is the target proof load-effect (deterministic value), L is the traffic live-load effect (random variable) and I is the dynamic contribution (random variable).

The dynamic load effect (impact) should be included in the target load (L_T) as part of the load model via the regular design procedure. Since comparable extreme values for the traffic load are considered, the design procedure to account for the dynamic loads is suitable here as well. Therefore, the impact (I) may be removed from the limit state function. In this way, the probabilistic analysis can be performed using recorded traffic loads without, or with minimal, dynamic contribution (e.g. WIM data).

Missing in the limit state function of Eq. (4) are model uncertainties. Our understanding of the translation from applied loads, in a test or from actual traffic, to the load effect is limited. The degree of uncertainty depends on the level of sophistication incorporated in the mechanical model. Additional uncertainty stems from the statistical modelling of the load effect – i.e. the assumed distribution functions. The variability of the traffic load may be split into a time-invariant (C_{0L}) and time-variant part (L) (fib 2016). By including model uncertainties, splitting the liveload variability and removing the dynamic contribution, the limit state function becomes:

$$Z = \theta_{IT} L_T - \theta_I C_{0I} L \tag{5}$$

An overview and description of the parameters in the suggested limit state function is provided in Table 3 .

The statistical properties of the random variables are based on general recommendations for probabilistic modelling (fib 2016; JCSS 2015). The coefficient of variation of the model uncertainty concerning the load effect produced in the proof load test (θ_{LT}) is based on the value of the model uncertainty related to the traffic load. Because the conditions are more controlled during a test, a lower value may seem more appropriate. However, when viewed as a resistance parameter, it should also cover the uncertainty associated with selecting the most critical locations to test.

3.2. Traffic load

In Lichtenstein (1993) the statistical description of the live load and impact (dynamic) load is based on Nowak (1993). It is recommended to use

Var.	Description	Distribution	Mean (m)	CoV (V) [-]
$ heta_{LT}$	Model uncertainty load effect produced in	Lognormal	1	0.1
	the test			
L_T	Load effect caused by proof loading	-	(varies)	-
	vehicle or frame			
$ heta_L$	Model uncertainty load effect produced	Lognormal	1	0.11
	by the traffic load			
C_{0L}	Time-invariant part of the traffic load	Lognormal	1	0.1
	variability			
L = M	Annual maximum of the traffic load effect	Gumbel	300-16000 kip:ft	0.02-0.09
	(moment, varies with span length)			
L = V	Annual maximum of the traffic load effect	Gumbel	50-400 kip	0.03-0.10
	(shear, varies with span length)			

Table 3: Overview of variables included in the suggested limit state function.

more recent data, preferably obtained from the measurement of axle loads at multiple locations and for a longer period of time (e.g. one year or more). Weight-in-motion (WIM) data is well-suited to obtain an accurate statistical representation of the traffic load effect.

In the Netherlands, WIM recording stations are positioned at several traffic-intense highway locations. Using WIM data from 2015 in the Netherlands, traffic simulations have been performed to obtain the maximum bending moment at midspan and the maximum shear force near the supports of a simply supported span. The Gumbel extreme value distribution is fitted to the data using the maximum likelihood estimation (MLE) method. A threshold value is chosen (probability of exceedance S = 0.25) to capture the, on the log-scale, linearly descending right tail of the distribution. Figure 2 shows the fitted distribution to the data points of the maximum bending moment of Dutch highway A27L lane 1 - the rightmost lane mostly occupied by trucks.

Because the weekly maxima are sufficiently uncorrelated, the Gumbel distribution may be converted to annual maxima by shifting the location parameter (μ) via $\mu_a = \mu_w + \beta_G \ln(52)$ where β_G is the scale parameter and 52 is the number of weeks in a year. Distributions have been fitted for various WIM datasets and span lengths. The analyzed roads shown a comparable

trend in mean and coefficient of variation with span length.

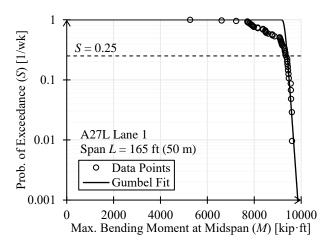


Figure 2: Gumbel fit of the load effect data points for the maximum bending moment at midspan of a simply supported span. Conversion factors: 1 ft = 0.305 m, $1 \text{ kip} \cdot \text{ft} = 1.35 \text{ kNm}$.

3.3. Influence of span length

The configuration of a bridge that is subjected to a proof load test is often different than the simply supported span for which the load effect was calculated, and the reliability analysis was performed. To overcome this limitation the target proof load is related to a load model via the proof load factor, see Eq. (1). In Lichtenstein (1993) the HS20 load model is used, but today the HL93 load model (AASHTO 2020) describes the traffic better. In addition to an HS20 truck or a (military)

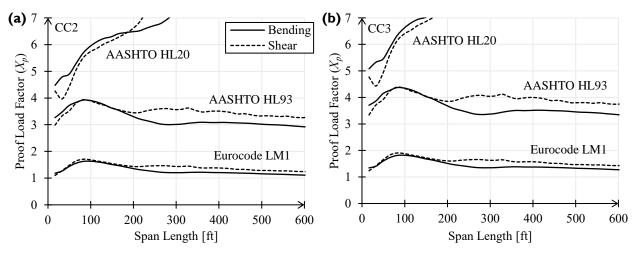


Figure 3: Relation between span length and target proof load factor considering unfactored load models in bending and shear: (a) consequence class 2 with annual reliability requirement $\beta = 3.4$ and (b) consequence class 3 with annual reliability requirement $\beta = 4.0$. Conversion factor: 1 ft = 0.305 m.

load tandem, the latter also includes a lane load (distributed load) that represents the other traffic present on the bridge. The HL93 load model is comparable to the Eurocode LM1 specification but has significantly lower loads. In Nowak et al. (2010) it is found that the Eurocode LM1 load effects are about a factor two higher than calculated using AASHTO HL93, owing to the higher unfactored loads in the traffic model. After applying (partial) factors the design load effect varies from country to country.

To study the relation between the span length and the target proof load factor, an example made with calculation is the improved probabilistic model and traffic data from the Netherlands. Per span length two probabilistic analyses are performed: one considering the bending moment at midspan, and one considering the shear force near the supports. Consequence class 2 and 3 of EN 1990 (CEN 2019) are considered with target annual reliability indices of 3.4 and 4.0, respectively. The distributed load of the Eurocode LM1 and the AASHTO HL93 load model are applied over a lane width of 3 m. The proof load factors following from the reliability analyses are displayed in Figure 3.

It is observed that the target proof load factor is considerably larger when using the AASHTO HS20 and HL93 load models in comparison to

Eurocode LM1. This follows from the relatively high unfactored load effect following from LM1. Because of the large discrepancy between unfactored load models, it is recommended to cautiously evaluate traffic models and statistical descriptions for application within the USA.

Another observation is the continuously increasing proof load factor with span length when the HS20 load model is used. This is because the load model only includes a single truck, whereas in reality many vehicles may be present on the bridge. The issue is overcome by the HL93 load model which also includes a distributed lane load. For both the Eurocode LM1 and the HL93 load model an almost constant factor is obtained over various spans. Only around 100 ft (30 m) a relatively large factor is required. This may be explained by the occurrence of long and heavy vehicles (an oversize load for which usually an exemption must be requested) that are not accurately represented by the load model.

4. DISCUSSION

4.1. Suggested Improvements

The suggested probabilistic model includes model uncertainties for both the actual live load (θ_L) and the live load produced in a proof load test (θ_{LT}) . Their statistical description has been

estimated and requires further refinement. Especially the uncertainty associated with the proof load test will need to cover different aspects depending on the application: how is the load applied, how many positions and lanes are tested, is bending or shear critical, etc. Because there are several remaining challenges, the probabilistic model and the results presented in this article should be viewed as indicative.

The traffic load analysis was performed using highway measurements obtained in the Netherlands; therefore, the resulting distributions have a limited applicability. By using the method followed in this article applicable distributions can be derived for different countries. For completeness also other configurations besides the single span case need to be considered (Casas and Gómez 2013).

In the MBE an adjustment to the target live-load of +15% is suggested in case one-lane load controls the load effect. This is a measure to counteract the more favorable two-lane traffic load description. An important assumption in the two-lane situation is that the bridge is able to redistribute the traffic load between its lanes. This is not always the case. The use of a multiple presence factor (MPF) calibrated on the basis of WIM data is recommended (Fu et al. 2013).

4.2. Remaining Challenges

Proof load testing is a valuable tool to demonstrate sufficient load-carrying capacity. However, the derivation of factors and rules to carry out a test that results in the desired reliability remains challenging.

The time-dependence of the structural reliability can be incorporated into a probabilistic analysis directly to deliver the point in time where the annual reliability is not sufficient anymore. An example of such a calculation is provided in De Vries et al. (2022). A future framework for proof load testing should be flexible in terms of which information is utilized. In some cases, bridge documentation, material data, traffic data or even proof load testing data on similar bridges may be available. Additional rules may be established on the basis of Bayesian inference to utilize

knowledge about the structure and its context (traffic loads, environment, geographical location, etc.).

By thinking about the bridge as a system of components, the question is how many components are tested in a proof load test. This may also depend on the type of bridge or failure mechanism being considered. In a (successful) proof load test, one can only observe that the system (i.e. the entire structure) carries the load. However, the load may not follow the expected load path and/or redistribution of forces can take place. For this reason, the proof load test result does not necessarily tell us something about the performance of a component.

5. CONCLUSION

The magnitude of the target load is directly related to the desired level of reliability. In the MBE (AASHTO 2018) the target load is obtained through application of the live-load model multiplied by a factor for proof load testing that can also include bridge-specific adjustments (X_{pA}). In this way, the target proof load can be easily calculated for any bridge or viaduct under consideration. The background report by Lichtenstein (1993) was studied to uncover the underlying probabilistic model. The calculations resulting in the basic value of $X_p = 1.4$ as used in the MBE have been reproduced with success.

Although a method based on the probabilistic analysis of live-load alone (such as the MBE method) is practical, several challenges remain: influence of time-dependent effects, reliability of stop criteria, usage of information about the structure and the importance of system-level assessment. Verifying the reliability of a bridge or viaduct through proof load testing is markedly different from the design process.

The main idea behind the MBE method (i.e. the resistance is at least equal to the self-weight and the applied live-load in the test) remains valuable and therefore suggestions for improvement have been provided. In summary, the improvements entail including model uncertainties in the probabilistic model, updating the traffic load description and adopting the

appropriate live-load model. With a probabilistic analysis it was shown that live-load models HL93 and Eurocode LM1 for both bending and shear can provide reasonably constant proof load factors over a large range of span lengths.

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7. REFERENCES

- AASHTO (2018). "The manual for bridge evaluation." *Standard, 3rd Edition. Washington, D.C., USA.*
- AASHTO (2020). "LRFD Bridge Design Specifications." *Standard*, *9th edition*.
- Alampalli, S., Frangopol, D. M., Grimson, J., Halling, M. W., Kosnik, D. E., Lantsoght, E. O. L., Yang, D. Y., and Zhou, E. (2021). "Bridge Load Testing: State-of-the-Practice." *Journal of Bridge Engineering*, 26(3).
- Alampalli, S., Frangopol, D. M., Grimson, J., Kosnik, D., Halling, M., Lantsoght, E. O. L., Weidner, J. S., Yang, D. Y., and Zhou, Y. E. (2019). "Primer on Bridge Load Testing." *Transportation Research Board, TRB Circular E-C257*.
- Casas, J. R., and Gómez, J. D. (2013). "Load rating of highway bridges by proof-loading." *KSCE Journal of Civil Engineering*, 17(3), 556-567.
- CEN (2019). "Eurocode 0: Basis of structural design." Standard, EN 1990+A1+A1/C2:2019.
- De Vries, R., Lantsoght, E. O. L., Steenbergen, R. D. J. M., and Fennis, S. A. A. M. (2022). "Reliability assessment of existing reinforced concrete bridges and viaducts through proof load testing." *Proceedings to the IABMAS conference, Barcelona, Spain.*
- Faber, M. H., Val, D. V., and Stewart, M. G. (2000). "Proof load testing for bridge assessment and upgrading." *Engineering Structures*, 22, 1677-1689.
- fib (2016). "Partial factor methods for existing concrete structures." *Fédération internationale*

- du béton, Bulletin 80, Recommendation, Task Group 3.1.
- Fu, G., Liu, L., and Bowman, M. (2013). "Multiple presence factor for truck load on highway bridges." *Journal of Bridge Engineering*, 18(3), 240-249.
- Grigoriu, M., and Hall, W. B. (1984). "Probabilistic models for proof load testing." *Journal of Structural Engineering*, 110(2), 260-274.
- JCSS (2015). "Probabilistic model code."
- Lantsoght, E. O. L., van der Veen, C., de Boer, A., and Hordijk, D. A. (2017). "State-of-the-art on load testing of concrete bridges." *Engineering Structures*, 150, 231-241.
- Lichtenstein, A. G. (1993). "Bridge rating through nondestructive load testing." *Technical Report, NCHRP Project 12-28(13)A.*
- Lin, T. S., and Nowak, A. S. (1984). "Proof loading and structural reliability." *Reliability Engineering*, 8, 85-100.
- Medha, K., Schmidt, J. W., Sørensen, J. D., and Thöns, S. (2019). "A decision theoretic approach towards planning of proof load tests." *13th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP)*.
- NCHRP (1998). "Manual for bridge rating through load testing." *Transportation Research Board* (TRB), Research Results Digest, No. 234.
- Nowak, A. S. (1993). "Live Load Model for Highway Bridges." *Structural Safety*, 13(1-2), 53-66.
- Nowak, A. S., Lutomirska, M., and Sheikh Ibrahim, F. I. (2010). "The development of live load for long span bridges." *Bridge Structures*, 6(1), 73-79.
- Rackwitz, R., and Schrupp, K. (1985). "Quality control, proof testing and structural reliability." *Structural Safety*, 2, 239-244.
- Schmidt, J. W., Thöns, S., Kapoor, M., Christensen, C. O., Engelund, S., and Sørensen, J. D. (2020). "Challenges related to probabilistic decision analysis for bridge testing and reclassification." *Frontiers in Built Environment*, 6.
- Zhang, W.-H., Lu, D.-G., Qin, J., Thöns, S., and Faber, M. H. (2021). "Value of information analysis in civil and infrastructure engineering: A review." *Journal of Infrastructure Preservation and Resilience*, 2.