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Delft Aerospace Computational Science

Development of a constitutive model for self-healing materials

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Table of Contents

Preface

This report is the result of a pilot-study towards the numerical simulation of self-healing materials. At the moment this project is conducted the development of self-healing materials is in an early stage. At the chair of Engineering Mechanics of the faculty of Aerospace Engineering, Delft University of Technology, no numerical model has been developed yet for this new kind of materials. This project is a first exploration of the possibilities to simulate the behaviour of self-healing materials by numerical models.

This exploration has been done by developing a constitutive law that adequately describes the mechanical behaviour of these materials. Subsequently this new constitutive law has been tested in a 1-dimensional test case and in a finite element setting.

1. Introduction to the concept of self healing materials

A self-healing material is a material that has the ability to recover its strength, stiffness and fracture toughness after a crack has occurred. Each material can have its own methodology to accomplish the self-healing capability. The development of self-healing materials is at an early stage and at present only a small range of materials with self-healing capabilities exists.

A natural example of a material with self-healing capabilities is concrete. When the several components of concrete have not been mixed with the right ratio or when the mixing has not been executed perfectly, lumps of unhydrated material still exist in the hardened concrete. When a crack opens and contacts humid air, hydratation of the unhydrated material can occur and the crack is repaired.

An example of a human-created self-healing material can be found by looking at a polymer composite developed by the University of Illinois [1]. Two types of microcapsules, containing either a healing agent or a catalyst, are embedded in the polymer material. When a crack meets a healing agent capsule, the stress concentration at the crack tip causes rupture of this capsule and the healing agent is released into the crack by capillary action. When the healing agent contacts the catalyst, polymerization occurs which binds the two crack faces, see Fig. 1.

It is noted that in both examples the healing agent is embedded in the material and is hereby situated local to each occurring crack. Furthermore the self-healing process resembles in both cases and can be separated into three steps. A crack forms in the matrix (step 1). The crack initiates the self-healing process, healing agent is released into the crack by means of a transport process and it contacts a catalyst (step 2). The catalyst causes the healing agent to cure and mechanical properties are (partially) recovered (step 3). The constitutive model to be developed in this report will adhere to the above described process. The first step can be simulated by existing numerical models. This report will focus on the simulation of the third step.

Current efforts in the field of self-healing materials are aimed at working out these concepts and the development of new self-healing concepts. The concept of a microcapsule containing a healing agent is probably the most promising at this moment since the idea is relatively simple and easy to implement. Possible future developments are healing agents that find their way to the crack by means of a diffusive process.

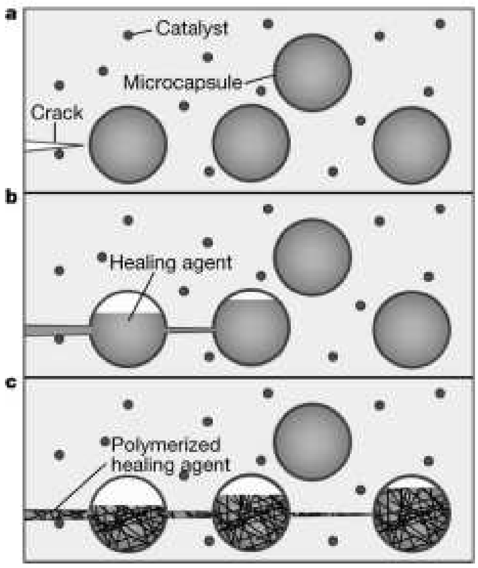

Figure 1: Illinois' self-healing concept for polymer materials [1].

2. Aim of the project

The project aims at creating a constitutive model that has the ability to simulate the self-healing capability of a material. The current class of damage models describes the degradation of mechanical properties after some stress threshold has been passed. The self-healing capability now will come as an extension to the damage model; the mechanical properties of the material that have been degraded due to the pass of a stress threshold are recovered after the self-healing capability has been initiated.

The new model is aimed to suit a large variety of materials, e.g. concrete and polymers. The exact healing behaviour (described by a so called healing function) can be chosen appropriately to the material concerned.

The self-healing constitutive model is developed already at this early stage of the development of the self-healing concept; the development of the (physical) material itself has hardly started yet. Before the self-healing materials themselves have been developed, simulations can already be made with a self-healing material finite element model. In this way the development of a self-healing material and the simulation of its properties and behaviour can be done simultaneously. The development of a self-healing material can now benefit from finite element simulations. A finite element analysis could point out how the different parameters for a new to develop material should be chosen to make it effective for its purpose.

The assumptions made for this project are listed below.

1. The constitutive model is 1-dimensional (Mode I).

2. The constitutive model allows healing only once.

3. As mentioned in chapter 1 the self-healing process will be assumed to consist of the following three steps: a crack forms in the matrix (step 1), the crack initiates the self-healing process, healing agent is released into the crack by means of a transport process and it contacts a catalyst (step 2) and the catalyst causes the healing agent to cure (step 3). The current class of damage models describes the first step. The extension of the current class of damage models, developed in this report, will describe the third step; the curing of the material and the subsequent recovery of mechanical properties. The second step is not considered in this project. It is assumed that this step took place without any problems.

   The second step determines if and when the self healing capability is initiated, if the healing agent is distributed well through the crack, if it does contact the catalyst and if curing does not start before the healing agent has been distributed all through the crack. Mechanical theory in combination with fluid dynamics theory will be needed to simulate this second step. To circumvent the missing of the second step the start of the third step is initiated by a time-input t_r which represents the moment that healing agent and catalyst contact and consequently curing is started.

3. Development of self-healing constitutive model (Mode I)

In this chapter the new constitutive model, including self-healing capability, will be developed. In the first section the existing class of damage models is considered, since the new model will be based on this model. In the second section the new constitutive model is developed. The constitutive model of the first section will be extended here. In the third section the properties of the new constitutive model are considered.

3.1 Description of current class of damage models

The new constitutive model that is developed in this project will be an extension of the existing damage model (see e.g. [2]). This original damage model is the subject of this section. Since the new constitutive law to be developed will be 1-dimensional (Mode I), the treatment of the damage model will also be limited to 1 dimension.

Existing damage models describe the degradation of material properties as result of a crack. This degradation of mechanical properties is expressed by a damage parameter d. Different approaches exist for the implementation of this damage model in a finite element code. The two approaches most used are the continuum damage model and the discrete damage model. The continuum damage model smears out the crack; the crack is considered to be a continuum with degrading mechanical properties. The discrete damage model lumps the degrading mechanical properties into a cohesive zone; an interface element. The interface elements have an initial thickness of zero and are introduced in between of the continuum elements that model the bulk material. When an ultimate stress level has been exceeded, a discrete crack appears.

The concept for simulating the self-healing capability of self-healing materials, as developed in the next section of this report, can be applied to the complete class of damage models as described above. In the remainder of this report only the most commonly used damage model, the discrete damage model will be considered however.

The discrete damage model is described as follows. As long as the ultimate stress level τ_{ult} is not exceeded the interface elements have an elastic dummy stiffness K. After the ultimate stress level τ_{ult} has been exceeded the constitutive law of the interface element is described by the following formula:

$$\tau = (1 - d)Kv \tag{3.1}$$

where the parameter K is the dummy stiffness, d is the damage and v is the crack opening, also called jump in a finite element consideration. The damage d is a function of evolution of the crack opening v, i.e. $d = d(\text{history of } v)$. This dependency of d on the history of jump v is implemented as follows. An intermediate parameter κ_{eq} is introduced, which is a function of jump v and forms a measure of this jump v. A history parameter $\kappa_u = \max_{\tilde{t} \leq t} \kappa_{eq}(\tilde{t})$ stores the largest κ_{eq} obtained at time t. The damage d is a monotonic increasing function of κ_u with $lim_{\kappa_u \to \infty} d = 1$. In Fig. 2 a typical plot of traction vs. jump for the damage model is shown. For this plot the jump is monotonically increasing. The damage law used is as follows:

$$d = 1 - exp^{-k(\kappa_u - \kappa_0)} \tag{3.2}$$

It is noted that the term $(1 - d)K$ in eq. (3.1) can be considered as the effective stiffness after τ_{ult} has been exceeded and this effective stiffness decreases as the crack opening increases. Secondly it is noted that in reality the dummy stiffness K should be infinite before τ_{ult} is exceeded, since the crack should not open before τ_{ult} is exceeded. This however would give computational problems (ill-conditioning of the global matrix and stress oscillations). Therefore the interface element has been given a dummy stiffness of the same order as the stiffness of the continuum elements. The small opening of the interface element that occurs because of this reduced stiffness before τ_{ult} is exceeded gives a small inaccuracy which is permitted in order to have numerical stability.

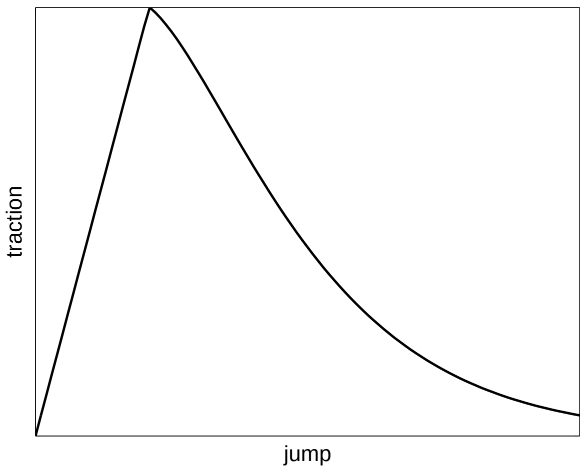

Figure 2: Traction plot illustrating the damage model.

3.2 Development of self-healing constitutive model

This section will describe the extension of the original damage model. This will result in a new constitutive law for materials with self-healing capability, eq. (3.7).

A typical development in time for the interface element to develop is as follows. The interface element undergoes a certain load path or displacement path, causing damage to the interface element.

At a certain time t_r (indicating that healing agent and catalyst have contacted and step 3 starts) curing of the healing agent is initiated. The damage of the interface element at this instant t_r is called d_r. This parameter d_r is the fraction of stiffness that has been lost at t_r. The self-healing model has been created such that this part of the stiffness will be recovered up to a factor r, e.g. $r = 0.75$.

The recovery of stiffness happens gradually as a function of time and is specified by the healing function $h(t)$, where $h(t)$ equals zero if $t = 0$ and $\lim_{t\to\infty} h(t) = r$. This healing function can be chosen appropriate to a particular healing agent. An example of a healing function with $r = 1$ can be found in Fig. 3. The formula for this function reads as follows:

$$h(t) = r\left(\frac{1}{\left(\frac{1}{2}+\frac{1}{2}p\right)\pi}\right)\left(\arctan\left(\frac{2\tan\left(\frac{p\pi}{2}\right)}{t_k}(t-0.5t_k)\right)+\frac{p\pi}{2}\right) \tag{3.3}$$

where

$$p = \frac{k}{2-k}$$

The parameter k is the percentage chosen as explained below, e.g. $t_k = t_{0.975}$. Specific parameters for the healing function are the maximum recovery r and the heal time $t_{0.975}$ that it takes to achieve 97.5 per cent of the maximum recovery (each other percentage can be chosen here).

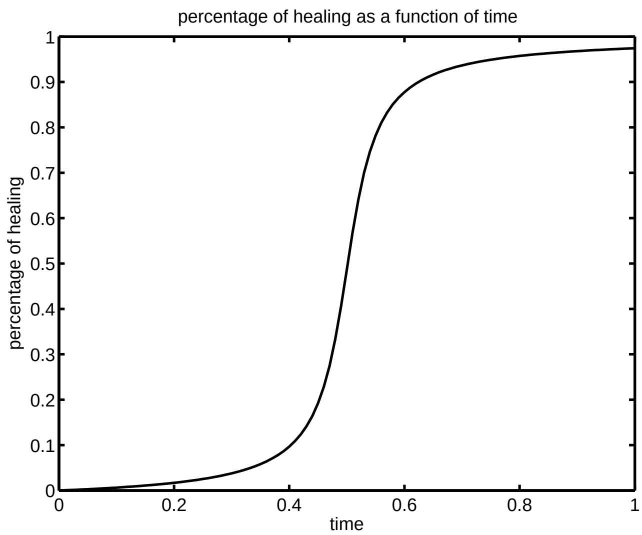

Figure 3: Heal function with $r = 1$ and $t_{0.975} = 1s$.

We divide up the stiffness in two contributions p_1 and p_2, where $p_2 = d_r$ (the part of the stiffness that has been lost at t_r and will be recovered) and $p_1 = 1 - d_r$ (the part of the stiffness that has not been lost at t_r). These two parts have stiffness C_1 and C_2 respectively and both contribute to the total stiffness C:

$$p_1C_1 + p_2C_2 = C \tag{3.4}$$

Remember that the stiffness of the undamaged material equals the value K. With this notation we can follow the development of the stiffness of both parts and especially focus on the stiffness of p_2. At the instant t_r that healing starts, p_1 has by definition its full stiffness, $C_1 = K$. The stiffness contribution from p_2 at this moment is by definition zero, $C_2 = 0$, since this is the part that has lost its stiffness at t_r. As time continues after t_r the contribution of the stiffness of part p_1 keeps following the usual damage laws. Its behaviour is indifferent from when there would be no healing process and the original damage model would be used instead. The contribution of part p_2 starts to increase from zero after t_r. This stiffness contribution follows the aforementioned healing function and so $C_2 = h(t)K$. If no new damage occurs $\lim_{t\to\infty} C_2(t) = rK$

When damage is repaired by means of a healing agent there is an addition of material. This is incorporated by the model. Consider a micro crack at time t_r that has an opening v_r at this instance, i.e. $v_r = v(t_r)$. This opening is filled with healing agent at t_r. The added material causes the equilibrium jump to shift (the equilibrium jump is the jump v for which the stress τ is zero). The parameter v_r is found back in the stress-strain equation at the stress contribution of p_2. See e.g. eq. (3.5).

If no more damage would occur after t_r the traction as a function of the Mode I displacement would be:

$$\tau = p_1 C_1 v + p_2 C_2 (v - v_r) \tag{3.5}$$

Substituting $p_1 = (1 - d_r)$, $p_2 = d_r$, $C_1 = K$ and $C_2 = h(t)K$ gives:

$$\tau = (1 - d_r)Kv + d_r f(t - t_r)K(v - v_r) \tag{3.6}$$

In general the interface element will obtain more damage after t_r. Therefore formula (3.6) has to be adapted so to include the occurrence of this increase of damage. This then results in the following equation:

$$\tau = (1 - d_1)Kv + d_r f(t - t_r)(1 - d_2)K(v - v_r) \tag{3.7}$$

where

$$d_1 = d_1\Big(\kappa_{u,1}(v)\Big)$$

$$d_2 = d_2\Big(\kappa_{u,2}(v - v_r)\Big)$$

where $\kappa_{u,1}$ and $\kappa_{u,2}$ correspond to part p_1 and p_2 respectively. Note that the damage parameter d_1 is the damage parameter that already existed before healing started. Its behaviour as a function of $\kappa_{u,1}$ is unaltered by the start of the self healing process. Note that $d_1(t_r) = d_r$. The damage parameter d_2 arises when healing starts. Its initial value is zero. It is noted that part p_2 can have another damage function as the original material, since the healing agent could have other properties (another τ_{ult} and another fracture toughness G_c) than the original material.

3.3 Properties of the self-healing constitutive model

The properties of the constitutive model for materials with self-healing capability, expressed by eq. (3.7), are considered. Important properties of the constitutive model are the dummy stiffness and maximum stress level τ_{ult} of the virgin material, its fracture toughness and the damage. For a finite element analysis the consistent tangent is important.

3.3.1 Dummy stiffness and maximum stress level

The dummy stiffness of the virgin material, which is applicable before damage occurs, as well as its maximum stress level, both are unaltered when the existing damage model is replaced by the self-healing constitutive model.

3.3.2 Fracture toughness

When damage occurs, the fracture toughness (defined as the deformation energy the interface element can take before fracture occurs) decreases. The self-healing capability of the material however recovers the fracture toughness. We consider the remaining fracture toughness at an instant of time and denote it by $G_c(t)$. The fracture toughness of the virgin material is denoted by $G_{c,1}$ and the fracture toughness of part p_2 is denoted by $G_{c,2}$. This value in general differs from $G_{c,1}$, since the healing agent can have other properties than the bulk material. When the fracture toughness $G_c(t)$ at t_r has decreased to $G_{c,1}(t_r)$, the self-healing process recovers the fracture toughness G_c up to:

$$G_c(t) = G_{c,1}(t_r) + d_r f(t - t_r) G_{c,2} \tag{3.8}$$

where it is assumed that no damage occurs between t_r and t. The maximum fracture toughness that can be obtained when time goes to infinity and no new damage occurs amounts:

$$G_c(t) = G_{c,1}(t_r) + d_r r G_{c,2} \tag{3.9}$$

3.3.3 Damage

Now the material model has been modified, it is good to repeat the definition of damage. It is defined as the fraction of stiffness that has been lost. The damage for the extended damage model, expressed by eq. (3.7), is evaluated by the following formula:

$$\begin{aligned} d = d_1 \qquad & t \le t_r \\ d = d_1 - d_r \cdot h(t) \cdot (1 - d_2) \qquad & t \ge t_r \end{aligned} \tag{3.10}$$

This formula for damage can be verified by using the definition of damage in combination with eq. (3.7):

$$\begin{aligned} d &= 1 - \left(\frac{\frac{d\tau}{dv}\big|_{\text{no damage increase}}}{K} \right) \\ &= 1 - \frac{(1-d_1)\cdot K + d_r \cdot h(t) \cdot (1-d_2) \cdot K}{K} \\ &= d_1 - d_r \cdot h(t) \cdot (1 - d_2) \end{aligned}$$

3.3.4 Consistent tangent

The consistent tangent can be calculated from eq. (3.7) and equals:

$$\frac{\mathrm{d}\tau}{\mathrm{d}v} = (1-d_1)K + \frac{\mathrm{d}d_1}{\mathrm{d}\kappa_{u1}}\frac{\mathrm{d}\kappa_{u1}}{\mathrm{d}v}Kv + d_r h(t)(1-d_2)K + d_r h(t)\frac{\mathrm{d}d_2}{\mathrm{d}\kappa_{u2}}\frac{\mathrm{d}\kappa_{u2}}{\mathrm{d}v}K(v - v_r) \tag{3.11}$$

where it is important to note that $\frac{\mathrm{d}\kappa_{u1}}{\mathrm{d}v}$, $\frac{\mathrm{d}\kappa_{u2}}{\mathrm{d}v}$ are only nonzero when $\kappa_{eq,1}$, $\kappa_{eq,2}$ are surpassing the value $\kappa_{u,1}$, $\kappa_{u,2}$ resp. Alternatively it can be said that the terms $\frac{\mathrm{d}\kappa_{u1}}{\mathrm{d}v}$, $\frac{\mathrm{d}\kappa_{u2}}{\mathrm{d}v}$ are only nonzero when their respective damage d_1, d_2 is increasing.

4. Examples

In this chapter examples are given of mechanical problems that include the use of interface crack elements. The constitutive model as expressed by eq. (3.7) is used to describe the mechanical behaviour of these interface crack elements. In the first section three tests are performed on a one-dimensional model. This shows the behaviour of the self-healing constitutive law in an integration point for a finite element analysis. In the second section the implementation of the new constitutive law in a finite element code is tested.

4.1 One-dimensional examples

In this subsection three one-dimensional examples are shown to demonstrate the performance of the new constitutive model. The test configuration is the same for all three examples but the load case differs for each example. The test configuration and its parameters are shown in Fig. 4. It consists of one interface element together with one elastic material element on top of the interface element and another one below. The lower element is fixed. The stiffness of the two linear elastic elements has been set rather high in comparison to the dummy stiffness of the interface element in order to see the effect of the interface element more pronounced. Both fracture toughnesses $G_{c,1}$ and $G_{c,2}$ of the interface element have been given the same value. A plot showing the applied load vs. displacement for the test configuration without healing can be found in Fig. 5.a.

For each example a figure is shown containing the prescribed displacement of node 4 as a function of time (Fig. a), the development of damage in time (Fig. b), the traction at node 4 vs. time (Fig.

c) and the traction vs. displacement at node 4 (Fig. d). In each example the healing starts at $t = 1\ s$ and the heal time $t_{0.975}$ has been chosen 1 s. The maximum recovery r has been set equal to 0.75 in the first and second example. In the third example this value has been set equal to 1 (i.e. full recovery). The healing function that is used for example 3 is shownhttp://www.novell.com/linux/ at Fig. 5.b. The healing function for example 1 and 2 is this same function but with $r = 0.75$.

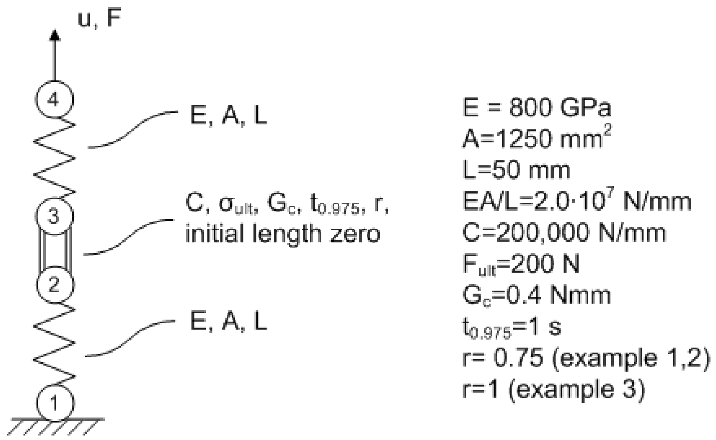

Figure 4: Configuration for example 1 to 3.

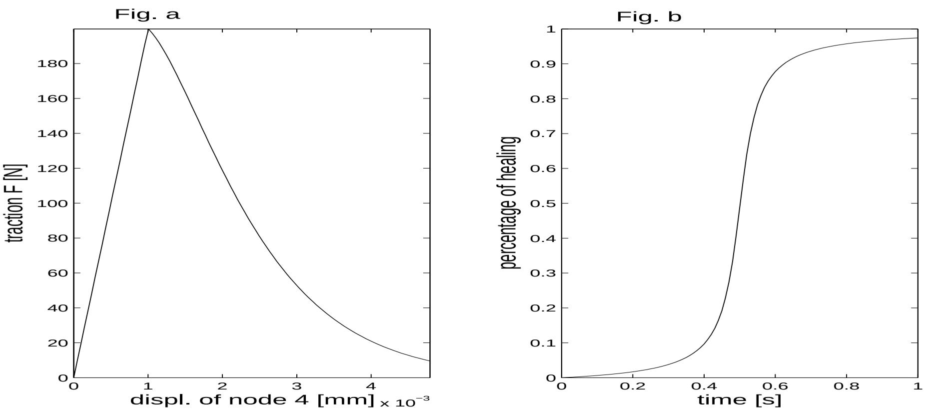

Figure 5: Plots illustrating behaviour of test configuration.

4.1.1 Example 1

In this example the displacement of node 4 is increased from zero at $t = 0\ s$ to the value that corresponds to a damage of the interface element of 0.99 at $t = 1\ s$ (see Fig. 6a, b). Note that a damage of 1.0 can never be reached since the damage function approaches 1.0 asymptotically. Between $t = 1\ s$ and $t = 2\ s$ the displacement is kept constant and healing is allowed to start. The damage now decreases and after 1 second of healing (at $t = 2\ s$) the damage has almost reached $1 - r$, which in this case equals 0.25. At $t = 2\ s$ the displacement at node 4 again increases. This causes the damage to increase again to almost 1 as from a short period after $t = 2\ s$. When looking at the traction plots (Fig. 6c, d) it is seen that when the material has almost fully healed, it can again take a considerable load. It has regained its stiffness up to a factor 0.75. Note that both the ultimate stress and the fracture toughness is approximately 75 percent of the original value when healing has almost completed (i.e. at $t = 2\ s$)

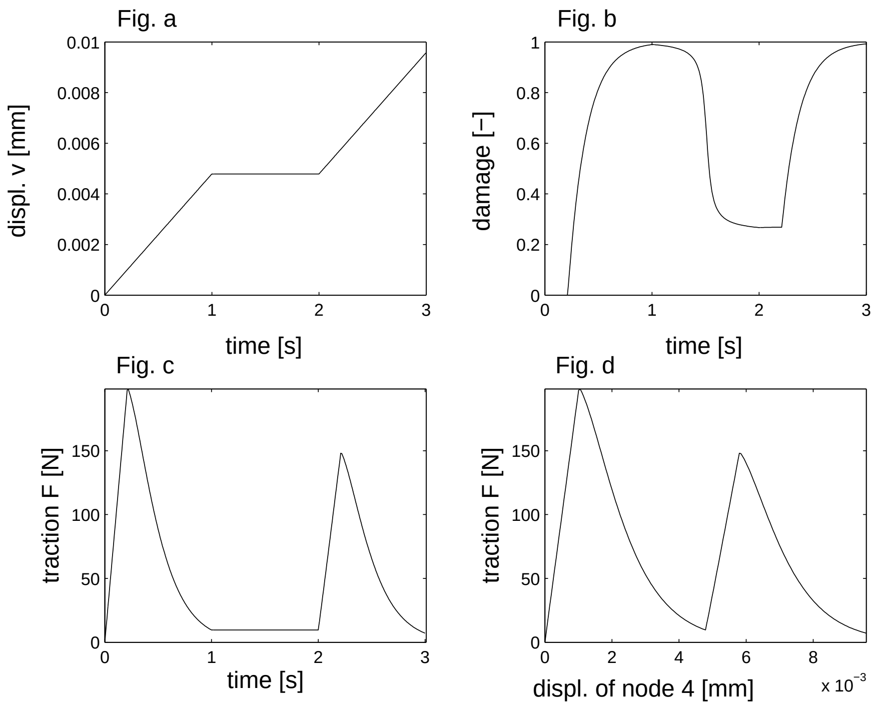

Figure 6: Plots for example 1.

4.1.2 Example 2

In this example the displacement of node 4 is increased from zero at $t = 0$ s to the value that corresponds to a damage of the interface element of 0.99 at $t = 1$ s (see Fig. 7a). At $t = 1$ s the displacement is kept constant and healing is allowed to start. The damage decreases to $1 - r$ $(= 0.25)$ at $t = 2$ s (see Fig. 7b). At $t = 2$ s the displacement is taken back to zero. This causes large compression forces to develop (see Fig. 7c, d). No more damage develops, which results from the choice of damage function, which is in this case (3.2). Note that the equilibrium jump v_r of material fraction p_2 is about $4.8 \cdot 10^{-3}$ mm (which approximately equals the input displacement at $t = 1$ s). This positive equilibrium jump causes the compression forces to develop for displacements of node 4 below $4.8 \cdot 10^{-3}$ mm.

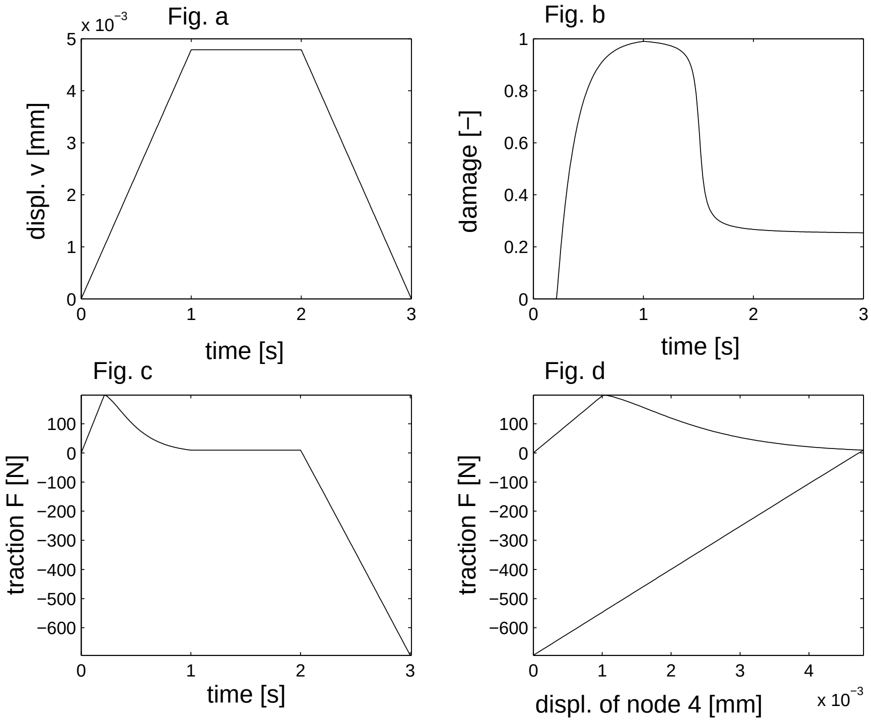

Figure 7: Plots for example 2.

4.1.3 Example 3

In this example the displacement is increased from zero to a value that corresponds to a damage of the interface element of 0.5 at $t = 1$ s. At $t = 1$ s healing is allowed to start and meanwhile the displacement is decreased to zero at $t = 2$ s. It is kept at zero between $t = 2$ s and $t = 3$ s (see Fig. 8a). Since r has been chosen equal to 1 the damage reduces to zero (see Fig. 8b). Looking at the traction plots (Fig. 8c, d), it can be seen that the traction goes up at $t = 0$ s till the maximum force F_{ult} and then decreases till $t = 1$ s under the creation of damage. At $t = 1$ s the damage has increased to 0.5. When from $t = 1$ s till $t = 2$ s the displacement is taken back to zero and healing starts, we see the curve initially follows the dashed line n^o 1, which is the line it would follow when no healing is applied. However between $t = 1$ s till $t = 2$ s the material fraction p_2 increases its stiffness from zero to full stiffness K and therefore the total stiffness increases from $0.5K$ to K. Therefore the curve initially following line n^o 1 gradually starts following the steeper line n^o 2, which indicates that the stiffness increases to K. Also note that the equilibrium jump v_r (where the traction is zero) has shifted from zero to a value slightly below one. The interesting phenomenon between $t = 1$ s and $t = 2$ s can be seen more detailed in Fig. 9. When after $t = 3$ s the displacement is again increased we see two kinks; the first one appearing when material fraction p_1 starts to increase its damage and a second kink when the material fraction p_2, having an equilibrium jump larger than p_1, starts to have damage (p_1 has an equilibrium jump of zero and p_2 has an equilibrium jump of appr. $1.7 \cdot 10^{-3}$).

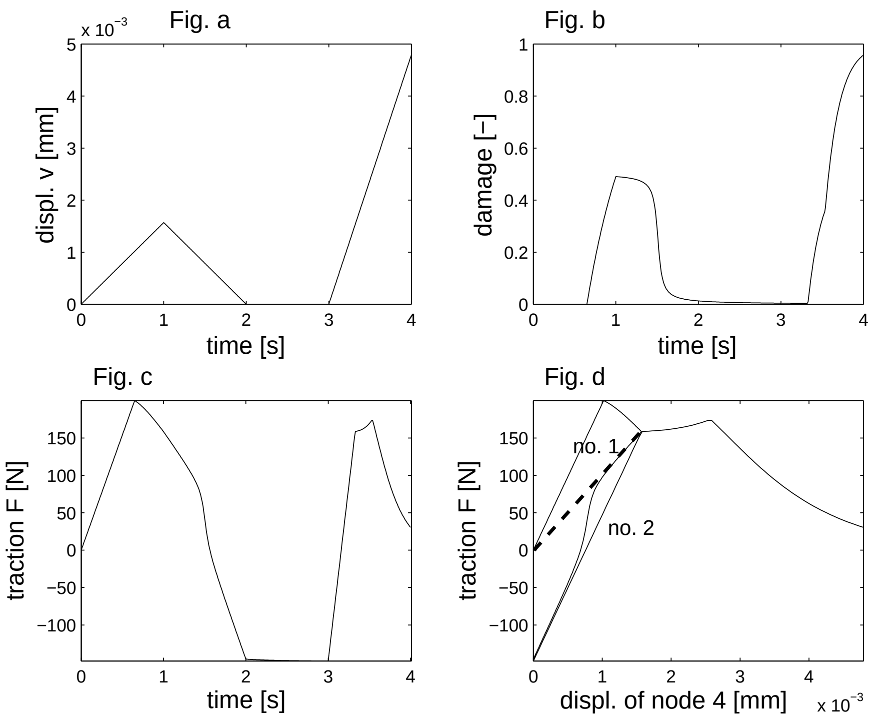

Figure 8: Plots for example 3.

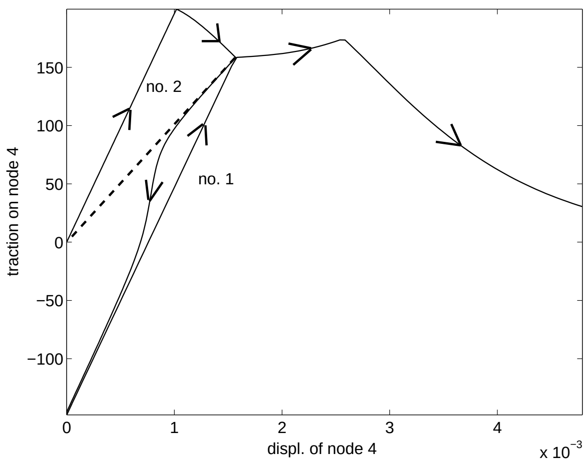

Figure 9: Detailed plot for example 3.

4.2 Example of a finite element computation

In this section the results of implementing the constitutive law (3.7) in a finite element analysis code are discussed. It is verified if the new constitutive law, when implemented, works as intended. The implementation is tested for a slender beam with many degrees of freedom. This test configuration and the material parameters have been copied from ref. [3] where it was tested for delamination (no healing is applied). It is verified if the implementation gives the same results as in ref. [3] when again no healing is initiated. This verifies if the new constitutive law (3.7) works unaltered w.r.t. the original damage law when there is no healing. Subsequently this same test configuration is again tested but for this second test healing is initiated after some period of time. The results of this test show the performance of the new constitutive law (3.7) in a realistic test case.

The finite element analysis code in which the new constitutive law is implemented is the Jem/Jive based code of the chair Engineering Mechanics of the Faculty of Aerospace Engineering.

The test configuration consists of a slender beam of 10 mm long, 1 mm high and an initial crack of 1 mm, as is shown in Fig. 10. A peel test is conducted on it. The material properties copied from [3] are: Young's modulus of the bulk material $E_{bulk} = 100\ MPa$, poisson ratio of the bulk material $\nu_{bulk} = 0.3$, dummy stiffness of the interface element $C = 100\ MPa$, fracture toughness of the interface element $G_c = 0.1\ Nmm^{-1}$ and tensile strength of the interface element $\sigma_t = 1\ MPa$.

For the first test case the displacement as a function of time is shown in Fig. 11.a, which resembles the displacement applied in the test of [3]. The force vs. displacement plot for this test case are shown in Fig. 12. The plot resembles the solution in [3]. This result verifies the correctness of the constitutive law and the implementation of it in the finite element analysis code.

For the second test case the displacement as shown in Fig. 11.b is applied. The healing process is initiated at $t = 2/s$. The heal time has been taken $t_{0.975} = 1\ s$ and the maximum recovery has been taken $r = 0.75$. The results are shown in Fig. 13. The upper branch of the curve corresponds to the traction between $t = 0\ s$ and $t = 1\ s$ (branch 1), the lower branch of the curve corresponds to the traction between $t = 1\ s$ and $t = 2\ s$ (branch 2) and the middle branch of the curve corresponds to the traction between $t = 3\ s$ and $t = 4\ s$ (branch 3). It is seen that branch 1 of the curve of Fig. 13 equals the curve of Fig. 12. The situation is indifferent from the first test case. When between $t = 1\ s$ and $t = 2\ s$ the displacement is taken back to zero we see from branch 2 that the traction goes back to zero. After one second of healing (between $t = 2\ s$ and $t = 3\ s$) the peel test is repeated. The third branch of the curve resembles the first branch, but now the traction is on a lower level. In order to draw conclusions on the influence of healing on the traction one should compare the third branch of the curve (the second peel test, damage recovered by 75 per cent) with the first branch of the curve (material that starts without any damage) and the second branch of the curve (material that has a damage history). It is seen that the third branch lies between these two branches, although considerably closer to the first branch than to the second branch. The closer r will be taken to one, the closer the third branch will approach the first branch.

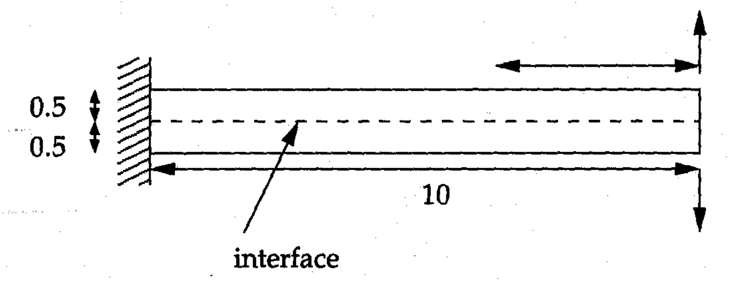

Figure 10: Configuration of the slender beam.

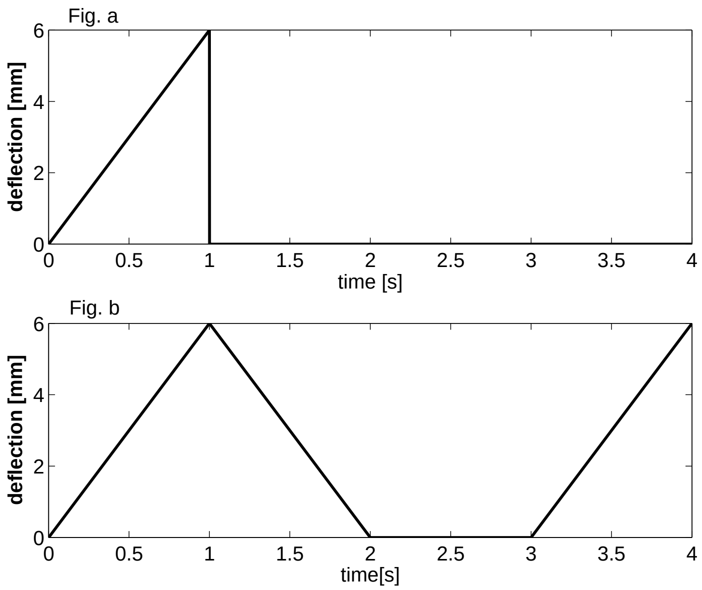

Figure 11: Prescribed displacements for slender beam test.

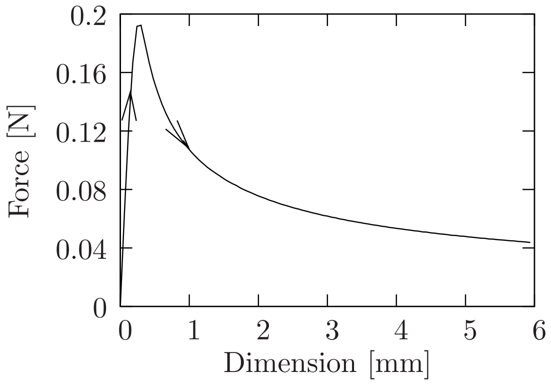

Figure 12: Results of slender beam testcase 1.

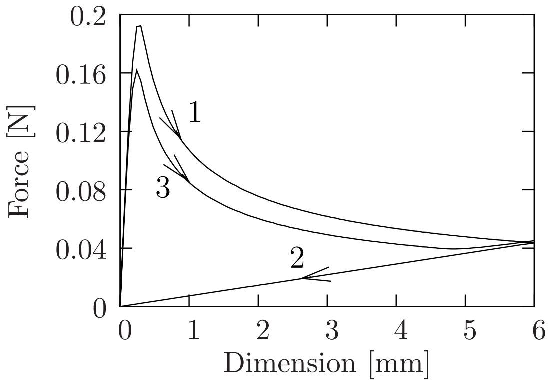

Figure 13: Results of slender beam testcase 2.

5. Conclusions

The goal of this project was to formulate a constitutive law for materials with self-healing capability. This has been achieved and resulted in eq. (3.7). This constitutive law is to be used for interface crack elements which are positioned in between of elements of the bulk material. The model describes the recovery of the mechanical properties of the material; the elastic stiffness and the fracture toughness. Their values increase after the self-healing process has started.

The constitutive model is an extension of the damage model for interface crack elements. The original constitutive model incorporated the use of one damage parameter per integration point. The new constitutive model incorporates two damage parameters per integration point. The behaviour of the healing agent is modelled by a healing function, which is in eq. (3.7) denoted by $h(t)$. It is characterised by the maximum recovery r and the heal time $t_{0.975}$.

The new constitutive model described in this report is part of a larger model. The self-healing process is assumed to consist of three steps: a crack forms in the matrix (step 1), the crack initiates the self-healing process, healing agent is released into the crack by means of a transport process and it contacts a catalyst (step 2) and the catalyst causes the healing agent to cure (step 3). The current class of damage models describes the first step. The new constitutive model, which is an extension of the current class of damage models, describes step 1 as well as step 3 however. To circumvent the missing of the second step in the simulation, the initiation of the curing process is done by a time-input and it is assumed that step 2 is took place without problems.

The examples in section 4.1 show that the model behaves as intended. Before the healing process starts, the model behaves exactly like the damage model. After the healing process has started the lost stiffness is recovered up to a factor r. Also the fracture toughness is recovered.

The new constitutive model has also been tested in a finite element analysis code on a slender beam. A peel test was conducted with interface elements placed halfway the thickness of the beam. It was seen that the beam regained a considerable part of its resistance against peeling when the maximum recovery r was chosen equal to 0.75.

Future work consists of adapting the constitutive law to a 3-dimensional law. The current constitutive law only accounts for Mode I behaviour, a complete model should also include Mode II behaviour however. This extension should start again by considering the original damage law, but now the 3-dimensional model. Furthermore the aforementioned step 2 has to be modelled, which involves fluid dynamics theory. This would complete the model of self-healing materials.

If the behaviour of the model is not found satisfactory, a possible extension consists of making the equilibrium jump parameter v_r variable instead of fixed. The parameter v_r should follow a creep law $\dot{\boldsymbol{v}}_{\boldsymbol{r}} = \boldsymbol{G}\boldsymbol{\tau}$. This adaptation would take into account that as long as the healing agent has not solidified the equilibrium jump can change and follow the opening or closing of the crack.

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