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DOI
10.1002/we.2914

Publication date
2024

Document Version
Final published version

Published in
Wind Energy

Citation (APA)
Bierbooms, W. A. A. M., & Veldkamp, D. (2024). Probabilistic treatment of IEC 61400-1 standard based extreme wind events. *Wind Energy*, *27*(11), 1319-1339. https://doi.org/10.1002/we.2914

Received: 15 September 2023 | Revised: 23 January 2024 | Accepted: 16 April 2024

DOI: 10.1002/we.2914

RESEARCH ARTICLE

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Probabilistic treatment of IEC 61400-1 standard based extreme wind events

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Funding information
There was no external funding for this work.

Abstract

The procedure to estimate the amplitude of the special wind events according to IEC 61400-1 as proposed by Larsen is reviewed. Corrections are specified that yield larger amplitudes for the extreme operating gust (EOG) and the extreme coherent gust with simultaneous direction change (ECD). For the ECD case, distributions for longitudinal and lateral gust amplitudes are derived and applied in simulations to derive the load distribution, from which the 50-year extreme load can be found. Results are compared with the calculation with the conventional ECD: In the example calculation, the 50-year values of both blade root bending moment and tip deflection are smaller than the conventional values.

KEYWORDS

extreme loads, gust amplitudes, IEC, joint events, return value

1 | INTRODUCTION

In wind turbine design, all loads that occur in the turbine's life are considered: both fatigue loads and ultimate loads. Currently, International Electrotechnical Committee (IEC) 61400-1[1] prescribes the characteristic turbulence to obtain fatigue loads and extreme deterministic wind events to estimate the ultimate loads. These events are as follows: extreme operating gust (EOG), extreme direction change (EDC), extreme wind shear (EWS), and extreme coherent gust with direction change (ECD). The gusts are taken to be proportional to the characteristic (90% fractile) 1-point turbulence. They are deterministic with the wind speed being 100% coherent over the rotor plane; there is a compensation factor for rotor size to account for cancellation of uncorrelated high-frequency speed fluctuations.

An alternative for the deterministic approach is constrained simulation, which embeds a given wind speed change in a turbulent wind field.[2,3] This approach is fully physical but more difficult to use because of its stochastic nature. Nevertheless, IEC 61400 already allows the use of random turbulence fields instead of the EOG.

An intermediate approach is to stick to the deterministic gust but use a probability distribution to set the amplitude. We will discuss this new method, here denoted as the"gust distribution method" and compare it to the IEC method to establish the 50-year load. A strong point of this probabilistic treatment is that the loads are extrapolated instead of gust amplitudes; after all, the 50-year load is of interest and not so much the gust amplitude. Both in terms of fidelity and required computational effort, the gust distribution method procedure can be regarded as in between a full probabilistic approach (via constrained simulation) and a purely deterministic one (IEC).

For the determination of the gust amplitude, the Larsen procedure[4–6] will be applied. This procedure will be presented in Section 2. We will focus on the EOG to show the principle of the derivation of gust size and on the ECD to address the issue of simultaneity of longitudinal and lateral wind speed excursions. We discovered two mathematical errors in Larsen's method. This paper aims to correct those and elaborate on the consequences of that. The derived correct expressions will be indicated by "Corrected Larsen" and are described in Section 3. Section 4 addresses the probabilistic treatment: The distribution of the gust amplitude is used to establish the distributions of extreme blade root bending moment and tip deflection. A demonstration of the gust distribution method is given in Section 5. For the load calculations involved, we use the Goldwind GW131-2300 turbine.

2 | LARSEN'S PROCEDURE

Larsen's procedure[4–6] is based on the assumption that each special wind event ("gust") is a speed excursion that happens naturally in a random turbulence field. With various manipulations (discussed below), a Gumbel distribution for the dimensionless annual extreme $\zeta_m = u_{em}/\sigma_u$ (u_{em} is the wind speed deviation from the mean and σ_u is the standard deviation) is obtained as follows:

$$F(\zeta_m) = \exp\left(-\exp\left(-\frac{\zeta_m}{2C(z)} + \ln \kappa T\right)\right) = \exp\left(-\exp\left(-\frac{\zeta_m - m}{\beta}\right)\right) \tag{1}$$

The parameter m is the location parameter and the mode of the Gumbel distribution, while β is the scale parameter. If the value of the cumulative distribution function (cdf) is set, say $F(\zeta_m) = 0.98$, the dimensionless extreme excursion ζ_m is found with the following:

$$\zeta_m(F) = m - \beta \ln(-\ln F) \tag{2}$$

The 'real' extreme excursion in m/s is as follows:

$$V_{gust} = f_\tau f_D \sigma_1 \zeta_m \tag{3}$$

where V_{gust} is the amplitude of a deterministic IEC-type gust, σ_1 is the IEC characteristic turbulence (90% quantile) at speed V, and f_D and f_τ are reduction factors for rotor size and for gust rise time. Hence, the speed change is ζ_m times the effective turbulence experienced by the rotor.

For clarity, here we bring Larsen's procedure back to seven main steps. In addition, each step is visualized with time series and probability density function (pdf) in Figure E.2 in Appendix E. We found two errors in Larsen's method, which we discuss in Section 3; in this section, we present the equations as given by Larsen.

Step 1. It is assumed that turbulent velocity excursions $u_e(t)$ (in m/s) from the 10-min mean level are not Gaussian but Gamma distributed (at least) in the tail of the distribution with pdf[*]:

$$f_{u_e}(u_e) = \frac{1}{2}\frac{1}{\sqrt{2\pi C(z)\sigma_u |u_e|}} \exp\left(-\frac{|u_e|}{2C(z)\sigma_u}\right) = \frac{x^{k-1}}{\theta^k \Gamma(k)} \exp\left(-\frac{x}{\theta}\right) \tag{4}$$

where

$$k = \frac{1}{2} \quad \theta = 2C(z)\sigma_u \quad u_e = u(t) - \bar{U} \tag{5}$$

The shape factor $k = 1/2$, the scale factor is $\theta = 2C(z)\sigma_u$ with $C(z)$ a nondimensional factor which is terrain and height dependent, and σ_u the standard deviation of u. With the factor $C(z)$, the distribution can be fitted to the tails of the pdf of measured turbulence; Larsen[6] gives values for C dependent on height and terrain type, $C \approx 0.4$.

Step 2. The velocity excursion u_e is transformed into a standard Gaussian variate η with the following:

$$\phi(\eta) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\eta^2}{2}\right) \tag{6}$$

where

$$\eta = \text{sign}(u_e)\sqrt{\frac{|u_e|}{C\sigma_u}}. \tag{7}$$

*In principle, the fit depends on wind speed, but apparently, one fit is done on all $(u_e(t) - U)/U$.

Step 3. The local maxima of time series of η are considered. It is assumed that the spectrum of η is Kaimal as specified in the IEC standard.[1] The result of the transformation in step 2 is that we have a standard normal process that has been studied extensively in the past (see Papoulis[7] section 11-4 on the level crossing problem). The analytical expression for the distribution of local maxima η_e was first derived by Rice[8] (see also Cartwright & Longuet-Higgins[9] and Davenport[10]). The pdf of the local extremes is (ϕ and Φ are the standard normal pdf and cdf):

$$f_{\eta_e}(\eta_e) = \delta\phi\left(\frac{\eta_e}{\delta}\right) + \sqrt{1-\delta^2}\,\eta_e e^{-\frac{\eta_e^2}{2}}\Phi\left(\frac{\sqrt{1-\delta^2}}{\delta}\eta_e\right) \tag{8}$$

This distribution has only one parameter, the bandwidth δ:

$$\delta = \sqrt{\frac{m_0 m_4 - m_2^2}{m_0 m_4}} \tag{9}$$

which is a function of the spectral moments m_i of the signal, given by[†]:

$$m_i = \int_0^{f_{co}} f^i S(f)\,df \tag{10}$$

For a narrow banded spectrum, δ is small and for broadband spectra δ approaches unity.

Step 4. Next, the global extreme is considered by looking at the largest extreme in period T (or equivalently: of N local maxima). Note that this largest value in period T is not constant but a variate as well. The cdf of this variate is obtained by taking the power N of the original cdf (so implicitly assuming that the local extremes are independent). For extremes, we are interested in large values of N, and for large N, an analytical expression for the probability density of global extreme η_{em} is available:

$$f_{\eta_{em}}(\eta_{em}) = \eta_{em}\exp\left(-\exp\left(-\frac{\eta_{em}^2}{2} + \ln N\sqrt{1-\delta^2}\right) - \frac{\eta_{em}^2}{2} + \ln N\sqrt{1-\delta^2}\right) \tag{11}$$

Step 5. The distribution of ζ_m ($= u_{em}/\sigma_u = C\eta_{em}^2$) is obtained by the inverse transformation (see Equation (7)); ζ_m is the largest of N local maxima:

$$f_{\zeta_m}(\zeta_m) = \frac{1}{2C}\exp\left(-\exp\left(-\frac{\zeta_m}{2C} + \ln N\sqrt{1-\delta^2}\right) - \frac{\zeta_m}{2C} + \ln N\sqrt{1-\delta^2}\right) \tag{12}$$

Larsen introduces the "expected rate of local maxima" κ, which is defined by the equivalence (T is the available time):

$$\kappa T = N\sqrt{1-\delta^2} \tag{13}$$

Hence, we get the following:

$$f_{\zeta_m}(\zeta_m) = \frac{1}{2C}\exp\left(-\exp\left(-\frac{\zeta_m}{2C} + \ln\kappa T\right) - \frac{\zeta_m}{2C} + \ln\kappa T\right) \tag{14}$$

which is the pdf that corresponds to the Gumbel cdf in Equation (1) with location and scale parameters:

$$m = 2C\ln\kappa T \quad \beta = 2C \tag{15}$$

Step 6. We are interested in the characteristic event, which IEC takes as the event with 50-year return period. If we consider the Gumbel distribution of the 50-year event, we can *define* the characteristic event as the most likely event (the mode) of the distribution of ζ_m. For a Gumbel distribution, the mode is equal to the location parameter, so

† In Larsen and Hansen[6], analytical expressions are provided for the moments of the Kaimal spectrum as function of cut-off frequency f_{co}, length scale, and mean wind speed. There is a typo in Equation (45) in Larsen[5]: The third term should have a factor of 3/2 instead of 2/3.

$$\mathrm{mode}(\zeta_m) = m = 2C\ln\kappa T \tag{16}$$

Here T is the available time in which maxima occur (see below) and κ is the expected rate of local maxima:

$$\kappa = \frac{N}{T}\sqrt{1-\delta^2} = \frac{N}{T}\frac{\nu_0}{\nu_p} \tag{17}$$

with ν_0 the frequency of zero upcrossings and ν_P the frequency of local peaks:

$$\nu_0 = \sqrt{\frac{m_2}{m_0}} \quad \nu_P = \sqrt{\frac{m_4}{m_2}} \tag{18}$$

For the number N of local maxima, Larsen[5] gives two options:

1. All local maxima
2. Only the local maxima above a threshold of 2 times the standard deviation

Larsen and Hansen[6] use the second option; the number N is (Rice,[8] Equations 3.6–11):

$$N = \nu_0 T e^{-\frac{1}{2}\eta_t^2} \tag{19}$$

which for the EOG leads to (with threshold $\zeta_t = 2$):

$$\kappa = \nu_0 e^{-\frac{1\zeta_t}{2C}}\frac{\nu_0}{\nu_P} = e^{-\frac{1}{C}}\sqrt{\frac{m_2^3}{m_0^2 m_4}} \tag{20}$$

For the ECD, the number N is taken as (Equation B.6 in Larsen[5]):

$$N = T\sqrt{\frac{m_{u4}}{m_{u2}}}\sqrt{\frac{m_{v4}}{m_{v2}}} \tag{21}$$

With subscript u indicating the longitudinal direction and v the lateral direction. One may recognize the above expression as the product of the two peak frequencies. Substitution leads to the following:

$$\kappa = \sqrt{\frac{m_{u2}}{m_{u0}}}\sqrt{\frac{m_{v4}}{m_{v2}}} \tag{22}$$

for u-excursions and likewise for v-excursions. In the graphs in the next section, values found with Equations (20) and (22) will be indicated by "Larsen."

In Larsen's approach, T is the time in 50 years that the wind speed is in a 2 m/s interval around some average wind speed V. Alternatively, one can take T to be the time in 1 year in this interval and establish the 98% fractile of the distribution of the 1-year maxima: This gives the same result (see Appendix C).

Step 7. The physical speed excursion is found by putting the result of step 6 (the maximum ζ_m) into Equation (3).

3 | CORRECTED LARSEN PROCEDURE

When applying Larsen's procedure as described in the previous section, we discovered that there are two expressions that are mathematically inconsistent; this invalidates the results shown in Larsen and Hansen.[6] Here we present the correct expressions, which we denote "Corrected Larsen." Next, the comparison is made to the IEC extreme events.

EOG. In Larsen's procedure, only local maxima larger than two times the standard deviation are considered. In our view, this is unnecessary; when looking at global maxima, values above the threshold are automatically obtained. One is of course allowed to choose to use a threshold, but the way it is done in Larsen[5] is not correct. In order to show this, it is necessary to go into the details of Step 4: The maximum x_T within time period T of some variate x with distribution $F_x(x)$ is considered. The distribution of x_T is given by $F_x^N(x)$ with N the number of occurrences of x within T. In case one is only interested in values above threshold t, a new variate y can be introduced which consists of all values of x larger than t. The distribution $F_y(y)$ is a truncated version of F_x. Similarly, the distribution of y_T, the maximum of y within T, equals F_y^M with M equal to the number of y-values within T; M is typically much smaller than N. In Larsen,[5] erroneously F_x^M is used instead of F_y^M. In Appendix A, it is shown that taking a threshold into account does not make a difference for the statistics of wind extremes.

When taking the Rice distribution (Step 3), the correct number of local extremes to consider is the number of local peaks (first option in Step 6):

$$N=\nu_P T \tag{23}$$

Leading to the following:

$$\kappa=\nu_0 \tag{24}$$

Equation (11) simplifies to the following:

$$f_{\eta_{em}}(\eta_{em})=\eta_{em}\exp\left(-\exp\left(-\frac{\eta_{em}^2}{2}+\ln\nu_0 T\right)-\frac{\eta_{em}^2}{2}+\ln\nu_0 T\right) \tag{25}$$

With

$$\kappa T=\nu_0 T=e^{\ln\nu_0 T} \tag{26}$$

Hence, the cumulative density function is (Davenport,[10] section 11) as follows:

$$F_{\eta_{em}}(\eta_{em})=\exp\left(-\exp\left(-\frac{\eta_{em}^2}{2}+\ln\nu_0 T\right)\right) \tag{27}$$

The frequency of local peaks ν_P does not enter in the final result, but instead we see the frequency of zero upcrossings ν_0.

In Figure 1, it is seen that the new value for hilly terrain (Larsen corrected with $\kappa=\nu_0$, red solid line) is approximately 40% higher and more or less fits the 50-year IEC 61400-1 Ed. 2 value with $\beta=6.4$ around 15 m/s (solid black line with open circles) (IEC,[11] Stork[12]). All values are calculated with the characteristic 90% fractile turbulence based on $I_{ref}=0.16$. It would be more consistent to include the turbulence distribution or use the median instead of the 90% value; this would probably take down the "Larsen corrected" values somewhat. It is somewhat of a mystery that hilly terrain creates larger gusts than flat terrain without this being reflected in the turbulence. The EOG gust size V_{gust} is now defined to be smaller (IEC 61400-1 Ed. 4[1]):

$$V_{gust}=f_D\beta\sigma_1 \quad f_D=\frac{1}{1+0.1D/\Lambda_1} \quad \sigma_1=I_{ref}(0.75V+5.6) \tag{28}$$

With $\beta=3.3$ rather than $\beta=6.4$. It is not a 50-year gust but the worst gust that is supposed to coincide with operation events such as starts and stops; in fact, it is even smaller than the previously defined 1-year gust ($\beta=4.8$).

ECD. The ECD is a joint event with a simultaneous excursion in longitudinal direction and lateral direction. It is clear that Equation (22) cannot be correct since it has dimension s^{-2}—recall that κ is a frequency and must have dimension s^{-1} (= Hz). In Appendix B, we show that for the longitudinal excursions, the number of joint local maxima (u_e,v_e) that are at most τ s apart is (subscript m indicates "maximum"):

$$N_{uv}=2\tau\nu_{mu}\nu_{mv}T=2\tau\nu_{mv}N_{mu}=2\tau\nu_{mu}N_{mv} \tag{29}$$

where ν_{mu} and ν_{mv} are the frequencies of the maxima. The distribution for both u and v becomes:

$$f_{Gu}(x)=\frac{1}{\beta}\exp\left(-\exp\left(-\frac{x-m}{\beta}\right)-\frac{x-m}{\beta}\right) \tag{30}$$

With for u excursions:

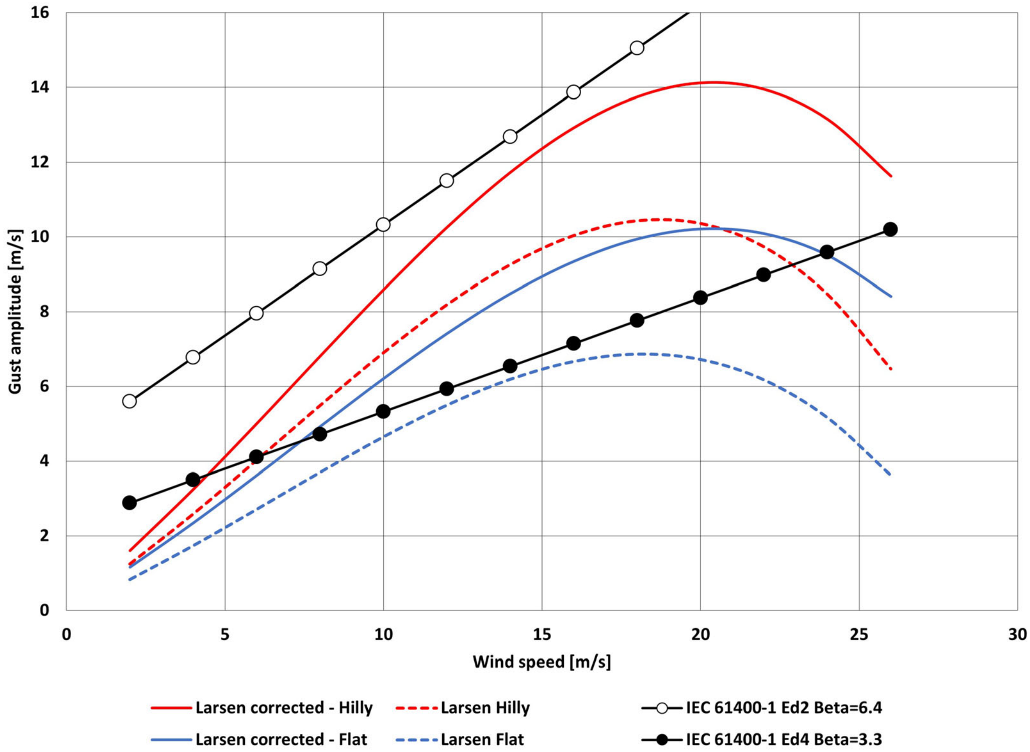

FIGURE 1 EOG: 50-year wind speed change as function of wind speed for hilly and flat terrain with $I_{ref} = 0.16$. The IEC EOG constants are $\beta = 6.4$ and $\beta = 3.3$ and the IEC rotor size reduction is $f_\tau = 1/(1+0.1D/\Lambda) = 0.77$. *Note*: Larsen's values are calculated with $V_{ave} = 6.65$ m/s instead of $V_{ave} = 7.5$ m/s as stated in his paper (see Laren & Hansen,[6] Figure 6). Since the average wind speed influences the wind speed distribution and hence the time T in each 2 m/s interval and the values of the extremes (see Equation (1)), for comparison, we also use $V_{ave} = 6.65$ m/s here. EOG, extreme operating gust.

$$x = u \quad \beta = 2C \quad m = 2C\ln\kappa T \quad \kappa = 2\tau\sqrt{\frac{m_{u2}m_{v4}}{m_{u0}m_{v2}}} \tag{31}$$

And for v,

$$x = v \quad \beta = 2C \quad m = 2C\ln\kappa T \quad \kappa = 2\tau\sqrt{\frac{m_{v2}m_{u4}}{m_{v0}m_{u2}}} \tag{32}$$

The distribution for u and v is the same as in the one-dimensional case, except that the mode m is shifted to the left (towards smaller amplitudes) because the number of peaks in u is reduced to the number of peaks that have a v-peak close by and hence is multiplied by probability $2\tau\nu_{vP}$: only simultaneous peaks are of interest. The same holds for the v distribution (with switched indices). Naturally, if the time interval 2τ is larger, the number of (almost) simultaneous events becomes larger; the effect is shown in Figures 2 and 3. Both 50-year u-amplitude and the v-amplitude increase; the wind direction change is not affected much. Larsen's expression for κ misses the time interval 2τ; it is incorrect with respect to dimensions, but it may be regarded as having $\tau = 0.5$ s (in which case it would correct). This may result in gusts that are too small (see Figure 2 and Appendix B for details). Larsen's values may still be conservative because it is assumed that the 50-year values of the speed excursions in u and v occur simultaneously. However, since excursions in u and v are independent, the 50-year extreme value of u may also coincide with any other value of v rather than the 50-year value.

4 | PROBABILISTIC TREATMENT

Larsen's procedure constructs the probability distribution of the gust amplitude conditional on wind speed (and the associated speed interval) and gives the 50-year extreme value (exceedance probability 0.02).

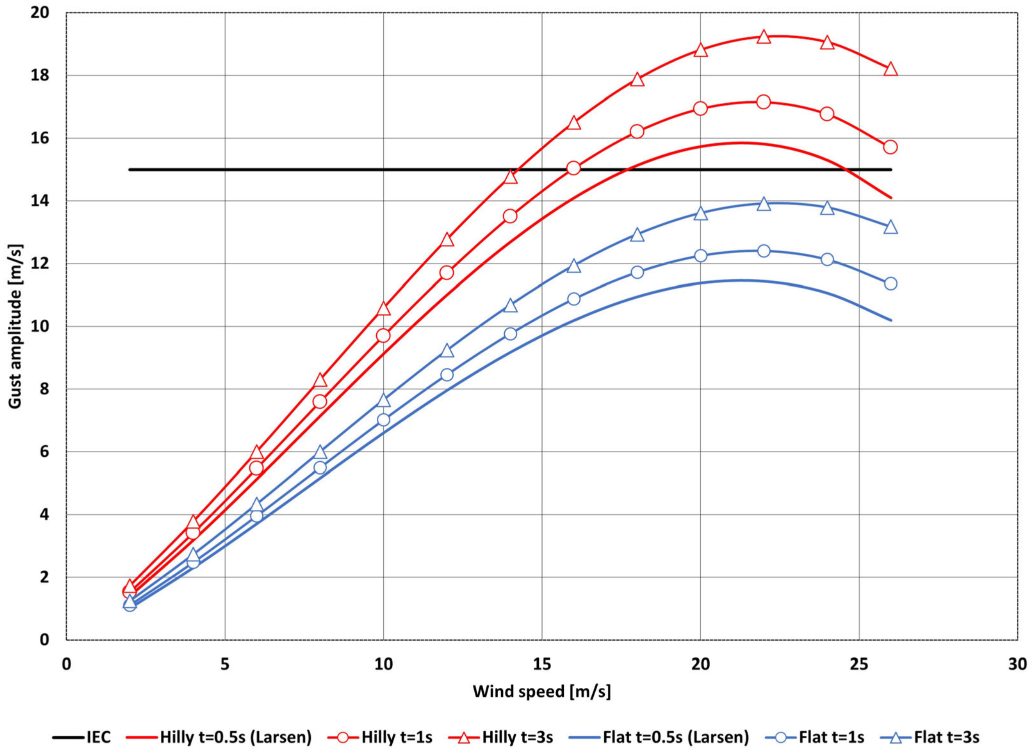

FIGURE 2 ECD: 50-year wind speed change as function of time difference τ according to Corrected Larsen ($V_{ave} = 7.5$ m/s, $I_{ref} = 0.16$). Original Larsen (solid lines) effectively uses $\tau = 0.5$ s (compare to Larsen & Hansen[6] Figure A.2). ECD, extreme coherent gust with direction change.

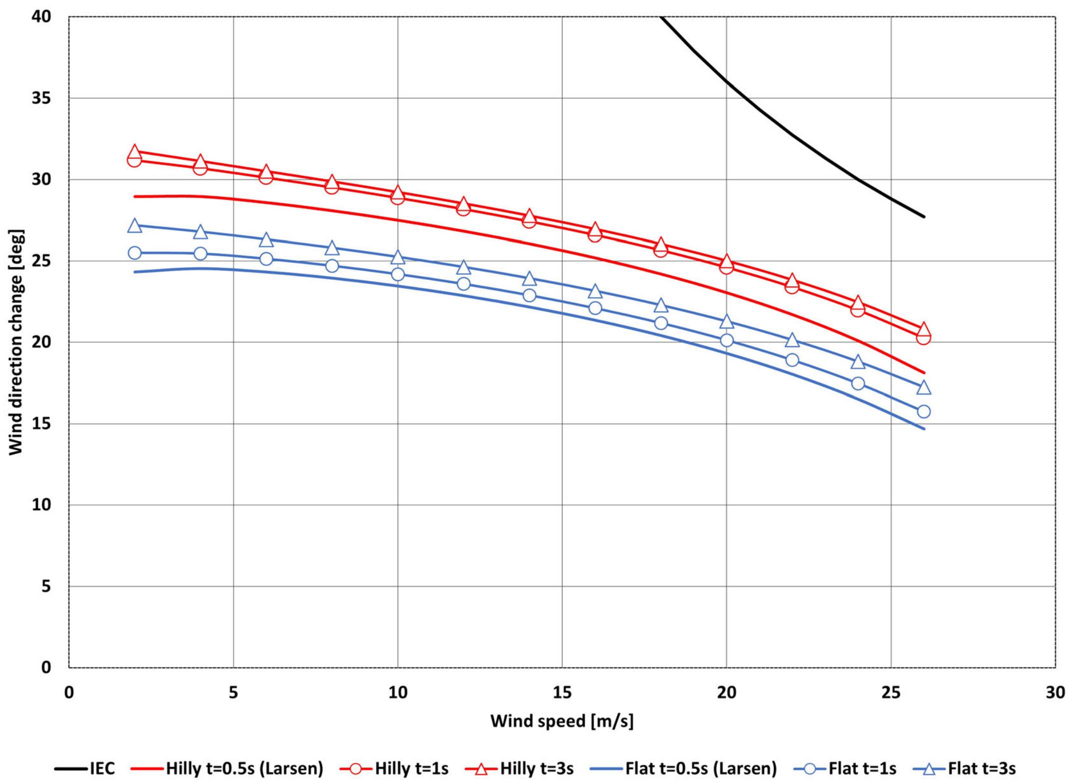

FIGURE 3 ECD: 50-year direction change as function of time difference τ according to corrected Larsen ($V_{ave} = 7.5$ m/s, $I_{ref} = 0.16$). Original Larsen (solid lines) effectively uses $\tau = 0.5$ s (compared with Larsen & Hansen[6], Figure B.1). ECD, extreme coherent gust with direction change.

The simplest way to introduce Larsen's gusts into the IEC calculation is replacing the IEC deterministic gusts by the 98% quantile values for each wind speed (the red solid line in Figure 1); The calculation goes through as before.

An alternative is to use the conditional gust distributions at each wind speed to find the load distribution over all wind speeds. If we only consider one interval ΔV_j, we know the conditional distribution of the annual (dimensionless) extreme gust amplitude ζ (see Equation (1); for readability, we only use gust amplitude and we drop the subscript m here):

$$P(\zeta_i, V_j) = P(\zeta \in \Delta\zeta_i \mid V \in \Delta V_j) \approx f_{Gu}(\zeta, m, \beta, T(\Delta V_j))\,\Delta\zeta \tag{33}$$

Obviously the probability of the associated load $L(\zeta_i, V_j)$ is the same:

$$P(L(\zeta_i, V_j)) = P(\zeta_i, V_j) \tag{34}$$

If we know the probabilities, we can sort the loads in ascending order and construct the cdf for each wind speed V_j:

$$F_j(L) = F(L|V_j) = P(L' \le L|V_j) \tag{35}$$

The probability that the extreme load $L' \le L$ for all wind speeds V_j is the product of all conditional cdfs:

$$P(L' \le L) = P(L' \le L|V_1) \cdot \ldots \cdot P(L' \le L|V_j) \cdot \ldots \cdot P(L' \le L|V_n) \tag{36}$$

or

$$F_{1year}(L) = \prod_{j=1}^{n} F_j(L, V_j) \tag{37}$$

Another possibility is to use the gust distributions for all wind speeds to find the full load distribution directly. Repeat the steps below M times (Monte Carlo method):

1. Draw a random wind speed V
2. Draw a random gust amplitude ζ
3. Convert to real gust amplitude $Vgust$
4. Do a load simulation with $(V, Vgust)$
5. Extract the extreme load L.

Each sample has probability $P = 1/M$. In this method the time T that goes into the Gumbel distribution of the gust amplitude does *not* depend on the wind speed distribution but must be set to a fixed value. If we choose $T = 1$ h, we will get the distribution of hourly extremes. From this, the hourly cdf can be constructed by sorting the loads. Because there are $N = 365.25 \cdot 24 = 8766$ random draws every year, the annual cdf is as follows:

$$F_{1year}(L) = F_{1h}^{N}(L) \tag{38}$$

with $N = 8766$. Note that the Gumbel distribution is the limiting case for the largest of many extremes (occurring over a long time), so the time T must probably not be too small. Alternatively, we can go through all combinations of the parameters (V, ζ) on a predefined grid. The probability of (the load at) a particular grid point given by a wind speed V and dimensionless gust amplitude ζ:

$$P(V, \zeta) \approx f_V(V)\, f_\zeta(\zeta)\, \Delta V \Delta\zeta \tag{39}$$

The construction of the (hourly) cdf of the loads is done as before in the Monte Carlo approach, except that the probabilities are different. IEC (and Larsen) is essentially assuming that the load is a monotonous function of the gust amplitude, and one only needs to do a few calculations with the 50-year wind event to find the 50-year load. The advantage of working out the load distribution with the gust distribution method is that it is guaranteed that the correct 50-year load is obtained, even if the load is some nonmonotonous function of the gust.

5 | LOADS CALCULATED WITH ECD

For an example calculation with ECD, we use the Goldwind GW131 turbine of which the main characteristics are as follows: rated power 2300 kW at 9 m/s, rotor diameter 131 m, hub height 125 m, and rotor speed 5–12.3 rpm. For the simulations, we use the Flex5 aeroelastic code, which relies on the mode shape approach for fast calculations: we use eight bending modes and one torsion mode for the blades and 2×2 modes for the tower. In the calculation, we assume that the wind regime is IEC class II with average wind speed $V_{ave} = 8.5$ m/s and turbulence class A with $I_{ref} = 0.12$. The wind is deterministic, and average wind speed and reference turbulence only enter the calculation indirectly by influencing the distribution of the ECD amplitude.

5.1 | IEC method

According to IEC 61400-1 Table 2[1], the speed increase for the ECD is 15 m/s and the change in direction is $\Delta V_{dir} = \pm 720/V$. The ECD must be investigated for the interval $[V_{rat}-2, V_{rat}+2]$, but here we consider all wind speeds $6 \le V \le 15$ m/s. The representative load case at some wind speed is the average of the extreme values found for simulations with different rotor azimuth starting angles. Results are given in Figures 4 and 5.

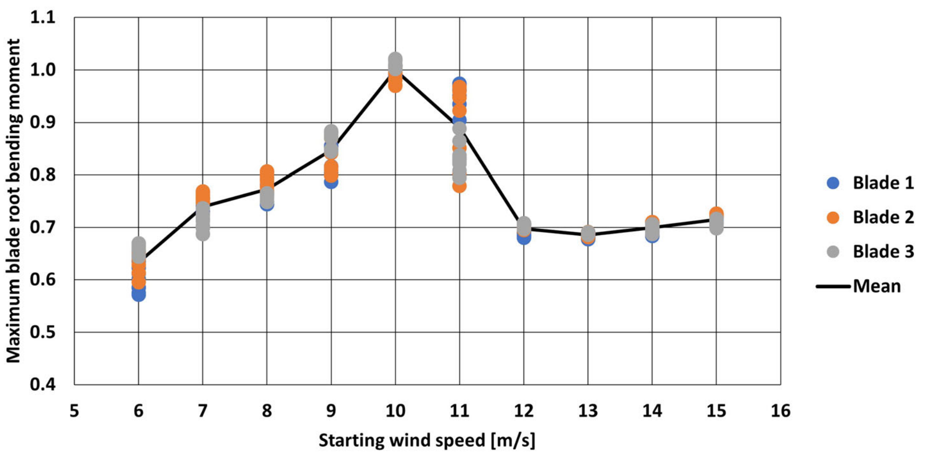

FIGURE 4 Maximum blade root bending moment under extreme coherent gust with direction change (ECD) as function of initial wind speed according to IEC 61400-1[1] (normalized with the average maximum bending moment at 10 m/s).

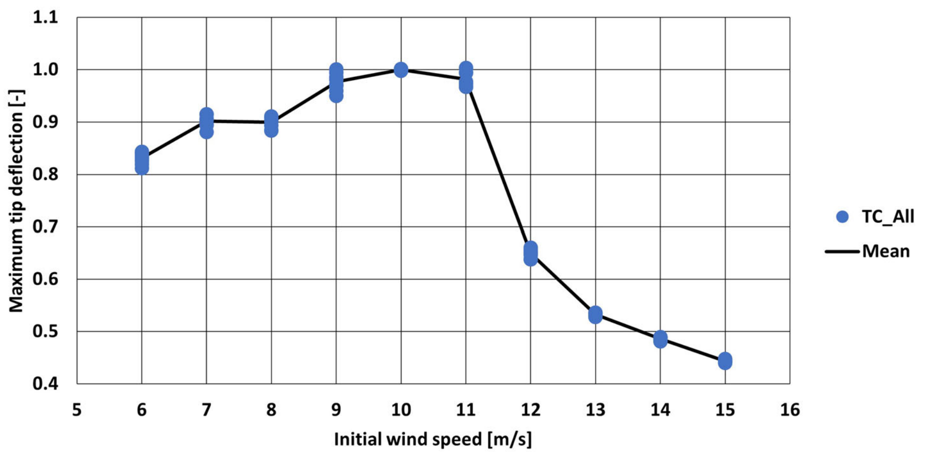

FIGURE 5 Maximum tip deflection at tower passage under extreme coherent gust with direction change (ECD) as function of initial wind speed according to IEC 61400-1[1] (normalized with the average maximum tip deflection at 10 m/s).

5.2 | Gust distribution method

To get the load distribution, we calculate the maximum combined blade root bending moment M_{xy} for all points on a 3D grid with wind speed (in m/s) $V = [4:1:20]$ and dimensionless excursions $\zeta_u = \zeta_v = [1:0.5:10]$. In total, there are $17 \cdot 19 \cdot 19 = 6137$ grid samples. The probability for each sample (given wind speed V_j and wind speed interval ΔV_j) is as follows:

$$P(V,\zeta_u,\zeta_v) \approx f_{\zeta u}(\zeta_u) f_{\zeta v}(\zeta_v) \Delta\zeta_u \Delta\zeta_v \tag{40}$$

where f is the pdf, the wind speed interval $\Delta V = 1$ m/s and the dimensionless speed excursion interval $\Delta\zeta_u = \Delta\zeta_v = 0.5$. Because the grid is large, we will see loads with low probabilities for unlikely combinations: The exceedance probability graph extends down to approximately $p_{50} = 3 \times 10^{-4}$. The cdf of the annual load extreme is found by multiplication of the cdfs for individual wind speeds.

For each combination (V,ζ_u,ζ_v), the actual wind speed changes are calculated with Equation (3) with $C = 0.4$ and $I_{ref} = 0.12$. For ζ_u and ζ_v the probability distributions are recalculated for each wind speed V. For the parameter κ in the probability distributions of the ζ_u and ζ_v gust amplitudes, a maximum difference in start time of $\tau = 0.5$ s is used (see Equations (29)–(32)); in the load calculations, gusts are assumed to start simultaneously. After the calculation, the loads are sorted and the cumulative distribution established. In this example, we have the following settings:

- Wind regime used is Class II with $V_{ave} = 8.5$ m/s and $k = 2$
- The turbulence (used as input for the gusts) is set according to IEC with $I_{ref} = 0.12$ (but the wind field has no turbulence)
- Only positive wind direction changes are considered (wind direction changes from North to East)
- Speed and direction changes are simultaneous.

The annual maximum loads are given by the light blue line in Figure 6 and its Weibull fit (in dark blue). The 50-year load (or characteristic load) is the load that has exceedance probability:

$$P(S \geq S_{50}) = 0.02 \tag{41}$$

which is once every 50 years. The load is found at the intersection of the dark blue curve and the horizontal red dotted line; the characteristic load S_{char} is further indicated by the vertical black dotted line. The load is $S_{char} = 0.84$, which is 16% smaller than the conventional load found with the

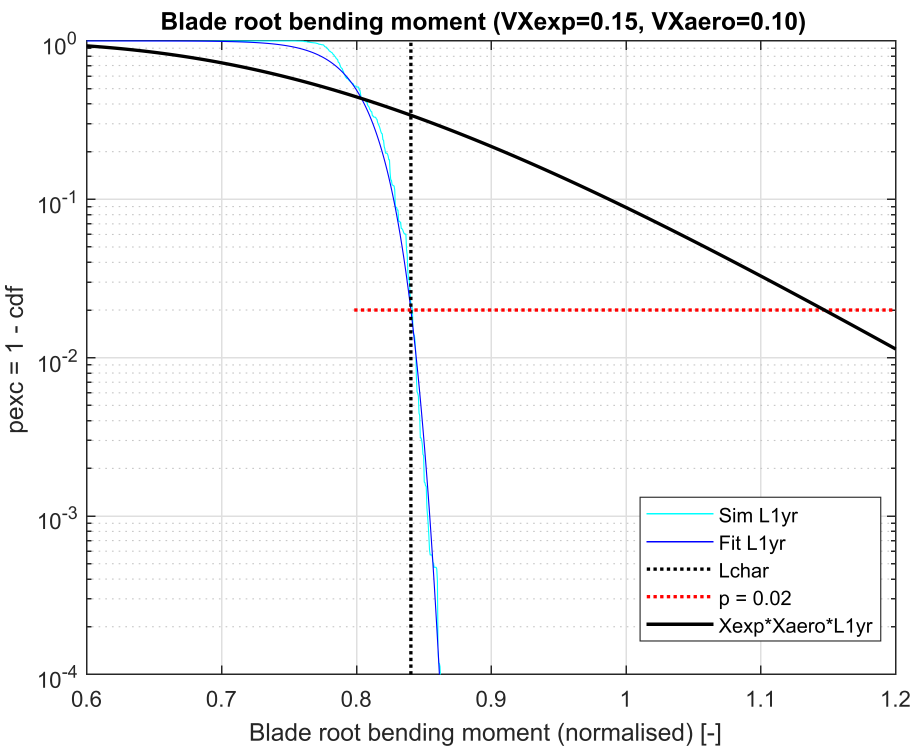

FIGURE 6 Distribution of blade root bending moment with probabilistic calculation.

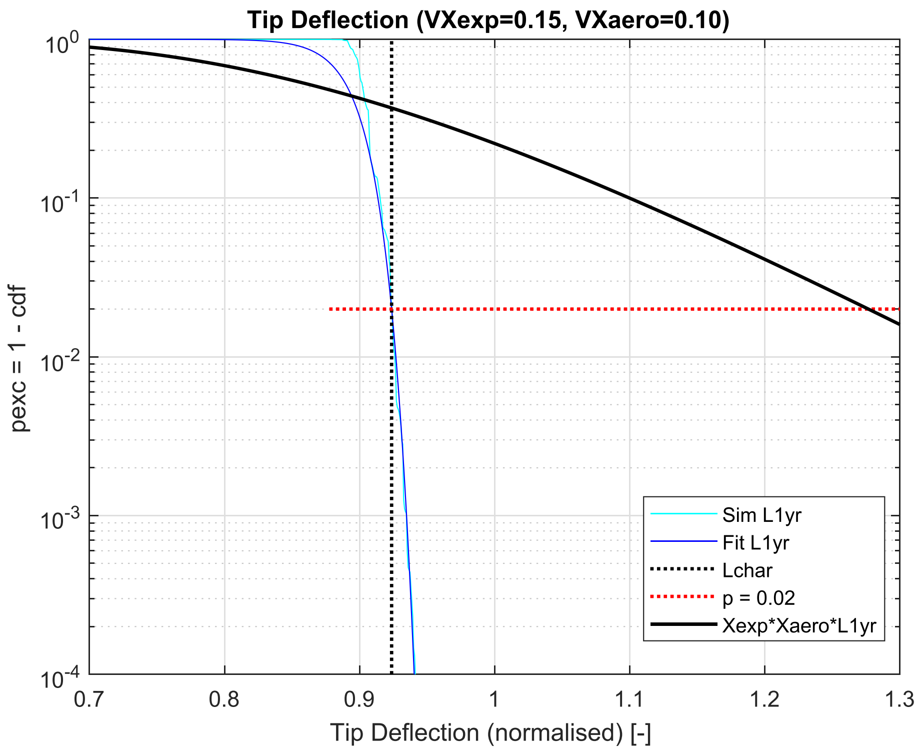

FIGURE 7 Distribution of maximum tip deflection with probabilistic calculation.

IEC 50-year gust. However according to the background document to IEC 61400-1[1] (see Dalsgaard Sørensen & Stensgaard Toft[13]), for probabilistic design, we must apply additional uncertainty to the distribution of the simulated annual extreme loads. As example, we use the following:

$$S_X = X_{exp} X_{aero} S \tag{42}$$

where X_{exp} is the exposure correction accounting for uncertainty in wind conditions at the turbine site (lognormal distribution with $COV = 0.15$) and X_{aero} is the uncertainty in the aerodynamic model, for example, in the values of the lift coefficient (Gumbel distribution with $COV = 0.10$). We can transform the blue line by multiplying each load with a random factor or we can use an analytical approach to transform the curve. Doing this leads to (intersection of black curve and red dotted line in Figure 6) the following:

$$S_{X,char} = 1.15 = 1.36\, S_{char} \tag{43}$$

This is close to the load factor that must be applied according to IEC 61400-1 (1.35) which should account for uncertainty in conventional design. It is encouraging that there is a good fit, but it must be stressed that this is just one example and it may be a coincidence; Results of probabilistic calculations tend to be sensitive to the choices made.

For the tip deflection, we find similar numbers: the 50-year deflection without additional uncertainty is 0.93 and with uncertainty 1.28 (see Figure 7).

6 | CONCLUSIONS

The method proposed by Larsen is reviewed; it is found that the calculation of the frequency of peak events is not correct. If the correct frequency of maxima is used, the speed excursions of longitudinal turbulence become significantly larger (Figure 2).

The exceedance probability curves of the maximum blade root bending moment and the maximum tip deflection under ECD are established by doing calculations on a grid with points (V, ζ_u, ζ_v). In the example calculation, the blade root bending moment is 84% of the conventional value and the tip deflection 93%.

If uncertainties for exposure and aerodynamics are incorporated, the 50-year load increases by approximately 1.35 which happens to be the usual load factor in IEC 61400-1. Hence, the consistency of probabilistic calculation according to the background document [13] is confirmed (at least within the simple framework used here).

Future research should include validation of the character of turbulence (for prediction of extreme excursions can it be assumed to be Gaussian or not?) and verification of the gust distribution method with long-term load measurements.

ACKNOWLEDGMENTS

We thank the two reviewers for useful comments and Mr Michiel Zaayer (TU Delft) for pointing out an inconsistency in the probabilistic calculation in Section 4.

PEER REVIEW

The peer review history for this article is available at https://www.webofscience.com/api/gateway/wos/peer-review/10.1002/we.2914.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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How to cite this article: Bierbooms W, Veldkamp D. Probabilistic treatment of IEC 61400-1 standard based extreme wind events. *Wind Energy*. 2024;1-21. doi:10.1002/we.2914

APPENDIX A: USE OF A THRESHOLD

Larsen[5] gives two options for the number of local extremes to be used (see Figure A.1):

1. All local maxima
2. Only the local maxima above a threshold $t = 2\sigma$

It will be shown that both options lead to the same result. To demonstrate that, it is convenient to consider the exceedance probability $Q = 1 - F$. We start with the first option.

Without a threshold. The exceedance probability Q_N of the largest of N values of the local maxima η_e can be approximated by (for large values of η_e) the following:

$$Q_N(\eta_{em}) = 1 - F_N(\eta_{em}) = 1 - F^N(\eta_{em}) = 1 - (1 - Q(\eta_{em}))^N \approx 1 - (1 - NQ(\eta_{em})) \approx NQ(\eta_{em}) \tag{A1}$$

With η_{em} the global maximum.

With a threshold. The variate of the second option will be indicated by ϵ. Its distribution is given by a truncated Rice cdf F_t which can be derived from the original cdf F (for arguments larger than t):

$$F_t(\epsilon) = \frac{F(\epsilon) - F(t)}{1 - F(t)} \tag{A2}$$

And thus, the exceedance probability Q_t is as follows:

$$Q_t(\epsilon) = 1 - F_t(\epsilon) = \frac{1 - F(\epsilon)}{1 - F(t)} = \frac{Q(\epsilon)}{Q(t)} \tag{A3}$$

For large values, the exceedance probability Q can be approximated by (Equation (6.6) in Cartwright and Longuet-Higgins[9]) as follows:

$$Q(\eta) = \sqrt{1 - \delta^2}\, e^{-\eta^2/2} \tag{A4}$$

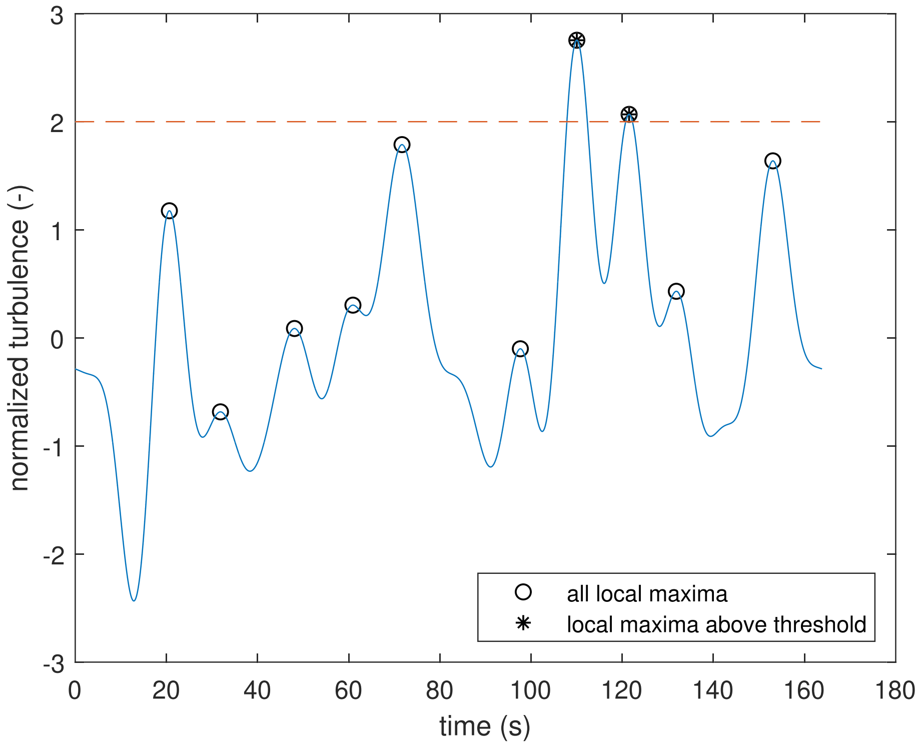

FIGURE A.1 Wind signal with threshold.

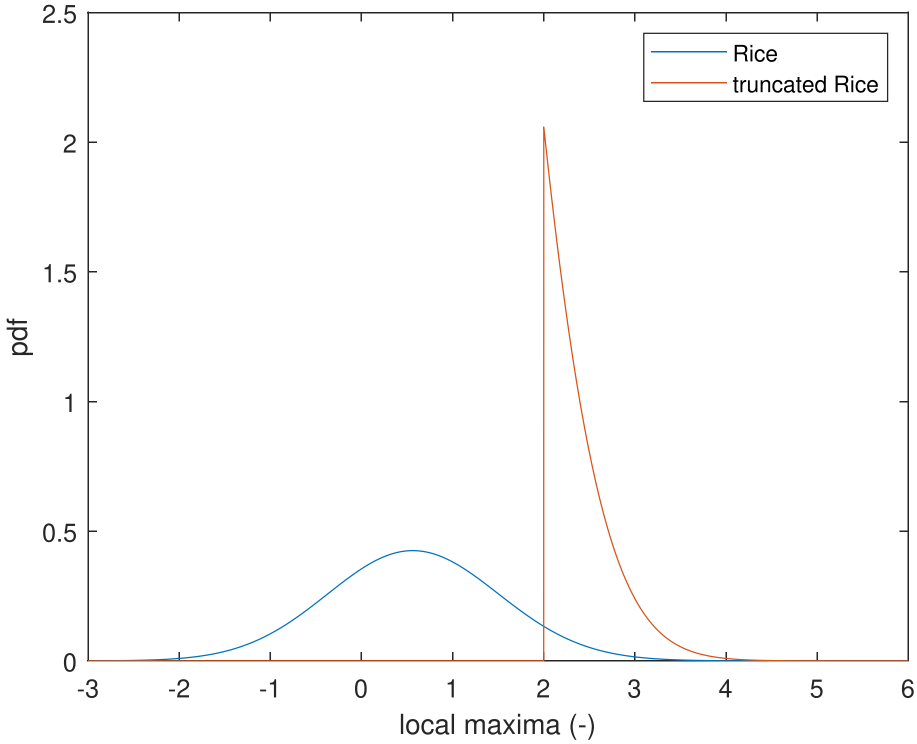

FIGURE A.2 Rice distributions: normal and truncated.

The number M within a period T that the threshold is exceeded (level crossing) is given by Rice[8] (Equations 3.6–11):

$$M = \nu_0 T e^{-t^2/2} \tag{A5}$$

The exceedance probability Q_{tM} of the largest of M values of ϵ can be approximated by (generalization of Equation (A1)) the following:

$$Q_{tM}(\varepsilon_m) \approx M Q_t(\varepsilon_m) \tag{A6}$$

With ϵ_m being the global maximum. With the aid of Equations (A3), (A4), and (17), we obtain

$$Q_{tM}(\epsilon_m) \approx \nu_0 T e^{-t^2/2} \frac{Q(\epsilon_m)}{\sqrt{1-\delta^2}} e^{t^2/2} \approx \nu_P T Q(\epsilon_m) \approx N Q(\epsilon_m) \tag{A7}$$

The same result is obtained as in case of the option "Without a threshold"; see Equation (A1)

APPENDIX B: FREQUENCY OF JOINT EVENTS

The probability of a wind speed V in between U and $U + dU$ is given by the following:

$$P(U < V < U + dU) = f(U)dU \tag{B1}$$

with f being the Weibull density (usually). The probability that the wind speed V is exactly equal to some specific value (i.e., with a range $dU = 0$) is equal to 0. In the same way, the probability of events (like local maxima of two independent random variables) happening exactly simultaneously is equal to 0. This implies that we have to introduce a fixed distance in time τ for which we consider events to be simultaneous; one may think of a distance of (say) 1 s. The expression for the frequency of a local maxima (peaks) of a Gaussian random variable was derived by Rice[8]:

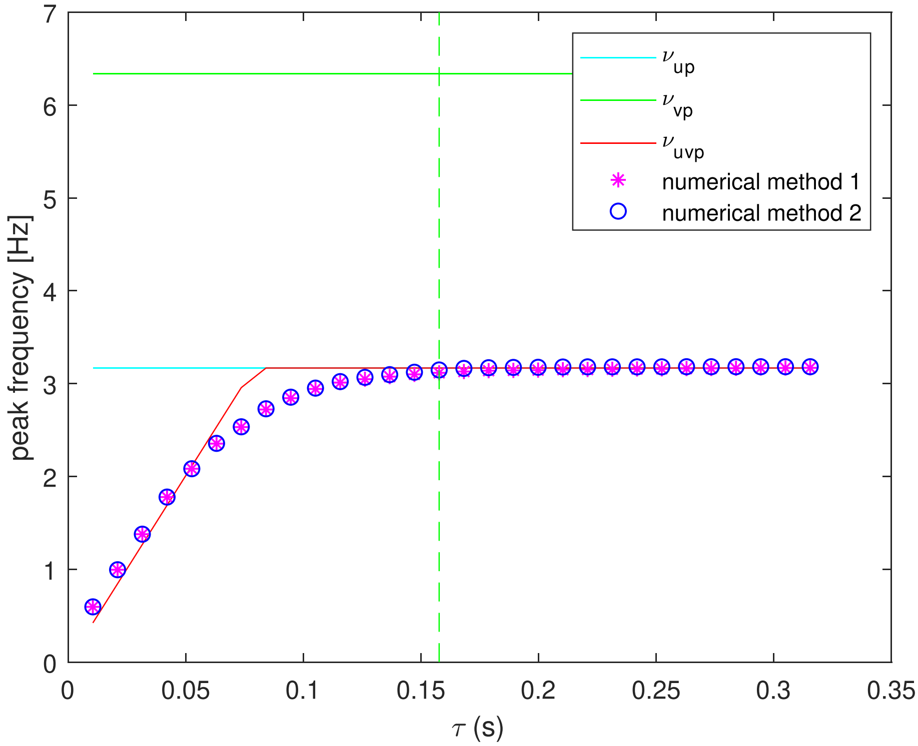

FIGURE B.1 Mean frequency of joint local maxima as function of time difference τ (red line); the green dashed line corresponds to $\tau = 1/\nu_{vp}$. *Note*: (1) To obtain different values of the peak frequencies, a cut-off frequency (of the Kaimal spectrum) of 5 and 10 Hz has been applied for the *u* and *v* component respectively. (2) Larsen's expression cannot be shown because it has no time constant τ.

$$\nu_p = \sqrt{\frac{m_4}{m_2}} \tag{B2}$$

For a small time interval Δt, the probability of a local maximum of u inside that interval is as follows:

$$P_{up} = \Delta t \sqrt{\frac{m_{u4}}{m_{u2}}} \tag{B3}$$

This can be understood as follows. If the frequency of maxima is ν_p [Hz], there are $\nu_p T$ maxima in time T. If we take a smaller time interval Δt, the average number of maxima within that interval is as follows:

$$P_{up} = \Delta t / T(\nu_p T) = \Delta t \nu_p. \tag{B4}$$

Because $\nu_p \Delta t < 1$, we can view this number as a probability. To have a joint event, a local maximum of v should be within a distance τ of the local maximum of u. Suppose the maximum for u occurs at $t(u_{max})$, then for simultaneity, we must have the following:

$$t(u_{max}) - \tau \leq t(v_{max}) \leq t(u_{max}) + \tau \tag{B5}$$

Therefore, we have to consider a time interval of 2τ. The probability of a local maximum of v inside such an interval is as follows:

$$P_{vp} = 2\tau \sqrt{\frac{m_{v4}}{m_{v2}}} \tag{B6}$$

The events are independent so the probability of a local maximum of v being within a distance τ of the u-maximum is as follows:

$$P_{uvp} = P_{up}P_{vp} = \Delta t\sqrt{\frac{m_{u4}}{m_{u2}}}2\tau\sqrt{\frac{m_{v4}}{m_{v2}}} \tag{B7}$$

This leads to the frequency of the joint events:

$$\tau \ll \frac{1}{\nu_{vp}} : \nu_p = \lim_{\Delta t \to 0} \frac{P_{uvp}}{\Delta t} = 2\tau \nu_{up}\nu_{vp} = 2\tau\sqrt{\frac{m_{u4}}{m_{u2}}}\sqrt{\frac{m_{v4}}{m_{v2}}} \tag{B8}$$

This relation holds for small τ, assuming $\nu_{up} \le \nu_{vp}$ (and vice-versa). For large values of τ, the mean frequency will be equal to the upper limit:

$$\tau \gg \frac{1}{\nu_{vp}} : \nu_p = \nu_{up} = \sqrt{\frac{m_{u4}}{m_{u2}}} \tag{B9}$$

This upper limit makes sense since the frequency of joint events can never be larger than the smallest of the frequencies of the individual events. The above relation is shown in Figure B.1. The theoretical value is compared with a count: After random wind signals have been generated for *u* (longitudinal) and *v* (lateral), the maxima are extracted and the time at which they occur. It is now a matter of going through all *u*-maxima and checking if there is a *v*-maximum within distance τ (i.e., $|t(v_{max}) - t(u_{max})| < \tau$). The frequency $f(\tau)$ is the number of combined maxima divided by the total time.

APPENDIX C: : MODE AND RETURN VALUE FOR GUMBEL DISTRIBUTION

If the distribution of annual extremes is known, some characteristic value must be chosen for design. The IEC definition is '[the value that has] an annual probability of exceedance of $1/N$ ("return period": *N* years)' (IEC,[1] definition (3.19)). This presents a problem with the $N = 1$ year return period: According to the definition, the exceedance probability would be 1 and hence the extreme would be zero. In his article, Larsen[6] estimates the most likely value (the mode of the distribution). We briefly discuss why this approach is correct.

The final distribution (step 5) is a Gumbel distribution. Let us consider the distribution of the gusts for one wind speed interval, for example, $\Delta V = 9 - 11$ m/s. Then the total time T_1 in 1 year is given by the chosen wind speed distribution ($T_1 = 365 \cdot 24 \cdot 3600 \cdot (F(11) - F(9))$), and the Gumbel cdf of the annual extreme ζ_m is (compared with Equation (2)):

$$F_1(\zeta_m) = \exp\left(-\exp\left(-\frac{\zeta_m}{2C} + \ln\kappa T_1\right)\right) \tag{C1}$$

with terrain constant *C* and frequency of local extremes κ. We are interested in the T_N-years value, for which

$$F_1(\zeta_m) = 1 - \frac{T_1}{T_N} = \exp\left(-\exp\left(-\frac{\zeta_m}{2C} + \ln\kappa T_1\right)\right) \tag{C2}$$

Solving for ζ_m yields:

$$\zeta_m = 2C\left(\ln\kappa T_1 - \ln\left(-\ln(1 - \frac{T_1}{T_N})\right)\right) \approx 2C\ln\kappa T_N \tag{C3}$$

We recognize the mode of the distribution of the T_N-years maxima:

$$F_N(\zeta_m) = \exp\left(-\exp\left(-\frac{\zeta_m}{2C} + \ln\kappa T_N\right)\right) \tag{C4}$$

Hence, instead of calculating the $F_1 = 0.98$ value of the distribution of 1-year extremes, we can take the mode of the distribution of 50-year extremes. Check: Some representative numbers are a follows: $V_{ave} = 6.65$ m/s, $V = 10$ m/s, $\Delta V = 2$ m/s, $C = 0.466$ (hilly), $T_1 = 3.8 \times 10^6$ s, $T_{50} = 1.9 \times 10^8$ s, $\nu_0 = 0.013$ Hz. This yields for the dimensionless excursions using the 98% fractile of F_1 and the mode of F_{50}, respectively (without reduction factors for rotor size and gust rise time):

$$\zeta_m(F_1) = 13.709 \quad \zeta_m(F_{50}) = 13.719 \tag{C5}$$

The difference is less than 0.1%; hence, it is reasonable to use the mode as the characteristic value. The second edition of IEC 61400-1[11] gives gust sizes for the 1-year return time and for the 50-year return time as (dimensionless values) follows:

$$g_1 = 4.8 \quad g_{50} = 6.4 \tag{C6}$$

If we assume that g_1 is the mode of the distribution of 1-year extremes and g_{50} is the 98% value, then we can derive that this distribution is as follows:

$$F_1(g) = \exp\left(-\exp\left(-\frac{g-m_1}{\beta}\right)\right) \tag{C7}$$

with $m = g_1 = 4.8$ and $\beta = 0.410$. Now the distribution of the N-years extremes is related to the distribution of 1-year extremes as follows (assume independence):

$$F_N(g) = F_1^N(g) = \left(\exp\left(-\exp(-\frac{g-m_1}{\beta})\right)\right)^N = \exp\left(-\exp\left(-\frac{g-(m_1+\beta\ln N)}{\beta}\right)\right) \tag{C8}$$

Which means that the distribution is shifted to the right with the new location parameter m_N:

$$m_N = m_1 + \beta \ln N \tag{C9}$$

This is shown in Figures C.1 and C.2 for $N = 50$. As expected, the horizontal shift in gust size is $\beta \ln N = 1.6$ and the vertical shift in exceedance probability for large g is a factor of 50.

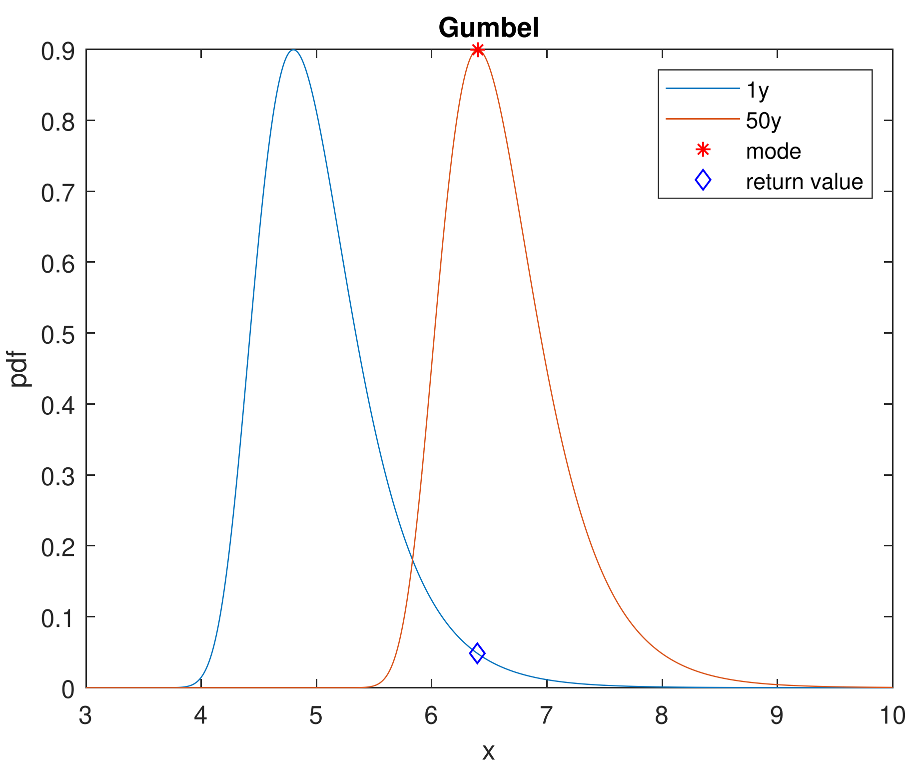

FIGURE C.1 Gumbel probability density.

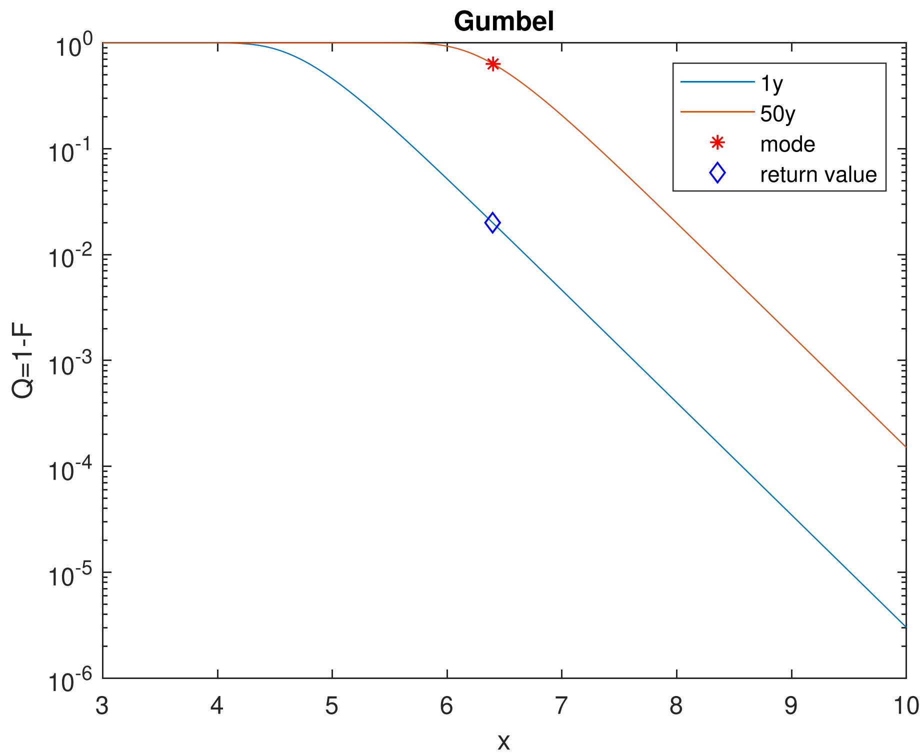

FIGURE C.2 Gumbel exceedance probability.

APPENDIX D: : GAUSSIAN TURBULENCE

In the Larsen model, it is assumed that turbulent velocity excursion is not Gaussian but Gamma distributed. If we assume that turbulence *is* Gaussian (something that is commonly done in a fatigue analysis), we can still use the Larsen model but now without the transformations in steps 2 and 5. The cdf corresponding to the pdf in Equation (11) is as follows:

$$F_{\eta_{em}}(\eta_{em}) = \exp\left(-\exp\left(-\frac{\eta_{em}^2}{2} + \ln(N\sqrt{1-\delta^2}\right)\right) \tag{D1}$$

The T-return value can be obtained by inverting the cumulative distribution corresponding to Equation (D1). The T-return value η_{eT} equals (Cartwright and Longuet-Higgins,[9] Equation (6-6)):

$$\eta_{eT} = \sqrt{2\ln(N\sqrt{1-\delta^2})} = \sqrt{2\ln \nu_0 T} \tag{D2}$$

Hence, the T-return value is (almost) equal to the mode of the distribution of the largest of N local maxima. Another choice as characteristic value could be the mean value; due to the asymmetry of the distribution, the mean value is larger than the mode (Cartwright Equation (6–14),[9] Davenport p190[10]):

$$\overline{\eta_{em}} = \sqrt{2\ln \nu_0 T} + \frac{\gamma}{\sqrt{2\ln \nu_0 T}} \tag{D3}$$

Here, $\gamma \approx 0.5772$ is the Euler–Mascheroni constant. This expression is often used in wind engineering applications. Davenport[10] also provides the standard deviation:

$$\sigma_{\eta_{em}} = \frac{\pi}{2\sqrt{3\ln\nu_0 T}} \tag{D4}$$

Larsen's (corrected) expression for the mode (or 50-year value) is (Equation (16)):

$$\text{mode}(\zeta_m) = m_L = 2C\ln\nu_0 T \tag{D5}$$

And the mode based on Gaussian turbulence is (Equation (D2)):

$$\text{mode}(\zeta_m) = m_R = \sqrt{2\ln\nu_0 T} \tag{D6}$$

The ratio of the Larsen and the Rice number is as follows:

$$\frac{m_L}{m_R} = C\sqrt{2\ln\nu_0 T} \tag{D7}$$

As an example, take $V_{ave} = 6.65$ m/s, $V = 10$ m/s, $\Delta V = 2$ m/s, $\nu_0 = 0.013, T_{50} = 1.9 \times 10^8, C = 0.337$ (flat terrain), then the ratio is 1.83. It is not clear whether the non-Gaussianity of turbulence is enough to explain the difference. In case one wants to apply the Corrected Larsen procedure, it is essential that it is first determined by site measurements whether the local turbulence is better described by a Gamma distribution or by a Gaussian one. See Figure D.1.

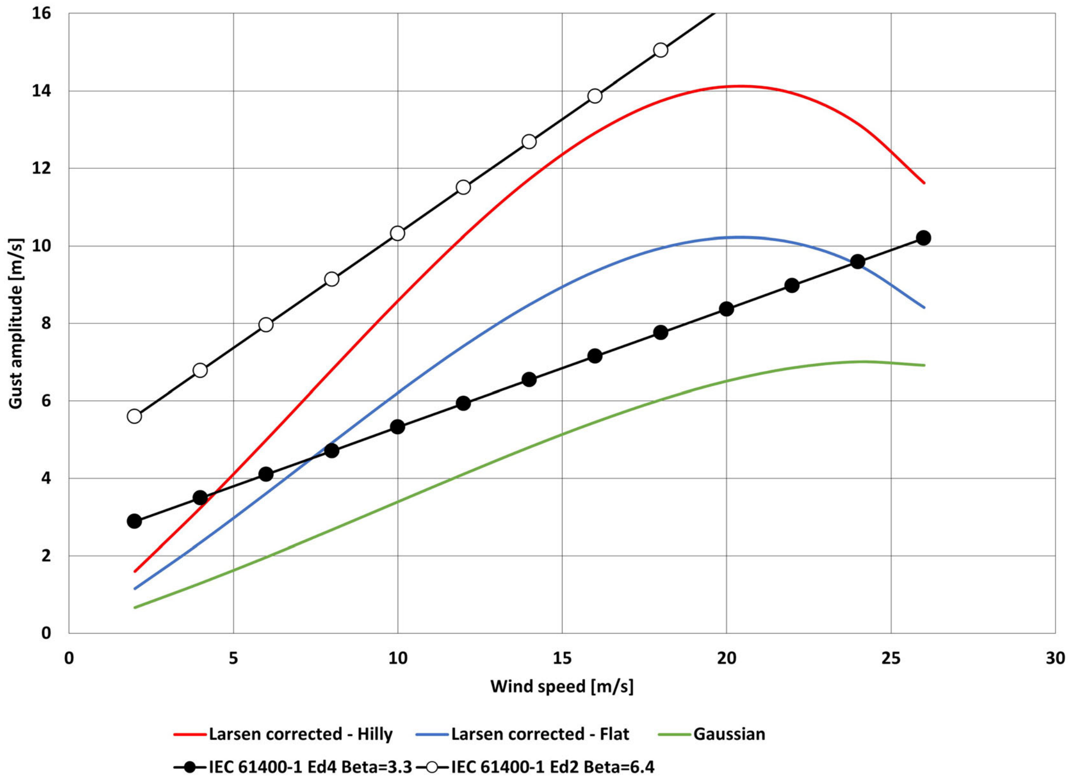

FIGURE D.1 Figure 1 with Gaussian mode added (green).

APPENDIX E: : VERIFICATION OF INTERNAL CONSISTENCY OF THE LARSEN PROCEDURE

In this appendix, we verify whether Larsen's (corrected) method is internally consistent. For the test, we generated 10^5 turbulent time series of $T = 6 \times 10^4$ s each (about 17 h) with a time step of 0.1 s. The turbulence is based on the Kaimal spectrum and made to be Gamma distributed according to Equation (4). Each time series has one global maximum, and the histogram of the 10^5 global maxima is shown in Figure E.1, together with the mode estimates according to Equation (16). In the "corrected Larsen" method, $\kappa T = \nu_0 T = 0.0154 \cdot 6 \times 10^4 = 923$, while in the original Larsen method, $\kappa T = 5.77 \times 10^{-4} \cdot 6 \times 10^4 = 35$. The mode according to "corrected Larsen" equals 7.68 m/s (red line) and coincides with the mode of the histogram; it deviates less than 0.2% from the exact value of 7.69 derived with the Gumbel distribution arising from the parent Rice distribution. For completeness and clarification, the time series of each of the steps from Section 2 are given in Figure E.2 as well as the corresponding histograms and distributions. Figure E.2 has time series on the left and histograms on the right. Note that in Step 4, the exact distribution (green dotted line) coincides with "Davenport" (Equation (11)).

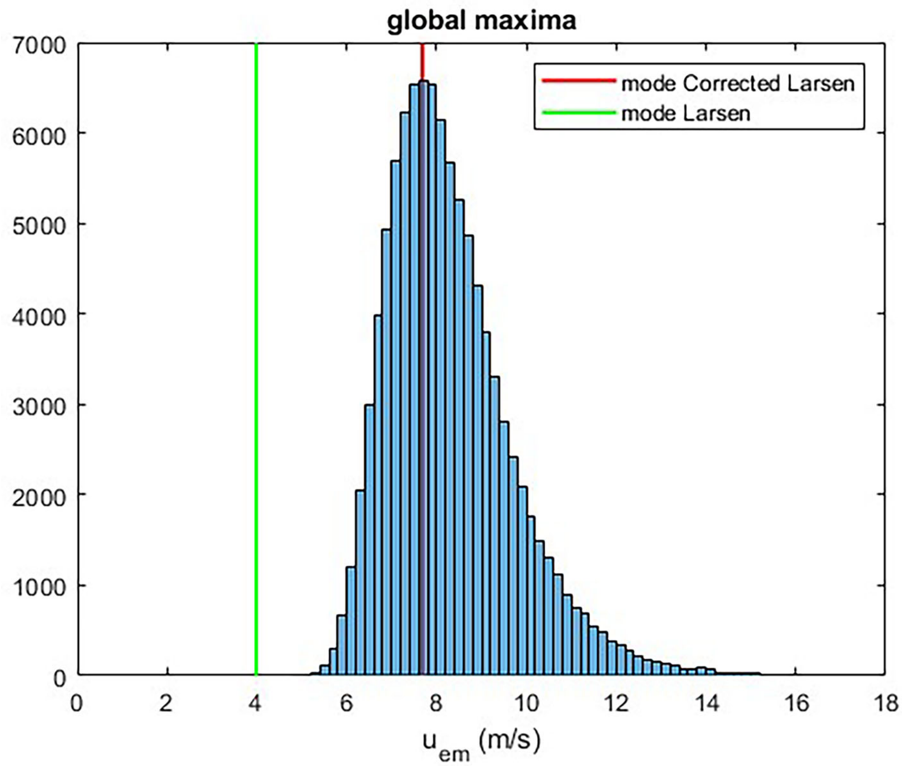

FIGURE E.1 Distribution of 10^5 global maxima ($T = 6 \times 10^4$ s, $U = 10$ m/s, $C = 0.4, f_{co} = 0.05$ Hz; this leads to $\delta = 0.89, \sigma = 1.41$ m/s, $\nu_0 =$ 0.0154 Hz, $\nu_P = 0.0337$ Hz, and $N = \nu_P T = 2021$).

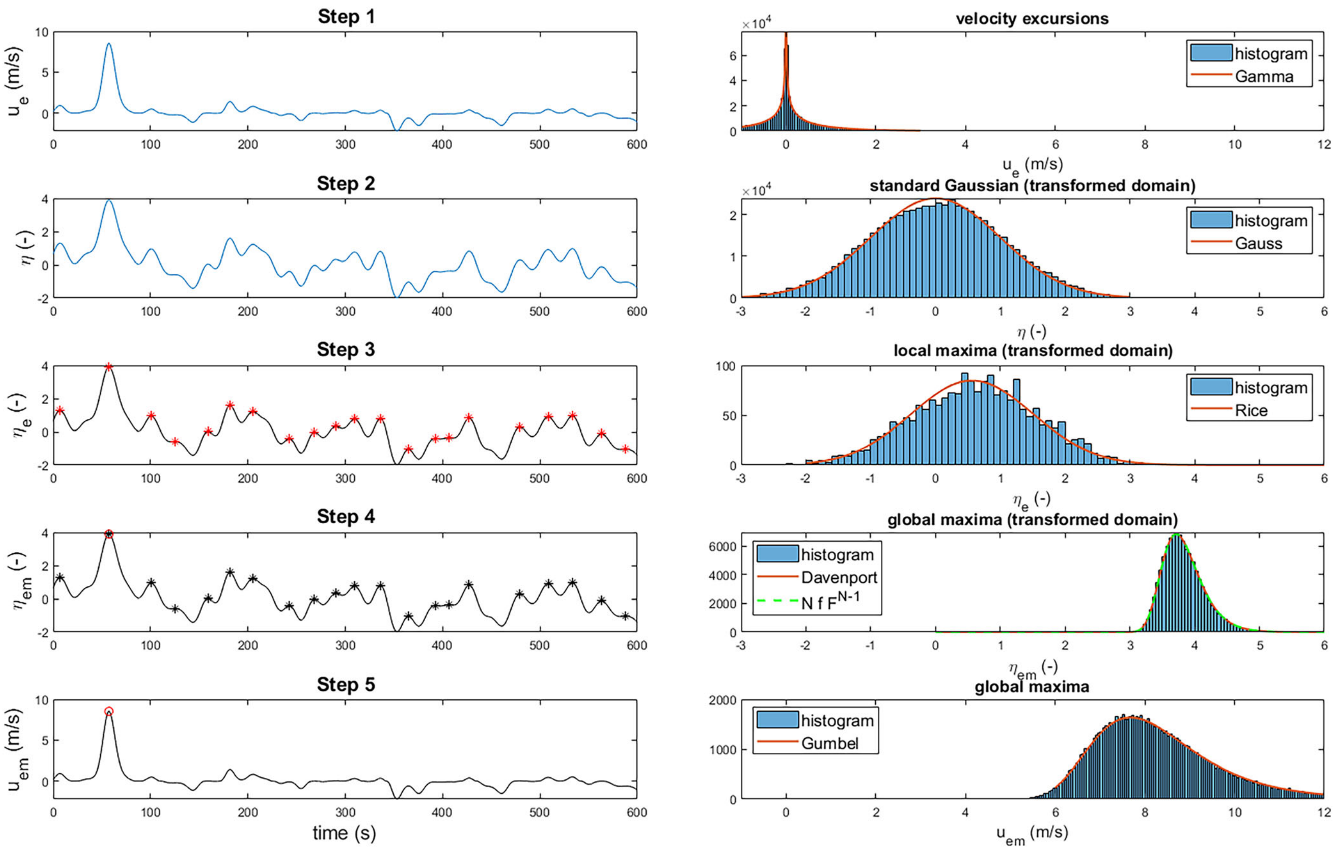

FIGURE E.2 Verification of Larsen's method. Left: time series of 60 s containing the global maximum, right: histograms. From top to bottom: (1) Original Gamma distributed signal and pdf of all values. (2) Signal transformed to Gaussian. (3) Local maxima of the Gaussian signal. (4) Global maxima of the Gaussian signal (Davenport distribution). (5) Signal back transformed; Gumbel distribution of global maxima.