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Determination of the extreme value in the response of wind turbines by means of constrained stochastic simulation

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Via so-called constrained stochastic simulation gusts can be generated which satisfy some specified constraint. In this paper it is used in order to generate specific wind gusts which will lead to local maxima in the response of wind turbines. By performing many simulations (for given gust amplitude) the conditional distribution of the response is obtained. By a weighted average of these conditional distributions over the probability of the gusts the overall distribution (for given mean wind speed) of the response is determined. The probabilistic method is demonstrated on basis of a linearized model of a stall regulated wind turbine. By considering a linear model the proposed probabilistic method could be validated: the determined 50 year response value corresponds with the theoretical value (based on Rice).

1. Introduction

The objective of this paper is to find the extreme response of wind turbines under stochastic wind loading. The (aero)dynamics of a wind turbine is in general far from linear and for the determination of the loading a wind field over the entire rotor disc has to be considered. As a result the 50 years response can not be assumed to happen during the 50 year wind.

In order to arrive at the 50-years extreme response of wind turbines it would be ideal to have available the wind data, at the specific location of the wind farm, over a period of say 500 year and unlimited computational power. The 50-years response could than be determined on basis of simple statistical analysis of the simulated response. Both conditions do of course not apply in practice. Instead in standards some deterministic gust shape is provided which should represent the 50 years extreme situation. However both the gust shape as well as amplitude is rather arbitrary. Furthermore the deterministic wind does not reflect the stochastic nature of turbulence. An alternative is to do simulations, as long as practical feasible, and extrapolate the results to the desired return period of 50 years applying extreme value theory.

Here we will present another alternative. In Ref. 1 so called constrained stochastic simulation is treated which allow to generate wind gusts which satisfy some specified constraint. E.g. one may generate time series around a local maximum with specified amplitude, or wind gusts which contain a prescribed velocity jump in a specified rise time. These wind gusts are embedded in a stochastic background in such a way that they are, in statistical sense, not distinguishable from real wind gust (with the same characteristics of the constraint).

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Constrained stochastic simulation enables us to limit the simulation to situations which contributes to the extreme response and skip all others. This saves a lot of computational time and limit the required extrapolation (and accompanying uncertainty) to the required return period.

In this paper we focus on the overall probabilistic method to come to the extreme response; constrained stochastic simulation is treated in detail in Ref. 1 and 6.

In order to present the probabilistic method to arrive at the 50 years response we will use a linearized dynamic model of a wind turbine which is excited by turbulence. The advantage of considering a linear system is that a theoretical analysis of the extreme loading is available (valid for normal random variables) which makes a validation of the probabilistic method possible (since the response of a linear system to a Gaussian input will also be Gaussian).

2. Reference system and theoretical 50 years response value

As reference system a generic fixed speed stall regulated wind turbine is taken. It is a 3-bladed 1 MW turbine with a rotor diameter of 51 m and a hub height of 55 m. For the determination of the extreme loads all mean wind speeds should be considered. Since we want to demonstrate the probabilistic method the calculations are performed for just one mean wind speed, 15 m/s. As example for the loading the blade root flapping moment is considered, but any other load signal could have been taken as well. Simulations with a random wind speed are performed with the Bladed software package. Just for convenience a uniform wind field (constant over the rotor disc) is taken. A linear system of the stall turbine (i.e. the transfer function between the wind input and the blade root flapping moment) is obtained by application of a system identification toolbox, Ref. 7. For this purpose the periodic excitations (rotor imbalance, gravity, tower shadow, wind shear) are set to zero.

Concerning the extreme loading of wind turbines we are primarily interested in the level of the response with a certain return period, since this determines together with the statistics of the structure strength the failure rate. For wind turbines it is common to consider a 50 year return period, so we are looking for the response level which is on average exceeded once in 50 year. For a normal random variable the mean level (up)crossing frequency of level r_n is given by Ref. 2:

$$\nu_n = \nu_0 e^{-\frac{r_n^2}{2\sigma^2}} \tag{1}$$

with $\nu_0=\sqrt{(\lambda/\sigma)}$ the mean zero crossing frequency and λ the second order spectral moment

From Eq. (1) the 50 years response r_{50} can easily be determined:

$$r_{50} = \sigma\sqrt{2\log(\nu_0 T_{50})} \tag{2}$$

with T_{50} the number of seconds in 50 year, i.e. $\nu_{50}=1/T_{50}$

The parameters ν_0 and σ can be determined from the spectral properties of turbulence (more specific: the auto correlation function) and the transfer function of the wind turbine. For the reference system the 50 years response of the blade root flapping moment turns out to be equal to 0.268 MNm (6.5 σ).

3. A probabilistic method to find the extreme response

The proposed probabilistic method is based on conditional distributions. The gust amplitudes of the stochastic wind input of the wind turbine will be denoted as random variable $\boldsymbol{x}$ and the local maxima of the response as random variable $\boldsymbol{y}$. The marginal densities are $f_x(x)$ and $f_y(y)$; the joint density is $f(x,y)$ and $f_c(y) = f(y\,|\,x)$ is the conditional density of $\boldsymbol{y}$ upon observing $\boldsymbol{x}$=x. The following well-known relations exist:

$$f_x(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \tag{3}$$

$$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \tag{4}$$

$$F_x(x) = \int_{-\infty}^{x} f_x(\alpha)\,d\alpha \tag{5}$$

$$F_y(y) = \int_{-\infty}^{y} f_y(\beta)\,d\beta \tag{6}$$

$$F(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(\alpha,\beta)\,d\alpha\,d\beta \tag{7}$$

$$f_c(y) = \frac{f(x,y)}{f_x(x)} \tag{8}$$

$$F_c(y) = \int_{-\infty}^{y} f_c(\beta)\,d\beta \tag{9}$$

Combination leads to:

$$\begin{aligned} F_y(y) &= \int_{-\infty}^{y}\int_{-\infty}^{\infty} f(\alpha,\beta)\,d\alpha d\beta = \int_{-\infty}^{y}\int_{-\infty}^{\infty} f_c(\beta)\, f_x(\alpha) d\alpha d\beta \\ &= \int_{-\infty}^{\infty} F_c(y)\, f_x(\alpha) d\alpha \approx \sum F_c(y)\, n_x \end{aligned} \tag{10}$$

with n_x the probability ('fraction of time') that a local maximum is within the discretized amplitude intervals of random input $\boldsymbol{x}$

So the distribution of response $\boldsymbol{y}$ can be obtained through a weighted summation of the conditional distributions. The conditional distributions can be determined by simulations of wind gusts which are obtained by constrained stochastic simulation, say 100-1000 simulations for each amplitude. The amplitude of the wind gusts may be varied from say 1σ to 5σ. In case the wind input are maximum amplitude gusts the probability n_x can be derived from the density of local maxima, Ref. 2. For extreme rise time gusts the required expression can be found in Ref. 1. In this paper gusts are used which leads to a local maximum in the response. For these gusts an analytical expression for the gust probability is given in Ref. 6 (Eq. (D.11)).

The advantage of the here presented method is that through constrained random simulation gusts with any required high amplitude can be generated. So by just a limited number of simulations extreme responses are obtained. The method has already been used for the determination of the extreme loading of offshore structures, e.g. Ref. 4, and wind turbines Ref. 5. Eq. (10) is used in order to obtain the distribution of the local maxima in the response. To this end the conditional distributions F_c are fitted to some theoretical distribution (EVD or 3 parameter Weibull). The long term response, say 50 year, is obtained via:

$$F_{50}(y) = F_y(y)^{N_{50}} \tag{11}$$

with N_{50} the number of local maxima in 50 years; it is assumed that the local maxima are independent.

The method has been validated by comparison with results from long simulations, up to 50 times 40 min. (Ref. 5) or 100 times 3 hour (Ref. 4). Also some practical problems have been attacked as e.g. the required number of simulations for each amplitude and the amplitude range. However up to now no verification have been done based on theoretical results; since we deal with a linear system such a verification is now possible.

A more fundamental problem inherent to constrained stochastic simulation is that there can be no one-to-one relation between local maxima in the input and the response, since the number of local maxima in the input will generally differ from that of the response, Ref. 2. So it is unclear which local maximum in the response should be associated to a prescribed gust input. For large amplitude gusts this will not be a problem; a large gust will result into a large response so one may just pick the maximum value in the response, Fig. 1.

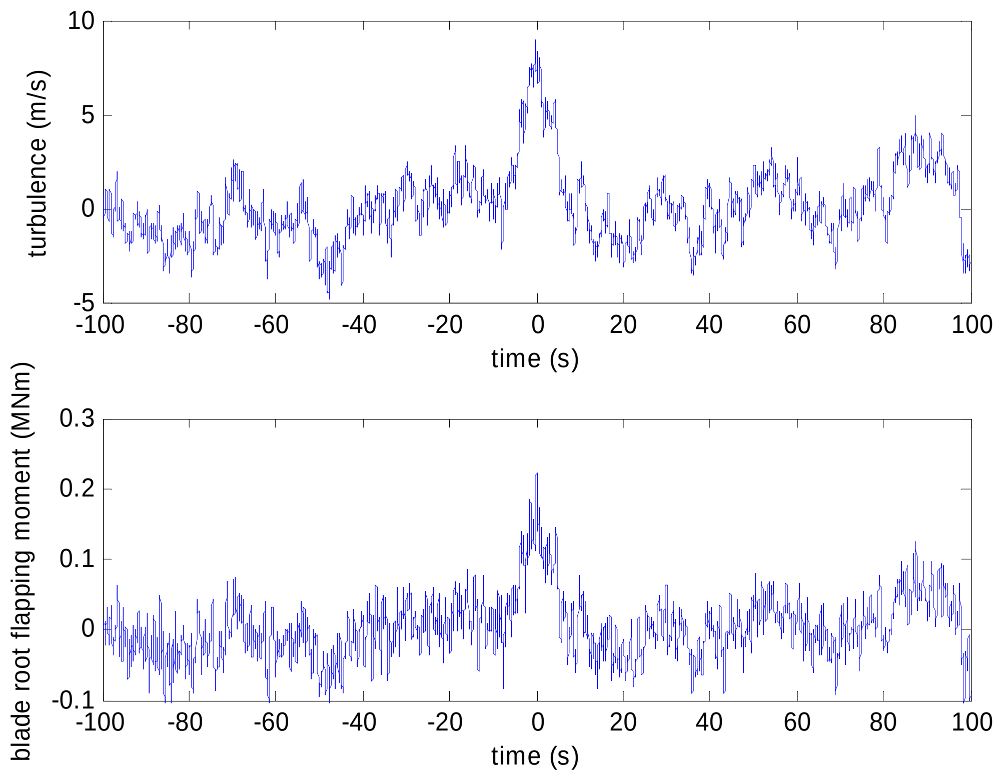

Figure 1. Example of constrained stochastic simulation; 5σ gust at t=0 s
Top: turbulence (input)
Bottom: blade root flapping moment (response).

For gust amplitudes up to say 3σ the situation is different, see Fig. 2. Since the constrained wind gusts are embedded in a stochastic background one may expect variations of order of 3σ due to the background alone. It is now not obvious which response level should be associated with the prescribed gust. This means that it is not possible to determine the conditional distributions for these gusts. In previous work it is not clear how this problem is treated. Fortunately the contribution of modest gusts to the tail of the response distribution will be limited. It is therefore more natural to rewrite Eq. (10) in terms of the exceedance probability:

$$G_y(y) = 1 - F_y(y) = 1 - \int_{-\infty}^{\infty} F_c(y) f_x(\alpha) d\alpha = \int_{-\infty}^{\infty} (1 - F_c(y)) f_x(\alpha) d\alpha$$
$$\int_{-\infty}^{\infty} G_c(y) f_x(\alpha) d\alpha \approx \sum G_c(y) n_x \tag{12}$$

In the next section it will be investigated if we can use constrained stochastic simulations for the prediction of the 10-min. maxima in the response rather than for all local maxima. By doing so a one-to-one relation is established between input and response. Another advantage is that in that case, as will be shown, the contribution of modest gusts on the final result is negligible.

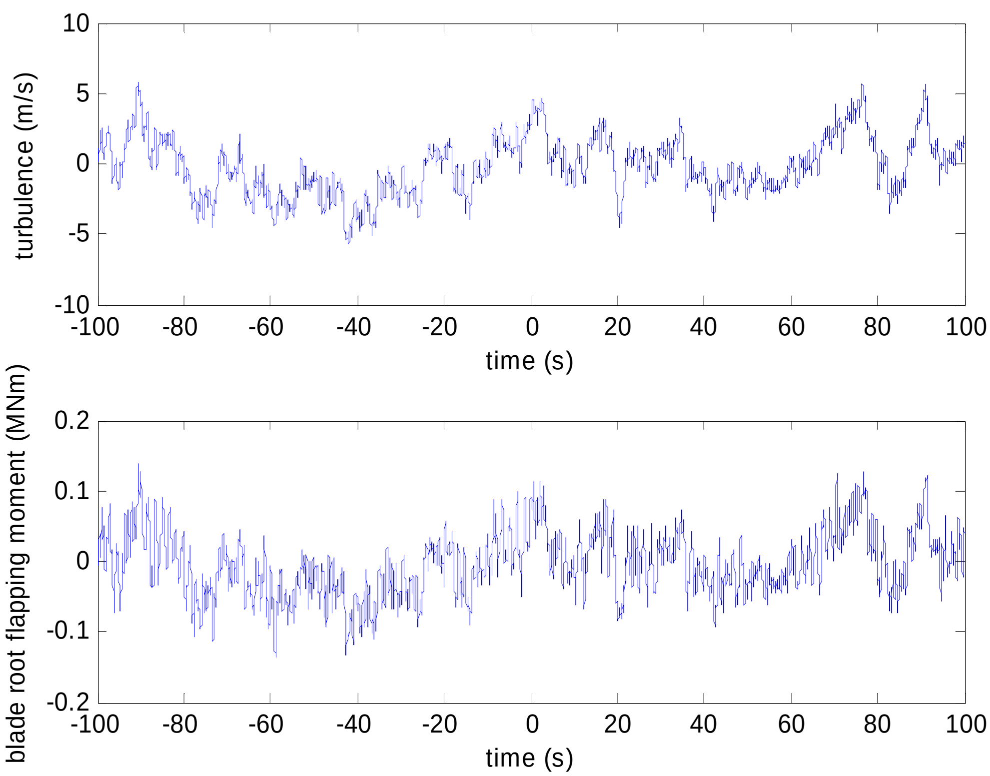

Figure 2. Example of constrained stochastic simulation; 2σ gust at t=0 s.

4. Application of 10-min. maxima for the probabilistic method

In this Section it will be investigated if 10-min. maxima, rather than all maxima, can be used in the probabilistic method outlined in the previous Section. This will solve the indicated problems.

For the reference wind turbine 50 years of simulation is performed, consisting out of 2628000 time series of 10-min. each. For each 10-min. interval the maximum input value as well as maximum response is determined. In Figure 3 a close up is shown for the largest maxima together with the 1 hour and 1 day maxima. It turns out that almost all 10-min. maxima are also the 1 hour and 1 day maximum, so the maxima of input and response seems to occur simultaneous (are dependent).

The joint distribution F of all local maxima, Eq. (7), and joint distribution F_p for the maxima x_p and y_p during period p is:

$$F_p(x,y) = F(x,y)^{N_p} \tag{13}$$

with N_p the number of local maxima (of the input) in time period p

We will now examine if we can express $F_{yp}(y)$ in terms of conditional distributions like Eq. (10). The joint density f_p is given by:

$$f_p(x,y) = \frac{\partial^2 F_p(x,y)}{\partial x \partial y} \tag{14}$$

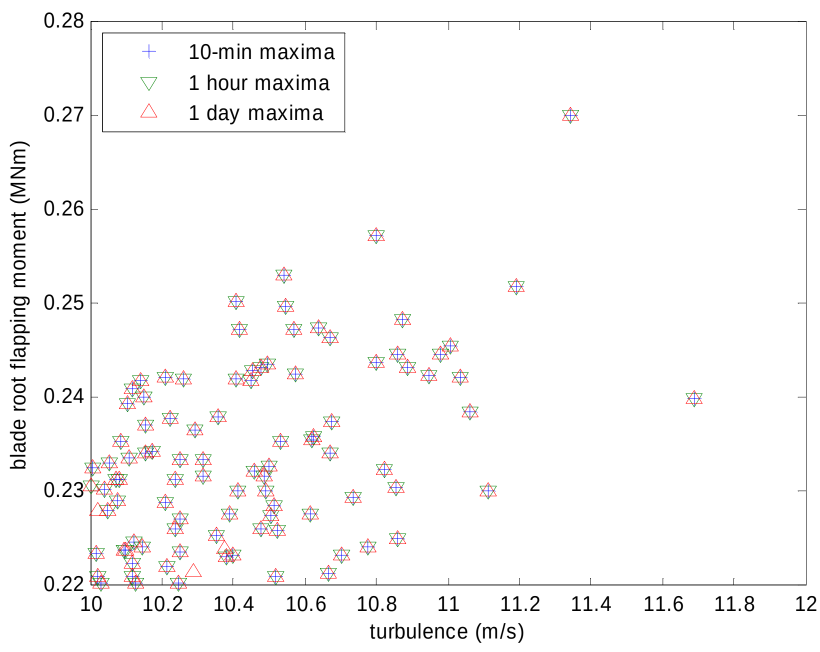

Figure 3. The 10-min, 1-hour and 1 day maxima in response and input.

From Eq. (13) we obtain:

$$\frac{\partial F_p(x,y)}{\partial x} = N_p\, F(x,y)^{N_p-1} \frac{\partial F(x,y)}{\partial x} \tag{15}$$

and

$$\begin{aligned}\frac{\partial^2 F_p(x,y)}{\partial x \partial y} &= N_p(N_p-1)\, F(x,y)^{N_p-2} \frac{\partial F(x,y)}{\partial x}\frac{\partial F(x,y)}{\partial y} + \\ &N_p\, F(x,y)^{N_p-1} \frac{\partial^2 F(x,y)}{\partial x \partial y}\end{aligned} \tag{16}$$

The (first order) partial derivatives have an upper bound given by:

$$\frac{\partial F(x,y)}{\partial x} = \int_{-\infty}^{y} f(x,\beta)\, d\beta < \int_{-\infty}^{\infty} f(x,y)\, dy = f_x(x) \tag{17}$$

and a similar expression for the partial derivative to y. For large x and y the densities $f_x(x)$ and $f_y(y)$ go to 0 while F(x,y) is approaching 1, thus the 1st term at the right hand side of Eq. (16) can be neglected. So we have:

$$f_p(x,y) \approx N_p\, F(x,y)^{N_p-1} f(x,y) \tag{18}$$

Similar to Eq. (8) the conditional density for the maximum per period is given by:

$$f_{cp}(y) = \frac{f_p(x,y)}{f_{xp}(x)} \tag{19}$$

Assuming that the maxima per period are independent the marginal distribution F_{xp} equals:

$$F_{xp}(x) = F_x(x)^{N_p} \tag{20}$$

so the density f_{xp} is given by:

$$f_{xp}(x) = \frac{dF_{xp}(x)}{dx} = N_p\, F_x(x)^{N_p-1} \frac{dF_x(x)}{dx} = N_p\, F_x(x)^{N_p-1} f_x(x) \tag{21}$$

For large values of y, $F(x,y) \approx F_x(x)$, thus:

$$f_{xp}(x) \approx N_p\, F(x,y)^{N_p-1} f_x(x) \tag{22}$$

Finally, combination of Eq. (18), (19) and (22) leads to:

$$f_{cp}(y) \approx \frac{N_p\, F(x,y)^{N_p-1} f(x,y)}{N_p\, F(x,y)^{N_p-1} f_x(x)} = \frac{f(x,y)}{f_x(x)} = f_c(y) \tag{23}$$

So, the conditional densities obtained via constrained stochastic simulation can also be used for the determination of the distribution of the maxima of the response over some time period, as long as we are interested in the tail of the distribution:

$$F_{yp}(y) = \int_{-\infty}^{\infty} F_{cp}(y)\, f_{xp}(\alpha)\, d\alpha \approx \int_{-\infty}^{\infty} F_{c}(y)\, f_{xp}(\alpha)\, d\alpha \approx \sum F_c(y)\, n_{xp} \quad \text{for large y} \qquad (24)$$

with n_{xp} the probability ('fraction of time') that a maximum over period p is within the discretized amplitude intervals. This probability is given by:

$$n_{xp} = F_{xp}(x_{upp}) - F_{xp}(x_{low}) \qquad (25)$$

with F_{xp} the distribution of the maximum of period p, Eq. (20), and x_{upp} and x_{low} the upper and lower bound resp. of the amplitude intervals.

In Section 5 this equation will be applied for the reference case and compared with the theoretical result.

In this Section it is assumed that the constraint gusts concerns maximum amplitude gusts. In fact the method can be applied for any constraint gust as long as there is a clear correlation between input and response (like Fig. 3). If this is not the case (e.g. by considering troughs instead of crests) the here presented method makes no sense since the conditional distributions F_c for these kind of gusts will be more or less equal to the required distribution F_y of the response (so there is no advantage compared to normal, unconstrained, random simulations).

The method can be applied for any desired time period p. For determination of the extreme response of wind turbines different mean wind speeds (and possibly different turbulence intensities) have to be considered. In this respect it is most convenient to consider 10-min. (or 1 hour) periods since that is the common interval for mean wind speeds.

5. Determination of the 50-years response based on constrained simulations

As mentioned before, constrained stochastic simulation allows generation of wind gusts which satisfy some specified constraint. E.g. one may generate time series around a local maximum with specified amplitude. Here a rather special kind of constrained simulation is considered. Gusts are generated which will lead to a local extreme (of unspecified value) in the response of the wind turbine. Details including analytical expressions of this kind of gusts can be found in Ref. 6. In order to generate such gusts the turbulence spectrum as well as the transfer function of the linearized wind turbine model should be known.

Gusts have been used with 5 amplitude levels (from 3σ up to 7σ; 1000 simulations for each amplitude). The results of the constrained simulations are depicted in Fig. 4. Please note that constrained simulation is still stochastic; i.e. 1000 simulations leads to 1000 different values of the maximum response (from which a distribution can be constructed). The range of amplitudes is chosen such that small gust amplitudes up to 3σ are excluded (see discussion at the end of Section 3) and that the maximum value which can be expected in 50 years is included. The latter can be estimated to be 6σ or 6.5σ and follows from the inverse normal distribution $N^{-1}(1-1/n_{50},0,\sigma)$ with n_{50} the number time points in 50 years (n_{50}=1.6e9 or 3.2e10 for a sampling rate of

1 Hz and 20 Hz resp.). The empirical distributions from the constrained stochastic simulations is given by $F_e=i/(N+1)$ with i=1 to N and N=1000.

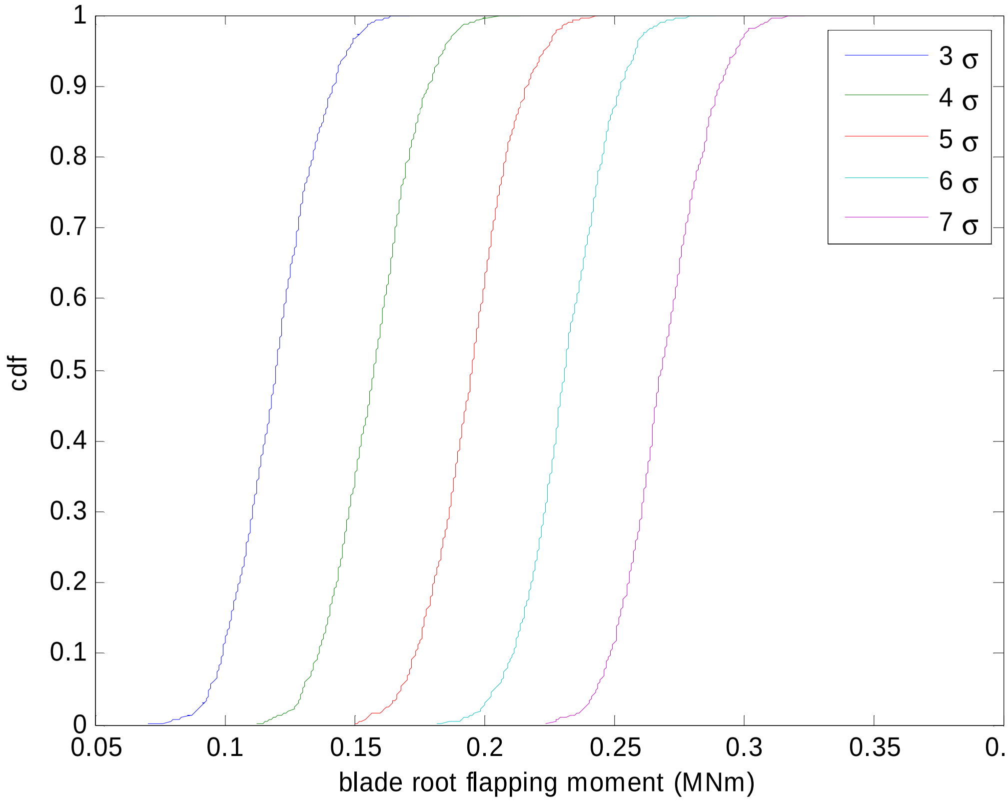

Figure 4. The distributions of the maxima in the response obtained via constrained stochastic simulation for several gust amplitudes.

The overall distribution of the response can now be calculated via the weighted summation of the 5 conditional distributions, Eq. (24). In order to do this calculation it must be possible to evaluate the conditional distributions over the range of response level y of interest. Here we apply a straightforward scheme:

$$
\begin{aligned}
&y \le y^*_{\min}: && F_c(y)=0\\
&y^*_{\min} < y \le y_{\min}: && F_c(y)=H_\gamma(y)\\
&y_{\min} < y \le y_{\max}: && F_c(y)=F_e(y)\\
&y_{\max} < y \le y^*_{\max}: && F_c(y)=H_\gamma(y)\\
&y > y^*_{\max}: && F_c(y)=1
\end{aligned}
\tag{26}
$$

with y_{min} and y_{max} the 10th and 90th percentile of the response from the constrained simulation (for a given amplitude) and y^*_{min} and y^*_{max} the estimated endpoints (left and right resp.) of the distribution. As fit function the generalized Pareto distribution (GP) is used:

$$H_{\gamma}(y) = 1 - (1+\gamma y)^{-1/\gamma} \quad (27)$$

with γ the extreme value index.

So in total five regions are distinguished. For values in the range of the constrained simulations the value is determined from (linear) interpolation of the empirical distribution F_e. For values larger than this range, extrapolation is necessary. It is possible that he response is bounded. This implies that the GP can have a negative extreme value index and thus (right) endpoint y^*_{max}:

$$y^*_{\max} = -\frac{a}{\gamma} + b \quad (28)$$

Above y^*_{max}, F_c is set to 1. For the left tail the minima are fitted to a GP (for this purpose it is common practice to do a multiplication by -1 so maxima are obtained and the same methods / routines can be used).

As example of the fit procedure (upper tail) we present the results for the constrained simulations with amplitude 5σ. The moment estimators for the GP parameters are taken from Ref. 3. In Fig. 5 the estimated parameters are shown as function of k (the number of order statistics). The values for k=100 have been used onwards.

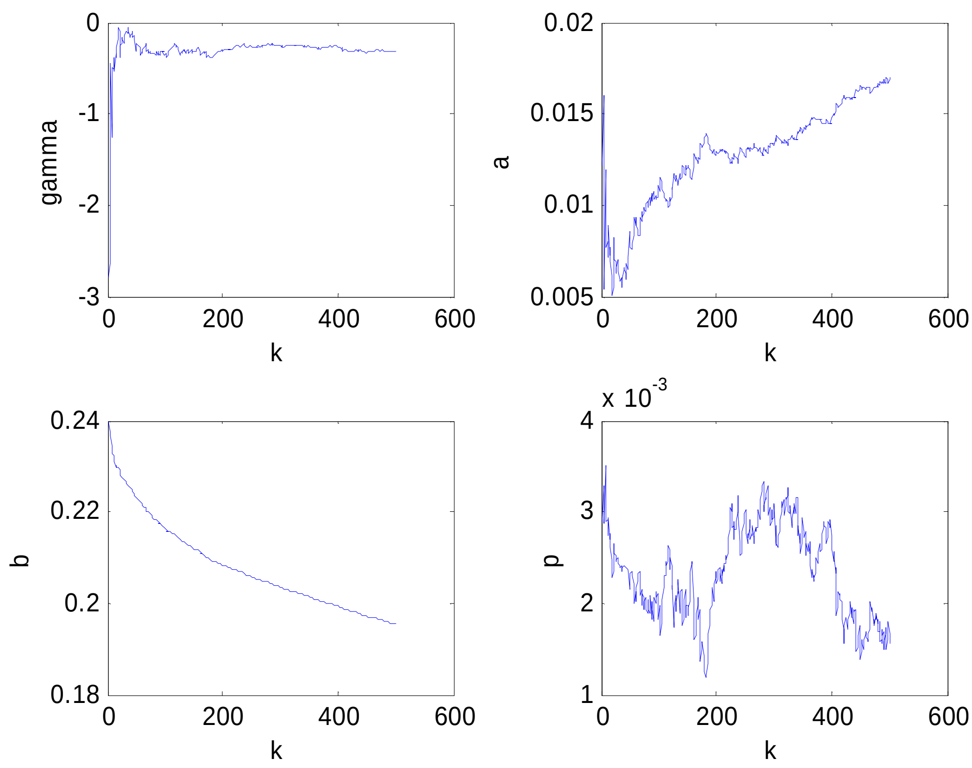

Figure 5. Estimated GEV parameters (extreme value index γ, scale parameter a and location parameter b) for the constrained simulations for gust amplitude 5σ and the resulting tail estimate p=1-F.

The resulting fit is shown in Fig. 6; in order to present the tail behavior also $–\log(1-F_c)$ as well as $\log(F_c)$ is plotted. It turns out, for this particular case, that the estimate of the right endpoint is just a little bit larger than the largest value obtained from the constrained simulations.

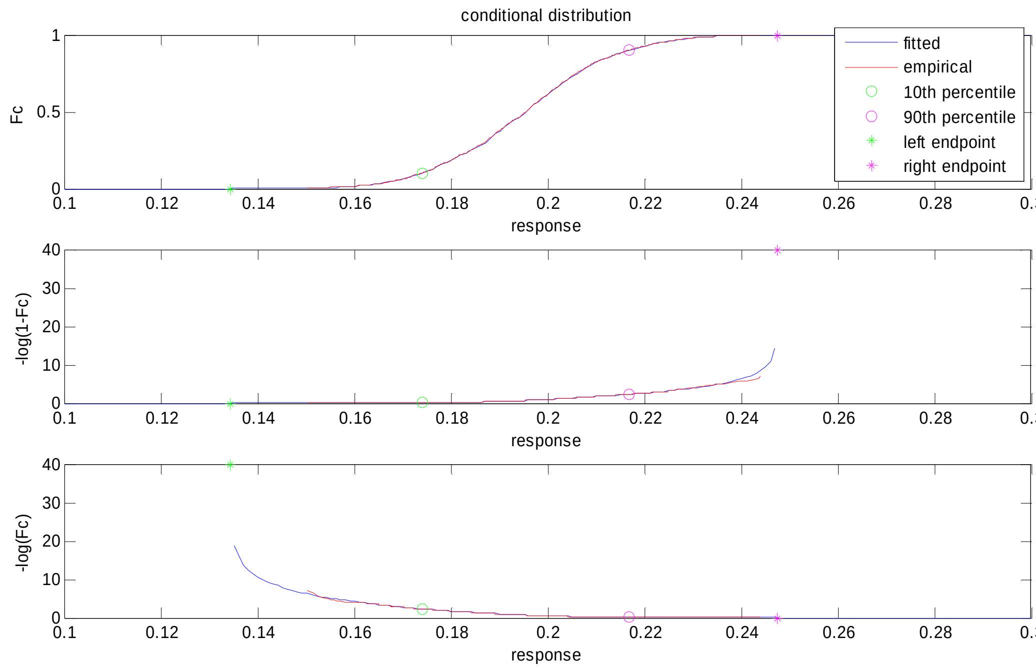

Figure 6. The conditional distribution obtained via constrained stochastic simulation; identical to Fig. 4 (amplitude 5σ) and the extrapolation of the tails (middle: right tail; bottom: left tail).

Finally the distribution of the response can be estimated by use of Eq. (24), see Fig. 7. The estimated 50 years value equals 0.276 MNm (this is the value for which the distribution equals $1-1/T_{50}$ with T_{50}=2628000 the number of 10-min. series in 50 years). This is in agreement with the theoretical value of r_{50}=0.268 MNm.

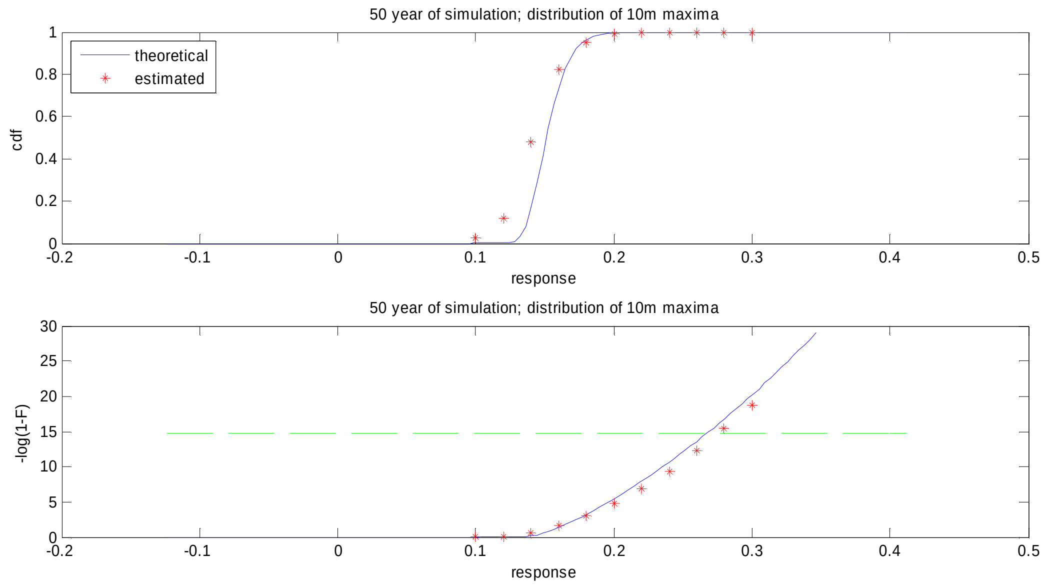

Figure 7. Comparison between the estimated and theoretical distribution of the 10-min. maxima in the response. The horizontal green dashed line indicates the 50-years return period.

The result can be improved by doing the constrained stochastic simulations for more gust amplitudes. To this end the contribution of each gust amplitude to the summation of Eq. (24) is shown in Fig. 8, for y=0.268 MNm (50 years value). It turns out that the final result is in fact dominated by the 6.5σ gust. This also justifies our claim that gusts up to 3σ (which can not be generated via constrained simulation) are negligible.

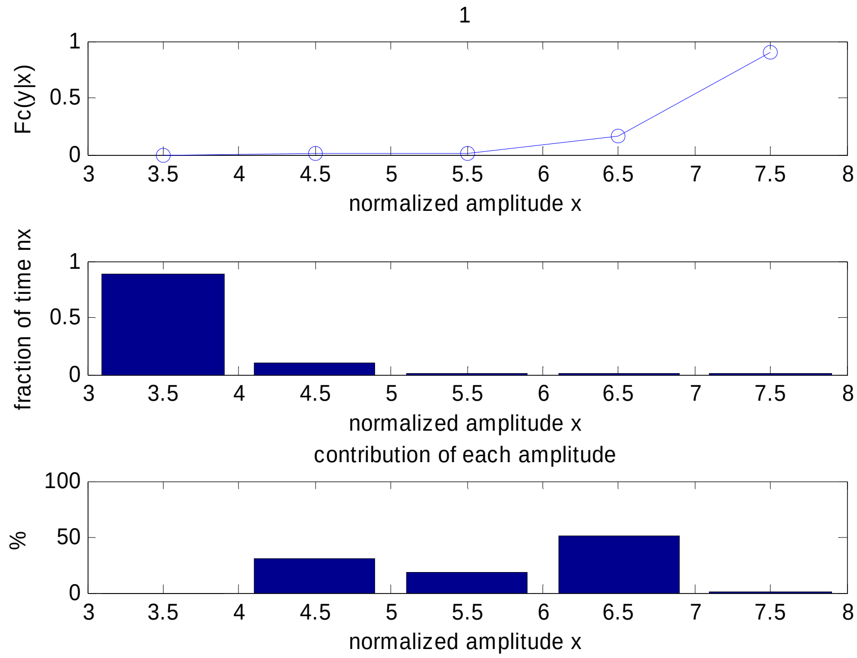

Figure 8. Top: The value of the conditional distribution (exceedance probalitity) for y=0.268 MNm for gust amplitudes 3.5σ-7.5σ (with a step of 0.5σ).
Middle: the fraction of time n_{xp} for each gust amplitude.
Bottom: Contribution of each gust amplitude to the tail estimation of the response (i.e. the normalized product of the values of the top and middle graph).

Based on Fig. 8 we redo the calculations for gust amplitudes in between 4σ and 7σ in steps of 0.25σ (13 in total), see Fig. 9. This leads to an estimate of the 50 years response value of 0.271 MNm which is only 1% larger than the theoretical one.

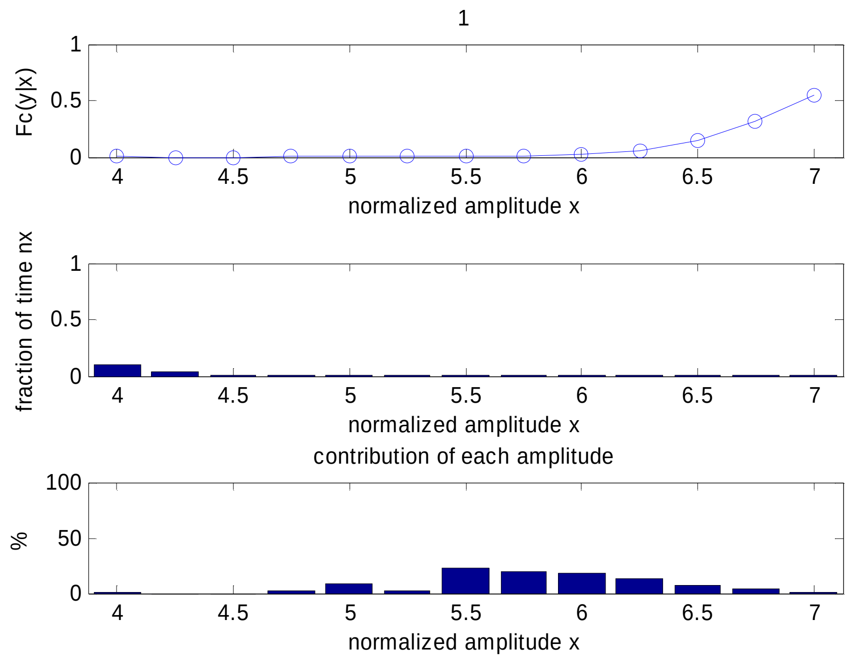

Figure 9. Top: The value of the conditional distribution (exceedance probalitity) for y=0.268 MNm for gust amplitudes 4σ-7σ (with a step of 0.25σ).
Middle: the fraction of time n_{xp} for each gust amplitude.
Bottom: Contribution of each gust amplitude to the tail estimation of the response (i.e. the normalized product of the values of the top and middle graph).

6. Concluding remarks

The shape of the histograms (bottom Fig. 8 and 9) is determined by the product of the conditional distribution (for given response level and gust amplitude as parameter; top graphs of Fig. 8 and 9) and the density of the gust amplitudes. Assuming that the input and response are correlated (see Fig. 3) it can be expected that the exceedance probability increases with gust amplitude, as can be seen in Fig. 8 and 9 (top). The density of gust amplitudes f_{cp} is determined by the combination of the density of the gust amplitudes (Eq. (D.11) of Ref. 6) and Eq. (20) with N_p the number of local maxima in 10-min.: N_p=2032 for the reference case. The density, which is slightly non-Gaussian, is plotted in Fig. 10 (the corresponding histogram for amplitude ranges are depicted in the middle graphs of Fig. 8 and 9). As a result a bell shape histogram can be anticipated.

The contributions of each gust amplitude to the estimation of the tail probability, as shown in Fig. 8 and 9 bottom, provides a rational base for the determination of the range of gust amplitudes as well as the required discretization. This may be preferred to base the decision on e.g. the 99 percentile of the estimated response distribution (either by comparison with normal, i.e. unconstrained, simulations or considering the convergence).

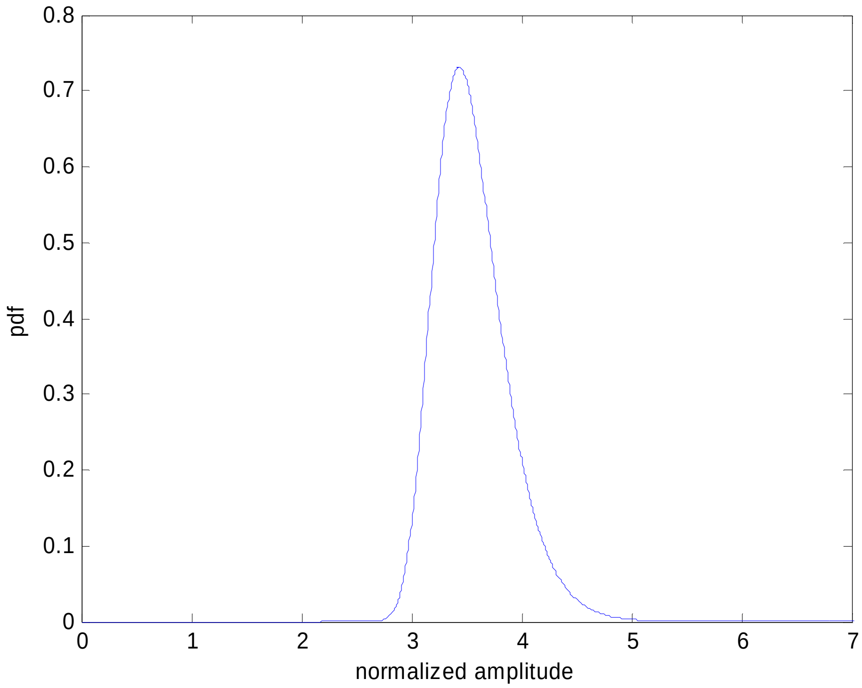

Figure 10. The density of the 10-min. maxima of the gusts amplitudes.

In order to demonstrate the validity of Eq. (24) which is based on some assumptions we have done the same analysis for the 10-min. gust amplitudes as above in case of 1 day maxima; the result is shown in Fig. 11. As expected the estimate is poor for small values of the response value but very good for the tail. In fact the 50 years estimate is 0.270 MNm, so even better than the estimate based on 10-min. maxima.

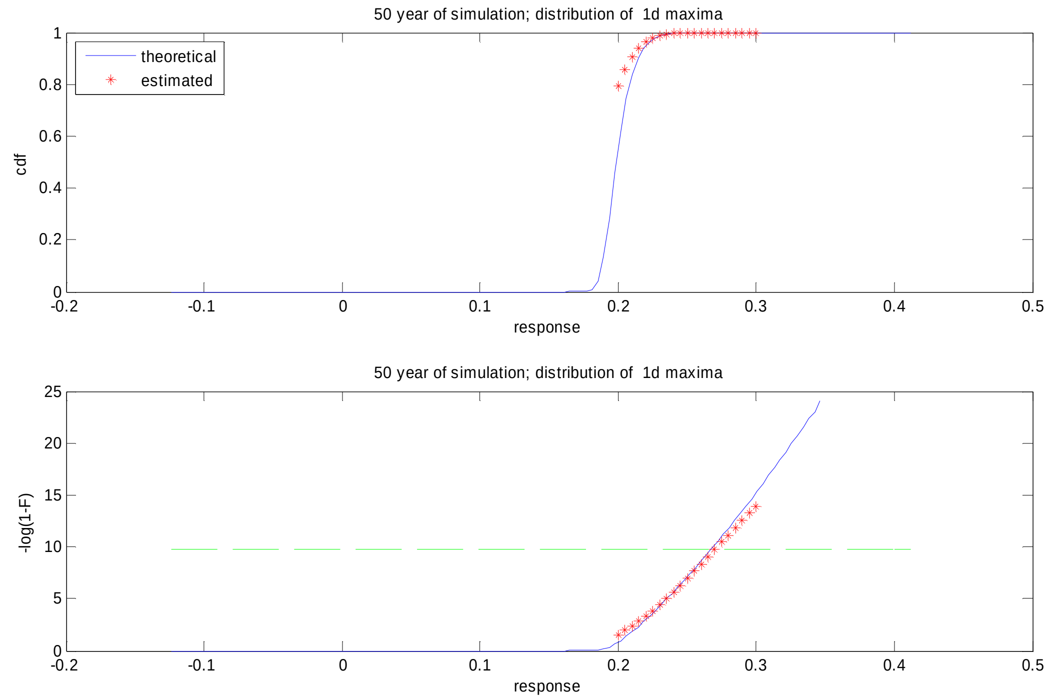

Figure 11. Comparison between the estimated and theoretical distribution of the 1-day maxima in the response.

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