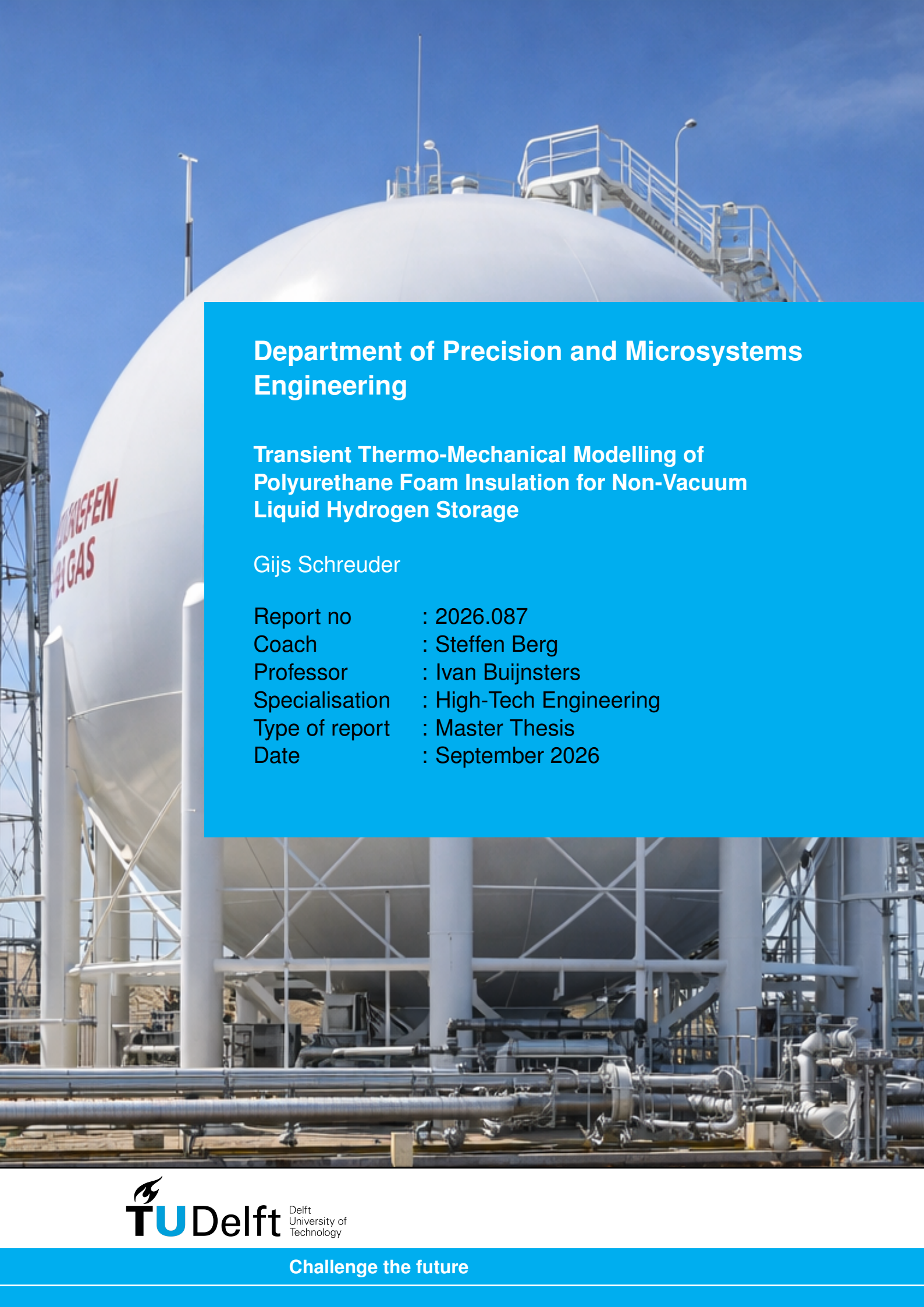




Department of Precision and Microsystems Engineering

Transient Thermo-Mechanical Modelling of Polyurethane Foam Insulation for Non-Vacuum Liquid Hydrogen Storage

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DELFT UNIVERSITY OF TECHNOLOGY

MASTER THESIS

Transient Thermo-Mechanical Modelling of Polyurethane Foam Insulation for Non-Vacuum Liquid Hydrogen Storage

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Preface

This thesis, *Transient Thermo-Mechanical Modelling of Polyurethane Foam Insulation for Non-Vacuum Liquid Hydrogen Storage*, marks the completion of my Master of Science in Mechanical Engineering, track High-Tech Engineering, at Delft University of Technology. The research was carried out between November 2025 and September 2026 at the Energy Transition Campus Amsterdam of Shell Global Solutions International B.V. The subject of the project appealed to me for two reasons. The first is the role that liquid hydrogen may come to play in the energy transition. Storing hydrogen in liquefied form is one of the few routes to holding and moving it in the quantities a future energy system would demand, and the tanks that make this possible are only now being designed at industrial scale. The second is that this is an open engineering problem rather than a closed academic one. The insulation concept studied here is being developed and tested while it is still being understood, and the cracking observed during laboratory thermal cycling was the starting point of this work rather than one of its results. Working on a question to which the industry does not yet have the answer, with access to experimental observations and a representative tank geometry, is what made the project attractive to me. The result is a finite element model that follows the stress and elastic strain in the foam through cool-down, cold hold and warm-up, and that identifies where and when the insulation is most at risk of damage. The thesis is written for readers with a background in mechanical or materials engineering. A working knowledge of the finite element method is assumed, but no prior familiarity with cryogenic insulation is required. I hope this work offers a useful step towards foam insulation systems that can be relied upon at cryogenic temperature.

Acknowledgements

I would really like to thank my supervisors, Ivan Buijnsters at Delft University of Technology and Steffen Berg at Shell, for their guidance and constructive feedback throughout this project. Their questions consistently pushed me to substantiate what I claimed rather than to assert it, and the thesis is considerably better for it. Within Shell, I also want to thank Evren Unsal for the close guidance during my time at the Energy Transition Campus, and Casimir van Doorne for the perspective of the hydrogen team. This research rests on what was made available to me. The willingness to share an unresolved problem openly shaped both the questions this thesis asks and the way it answers them.

Abstract

Large-scale liquid hydrogen storage motivates non-vacuum insulation concepts for double-walled storage tanks in which rigid polyurethane foam is bonded directly to the inner steel shell, behind a passive perlite layer. During filling of the tank with liquid hydrogen, the insulation material is cooled from ambient temperature towards 20 Kelvin, at which hydrogen liquefies at atmospheric pressure. Cracking sounds reported during laboratory thermal cycling, predominantly on warm-up, suggest potential mechanical failure of the polyurethane insulation material. The mechanical integrity is therefore a governing design concern, yet the stress and elastic strain developing within the polyurethane foam during these temperature transients remain insufficiently quantified. This thesis investigates how transient thermal gradients and constrained thermal contraction generate stress and elastic strain concentrations in bonded cryogenic foam insulation.

A coupled transient thermo-mechanical finite element model was built in ANSYS Mechanical APDL, using an axisymmetric formulation for a homogenised, isotropic, linear elastic foam with temperature-dependent conductivity and stiffness, and thermal contraction referenced to the stress-free state at 293 Kelvin. Three configurations were analysed, progressing from an idealised foam–steel corner geometry to an experiment-specific assembly and a spherical tank model, so that the simplest model provides a reference for the more complex ones. The cold region was driven from 293 Kelvin to 20 Kelvin along a smooth, monotonically decreasing ramp, held cold and warmed to 100 Kelvin, with cooling-rate, cooling-path, warm-up-rate and material-property sensitivity cases. The mechanical formulation is linear elastic and contains no damage law: failure is assessed in a separate post-processing step, so the results are risk indicators rather than validated failure predictions. Within that scope, the model does identify where and when cracking is most likely, through two criteria: the first principal stress against the cryogenic tensile strength, and the first principal elastic strain against the strain-to-failure. The strain criterion governs, and it localises the risk to the bonded foam–steel interface and its geometric discontinuities at the end of cool-down.

Two mechanisms produce this loading. The first is a stationary tensile concentration at the foam–steel interface, whose stress magnitude is set by the product of foam stiffness, expansion mismatch and total temperature drop, and locally amplified at the corner. The second is an internal tensile peak that travels outward with the cooling front and is largest at the end of cool-down. The mismatch sets the amplitude of both the interface concentration and the internal peak, whereas the foam stiffness scales only the stress and leaves the elastic strain unchanged, making the problem strain-controlled. The severity then depends on the accumulated temperature drop rather than on the cooling path: neither slower nor stepwise cooling reduced the peaks, and warm-up peaks nowhere substantially exceeded cool-down at matched cold-side temperatures. Both features also persist in the corner-free spherical tank. Reducing the foam–substrate expansion mismatch is therefore a more effective mitigation than adjusting the cooling protocol. The observed warm-up cracking cannot be reproduced by an elastic model and most plausibly reflects history-dependent damage initiated at the cold state. Future work should add a cohesive-zone formulation and characterise the foam down to 20 Kelvin.

Contents

Preface	i
Acknowledgements	i
Abstract	ii
Nomenclature	viii
1 Introduction	1
1.1 Liquid Hydrogen Storage	2
1.2 Insulation Challenges	3
1.3 Thermo-Mechanical Integrity	4
1.4 Thesis Outline	4
2 Literature Review	5
2.1 Insulation Requirements	6
2.2 Polyurethane Foam Insulation	7
2.2.1 Limitations of Vacuum-Based Insulation	8
2.2.2 Motivation for Polyurethane Foam Insulation	8
2.3 Microstructure and Installation Effects	9
2.3.1 Cellular Structure and Density	9
2.3.2 Spray Application, Anisotropy and Defects	9
2.3.3 Aging and Moisture Effects	11
2.4 Damage Mechanisms and Behaviour	11
2.4.1 Thermal Response and Effective Conductivity	11
2.4.2 Mechanical Behaviour and Property Uncertainty at Low Temperature	12
2.4.3 Thermal Contraction and CTE Mismatch	13
2.4.4 Transient Temperature Gradients and Constrained Contraction	14
2.4.5 Interface Damage, Cracking and Stress Concentrations	15
2.5 Theoretical Framework	15
2.5.1 Transient Heat Conduction and Temperature Gradients	16
2.5.2 Thermal Contraction and Strain Decomposition	17
2.5.3 Constrained Contraction and Stress Formation	18
2.6 Modelling Approaches	20
2.7 Research Gap and Questions	21
3 Research Methodology	22
3.1 Finite Element Modelling Approach	23
3.2 Modelling Strategy	24
3.3 Geometry and Configurations	26
3.4 Element Type and Formulation	29
3.5 Material Properties	30
3.6 Initial and Boundary Conditions	33

3.7	Mesh Strategy	38
3.8	Numerical Setup	41
3.9	Modelling Scenarios	42
3.10	Post-Processing	44
3.11	Verification and Validation	49
3.11.1	Mesh Sensitivity: Peak and Path-Based Convergence	49
3.11.2	Time-Step and Boundary-Condition Checks	52
3.11.3	Long Cold-Hold and Near-Steady-State Check	53
3.11.4	Analytical Thermal Check	54
3.12	Assumptions and Limitations	56
4	Results and Discussions	58
4.1	Baseline Cool-Down Response	59
4.2	Internal Strain Interpretation	65
4.3	Thermal Mismatch and Gradients	69
4.4	Cooling Protocol Sensitivity	73
4.5	Warm-Up Response	75
4.6	Experiment-Specific Model	78
4.7	Spherical Tank Geometry	80
4.8	Failure Relevance	83
5	Conclusions & Recommendations	86
5.1	Conclusions	87
5.1.1	Material Behaviour	87
5.1.2	Stress and Strain Development	87
5.1.3	Damage Interpretation and Experimental Comparison	88
5.1.4	Answer to the Main Research Question	88
5.2	Recommendations	89
5.2.1	Modelling Recommendations	89
5.2.2	Experimental Recommendations	90
5.2.3	Design Recommendations	90
	References	92
A	Stress State and Principal Stresses	98
A.1	The Stress State at a Point	99
A.2	Principal Stresses and Principal Strains	99
B	Additional Plots	101
C	ANSYS Scripts	108
C.1	Overview	109
C.2	Simplified Mechanism Model	110
C.3	Experiment-Specific Model	115
C.4	Spherical Tank Model	121
C.5	Thermal Load Variations	127
C.6	Material Property Variations	132
C.7	Mechanical Boundary Conditions	134
C.8	Mesh Definitions	135
C.9	Post-Processing Procedure	137
C.10	Analytical Thermal Benchmark	144

List of Figures

1.1	Growth in LH ₂ storage tank size	2
2.1	Heat ingress and boil-off gas generation in an LH ₂ tank	7
2.2	Multiscale representation of rigid polyurethane foam architecture	10
2.3	Temperature-dependent compressive stress–strain curves of polyurethane foam .	13
2.4	Thermal expansion of rigid PU foam vs. temperature	14
2.5	Schematic illustration of the two main constraint mechanisms	20
3.1	The three model configurations used in this study	25
3.2	Dimensioned cross-section of the simplified foam-metal geometry	27
3.3	Axisymmetric cross-sections of the experiment-specific and spherical tank models	28
3.4	Cross-section of the experiment-specific geometry	29
3.5	Boundary conditions applied to the simplified mechanism model	35
3.6	Boundary conditions applied to the experiment-specific and spherical tank models	35
3.7	Imposed cold-region temperature profiles	36
3.8	Full imposed thermal history for the warm-up cases	37
3.9	Mapped foam mesh at two refinement levels	39
3.10	Extraction paths used for post-processing	46
3.11	Mesh sensitivity of peak foam response	50
3.12	Path-based mesh convergence along the diagonal path	51
3.13	Path-based mesh convergence along the lower radial path	51
3.14	Time-step sensitivity along the lower radial path	52
3.15	Boundary-condition sensitivity along the lower radial path	53
3.16	Long cold-hold check along the lower radial path	54
4.1	Temperature contours in the simplified mechanism model	60
4.2	Through-thickness temperature profiles during cool-down	60
4.3	Displacement magnitude during the baseline cool-down	61
4.4	First principal stress contours during cool-down	62
4.5	First principal stress along the lower radial path	62
4.6	First principal elastic strain along the lower radial path	64
4.7	First principal elastic strain contours during cool-down	64
4.8	Baseline cool-down fields in the simplified mechanism model	65
4.9	Elastic strain components along the lower radial path	67
4.10	Thermal strain components along the lower radial path	67
4.11	First principal stress direction vectors during cool-down	69
4.12	First principal elastic strain direction vectors during cool-down	69
4.13	First principal elastic strain and temperature along the lower radial path	70
4.14	CTE sensitivity along the lower radial path	71
4.15	Young’s modulus sensitivity along the lower radial path	72
4.16	Cooling-rate sensitivity along the lower radial path	74

4.17	Stepwise cooling sensitivity along the lower radial path	75
4.18	Cool-down and slow warm-up along the lower radial path	76
4.19	Warm-up-rate sensitivity along the lower radial path	77
4.20	Experiment-specific model response along the radial foam path P1	79
4.21	Evolution of the first principal elastic strain field in the experiment-specific model	79
4.22	Spherical tank field results	81
4.23	Spherical tank radial foam-path response	81
4.24	Global maximum in the foam during cool-down and cold hold	83
A.1	Stress state at a point in general Cartesian coordinates	99
B.1	Comparison of free-mesh refinement levels	102
B.2	Comparison of alternative mesh-control strategies	102
B.3	Mapped-mesh comparison along the upper radial path	103
B.4	Mapped-mesh comparison along the lower radial path	103
B.5	Temperature profile in the one-dimensional analytical benchmark	104
B.6	Comparison between cool-down and fast warm-up along the lower radial path . .	104
B.7	Axial path response at selected times	105
B.8	Comparison between cool-down and slow warm-up along the axial path	105
B.9	Experiment-specific model response along the axial foam path P2	106
B.10	First principal elastic strain and stress along the diagonal path	107
B.11	First principal elastic strain and stress along the upper radial path	107

List of Tables

2.1	Comparison of CGH ₂ and LH ₂	6
3.1	Overview of the three model configurations, their purpose and main output. . .	25
3.2	Constant material properties and data sources.	31
3.3	Stainless steel 304 temperature-dependent properties	33
3.4	Copper and aluminium temperature-dependent properties	33
3.5	Boundary conditions for the three model configurations	34
3.6	Mapped foam mesh levels and simulation run times	39
3.7	Load-step and time-stepping scheme for the baseline thermal protocol.	42
3.8	Simulation scenarios and simulation cases, grouped by analysis cluster.	43
3.9	Extraction paths and coordinates	45
3.10	Analytical thermal check for the one-dimensional foam-slab benchmark	55
3.11	Principal modelling assumptions and their implications for interpretation.	57
B.1	Mesh convergence along the diagonal path	105
B.2	Mesh convergence along the lower radial path	106
C.1	Mapping between simulation cases and script sections	109
C.2	Cool-down durations and load-step boundaries for the cooling-rate cases.	128

C.3 Warm-up durations, end times and time-step settings. 130

C.4 Division and element counts for the mapped mesh levels 136

C.5 Steel element size used with each mapped mesh level. 136

C.6 Free-mesh variants used in the convergence comparison. 137

Nomenclature

Latin symbols

Symbol	Description	Unit
c	Longitudinal elastic wave speed in the foam	m s^{-1}
c_p	Specific heat capacity	$\text{J kg}^{-1} \text{K}^{-1}$
E	Young's modulus	Pa
E_f	Young's modulus of the foam	Pa
E_1, E_2, E_3	First, second and third principal strain ($E_1 \geq E_2 \geq E_3$)	–
G	Shear modulus	Pa
I_σ	Maximum-principal-stress failure index	–
I_ε	Maximum-principal-strain failure index	–
k	Thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$
k_{eff}	Effective thermal conductivity of the foam	$\text{W m}^{-1} \text{K}^{-1}$
k_g	Gas-conduction contribution to k_{eff}	$\text{W m}^{-1} \text{K}^{-1}$
k_r	Radiative contribution to k_{eff}	$\text{W m}^{-1} \text{K}^{-1}$
k_s	Solid-conduction contribution to k_{eff}	$\text{W m}^{-1} \text{K}^{-1}$
L	Foam layer thickness, characteristic conduction length	m
L_0	Reference length at the stress-free reference temperature	m
$\mathbf{q}$	Heat flux vector	W m^{-2}
r	Radial coordinate (global X-direction)	m
R	Fillet radius of the foam–steel corner	m
S_1, S_2, S_3	First, second and third principal stress ($S_1 \geq S_2 \geq S_3$)	Pa
S_x, S_y, S_z	Radial, axial and hoop stress component	Pa
S_{xy}	In-plane shear stress component	Pa
SF	Safety factor, reciprocal of the governing failure index	–
t	Time	s
t_{cool}	Imposed cool-down duration	s
t_{diff}	Thermal diffusion time scale across the foam layer	s
t_{mech}	Mechanical (elastic-wave) time scale	s
T	Temperature	K
T_0	Initial temperature	K
T_{ref}	Stress-free reference temperature (293 K)	K
u_r	Radial displacement	m
UX, UY	Radial and axial displacement degree of freedom	m
x	Position along the one-dimensional benchmark path	m
z	Axial coordinate (global Y-direction, axis of symmetry)	m

Greek symbols

Symbol	Description	Unit
α	Coefficient of thermal expansion	K^{-1}
α_{sec}	Secant coefficient of thermal expansion relative to T_{ref}	K^{-1}
α_{th}	Thermal diffusivity, $\alpha_{\text{th}} = k/(\rho c_p)$	$\text{m}^2 \text{s}^{-1}$
$\Delta\alpha$	Foam-metal expansion mismatch, $\alpha_{\text{foam}} - \alpha_{\text{metal}}$	K^{-1}
$\Delta L/L_0$	Relative thermal expansion	—
Δt	Time-step size	s
ΔT	Temperature change relative to T_{ref} , $T - T_{\text{ref}}$	K
δ_{ij}	Kronecker delta	—
ε	Strain	—
ε_1	First (maximum) principal elastic strain	—
ε_{el}	Elastic strain	—
ε_f	Strain-to-failure of the foam	—
ε_{pl}	Plastic strain	—
ε_{th}	Thermal (stress-free) strain	—
ε_{tot}	Total strain	—
$\varepsilon_r, \varepsilon_z, \varepsilon_\theta$	Radial, axial and hoop strain	—
γ_{rz}	In-plane engineering shear strain	—
λ	Eigenvalue of the stress tensor	Pa
ν	Poisson's ratio	—
ρ	Density	kg m^{-3}
σ	Cauchy stress tensor	Pa
σ_1	First (maximum) principal stress	Pa
σ_{UTS}	Ultimate tensile strength	Pa
$\sigma_r, \sigma_z, \sigma_\theta$	Radial, axial and hoop stress	Pa
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	Normal stress components	Pa
τ_{rz}	In-plane shear stress	Pa
$\tau_{xy}, \tau_{yz}, \tau_{xz}$	Shear stress components	Pa
θ	Hoop (circumferential) direction	—
ω	Angular frequency of the sine-squared temperature curve, $\pi/(2t_{\text{cool}})$	rad s^{-1}
∇T	Temperature gradient	K m^{-1}

Abbreviations

Abbreviation	Description
APDL	ANSYS Parametric Design Language
BoG	Boil-off gas
CGH2	Compressed gaseous hydrogen
CTE	Coefficient of thermal expansion
FE	Finite element
LH2	Liquid hydrogen

Abbreviation	Description
NIST	National Institute of Standards and Technology
OFHC	Oxygen-free high-conductivity (copper)
PU	Polyurethane
PUF	Polyurethane foam
RRR	Residual resistivity ratio
SEM	Scanning electron microscopy
SI	Système International d'Unités
TRC	Thermodynamics Research Center (NIST)
UNS	Unified Numbering System
UTS	Ultimate tensile strength

ANSYS field labels, element types and commands

Label	Description
ALPX	Secant coefficient of thermal expansion input
ANTYPE, TRNOPT	Analysis-type and transient-solution-option commands
EPEL1, EPEL3	First and third principal elastic strain
EPELX, EPELY, EPELZ	Radial, axial and hoop elastic strain components
EPELXY	In-plane shear elastic strain
EPTHX, EPTHY, EPTHZ	Radial, axial and hoop thermal strain components
ESIZE	Global element-size command
EX	Young's modulus input
KBC	Load-application key; KBC,0 applies a ramped load
KEYOPT	Element option key
LCCAT	Line-concatenation command
LESIZE	Line element-division command
PLANE55	Two-dimensional thermal-only solid element
PLANE223	Two-dimensional coupled-field (structural–thermal) solid element
POST26	Time-history postprocessor
S1, S3	First and third principal stress
SMRTSIZE	Smart-size free-meshing command
TEMP	Temperature degree of freedom

Chapter 1

Introduction

Section	Page
1.1 Liquid Hydrogen Storage	2
1.2 Insulation Challenges	3
1.3 Thermo-Mechanical Integrity	4
1.4 Thesis Outline	4

THIS INTRODUCTORY CHAPTER presents the role of liquid hydrogen storage in future large-scale hydrogen infrastructure. Section 1.1 introduces liquid hydrogen as a storage route and explains why storage at cryogenic temperature is technically challenging. Next, Section 1.2 discusses the insulation challenge for large stationary liquid hydrogen tanks and motivates the use of polymeric foam insulation. Section 1.3 introduces the thermo-mechanical integrity problem of foam insulation under cryogenic thermal loading. Finally, Section 1.4 outlines the structure of the thesis.

1.1 Liquid Hydrogen Storage

Hydrogen is widely considered to be a promising energy carrier for future low-carbon energy systems. It can be produced from renewable electricity, transported over long distances, and used in sectors where direct electrification is difficult. However, hydrogen has a low volumetric energy density under ambient conditions, making storage and transport challenging on a large scale [1, 2]. Liquid hydrogen (LH₂) provides a more compact storage route by increasing the volumetric density of hydrogen through liquefaction. At atmospheric pressure, liquefaction reduces the required storage volume by approximately a factor of 800-850 compared to gaseous hydrogen [1]. This makes it attractive for large-scale storage, long-distance transport, and future international hydrogen supply chains [2]. For stationary energy infrastructure, this is particularly relevant because storage volumes are expected to increase significantly in the future, as shown in Figure 1.1 [3, 4].

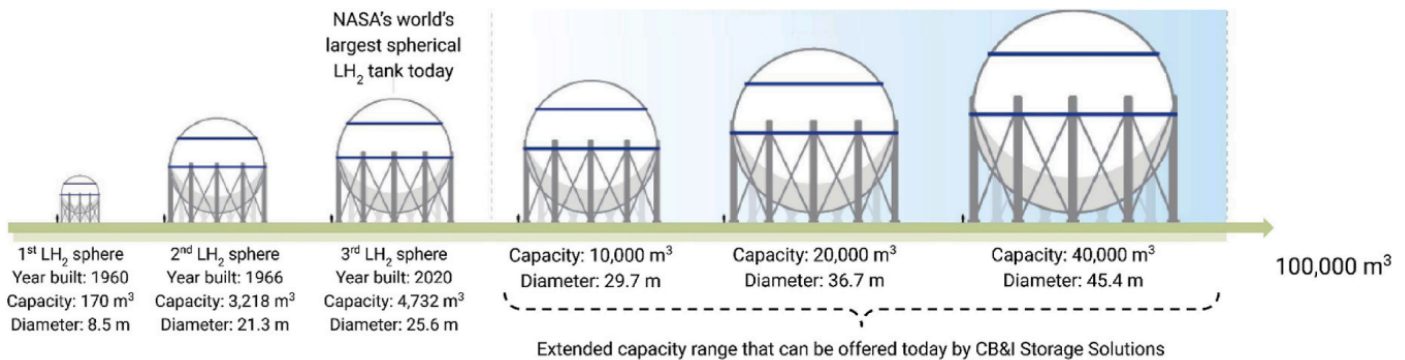


Figure 1.1: Growth in the size of liquid hydrogen cryogenic storage tanks [5].

The main challenge of LH₂ storage is its extremely low boiling temperature. At atmospheric pressure, hydrogen becomes liquid only when cooled to approximately 20 K [6]. At this temperature, even small amounts of heat ingress can cause vaporization, pressure build-up, and boil-off. Boil-off refers to the loss of stored liquid hydrogen as it vaporizes as a result of heat ingress from the surroundings. Therefore, the design of LH₂ storage tanks is strongly governed by the need to minimize heat transfer from the ambient environment to the cryogenic liquid [2, 3]. Large-scale LH₂ storage also introduces practical and structural challenges. Existing tank and insulation concepts have been largely developed for smaller or aerospace-oriented systems, where controlled manufacturing and high-performance vacuum insulation are more

feasible. However, future stationary tanks may require much larger storage capacities and must be designed for long-term reliability, scalability, constructability, and maintainability [4]. This motivates the development of insulation systems that are not only thermally efficient but also robust under realistic operating conditions.

1.2 Insulation Challenges for Large-Scale LH₂ Tanks

The primary function of a cryogenic insulation system is to reduce the ingress of heat into the tank. In LH₂ storage systems, this is especially important because the low boiling temperature and limited latent heat of vaporization make the stored liquid highly sensitive to external heat loads [1]. Heat ingress directly increases boil-off rate and can therefore reduce storage efficiency, increase operational losses, and complicate pressure management inside the tank. Traditional LH₂ storage tanks generally rely on vacuum-based insulation systems, such as multilayer insulation or evacuated powder insulation. These systems can achieve very low heat fluxes when the vacuum quality is maintained [3]. However, as tank size increases, maintaining a high-quality vacuum over large annular spaces becomes increasingly complex. The scalability of vacuum insulation therefore introduces significant practical and economic challenges [4, 7]. The performance of vacuum insulation may also be affected by installation defects, moisture ingress, outgassing, or partial vacuum degradation [8]. For large stationary LH₂ tanks, bulk-fill insulation materials are also commonly used in the annular insulation space [9]. Expanded perlite is one of the conventional choices because it is low-cost, lightweight, easy to install in large annular spaces, and provides acceptable thermal performance under vacuum or moderate-vacuum conditions [10]. These limitations of vacuum-based insulation motivate the investigation of non-vacuum or reduced-vacuum insulation concepts for future large-scale LH₂ storage tanks. Such concepts generally accept a higher baseline heat ingress in exchange for improved constructability, robustness, lower cost, and greater tolerance to operational degradation [9, 11]. In this context, polymeric foam insulation, particularly rigid polyurethane foam, is a promising candidate [11, 12]. Polyurethane foam has a low density, relatively low thermal conductivity, and can be applied to complex geometries and penetrations [12–14].

In addition, polyurethane-based insulation is generally much cheaper than vacuum-based insulation systems, making it attractive for large-scale stationary storage applications where cost and scalability are important design drivers [9]. However, the use of polymeric foam in cryogenic LH₂ storage is not governed by thermal performance alone. Because the foam is exposed to large temperature changes, thermal gradients, and mechanical constraints, its thermo-mechanical behaviour becomes a critical design concern [15]. The practical concept considered in this thesis is a layered tank insulation approach, consisting of a polyurethane foam layer and a perlite insulation layer. The foam is therefore the mechanically critical layer in this study, because it is bonded to the cold structure. The two layers are combined because they are complementary: the polyurethane foam is bonded directly to the cold inner shell, where it provides a high thermal resistance per unit thickness and keeps the foam-perlite interface above the condensation temperature of the purge gas, while the perlite is much cheaper, non-combustible and can simply be poured to fill the large remaining annular volume without introducing thermal stresses. The sandwich therefore limits the expensive, mechanically loaded foam to a thin layer at the cold side and uses perlite for the bulk of the insulation thickness.

1.3 Thermo-Mechanical Integrity Problem in Foam Insulation

Although polymeric foam insulation is promising for large-scale LH_2 storage, its mechanical integrity under cryogenic conditions remains uncertain [15, 16]. At low temperatures, polyurethane foams generally become stiffer and less ductile, which increases their susceptibility to brittle damage [14, 17]. In addition, foam properties are influenced by density, cellular morphology, anisotropy, and manufacturing-induced inhomogeneities [18–20]. During cool-down from ambient temperature to approximately 20 K, strong transient temperature gradients can develop through the insulation layer. The colder regions of the foam contract earlier than the warmer regions, leading to non-uniform deformation through the insulation thickness [15, 16]. This effect is particularly important during transient operation, because the most critical mechanical state may occur before thermal equilibrium is reached.

The problem is further intensified by the interaction between the foam and adjacent metallic components. Polyurethane foam and metals have different coefficients of thermal expansion and different temperature-dependent stiffness. When the foam is bonded to, or constrained by, a metallic structure, it cannot contract freely during cooling. This constrained thermal contraction can generate internal stresses, elastic strain concentrations, and interfacial stresses [15, 21]. Experiments performed in cryogenic storage tank design also indicate that damage may develop during thermal transients, suggesting that transient temperature gradients and constrained contraction are key mechanisms in the mechanical response of cryogenic foam insulation. These experimental observations form the direct motivation for the present study. However, there is still limited quantitative understanding of how temperature gradients, foam–metal property mismatch, boundary constraints and local geometry interact to produce stress and elastic strain concentrations inside the foam [22].

1.4 Thesis Outline

The structure of this thesis is as follows. Chapter 2 presents the background, the literature study and the theoretical framework required to analyse this problem. Because the research questions follow directly from the gaps identified in the literature, they are not stated in this introductory chapter but are formulated at the end of Chapter 2, once the relevant mechanisms and the limitations of existing work have been established. The chapter first focuses on the physical and practical challenges associated with LH_2 insulation to establish the problem context, and then introduces the thermo-mechanical theory and modelling principles that translate the mechanisms identified in the literature into the framework used to interpret the finite element results, before identifying the research gaps and formulating the research questions. Chapter 3 describes the research methodology. The model configurations, finite element geometry, material properties, thermal and mechanical boundary conditions, mesh strategy, simulation cases, post-processing methods and numerical verification are presented. Chapter 4 presents and discusses the results of the numerical simulations, baseline cool-down behaviour, sensitivity studies, warm-up response, comparison with experimental observations and relevance for representative tank geometries. Finally, Chapter 5 summarises the main conclusions and provides recommendations for further modelling, experimental validation and the design of cryogenic foam insulation systems.

Chapter 2

Literature Review

Section	Page
2.1 Insulation Requirements	6
2.2 Polyurethane Foam Insulation	7
2.3 Microstructure and Installation Effects	9
2.4 Damage Mechanisms and Behaviour	11
2.5 Theoretical Framework	15
2.6 Modelling Approaches	20
2.7 Research Gap and Questions	21

THE PURPOSE OF THIS CHAPTER is to review the literature most relevant to polyurethane foam insulation for liquid hydrogen storage. It discusses LH_2 insulation requirements, the motivation for non-vacuum foam insulation, foam microstructure and installation effects, thermo-mechanical damage mechanisms, and existing modelling approaches. The chapter concludes by identifying the research gap and deriving the research questions.

2.1 Liquid Hydrogen Insulation Requirements

Liquid hydrogen storage imposes strict thermal requirements on the storage tank design because hydrogen must be maintained at approximately 20 K at atmospheric pressure to remain in the liquid state. Although ingress of heat from the environment cannot be avoided completely, it must be minimised to limit vaporisation, pressure rise, and boil-off losses [1, 3, 6]. As shown in Table 2.1, LH_2 has distinct cryogenic thermodynamic properties, which directly influence insulation requirements, pressure management, and boil-off behaviour. Consequently, insulation performance is one of the dominant design drivers for LH_2 storage tanks.

Table 2.1: Comparison of compressed gaseous hydrogen (CGH_2) and liquid hydrogen (LH_2) [2].

Property	CGH_2	LH_2
Temperature ($^{\circ}\text{C}$)	25	−253
Pressure (bar)	690	1
Ortho/para hydrogen fraction	0.75 / 0.25	0.01 / 0.99
Mass density (kg m^{-3})	38.88	70.94
Molar density (kgmol m^{-3})	19.29	35.19
Mass enthalpy (kJ kg^{-1})	4639	−250.2
Mass entropy ($\text{kJ kg}^{-1} \text{K}^{-1}$)	43.33	8.25
Heat of vaporization (kJ kg^{-1})	—	445.8
Specific volume ($\text{m}^3 \text{kgmol}^{-1}$)	0.052	0.028

Boil-off is not caused by external heat ingress alone. Liquid hydrogen is also affected by ortho-para hydrogen conversion, in which the higher-energy ortho-hydrogen state gradually converts to the lower-energy para-hydrogen state at cryogenic temperature. This conversion releases heat and can therefore contribute significantly to boil-off if insufficient conversion is achieved during liquefaction [1, 2]. External heat ingress remains particularly important because liquid hydrogen has a low boiling temperature and a limited latent heat of vaporisation compared with many other cryogenic liquids. Even relatively small heat fluxes can therefore lead to measurable boil-off over long storage durations [3]. From a system perspective, this affects not only product losses, but also pressure management, venting strategy, safety considerations and overall storage efficiency. The insulation system must therefore reduce heat transfer through the tank wall, supports, penetrations and insulation space as much as practically possible.

As tank size increases, the thermal and structural design problem changes. Larger tanks are thermally attractive because the surface-to-volume ratio decreases with increasing tank size. This means that the relative heat ingress per unit of stored hydrogen can decrease for larger

tanks, which is beneficial for long-duration storage [1]. However, this thermal scaling advantage is accompanied by practical and structural challenges. Large stationary tanks often contain more penetrations and require more complex support structures than smaller shop-fabricated tanks [4, 7].

This scale-up changes the requirements imposed on the insulation system. For small and medium-sized tanks, insulation concepts can often be installed under controlled manufacturing conditions and combined with high-quality vacuum systems. For large stationary tanks, however, constructability, robustness, inspection, maintenance, costs and long-term degradation become increasingly important. An insulation system for future large-scale LH_2 storage must therefore be evaluated not only by its ideal thermal performance, but also by its sensitivity to installation quality, operational degradation and mechanical reliability under cryogenic thermal cycling.

The relation between heat ingress and boil-off is illustrated schematically in Figure 2.1. Heat entering through the insulation system, supports or penetrations increases the thermal energy of the tank contents and can lead to evaporation of the liquid phase. The resulting boil-off gas contributes to pressure build-up and must either be stored, reliquefied, used or vented depending on the tank design and operating strategy [1, 6]. This illustrates why insulation performance is not only a thermal design parameter, but also directly linked to pressure management, operational losses and system reliability.

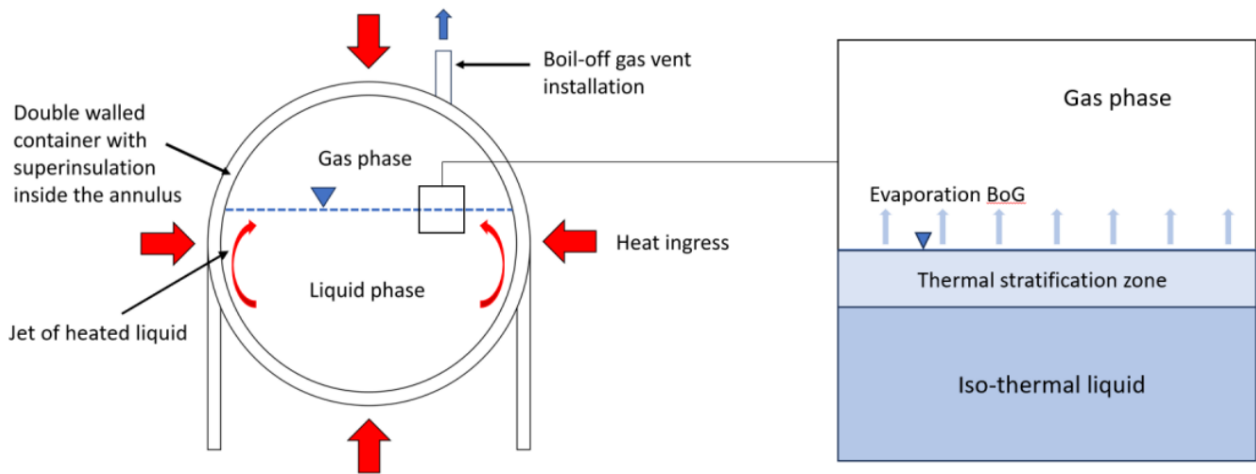


Figure 2.1: Schematic representation of heat ingress and boil-off gas (BoG) generation in an LH_2 storage tank [6].

2.2 Polyurethane Foam as Non-Vacuum Cryogenic Insulation

Traditional LH_2 storage tanks generally rely on vacuum-based insulation systems. These concepts aim to reduce gaseous conduction and convection by evacuating the annular space between the inner and outer tank wall. Additional insulation materials are then used to limit solid conduction and radiative heat transfer. Common vacuum-based systems include multilayer insulation, evacuated perlite and evacuated glass bubbles [3, 9, 10].

2.2.1 Limitations of Vacuum-Based Insulation

The multilayer insulation can achieve extremely low effective thermal conductivity under high-vacuum conditions and is therefore widely used in small-scale cryogenic systems and aerospace applications [1]. Bulk-fill materials such as perlite and glass bubbles are more practical for larger tank annular spaces and have been used in stationary cryogenic storage systems [3, 10]. Under intact vacuum conditions, these systems can provide very low heat ingress and have demonstrated long-term operational performance in LH₂ storage applications.

However, the performance of vacuum-based insulation depends strongly on maintaining vacuum integrity. This becomes increasingly challenging as tank size increases. Large annular spaces require longer pump-down procedures, careful leak detection, moisture control and continuous monitoring during operation [4, 7]. Even partial degradation of the vacuum can increase gaseous heat transfer and lead to higher boil-off rates. In addition, vacuum-based insulation systems can be expensive to manufacture, install and maintain at large scale. Large vacuum-insulated tanks also introduce structural challenges. The outer shell must withstand atmospheric pressure acting over a large surface area, which can require additional stiffening and increase construction complexity [3]. Furthermore, penetrations, supports and local discontinuities may create thermal bridges or regions where insulation performance is difficult to maintain. These issues reduce the attractiveness of high-quality vacuum conditions for future infrastructure-scale LH₂ storage.

These limitations motivate the investigation of insulation concepts that are less sensitive to vacuum quality, cheaper, and more tolerant of construction imperfections and long-term operational variability. Non-vacuum and reduced-vacuum insulation concepts generally accept higher baseline heat ingress compared with ideal high-vacuum systems, but may offer advantages in robustness, cost, scalability and maintainability [9, 11].

2.2.2 Motivation for Polyurethane Foam Insulation

Polymeric foam insulation, particularly rigid polyurethane foam, is a promising candidate for this type of insulation concept. Rigid polyurethane foam consists of a polymer matrix containing a high fraction of closed gas-filled cells. This cellular structure gives the material a combination of low density, relatively low thermal conductivity and sufficient mechanical stiffness for insulation applications [12, 14, 18]. Therefore, polyurethane foam has been used in several cryogenic insulation systems, including applications in aerospace [13, 19, 23].

A practical advantage of polyurethane foam is that it can be spray-applied directly to complex surfaces, local penetrations, and structural details without the need for mechanically fastened panels or a fully enclosed vacuum jacket [12, 13]. This is particularly relevant for large-scale tank systems, where penetrations, supports and local geometric complexity make insulation installation more difficult. Foam can also provide a continuous insulation layer, which may reduce local discontinuities and improve structural continuity compared with layered systems containing many internal interfaces. Compared with vacuum-based insulation, polyurethane foam is less dependent on maintaining a high-quality vacuum. This makes it attractive for non-vacuum or reduced-vacuum insulation concepts where robustness and scalability are prioritized over the lowest possible heat flux [9, 11]. From a practical perspective, foam insulation is also lightweight, relatively easy to apply and lower in cost than complex high-vacuum insulation systems at large scale.

Nevertheless, polyurethane foam insulation also has important limitations. Its thermal conductivity is higher than that of high-vacuum multilayer insulation, which means that larger insulation thicknesses may be required to achieve acceptable heat ingress [9, 14]. In addition, the mechanical behaviour of polyurethane foam changes significantly at cryogenic temperatures. Cooling increases stiffness and reduces ductility, while thermal contraction mismatch with adjacent metallic structures can generate internal stresses [17, 18]. As a result, the attractiveness of polyurethane foam is not determined by thermal performance alone, but also by whether it can maintain mechanical integrity during cryogenic thermal transients.

2.3 Microstructure, Manufacturing and Installation Effects

Polyurethane foam insulation is not a perfectly homogeneous material. Its thermal and mechanical behaviour are governed by the cellular structure, the manufacturing process, and the quality of installation. These factors are particularly relevant for cryogenic applications because local defects, density variations and anisotropy can influence both heat transfer and stress development during cooling.

2.3.1 Cellular Structure and Density

Rigid polyurethane foam derives its insulation performance from its closed-cell morphology. The gas-filled cells reduce the amount of solid polymer available for heat conduction and suppress convective heat transfer within the material. At the same time, the polymer cell walls and struts provide the mechanical skeleton of the foam [18, 24]. The resulting macroscopic properties therefore depend strongly on microstructural features such as cell size, cell shape, closed-cell content, wall thickness and density [20, 24, 25].

Foam density is one of the most important design parameters. Lower-density foams generally reduce solid conduction because less polymer material is present to form conductive pathways. However, reducing density also decreases stiffness, compressive strength and resistance to mechanical damage [18]. Higher-density foams provide greater mechanical resistance, but generally increase solid-phase heat conduction. This creates a fundamental trade-off between thermal insulation efficiency and mechanical integrity.

Cellular morphology also affects the mechanical response. Variations in cell size, wall thickness and local density can create spatial variations in stiffness and strength. This variation is illustrated in Figure 2.2. Under cryogenic cooling, these variations may lead to non-uniform deformation and local stress concentrations.

2.3.2 Spray Application, Anisotropy and Defects

Manufacturing and installation strongly influence the final foam structure. Spray-applied polyurethane foam is attractive for large cryogenic insulation systems because it can be applied directly to curved surfaces, penetrations and local structural details [12, 13]. This can reduce installation complexity and cost, and can create a continuous insulation layer around complex geometries. Alternative foam installation approaches include prefabricated foam panels or bonded foam blocks, which offer better control over density, thickness and curing conditions,

but introduce joints, interfaces and additional installation steps that may become critical under cryogenic thermal cycling [23]. The main disadvantage of spray application is that the final foam quality depends strongly on the application process: variations in spray distance, rise direction, substrate temperature and surface preparation can introduce spatial variations in density, thickness, adhesion quality and cell orientation [13]. As a result, spray-applied foam insulation may contain spatially varying material properties even when the nominal formulation is constant.

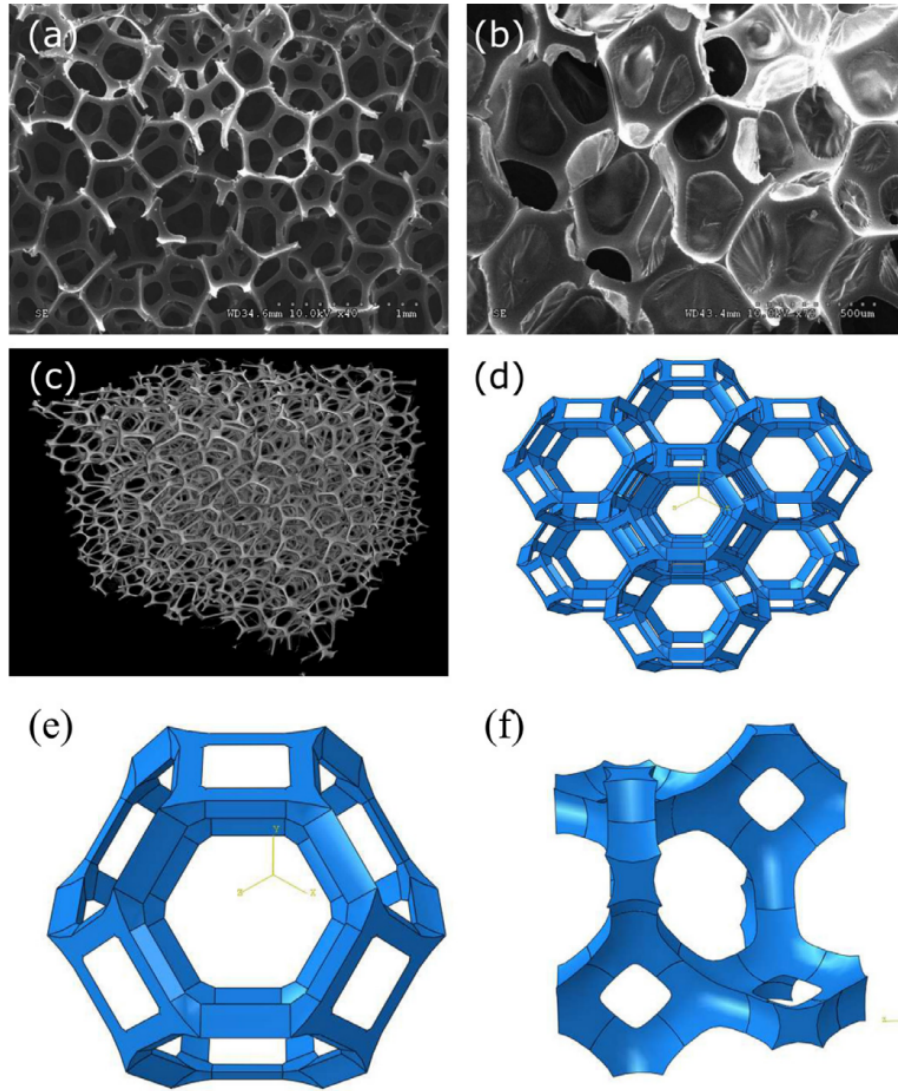


Figure 2.2: Multiscale representation of rigid polyurethane foam cellular architecture: (a) and (b) SEM images, (c) X-ray microtomography, (d) idealized repeated unit cell, (e) KC1 (tetrakaidecahedral unit cell), and (f) KC2 [24].

One important effect of this is anisotropy. During foam expansion, cells may elongate in the foam rise direction. This introduces direction-dependent thermal and mechanical properties, including stiffness, strength and thermal contraction behaviour [18, 25]. In cryogenic insulation systems, the orientation of the foam rise direction relative to the tank wall can therefore influence the stress state during cool-down. Directional properties are especially relevant near the foam-metal interface, where mechanical constraint and thermal gradients are strongest. Density gradients can also arise during manufacturing. Near the substrate, foam expansion

may be constrained and heat transfer to the metal surface can influence curing conditions. This can lead to local variations in cell size, packing density and stiffness through the foam thickness [25]. Since mechanical properties scale strongly with density, such gradients may contribute to non-uniform stress distributions during cryogenic contraction.

Installation-induced defects are particularly important for mechanical reliability. Voids near the substrate, weak adhesion regions, local thickness variations and incomplete bonding can act as stress concentrators during cool-down [13]. Experimental studies on cryogenic foam insulation have shown that cracking is often initiated by local defects or weak interfacial regions rather than by uniform failure of the bulk foam [18, 19]. This means that average foam properties alone are insufficient to fully describe cryogenic reliability.

2.3.3 Aging and Moisture Effects

After installation, polyurethane foam properties can evolve over time. In closed-cell foams, blowing agents may gradually diffuse out of the cells while atmospheric gases diffuse inward. This changes the internal cell gas composition and can increase the effective thermal conductivity over long storage durations [26–28]. Although this process is mainly relevant for long-term thermal performance, it can also influence local temperature gradients and therefore the thermo-mechanical loading of the insulation.

Moisture uptake is another important degradation mechanism. Water ingress can increase thermal conductivity and may cause additional problems under cryogenic conditions if absorbed moisture freezes inside pores, cracks or weak regions of the foam [23, 29]. Freezing and thawing during thermal cycles can promote local damage and may accelerate microcrack formation. Surface degradation due to environmental exposure may also contribute to crack initiation under repeated thermal loading [12].

2.4 Cryogenic Thermo-Mechanical Behaviour and Damage Mechanisms

Polyurethane foam insulation must perform thermally and mechanically at the same time, and at cryogenic temperature these two aspects cannot be separated. The thermal properties of the foam determine the temperature field inside the insulation layer, the mechanical properties determine how that field is converted into stress and elastic strain, and damage arises where the two interact under mechanical constraint. This section follows that sequence: it first describes the thermal response and effective conductivity, then the mechanical behaviour at low temperature, and finally the thermo-mechanical damage mechanisms that result when constrained contraction, transient temperature gradients and local geometry act together.

2.4.1 Thermal Response and Effective Conductivity

In closed-cell polyurethane foam, heat transfer occurs through several mechanisms: solid conduction (k_s) through the polymer matrix, gas conduction (k_g) inside the cells, and radiation (k_r) across the pore space [30–33]. Natural convection is generally negligible in intact closed-cell

foams because the cell dimensions are small [30]. The effective thermal conductivity (k_{eff}) is therefore commonly interpreted as the combined contribution of solid conduction, gas conduction and radiation, as given in Equation 2.1.

$$k_{\text{eff}} = k_s + k_g + k_r \quad (2.1)$$

At ambient temperature, gas conduction often forms a major contribution to the total heat transfer in closed-cell foams. However, this contribution decreases significantly at cryogenic temperatures. As the temperature decreases, the thermal conductivity of the cell gas is reduced and the gas pressure inside the cells decreases [30, 31]. As a result, solid conduction through the polymer skeleton becomes relatively more important at low temperature. This temperature dependence means that thermal conductivity data obtained at room temperature cannot be directly extrapolated to liquid hydrogen conditions. Experimental measurements on rigid polyurethane foam show that the effective thermal conductivity can decrease strongly when cooling from room temperature to approximately 20 K, particularly under reduced-pressure or evacuated conditions [30]. For liquid hydrogen storage, this is beneficial from a thermal insulation perspective. However, it also means that the temperature field inside the foam is highly temperature-dependent and may evolve differently during cool-down than would be expected from constant-property assumptions.

For thick insulation layers, the low thermal conductivity of polyurethane foam also promotes the formation of transient temperature gradients. During cool-down, the side of the foam close to the cryogenic surface cools first, while regions further away may remain significantly warmer for a longer time. This delayed thermal response is important because it creates spatially non-uniform thermal contraction.

2.4.2 Mechanical Behaviour and Property Uncertainty at Low Temperature

Rigid polyurethane foams generally become stiffer and stronger as temperature decreases [16, 34–36]. This behaviour is caused by the reduced mobility of the polymer chains at low temperature. As the polymer matrix approaches a glassy state, molecular motion becomes increasingly restricted, which increases the elastic modulus and resistance to deformation. Experimental studies report increases in tensile strength and compressive strength when cooling polyurethane foams from room temperature to cryogenic temperatures [17, 37]. The temperature-dependent deformation behaviour of rigid polyurethane foams under compression is illustrated in Figure 2.3.

However, this increase in stiffness and strength is accompanied by a reduction in ductility. At cryogenic temperatures, polyurethane foams generally show lower elongation at break and reduced strain-to-failure compared with room temperature behaviour [35]. This means that the material may sustain higher stresses, but has less ability to accommodate deformation without damage. Consequently, low-temperature operation can shift the mechanical response towards a more brittle failure mode. This brittle response is particularly relevant for cryogenic insulation systems. During cool-down, the foam is subjected to thermal contraction and constraint from adjacent structural components. If the foam becomes stiffer while its strain capacity decreases, local stress and elastic strain concentrations become more critical. Although the foam may become stronger in a conventional mechanical test, it also becomes less tolerant to imposed

thermal strain.

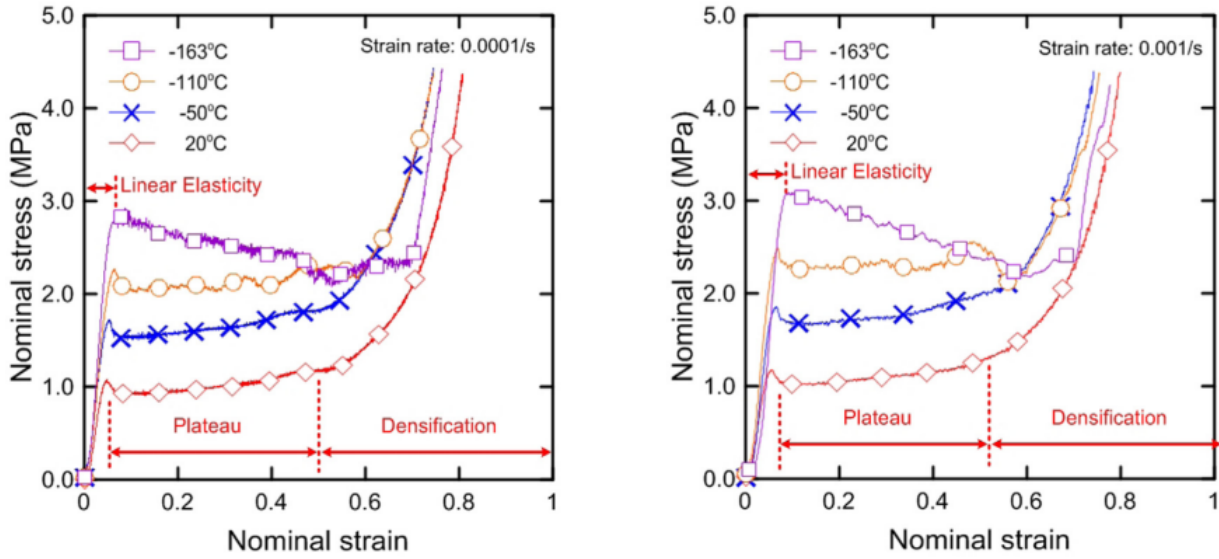


Figure 2.3: Temperature-dependent compressive stress–strain curves of polyurethane foam (PUF) at strain rates of (a) 10^{-4} s^{-1} and (b) 10^{-3} s^{-1} [37].

The magnitude of these properties depends strongly on density: higher-density foams have thicker cell walls and a larger solid polymer fraction, and are correspondingly stiffer and stronger [18, 38, 39]. Together with the anisotropy discussed in Section 2.3.2, this makes material property selection uncertain, since reported mechanical properties depend on foam formulation, density, cell morphology, specimen orientation, test method and temperature [18, 40]. Reliable material data at temperatures approaching 20 K are moreover limited compared with data at room temperature or liquid nitrogen temperature.

2.4.3 Thermal Contraction and CTE Mismatch

One of the main drivers of thermo-mechanical damage is differential thermal contraction. Polyurethane foam generally has a higher coefficient of thermal expansion than metallic tank materials such as aluminium or steel [21, 41]. During cooling from ambient temperature to cryogenic conditions, the foam therefore tends to contract more than the adjacent metal. If the foam were completely free, this contraction would mainly result in deformation. However, in practical insulation systems the foam is often bonded to or constrained by a metallic substrate, so free contraction is prevented.

When the foam and metal are bonded, both materials must remain compatible at the interface. In this context, compatibility means that the displacement field is continuous across the bonded interface, so the foam and metal cannot slide or separate relative to each other. This compatibility requirement converts part of the thermal contraction mismatch into mechanical strain and stress. As a result, thermal expansion mismatch can generate tensile and shear stresses within the foam and at the foam-metal interface [15, 21, 42]. This mechanism is particularly important during large temperature changes, because the mismatch strain scales with the difference in thermal expansion coefficient and the temperature drop.

In a simplified sense, the magnitude of thermally induced stress increases with elastic modulus,

thermal expansion mismatch and temperature change [21]. This relation explains why cryogenic stiffening can amplify thermal stress development. As the foam becomes stiffer during cooling, a given imposed thermal mismatch produces higher stress. At the same time, the foam becomes less ductile, reducing its ability to accommodate the imposed strain without cracking. Thermal contraction and CTE mismatch therefore form a fundamental mechanism for damage initiation in cryogenic foam insulation. The temperature-dependent thermal expansion behaviour of rigid polyurethane foams is illustrated in Figure 2.4.

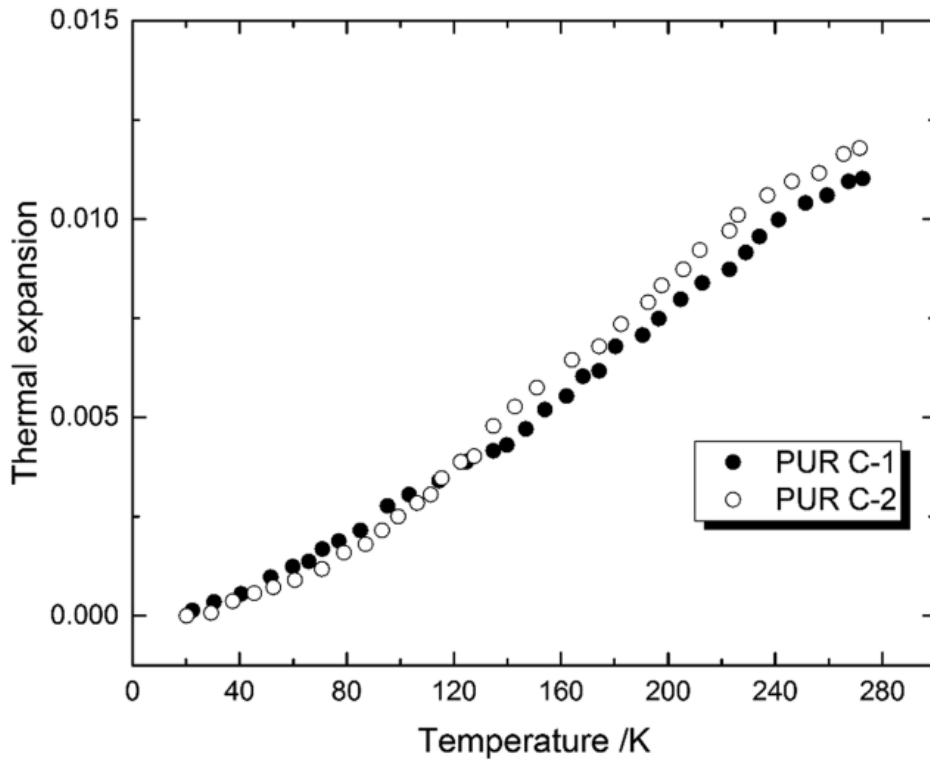


Figure 2.4: Thermal expansion of closed-cell rigid polyurethane foams as a function of temperature in the range 20–270 K [21].

The mismatch problem is further complicated by the cellular structure of the foam. The effective thermal expansion of polyurethane foam is influenced not only by the polymer matrix, but also by density, cell geometry, anisotropy and internal cell-gas effects [21, 38, 39]. Therefore, the contraction behaviour of foam insulation cannot be treated as a simple isotropic material property without uncertainty.

2.4.4 Transient Temperature Gradients and Constrained Contraction

Thermal contraction mismatch defines the potential for stress development, but the most critical stress state often occurs during transient thermal loading. During cool-down, the region of the foam closest to the cold surface cools and contracts first, while the outer region remains warmer and less contracted. This creates a through-thickness incompatibility in thermal strain [15, 16]. The foam is therefore not only constrained by the metal substrate, but also internally constrained by neighbouring foam regions at different temperatures. This transient effect is especially relevant for thick insulation layers, although the influence of thickness is not necessarily one-directional. A thicker layer can maintain a through-thickness temperature

difference for a longer time, while also reducing the average gradient for a given temperature difference and providing more elastic compliance to accommodate differential contraction [43].

The colder region of the insulation has a higher stiffness and tends to contract more, while the warmer region still behaves more compliantly and remains closer to its original dimensions. This spatial variation in temperature-dependent stiffness and thermal strain produces a time-dependent, multi-axial stress state. Several cryogenic insulation studies indicate that peak thermo-mechanical stresses may occur during cool-down rather than during steady cryogenic hold [15, 16]. This is important because steady-state thermal analyses alone cannot fully capture the critical loading condition. Warm-up can also be mechanically relevant. During heating, the temperature gradient reverses and previously cold regions expand while other regions remain colder. This may reopen, close or further propagate existing damage depending on the local stress state and constraint conditions [15]. For this reason, both cool-down and warm-up should be considered when assessing the reliability of cryogenic foam insulation.

2.4.5 Interface Damage, Cracking and Stress Concentrations

The foam-metal interface is one of the most critical regions in cryogenic insulation systems. Because the foam and metal contract differently, the interface must transfer shear and normal stresses during cooling. These stresses can promote debonding, interfacial cracking or local failure of the foam close to the substrate [15, 42]. Even when the bulk foam properties are acceptable, weak adhesion or local defects at the interface can dominate the mechanical reliability of the insulation system. Local geometry can further increase stress concentrations. Corners, thickness transitions, penetrations and other discontinuities restrict free deformation and disturb the stress field. Experimental studies on cryogenic insulation systems report that cracking and debonding often occur near such geometric discontinuities, where constraint prevents uniform contraction [15, 44, 45].

Cracking in polyurethane foam is also influenced by the cellular microstructure. At low temperature, the foam becomes stiffer and more brittle, while cell walls and struts provide local paths for crack initiation and propagation [17, 46]. Microstructural defects, density gradients and anisotropy can therefore influence where damage initiates, even if the global loading is governed by thermal contraction and temperature gradients. Under repeated thermal cycling, microcracks may accumulate and contribute to progressive degradation [47]. Damage formation can have consequences beyond local mechanical failure. Cracks, debonded regions and interfacial gaps can create additional heat-transfer paths and allow gas or moisture migration toward colder regions [23, 48], which may increase local heat ingress, promote moisture accumulation or frost formation, and further degrade the insulation performance over time [16]. Thermally induced cracking is therefore a coupled problem: mechanical damage may be initiated by thermal loading, while the resulting damage can reduce the thermal insulation function and increase the risk of progressive degradation.

2.5 Theoretical Framework

The previous sections describe the thermal and mechanical behaviour of polyurethane foam and the resulting damage mechanisms in qualitative terms. This section formalises those mechanisms in the relations used later to build and interpret the model: transient heat conduction

and the moving temperature gradient, the decomposition of thermal and elastic strain, and the stress that arises when contraction is constrained. The definitions of the principal stress and strain measures used to report the results are given in Appendix A.

2.5.1 Transient Heat Conduction and Temperature Gradients

In the present thermo-mechanical formulation, the temperature field is treated as the driving field for the mechanical response. The local temperature determines both the free thermal contraction and the temperature-dependent material properties of the foam. To first order, the coupling is therefore one-way: transient temperature gradients generate stress and elastic strain, whereas the resulting deformation is assumed not to significantly alter the heat-conduction problem. The starting point is Fourier's law, which states that the local heat flux is proportional to the negative temperature gradient [49]:

$$\mathbf{q} = -k(T)\nabla T \quad (2.2)$$

where $\mathbf{q}$ is the heat flux vector (W m^{-2}), $k(T)$ is the temperature-dependent thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$), and ∇T is the temperature gradient (K m^{-1}). The negative sign expresses that heat flows from regions of high temperature to regions of low temperature. Combining Fourier's law with the conservation of thermal energy for a solid without internal heat generation yields the transient heat-conduction equation [49]:

$$\rho c_p(T) \frac{\partial T}{\partial t} = \nabla \cdot (k(T)\nabla T) \quad (2.3)$$

where ρ is the density (kg m^{-3}), $c_p(T)$ is the temperature-dependent specific heat capacity ($\text{J kg}^{-1} \text{K}^{-1}$), t is time (s), and T is temperature (K). For the cryogenic foam considered here, k , c_p and, to a smaller extent, ρ all vary significantly between ambient temperature and 20 K, so they cannot be treated as constants over the temperature range of a cool-down [31, 35]. When the material properties are approximately constant over a limited temperature interval, Equation 2.3 reduces to the classical form [49]:

$$\frac{\partial T}{\partial t} = \alpha_{\text{th}} \nabla^2 T, \quad \alpha_{\text{th}} = \frac{k}{\rho c_p} \quad (2.4)$$

where α_{th} is the thermal diffusivity ($\text{m}^2 \text{s}^{-1}$). The thermal diffusivity governs how rapidly a temperature disturbance propagates through the material: a high diffusivity allows the temperature field to adjust quickly, whereas a low diffusivity causes the temperature change to penetrate slowly. Rigid polyurethane foam combines a very low thermal conductivity with a low density and a moderate specific heat, which results in a low thermal diffusivity. Physically, this is precisely the property that makes the foam an effective insulator, but it is also the property responsible for the steep, slowly moving temperature fronts that develop during cool-down.

During cool-down, a decreasing temperature is imposed on the boundary of the foam that is in contact with the cooled component. Because the diffusivity is low, the cold region near this boundary cools rapidly while material further away remains close to its initial temperature for a considerable time. A cooling front therefore propagates into the foam thickness rather than

the whole layer cooling uniformly. For a thick insulation layer, this means that a pronounced through-thickness temperature difference exists for a significant part of the transient, which is the origin of the transient thermal gradients discussed qualitatively in Section 2.4.4. The characteristic time scale over which heat diffuses across a layer of thickness L can be estimated from the diffusion length scaling:

$$t_{\text{diff}} \approx \frac{L^2}{\alpha_{\text{th}}} \quad (2.5)$$

which shows that the time required to reach thermal equilibrium grows with the square of the layer thickness [49].

The temperature gradient is not only the driving quantity for heat flux in Equation 2.2, it is also the quantity that directly couples the thermal problem to the mechanical response. The magnitude of the gradient is largest where the temperature changes most steeply over a short distance, which during cool-down means close to the boundary where the cooling is imposed. How this spatial and temporal variation in gradient magnitude translates into mechanical constraint is developed in Section 2.5.3, and Chapter 4 identifies where this proves most significant in the specific model geometries considered. The local cooling rate is related to the divergence of the heat flux through the energy balance of Equation 2.3. A region where more heat leaves than enters cools down and the steeper the surrounding gradient, the faster the local temperature changes. Because the gradient is steepest near the cold boundary at early times and then flattens as the foam approaches equilibrium, the gradient is inherently a transient quantity: its magnitude and its location both move through the foam during cool-down and warm-up. This moving gradient is central to the mechanism investigated in this study.

It is therefore useful to distinguish two limiting situations. In the steady cryogenic state, the whole foam has reached a low and nearly uniform temperature, the temperature gradient is small, and the contraction is approximately uniform. In the transient state, a large gradient exists across the thickness, the cold side has contracted substantially while the warm side has not, and the resulting incompatibility in contraction is large. This is consistent with the observation reported in Section 2.4.4 that damage tends to occur during transient cool-down and warm-up rather than during steady cryogenic hold. The transient temperature gradient is thus the link between the purely thermal analysis of this section and the mechanical analysis that follows.

2.5.2 Thermal Contraction and Strain Decomposition

A change in temperature causes a stress-free change in the dimensions of a material. For an unconstrained, isotropic solid this dimensional change is quantified by the coefficient of thermal expansion (CTE), α . If the CTE is treated as constant over the temperature interval, the free unconstrained thermal strain associated with a temperature change from a reference temperature T_0 to a temperature T is:

$$\varepsilon_{\text{th}} = \alpha (T - T_0) = \alpha \Delta T \quad (2.6)$$

where ε_{th} is the thermal strain and $\Delta T = T - T_0$. For cooling, ΔT is negative, so the thermal strain is negative and the material contracts. In the present problem the foam is cooled from

approximately 293 K to about 20 K, so $\Delta T \approx -273$ K, which represents a very large thermal load. The coefficient of thermal expansion of polyurethane foam is itself temperature-dependent and generally decreases as the temperature is lowered [21, 38]. When this dependence is taken into account, the free thermal strain is obtained by integrating the CTE over the temperature path rather than using a single constant value:

$$\varepsilon_{\text{th}}(T) = \int_{T_0}^T \alpha(T') dT' \quad (2.7)$$

Equation 2.7 reduces to Equation 2.6 when α is constant. In full tensor form, and including in principle a plastic contribution, the additive small-strain decomposition is [50, 51]:

$$\boldsymbol{\varepsilon}_{\text{tot}} = \boldsymbol{\varepsilon}_{\text{el}} + \boldsymbol{\varepsilon}_{\text{th}} + \boldsymbol{\varepsilon}_{\text{pl}} \quad (2.8)$$

For a highly cross-linked rigid polyurethane foam, any significant inelastic deformation at cryogenic temperature is likely to be associated with damage mechanisms such as cell-wall fracture, microcracking or local crushing, rather than with stable plastic flow. The total strain therefore reduces to the sum of the elastic and thermal parts where the two parts have fundamentally different roles. The thermal strain $\boldsymbol{\varepsilon}_{\text{th}}$ is the stress-free contraction or expansion, equal in all directions for an isotropic material, and it does not by itself generate stress. The elastic strain $\boldsymbol{\varepsilon}_{\text{el}}$ is the part associated with stress through Hooke's law. This distinction is essential because, in a constrained body, the total strain that is geometrically permitted can differ from the free thermal strain the material wants to undergo, and the difference must be accommodated as elastic strain. For a linear elastic isotropic material the elastic strain and stress are related by the generalised Hooke's law [50]:

$$\varepsilon_{xx,\text{el}} = \frac{1}{E} [\sigma_{xx} - \nu (\sigma_{yy} + \sigma_{zz})] \quad (2.9)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad G = \frac{E}{2(1 + \nu)} \quad (2.10)$$

where E is the Young's modulus, ν is the Poisson's ratio and G is the shear modulus. Analogous expressions hold for the yy and zz directions. Equation 2.9 makes explicit why a region can carry a large total strain while remaining nearly stress-free: if the total strain is almost entirely thermal, the elastic strain, and hence the stress, is small.

2.5.3 Constrained Contraction and Stress Formation

Thermal stress arises only when the contraction is wholly or partially prevented, that is, when the material is constrained. In that case part of the deformation that the material would undergo freely cannot occur and that suppressed deformation appears as elastic strain. This can be made explicit using the strain decomposition of Equation 2.8. Rearranging gives the elastic strain as the difference between the total geometrically admissible strain and the free thermal strain:

$$\boldsymbol{\varepsilon}_{\text{el}} = \boldsymbol{\varepsilon}_{\text{tot}} - \boldsymbol{\varepsilon}_{\text{th}} \quad (2.11)$$

Consider the idealised limiting case of a bar that is fully restrained between two rigid supports so that its total strain is forced to be zero, $\varepsilon_{\text{tot}} = 0$. On cooling, the bar would contract freely by $\varepsilon_{\text{th}} = \alpha\Delta T$, but this is prevented. From Equation 2.11, the elastic strain is then $\varepsilon_{\text{el}} = -\varepsilon_{\text{th}} = -\alpha\Delta T$. Through Hooke's law for a uniaxial state, the resulting stress (σ) is:

$$\sigma = E_f \varepsilon_{\text{el}} = -E_f \alpha \Delta T \quad (2.12)$$

where E_f is the Young's modulus. Equation 2.12 is the classical expression for fully constrained thermal stress [51]. For cooling, $\Delta T < 0$, so the stress becomes positive, corresponding to tension. This is the expected result for a body that is prevented from contracting develops tensile stress. It is a limiting one-dimensional case rather than the equation solved by the model. Equation 2.12 assumes uniform temperature change, constant material properties and complete restraint. In the actual model, the temperature field, $E_f(T)$, $\alpha(T)$, displacement field and stress state vary in space and time, and the solution follows from the coupled heat-conduction equation, mechanical equilibrium, compatibility and the imposed boundary conditions. The expression is therefore used only as a scaling argument, showing that thermally induced stress increases with stiffness E_f , thermal expansion coefficient α , and temperature change ΔT .

In the real insulation system, the constraint is not provided by rigid supports but by two coupled mechanisms. The first is the constraint imposed by the adjacent metallic component. As described in Section 2.4.3, the mismatch $\Delta\alpha = \alpha_{\text{foam}} - \alpha_{\text{metal}}$ is accommodated as elastic strain in both materials, concentrated near the interface. By analogy with Equation 2.12, the magnitude of the interface stress scales with $E_f \Delta\alpha \Delta T$, so with the stiffness, the CTE mismatch and the temperature drop [42, 45].

The second mechanism is internal to the foam and is the direct consequence of the transient temperature gradient of Section 2.5.1. This through-thickness incompatibility in thermal strain produces elastic strain and stress even in the absence of an external boundary constraint. The two mechanisms act together: the foam-metal mismatch provides a constraint at the interface, while the moving internal gradient provides a constraint within the bulk foam. Two other effects modify the scaling of Equation 2.12 and are relevant to the interpretation of the results. First, the foam is far more compliant than the metal, so the elastic strain is taken up predominantly by the foam rather than by the metal so the soft material accommodates most of the mismatch. Second, local geometry concentrates the constraint, as described in Section 2.4.5, so that the stress and elastic strain fields are locally amplified relative to the smooth-interface value [15]. The two mechanisms are illustrated schematically in Figure 2.5.

In summary, the temperature field imposes a thermal strain through Equations 2.6 and 2.7. Where this thermal strain is constrained, by the foam-metal mismatch, by the internal temperature gradient, or by local geometry, the difference between the admissible total strain and the free thermal strain appears as elastic strain and generates stress. The full multiaxial form of these relations, and the definitions of the principal stress and strain measures used to report the results, are given in Appendix A.

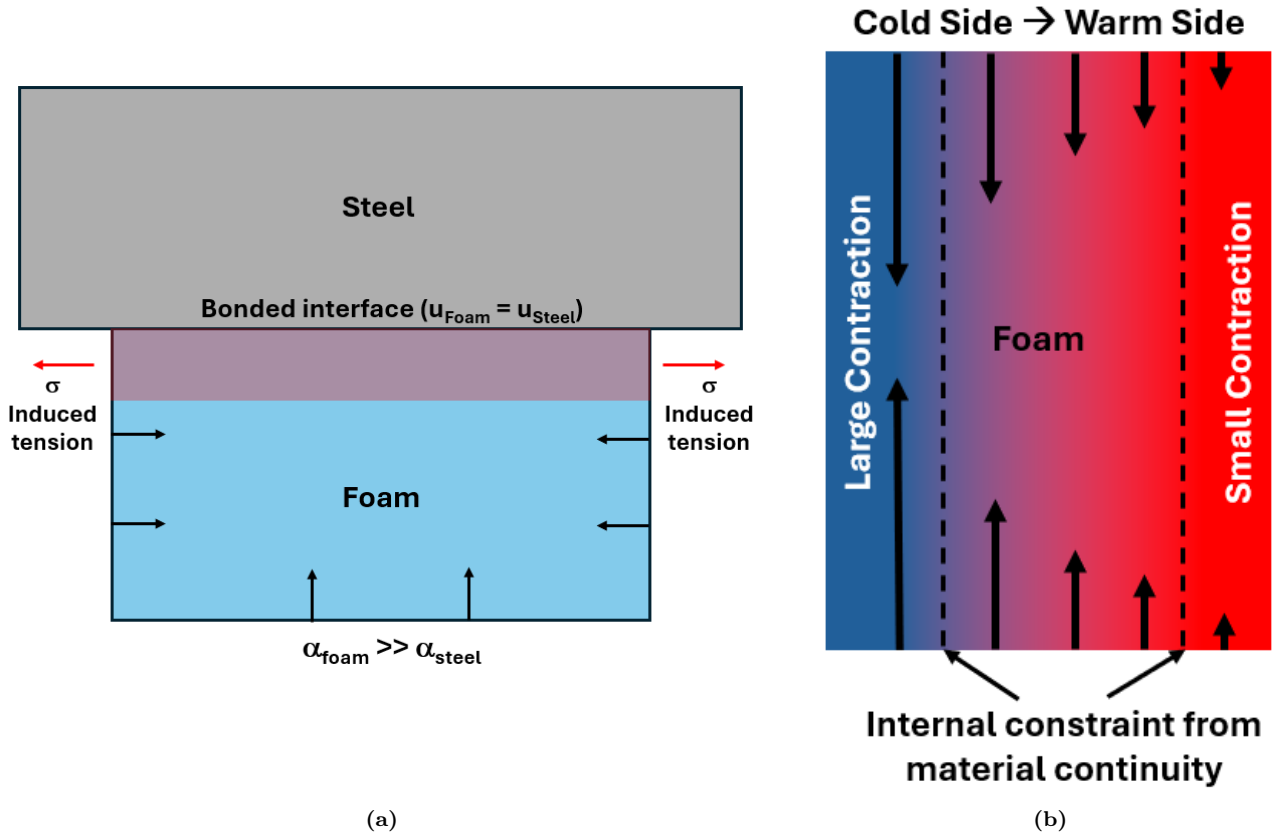


Figure 2.5: Schematic illustration of the two main constraint mechanisms during cool-down: (a) foam-metal thermal expansion mismatch and (b) internal constraint caused by a transient temperature gradient.

2.6 Modelling Approaches

At the microstructural or cell scale, polymeric foams can be modelled using image-based or idealised unit-cell approaches that explicitly represent the cellular geometry. This scale resolves features such as cell walls, struts, cell size and local morphology, but is limited to small representative volumes [22, 52, 53]. At the structural or component scale relevant to tank insulation, the foam is instead treated as a homogenised continuum with effective properties such as density, elastic modulus, Poisson's ratio and thermal expansion coefficient [54, 55].

This simplification enables the analysis of larger geometries and system-level stress distributions, but it does not explicitly capture cell-level defects, anisotropy, density gradients or microstructural failure mechanisms. For cryogenic insulation systems, finite element modelling is used to analyse temperature distributions, thermal stresses and structural response under realistic boundary conditions [56, 57]. Transient thermo-mechanical models are especially relevant because cool-down and warm-up can involve large temperature gradients, changing material properties and constrained thermal contraction at the same time. However, the predictive capability of such models remains limited by uncertain temperature-dependent material data, simplified boundary conditions and the absence of validated failure criteria for cryogenic foam damage. Even in simplified continuum models, the problem remains challenging because the response is governed by several coupled effects at once: transient heat conduction, temperature-dependent stiffness and contraction, foam-metal constraint, local geometry and uncertain cryogenic material data. Small changes in boundary conditions, interface assumptions or material

properties can therefore change the predicted stress and elastic strain fields, which limits the model's predictive capability for damage initiation [22]. Therefore, modelling is most reliable when used to identify thermo-mechanical mechanisms and critical regions, rather than to directly predict crack initiation or propagation.

2.7 Research Gap and Research Questions

The literature reviewed in this chapter shows that polyurethane foam insulation is a promising candidate for non-vacuum LH₂ storage, but that its mechanical reliability under cryogenic thermal loading remains insufficiently understood. In particular, transient temperature gradients, foam-metal thermal expansion mismatch, mechanical constraint and local geometry may interact to produce stress and elastic strain concentrations in the foam. Although material properties and insulation concepts have been studied experimentally, the coupled transient thermo-mechanical response of foam insulation during cool-down and warm-up remains insufficiently quantified [15, 22]. This thesis addresses this gap using a coupled transient thermo-mechanical finite element model. The model is not intended to predict crack initiation or propagation directly, but to identify critical mechanisms, critical regimes, loading phases and regions of elevated stress and elastic strain. The main research question is formulated as follows:

How do transient thermal gradients and constrained thermal contraction generate stress and elastic strain concentrations in polymeric foam insulation for non-vacuum liquid hydrogen storage systems?

To answer this question, the following subquestions are addressed:

Material behaviour

- *Which temperature-dependent material properties are most relevant for representing the thermo-mechanical response of polyurethane foam insulation under cryogenic loading?*
- *How can these properties be selected and incorporated into a transient thermo-mechanical FE model?*

Stress and strain development

- *How do transient temperature gradients, foam-metal thermal expansion mismatch and mechanical constraints affect the evolution of stress and elastic strain during cool-down and warm-up?*
- *Which spatial regions, foam-metal geometric features and temperature loading phases are most critical for stress and elastic strain concentration in the foam?*

Damage interpretation and experimental comparison

- *How do the predicted stress and elastic strain concentrations compare qualitatively with experimentally observed cracking during thermal transients?*
- *To what extent can the predicted stress and elastic strain concentrations from the models be used to identify thermo-mechanical risk regions in representative foam-insulated LH₂ tank geometries?*

Chapter 3

Research Methodology

Section	Page
3.1 Finite Element Modelling Approach	23
3.2 Modelling Strategy	24
3.3 Geometry and Configurations	26
3.4 Element Type and Formulation	29
3.5 Material Properties	30
3.6 Initial and Boundary Conditions	33
3.7 Mesh Strategy	38
3.8 Numerical Setup	41
3.9 Modelling Scenarios	42
3.10 Post-Processing	44
3.11 Verification and Validation	49
3.12 Assumptions and Limitations	56

THIS CHAPTER describes the construction and analysis of the coupled transient thermo-mechanical finite element models used in this study. The methodology is organised around three model configurations. The simplified mechanism geometry, the experiment-specific model and the simplified spherical tank model. The chapter presents the geometry, material properties, boundary conditions, mesh strategy, numerical setup, simulation matrix, post-processing approach and modelling limitations.

3.1 Finite Element Modelling Approach

The thermo-mechanical problem defined in Section 2.5 couples transient heat conduction to the resulting stress and strain field through temperature-dependent material properties and constrained thermal contraction. Closed-form analytical solutions of these coupled equations exist only for simple, idealised geometries. For the irregular geometries, multiple material interfaces and temperature-dependent properties considered here, no analytical solution is available, and a numerical method is required to obtain approximate solutions on the actual model geometry.

The finite element method is adopted in this study because it is particularly well suited to solid-mechanics problems involving irregular geometries, material interfaces and localised stress concentrations. The analyses are performed in ANSYS Mechanical [58], a standard commercial package whose solvers have been extensively verified and are widely used in industry and research. It provides all functionality required here, coupled transient thermal, structural analysis, temperature-dependent material properties and bonded interfaces between dissimilar materials, and enough scripting flexibility to define the property tables, thermal load histories and post-processing paths of this study. In the finite element method, the continuous domain is subdivided into a mesh of small elements connected at nodes. The temperature and displacement fields are approximated within each element by interpolation functions, and the governing equilibrium and heat-conduction equations are enforced in a weak integral form.

The analysis in this study is thermally transient but mechanically quasi-static. The only quantity for which time dependence is physically significant is the temperature field, which evolves according to the transient heat-conduction equation of Section 2.5.1. The mechanical response is obtained as a sequence of equilibrium states: at each time step, the stress and strain field is the field that satisfies mechanical equilibrium under the temperature distribution present at that instant. This is justified because the mechanical response propagates through elastic waves whose time scale is orders of magnitude shorter than the thermal diffusion time scale that drives the load. Any mechanical transient therefore decays effectively instantaneously relative to the thermal transient, so that the structure reaches equilibrium at each instant and follows the evolving temperature field.

This separation of time scales can be verified using a simple scaling argument. The thermal diffusion time scale across the foam layer of thickness $L = 0.15$ m is estimated from Equation 2.5 as $t_{\text{diff}} \approx L^2/\alpha_{\text{th}}$, where the thermal diffusivity of the foam is $\alpha_{\text{th}} = k/(\rho c_p) \approx 5 \times 10^{-7}$ m² s⁻¹. This gives $t_{\text{diff}} \approx 4.5 \times 10^4$ s. The mechanical time scale is set by the propagation of elastic waves through the foam. As an order-of-magnitude estimate, the longitudinal wave speed is taken as $c \approx \sqrt{E/\rho}$, giving $c \approx 390$ m s⁻¹ for the foam at room temperature. The corresponding

mechanical time scale is therefore $t_{\text{mech}} = L/c \approx 4 \times 10^{-4}$ s [58, 59]. The ratio

$$\frac{t_{\text{diff}}}{t_{\text{mech}}} \approx 10^8$$

confirms that mechanical disturbances decay many orders of magnitude faster than the thermal field evolves. The assumption of mechanical quasi-static equilibrium at each thermal time step is therefore justified. Consequently, the overall time scale of the problem is set by the thermal transient.

3.2 Modelling Strategy and Scope

The numerical work in this study has three distinct objectives, which are best addressed using separate finite element models:

- **Simplified mechanism model:** an idealised model used to isolate the governing thermo-mechanical mechanisms as cleanly as possible.
- **Experiment-specific model:** a model that reflects the experiment by including its specific multi-material assembly and measured cooling behaviour.
- **Simplified spherical tank model:** a representative tank-geometry model used to examine how the identified mechanisms carry over towards full-scale application.

The three model configurations are shown in Figure 3.1. The models progress from an idealised geometry to more representative configurations. This layered strategy supports verification, because the mechanisms established in the simplest model provide the reference against which the behaviour of the more complex geometries is interpreted [60]. The solution is linear elastic and contains no damage law, and any failure assessment is performed as a separate post-processing step. The models therefore yield risk indicators rather than validated failure predictions. Table 3.1 summarises the purpose, main output and results sections associated with each model.

All three models share the same modelling basis: a homogenised, isotropic, linear elastic continuum representation of the foam with temperature-dependent properties [22], a coupled transient thermo-mechanical formulation, and the axisymmetric assumption. The simplified mechanism model is the basis of the study and produces the large majority of the results. Its reduced, controlled geometry is inexpensive to solve, so it carries the bulk of the numerical program and provides the reference response against which the other two configurations are interpreted. It is deliberately not intended to reproduce the experiment, but to isolate the governing mechanisms under conditions that can be varied systematically: transient cooling through the insulation layer, constrained thermal contraction, foam-metal thermal expansion mismatch, and the concentration produced by a geometric discontinuity.

Table 3.1: Overview of the three model configurations, their purpose and main output.

Model	Purpose	Main output
Simplified mechanism model	Primary thermo-mechanical analysis: mechanism isolation, numerical verification, sensitivity and cooling/warm-up studies.	Temperature fields, stress and elastic strain paths, and component-wise strain decomposition.
Experiment-specific model	Mechanical interpretation of the real configuration, focused on the warm-up phase in which cracking was reported.	Stress and elastic strain evolution during experimental cool-down and warm-up, and qualitative comparison with reported cracking.
Simplified spherical tank model	Geometry comparison: influence of a smoother tank geometry, realistic foam thickness and the two-layer foam-perlite insulation build-up.	Stress and elastic strain distribution in a representative tank geometry.

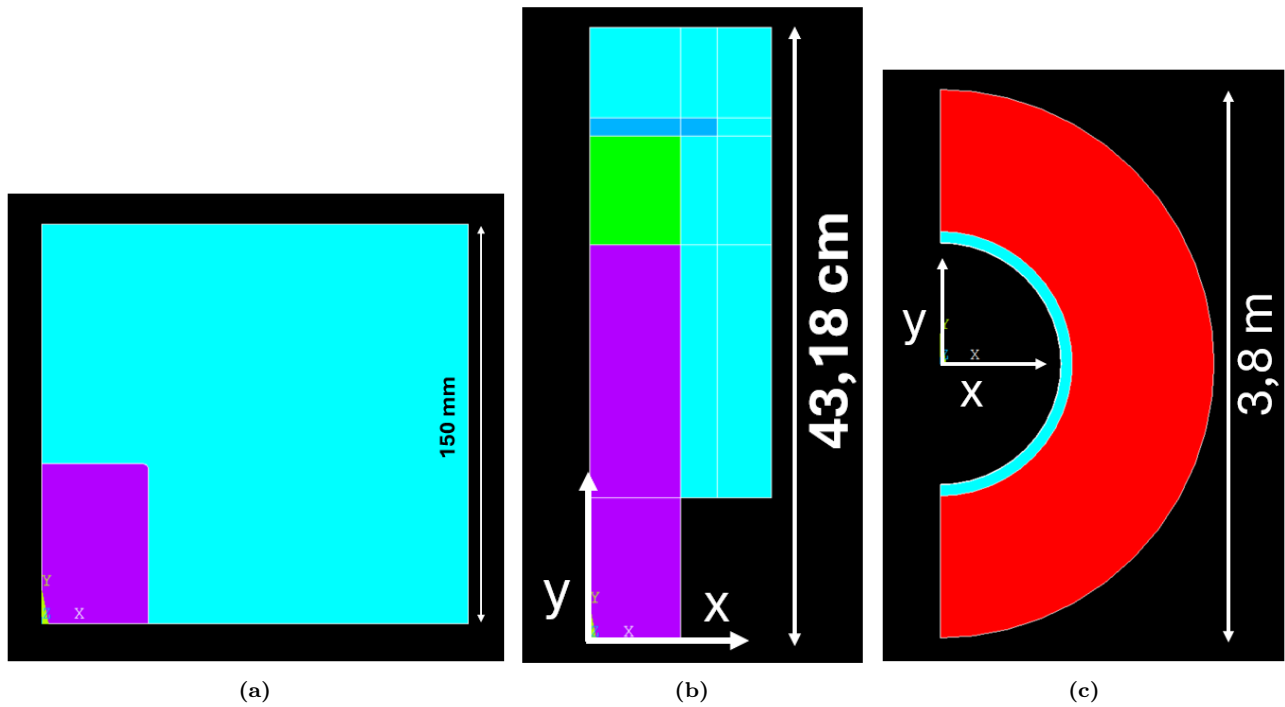


Figure 3.1: The three model configurations used in this study: (a) simplified mechanism model, (b) experiment-specific model, and (c) simplified spherical tank model. Colours denote foam (light blue), steel (purple), copper (green), aluminium (dark blue), and perlite (red). More detailed model geometry and dimensions are given in Figures 3.2 and 3.3.

The two remaining configurations are used more narrowly and support this interpretation rather than introducing independent analyses. The experiment-specific model is framed more cautiously, because its thermal loading is a prescribed temperature-time profile approximating the measured evolution rather than a thermal field solved from first principles. It therefore evaluates the mechanical consequences of an experimentally representative transient, and the resulting

comparison with the reported cracking is qualitative rather than a validation, as discussed in Section 3.11. The spherical tank model has a smooth and continuous foam-metal interface without the corner, so comparing its response with that of the mechanism model isolates the influence of tank-scale geometry, realistic foam thickness and the two-layer foam-perlite insulation build-up. Perlite is present only in the spherical tank model, where it represents the second insulation layer of the real concept. In the simplified mechanism and experiment-specific models the perlite is not included, since these configurations reproduce laboratory-scale specimens in which only the foam layer is present. The geometries and dimensions of the models are specified in Section 3.3.

3.3 Geometry and Model Configurations

All three model configurations are implemented as two-dimensional axisymmetric cross-sections, following the coordinate convention discussed in Section 3.4: the X-direction is the radial direction r , the Y-direction is the axial direction z and coincides with the axis of symmetry, and the circumferential, or hoop, direction is represented implicitly through the axisymmetric formulation. Each geometry is therefore defined by a planar cross-section that is revolved about the Y-axis to generate the full three-dimensional body. This section specifies the geometry, dimensions and material regions of each configuration.

Simplified Foam-Metal Geometry

The simplified mechanism geometry consists of a 304 stainless steel, UNS S30400, block [61] surrounded on two sides by a rigid polyurethane foam layer, forming an L-shaped foam region. The cross-section is rectangular: the steel block occupies the lower-left region, extending 40 mm in the radial direction and 60 mm in the axial direction, and the foam fills the remainder of a 160 mm radial by 150 mm axial domain. This leaves a foam thickness of 90 mm above the steel and 120 mm radially outboard of it. The dimensions are shown in Figure 3.2.

The steel and the foam share a bonded interface along the top and right faces of the steel block. These two faces meet at a corner in the foam, located at the radial and axial coordinates (40 mm, 60 mm). A perfectly sharp corner produces a theoretical stress singularity, at which the local stress does not converge under mesh refinement. This is discussed in more detail in Section 3.7. To manage this, the corner is geometrically rounded with a fillet radius R , and three variants were examined: a sharp corner, a small radius of $R = 2$ mm, and a larger radius of $R = 10$ mm, corresponding to 5% and 25% of the steel radial width, respectively. The final mechanism model adopts $R = 2$ mm. This radius is small enough to preserve the local character of a near-sharp re-entrant transition while avoiding a perfectly sharp mathematical singularity. The value of $R = 2$ mm was selected as the smallest radius for which a valid mapped mesh could still be generated around the fillet, since smaller radii did not mesh without errors at the transition between the straight interface and the rounded corner. The geometry therefore deviates as little as possible from the original sharp corner while remaining numerically tractable.

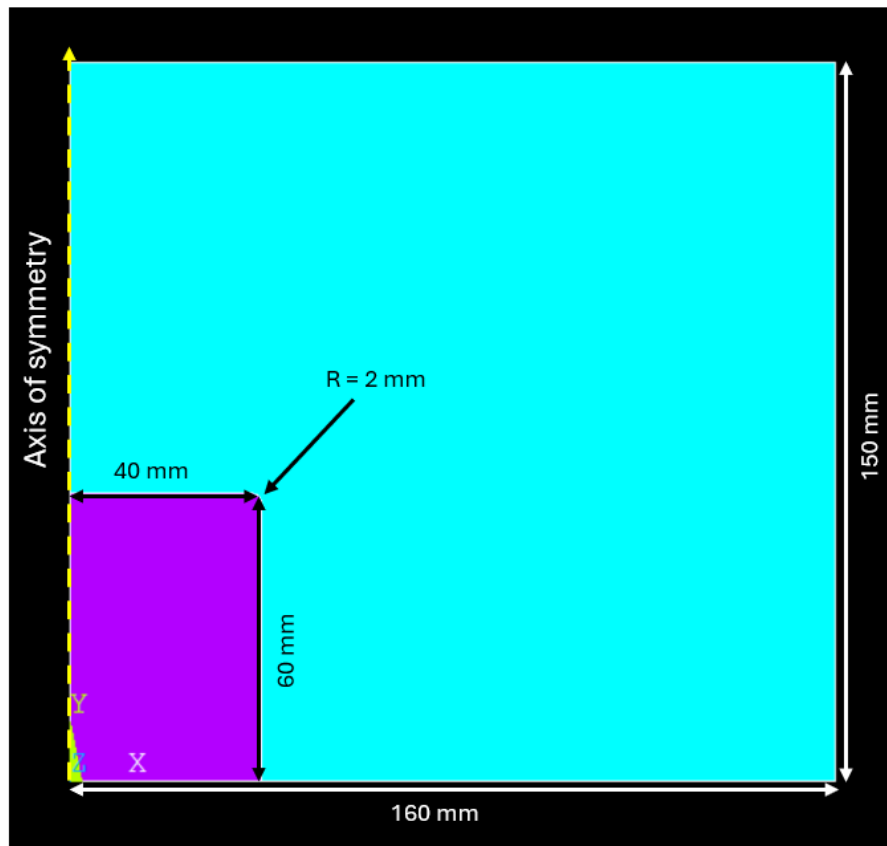


Figure 3.2: Dimensioned cross-section of the simplified foam-metal geometry, showing the steel block (purple), the surrounding L-shaped foam region (light blue), and the rounded corner with fillet radius R .

Experiment-Specific Geometry

The experiment-specific geometry reproduces the laboratory configuration as an axisymmetric cylindrical assembly. It comprises four material regions: a stainless steel body, a copper cold-head region, an aluminium heater puck, and the surrounding polyurethane foam layer. The copper region represents the cold head through which the assembly is cooled, and the foam is modelled as a continuous layer bonded to the metallic assembly. The dimensions are taken from the available geometry. The corresponding model dimensions are shown in Figure 3.3. Along the axis, the stainless steel extends over the first 11 inches, or 279.4 mm, of which the lowest 4 inches, or 101.6 mm, is bare and not covered by foam. This is followed by the copper cold head over 11–14 inches, corresponding to 76.2 mm, and the aluminium heater puck over 14–14.5 inches, corresponding to a thickness of 12.7 mm. The steel and copper share a common outer radius of 2.5 inches, or 63.5 mm. The aluminium puck extends to a larger radius of 3.5 inches, or 88.9 mm, and the foam forms a continuous layer out to an outer radius of 5.0 inches, or 127 mm, covering the assembly from an axial position of 4 inches up to the end of the model at 17 inches, or 431.8 mm.

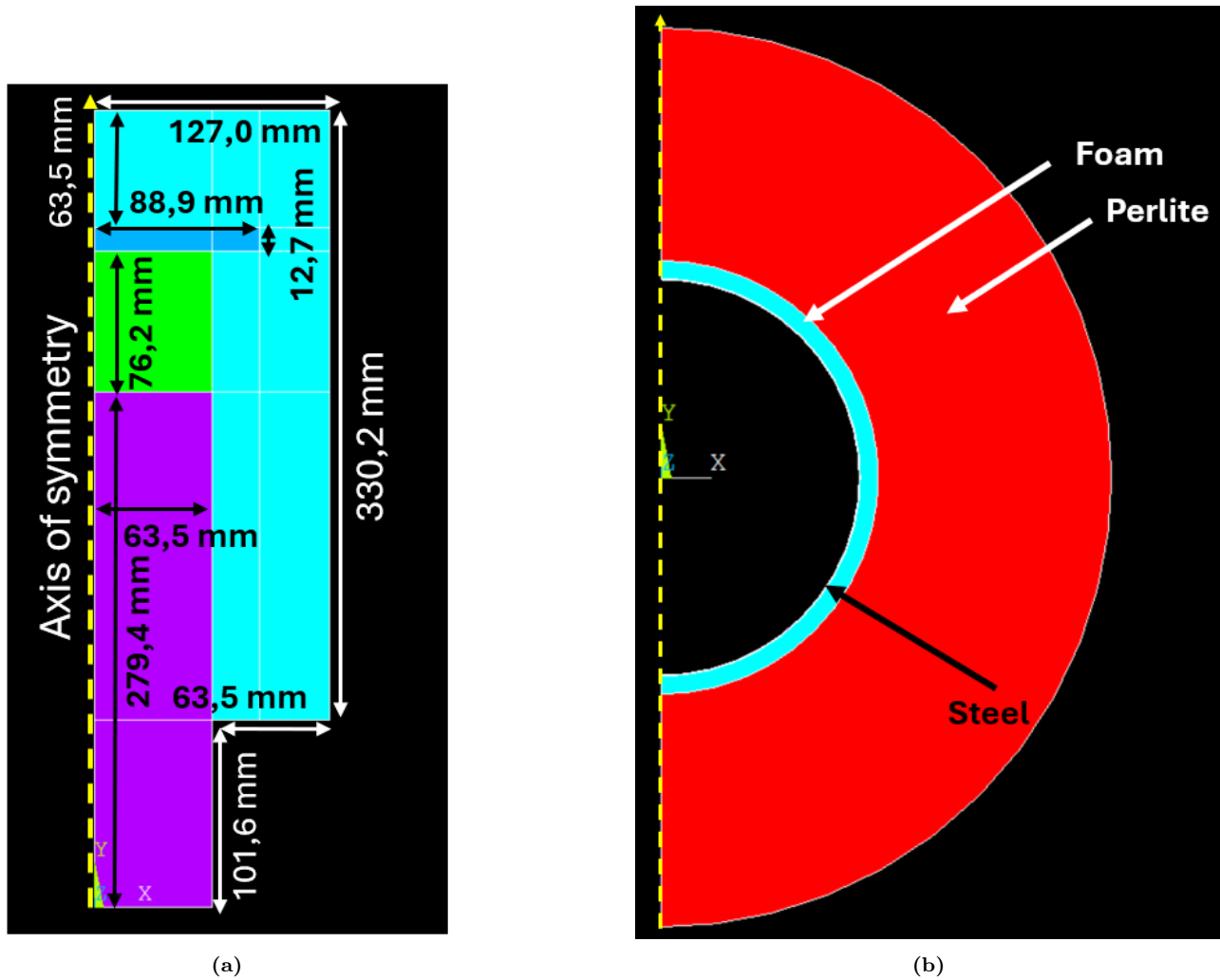


Figure 3.3: Axisymmetric cross-sections of (a) the dimensioned experiment-specific geometry and (b) the representative spherical tank geometry with the material regions indicated. Colours denote foam (light blue), steel (purple), copper (green), aluminium (dark blue), and perlite (red).

Spherical Tank Geometry

The spherical tank geometry is a tank-scale configuration of the type considered for large stationary LH₂ storage. It is modelled as a spherical inner shell surrounded by two concentric insulation layers, and is generated by revolving a semicircular cross-section about the axial Y-axis. Three material regions are included: the inner stainless steel shell, a polyurethane foam layer bonded to its outer surface, and a perlite layer outside the foam as the second insulation layer.

The configuration is characterised here by the proportions that govern the mechanical response. The radius of curvature of the shell is more than an order of magnitude larger than any geometric feature of the mechanism model, so the foam-metal interface is smooth and effectively free of local discontinuities. The foam layer is thicker than in the mechanism model and the perlite layer is substantially thicker than the foam, so that the ambient boundary condition is applied at the outer perlite surface rather than at the foam surface. The outer steel shell of the full double-walled tank is not included. Only the inner shell and its surrounding insulation are modelled. The overall geometry is shown in Figure 3.3.

3.4 Element Type and Axisymmetric Thermo-Mechanical Formulation

All three model configurations are analysed as two-dimensional axisymmetric cross-sections. Because the geometry, material properties, thermal loading and mechanical constraints are essentially independent of the circumferential direction, the three-dimensional problem can be reduced to a two-dimensional axisymmetric problem [62]. In this formulation, a two-dimensional cross-section is defined in a plane containing the axis of symmetry, and the full body is represented by rotating this cross-section about that axis. This is appropriate because each configuration, the foam-metal mechanism geometry, the cylindrical experiment assembly and the spherical tank shell, is rotationally symmetric about that axis, and it retains the dominant radial and axial response while substantially reducing the computational cost relative to a full three-dimensional model. The coordinate convention follows the solver requirement that the axis of symmetry coincides with the global Y-axis [63]. The convention is illustrated in Figure 3.4 for the experiment-specific configuration.

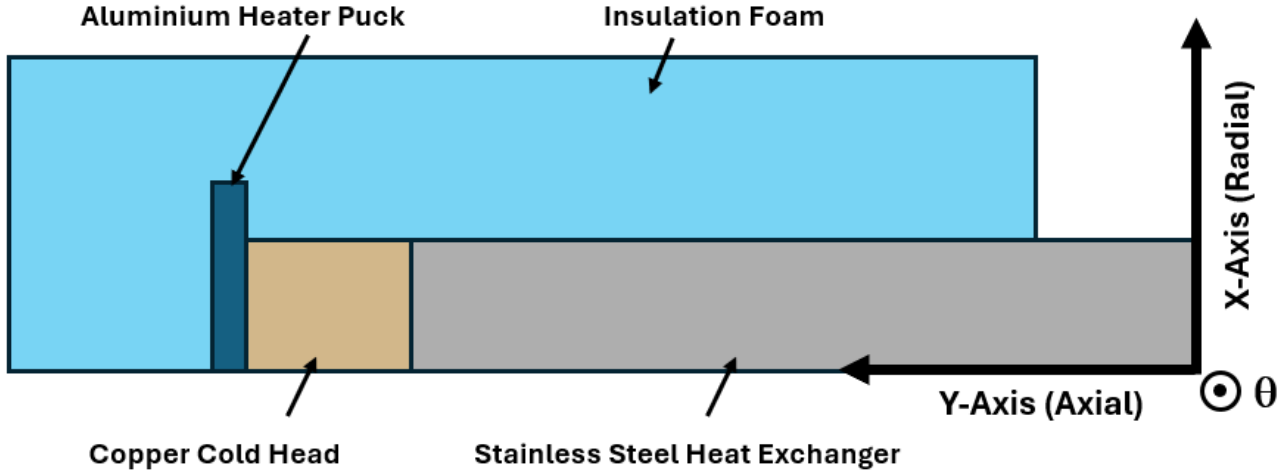


Figure 3.4: Cross-section of the experiment-specific geometry, showing the axisymmetric coordinate convention: X is the radial direction (r), Y is the axial direction (z) and coincides with the axis of symmetry, and Z is the hoop direction (θ).

A distinctive feature of the axisymmetric formulation, and the main reason it differs from a plane analysis, is the hoop strain. Even though no degree of freedom exists in the hoop direction, a radial displacement u_r necessarily changes the circumference of the corresponding ring of material, producing a non-zero hoop strain [59, 62]:

$$\varepsilon_\theta = \frac{u_r}{r} \quad (3.1)$$

The axisymmetric strain state therefore comprises four components: the radial strain ε_r , the axial strain ε_z , the hoop strain ε_θ , and the in-plane shear strain γ_{rz} . Correspondingly, the stress state comprises the radial stress σ_r , the axial stress σ_z , the hoop stress σ_θ , and the shear stress τ_{rz} [62]. This is the key difference from a plane-strain analysis, in which the out-of-plane strain is set to zero. In the axisymmetric case, the out-of-plane hoop strain is generally not zero, but is determined by the radial displacement through Equation 3.1. The radial-axial cross-section therefore captures the dominant thermo-mechanical mechanisms: differential contraction in the

radial direction, through-thickness gradients, and the constraint at the foam-metal interface. All models were built in ANSYS Mechanical APDL. The simplified and experiment-specific models use an identical element type and formulation, so that the temperature, stress and elastic strain fields can be compared consistently between these two configurations. The spherical tank model uses the same formulation for the steel and foam regions, but introduces a second element type for the perlite layer, as described below.

The foam and metallic regions are discretised with the coupled-field element PLANE223, an eight-node quadratic quadrilateral element with midside nodes [63]. This element was selected because the present problem requires the transient temperature field and the thermo-elastic response to be evaluated on the same axisymmetric mesh. PLANE223 supports structural-thermal coupled-field analysis, including the displacement degrees of freedom UX and UY and the temperature degree of freedom $TEMP$, which are activated using $KEYOPT(1) = 11$ [64]. This makes the element appropriate for thermal-stress analyses in which temperature-dependent material properties and thermal expansion are included directly in the same formulation. Using a single coupled-field element also avoids transferring temperatures from a separate thermal mesh to a structural mesh. The spherical tank model additionally employs the thermal-only element PLANE55 ($KEYOPT(3) = 1$) for the perlite annulus. Since perlite is a loose granular fill that transmits neither tensile nor shear stress, it is treated as mechanically passive. Only heat conduction through the perlite layer is modelled. The foam-perlite interface consequently acts as a free mechanical boundary. In all models, the axisymmetric option is activated through $KEYOPT(3) = 1$. The analysis uses small-strain kinematics and geometric nonlinearity is not activated. The solution is nonlinear only because the material properties are temperature-dependent and the thermal and structural fields are coupled, not because of large deformation.

3.5 Material Properties

Temperature-dependent thermal and mechanical properties were assigned to the foam and to the metallic components. The analysis requires the following material properties: density ρ , thermal conductivity k , specific heat capacity c_p , Young's modulus E , Poisson's ratio ν and coefficient of thermal expansion α . All materials are modelled as isotropic. Properties that vary with temperature are supplied as tabulated data and interpolated linearly between the tabulated points. To avoid extrapolation outside the tables during the 293-20 K solve, each temperature-dependent table is anchored at or below the minimum solve temperature of 20 K and at or above 293 K.

All coefficients of thermal expansion are defined as secant values relative to the stress-free reference temperature $T_{\text{REF}} = 293$ K, consistent with the convention of ANSYS Mechanical APDL and with the thermal-strain definition discussed in Section 2.5.2. This is required because ANSYS computes the thermal strain from the total expansion relative to the stress-free reference temperature, rather than from an instantaneous tangent coefficient at the current temperature. Therefore, when expansion data are available relative to a reference temperature, they must be converted to an effective secant coefficient, defined as [65]:

$$\alpha_{\text{sec}}(T) = \frac{\varepsilon_{\text{th}}(T)}{T - T_{\text{ref}}} = \frac{\Delta L/L_0}{T - T_{\text{ref}}} \quad (3.2)$$

Using secant values ensures that the tabulated ALPX values reproduce the correct total thermal

strain between T_{ref} and the current temperature. The constant properties and data sources are summarised in Table 3.2.

Table 3.2: Constant material properties and data sources.

Material	Model(s)	ρ [kg m ⁻³]	ν [-]	k [W m ⁻¹ K ⁻¹]	c_p [J kg ⁻¹ K ⁻¹]	Source
Polyurethane foam	all		0.30	T-dep.	T-dep.	[66]
Stainless steel 304	all	7800	0.29	15.12	470.5	[61]
OFHC copper, RRR = 100	experiment	8940	0.343	396.9	389.1	[67, 68]
Aluminium 6061-T6	experiment	2700	0.33	154.3	942.9	[69]

Polyurethane Foam

The rigid polyurethane foam is represented as a homogenised, isotropic, linear elastic continuum with temperature-dependent properties. The cell-level morphology, spray-induced anisotropy and local density gradients are therefore not modelled explicitly, but enter only through effective macroscopic properties [22, 55]. This is the standard approach for analysing macro-scale temperature, displacement, stress and elastic strain fields in a system-level configuration at acceptable computational cost [22]. Because the foam operates far below its glass transition temperature throughout the cryogenic range considered, its response is dominated by the stiff, brittle behaviour: below approximately 110 K the compressive cell-wall-buckling plateau largely disappears and the tensile response becomes glassy and linear elastic up to fracture, with negligible plastic reserve [51, 70]. This does not by itself make the response linear up to failure, as the reported stress–strain curves are non-linear and damage can initiate before final fracture [36, 44]. A linear elastic description up to a failure threshold is therefore adopted not as an exact representation of the material, but as the appropriate level of modelling fidelity given the available data, since the multiaxial, temperature- and rate-dependent data required for a plasticity or damage model are not available for this foam at cryogenic temperature. The foam is modelled at a density representative of spray-applied rigid polyurethane cryogenic insulation.

The Young’s modulus is based on direct measurements by Stirna et al. [17] on spray polyurethane foam, measured perpendicular to the rise direction. These data report an increase of roughly a factor of three between 293 K and 77 K, linearly extrapolated to 20 K because Stirna et al. [17] does not provide measurements below 77 K. This extrapolation is conservative relative to the near-plateau stiffening behaviour observed below 77 K by Arvidson and Sparks [36] for rigid polyurethane foam of comparable grade, which suggests that the actual modulus at 20 K is unlikely to be substantially higher than the linearly extrapolated value. Over the 293–20 K range, the conductivity decreases substantially and the specific heat falls by more than an order of magnitude, both consistent with the behaviour reported for closed-cell rigid polyurethane foam at cryogenic temperature. The thermal diffusivity that results from these properties is low throughout the range, which is the property responsible for the slowly moving cooling front.

The coefficient of thermal expansion is based on the NIST TRC cryogenic properties database [71]

and Yang et al. [38], following a four-step selection procedure. First, the NIST TRC database [71] provides cumulative linear expansion polynomials for rigid polyurethane foam at two densities, 32.04 and 64.07 kg m⁻³, both measured parallel to the foam rise direction. Linear interpolation to the modelled density gives a secant CTE in the parallel direction at 20 K. Second, Yang et al.[38] report that the CTE perpendicular to the foam rise direction is consistently 25-30% higher than the parallel value for rigid closed-cell polyurethane foam. This factor is incorporated into the calculation. Third, because the foam in the experiment is bonded to metallic components and is predominantly constrained perpendicular to the rise direction, the perpendicular CTE is selected as the more representative value for the isotropic scalar ALPX used in the model. Fourth, the intermediate temperature points follow the shape of the NIST TRC linear expansion curve, affinely scaled so that the curve passes exactly through the two independently justified anchor points.

Steel, Aluminium, Copper and Perlite

The metallic components are not the primary damage material of this study, but their stiffness, thermal conductivity and thermal expansion govern both the thermal boundary response and the mechanical constraint imposed on the foam. Stainless steel is present in all three models: in the simplified mechanism model it represents a generic constraining metal block, in the experiment-specific model it represents the stainless steel body, and in the spherical tank model it represents the inner shell. Copper and aluminium appear only in the experiment-specific model, where the OFHC copper represents the cold head to which the cooling is applied and the aluminium 6061-T6 represents the heater puck.

For all metals the thermal conductivity and specific heat were taken as constant at their 293 K values, as given in Table 3.2, whereas the Young's modulus and the secant CTE (Aluminium) were retained as temperature-dependent, as given in Tables 3.3 and 3.4. This is a justified simplification because the metals are one to three orders of magnitude more conductive than the foam, so they remain effectively isothermal and the temperature dependence of their k and c_p has a negligible effect on the through-thickness thermal field, which is controlled almost entirely by the foam. The steel modulus and CTE are taken from the NIST 304-stainless cryogenic dataset [61], and the copper and aluminium properties from the corresponding NIST cryogenic curve fits and an article [67, 69, 72].

Table 3.3: Stainless steel 304 temperature-dependent Young’s modulus and secant coefficient of thermal expansion [61].

T [K]	E [GPa]	α_{sec} [10^{-5} K^{-1}]
0	210.1	1.099
20	210.1	1.099
40	211.7	1.177
77	214.4	1.296
100	213.7	1.354
150	210.9	1.444
200	207.2	1.496
250	203.3	1.523
293	199.5	1.551
300	199.5	1.551

Table 3.4: Copper and aluminium temperature-dependent Young’s modulus and secant coefficient of thermal expansion [67, 69, 72]

Quantity	20 K	293 K	300 K
Cu E [GPa]	136.95	126.097	126.097
Al E [GPa]	77.8084	70.3586	70.3586
Al α_{sec} [10^{-5} K^{-1}]	1.522	2.2642	2.2642

Finally, the perlite layer in the spherical tank model is treated as a mechanically passive granular fill, reflecting its physical nature as a loose, unbonded particulate material. As a granular fill, it cannot transmit tensile stress and provides no shear coupling at its boundary with the foam, so the outer foam surface behaves as an effectively free mechanical boundary. This ensures that the ambient boundary condition of 293 K is applied at the outer perlite surface rather than directly at the foam surface, which would underestimate the insulation thickness and overestimate the thermal gradient across the foam. Because the perlite is modelled as a thermally conducting but mechanically passive layer, its role in this study is to move the ambient boundary condition away from the foam outer surface, which reduces the temperature gradient across the foam compared with the mechanism model.

3.6 Initial and Boundary Conditions

This section describes the initial conditions and the thermal and mechanical boundary conditions used in the three model configurations. The thermal boundary conditions define the transient temperature field that drives thermal contraction, while the mechanical boundary conditions determine how much of this contraction is converted into elastic strain and stress. In all three models, the initial temperature is uniform at 293 K, which coincides with the stress-free reference temperature $T_{\text{REF}} = 293 \text{ K}$, so that the initial state carries no thermal strain and no stress. The thermal load is then imposed as a prescribed temperature-time curve on the cold region of each model, while the exposed outer surfaces are held at the ambient temperature of 293 K and the remaining faces are treated as adiabatic or symmetry boundaries, as appropriate. Table 3.5 summarises the conditions for the three models. The applied boundary conditions are visualised in Figures 3.5 and 3.6. Orange triangle symbols indicate the thermal boundary conditions. The symbols applied to the cold region represent the imposed cooling curve, corresponding to the steel block in the mechanism model, the copper cold head in the experiment-specific model and the inner steel shell in the spherical tank model. The orange symbols along the outer foam or perlite boundaries represent the ambient temperature condition of 293 K. Blue arrow symbols indicate the mechanical boundary conditions used to

remove rigid-body motion.

Table 3.5: Initial, thermal boundary and mechanical boundary conditions for the three model configurations.

Model	Thermal conditions	Mechanical constraints
Simplified mechanism	Initial temperature: 293 K Cooled region: entire steel block Ambient boundary: top and right outer faces of the foam	$U_X = 0$ on the symmetry axis, $X = 0$ $U_Y = 0$ on the base line, $Y = 0$
Experiment-specific	Initial temperature: 293 K Cooled region: copper cold head Ambient boundary: exposed foam surfaces and bottom steel surface	$U_X = 0$ on the Y-axis $U_Y = 0$ over the entire steel base
Spherical tank	Initial temperature: 293 K Cooled region: entire inner steel shell Ambient boundary: outer perlite surface	$U_X = 0$ on the Y-axis $U_Y = 0$ at a single node at the outermost point of the rotation axis

Cool-Down Boundary Condition

The cool-down is imposed as a smooth sine-squared temperature-time curve:

$$T(t) = T_0 - \Delta T \sin^2(\omega t), \quad \omega = \frac{\pi}{2t_{\text{cool}}} \quad (3.3)$$

applied over the baseline cool-down duration $t_{\text{cool}} = 2100$ s, with $T_0 = 293$ K and $\Delta T = 273$ K, so that the cold region falls from 293 K to 20 K. The baseline sine-squared curve and the additional cooling-protocol variations used later in the study are shown in Figure 3.7. The sine-squared form is used because it starts and ends with zero rate and is continuously differentiable, which both approximates the shape of the cool-down and avoids the numerical shock of an instantaneous temperature jump. The curve is applied as a ramped load within each load step, and the cool-down is divided into successive load steps so that the time-step size can be refined during the steepest part of the curve. The corresponding time-integration settings are given in Section 3.8.

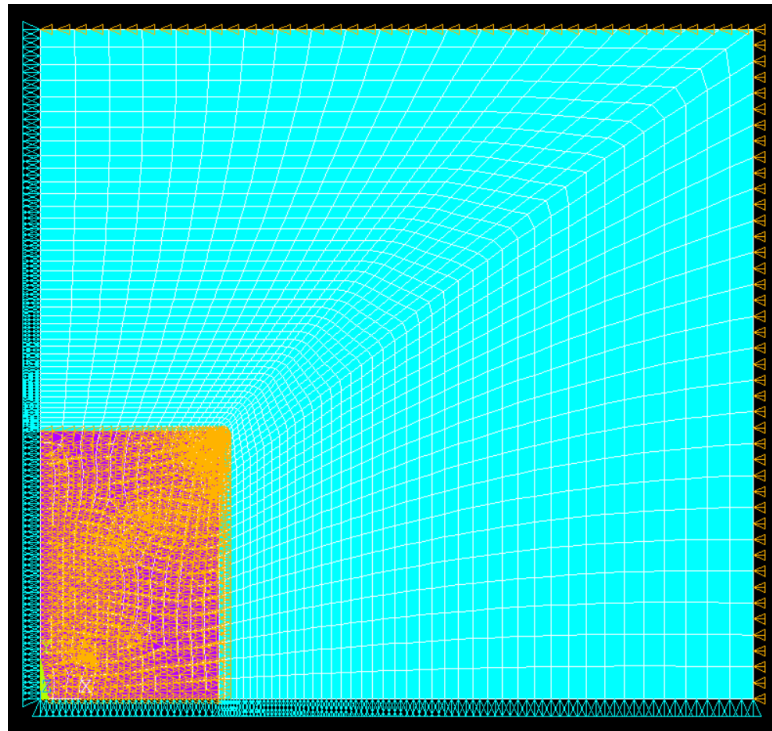
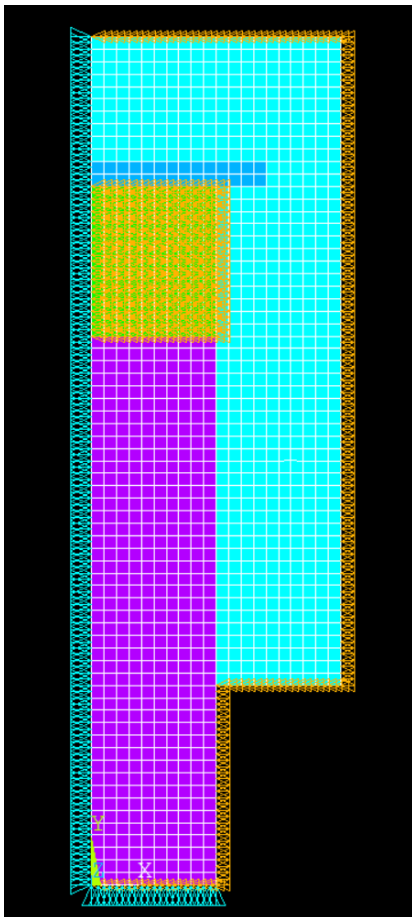
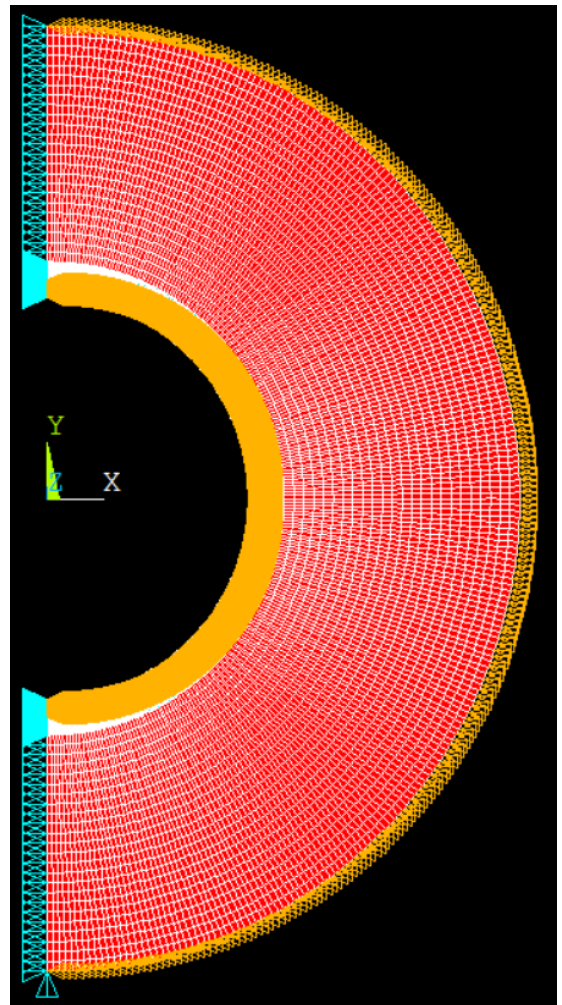


Figure 3.5: Thermal and mechanical boundary conditions applied to the simplified mechanism model.



(a)



(b)

Figure 3.6: Thermal and mechanical boundary conditions applied to (a) the experiment-specific model and (b) the spherical tank model.

The region to which this curve is applied differs by model, reflecting the physical cold source in each case. In the simplified mechanism model, the curve is imposed on the entire steel block, so that the steel is treated as an isothermal cold body following the imposed profile and the transient gradient of interest develops within the foam. In the experiment-specific model, the curve is imposed only on the copper cold head, representing the cryocooler contact, so that the surrounding stainless steel and aluminium respond thermally rather than being driven. In addition to the continuous baseline, a stepwise cool-down protocol is used with the mechanism model to test whether the response depends on the cooling path rather than only on the total temperature drop. This protocol reaches 20 K through three sine-squared segments of 700 s each, separated by two isothermal holds of 120 s at approximately 225 K and 88 K, spanning the same 293–20 K range. The influence of the overall cooling rate is examined through additional cool-down durations, defined in the simulation Scenarios in Section 3.9.

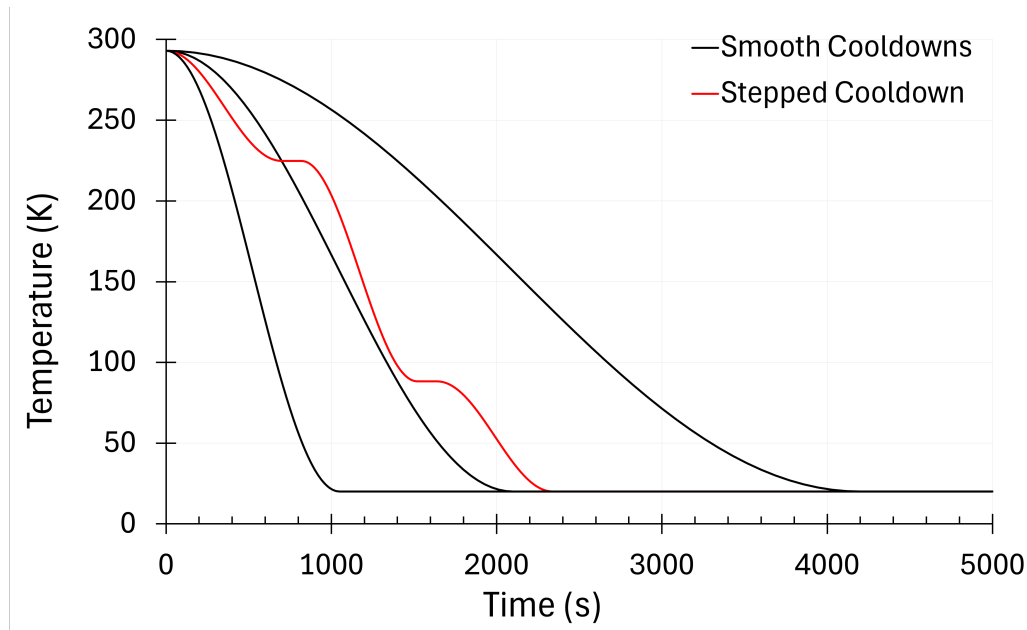


Figure 3.7: Imposed cold-region temperature profiles, showing smooth sine-squared cool-downs from 293 K to 20 K over three durations and a three-segment stepped cool-down with intermediate holds.

Warm-Up Boundary Condition

The warm-up simulations are performed as a continuation of a preceding cool-down and cold-hold period, rather than from an artificial uniform cold initial condition, so that the temperature history entering the warm-up is physically consistent. After the baseline cool-down to 20 K, the cold region is held at 20 K for one hour, represented by a 3600 s hold, to allow the foam to approach a near-uniform cold state and to reproduce the duration used at the experiment. After that the warm-up curve is applied. This sequence is used because the experimental observations indicate that audible cracking occurs mainly during warm-up, so cool-down alone cannot capture the loading condition of interest.

The warm-up is imposed with the same sine-squared form, rising from 20 K by $\Delta T = 80$ K to 100 K, so that it represents the real warm-up phase up to 100 K. Two warm-up rates are considered to bracket the experimentally relevant behaviour: a slower passive warm-up over 3420 s and a faster warm-up over 720 s, see Figure 3.8. Both are applied to the same cold region as the cool-down, with the outer surfaces held at 293 K throughout. The warm-up range and

rates are chosen to represent the experiment warm-up phase. The warm-up cycle is applied to both the simplified mechanism model and the experiment-specific model.

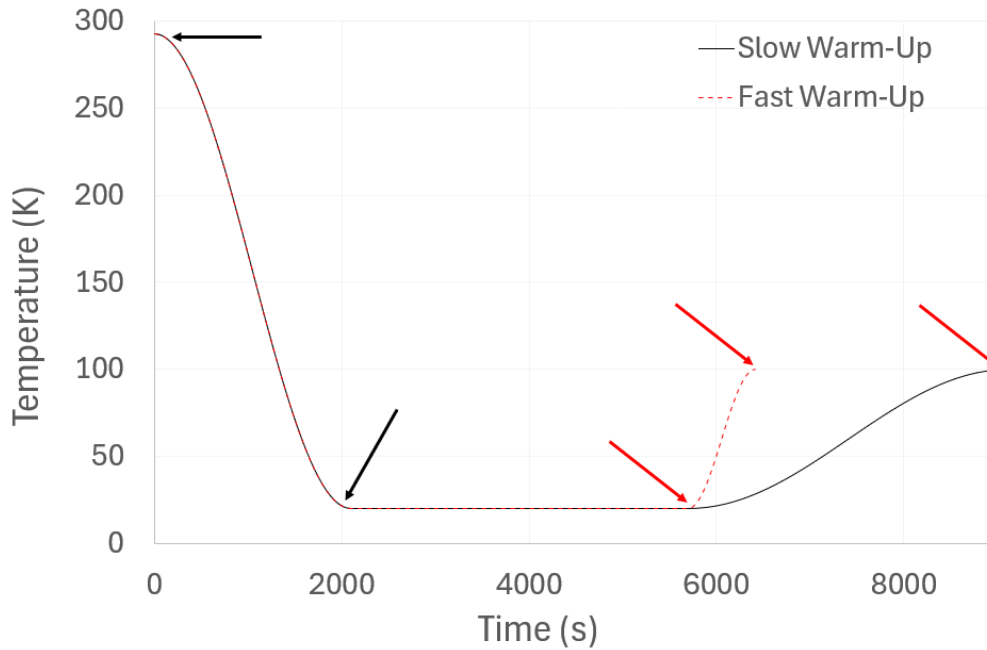


Figure 3.8: Full imposed thermal history for the warm-up cases: cool-down extracted from the finite element model from 293 K to 20 K (black arrows), cold hold of one hour at 20 K, and warm-up from 20 K to 100 K (red arrows) for the fast 720 s and slow 3420 s warm-up rates.

Mechanical Constraints and Thermal Interfaces

The mechanical constraints enforce the axisymmetric condition and remove rigid-body motion without preventing overall contraction, so that the computed stresses arise from material mismatch, transient gradients and local geometry rather than from the external supports. The symmetry axis is constrained radially, $UX = 0$, and the base is constrained axially, $UY = 0$ for the simplified and experimental model. In the experiment model, the axial constraint is applied over the entire steel base rather than at a single point, following the boundary-condition specification. For the spherical tank model, the symmetry axis is constrained radially, $UX = 0$, and a single node at the outermost point of the rotation axis is constrained axially, $UY = 0$. The constraints are intended to minimise artificial restraint while preventing rigid-body motion. Their influence on the internal stress and strain field is assessed through a boundary-condition sensitivity check. All material interfaces are modelled as perfectly bonded. The regions share merged nodes at their common boundaries, so displacement compatibility and thermal continuity are enforced directly across each interface. This represents an idealised, intact insulation system. There is no interfacial thermal contact resistance, and no debonding, sliding or crack opening is permitted during the simulation. This is consistent with the scope of the model, which quantifies the stress and elastic strain that a bonded system would experience rather than simulating interface failure itself.

3.7 Mesh Strategy and Numerical Verification Approach

The mesh strategy was chosen to capture the main thermal gradients and stress and strain distributions with sufficient numerical stability. Refinement was concentrated at the foam-metal interface, where the strongest thermal gradients and the highest stress and elastic strain concentrations occur. At the same time, the mesh was optimised to maintain computational efficiency by balancing solution accuracy with simulation time. This section describes the meshing approach, the mesh levels used for the convergence study, the treatment of the corner peak, the quantities used to assess convergence, and the coarser mesh used for vector-plot interpretation. The mesh construction specific to the experiment-specific and spherical tank models is described at the end of the section.

Mapped Mesh and Mesh Levels

ANSYS offers two meshing approaches for the foam region: a free mesh, in which the element size is controlled globally through smart-size and element-size settings and the mesher fills the domain automatically, and a mapped mesh, in which the number of divisions is prescribed on each bounding line and a structured quadrilateral mesh is swept across the region. Both approaches were investigated for the foam. They were first compared using the maximum stress and strain at the foam-metal interface as the refinement was increased, to see how these peak quantities behaved under mesh refinement.

Neither approach produced a cleanly converging peak, because in both meshes the transition between the straight interface and the rounded corner remains a highly localised refinement-sensitive region. The mapped mesh, however, gave the more regular and better-behaved response of the two and showed an acceptable degree of convergence. It was therefore selected as the meshing approach for all simulations of the simplified mechanism model, with the supporting comparison presented in Section 3.11.1. The mapped mesh has the additional advantage of controlled, reproducible element sizing and regular element shapes near the interface, which is essential for extracting stress and strain along consistent physical paths.

The mapped mesh is generated by first prescribing the number of divisions on each bounding line using `LESIZE` and then concatenating the multi-segment interface and outer boundaries into single logical lines using `LCCAT`, so that opposite sides carry a compatible number of divisions and a structured quadrilateral mesh can be swept across the foam region. Graded divisions are used on the interface and outer-boundary lines to concentrate elements near the interface, where the thermal gradient and the mechanical constraint are strongest. The steel region was meshed separately with a free mesh, controlled through the smart-size and element-size settings, because its geometry, including the rounded corner, is not directly compatible with the same mapped topology as the foam. The interface line divisions were kept compatible where merged nodes were required, so that the bonded foam-metal interface remained conformal.

To assess mesh sensitivity, seven mapped mesh levels were generated for the foam by progressively refining the number of divisions on the bounding lines. The mesh levels are numbered from fine to coarse, with Level 1 being the finest mesh and Level 7 the coarsest mesh. The resulting node and element counts and the associated run times are listed in Table 3.6. The refinement spans a wide range, from a coarse mesh of a few hundred elements to a fine mesh of over ten thousand elements, so that the stability of the path-based quantities can be evaluated across almost two orders of magnitude in element count. See Figure 3.9 for coarse (level 5) and

fine mesh (level 2). The selection of the production mesh level from this study, based on the stability of the path-based response against run time, is presented in Section 3.11.1.

Table 3.6: Mapped foam mesh levels with corresponding node counts, element counts and approximate simulation run times.

Mesh level	Nodes	Elements	Run time
1	32 865	10 816	~75 min
2	21 505	7 056	~29 min
3	13 333	4 356	~11 min
4	5 461	1 764	~7 min
5	2 821	900	~4 min
6	1 281	400	~3 min
7	645	196	~2 min

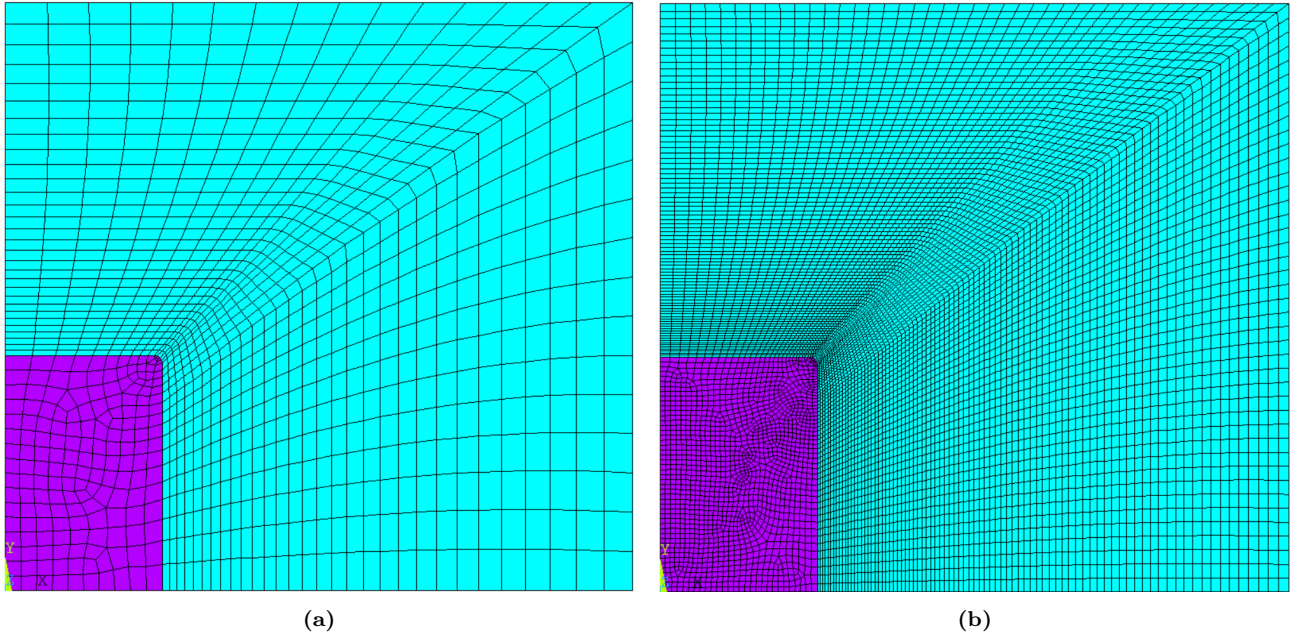


Figure 3.9: Mapped foam mesh with the free-meshed steel region, shown at two refinement levels: (a) coarser mesh, Level 5, and (b) finer mesh, Level 2. The graded division concentrates elements toward the foam–steel interface and the rounded corner.

Treatment of the Corner

The corner at the foam-metal interface is a genuine physical risk region, but it is also the location where the numerical solution is least reliable, because a sharp corner produces a theoretical stress singularity at which the local peak does not converge under mesh refinement. As described in Section 3.3, the corner is therefore geometrically rounded with a small fillet radius of $R = 2$ mm. Even with this rounding and with local refinement, the exact stress and elastic strain maximum at the transition from the horizontal straight section to the inclined section remains sensitive to mesh resolution. The corner maximum was therefore not used as the primary convergence criterion. Instead, the residual local stress at the corner is treated as

a mesh-sensitive stress concentration and reported only as a qualitative local indicator, while all convergence assessment and quantitative conclusions are based on the path-based quantities extracted in the foam away from the immediate corner region.

Path-Based Convergence Quantities and Visualisation Mesh

Convergence was evaluated using fixed physical paths through the foam. Along these paths the extracted quantities were temperature, the first principal stress S_1 and the first principal elastic strain $EPEL_1$. Convergence was then judged by whether these path profiles remain stable between successive mesh levels. This approach evaluates whether the physically relevant stress and elastic strain distribution in the foam is stable under mesh refinement, rather than whether the singular corner value converges to a single number. It is the appropriate criterion for this problem, because the conclusions of the study rest on the stress and strain distribution and its evolution, not on the absolute corner peak. The path definitions used for both convergence assessment and results extraction are given in Section 3.10.

For the interpretation of stress and strain directions through vector plots, an additional coarser model was used with the same element type, material data, boundary conditions and thermal load, but with a sharp foam–steel corner instead of the round corner and a deliberately coarse mapped mesh (Appendix C). The coarse mesh was used only to make the vector directions legible and was not used for peak values or for any convergence assessment.

Mesh Construction for Experiment and Spherical Tank Models

The experiment-specific and spherical tank geometries were also meshed with mapped quadrilateral elements, using a construction adapted to each geometry. For the experiment-specific model, a clean, T-intersection-free mapped mesh was obtained by building the geometry from a set of simple rectangular sub-areas that were subsequently glued into a single connected domain, rather than from a single multi-sided outline. This is a purely constructional step and does not alter the modelled dimensions. For the spherical tank model, the arc-shaped interfaces were concatenated in the same way as for the mechanism model. This allowed a structured mesh through the steel, foam and perlite layers to be generated, with a coarser radial division in the mechanically passive perlite layer. One geometric simplification is used in the spherical tank mesh. The physical wall thickness of the inner steel shell is very small relative to the insulation layers, which, combined with the fine circumferential mesh required around the shell, produces steel elements with extreme aspect ratios and shape-quality violations. The steel wall was therefore modelled with an artificially increased thickness. This is acceptable because the entire steel shell is imposed on a uniform cooling curve, so that no thermal gradient is solved within the steel and its exact wall thickness has no influence on the temperature field transmitted to the foam. Since the entire steel shell is assigned a uniform prescribed temperature, the shell deforms as a kinematically determined body whose radial displacement at the interface is set entirely by the imposed thermal strain. In the axisymmetric formulation, wall thickness does not enter the interface displacement, so the artificial increase in steel wall thickness has no mechanical consequence for the foam response. The physically relevant quantities, the inner shell radius, the foam thickness and the perlite thickness, are retained at their actual values.

3.8 Numerical Setup and Solution Procedure

All simulations were performed in ANSYS Mechanical APDL using a consistent SI unit system: metres, kilograms, seconds, kelvin and pascals. The workflow follows three standard stages. In preprocessing, the geometry is defined, the coupled-field element and temperature-dependent material properties are assigned, and the mesh is generated. In the solution stage, the time-dependent thermal boundary conditions are applied and the coupled transient thermo-mechanical response is solved. In post-processing, the temperature, displacement, stress and strain fields are extracted along the paths and fixed points defined in Section 3.10. Within each model configuration, the same element formulation, material data, production mesh and solver settings are used across comparable cases, so that differences between cases can be attributed to the imposed thermal loading or parameter variation.

The analysis is a coupled-field transient solution, using `ANTYPE,TRANS` and `TRNOPT,FULL`. As discussed at the start of this chapter, although the analysis type is transient, the problem is thermally transient but mechanically quasi-static: the only physically significant time dependence is the evolution of the temperature field, while the mechanical response follows this field as a sequence of equilibrium states. This holds because the mechanical (elastic-wave) time scale is orders of magnitude shorter than the thermal diffusion time scale, so that any mechanical transient decays effectively instantaneously and mechanical equilibrium is reached at each time step. Because the material properties are temperature-dependent and the thermal and structural fields are coupled, the problem is nonlinear even though the kinematics are small-strain and the material is linear elastic (Section 3.5). At each substep (See Appendix C.2), ANSYS evaluates the material properties at the current nodal temperature by linear interpolation, and equilibrium is obtained with a full Newton–Raphson scheme [58], an iterative procedure that re-forms the tangent stiffness matrix at every iteration, allowing up to 80 equilibrium iterations per substep. Convergence was monitored primarily using the heat-flow residual, with an L2-norm tolerance of 0.1% (See Appendix C.2), because the response is driven by the transient thermal field [63].

Automatic time stepping was enabled primarily to resolve the transient thermal part of the coupled analysis. The strongest numerical time-scale requirement arises from the changing temperature gradients during cool-down and warm-up. At each thermal substep, the corresponding displacement, stress and elastic strain fields are solved from the coupled thermo-mechanical equilibrium equations. The initial, minimum and maximum time-step sizes were prescribed per load step. The loading within each load step is applied as a ramp, using `KBC,0`, consistent with the smooth temperature-time curves of Section 3.6. The transient is divided into successive load steps so that the time-step size can be reduced during the steepest part of the cool-down and relaxed during the near-steady cold hold, as summarised in Table 3.7. The cool-down from 293 K to 20 K spans 2100 s and uses the finest time steps over its steepest interval, from 600 s to 1500 s. The subsequent hold at 20 K uses coarser steps (See Appendix C.2).

The stepwise cool-down of Section 3.6 follows the same principle, using six sequential load steps, consisting of three cooling segments, two intermediate holds and a final hold, within one analysis. Because the same solver settings (See Appendix C.2), convergence controls and production mesh were applied to the baseline, cooling-rate, stepwise and warm-up cases, differences between cases can be attributed to the imposed thermal loading rather than to changes in the numerical setup.

Table 3.7: Load-step and time-stepping scheme for the baseline thermal protocol.

Load step	Time interval [s]	Δt init/min/max [s]	Purpose
1a	0–600	5 / 1 / 10	Cool-down, early stage
1b	600–1500	2 / 0.5 / 5	Cool-down, steep temperature interval
1c	1500–2100	5 / 1 / 10	Cool-down, approach to 20 K
2	2100–5700	15 / 5 / 40	Cold hold at 20 K, approximately 1 h
3a, fast warm-up	5700–6420	5 / 1 / 10	Warm-up from 20 K to 100 K over 720 s
3b, slow warm-up	5700–9120	20 / 5 / 40	Warm-up from 20 K to 100 K over 3420 s

Note: The Δt values denote the initial, minimum and maximum time-step sizes.

3.9 Modelling Scenarios and Simulation Cases

A set of modelling scenarios was defined to separate the effects of material-property sensitivity, cooling-rate sensitivity, warm-up response, numerical verification and geometry transferability. All cases share the same primary outputs: the temperature field, the first principal stress S_1 , the first principal elastic strain EPEL1 and the directional radial, axial, hoop and shear stress and strain components, with case-specific outputs noted in the subsections below. The complete list is given in Table 3.8.

Baseline Cool-Down and Cooling-Rate Cases

The baseline cool-down is the reference transient case, using the reference material set and the continuous sine-squared cool-down from 293 K to 20 K over 2100 s. It establishes the basic evolution of the temperature field, displacement field, stress distribution and elastic strain distribution before any parameter is varied, and it provides the reference against which all sensitivity cases are compared. Its outputs are the temperature contours and cooling front, the displacement field, and the stress and elastic strain distributions and paths. The cooling-rate cases evaluate how a faster or slower cool-down changes the magnitude of the resulting stress and elastic strain concentrations. Two rates are compared against the 2100 s baseline: a fast cool-down to 20 K in 1050 s and a slow cool-down in 4200 s. Only the imposed cool-down duration is varied. The geometry, material properties and mechanical constraints are unchanged. The main outputs are the peak stress/elastic strain and the path profiles.

Table 3.8: Simulation scenarios and simulation cases, grouped by analysis cluster.

Cluster	Case	Model	Varied / imposed parameter	Purpose
Reference	Baseline cool-down	Mechanism	Reference cool-down, 293–20 K in 2100 s	Establish reference temperature, stress and strain fields.
Verification	Boundary-condition comparison	Mechanism	Modified mechanical support conditions	Assess whether the internal strain feature is a constraint artefact.
	Numerical verification	Mechanism	Mesh level, time step and hold length	Verify mesh and time-step stability and the near-steady state.
	Analytical thermal check	Mechanism/ spherical tank	One-dimensional thermal-resistance or transient conduction estimate	Check whether the simulated thermal gradients and temperature evolution are physically reasonable.
Material-property sensitivity	CTE sensitivity	Mechanism	Foam CTE scaled by 0.5×, 1.5× and 2×, and equal foam–metal CTE	Isolate the contribution of CTE mismatch to the strain and stress feature.
	Young’s modulus sensitivity	Mechanism	Foam $E(T)$ scaled by 0.5×, 1.5× and 2×	Quantify the influence of foam stiffness on stress and strain.
Cooling-protocol sensitivity	Cooling-rate cases	Mechanism	Cool-down duration of 1050 s, baseline 2100 s and 4200 s	Isolate the transient-gradient effect of cooling rate.
	Stepwise cooling	Mechanism	Intermediate holds at 224.75 K and 88.25 K	Test whether staged cooling changes the peak stress and strain.
Warm-up response	Warm-up cases	Mechanism	Warm-up rate of 720 s and 3420 s, 20–100 K	Assess warm-up response and compare to cool-down stress and strain.
Model transfer	Experimental interpretation	Experiment	Approximating cool-down and warm-up profile imposed on the copper cold head	Qualitative comparison with reported warm-up cracking sounds.
	Spherical tank comparison	Spherical tank	Tank geometry, foam thickness and perlite layer	Assess transfer of the mechanisms to a tank geometry.
Failure relevance	Failure-relevance assessment	Mechanism	Peak S_1 and EPEL1 compared with foam limits in post-processing	Identify regions and loading phases with elevated damage risk.

Warm-Up Cases

The warm-up cases were included because the experimental cracking was reported mainly during warm-up rather than during cool-down. Each warm-up case begins from the cooled state after a cold hold of one hour, and then applies either a slower passive warm-up of 3420 s or a faster warm-up of 720 s from 20 K to 100 K, so that the reported cracking range of roughly 60-80 K is covered. The main outputs are the time evolution of the peak stress and elastic strain and their location, related to the temperature range in which cracking was reported.

Material-Property Sensitivity, Verification and Failure-Relevance Cases

Beyond the cooling-protocol cases, several additional cases isolate specific mechanisms and establish numerical reliability. The CTE-sensitivity cases vary the foam-metal thermal-expansion mismatch. The foam coefficient of thermal expansion is scaled to $0.5\times$, $1.5\times$ and $2\times$ its reference values, and a diagnostic case is included in which the foam and metal are assigned equal coefficients of thermal expansion. The equal-CTE case in particular tests how much of the internal strain feature is driven by the mismatch rather than by the temperature gradient alone. The Young's-modulus-sensitivity cases scale the temperature-dependent foam modulus to $0.5\times$, $1.5\times$ and $2\times$ its reference values, to quantify how strongly the foam stiffness governs the stress and elastic strain.

The boundary-condition comparison modifies the mechanical constraints attached to the foam to assess whether the internal strain feature persists when the support conditions are modified, and therefore whether it is likely to be a thermo-mechanical response rather than solely a constraint artefact of the applied supports. The long cold-hold or near-steady-state case extends the hold at 20 K well beyond the baseline, to verify that the through-thickness gradient and the transient peaks decay towards a lower, near-uniform steady state. Finally, the failure-relevance assessment is a post-processing case rather than a separate solve where the peak first principal stress and elastic strain from the cool-down and warm-up cases are compared against the extrapolated foam strength and strain limits to identify the regions and loading phases of greatest damage risk.

An analytical thermal check is performed to provide an independent reference for the simulated thermal response. The purpose of this calculation is not to reproduce the full coupled thermo-mechanical model, but to assess whether the order of magnitude of the temperature gradients and heat-transfer behaviour is physically reasonable.

3.10 Post-Processing

The post-processing is used to extract both global field quantities and local path-based quantities from the model. The main objective of the study is to identify where and when the stress and elastic strain concentrations develop in the foam. Global contour plots of temperature, first principal stress and first principal elastic strain were generated for orientation, while the quantitative interpretation was based on fixed physical paths through the foam. All path data were exported for further plotting and comparison.

Path Definitions

Fixed physical paths were defined through the foam to extract temperature, stress and strain and to compare them between mesh levels and simulation cases. Each path is defined at a fixed physical location so that it represents the same line in every simulation. Because the paths are picked on existing nodes, their end coordinates differ marginally between mesh levels. The deviation is at most a few millimetres over a 120 mm path and does not affect the comparison. To avoid extracting values across the material discontinuity at the foam-metal interface, only the foam was selected before each path was defined and mapped. Values at the first interface point are nevertheless interpreted cautiously because they lie at a material discontinuity.

For the mechanism model, four paths were used (Figure 3.10a): a diagonal path running from the corner to the outer top corner of the foam, two horizontal radial through-thickness paths at different axial heights to capture how the internal stress and strain change with distance from the corner, and one vertical axial through-thickness path in the foam above the steel. Separate paths were defined for the experiment-specific and spherical tank models (Figure 3.10b), following the same principle. In the spherical tank model, one radial path lies entirely within the foam, while a second extends through both the foam and the perlite to the outer boundary. The paths, except from the spherical paths, are listed in Table 3.9.

Table 3.9: Extraction paths, with coordinates given as radial position r and axial position z in metres.

Model	Path	Start (r, z)	End (r, z)	Type / purpose
Mechanism	Diagonal	(0.0394, 0.0594)	(0.1600, 0.1500)	From the corner to the outer top corner
Mechanism	Radial, upper	(0.0400, 0.0580)	(0.1600, 0.0571)	Through-thickness, near corner height
Mechanism	Radial, lower	Level 2 (0.0400, 0.0410), Level 4 (0.0400, 0.0412)	Level 2 (0.1600, 0.0411), Level 4 (0.1600, 0.0429)	Through-thickness, away from corner
Mechanism	Axial	Level 2 (0.0337, 0.0600), Level 4 (0.0338, 0.0600)	Level 2 (0.0343, 0.1500), Level 4 (0.0305, 0.1500)	Through-thickness, above the steel
Experiment	Path P1	(0.0635, 0.3175)	(0.1270, 0.3175)	Radial, through-thickness through the foam
Experiment	Path P2	(0.0699, 0.3683)	(0.0699, 0.4318)	Axial, through the foam above the puck

Because the paths are fixed at these physical locations, they do not necessarily pass through the point at which the stress or elastic strain attains its absolute maximum at a given instant. The path profiles therefore represent the through-thickness response at a defined location rather than an extraction of the global maximum, which is obtained separately from the field results.

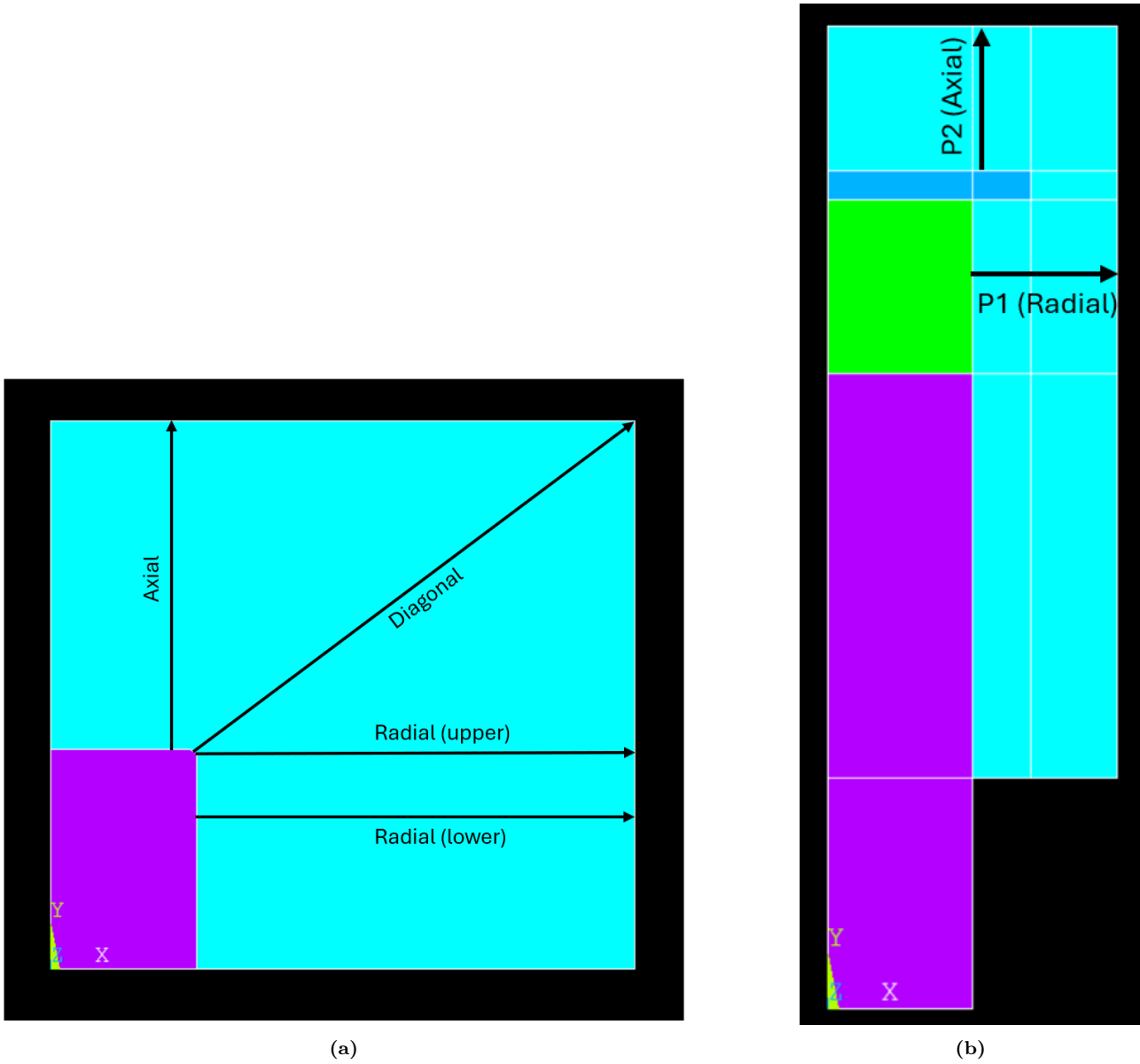


Figure 3.10: Extraction paths used for post-processing, shown on the model geometries: (a) the four mechanism-model paths and (b) the two experiment-model paths.

Stress and Strain Extraction

The first principal stress S_1 was used as the main tensile stress indicator, since tensile failure dominates in brittle foam, and the first principal elastic strain EPEL1 was used as the main elastic strain indicator. Along each path these were extracted together with the directional stresses, S_x , S_y and S_z , the directional elastic strains, EPELX, EPELY and EPELZ, the in-plane shear components, S_{xy} and EPELXY, and, where relevant, the ordered principal values, S_1 - S_3 and EPEL1-EPEL3. The interpretation combined S_1 and EPEL1 with the directional components, because the principal direction can rotate along a path and neither principal quantity alone fully describes the state, as discussed in Appendix A. The maximum foam stress and strain were identified by querying the foam nodal maximum at several time instants on the finest Level 1 mapped mesh. Rather than tracking a single fixed point in time, the stress and strain evolution was reconstructed by extracting the full path profiles at successive time

instants, which was found to be more robust than the /POST26 time-history extraction for these quantities. For the cool-down and subsequent cold-hold sequence, the profiles were extracted at 100, 500, 1000, 2000, 4000 and 5000 s to represent the complete transient. For the warm-up, profiles were extracted at time instants at which the imposed cold-region temperature reached selected values also used during cool-down, for example 40, 60 and 80 K, so that the mechanical state during warm-up can be compared with the cool-down state at the same temperature of the steel/copper. The time-history postprocessor was used only for the steel/copper temperature, extracted at a representative steel node, which is adequate because the entire steel/copper region cools uniformly.

Component-Wise Strain Decomposition

Because the first principal elastic strain direction can rotate along a path, the internal strain feature cannot be explained by the principal strain alone and was analysed through its directional components. Along the through-thickness paths, the radial, axial, hoop and shear elastic strains were extracted separately, so that the directional origin of the feature could be identified. In addition, the thermal strain components EPTX, EPTY and EPTZ were mapped directly onto the paths from the solution and compared with the elastic strain, in order to distinguish the stress-free thermal contraction from the mechanically relevant elastic strain. This separation is essential because a region can carry a large total strain while remaining nearly stress-free if that strain is predominantly thermal, as discussed in Section 2.5.2.

Vector Plots and Field Animations

Vector plots were used to visualise the directions of the principal stresses and principal elastic strains, primarily to interpret the rotation of the principal directions along the paths and in time. Because a dense mesh produces overlapping with unreadable arrows, the vector plots were generated on the coarser visualisation mesh and were used for qualitative interpretation only, not for peak values. For each model, contour animations of the temperature, first principal stress and first principal elastic strain in the foam were produced over the full transient, using a fixed colour scale so that the magnitude can be compared consistently across comparable cases and time steps. These animations were used to follow the development, movement and decay of the concentrations during cool-down, cold hold and warm-up, and selected frames are included in the results chapter to illustrate the evolution.

Experimental Temperature Input and Qualitative Cracking Comparison

The experimental comparison is based on the measured temperature data, which were recorded at the copper cold head, at the boundary between the copper and the steel. The measured cool-down is well represented by a sine-squared curve, which is the basis for the imposed thermal input used throughout this thesis. The measured warm-up behaviour was approximated with a sine-squared warm-up in the same way. Because the temperature history is imposed as an input rather than predicted by the model, this comparison is not treated as an independent thermal validation. The mechanical comparison is qualitative and the only experimental damage information is that audible cracking sounds were reported, without measured crack locations, in the approximate 60-80 K range during warm-up. The predicted stress and elastic

strain evolution, from both the mechanism model and the experiment-specific model under the imposed warm-up, is therefore compared qualitatively with this observation. So whether the predicted peaks fall in the same temperature range, whether the warm-up phase is more severe than the cool-down phase, and how the passive and faster warm-up cases differ.

Failure Indicators and Safety Factors

For cryogenic rigid polyurethane foam, no generally accepted failure criterion exists that is equivalent to the von Mises criterion commonly used for ductile metals. This is because the relevant failure mechanisms are not governed by stable plastic yielding, but by brittle cracking, cell-wall fracture, crushing and, in bonded systems, possible interfacial debonding. Experimental and modelling studies show that rigid polyurethane foams can be strongly temperature- and rate-dependent, and that they may crack at small tensile strains while showing different failure modes in compression [73, 74]. Failure is therefore controlled by a critical tensile stress or strain rather than by yielding [73].

A clear distinction is therefore made between the constitutive model used in the solution and the failure assessment performed afterwards. The model itself is based on the linear elastic constitutive description of Section 3.5, in which the solver computes the temperature, displacement, stress and elastic strain fields using temperature-dependent material properties, without any yield surface or damage law. Possible damage is assessed only in a post-processing sense, by interpreting selected stress and strain quantities as failure indicators and comparing them with available or extrapolated material limits [46, 75]. These indicators identify regions where damage risk may be elevated, rather than providing a deterministic prediction of crack initiation or failure time.

Because brittle materials are generally most sensitive to tensile opening, the maximum principal stress $S1$ is used as the primary scalar indicator for tensile cracking risk. The first principal elastic strain $EPEL1$ is considered as a supporting indicator, while shear and component stresses are examined where interface damage or mixed-mode loading may be relevant [46, 76]. These indicators are formalised through two complementary indices. Because a brittle material fails when the largest tensile stress reaches the ultimate strength, a maximum-principal-stress index is defined as [46, 75]

$$I_{\sigma} = \frac{\sigma_1}{\sigma_{UTS}(T)}, \quad (3.4)$$

where σ_1 is the maximum principal stress and $\sigma_{UTS}(T)$ is the temperature-dependent ultimate tensile strength. Because cryogenic failure also occurs at a critical elastic strain, a maximum-principal-strain index is defined as

$$I_{\varepsilon} = \frac{\varepsilon_1}{\varepsilon_f(T)}, \quad (3.5)$$

where ε_1 is the maximum principal elastic strain and $\varepsilon_f(T)$ is the temperature-dependent strain-to-failure [36, 77]. Failure is indicated when either index reaches unity. The governing safety factor is therefore taken as the reciprocal of the larger failure index, or equivalently as

$$SF = \min \left(\frac{\sigma_{\text{UTS}}(T)}{\sigma_1}, \frac{\varepsilon_f(T)}{\varepsilon_1} \right). \quad (3.6)$$

The strain index is included because the strain-to-failure becomes the binding limit at cryogenic temperature while the tensile strength rises [37]. Also because the principal strain is a direct kinematic output that, unlike stress, is independent of the uncertain foam modulus as the sensitivity study in Chapter 4 confirms. This is deliberately different from a true elastoplastic or damage constitutive model. Such models exist for rigid polyurethane foam, for example crushable-foam and unified creep-plasticity-damage formulations [55, 78, 79], but they require extensive multiaxial, temperature- and rate-dependent data that are not available for this foam at cryogenic temperature. Because the failure-relevant properties near 20 K must be extrapolated, any assessment based on such thresholds carries significant uncertainty. To conclude, both indices, and the material limits against which they are evaluated, are taken from the open literature on rigid polyurethane foam.

3.11 Verification and Validation

The credibility of a numerical model depends on both verification and validation. Verification concerns whether the equations are solved correctly within the chosen numerical implementation, while validation concerns whether the model represents the physical system with sufficient accuracy for its intended purpose [60]. In this study, verification is addressed through the mesh sensitivity, time-step and boundary-condition checks, the long cold-hold consistency check and the analytical thermal calculation as reported below.

The purpose of this first section is to establish that the field quantities used in the remainder of the chapter are numerically reliable enough to support the mechanistic interpretation. Verification is performed on the simplified mechanism model, since this is the primary analysis model and the two representative models copy its element formulation and solver settings. A central point is that the corner is a near-singular stress raiser whose local maximum does not fully converge under mesh refinement. The verification is therefore built around the path-based quantities defined in Section 3.10, which represent the physically relevant stress and elastic strain distribution in the foam, rather than around the singular corner value.

3.11.1 Mesh Sensitivity: Peak and Path-Based Convergence

To assess mesh sensitivity, three meshing strategies were compared using the maximum first principal stress and maximum first principal elastic strain in the foam as the convergence metric. The first strategy uses a mapped mesh in which the number of element divisions is prescribed on each bounding line, as described in Section 3.7, covering seven refinement levels from coarse to fine. The second uses a free mesh controlled by ANSYS smart-size settings, ranging from the coarsest to the finest possible smart-size level, `SMRTSIZE,5` to `SMRTSIZE,1`. The third combines the smart-size algorithm with a prescribed global element size, `ESIZE`, yielding two additional variants at element counts finer than the finest smart-size-only mesh. See Figures B.1 and B.2 for the second and third configuration. All mesh element counts and corresponding peak values are plotted in Figure 3.11.

As shown in the figure, the smart-size free mesh does not produce a cleanly converging peak. Even at the finest smart-size level, `SMRTSIZE,1`, the recovered stress and elastic strain are still rising without reaching a stable value. The two `ESIZE` variants lie approximately at the same level as each other, but consistently below the mapped mesh result. This indicates that free meshing, regardless of the refinement strategy used, under-resolves the localised concentration at the interface and corner even at high element counts. This is likely because the element shapes and grading near the interface cannot be controlled precisely. The mapped mesh gives the most regular and best-behaved response, monotonically approaching a higher asymptotic value. Because it also produces regular element shapes and consistent grading along the extraction paths, the mapped mesh was adopted for all mechanism-model simulations.

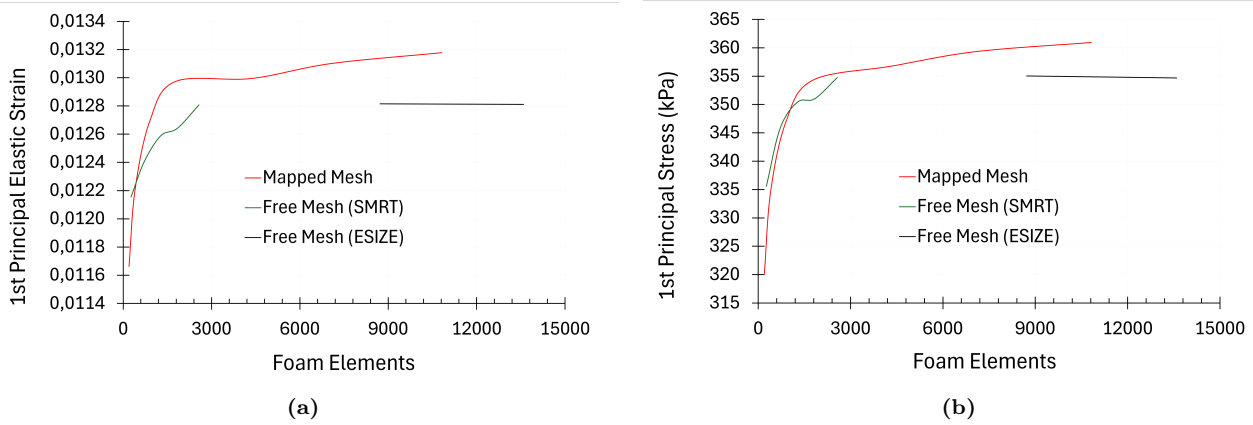


Figure 3.11: Mesh sensitivity of the peak response in the foam for mapped, smart-size and ESIZE-based mesh strategies: (a) first principal elastic strain and (b) first principal stress. Only the mapped mesh approaches a stable asymptotic value.

Consistent with the singular nature of the corner, the mapped peak is still rising slightly at the finest level rather than becoming fully flat, and is therefore best interpreted as a converging lower-bound estimate rather than a grid-independent value. The corner maximum is consequently reported only as a mesh-sensitive local indicator, as discussed in Section 3.7, and the quantitative conclusions of this chapter rest on the path-based quantities. The path-based response shows clear convergence for the mapped mesh. Figure 3.12 shows the first principal elastic strain along the diagonal path for mesh Levels 1 to 7, as defined in Table 3.6. Levels 1, 2 and 3 are essentially identical across the full path length, confirming that the path-based solution is almost grid-independent at these refinement levels. The corresponding relative differences with respect to the finest mesh are listed in Tables B.1 and B.2 for the diagonal and lower radial paths, respectively, confirming that Levels 1 to 3 remain within about 1% of the finest-mesh result over most of the path, rising to roughly 2–3% only at the outer boundary, where the absolute strain is small. The coarser levels, Levels 4 to 7, show some spread between approximately 0 and 0.04 m from the interface, where the near-corner gradient is under-resolved, but converge toward the finer-level result as the mesh is refined. The first principal elastic strain convergence plot for the radial-up path, and the first principal stress convergence plot for the radial-low path, are provided in Figures B.3 and B.4.

On this basis, mesh Level 2 was selected as the production mesh. Relative to the finest Level 1, it changes the recovered peak stress and elastic strain by less than about 1%, while reducing the run time by more than a factor of two, as shown in Table 3.6. Its path profiles are indistinguishable from those of Level 1. It therefore provides converged path quantities at

acceptable computational cost and is used for all subsequent mechanism-model results around the corner. This insensitivity to mesh refinement is even more pronounced along the lower radial path, which runs away from the corner at a greater axial distance, as defined in Section 3.10. As shown in Figure 3.13, virtually no variation between mesh levels is observed along this path. This has a practical consequence for the simulation matrix. For cases in which the primary output is a path-based quantity extracted away from the corner region, a coarser mesh of Level 4 can be used without introducing meaningful error, reducing computation time significantly. This applies to selected sensitivity and protocol cases discussed in Sections 4.3 to 4.5.

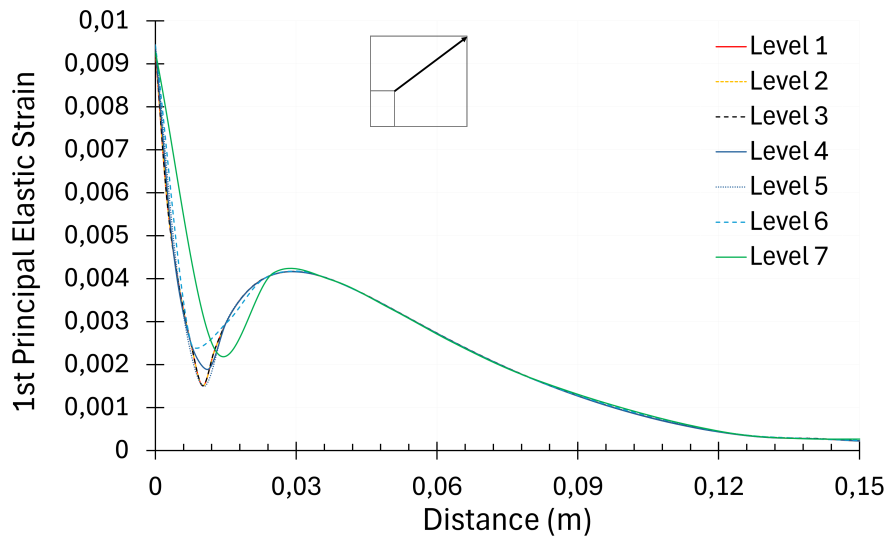


Figure 3.12: Path-based mesh convergence of the first principal elastic strain along the diagonal path for mesh Levels 1 to 7 at $t = 2000$ s. Levels 1 to 3 coincide across the full path length, confirming that the path-based solution is essentially grid-independent.

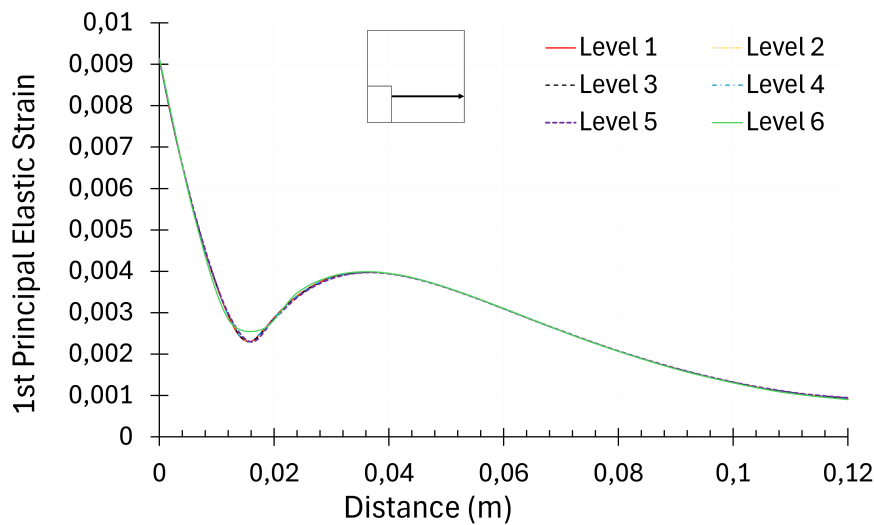


Figure 3.13: Path-based mesh convergence of the first principal elastic strain along the lower radial path for mesh Levels 1 to 6 at $t = 2000$ s. The lower radial path is virtually insensitive to mesh refinement, so a coarser mesh is sufficient for cases evaluated away from the corner.

3.11.2 Time-Step and Boundary-Condition Checks

The time-step sensitivity during cool-down was assessed using three situations where the first one is the reference scheme (Reference) described in Section 3.8, the second a coarser scheme (Larger Δt) with the same time intervals but larger time steps, and the last one a single-step scheme (1 Step) in which the full cool-down interval was applied at once. The first principal elastic strain along the lower radial path was then compared at $t = 1000$ s and $t = 2000$ s, as shown in Figure 3.14. The scripts for the different cool-down cases are provided in Appendix C.5. The three schemes are indistinguishable at both times, including the steep central part of the cool-down where the strongest transient gradients develop.

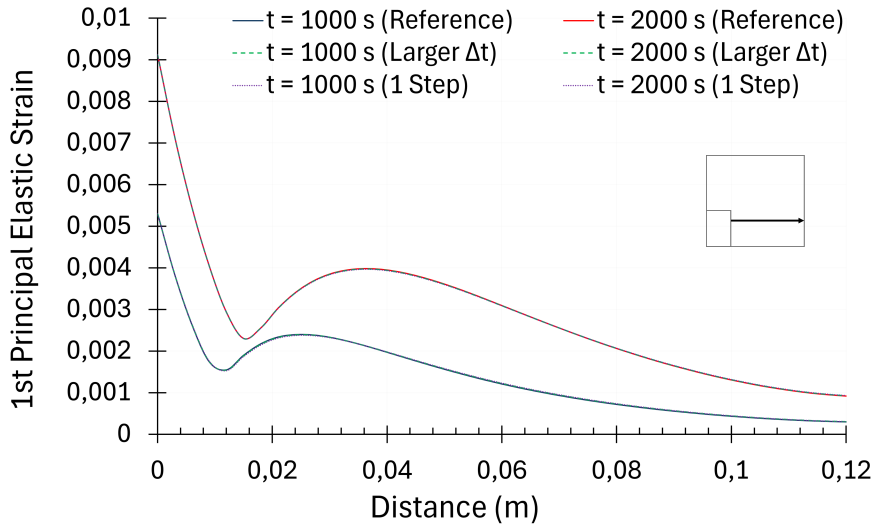


Figure 3.14: Time-step sensitivity of the first principal elastic strain along the lower radial path at $t = 1000$ s and $t = 2000$ s. Three time-stepping schemes are compared: the reference multi-step scheme (Reference), a coarser scheme using the same load-step intervals but larger time steps (Larger Δt), and a single-step scheme in which the full cool-down interval is applied at once (1 Step). The three schemes are indistinguishable, showing that the solution is not sensitive to time-step.

The coarser time-stepping is therefore adequate, and the solution is not limited by temporal discretisation. Despite this insensitivity, the reference multi-step scheme was retained for all production runs rather than the coarser or single-step alternative. The reason is that, in a nonlinear coupled-field transient analysis, using sufficiently small substeps ensures that the Newton–Raphson equilibrium iterations converge stably within each substep, as reflected by the heat-flow residual and force residual tracking below their convergence criteria throughout the solution [63]. Running with very large time steps can technically produce the same converged path result in simple cases, but it leaves less margin for the solver to bisect and recover if convergence becomes difficult at a particular substep, particularly during the steepest part of the imposed temperature curve.

When looking at the boundary conditions, the influence of the mechanical constraints was assessed by comparing three support cases along the lower radial path. The reference case applies axial constraints on the foam base and radial constraints on the foam left edge. In the first relaxed case, denoted No Bottom, the axial constraint on the foam base is removed while the left-edge constraint is retained. In the second relaxed case, denoted No Boundary, both the foam base and left-edge constraints are removed, leaving only the minimal supports required to

suppress rigid-body motion. The comparison is shown in Figure 3.15. Because the lower radial path runs horizontally and the foam base constraint is axial, removing the base constraint has a more direct influence on the path result than removing the left-edge constraint, which acts further away. This is reflected in the figure, where the difference between the No Bottom and No Boundary cases is negligible, while both relaxed cases show a modest reduction in amplitude relative to the reference. Importantly, the shape of the strain profile and the location of both the near-interface concentration and the internal strain increase are preserved across all three cases. The mechanical constraints therefore influence the amplitude of the response but not its character. The reference boundary conditions, selected and justified in Section 3.6 as the most representative of the physical system, are therefore retained for all production runs. The conclusion is that the internal strain feature is a genuine thermo-mechanical response driven by constrained differential contraction rather than an artefact of the applied supports.

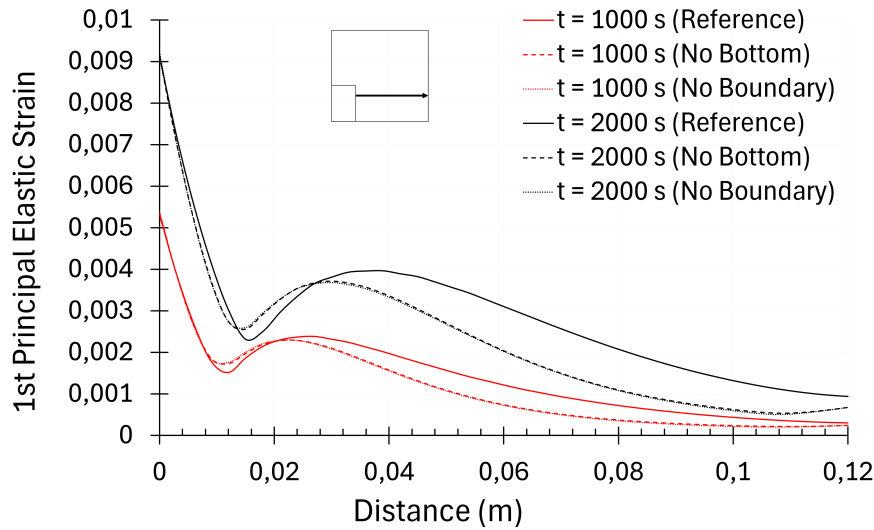


Figure 3.15: Boundary-condition sensitivity of the first principal elastic strain along the lower radial path.

Three support cases are compared: the reference case with axial constraints on the foam base and radial constraints on the foam left edge (Reference), the case with the base constraint removed (No Bottom), and the case with both the base and left-edge constraints removed (No Boundary). This leaves only the minimal supports required to suppress rigid-body motion. Relaxing the supports reduces the amplitude slightly but preserves the shape and location of both features, confirming that the internal strain peak is a genuine thermo-mechanical response rather than a constraint artefact.

3.11.3 Long Cold-Hold and Near-Steady-State Check

To separate the transient response from the residual cold state, the baseline cool-down was followed by an extended hold at 20 K, and the first principal stress and elastic strain along the lower radial path were extracted at successive times up to $t = 11000$ s. The results are shown in Figure 3.16. Between $t = 1000$ s and $t = 4000$ s, the profiles still differ noticeably, both in the near-interface peak and in the internal strain increase further into the foam. By $t = 6000$ s and $t = 7000$ s, the difference has become very small, and by $t = 11000$ s, the profile is almost indistinguishable. The field therefore approaches a near-steady cold state after approximately $t = 6000$ s, which corresponds reasonably well to the approximately one-hour cold hold used in the experiment. Some slow evolution remains visible at later times, including up to $t = 11000$ s, but these changes are small compared with the earlier transient response. The extended hold

therefore supports the interpretation that the solution is approaching a stable cold state, rather than drifting numerically over the simulated hold period.

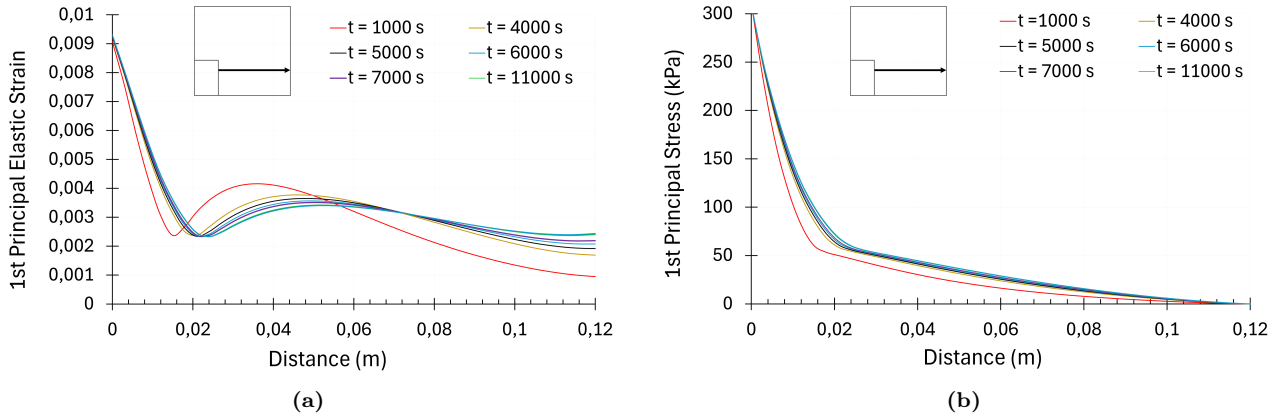


Figure 3.16: Long cold-hold check along the lower radial path, showing the evolution up to $t = 11000$ s of (a) first principal elastic strain and (b) first principal stress. The response becomes near-steady after approximately 6000 s, and the mid-transient peak clearly exceeds the residual cold-state level.

A residual stress and strain state remains at steady state because the fully cooled foam stays constrained by the foam-metal expansion mismatch even when the temperature field has become nearly uniform. Crucially, the near-interface strain peak at $t = 1000$ s, which falls mid-transient during cool-down, is substantially higher than the steady-state level reached at $t = 6000$ s and beyond. This confirms that the most severe mechanical loading away from the interface occurs during the transient cool-down rather than at steady cryogenic hold, consistent with observations reported in the literature for cryogenic foam insulation systems [15, 16].

3.11.4 Analytical Thermal Check

An analytical thermal check was performed using a separate one-dimensional foam-slab benchmark and is shown in Figure B.5. The benchmark was based on the simplified mechanism model, but the comparison was made only across a straight foam layer to the right of the steel, from the foam-steel interface to the outer foam boundary. The foam thickness was therefore $L = 0.12$ m. The constant representative foam properties were used, taken at the mid-range temperature of the tabulated model input, giving a thermal diffusivity of

$$\alpha_{th} = \frac{k}{\rho c_p} = 6.94 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}. \quad (3.7)$$

The left boundary was prescribed at 20 K, the outer foam boundary at 293 K, and the initial temperature was 293 K everywhere. This setup corresponds to a classical one-dimensional transient conduction problem with a closed-form analytical solution. The same case was solved in ANSYS using PLANE223, plane 2-D behaviour, constant material properties and stepped thermal boundary conditions. The analytical temperature profiles were compared with the ANSYS path temperatures at the same positions and times. As shown in Table 3.10, the agreement is very close, with mean temperature differences below 0.25 K for all extracted time instants. The largest local difference occurs at $t = 100$ s close to the imposed cold boundary, where the temperature gradient is steepest, but it remains only 1.73 K. Normalised by the

imposed temperature range of the benchmark, $\Delta T = 273$ K, the mean difference corresponds to less than 0.1% and the largest local difference to approximately 0.6%. This confirms that the transient heat-conduction implementation, time stepping and temperature extraction reproduce the expected analytical behaviour for a case in which an exact solution exists.

Table 3.10: Comparison between the analytical temperature profile, $T_{\text{analytical}}$, and the ANSYS temperature profile, T_{ANSYS} , for the one-dimensional foam-slab benchmark, evaluated at the diffusion-length position $x = \sqrt{\alpha_{\text{th}} t}$. The maximum and mean absolute differences are computed over the full temperature profile along the path, not only at this position.

Time [s]	Diffusion length [m]	$\sqrt{\alpha_{\text{th}} t}$	$T_{\text{analytical}}$ [K]	T_{ANSYS} [K]	Max. abs. diff. [K]	Mean abs. diff. [K]
100	0.0083		162.10	163.50	1.73	0.22
500	0.0186		162.10	162.78	0.66	0.19
1000	0.0263		162.10	162.62	0.53	0.21
2100	0.0382		162.15	162.55	0.41	0.21
5000	0.0589		170.12	170.53	0.40	0.25

The analytical estimate also confirms the physical time scale of the full mechanism model. The characteristic diffusion time across the 0.12 m foam layer is

$$t_{\text{diff}} \approx \frac{L^2}{\alpha_{\text{th}}} = \frac{0.12^2}{6.94 \times 10^{-7}} \approx 2.08 \times 10^4 \text{ s.} \quad (3.8)$$

or approximately 5.8 h, which is much longer than the 2100 s baseline cool-down. At $t = 2100$ s, the diffusion length is only about 0.038 m, much smaller than the full foam thickness. The analytical check therefore predicts a moving cooling front and a strong through-thickness temperature gradient rather than uniform cooling of the foam. This is consistent with the temperature fields and path profiles observed in the full thermo-mechanical model. Together, these checks show that the path-based temperature, stress and elastic strain fields of the mechanism model are sufficiently stable for the interpretation that follows.

Validation

The analytical check of Section 3.11.4 applies to a one-dimensional foam slab, because closed-form solutions exist only for such idealised configurations. For the corner region of the mechanism model, the multi-material experiment-specific geometry and the tank-scale spherical tank model, no analytical reference solution is available. Confidence in these results therefore rests on the numerical verification shown in the beginning of this Section alone. Establishing their accuracy would require experimental validation, for which no suitable measurements were available in this study.

The only experimental data available are the measured cool-down and warm-up temperature histories used primarily as prescribed thermal input for the experiment-specific model, as described in Section 3.6. These temperature histories therefore cannot serve as an independent validation of the same thermal response. Instead, the thermal behaviour is assessed through the

analytical and consistency checks presented above, which confirm that the simulated temperature gradients and transient time scales are physically reasonable. The mechanical response is even more difficult to validate directly, because internal foam stresses and strains were not measured during the experiment. Predicted stress and elastic strain concentrations can be compared with the timing and temperature range of the reported cracking observations, but they cannot prove the exact crack location or failure time.

3.12 Modelling Assumptions and Limitations

The models used in this thesis are deliberately simplified, in line with their purpose and this is to identify the governing thermo-mechanical mechanisms, critical regions and critical loading phases, rather than to predict crack initiation or failure directly. The individual assumptions are consolidated here, together with their consequences for the interpretation of the results, in Table 3.11. These assumptions define the boundary within which the results of the following chapter should be read. Their consequences for specific results are discussed at the relevant points in Chapter 4, where each is examined against the corresponding results, and are drawn together in the conclusions of Chapter 5.

In addition to these modelling assumptions, two points guide how the stress and elastic strain fields themselves are read in Chapter 4. First, a near-zero first principal stress S_1 does not imply a stress-free state: S_1 reflects only the largest tensile principal stress, so the point may still be under substantial compression or a general multiaxial state. S_1 remains nonetheless the primary tensile-risk indicator [46, 75]. Second, because EPEL1 is derived from the full multiaxial strain state rather than from S_1 directly, its spatial distribution need not coincide with that of S_1 . For this reason, neither quantity is interpreted in isolation: both are read together with the individual stress and strain components, since the principal directions rotate through the domain and different components can dominate in different regions [50].

Table 3.11: Principal modelling assumptions and their implications for interpretation.

Assumption	Implication for interpretation
Homogenised foam continuum	Cellular structure, density gradients, spray-induced variation and local defects are not resolved. The model identifies macro-scale mechanisms, not individual cracks.
Isotropic material behaviour	Direction-dependent foam properties from spray application are not represented. A genuinely anisotropic foam may redistribute stress differently.
Linear elastic, small-strain response	The model describes the pre-damage thermo-elastic stress and strain state, but not post-damage redistribution, crack initiation or crack growth.
Perfect foam-metal bonding	No debonding, sliding or interfacial thermal resistance is included. Therefore, interfacial failure, which is a known damage mode, is not captured.
Metal driven by a prescribed cooling curve	In the simplified and spherical tank models, the metal cold region is treated as an imposed isothermal cold source. In the experiment-specific model, the copper cold head is prescribed and the surrounding metals respond thermally. Cryocooler contact and conjugate heat transfer are not resolved.
Axisymmetric and idealised geometry	Non-axisymmetric defects, circumferential cracking and installation imperfections cannot be represented. The mechanism geometry is idealised.
Idealised sine-squared thermal input	The imposed temperature curves approximate the available cold-head temperature data rather than reproducing the measured traces exactly.
No crack initiation or propagation	The stress and elastic strain fields are risk indicators, not predictions of crack location, path or failure time.
Mesh-sensitive corner peak	The absolute maximum at the corner is not over-interpreted. Quantitative conclusions rest on the path-based quantities.

Chapter 4

Results and Discussions

Section	Page
4.1 Baseline Cool-Down Response	59
4.2 Internal Strain Interpretation	65
4.3 Thermal Mismatch and Gradients	69
4.4 Cooling Protocol Sensitivity	73
4.5 Warm-Up Response	75
4.6 Experiment-Specific Model	78
4.7 Spherical Tank Geometry	80
4.8 Failure Relevance	83

THIS CHAPTER presents and discusses the results of the coupled transient thermo-mechanical simulations. The results are organised around the three model configurations introduced in Chapter 3. The chapter presents the baseline response, followed by sensitivity studies, warm-up analysis, experiment-specific interpretation, spherical tank comparison and failure-relevance assessment.

4.1 Baseline Cool-Down Response

The baseline cool-down is the reference case against which all sensitivity studies are compared. It uses the simplified mechanism model with the reference material set and the continuous sine-squared cool-down from 293 K to 20 K over 2100 s, as described in Section 3.6. This section describes the chain underlying the interpretation of the results, where the imposed cooling of the steel generates a transient temperature gradient in the foam. This gradient drives spatially non-uniform thermal contraction. Because the foam is bonded to the steel and cannot contract freely, this non-uniform contraction is converted into stress and elastic strain concentrations near the interface and within the foam. The temperature, displacement, stress and elastic strain fields are presented in turn. The internal strain feature that first appears here is introduced in this section, but its origin is analysed in Section 4.2.

Temperature Field and Cooling Front

The prescribed cooling curve is applied to the steel block, which is therefore treated as an isothermal cold body, while the outer foam boundaries are held at 293 K. Because the thermal diffusivity of the foam is low, as discussed in Section 2.5.1, the foam does not cool uniformly. Instead, a cooling front propagates inward from the foam-steel interface, so that the foam adjacent to the steel approaches the imposed temperature while the outer foam remains close to ambient temperature for a considerable time. This behaviour is shown in Figure 4.1. The through-thickness temperature profiles confirm this behaviour, shown in Figure 4.2, where the gradient is steepest near the interface at early times, between $t = 100$ s and $t = 500$ s, and flattens as cool-down proceeds.

Displacement Field

The displacement field shows a coherent contraction of the assembly toward the cold core, as shown in Figure 4.3 at $t = 2000$ s (amplified 50x). The dashed outline marks the undeformed geometry and the contraction is visible as the gap between this outline and the amplified deformed mesh, which is near-zero at the bonded corner adjacent to the steel and grows toward the free outer edges of the foam. The displacement magnitude increases with distance from the constrained axis and base and is largest in the outer foam, while the foam adjacent to the steel is held back by the bonded interface. Because the foam is several orders of magnitude more compliant than the steel, the imposed contraction mismatch is accommodated almost entirely within the foam. The displacement field itself is smooth and modest in magnitude. It is the spatial non-uniformity of this displacement field, with relatively free contraction in the outer

foam against suppressed contraction at the interface, that is the kinematic origin of the elastic strain discussed next.

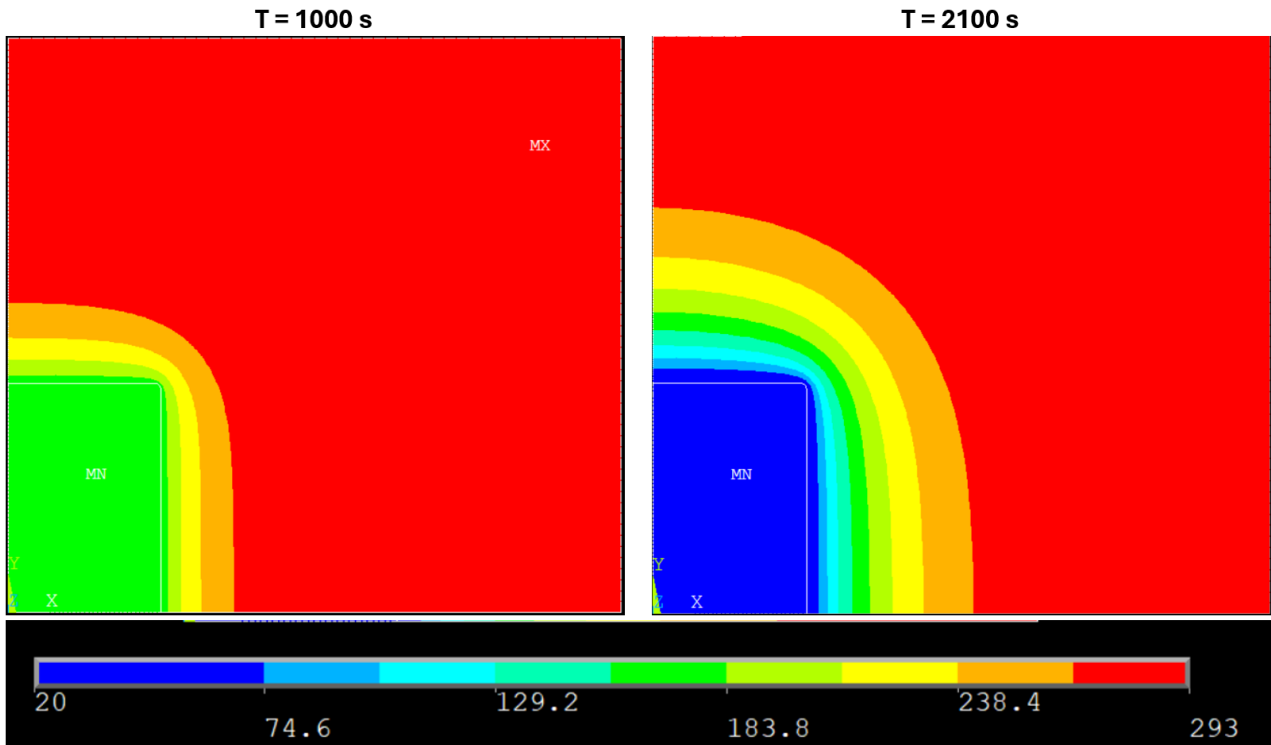


Figure 4.1: Temperature contours in the simplified mechanism model during cool-down, shown at $t = 1000$ s on the left and $t = 2100$ s on the right, with the corresponding temperature colour scale (in K). A cooling front propagates inward from the foam-steel interface, leaving the outer foam close to ambient.

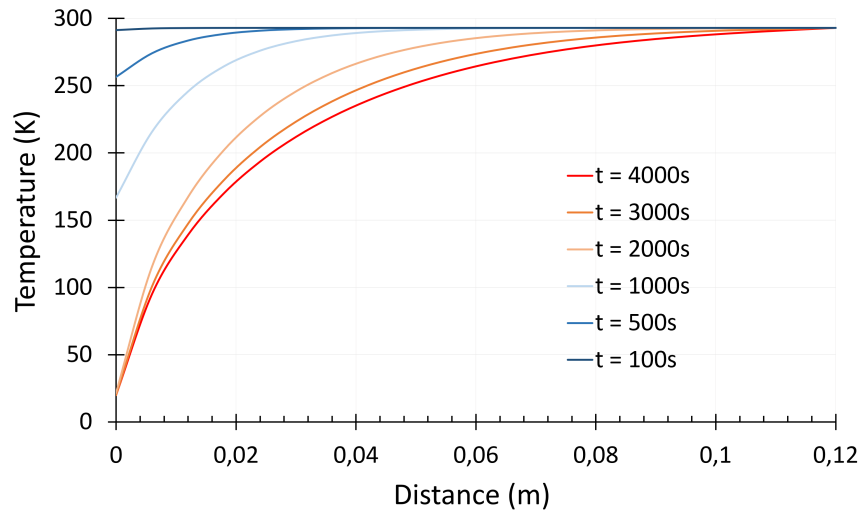


Figure 4.2: Through-thickness temperature profiles in the foam at successive times during cool-down ($t = 100$ – 4000 s), extracted from the finite element model along a path from the cold boundary ($x = 0$) to the outer surface ($x = 0.12$ m). The gradient is steepest near the interface at early times and flattens as cool-down proceeds, confirming that the thermal loading is inherently transient.

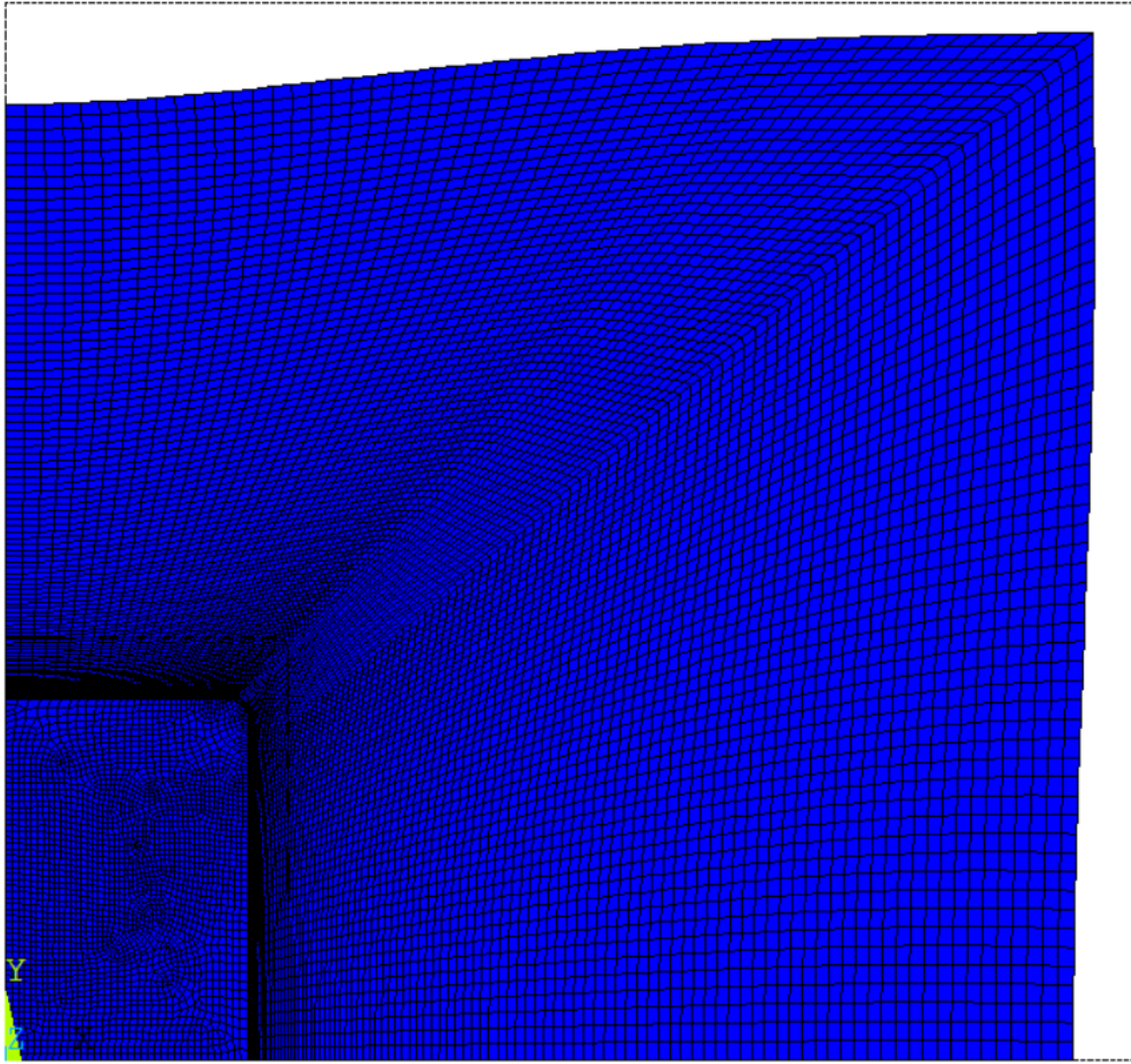


Figure 4.3: Displacement magnitude during the baseline cool-down at $t=2000$ s, showing contraction of the assembly toward the cold steel region (Amplified 50x). The contraction mismatch is accommodated almost entirely within the compliant foam.

Stress Distribution

The first principal stress S_1 is used as the primary indicator of tensile risk, since brittle foam is most vulnerable under tension, as discussed in Appendix A. The tensile stress is strongly concentrated along the foam-steel interface and reaches its maximum at the corner, where the bonded constraint and the local geometry coincide, visible as the concentrated band along the interface in Figure 4.4. Along the lower radial path, defined in Figure 3.10a, the tensile stress is highest at the interface, of the order of 300 kPa, and decays steeply within the first 20 mm before falling toward zero at the outer boundary, as shown in Figure 4.5. This interface concentration is the expected consequence of the foam-steel expansion mismatch under bonded constraint, whose magnitude scales with stiffness, CTE mismatch and temperature drop [21, 42]. The axial path through the foam above the steel shows the same qualitative behaviour, with a high tensile stress near the interface decaying toward the outer boundary, and is provided in Figure B.7 for completeness.

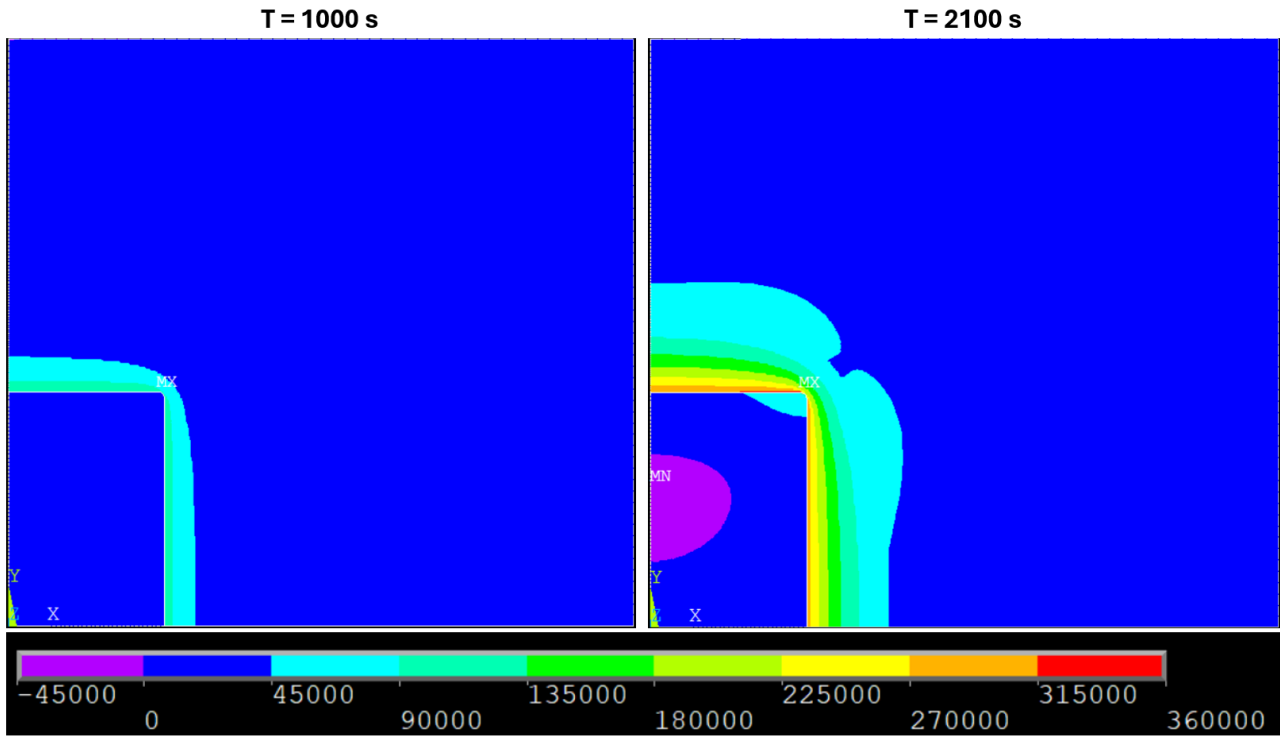


Figure 4.4: First principal stress contours during cool-down, shown at $t = 1000$ s in the left panel and $t = 2100$ s in the right panel, with the corresponding stress colour scale (in Pa). The tensile stress concentrates along the foam-steel interface and reaches its maximum at the corner, where the bonded constraint and the local geometry coincide.

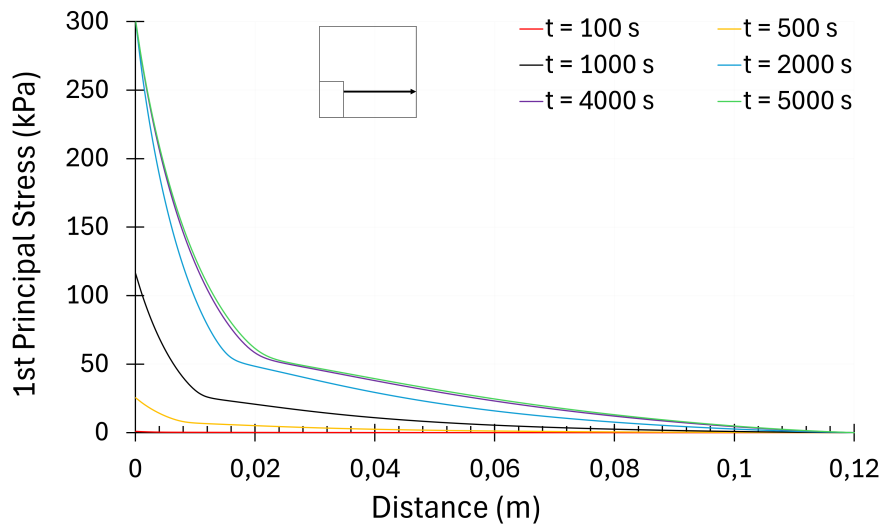


Figure 4.5: First principal stress along the lower radial path during cool-down. The tensile stress peaks at approximately 300 kPa at the interface and decays steeply within the first 20 mm of foam.

The stress along the diagonal and upper radial paths are given in Figures B.10b and B.11b. Along the upper radial path, which runs through the foam at the height of the corner, the interface stress reaches approximately 340 kPa against approximately 300 kPa along the lower radial path, consistent with its smaller distance from the corner. Along the diagonal path the interface stress is approximately 230 kPa and therefore lower rather than higher, because this path starts at the middle of the rounded corner whereas the stress maximum lies at the

transition between the rounded corner and the straight interface, as discussed in Section 3.7. The near-corner values along the diagonal path are consequently not interpreted quantitatively, but the comparison across the paths confirms that the tensile stress increases as the corner is approached.

Strain Distribution

The first principal elastic strain EPEL1 is used as the main strain measure because, unlike the total strain, it represents the mechanically relevant deformation associated with stress, whereas the stress-free thermal contraction is excluded. Near the interface, the elastic strain mirrors the stress field, with an elastic strain of approximately 0.009. This is shown in Figure 4.6. The elastic strain field is not confined to the interface, however. After decaying to a minimum a short distance into the foam, EPEL1 rises again to a second, internal increase before decaying toward the outer boundary. The same feature is present along the radial, diagonal and axial paths, as shown in the contour in Figure 4.7. This internal increase does not coincide with the tensile stress maximum and cannot be explained by the near-interface constraint alone. The elastic strain along the diagonal and upper radial paths are given in Figures B.10a and B.11a. Along the upper radial path the interface strain reaches approximately 0.013 against approximately 0.009 along the lower radial path, again reflecting its smaller distance from the corner, while along the diagonal path the interface strain is comparable to that of the lower radial path. The two-part structure of an interface concentration followed by a separate internal peak is recovered along both paths.

The feature also evolves distinctly in time. At $t = 100$ s, the elastic strain throughout the foam is still very low, as the cooling front has barely penetrated the foam. As cool-down progresses, both the near-interface concentration and the internal minimum and secondary increase grow in magnitude. Also, the location of the minimum and the internal peak are not fixed. They migrate outward through the foam, following the advancing cooling front. The internal peak reaches its maximum magnitude near the end of the imposed cool-down, around $t = 2100$ s. After this point, the internal increase decreases in amplitude and shifts further outward, while the near-interface strain remains approximately constant. This suggests that the two features have different physical origins, where the interface concentration is set by the total accumulated mismatch, while the internal feature is linked to the transient gradient. The precise mechanism behind this moving feature is analysed in the next section through a component-wise strain decomposition and vector plots. These two features are distinguished throughout this chapter because they differ in damage relevance. The interface concentration carries the highest stress and elastic strain and therefore governs the damage risk, as assessed in Section 4.8, whereas the internal peak is lower in magnitude but is the feature that responds to changes in the thermal protocol, as shown in Sections 4.4 and 4.5.

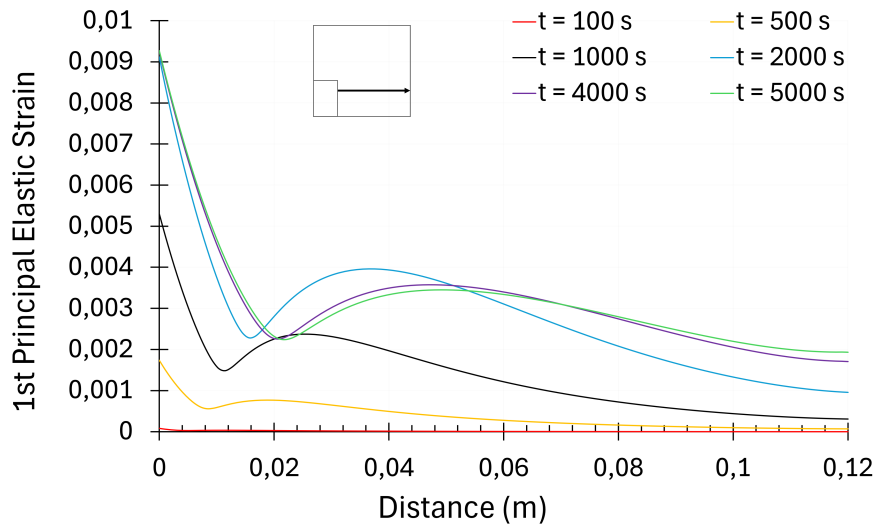


Figure 4.6: First principal elastic strain along the lower radial path during cool-down. The elastic strain shows a two-part structure, an interface concentration of approximately 0.009 and a separate internal peak that migrates outward as cool-down proceeds.

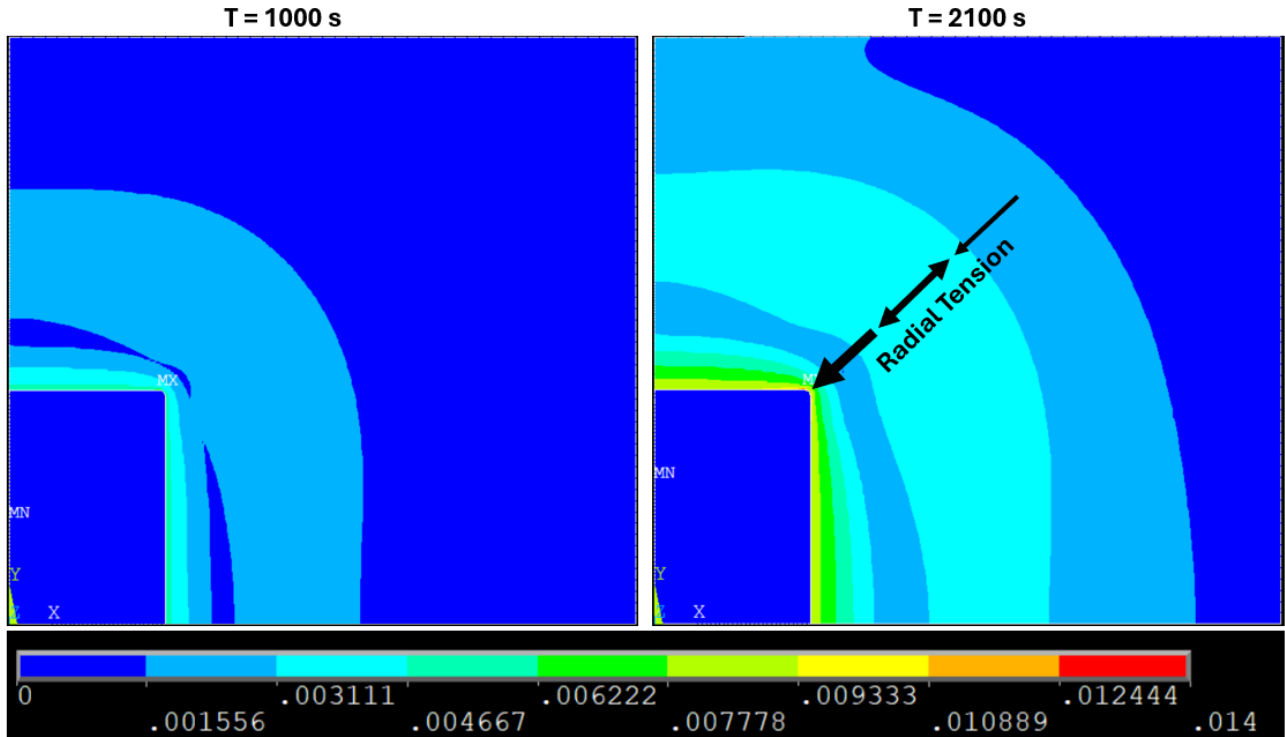


Figure 4.7: First principal elastic strain contours during cool-down, shown at $t = 1000$ s on the left and $t = 2100$ s on the right, with the corresponding elastic strain colour scale [-]. The elastic strain is not confined to the interface: a second, internal increase develops within the bulk foam. The arrows superimposed on the right panel illustrate the mechanism: the more strongly contracted inner foam and the less contracted outer foam place the intermediate foam in radial tension.

4.2 Internal Strain Increase and Component-Wise Interpretation

Section 4.1 established that the first principal elastic strain along the through-thickness paths does not decay monotonically away from the interface, but rises again to a second, internal increase that has no counterpart in the first principal stress maximum and that migrates outward with the cooling front. Figure 4.8 places the temperature, stress and elastic strain fields side by side at the end of cool-down to make this contrast visible before it is analysed quantitatively. The three panels share the same geometry and the same instant, so they can be read directly against one another. The stress follows the cold region closely: it is confined to a narrow band along the bonded interface and has decayed to near zero across most of the foam thickness. The elastic strain does not, remaining elevated over a substantially wider region that extends into foam which is still at intermediate temperature and carries almost no tensile stress. The contour view shows that this difference exists and roughly where it lies, but it cannot resolve the depth at which the elastic strain peaks or how that depth shifts as the transient proceeds.

Because EPEL1 is a scalar envelope of a principal direction that rotates through the domain, it cannot by itself reveal the directional origin of this feature. This section therefore decomposes the elastic strain into its Cartesian components, separates the mechanically relevant elastic strain from the stress-free thermal strain, and uses principal-direction vector plots, following the post-processing procedure of Section 3.10. The analysis is carried out on the lower radial path, which runs horizontally through the foam away from the near-singular corner and was shown in Section 3.11.1 to be essentially mesh-independent, so that the interpretation is not contaminated by the corner concentration.

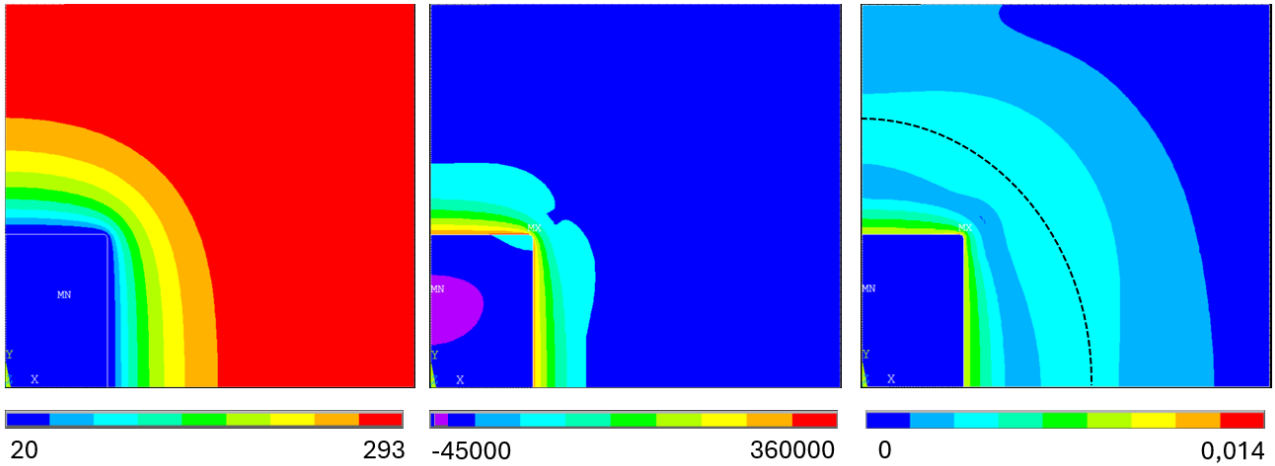


Figure 4.8: Baseline cool-down fields in the simplified mechanism model at $t = 2100$ s: (left) temperature in K, (center) first principal stress in Pa and (right) first principal elastic strain [-], shown over the foam thickness. The stress follows the cold region directly and concentrates along the bonded interface, whereas the elastic strain additionally shows a separate band further into the foam that coincides with the temperature front (dashed line), where the gradient is steepest and the stress has already decayed.

Observation in the First Principal Elastic Strain

Along the lower radial path shown in Figure 4.6, the first principal elastic strain shows the same two-part structure at every stage of the transient. A pronounced concentration at the

interface, a decay to a local minimum a short distance into the foam, and a second, internal increase before the final decay toward the outer boundary. This is for instance visible between the mid-cool-down state at $t = 1000$ s and a cold-hold state at $t = 4000$ s. Between these instants the internal peak grows in magnitude and moves outward, from approximately 0.025 m at $t = 1000$ s to approximately 0.035 m at $t = 4000$ s. It is significant that at $t = 4000$ s the imposed cold boundary has already reached 20 K, yet the internal peak continues to migrate outward. The feature therefore tracks the internal cooling front rather than the boundary temperature, which is a first indication that it is governed by the transient gradient and not by the total imposed temperature drop. Because EPEL1 reflects different behaviour in the two regions, the underlying principal direction must rotate along the path, which motivates the component-wise decomposition that follows.

Radial, Axial and Shear Strain Components

The Cartesian elastic strain components along the lower radial path (Figure 3.10a) are shown in Figure 4.9 for (a) $t = 1000$ s and (b) $t = 3000$ s. At the interface, the axial strain EPELY and the hoop strain EPELZ are tensile and comparable in magnitude, of the order of approximately 0.009 at $t = 3000$ s, while the radial through-thickness strain EPELX is compressive and the shear strain EPELXY is moderate. In this region, EPEL1 coincides with EPELY and EPELZ. This is the signature of the bonded-interface constraint, where the cold foam is prevented from contracting in the plane of the interface and is therefore held in a near-equibiaxial in-plane tension. Through elastic Poisson coupling, the in-plane tensile state produces a compressive elastic strain in the radial through-thickness direction, as discussed in Section 2.5.2 [50, 51]. The near-interface EPEL1 concentration is thus an in-plane axial-hoop tensile feature.

Moving outward, this picture inverts. The axial and hoop strains decay, with the hoop strain even reversing sign, whereas the radial strain EPELX rises from compression through zero to a tensile maximum, and the shear strain EPELXY rises to a maximum at the same location. Both maxima coincide with the second peak in EPEL1. The dominant tensile direction therefore switches from axial-hoop at the interface to radial, with a shear contribution, at the internal peak. Because the dominant component changes along the path, the principal axis rotates, which is precisely why the scalar EPEL1 traces an interface concentration, a minimum and a second peak without a single directional explanation.

The origin of the internal peak follows directly from this decomposition. The radial tension is the through-thickness differential-contraction constraint introduced in Section 2.5.3, where across the steep transient gradient, the colder and more strongly contracted inner foam restrains the warmer, less contracted outer foam, placing the intermediate foam in radial tension, as illustrated by the arrows superimposed on Figure 4.7. The shear contribution arises because the path lies at an axial offset from the corner. Consistent with this interpretation, the radial and shear peak grows and shifts outward between $t = 1000$ s and $t = 3000$ s, tracking the advancing front. The internal increase is therefore a genuine through-thickness mechanical response, distinct in both direction and physical origin from the in-plane interface concentration.

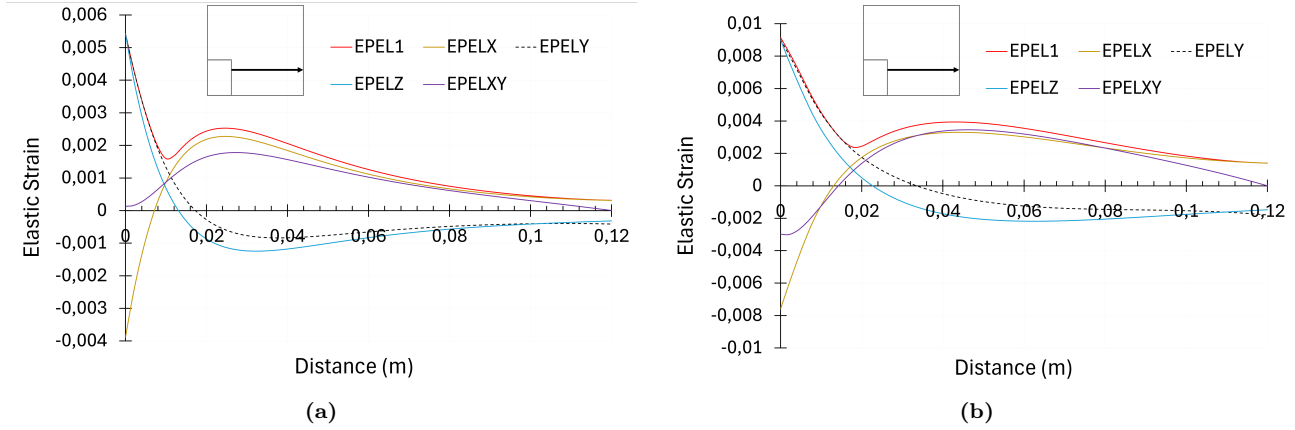


Figure 4.9: Cartesian elastic strain components along the lower radial path at (a) $t = 1000$ s and (b) $t = 3000$ s, showing the component-wise origin of the EPEL1 response. At the interface, the axial and hoop strains dominate in near-equibiaxial tension while the radial strain is compressive, identifying the concentration as an in-plane tensile feature.

Thermal Versus Elastic Strain

To confirm that the internal peak is a mechanically relevant, stress-bearing feature rather than an artefact of the thermal contraction, the thermal strain components are examined in Figure 4.10. The two thermal components EPTHX and EPTHY are indistinguishable at each instant in time, confirming that the thermal contraction is isotropic, as expected for an isotropic foam in which the thermal strain depends only on the local temperature, as discussed in Section 2.5.2. The thermal strain is large and monotonic, where of the order of approximately -0.007 at $t = 1000$ s at the interface and approximately -0.012 at $t = 3000$ s. The latter corresponds to the full secant contraction from 293 K to 20 K, decaying to zero where the foam remains near ambient temperature. But most importantly, there is no second peak in the graph.

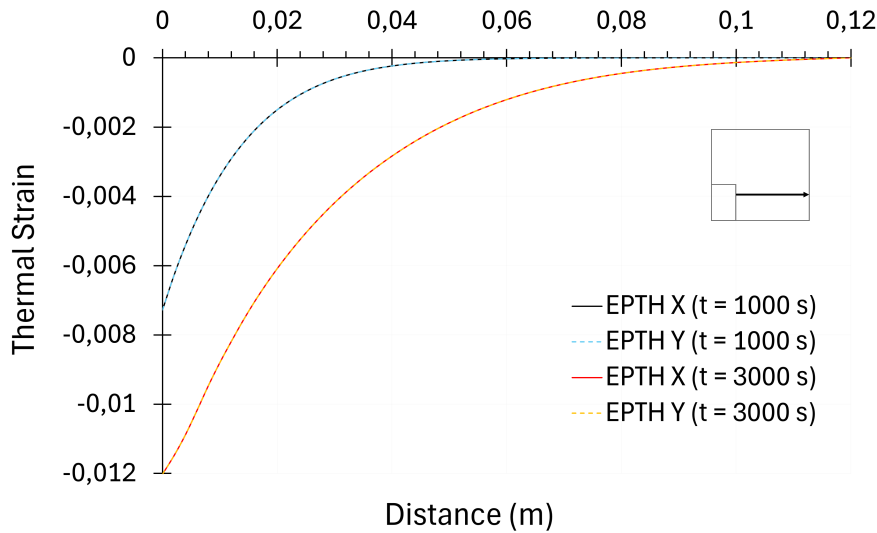


Figure 4.10: Thermal strain components along the lower radial path, showing the isotropic and monotonic thermal contraction field during cool-down. The thermal strain is isotropic and monotonic and contains no second peak, proving that the internal increase is a purely elastic, stress-bearing feature.

The contrast with the elastic strain is therefore decisive. The elastic strain is smaller in magnitude than the thermal strain near the interface, yet it is the quantity that carries the entire two-peak structure. This establishes two points. First, the internal increase is a purely elastic, stress-bearing feature, since it appears in the elastic strain but is entirely absent from the isotropic thermal field. Second, near the interface the large total strain is dominated by the stress-free thermal contraction, so the total strain would substantially overstate the mechanical loading there. This is exactly the reason the first principal elastic strain, rather than the total strain, is adopted as the mechanical indicator. The depth at which the thermal strain returns to zero marks the position of the cooling front, at approximately 0.05 m at $t = 1000$ s and approximately 0.11 m at $t = 3000$ s. The elastic second peak sits within the steep portion of this thermal profile, tying it directly to the moving transient gradient.

Vector-Plot Interpretation

The rotation of the principal directions derived from the component analysis is confirmed by the principal-direction vector plots. Figures 4.11 and 4.12 show the first principal stress and first principal elastic strain directions at $t = 200$ s, $t = 1000$ s and $t = 2000$ s. In the stress vector plot, the most notable development is the rotation of the vectors along the interface. At $t = 200$ s, the interface vectors are still oriented obliquely to the interface tangent, but by $t = 1000$ s they have rotated to align almost entirely along it. This reflects the build-up of the in-plane biaxial tension identified in the component-wise decomposition above. This state is not a shear state: the in-plane principal stresses are comparable, so the maximum in-plane shear stress, $(\sigma_1 - \sigma_2)/2$, remains small, in contrast to the internal peak, where the radial and shear components dominate. The distinction matters for failure, since the tensile and shear strengths of the foam are not expected to degrade in the same way at cryogenic temperature. In the elastic strain vector plot, the dominant development is the growth of vector magnitude with the advancing cooling front. At $t = 200$ s, only the vectors immediately adjacent to the interface are significant, by $t = 1000$ s and $t = 2000$ s, the zone of significant vectors expands progressively into the foam. This is consistent with the outward migration of the internal peak described in Section 4.1.

Both rotations are governed by the temperature field, but by different aspects of it. The interface rotation is set by the interface temperature and is therefore complete once the bond itself has become fully cold, whereas the rotation in the bulk follows the position of the advancing front, as the overlay in Figure 4.13 makes explicit. The pacing is thermal in both cases where the elastic-wave transit time across the foam is some eight orders of magnitude shorter than the thermal diffusion time, so the mechanical field is in equilibrium with the temperature field at every instant. The apparent motion of the vectors therefore reflects the diffusion-paced advance of the cooling front and not mechanical wave propagation. Consistent with Section 3.10, these vector plots were generated on the coarse visualisation mesh and are interpreted for direction only, not for magnitude.

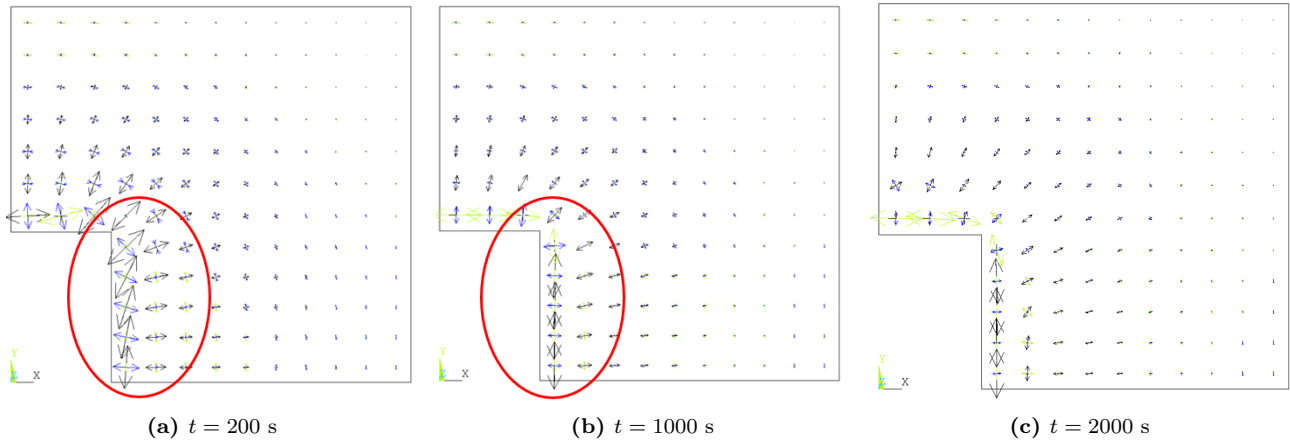


Figure 4.11: First principal stress direction vectors during cool-down. At $t = 200$ s (a, circled in red) the interface vectors are still oriented obliquely to the interface tangent, by $t = 1000$ s (b, circled in red) they have rotated to align almost entirely along it, confirming the build-up of in-plane biaxial tension.

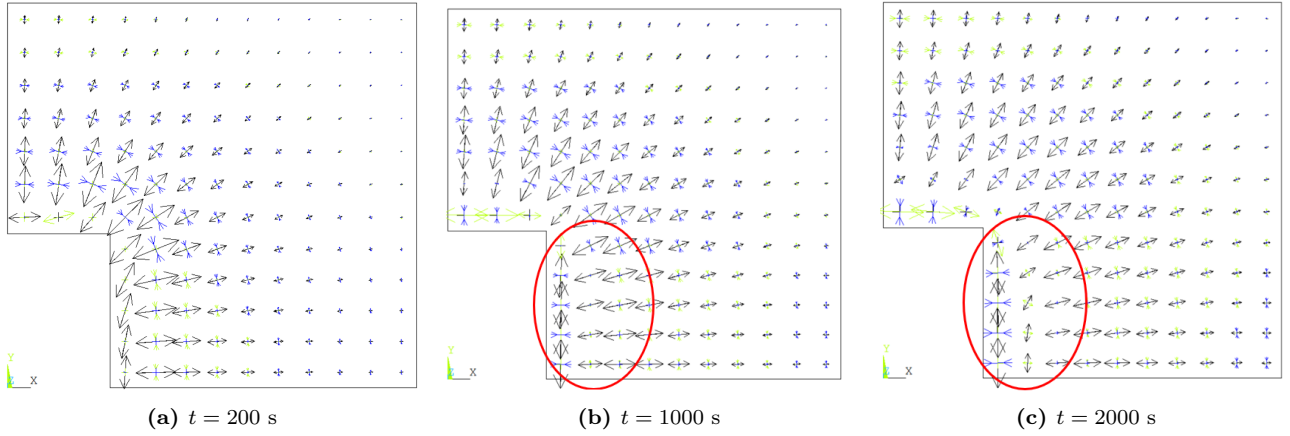


Figure 4.12: First principal elastic strain direction vectors during cool-down. The zone of significant elastic strain expands progressively into the foam, tracking the advancing cooling front. The red-circled region in (b) and (c) highlights this rotation directly. At $t = 1000$ s (b), the vectors within the circle are still oriented nearly horizontally, tangent to the interface, with only the vectors at the very top of the circle beginning to turn vertical. As the cooling front propagates further, by $t = 2000$ s (c) this vertical, radial orientation has developed over a much larger portion of the circled zone.

4.3 Role of Thermal Expansion Mismatch and Transient Gradients

Section 4.2 resolved the elastic strain response into an in-plane interface concentration and a through-thickness internal peak, and showed that the internal peak migrates outward with the cooling front. That analysis established what the features are and where they act, but not why they develop or what controls their magnitude. This section addresses that question by first overlaying the strain paths on the temperature field to locate the features relative to the moving gradient, and then using two targeted material-property sensitivity studies, defined in Section 3.9, that independently vary the foam coefficient of thermal expansion and the foam Young's modulus. Together, these isolate the contributions of the transient gradient, the foam-steel expansion mismatch and the material stiffness, and clarify their distinct roles. Unless

stated otherwise, all comparisons use the lower radial path at $t = 2000$ s, near the end of cool-down, where the internal feature is fully developed.

Temperature-Gradient Overlay with Strain Paths

Figure 4.13 shows the first principal elastic strain and the temperature along the lower radial path at successive times. At $t = 500$ s, the foam is only marginally cooled and the interface is still relatively warm, of the order of approximately 260 K. The cooling front has barely entered the foam, and the elastic strain is low across the whole path. From $t = 1000$ s onward, once the interface has become cold, the interface concentration is established at the bond, and a distinct internal peak appears in the interior.

The overlay makes the spatial correspondence explicit. Along the path, the elastic strain exhibits the interface concentration at the bond, a minimum in the fully cooled interior a short distance further out, and the second peak located within the steep region of the temperature profile, that is, at the moving thermal front. As cool-down proceeds and the front advances outward, from a front position of approximately 0.035 m at $t = 2000$ s to approximately 0.05 m at $t = 5000$ s, the internal peak moves outward with it, while the interface concentration remains anchored at the bond. The location of the internal feature is therefore set by the position of the transient gradient rather than by any fixed geometric feature.

The overlay also shows, however, that temperature alone does not explain the feature. The elastic strain is not a single-valued function of the local temperature. At a given temperature, the strain differs depending on whether that point lies at the interface, in the cooled interior, or at the advancing front. This indicates that the internal peak is organised by the moving gradient, but that its magnitude is governed by how the gradient interacts with the bonded constraint and the material mismatch, which the sensitivity studies below quantify.

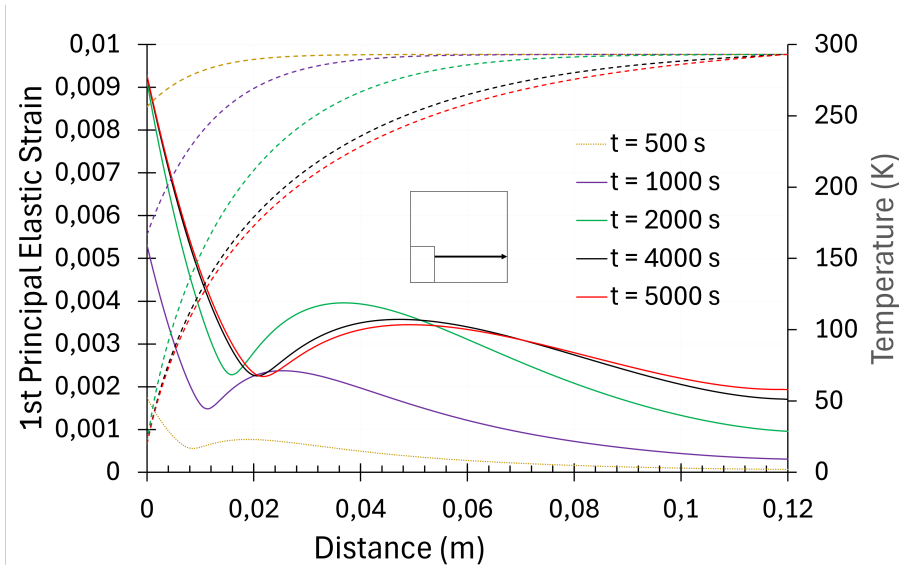


Figure 4.13: First principal elastic strain and temperature along the lower radial path during cool-down. The internal peak coincides with the steepest part of the temperature profile and migrates outward, tying the feature directly to the moving transient gradient.

Coefficient of Thermal Expansion Sensitivity

The dependence on the foam-steel expansion mismatch is examined by scaling the foam CTE to $0.5\times$, $1.5\times$ and $2.0\times$ its reference value, and by including a case in which the foam is assigned the same CTE as the steel, thereby removing the mismatch. Figure 4.14 shows that the influence on the first principal stress and elastic strain along the lower radial path is strong and monotonic. The interface stress increases from approximately 110 kPa for the $0.5\times$ case, through approximately 300 kPa for the reference case, to approximately 490 kPa and 680 kPa for the $1.5\times$ and $2.0\times$ cases, respectively. This scales approximately with the foam-steel mismatch $\Delta\alpha$ and is therefore consistent with the $E\Delta\alpha\Delta T$ scaling introduced in Section 2.5.3. Both the interface concentration and the internal peak in the elastic strain scale in the same way, confirming that both features are driven by constrained differential contraction rather than by geometry or temperature alone.

The equal-CTE case is the decisive test. Removing the contraction contrast reduces both the interface concentration and the internal peak to a small, nearly uniform residual, of the order of approximately 40 kPa in stress and approximately 0.002 in elastic strain at the interface. It does not eliminate them entirely. This shows that the foam-steel expansion mismatch is the dominant driver of both the interface concentration and the internal peak, while a minor residual response persists from the transient gradient acting on the foam's own contraction even in the absence of a mismatch. The internal peak is therefore not a numerical or purely geometric artefact, it is a differential-contraction feature whose amplitude is controlled by the CTE contrast, superimposed on a small gradient-driven background.

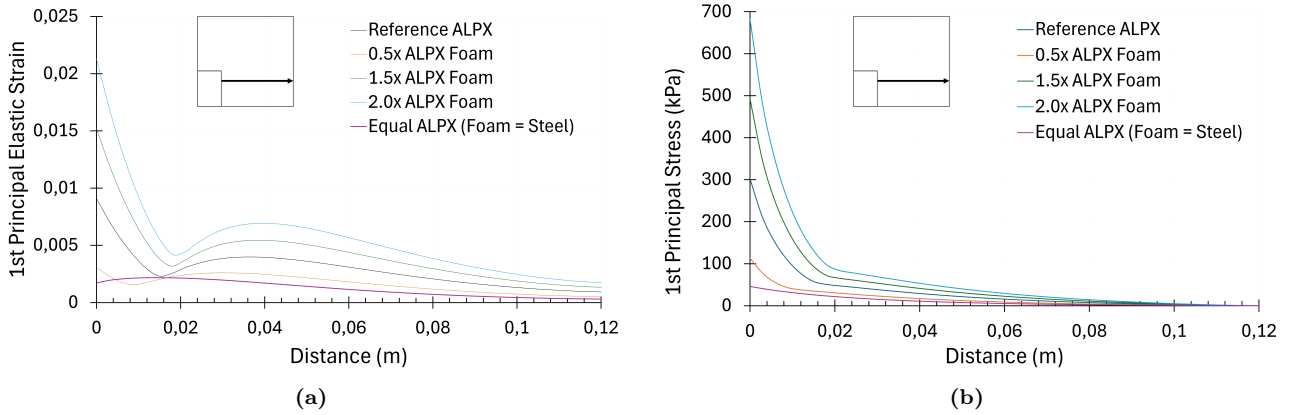


Figure 4.14: Effect of foam CTE scaling on (a) first principal elastic strain and (b) first principal stress along the lower radial path at $t = 2000$ s. Both the interface concentration and the internal peak scale with the foam-steel CTE mismatch and collapse to a small residual when it is removed, identifying constrained differential contraction as their dominant driver.

Young's Modulus Sensitivity

The role of the foam stiffness is examined by scaling the temperature-dependent foam modulus to $0.5\times$, $1.5\times$ and $2.0\times$ its reference value. The corresponding first principal stress and elastic strain along the lower radial path are shown in Figure 4.15. The two quantities respond in fundamentally different ways. The stress scales in direct proportion to the modulus, where halving the modulus halves the stress and doubling it doubles the stress over the entire path. The elastic strain, in contrast, is essentially unchanged, with the four cases indistinguishable

across the full path length. This is the signature of a strain-controlled thermal problem. Because the foam is several orders of magnitude more compliant than the steel, it accommodates almost all of the imposed contraction mismatch regardless of its own modulus, so the kinematic elastic strain field is fixed by the imposed contraction and the constraint geometry. The stress then follows through Hooke's law, $\sigma = E\varepsilon$ and therefore inherits the full modulus dependence while the strain does not.

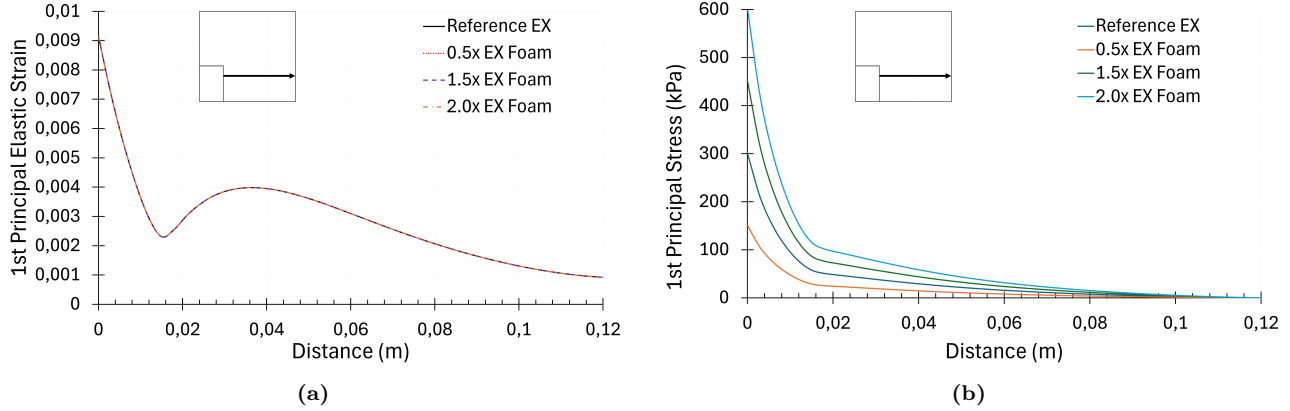


Figure 4.15: Effect of foam Young's modulus scaling on (a) first principal elastic strain and (b) first principal stress along the lower radial path at $t = 2000$ s. The stress scales in direct proportion to the modulus while the elastic strain is unchanged, the signature of a strain-controlled problem.

This distinction has a direct consequence for the interpretation of the results. The foam modulus at 20 K is one of the more uncertain inputs of the model, obtained by extrapolation of measurements available only down to 77 K. Because that uncertainty propagates into the stress but not into the elastic strain field, a strain-based indicator such as EPEL1 is considerably more robust to the modulus uncertainty than a stress-based indicator, and the location of the critical regions is unaffected by it.

Interpretation of the Moving Strain Feature

The three analyses together provide a complete physical account of the internal strain feature and separate the roles of the governing quantities. The transient temperature gradient organises the feature. It sets where and when the internal peak appears, which is why the peak is co-located with the moving front and migrates outward as cool-down proceeds, and why its location is independent of the foam modulus. The foam-steel expansion mismatch sets the amplitude. Scaling the foam CTE scales the feature, and removing the mismatch collapses it to a small residual. The foam modulus governs only the conversion of that constrained strain into stress, leaving the strain field itself unchanged.

The internal peak is therefore best understood as constrained differential contraction across the moving transient gradient. Within the steep-gradient zone, the colder, more strongly contracted inner foam restrains the warmer, less contracted outer foam, and this incompatibility is taken up as through-thickness elastic strain that travels with the front. The interface concentration is governed by the same mismatch but fixed in location by the foam-steel interface. Neither feature is explained by temperature, mismatch or stiffness in isolation. This interpretation identifies two physical levers for reducing the internal strain risk in a bonded foam insulation system: reducing the foam-substrate CTE contrast, and moderating the transient gradient

through the cooling rate. The influence of the cooling rate and of staged cooling is examined next. In terms of failure relevance, this internal peak is not the critical feature: it consumes only a small fraction of the cryogenic strain capacity, whereas the foam–steel corner consumes most of it (Section 4.8). Its significance is mechanistic rather than quantitative, since it identifies the transient gradient, and hence the cooling protocol, as a controllable driver of internal strain.

4.4 Cooling Protocol Sensitivity

Section 4.3 showed that the internal strain feature is not fixed to a material or geometric location, but travels with the moving temperature gradient, so that the peaks in strain and in temperature do not coincide. Because the position and steepness of that gradient are set by how fast the tank is cooled, this separation implies that the internal response should in principle be cooling-rate dependent. This raises the question of whether such a dependence is actually observed, and whether the cooling path could be used to reduce the internal strain, that is, whether the rate of cooling and the use of intermediate holds alter the transient gradient without changing the total temperature drop or the mismatch. This section examines these questions through two studies defined in Section 3.9: a cooling-rate variation and a stepwise cooling protocol. In both cases, all other parameters are held constant, and the profiles are compared at matched cold-region, steel, temperatures rather than at matched elapsed times, because at equal time a fast and a slow cool-down are at different stages of the transient. The purpose is a controlled numerical sensitivity study of the cooling path, not the derivation of an optimised operating protocol.

Cooling-Rate Sensitivity

For the cooling-rate sensitivity, three cool-down durations are compared. These durations are a fast cool-down to 20 K in 1050 s, the 2100 s baseline, and a slow cool-down in 4200 s. The imposed cold-region temperature histories are shown in Figure 3.7. To ensure a fair comparison, the path profiles are extracted at the times at which each cool-down reaches the same steel temperature: 100 K, an intermediate transient state, and 20 K, the fully cooled state. At 100 K, these times are $t = 668$ s, $t = 1333$ s and $t = 2673$ s for the fast, baseline and slow cases, respectively, all within 0.5 K of the target. At 20 K, they are $t = 1050$ s, $t = 2100$ s and $t = 4200$ s. The first principal stress and elastic strain along the lower radial path are shown in Figure 4.16.

At matched steel temperature, the near-interface concentration is largely rate-independent, because the foam at the interface has reached the same temperature and carries the same local mismatch in every case. The rate instead reshapes the internal feature. A faster cool-down leaves the foam less time to diffuse heat, so at a given steel temperature the cooling front is steeper and has penetrated less deeply. The internal strain peak is consequently higher and located closer to the interface. A slower cool-down produces a broader, more deeply penetrated front, so the internal peak is lower, shifted outward, and the elevated region extends further into the foam.

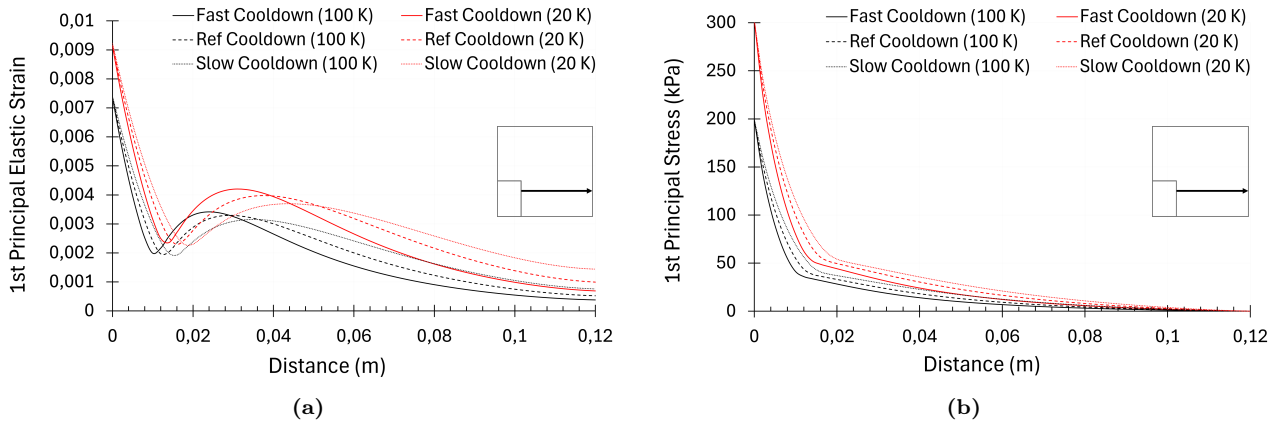


Figure 4.16: Cooling-rate sensitivity of (a) first principal elastic strain and (b) first principal stress along the lower radial path at matched steel temperatures. The cooling rate only redistributes the internal feature, sharpening it inward or broadening it outward, without substantially changing its peak magnitude.

The first principal stress, which decays monotonically from the interface, follows the same logic, where at matched steel temperature, it is lower and less deeply penetrated for a faster cool-down, reflecting the shallower cold front. It is worth noting that when the same cases are instead compared at a matched elapsed time, for instance $t = 4500$ s, this ordering reverses, because by that time a faster cool-down has cooled a greater depth of foam. This is precisely why the comparison is made at matched steel temperature. The differences between rates are, however, modest. Across the full path, the spread between the fast, baseline and slow curves at a given temperature is small compared with the difference between the 100 K and 20 K states. The response is therefore governed principally by the total temperature drop, that is, by how cold the interface has become and by the associated mismatch, and only secondarily by the rate at which that drop is applied. The cooling rate redistributes the internal feature, sharpening and drawing it inward for faster cooling or broadening and pushing it outward for slower cooling, without substantially changing its peak magnitude.

Stepwise Cooling

The stepwise protocol reaches 20 K through three sine-squared cooling segments separated by two isothermal holds, at approximately 225 K and 88 K, as shown together with the smooth profiles in Figure 3.7. The holds allow the foam temperature field to equilibrate partially before cooling resumes. The stepwise case is compared with the smooth baseline at three matched steel temperatures, 225 K, early in the cool-down, 105 K, in the middle-to-late stage, and 80 K, late in the cool-down. The temperatures are matched to within 0.02 K at the 225 K and 105 K points and within 0.16 K at 80 K. The resulting stress and elastic strain are shown in Figure 4.17.

The two protocols produce very similar responses, and where they differ the stepwise case is marginally more severe, not less. At the early, 225 K, and late, 80 K, comparison points, the stepwise case shows an internal increase shifted a little further outward, with a correspondingly slightly higher stress. At the intermediate 105 K point, the smooth and stepwise profiles almost coincide. The outward shift is consistent with the additional equilibration provided by the holds, which lets the front penetrate marginally deeper at a given steel temperature. Most importantly, the holds do not reduce the peak stress or elastic strain. Allowing the foam

to equilibrate before further cooling does not relax the response, because the feature simply re-forms as cooling resumes and the interface continues toward its final temperature. This is consistent with the mechanism established in Section 4.3, where the response is set by the accumulated mismatch and the final constraint rather than by the instantaneous cooling rate, so modifying the cooling schedule cannot remove it.

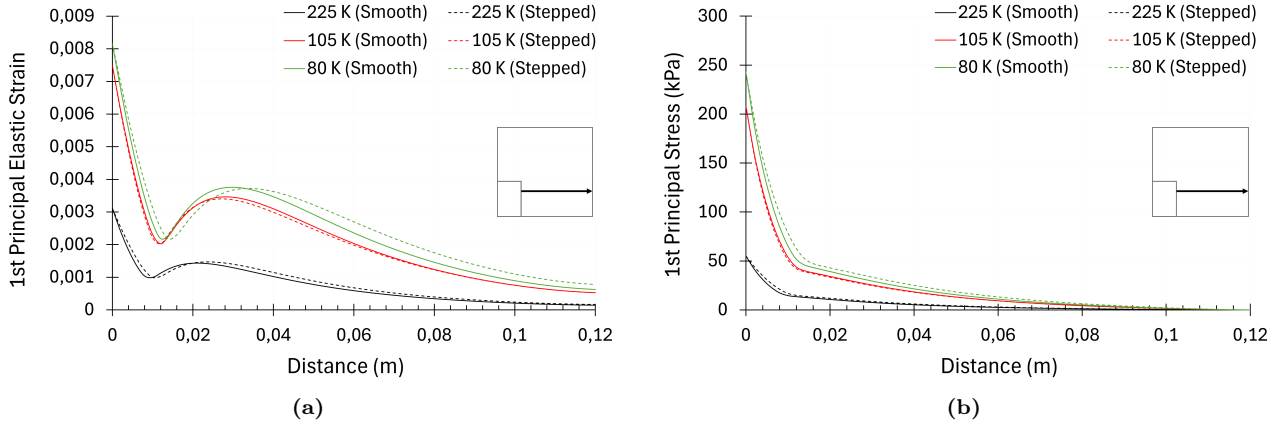


Figure 4.17: Effect of stepwise cooling on (a) first principal elastic strain and (b) first principal stress along the lower radial path at matched steel temperatures. Intermediate holds do not reduce the peak stress or elastic strain.

These results should be read as a numerical sensitivity study of the cooling path and not as a validated mitigation strategy, since a genuine assessment of operating procedures would require a damage model and experimental confirmation, which are outside the present scope. The warm-up phase, in which cracking was actually reported, is examined next in Section 4.5.

4.5 Warm-Up Response and Experimental Relevance

The warm-up phase is of particular interest because the experiment reported audible cracking mainly during warm-up, in the approximate 60-80 K range. The analysis so far has concentrated on cool-down, but this section examines the warm-up response of the simple mechanism model and, most importantly, compares it with cool-down at matched steel temperature to determine whether warm-up constitutes a more severe mechanical loading. Following Section 3.6, each warm-up begins from the cooled state after a one-hour cold hold at 20 K, so that the thermal history entering the warm-up is physically consistent rather than an artificial uniform cold state. The two warm-up cases from Figure 3.8 are evaluated, and all comparisons are made along the lower radial of Figure 3.10a.

Cool-Down, Cold Hold and Warm-Up Sequence

Figure 4.18 compares the first principal stress and elastic strain along the lower radial path for cool-down and slow warm-up at matched steel temperatures of 40 K, 60 K and 80 K. At the same steel temperature, the warm-up state differs fundamentally from the cool-down state, because the two are reached along opposite thermal paths. During cool-down at a given steel temperature, only a thin layer of foam adjacent to the interface has cooled, while the

bulk remains close to ambient. During warm-up at the same steel temperature, the foam has already been cooled throughout during the preceding hold, so a deep cold zone is present and the interface has become the warming boundary.

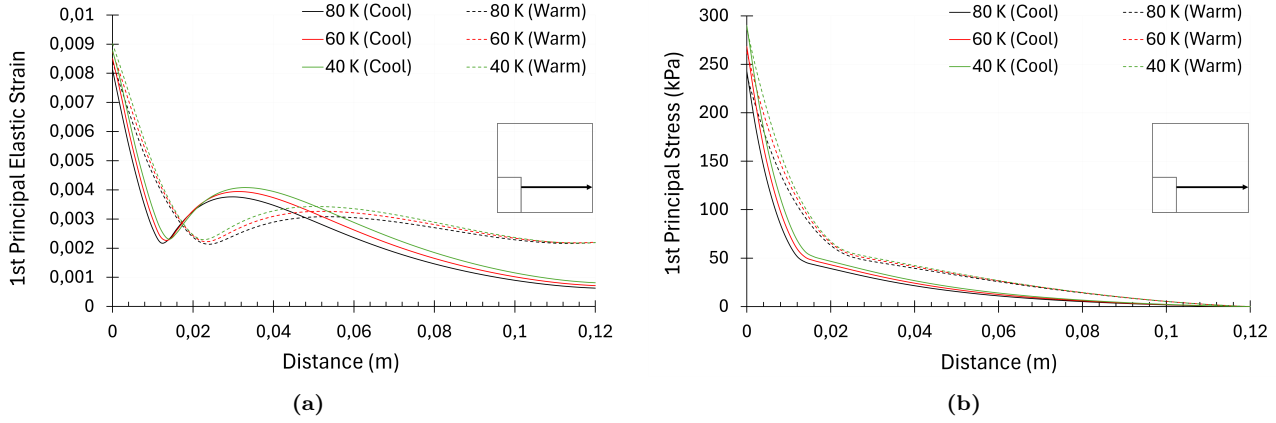


Figure 4.18: Comparison between cool-down and slow warm-up along the lower radial path at matched steel temperatures of 40 K, 60 K and 80 K: (a) first principal elastic strain and (b) first principal stress. At matched steel temperature, the warm-up state carries a deeper and broader loaded zone but not a higher strain, showing that the strain pattern is set by the preceding thermal path rather than by the instantaneous temperature.

First, the results in the strain graph (Figure 4.18) show a spatial redistribution between cool-down and warm-up. The near-interface concentration is of comparable magnitude in both phases, but is slightly higher during warm-up. This is consistent with the fact that, at the same cold-head temperature, the warm-up state retains a deeper and broader cold region in the foam. This region remains thermally contracted and therefore changes the constraint imposed on the near-interface foam, leading to a broader elastic-strain distribution. During warm-up, the internal feature is also shifted outward and broadened, so that elevated strain extends considerably deeper into the foam. The comparison therefore shows that the instantaneous steel temperature alone does not determine the strain pattern. Instead, the strain distribution depends on the through-thickness temperature field and therefore on the preceding thermal path.

The stress comparison (Figure 4.18) shows that at the interface the cool-down and warm-up profiles coincide at matched steel temperature, but away from the interface the warm-up stress is consistently higher and decays more slowly toward the outer boundary. This is a direct consequence of the deeper cold penetration during warm-up, where the bulk foam has already been cooled throughout during the preceding hold, so a larger cold volume remains contracted and continues to restrain the foam further from the interface, sustaining a higher tensile stress there compared with cool-down. The corresponding comparison for the faster warm-up is nearly identical and is provided in Figure B.6. The corresponding comparison along the axial path shows the same qualitative behaviour and is provided in Figure B.8.

Passive Versus Faster Warm-Up

In this section the two warm-up rates are compared with a slower passive warm-up over 3420 s and a faster warm-up over 720 s, as defined in Section 3.9. The profiles were extracted at matched steel temperatures of 40 K, 60 K and 80 K. The stress and elastic strain along the

lower radial path are shown in Figure 4.19. The two rates are almost indistinguishable. The faster warm-up produces a slightly higher internal increase, consistent with a slightly steeper transient gradient, but the near-interface concentration and the peak magnitudes are essentially unchanged. As in the cooling-rate study, the warm-up rate only redistributes the internal feature slightly and does not materially change the severity of the response. The warm-up rate is therefore not a decisive parameter for the mechanical loading in the model.

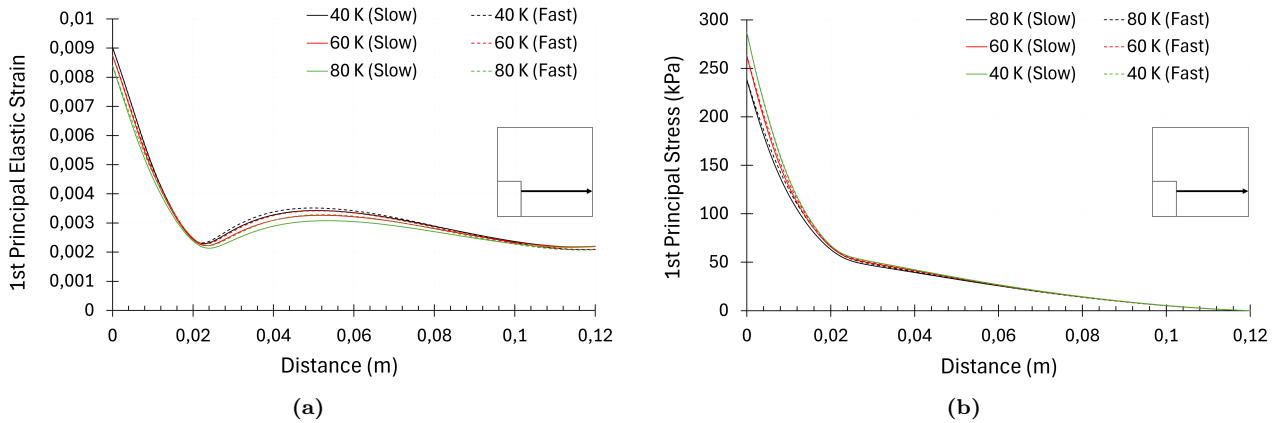


Figure 4.19: Warm-up-rate sensitivity of (a) first principal elastic strain and (b) first principal stress along the lower radial path at matched steel temperatures. The two warm-up rates are almost indistinguishable, so the warm-up rate is not a decisive parameter for the mechanical loading in the model.

Stress and Strain Peaks During Warm-Up and Experimental Relevance

The central question for the experimental comparison is whether warm-up produces higher stress and elastic strain than cool-down, which would provide a direct mechanical explanation for the reported warm-up cracking. As shown in Figure 4.18, the model does not support this straightforwardly. The governing peak values are comparable between the two phases: the near-interface values coincide, and although the internal feature is broader and penetrates deeper during warm-up, its peak magnitude is similar to the cool-down peak. Away from these peak locations, however, the warm-up stress is consistently higher than the cool-down value at the same position and decays more slowly toward the outer boundary. Furthermore, within the warm-up itself, the near-interface concentration is largest when the steel is coldest, early in the warm-up, and decreases as the steel warms. The model therefore does not predict a distinct mechanical maximum specifically in the reported 60-80 K band. That range corresponds to a state of high but already declining elastic loading. The linear-elastic model therefore does not, by itself, reproduce a warm-up-specific stress or strain peak that would single out warm-up as the more damaging phase.

This conclusion, however, rests on the near-interface values, which is precisely where the two phases are least distinguishable. Away from the bond the warm-up stress is consistently higher than the cool-down value at the same position, and the elevated strain region extends considerably deeper into the foam. The interface value is also the least certain quantity in the model, both because it lies in the mesh-sensitive region discussed in Section 3.7 and because the foam properties assumed there are the least representative, as discussed in Section 4.8. Once the interface point is set aside on those grounds, the comparison does identify warm-up as the more severely loaded phase over the bulk of the foam, so the difference between the two phases is better assessed from the spatial distribution of the fields than from the peak value alone.

This is an important result, and it must be interpreted within the scope of the model. The results identify that warm-up is a demanding transient phase, but the correspondence with the reported cracking remains qualitative. This distinction is carried into the experiment-specific comparison in the next section.

4.6 Experiment-Specific Model

The experiment-specific model is used for the limited purpose to check whether the thermo-mechanical mechanism established on the simplified mechanism model persists in the more representative geometry, and whether the predicted response is consistent with the reported cracking. As set out in Chapter 3, it is not a validation model. The same sine-squared cool-down and warm-up curves are used as for the mechanism model, so that the two models differ only in geometry and material assembly, not in thermal loading. Results are extracted along the two foam paths defined in Table 3.9, the radial path P1 and the axial path P2. Because the experiment-specific model uses the coarser assembly mesh with square-cornered material regions, the extracted profiles are somewhat less smooth than those of the mechanism model. This is a meshing artefact and does not affect the qualitative trends on which the interpretation rests.

Qualitative Comparison with Cracking Observations

The first principal stress and first principal elastic strain along the radial path P1 defined in Figure 3.10b are shown in Figure 4.20, comparing cool-down and warm-up at matched cold-head temperatures of 40 K, 60 K and 80 K. The axial path is shown in Figure B.9, but shows the same qualitative behaviour. The response reproduces the mechanism-model behaviour. On both paths, the elastic strain shows the same structure identified in Sections 4.1 and 4.2. The near-interface stress and strain are of comparable magnitude in both phases, and the governing peak values agree with the mechanism-model behaviour of Section 4.5. Away from the interface, however, the warm-up stress and elastic strain are higher than the cool-down values at the same cold-head temperature (except around 0.04 m). This reflects the deeper and broader cold penetration during warm-up. As in the mechanism model, this broader loaded region does not raise the governing peak, so the dominant difference between the phases remains a spatial redistribution rather than an increase in the peak severity.

The evolution of the elastic strain field over the full cool-down and warm-up sequence is shown as contours in Figure 4.21. The concentration develops at the foam-metal interface and at the corner near the aluminium puck, and migrates into the foam as the transient proceeds. The frames at $t = 1450$ s and $t = 6180$ s correspond to the moments when the cold head has reached approximately 80 K during cool-down and warm-up, respectively, and therefore represent the same thermal loading point as the 80 K curves in the path plots. Comparing these two frames confirms the spatial redistribution described above, where the concentration at $t = 6180$ s is slightly broader and shifted outward relative to $t = 1450$ s, while the peak magnitude indicated by the colour scale remains comparable between the two phases.

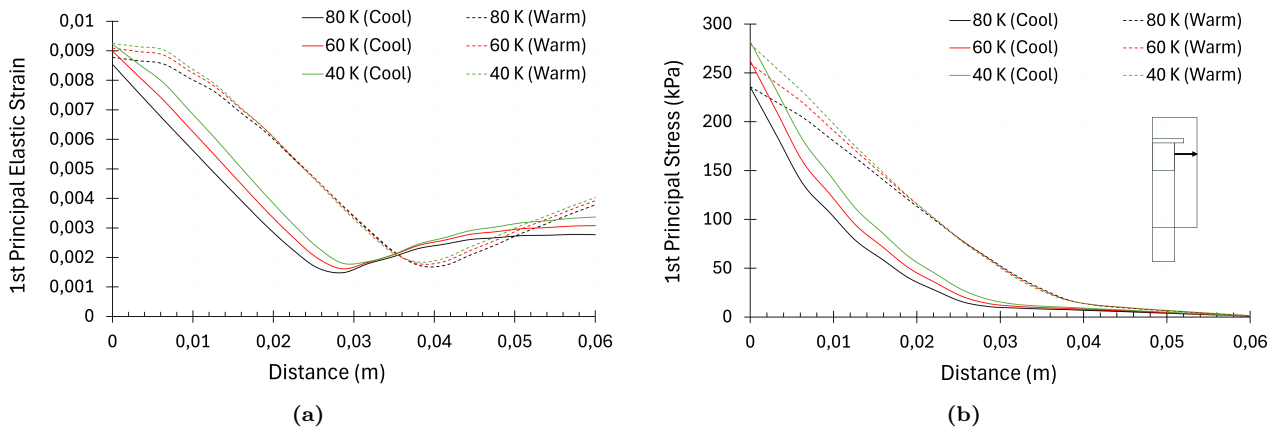


Figure 4.20: Experiment-specific model response along the radial foam path P1, comparing cool-down and warm-up at matched cold-head temperatures of 40 K, 60 K and 80 K: (a) first principal elastic strain and (b) first principal stress. The mechanism-model behaviour is reproduced, with warm-up again showing a spatial redistribution rather than an increase in severity.

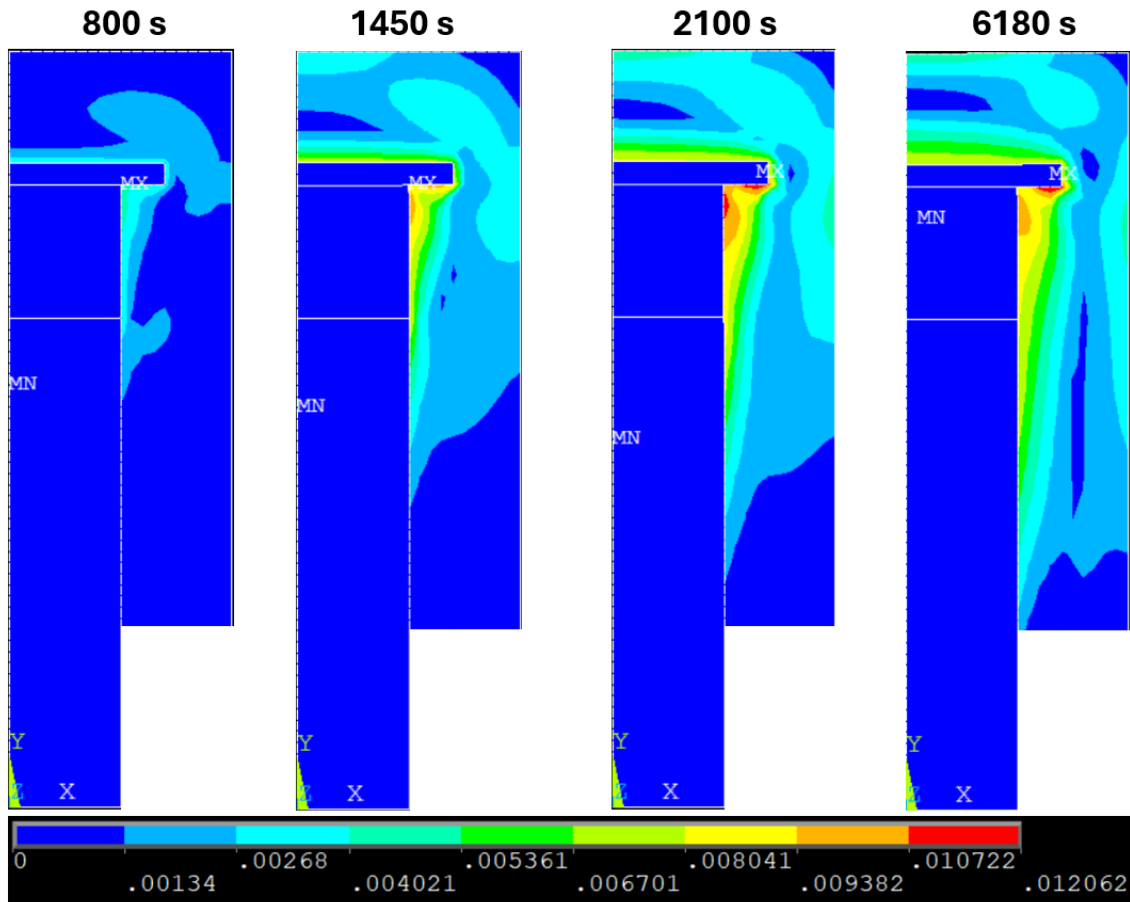


Figure 4.21: Evolution of the first principal elastic strain field in the experiment-specific model. The first three frames ($t = 800$, 1450 and 2100 s) show the cool-down phase, from an early transient state to the fully cold state. The last frame ($t = 6180$ s) shows the warm-up phase, at the cold-head temperature matched to the 1450 s cool-down frame (80 K). The concentration develops at the foam-metal interface and at the corner near the aluminium puck and migrates outward, with comparable peak magnitude at matched cold-head temperatures during cool-down and warm-up.

This provides a qualitative link between the imposed thermal sequence and the reported cracking. The model confirms that tensile stress and elastic strain concentrations do develop in the foam during the warm-up phase in which cracking was reported, and that they lie in the interface and near-corner regions. It does not, however, predict a distinct mechanical maximum specifically in the reported 60-80 K band. This result is conditional on the assumed temperature dependence of the material properties, particularly the foam Young's modulus and coefficient of thermal expansion, which are interpolated across a wide temperature range from limited data. If the real foam's property-temperature relationship differs from this assumption within the 60-80 K range, for instance through a sharper stiffening or CTE change than the model assumes, a distinct mechanical maximum could develop there that the present model would not capture. Consistent with the mechanism model in Section 4.5, the near-interface loading is highest when the cold head is coldest and declines as it warms. The experiment-specific model therefore supports the proposed thermo-mechanical mechanism as the origin of the loading, and confirms that it carries over to the real geometry, but it does not by itself single out warm-up as mechanically more severe than cool-down.

Limits of Mechanical Interpretation

The scope of this comparison must be stated explicitly. It cannot be treated as quantitative mechanical validation, for several reasons. The internal foam stresses and strains were not measured, so the predicted magnitudes cannot be checked against experiment. The crack location was not recorded and so the only available damage information is that audible cracking occurred during warm-up, without spatial resolution, so the model's identification of the interface and corner as critical regions cannot be confirmed directly. The temperature history is imposed as an input approximating the cold-head measurement, not predicted independently, so the comparison also does not constitute a thermal validation. Finally, the corner stress and strain in this model are mesh-sensitive and, unlike the mechanism model, were not subjected to a dedicated convergence study. The interpretation therefore rests on the path trends and the phase comparison rather than on absolute peak values. Within these limits, the experiment-specific model fulfils its intended role. It demonstrates that the mechanism isolated on the simplified geometry is reproduced in the real assembly under an experimentally representative thermal input, and that the predicted response is consistent with the timing and location one would associate with the observed cracking.

4.7 Spherical Tank Geometry Comparison

The spherical tank model is used to assess how the mechanisms established on the simplified mechanism model transfer to a representative tank geometry. As set out in Section 3.3, it is a geometry comparison rather than a design analysis of the actual tank. The spherical geometry is smooth and continuous, the foam layer is thicker than in the mechanism model, and a passive perlite layer forms the second insulation stage. Results are extracted along the radial foam path defined in Table 3.9, at the same time instants used for the mechanism model, so that the two can be compared directly.

Smooth Tank Geometry

The temperature, first principal stress and first principal elastic strain in the tank cross-section are shown in Figure 4.22 for $t = 2100$ s. The temperature field shows the cooling front passing through the foam, cold at the steel shell and near ambient toward the perlite, reproducing the moving-front behaviour of the mechanism model but now developing across a smooth, curved interface. The stress and elastic strain concentrate in a band in the foam adjacent to the steel shell and decay outward, with the maximum located at the smooth foam-steel interface rather than at any corner.

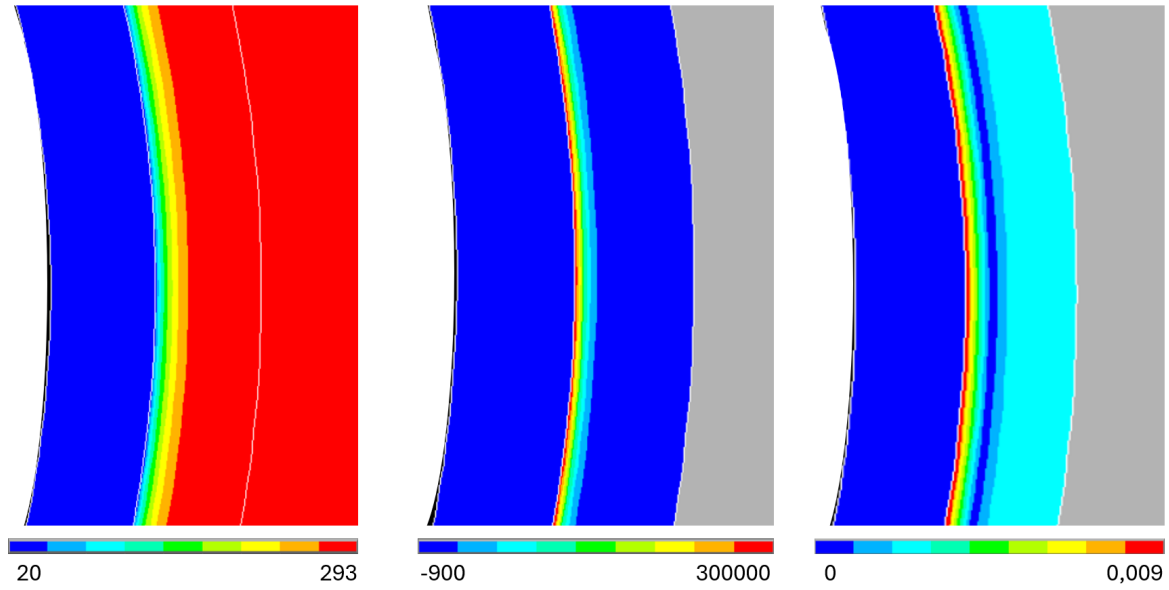


Figure 4.22: Spherical tank field results at $t = 2100$ s: (left) temperature (K), (center) first principal stress (Pa) and (right) first principal elastic strain. The maximum stress and strain occur at the smooth foam-steel interface.

The perlite layer carries no stress or elastic strain, consistent with its treatment as a thermally active but mechanically passive fill (Section 3.5). The path profiles along the foam are shown in Figure 4.23.

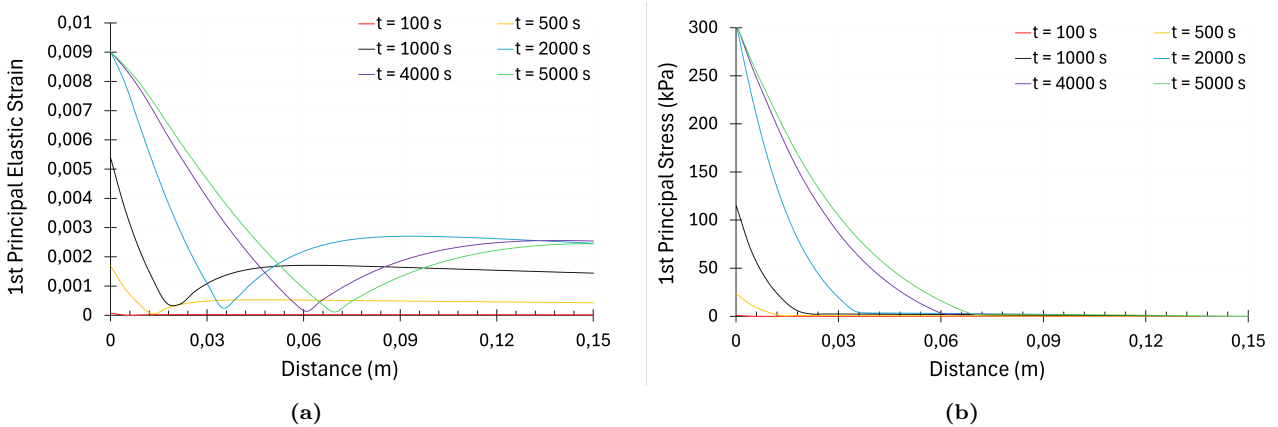


Figure 4.23: Spherical tank radial foam-path response: (a) first principal elastic strain and (b) first principal stress. The full two-peak response persists in a geometry without any corner, confirming that the internal peak is a general consequence of constrained differential contraction rather than a corner artefact.

They reproduce the full response identified on the mechanism model with a near-interface concentration, a decay to a minimum, an internal peak, and a final decay, with the internal peak migrating outward through the foam as the transient proceeds. Because the foam is thicker here, the internal peak has more distance to travel and is found progressively further from the interface at later times. The minimum between the two features falls close to zero, so the interface concentration and the internal peak are even more clearly separated than in the mechanism model.

Comparison with the Simplified Mechanism Model

Because the spherical tank profiles in Figure 4.23 are extracted at the same time instants as the mechanism-model profiles in Figure 4.5 and 4.6, the two can be compared directly. The following three results follow. First, the internal strain feature is present in the spherical tank geometry even though this geometry has no corner. This is evidence that the internal peak is not a corner artefact but a general consequence of constrained differential contraction across the moving transient gradient. The corner in the mechanism model fixes the location of the absolute maximum, but it is not responsible for the internal feature, which arises in the bulk foam regardless of the presence of a corner.

Second, the magnitude of the interface concentration transfers directly. The near-interface elastic strain is of the order of 0.009 and the near-interface stress is of the order of 300 kPa, comparable to the mechanism-model values, despite the different geometry, curvature and foam thickness. This is consistent with the finding of Section 4.3 that the interface concentration is governed by the foam-steel expansion mismatch rather than by the geometry, so that a smooth spherical interface and a flat bonded interface produce a similar mismatch-driven response.

Third, the internal peak is lower in the spherical tank, of the order of 0.0025 against roughly 0.004 in the mechanism model, while the interface concentration is essentially unchanged. This difference is informative. Because the material properties and the expansion mismatch are identical in both models, and the unchanged interface concentration confirms this, the reduced internal peak cannot be a material effect and must follow from the differences in geometry and thermal boundary. Here, three factors act in the same direction because the foam is thicker, so the through-thickness contraction gradient is spread over more material and the differential contraction per unit length is smaller. The larger radius of curvature of the spherical shell geometry, compared with the flat interface of the mechanism model, alters the way radial displacements generate circumferential strain contributions, which may moderate the internal peak independently of the foam thickness effect. In addition, the outer foam surface sits against the cooling perlite rather than being clamped at ambient, which lowers the sustained temperature difference across the foam later in the transient.

The interface concentration is unaffected by all three factors because it is set by the local fully cold mismatch, whereas the internal peak, which depends on the through-thickness gradient and its geometric constraint, is reduced and spread out. The comparison therefore separates a local geometric effect, the corner amplification, from the general through-thickness effects with the interface concentration and the moving internal peak. It indicates that a smooth, large-radius geometry and a greater foam thickness moderate the internal strain feature relative to the laboratory geometry. These observations identify regions and loading phases that warrant further attention, but they do not constitute a design assessment of the spherical tank.

4.8 Failure Relevance and Model Limitations

The preceding sections established where and when the model predicts elevated stress and elastic strain. This section relates those fields to the possibility of foam failure. Consistent with Section 3.10, this is a post-processing risk assessment, not a separate solve or a crack prediction. The computed peaks are compared with available cryogenic foam limits, the resulting margins are examined, and the response is classified as a risk indicator. At the end of this section, the bounding limitations are stated.

The assessment proceeds in three steps. The first paragraph extracts the failure criteria from the literature, namely the tensile strength and the strain-to-failure of rigid polyurethane foam at the modelled density and at cryogenic temperature. This step is necessary because no published low-temperature tensile data exist at the modelled density, so the limits must be constructed from the datasets that bracket it. The second paragraph applies these criteria to the simulated stress and elastic strain fields, establishing which criterion governs and at which locations the margin is smallest. The third paragraph states the limitations that bound the result.

Extraction of Failure Criteria from Literature Data

Figure 4.24 shows the global maximum first principal stress plot and first principal elastic strain plot against time, where both rise steeply during cool-down and plateau by approximately 2000 s. As shown, the mechanical loading reaches a near-steady cold state by the end of the cool-down transient. The global maxima occur at the foam-steel corner. The strain interior maximum located roughly 0.03-0.04 m from the interface, has a considerably lower maximum strain. These peaks are compared with tensile failure limits for the foam. No cryogenic data exist at the modelled density, so the limits are interpolated linearly in density between two primary low-temperature datasets that bracket it, both measured in tension at low temperature [36, 77]. The 295 K and 76 K limits were obtained by linear interpolation in density between the 32 and 64 kg m⁻³ longitudinal tensile data.

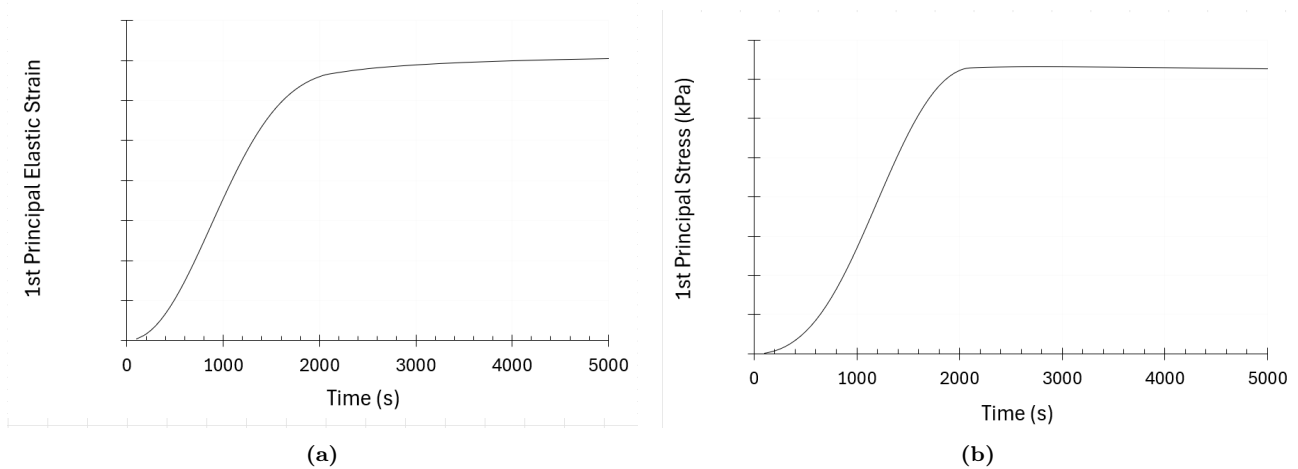


Figure 4.24: Global maximum in the foam during cool-down and cold hold: (a) first principal elastic strain and (b) first principal stress as a function of time. Both quantities rise steeply during cool-down and plateau by approximately 2000 s.

For the lower cryogenic limit, the 64 kg m⁻³ failure strain at 4 K was used as a conservative

lower bound, while the tensile strength was estimated by extending the approximately flat low-temperature 32 kg m^{-3} strength from 76 K and interpolating against the 64 kg m^{-3} value at 4 K. Two features of the underlying data matter for the comparison. The strain-to-failure is almost density-independent at cryogenic temperature, so the interpolated failure strain is robust. The tensile strength, in contrast, scales strongly with density, so the density-appropriate strength is well below that of the 64 kg m^{-3} foam alone, which tightens the stress margin. Because the model minimum temperature, 20 K, lies between the 4 K and 76 K data points, over which the tensile properties are nearly flat, the cold-state limits are taken at that near-flat level. This comparison is indicative only.

Application of the Failure Criteria

Applying the maximum-principal-stress and maximum-principal-strain indices defined in Section 3.10, and in Equations 3.4–3.6, to the cold state gives two results. First, the maximum-principal-stress criterion is not governing: at every location examined, including the corner, the computed first principal stress remains well below the interpolated cryogenic tensile strength, so the stress criterion alone does not indicate failure. Second, the maximum-principal-strain criterion governs. Its index is largest at the foam–steel corner, where a substantial part of the available cryogenic strain capacity is consumed and only a limited margin remains; this is the only location in the model at which the margin is marginal rather than comfortable. At the interior second peak the index is several times smaller and the margin is comfortable. The body of the foam is therefore clearly sub-critical, and the only marginal location is the foam–steel corner.

Because the interface concentration is set by the local mismatch at the interface temperature, it is reproduced at essentially the same level in every case examined in this chapter when compared at matched cold-side temperature. The protocol- and phase-dependent differences occur away from the interface, in the region where the margin is comfortable. Most of the variations studied in this chapter therefore act on the part of the response that is not damage-critical, which explains why none of the thermal protocols examined here reduces the governing margin.

Each index expresses the fraction of the available material capacity that is consumed at a given point, so that unity marks the limit and the safety factor is simply its reciprocal. That the strain index governs rather than the stress index is consistent with the sensitivity results of Section 4.3, in which the elastic strain proved essentially independent of the uncertain foam modulus while the stress is proportional to it and inherits that uncertainty in full. The location of the governing index is likewise consistent with the literature, in which cryogenic cracking and debonding initiate at geometric discontinuities and at the bondline during cool-down [15, 16], and with the interface cracking reported by the industrial partner. The indices should nonetheless be read as risk indicators rather than as design margins, since the governing value lies at a mesh-sensitive corner singularity and the underlying limits are interpolated rather than measured. The informative findings are therefore the location of the elevated values, the loading phase in which they occur, and their sensitivity to material properties and thermal protocol, not an absolute failure margin.

Limitations and Uncertainty

Several limitations bound this assessment. The governing indicator lies at the singular corner. Because the corner peak is still rising slightly under mesh refinement, its value is a converging lower-bound estimate rather than a grid-independent number, so the corner margin should be read as a non-conservative indicator of an elevated-risk location rather than as a design margin. The foam limits are interpolated both in density, from data at 32 and 64 kg m⁻³, and in temperature, from data at 4 and 76 K. The failure strain is robust because it is nearly density- and temperature-independent in this range, but the tensile strength is density-sensitive, and a lighter true foam would reduce the margins further. A further limitation concerns the foam properties assumed at the interface itself. The foam is modelled as a homogeneous continuum with a single set of effective properties, whereas spray-applied foam forms a denser and anisotropic layer against the substrate, as discussed in Section 2.3. The material immediately at the bond therefore has neither the density nor the stiffness assumed here, and its local strength is likely to differ from the bulk value used in the assessment. The governing index at the interface is consequently the least representative quantity in the assessment, and the most critical location in the real foam may lie just outside this denser surface layer, where the bulk properties assumed in the model are first attained, rather than exactly at the bond.

The most consequential limitation is the absence of a damage law, which is clearest in the warm-up response. The computed stresses arise from two mechanisms, one, a transient self-restraint stress that peaks while the through-thickness gradient is steepest and then relaxes, corresponding to the moving interior peak of Section 4.2. Second, an interface mismatch stress, $\sigma \approx E_f \Delta \alpha \Delta T$, that grows monotonically with the total temperature drop and plateaus at the cold state, as discussed in Section 4.3. The global maximum is governed by the second mechanism, which is why it occurs at the end of cool-down. On warm-up, this mismatch stress simply unloads as ΔT decreases, so the linear-elastic model predicts no higher peak loading during warm-up than during cool-down, as confirmed by the matched-temperature comparison of Section 4.5.

The greater cracking observed during warm-up by the industrial partner therefore cannot be explained by instantaneous elastic loading alone. Instead, it suggests history-dependent damage. Microcracks initiated at the corner and interface during the cold can become audible during the subsequent warm-up, even if the instantaneous elastic loading does not exceed the cool-down level [37, 47]. The present model captures only the pre-damage elastic state and contains no damage law or crack growth, so such history-dependent failure cannot appear in the results. Consistent with this, the elastic field at the corner retains the same component signs during warm-up as during cool-down, as discussed in Section 4.5, so no stress reversal occurs there. The corner behaviour is governed by the fixed bi-material geometry rather than by the direction of the temperature change. Capturing the warm-up cracking would require a damage or cohesive-zone model under cyclic loading [73], which is the principal recommendation for future work. Taken together with the modelling assumptions mentioned in Table 3.11, several limitations must be kept in mind. These include the homogenised and isotropic foam representation, the linear-elastic formulation, the perfectly bonded interface, the qualitative comparison and the idealised spherical tank geometry. The model can therefore identify the critical mechanisms, regions and loading phases associated with elevated thermo-mechanical loading. However, it cannot determine the exact crack initiation time, location or path. The implications of these findings for the research questions and for future modelling and experimental work are drawn together in Chapter 5.

Chapter 5

Conclusions & Recommendations

Section	Page
5.1 Conclusions	87
5.2 Recommendations	89

THIS FINAL CHAPTER concludes the thesis. Section 5.1 answers the research questions formulated in Section 2.7 on the basis of the results of Chapter 4. Section 5.2 presents recommendations for future modelling work, experimental work and design practice.

5.1 Conclusions

This thesis investigated how transient thermal gradients and constrained thermal contraction generate stress and elastic strain concentrations in rigid polyurethane foam insulation for non-vacuum liquid hydrogen storage. A coupled transient thermo-mechanical finite element model was developed and applied to a simplified mechanism model, the experiment-specific assembly and a representative spherical tank geometry. The main research question is answered through the three groups of subquestions defined in Section 2.7.

5.1.1 Material Behaviour

The temperature-dependent material properties that most strongly influence the thermo-mechanical response are the foam coefficient of thermal expansion and the foam Young's modulus. For the coefficient of thermal expansion, its mismatch with the metallic substrate is especially important. The low thermal conductivity and specific heat of the foam mainly govern the transient temperature field itself. The low thermal diffusivity produces a slowly moving cooling front and a persistent through-thickness temperature gradient, which defines the thermal loading, as shown in Sections 3.11.4 and 4.1.

The sensitivity studies of Section 4.3 showed that the problem is strain-controlled. The elastic strain field is set almost entirely by the CTE mismatch and the constraint geometry. It scales monotonically with the foam CTE and collapses to a small gradient-driven residual when the mismatch is removed. The foam Young's modulus then converts this strain into stress in direct proportion, without altering the strain field. These properties were successfully incorporated in a homogenised, isotropic, linear elastic continuum with tabulated temperature-dependent properties and a secant CTE convention, solved in a single coupled-field axisymmetric formulation. The numerical implementation was verified through mesh, time-step, boundary-condition and analytical thermal checks in Section 3.11. Because reliable foam data below 77 K are scarce and the 20 K modulus is extrapolated [17, 36, 77], the modulus-insensitive first principal elastic strain is the more robust mechanical indicator at cryogenic temperature.

5.1.2 Stress and Strain Development

The transient response of the bonded foam is governed by two coexisting mechanisms. The first is a stationary concentration at the foam-metal interface, where the bonded constraint converts the accumulated contraction mismatch into a near-equibiaxial in-plane, axial-hoop, tensile state, whose magnitude scales with $E_f \Delta \alpha \Delta T$ [21, 42]. It is anchored at the bondline and is locally amplified at the corner. The second is a moving internal peak of radial tension with a shear contribution, caused by constrained differential contraction across the transient

temperature gradient. It migrates outward with the cooling front and reaches its maximum near the end of cool-down, as shown in Sections 4.1 and 4.2.

The severity of both features is governed primarily by the total temperature drop and the mismatch, and only secondarily by the thermal path. Varying the cooling rate or introducing intermediate holds redistributes the internal feature without materially reducing the peak values, as shown in Section 4.4. During warm-up, the deeper cold penetration broadens the loaded region and shifts it outward, but at matched cold-side temperatures the warm-up peak stress and elastic strain nowhere substantially exceed their cool-down counterparts, as shown in Section 4.5. The most critical region is therefore the foam–metal interface, especially near geometric discontinuities. The most critical loading phases are the transient cool-down, during which the internal loading peaks, and the fully cold state, during which the interface loading plateaus.

5.1.3 Damage Interpretation and Experimental Comparison

Interpreted through the post-processing failure indicators of Section 3.10, the maximum-principal-strain criterion governs at cryogenic temperature. At the cold state, the interior strain peak retains a comfortable margin against the cryogenic failure strain, whereas the corner region consumes a large fraction of the available strain capacity and retains only a limited margin, as discussed in Section 4.8. The model therefore robustly localises the damage risk to the foam–metal interface and corner, which is consistent with the experimental literature on cryogenic foam insulation [15, 16] and with the interface cracking reported by the industrial partner. The correspondence with the experimental observations, however, remains qualitative and partial.

The predicted concentrations develop in the same phases and regions in which cracking was reported. The mechanism also transfers unchanged to the experiment-specific geometry, as shown in Section 4.6. However, the linear elastic model does not reproduce a warm-up-specific mechanical maximum in the reported 60–80 K range. The observed dominance of warm-up cracking therefore cannot be explained by instantaneous elastic loading alone and most plausibly reflects history-dependent damage, in which microcracks initiated at the cold, most brittle state become audible during the subsequent warm-up [37, 47].

Finally, the spherical tank comparison showed that the identified mechanisms transfer to a representative tank geometry. The internal peak persists in the absence of any corner, confirming that it is not a geometric artefact. The mismatch-driven interface concentration also transfers essentially unchanged to the spherical tank geometry. The internal feature is reduced in severity by the greater foam thickness, the smooth large-radius interface and the perlite boundary, as shown in Section 4.7. The model can therefore be used to identify thermo-mechanical risk regions in representative LH₂ tank geometries, provided its results are read as risk indicators rather than failure predictions. Additional factors that the present model cannot represent include the release of stored strain energy on reheating, differences in the acoustic detectability of cracking between the cold and warming states, and interfacial debonding.

5.1.4 Answer to the Main Research Question

Bringing these findings together, transient thermal gradients and constrained thermal contraction generate stress and elastic strain concentrations through a clear division of roles. The

transient gradient organises the response in space and time: it creates a moving cooling front that carries an internal through-thickness strain peak with it and determines when the loading develops. Constraint converts thermal strain into mechanical loading through three channels: the bonded foam–metal interface, the internal continuity between foam regions at different temperatures, and local geometry, which amplifies both. The foam–substrate CTE mismatch sets the amplitude of the response, and the temperature-dependent stiffness determines only how that constrained strain is converted into stress. The resulting critical condition is tensile strain at the bonded interface and its geometric discontinuities, governed by the accumulated temperature drop rather than by the rate at which it is applied.

These results carry a limited but useful implication for full-scale foam-insulated tanks. Such tanks should be assessed not only for their steady-state thermal performance but also for the transient thermo-mechanical loading during cool-down and warm-up, since the most severe elastic strain occurs during the transient rather than at steady cold hold. The regions of greatest concern are those where a steep thermal gradient, foam–substrate expansion mismatch and geometric constraint occur together. This is especially relevant at the foam–metal interface. Within this interface region, corners, thickness transitions and other discontinuities are of particular concern.

5.2 Recommendations

The recommendations follow from the limitations consolidated in Table 3.11 and Section 4.8, and are grouped into modelling, experimental and design recommendations.

5.2.1 Modelling Recommendations

The principal recommendation is to extend the model with a damage or cohesive-zone formulation under cyclic thermal loading [73, 78, 79]. The present linear elastic model captures the pre-damage state and cannot represent the history-dependent mechanism suggested by the warm-up cracking observations. A damage model would allow microcrack initiation during the cold state and its evolution during warm-up and repeated cycling to be represented explicitly. Closely related, the perfectly bonded interface should be replaced by a cohesive interface with finite strength. Where relevant, interfacial thermal contact resistance should also be included. This would allow debonding, a known dominant failure mode [15, 42], to be captured together with the thermal consequences of damage, since cracks and debonded regions can act as additional heat-transfer paths [18, 23].

A further recommendation concerns the material description itself. The present model uses temperature-dependent thermal conductivity, specific heat, thermal expansion and Young’s modulus, but the Poisson ratio is held constant, the modulus below 77 K is obtained by extrapolation, and the failure limits are applied as a single cryogenic value. A more elaborate description, with measured temperature-dependent stiffness, Poisson ratio and failure properties across the full 20–293 K range for the specific foam grade, would remove the largest remaining source of property uncertainty. Finally, the model should be extended to three dimensions and include orthotropic foam properties. This would allow spray-induced anisotropy, density gradients and non-axisymmetric defects to be represented [13, 18, 38]. These effects cannot be captured by the present axisymmetric isotropic formulation.

5.2.2 Experimental Recommendations

The largest source of uncertainty in the present failure assessment is the material data. A dedicated characterisation of the actual foam formulation is therefore recommended, using specimens representative of the as-installed density. The tests should extend down to 20 K and include Young's modulus, coefficient of thermal expansion, tensile strength and strain-to-failure. These properties should be measured both parallel and perpendicular to the rise direction. This would remove the density interpolation and temperature extrapolation that currently dominate the uncertainty in the safety factors [17, 36, 77].

In addition, future thermal-cycle experiments should include dedicated instrumentation. This would allow the qualitative comparison made in this thesis to be developed into a quantitative validation. Useful measurements include acoustic emission monitoring to determine the timing and approximate location of cracking, surface or embedded strain measurements near the foam-metal interface, and systematic post-test crack mapping. Repeated cool-down and warm-up cycling should be included to quantify progressive microcrack accumulation over the service life [47].

5.2.3 Design Recommendations

Within the scope of a risk-indicator model, three design implications follow. First, the foam-substrate CTE mismatch is the dominant lever on both the interface concentration and the internal strain feature. Reducing this mismatch through foam formulation or density selection is therefore the most effective mitigation strategy. This is more effective than adjusting the thermal protocol, since neither slower cooling nor staged cooling with intermediate holds reduced the peak response in the simulations discussed in Section 4.4.

Second, sharp corners and abrupt thickness transitions at bonded interfaces should be avoided or rounded with a sufficiently large radius. These geometric features should be minimised because the governing strain index was consistently located near such local geometry.

Third, the spherical tank comparison indicates that smooth, large-radius geometries with a generous foam thickness reduce the severity of the internal strain feature. This is favourable for tank-scale application. However, the mismatch-driven interface loading remains largely geometry-independent and should therefore drive the qualification of any bonded foam insulation system. Given the uncertainty in the cryogenic foam modulus, such qualification is best based on a strain-based criterion, comparing the first principal elastic strain against the temperature-dependent strain-to-failure, rather than on a stress-based criterion, as discussed in Section 4.3.

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Appendix A

Stress State and Principal Stresses

Section	Page
A.1 The Stress State at a Point	99
A.2 Principal Stresses and Principal Strains	99

A.1 The Stress State at a Point

At any material point the complete stress state is described by the Cauchy stress tensor, a symmetric second-order tensor with six independent components [50]:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \quad (\text{A.1})$$

where σ_{xx} , σ_{yy} and σ_{zz} are the normal stresses acting perpendicular to the coordinate planes, and τ_{xy} , τ_{yz} and τ_{xz} are the shear stresses acting parallel to them. Symmetry of the tensor follows from moment equilibrium and reduces the number of independent components from nine to six [50]. The numerical values of these components depend on the chosen coordinate system: rotating the coordinate axes changes the individual components even though the physical stress state is unchanged. The complete stress state at a point is shown in Figure A.1.

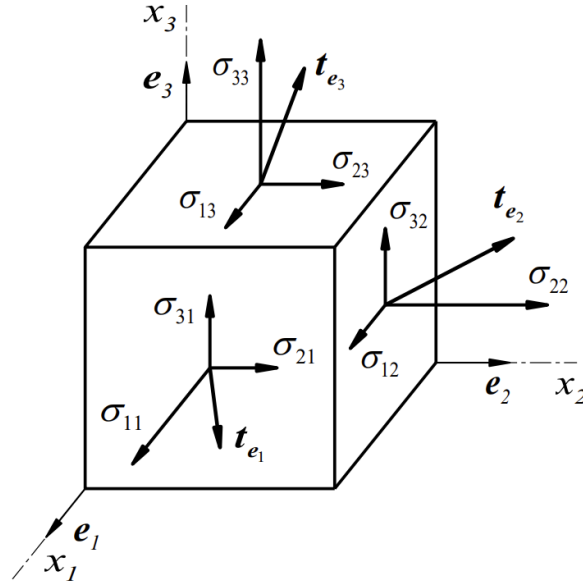


Figure A.1: Stress state at a point in general Cartesian coordinates x_1 , x_2 and x_3 . The nine stress components σ_{ij} acting on the faces of an infinitesimal element are shown, together with the traction vectors $\mathbf{t}_{\mathbf{e}_i}$ that represent the resultant stress vector on each face [80].

A.2 Principal Stresses and Principal Strains

Because the stress components are coordinate-dependent, it is useful to identify the orientation in which the description is simplest. For any stress state there exists an orientation in which all shear stresses vanish and only normal stresses remain. These normal stresses are the principal stresses, and the corresponding axes are the principal directions [50].

Mathematically, the principal stresses are the eigenvalues of the stress tensor, obtained from:

$$\det(\sigma_{ij} - \lambda \delta_{ij}) = 0 \quad (\text{A.2})$$

where λ denotes an eigenvalue and δ_{ij} is the Kronecker delta. The three eigenvalues are the principal stresses, conventionally ordered as:

$$S_1 \geq S_2 \geq S_3 \quad (\text{A.3})$$

so that S_1 is the algebraically largest, most tensile, and S_3 the algebraically smallest, most compressive, principal stress. It is important to note that smallest refers to sign and not to magnitude. So S_3 may be a large negative value representing the dominant compressive stress, while S_1 may be small or even negative if the entire stress state is compressive [50]. The principal directions are mutually perpendicular and vary from point to point, so the orientation of S_1 generally rotates throughout the domain. Consequently, comparing the magnitude of S_1 at one location with S_2 at another is not meaningful. The ordering of Equation A.3 holds locally at each individual point. Principal strains are defined in exactly the same way. The strain tensor is also symmetric, and its eigenvalues are the principal strains, ordered $E_1 \geq E_2 \geq E_3$ [50]. For an isotropic linear elastic material, the principal strain directions coincide with the principal stress directions.

Appendix B

Additional Plots

Section

Page

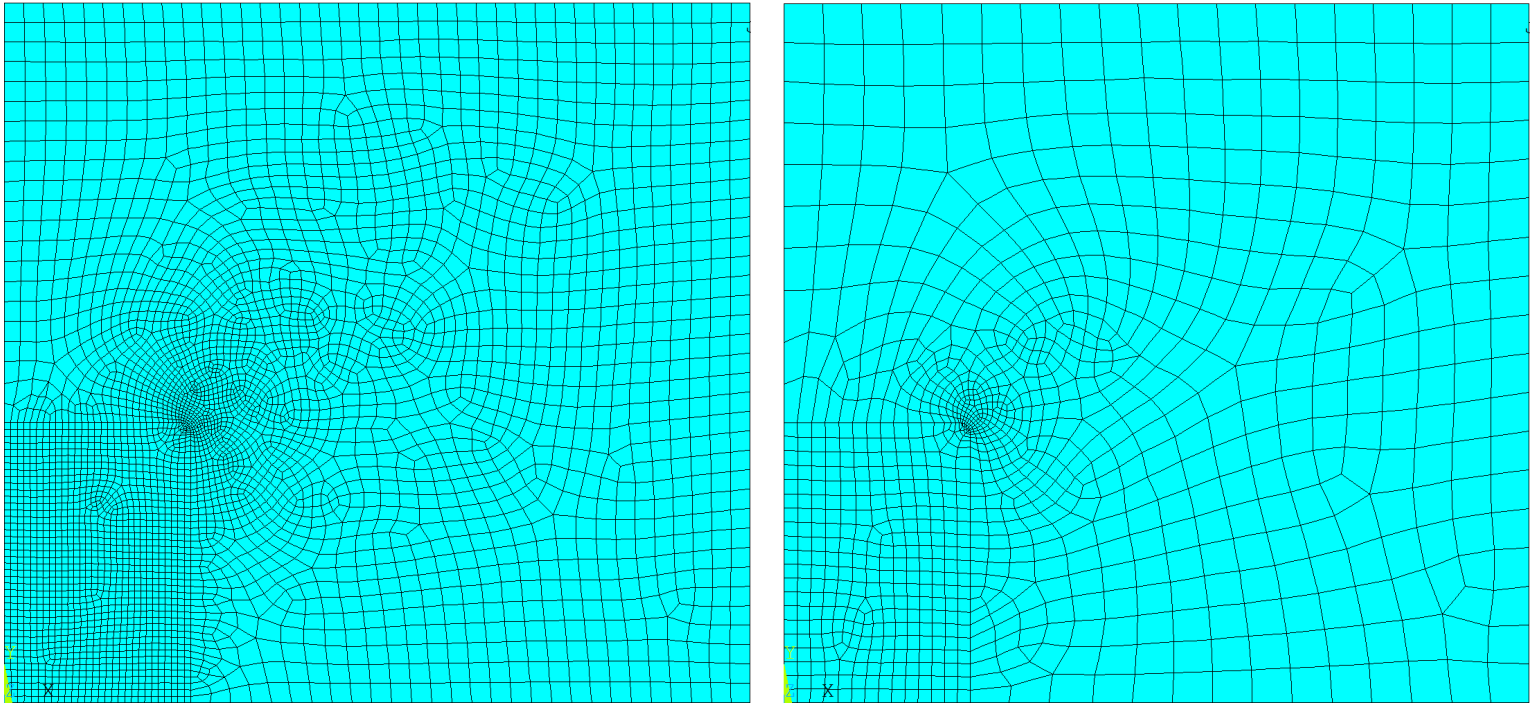


Figure B.1: Comparison of free-mesh refinement levels used in the mesh-sensitivity study: (a) free mesh level 1 and (b) free mesh level 4.

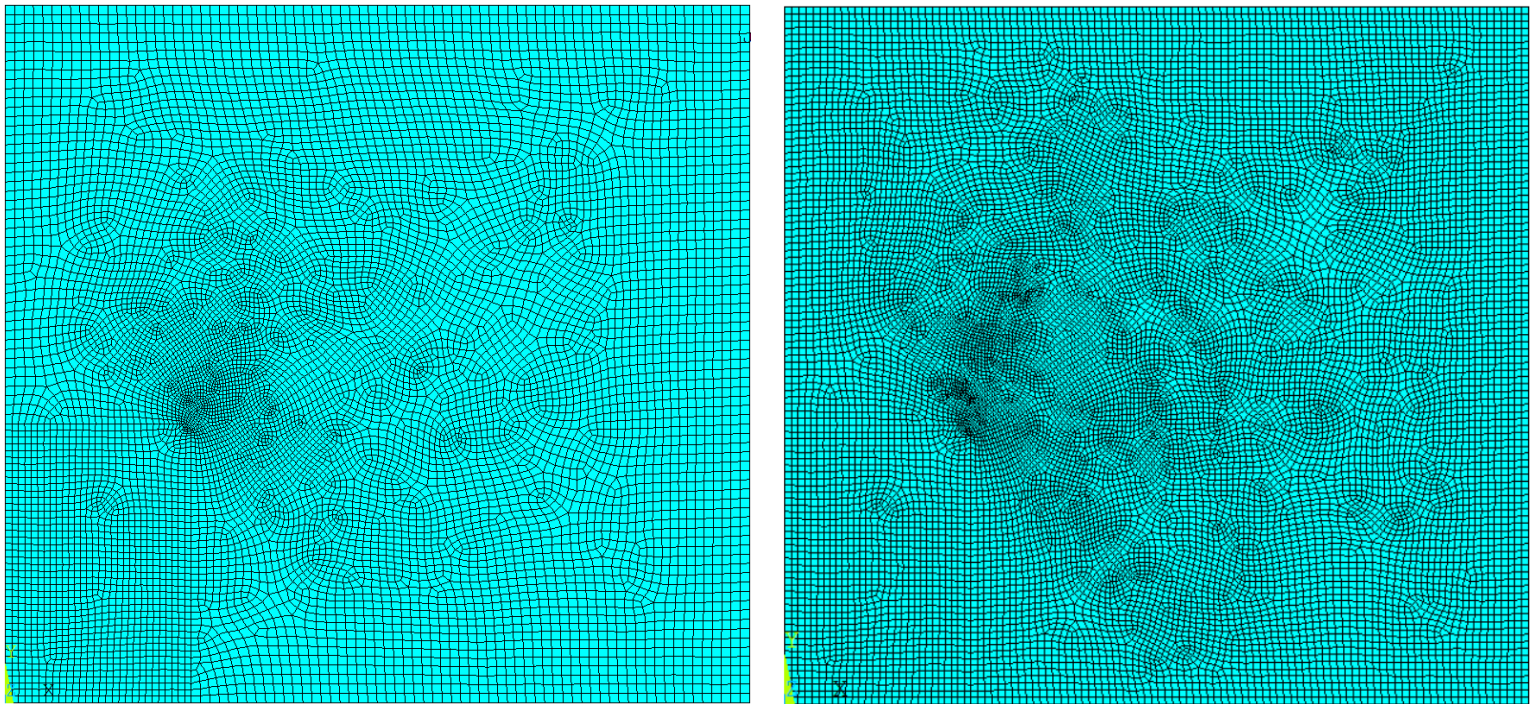


Figure B.2: Comparison of alternative mesh-control strategies used in the mesh-sensitivity study: (a) smart-size level 1 with a local ESIZE of 0.002 m and (b) ESIZE of 0.0015 m.

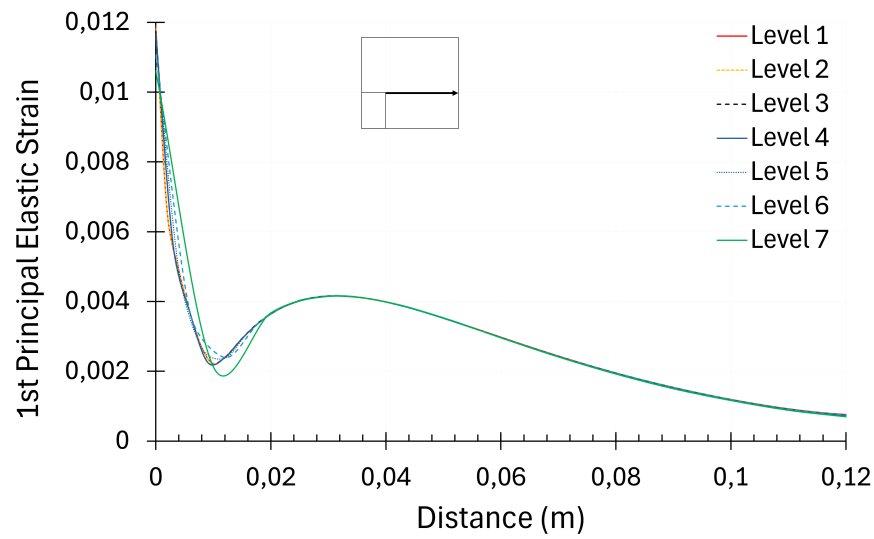


Figure B.3: Mapped-mesh comparison of the first principal elastic strain along the upper radial path for mesh Levels 1-7.

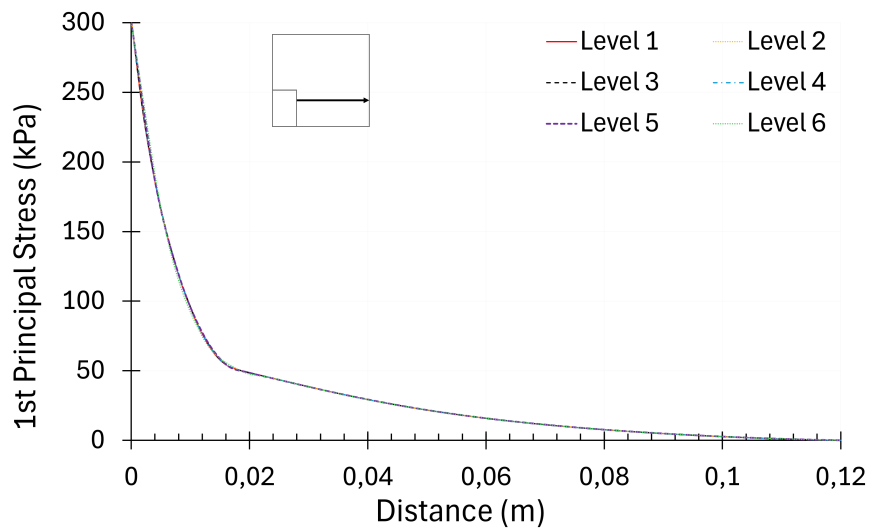


Figure B.4: Mapped-mesh comparison of the first principal stress along the lower radial path for mesh Levels 1-6.

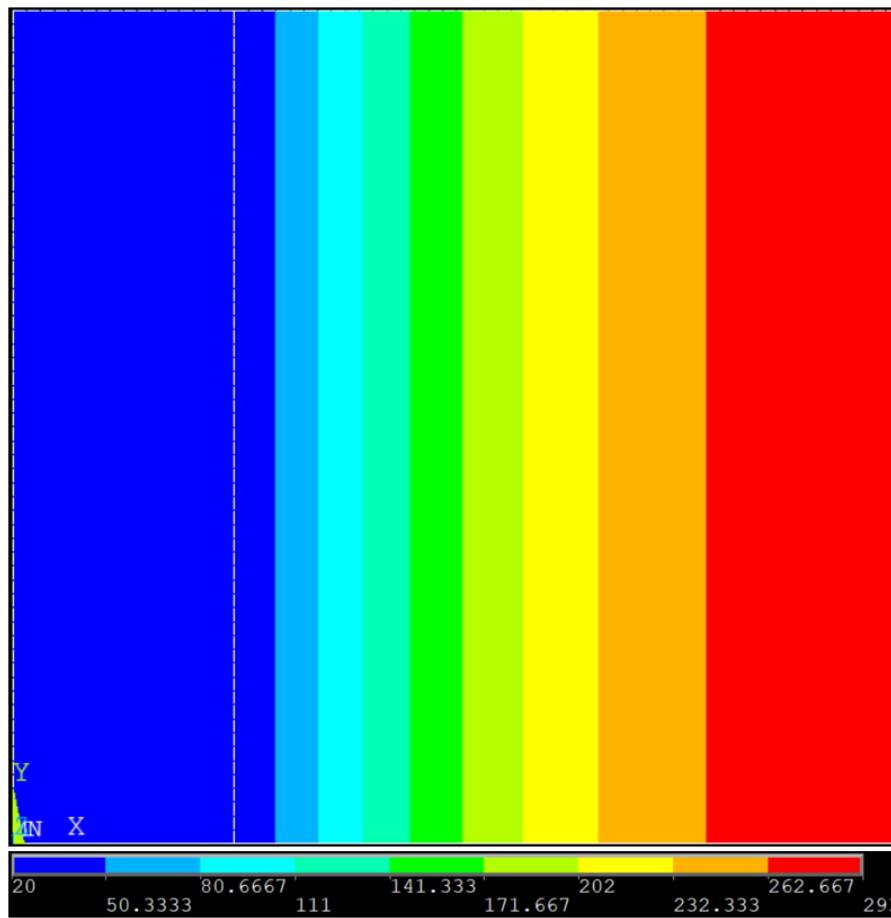


Figure B.5: Temperature profile in the one-dimensional analytical foam-slab benchmark at $t = 2100$ s. The profile illustrates the strong through-thickness temperature gradient that remains present at the end of the baseline cool-down.

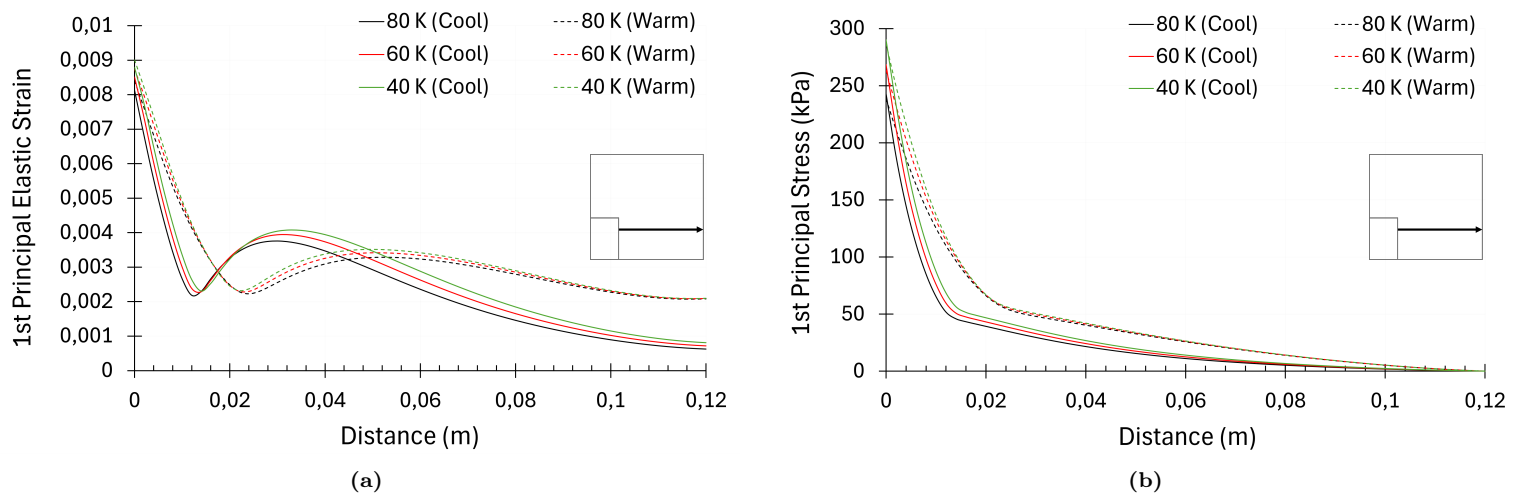


Figure B.6: Comparison between cool-down and fast warm-up along the lower radial path at matched steel temperatures of 40, 60 and 80 K: (a) first principal elastic strain and (b) first principal stress.

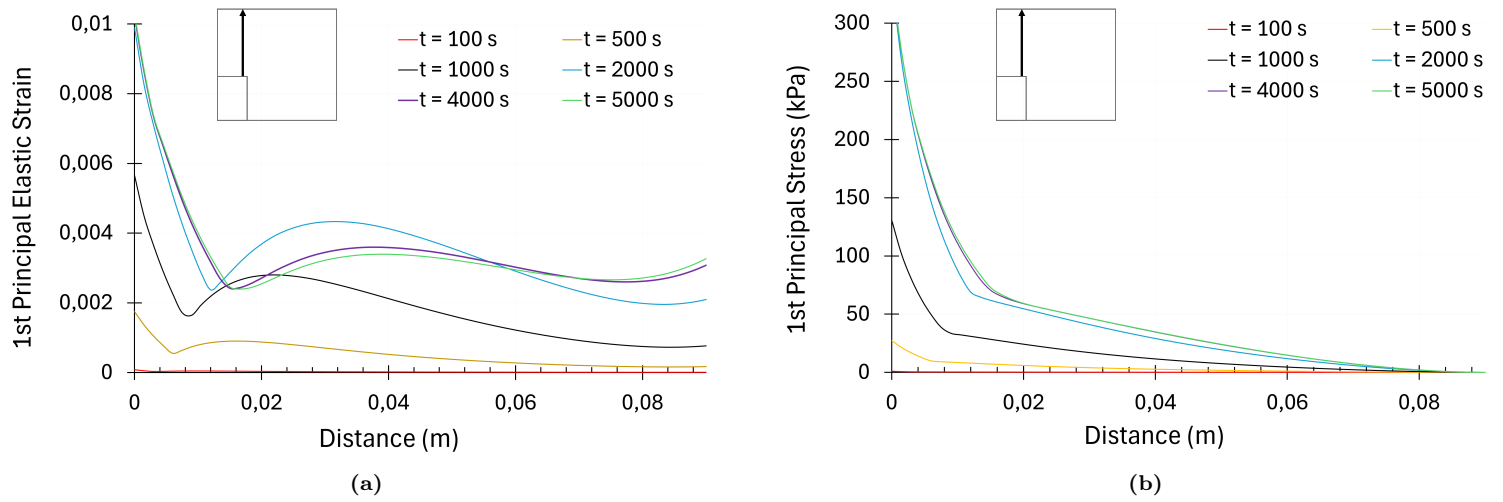


Figure B.7: Axial path response at selected times: (a) first principal elastic strain and (b) first principal stress along the axial path.

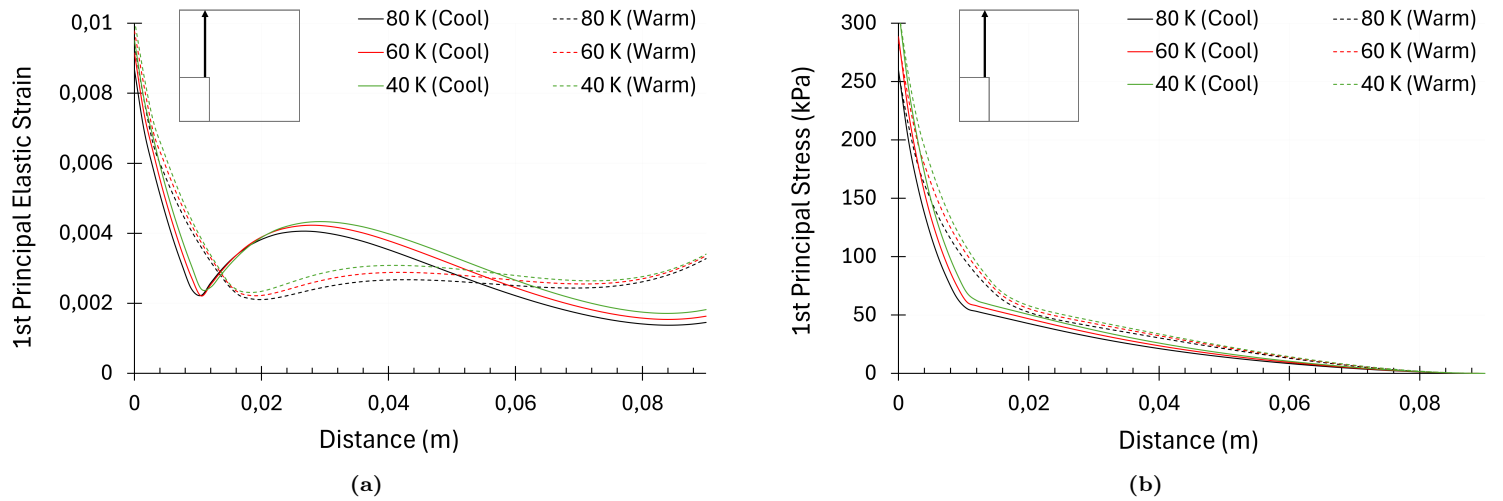


Figure B.8: Comparison between cool-down and slow warm-up along the axial path at matched steel temperatures of 40, 60 and 80 K: (a) first principal elastic strain and (b) first principal stress.

Table B.1: Relative difference in the first principal elastic strain along the diagonal path at $t = 2000$ s, relative to the finest mesh (10 816 elements, Level 1), at successive positions s along the path (See Figure 3.12).

Number of Elements	$s = 0.000$	$s = 0.010$	$s = 0.025$	$s = 0.050$	$s = 0.080$	$s = 0.151$
10 816	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7 056	0.55%	0.09%	0.01%	0.01%	0.00%	0.14%
4 356	1.25%	0.07%	0.01%	0.00%	0.00%	0.29%
1 764	1.12%	37.89%	0.36%	0.02%	0.06%	0.24%
900	1.77%	0.18%	0.14%	0.02%	0.24%	0.35%
400	1.65%	74.30%	1.49%	0.32%	0.02%	4.91%
196	0.14%	146.91%	0.50%	0.52%	0.83%	19.82%

Large relative differences at small s for the coarser meshes (1764 elements and fewer) reflect the steep, locally under-resolved gradient adjacent to the corner discussed in Section 3.7, rather than a lack of convergence of the path-based response used elsewhere in this study.

Table B.2: Relative difference in the first principal elastic strain along the lower radial path at $t = 2000$ s, relative to the finest mesh (10 816 elements, Level 1), at successive positions s along the path (See Figure 3.13).

Number of Elements	$s = 0.000$	$s = 0.012$	$s = 0.030$	$s = 0.050$	$s = 0.080$	$s = 0.120$
10 816	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7 056	0.56%	0.21%	0.10%	0.20%	0.46%	2.24%
4 356	1.38%	0.00%	0.10%	0.31%	0.62%	3.17%
1 764	1.79%	0.13%	0.14%	0.20%	0.41%	2.37%
900	3.72%	0.84%	0.29%	0.17%	0.36%	1.32%
400	6.93%	0.00%	0.12%	0.37%	0.15%	1.58%
196	10.82%	1.75%	0.10%	0.08%	1.49%	7.92%

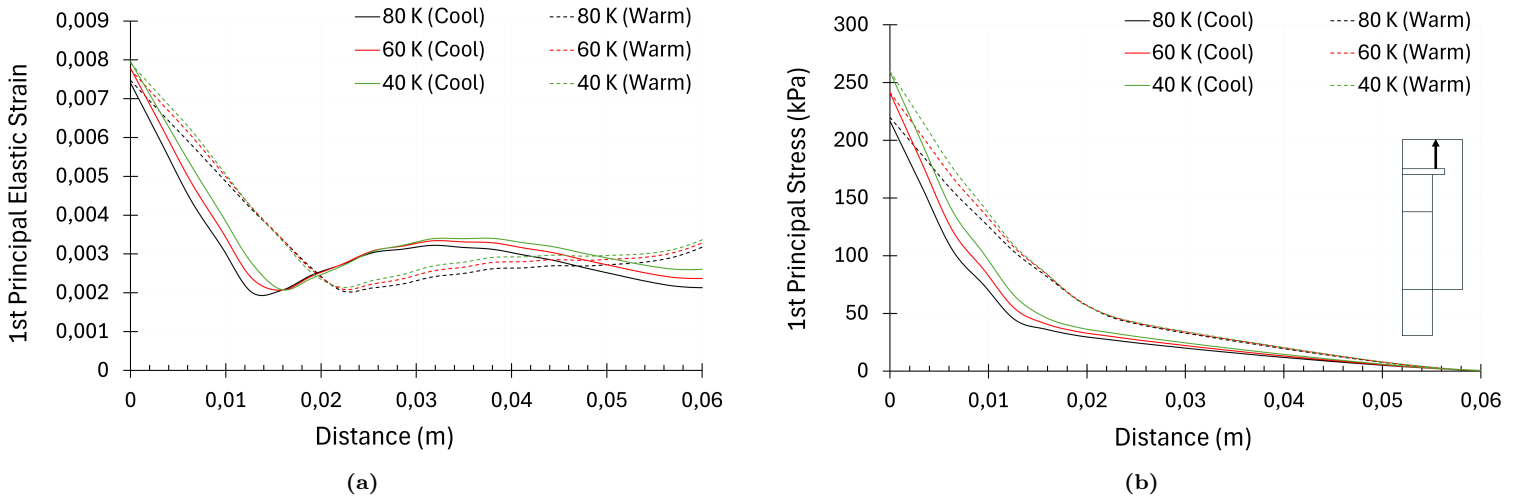


Figure B.9: Experiment-specific model response along the axial foam path P2, comparing cool-down and warm-up at matched cold-head temperatures of 40 K, 60 K and 80 K: (a) first principal elastic strain and (b) first principal stress. The axial path reproduces the behaviour found along the radial path P1, with the warm-up profiles shifted outward and elevated away from the interface while the near-interface values remain comparable between the two phases.

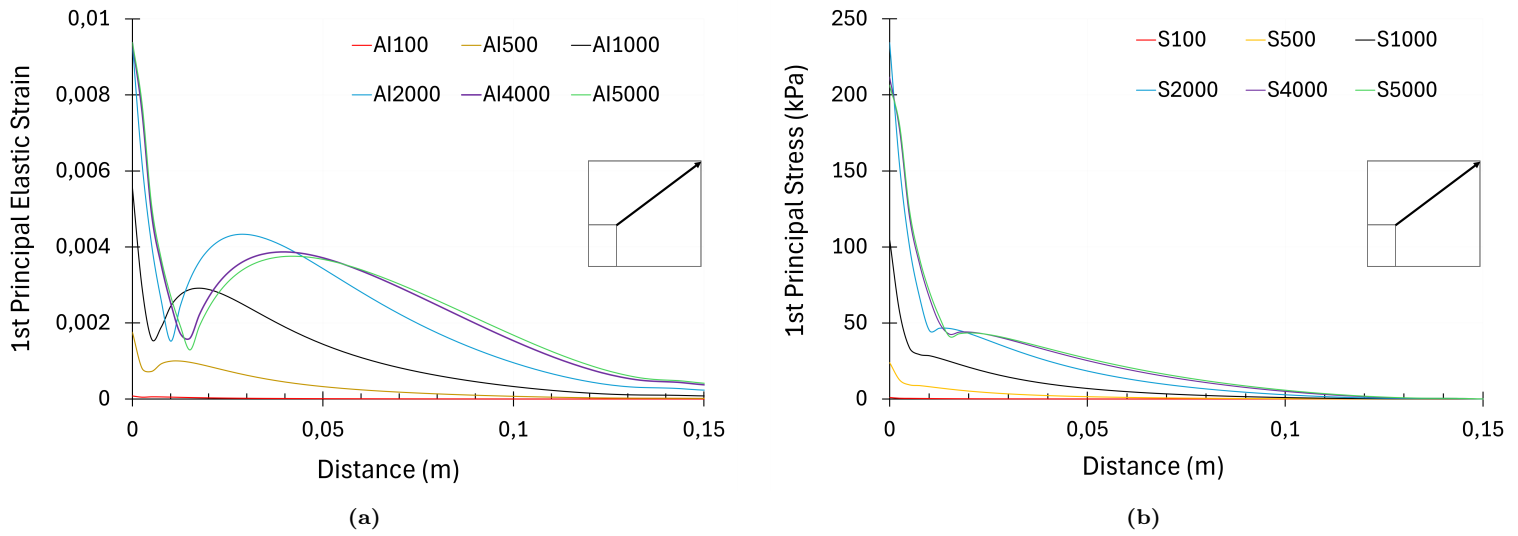


Figure B.10: (a) First principal elastic strain and (b) first principal stress along the diagonal path during cool-down. The path starts at the foam-steel corner and the same two-part structure of an interface concentration and a separate internal peak is recovered along this path.

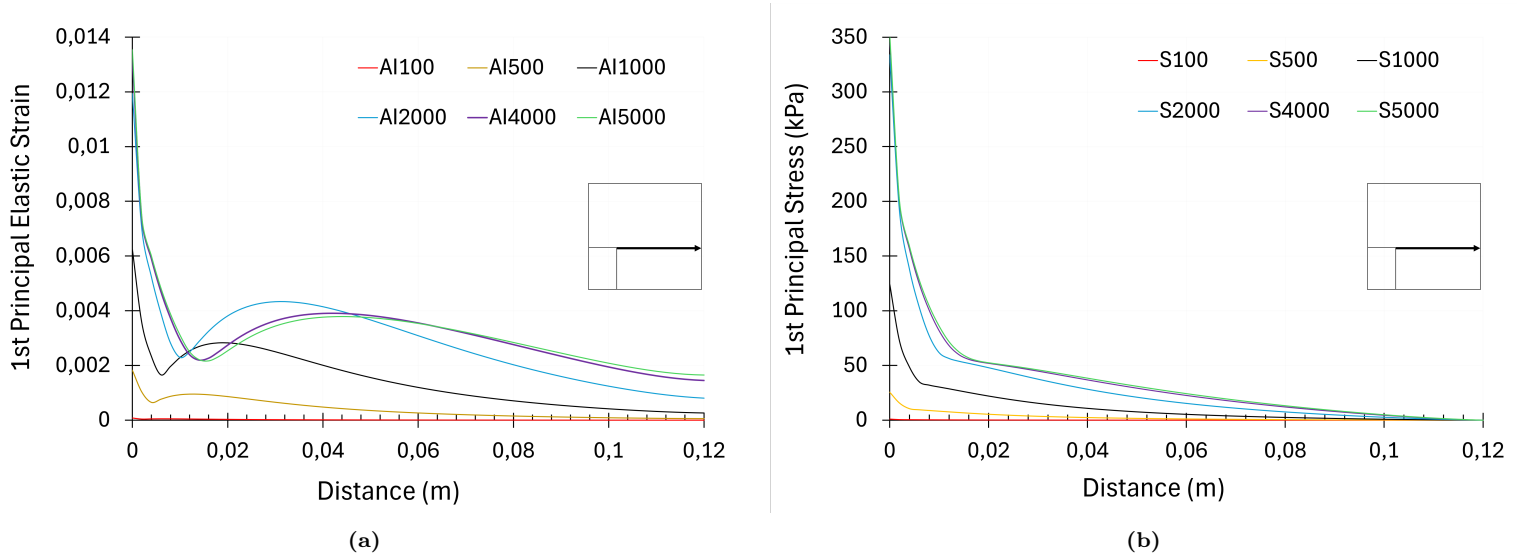


Figure B.11: (a) First principal elastic strain and (b) first principal stress along the upper radial path during cool-down. This path runs through the foam at the height of the corner and therefore allows the response close to the corner to be compared with that along the lower radial path, which lies further from it.

Appendix C

ANSYS Scripts

Section	Page
C.1 Overview	109
C.2 Simplified Mechanism Model	110
C.3 Experiment-Specific Model	115
C.4 Spherical Tank Model	121
C.5 Thermal Load Variations	127
C.6 Material Property Variations	132
C.7 Mechanical Boundary Conditions	134
C.8 Mesh Definitions	135
C.9 Post-Processing Procedure	137
C.10 Analytical Thermal Benchmark	144

C.1 Overview

All finite element models in this study were built and solved in ANSYS Mechanical APDL using input scripts. This appendix documents those scripts. The three model configurations introduced in Section 3.2 are each given in full: the simplified mechanism model in Section C.2, the experiment-specific model in Section C.3 and the spherical tank model in Section C.4. The one-dimensional benchmark used for the analytical thermal check is given in Section C.10. The spherical tank geometry and the foam and perlite material data are defined through named parameters, set in a separate input file that each model script reads in, so that the same scripts can be run for different foam grades and tank configurations.

The remaining simulation cases are variations on these baseline scripts. Rather than reproducing every case in full, only the blocks that differ from the corresponding baseline are listed, each preceded by a header stating which block it replaces and which cases it covers. Table C.1 maps every case of the simulation matrix to the sections that define it. All models share the same solver configuration: a coupled-field transient analysis (`ANTYPE,4,TRNOPT,FULL`) with automatic time stepping, a full Newton–Raphson scheme allowing up to 80 equilibrium iterations per substep, ramped load application (`KBC,0`) and convergence monitored on the heat-flow residual with an L_2 -norm tolerance of 0.1%. A consistent SI unit system is used throughout: metres, kilograms, seconds, kelvin and pascals.

Table C.1: Mapping between the simulation cases and the script sections of this appendix. Cases marked “baseline” use the corresponding full script without modification.

Case	Sections	Block that differs
Baseline cool-down	C.2	baseline
Fast cool-down (1050 s)	C.2, C.5	thermal load curve, load steps
Slow cool-down (4200 s)	C.2, C.5	thermal load curve, load steps
Stepwise cool-down	C.2, C.5	thermal load curve, load steps
Slow warm-up (3420 s)	C.2, C.5	warm-up load step
Fast warm-up (720 s)	C.2, C.5	warm-up load step
Long cold hold	C.2, C.5	cold-hold load step
Coarser time stepping	C.2, C.5	DELTIM settings
Single load step	C.2, C.5	load-step structure
CTE 0.5×, 1.5×, 2.0×	C.2, C.6	foam ALPX block
Equal-CTE diagnostic	C.2, C.6	foam ALPX block
E 0.5×, 1.5×, 2.0×	C.2, C.6	foam EX block
No Bottom / No Boundary	C.7	mechanical constraints
Mapped mesh Levels 1–7	C.2, C.8	line divisions, steel mesh
Free-mesh comparison	C.2, C.8	meshing strategy
Experiment-specific model	C.3	baseline
Experiment, slow warm-up	C.3, C.5	warm-up load step
Spherical tank model	C.4	baseline
Analytical thermal check	C.10	separate model
Failure-relevance assessment	C.9	post-processing only

C.2 Simplified Mechanism Model

The foam is meshed with a mapped mesh at the production refinement, mesh Level 2, while the steel region is free-meshed. For the foam, the four interface lines and the two outer foam lines are concatenated so that the region can be mapped as a single four-sided area.

Listing C.1: Complete input file for the simplified mechanism model, baseline cool-down, mesh Level 2.

```

1  ! =====
2  ! Thesis   : Transient Thermo-Mechanical Modelling of Polyurethane Foam
3  !           : Insulation for Non-Vacuum Liquid Hydrogen Storage
4  ! Author   : Gijs Schreuder, TU Delft
5  ! Model    : Simplified mechanism model (foam-steel, axisymmetric)
6  ! Case     : Baseline cool-down, 293 K -> 20 K in 2100 s, mesh Level 2
7  ! Solver   : ANSYS Mechanical APDL, coupled-field transient (PLANE223)
8  ! Units    : SI (m, kg, s, K, Pa)
9  ! =====
10
11 /CLEAR
12 /PREP7
13
14 *SET,R,0.002    ! Fillet radius at the foam-steel corner
15
16 ! 1. ELEMENT TYPE
17
18 ET,1,PLANE223
19 KEYOPT,1,1,11
20 KEYOPT,1,3,1
21
22 ! 2. GEOMETRY
23
24 K,1,0,0,0
25 K,2,0.04,0,0
26 K,3,0.04,0.06,0
27 K,4,0,0.06,0
28 K,5,0,0.15,0
29 K,6,0.16,0.15,0
30 K,7,0.16,0,0
31
32 ! Rounded corner
33 K,8,0.04-R,0.06,0
34 K,9,0.04,0.06-R,0
35 K,10,0.04-R,0.06-R,0
36
37 K,11,0.04-R+R*cos(45*acos(-1)/180),0.06-R+R*sin(45*acos(-1)/180),0
38
39 ! LINES
40
41 L,1,2          ! L1 bottom steel
42 L,9,2          ! L2 right steel/interface
43 L,8,4          ! L3 top steel/interface
44 L,4,1          ! L4 left steel
45 LARC,9,11,10,R ! L5 bottum half
46 LARC,11,8,10,R ! L6 top half
47 L,4,5          ! L7 left foam
48 L,5,6          ! L8 top foam
49 L,6,7          ! L9 right foam
50 L,2,7          ! L10 bottom foam

```

```

51
52 ! AREAS
53
54 AL,1,2,5,6,3,4
55 AL,7,8,9,10,2,5,6,3
56 NUMMRG,ALL
57 ALLSEL,ALL
58
59 ! 3. MATERIALS
60
61 ! Foam properties
62 /INPUT,foam_material,mac ! sets rho_f, k1-k11, cp1-cp11, E20, E293 and a1-a10
63
64 MP,DENS,1,rho_f
65 MP,PRXY,1,0.3
66
67 MPTEMP,,,,,,,,
68 MPTEMP,1,20,30,60,90,120,150
69 MPTEMP,7,180,210,240,270,300
70 MPDATA,KXX,1,1,k1,k2,k3,k4,k5,k6
71 MPDATA,KXX,1,7,k7,k8,k9,k10,k11
72 MPDATA,C,1,1,cp1,cp2,cp3,cp4,cp5,cp6
73 MPDATA,C,1,7,cp7,cp8,cp9,cp10,cp11
74
75 MPTEMP,,,,,,,,
76 MPTEMP,1,0,20,293,300
77 MPDATA,EX,1,1,E20,E20,E293,E293
78
79 MPTEMP,,,,,,,,
80 MPTEMP,1,0,20,77,100,150,200
81 MPTEMP,7,230,260,293,300
82 MPDATA,ALPX,1,1,a1,a2,a3,a4,a5,a6
83 MPDATA,ALPX,1,7,a7,a8,a9,a10
84
85 ! Steel
86
87 MP,DENS,2,7800
88 MP,PRXY,2,0.29
89 MP,KXX,2,15.12
90 MP,C,2,470.5
91
92 MPTEMP,,,,,,,,
93 MPTEMP,1,0,20,40,77,100,150
94 MPTEMP,7,200,250,293,300
95 MPDATA,EX,2,1,2.1010E+11,2.1010E+11,2.1170E+11,2.1440E+11,2.1370E+11,2.1090E+11
96 MPDATA,EX,2,7,2.0720E+11,2.0330E+11,1.9950E+11,1.9950E+11
97
98 MPTEMP,,,,,,,,
99 MPTEMP,1,0,20,40,77,100,150
100 MPTEMP,7,200,250,293,300
101 MPDATA,ALPX,2,1,1.0990E-05,1.0990E-05,1.1770E-05,1.2960E-05,1.3540E-05,1.4440E-05
102 MPDATA,ALPX,2,7,1.4960E-05,1.5230E-05,1.5510E-05,1.5510E-05
103
104 TREF,293
105
106 ! 4. MATERIAL ASSIGNMENTS
107
108 ! Steel

```

```
09 ASEL,S,AREA,,1
10 AATT,2,1,1
11 ALLSEL,ALL
12
13 ! Foam
14 ASEL,S,AREA,,2
15 AATT,1,1,1
16 ALLSEL,ALL
17
18 ! LINE DIVISIONS
19
20 ! Inner interface
21 LESIZE,2,,,48,4
22 LESIZE,5,,,4
23 LESIZE,6,,,4
24 LESIZE,3,,,28,4
25
26 ! Foam outer boundaries
27 LESIZE,7,,,84,4
28 LESIZE,8,,,42
29 LESIZE,9,,,42
30 LESIZE,10,,,84,4
31
32 ! Steel boundaries
33 LESIZE,1,,,28
34 LESIZE,4,,,48
35
36 ! 5. MESH (Mesh Level 2 (production mesh))
37
38 MSHAPE,0,2D
39 ESHAPE,2
40
41 ! STEEL MESH
42
43 SMRTSIZE,1
44 ESIZE,0.00005
45 MSHKEY,0
46 ASEL,S,AREA,,1
47 AMESH,ALL
48 ALLSEL,ALL
49
50 ! FOAM CONCATENATION
51
52 LSEL,S,LINE,,2
53 LSEL,A,LINE,,5
54 LSEL,A,LINE,,6
55 LSEL,A,LINE,,3
56 LCCAT,ALL
57 ALLSEL,ALL
58
59 LSEL,S,LINE,,8
60 LSEL,A,LINE,,9
61 LCCAT,ALL
62 ALLSEL,ALL
63
64 ! FOAM MAPPED MESH
65
66 MSHKEY,1
```



```

67 MSHAPE,0,2D
68 ESHAPE,2
69 ASEL,S,AREA,,2
70 AMESH,ALL
71 ALLSEL,ALL
72
73 ! MERGE + PLOT
74
75 NUMMRG,NODE
76 NUMMRG,ELEM
77 /GRAPHICS,FULL
78 ALLSEL,ALL
79 EPlot
80
81 ! 6. INITIAL CONDITION
82
83 IC,ALL,TEMP,293
84 FINISH
85
86 ! 7. BOUNDARY CONDITIONS
87
88 /PREP7
89 NSEL,S,LOC,X,0
90 D,ALL,UX,0
91 ALLSEL,ALL
92 NSEL,S,LOC,Y,0
93 D,ALL,UY,0
94 ALLSEL,ALL
95 FINISH
96
97 ! 8. THERMAL LOAD CURVE
98
99 *SET,T_start,293.0
100 *SET,dT,273.0
101 *SET,t_cool,2100.0
102 *SET,N,50
103 *SET,pi,acos(-1.0)
104 *SET,omega,pi/2.0/t_cool
105 *DIM,TCOOL,TABLE,N,1,1,TIME
106 *DO,i,1,N
107     TCOOL(i,0,1) = (i-1.0)/(N-1.0)*t_cool
108 *ENDDO
109 *DO,i,1,N
110     *SET,t_i,(i-1.0)/(N-1.0)*t_cool
111     TCOOL(i,1,1) = T_start - dT*(sin(omega*t_i))**2
112 *ENDDO
113
114 ! 9. SOLUTION
115
116 /SOLU
117 ANTYPE,4
118 TRNOPT,FULL
119 AUTOTS,ON
120 NROPT,FULL
121 NEQIT,80
122 KBC,0
123     temperature curves
124 CNVTOL,HEAT,1,0.001,2,1e-3

```

! Up to 80 Newton-Raphson iterations per substep

! Ramped loading, consistent with the smooth

! Heat-flow residual, L2-norm tolerance 0.1%

```
224
225 ! --- Load Step 1a:(0 -> 600s)
226 TIME,600
227 DELTIM,5,1,10
228 ASEL,S,AREA,,1
229 NSLA,S,1
230 D,ALL,TEMP,%TCOOL%
231 ALLSEL,ALL
232 NSEL,S,LOC,X,0.16
233 D,ALL,TEMP,293
234 ALLSEL,ALL
235 NSEL,S,LOC,Y,0.15
236 D,ALL,TEMP,293
237 ALLSEL,ALL
238 OUTRES,ALL,ALL
239 SOLVE
240
241 ! --- Load Step 1b:(600 -> 1500s)
242 TIME,1500
243 DELTIM,2,0.5,5
244 OUTRES,ALL,ALL
245 SOLVE
246
247 ! --- Load Step 1c:(1500 -> 2100s)
248 TIME,2100
249 DELTIM,5,1,10
250 OUTRES,ALL,ALL
251 SOLVE
252
253 ! --- Load Step 2:(2100 -> 5700s)
254 TIME,5700
255 ASEL,S,AREA,,1
256 NSLA,S,1
257 D,ALL,TEMP,20
258 ALLSEL,ALL
259 NSEL,S,LOC,X,0.16
260 D,ALL,TEMP,293
261 ALLSEL,ALL
262 NSEL,S,LOC,Y,0.15
263 D,ALL,TEMP,293
264 ALLSEL,ALL
265 DELTIM,15,5,40
266 OUTRES,ALL,ALL
267 SOLVE
268 SAVE
269 FINISH
270
271 ! 10. POST-PROCESSING
272
273 /POST1
274 SET, LAST
275 PLNSOL, TEMP
276 /DSCALE, 1, 1
277 PLNSOL, S, 1
278 FINISH
```

C.3 Experiment-Specific Model

The geometry reproduces the test assembly and is parameterised in inches, converted to metres at the top of the script. All regions use PLANE223 in axisymmetric mode, meshed with a uniform mapped mesh. The cooling curve is applied to the copper cold head only, which is selected by material number rather than by area. The script covers the full sequence of cool-down, one-hour cold hold and fast warm-up.

Listing C.2: Complete input file for the experiment-specific model, covering cool-down, cold hold and fast warm-up.

```

1  ! =====
2  ! Thesis   : Transient Thermo-Mechanical Modelling of Polyurethane Foam
3  !           Insulation for Non-Vacuum Liquid Hydrogen Storage
4  ! Author   : Gijb Schreuder, TU Delft
5  ! Model    : experiment-specific model (axisymmetric assembly)
6  ! Case      : Cool-down 2100 s + 1 h cold hold + fast warm-up (720 s, 20 -> 100 K)
7  ! Solver    : ANSYS Mechanical APDL, coupled-field transient (PLANE223)
8  ! Units     : SI (m, kg, s, K, Pa)
9  ! =====
10
11 /CLEAR
12 /PREP7
13
14 ! 1. ELEMENT TYPE
15
16 ET,1,PLANE223
17 KEYOPT,1,1,11
18 KEYOPT,1,3,1
19
20 ! 2. PARAMETERS
21
22 *SET,inch,0.0254
23
24 *SET,r_low,2.5*inch      ! steel/copper outer radius
25 *SET,r_al,3.5*inch      ! aluminum puck outer radius
26 *SET,r_foam,5.0*inch    ! foam outer radius
27
28 *SET,y0,0
29 *SET,yf,4.0*inch        ! foam starts here (bare steel below)
30 *SET,y1,11.0*inch       ! end of steel
31 *SET,y2,14.0*inch       ! end of copper
32 *SET,y3,14.5*inch       ! end of aluminum puck
33 *SET,y4,17.0*inch       ! end of model
34
35 *SET,tol,1e-6
36
37 ! 3. GEOMETRY
38
39 ! --- Metal parts ---
40 RECTNG,0,r_low,y0,yf    ! Stainless steel, bare section (Y:0-4")
41 RECTNG,0,r_low,yf,y1    ! Stainless steel, foam-covered section (Y:4-11")
42 RECTNG,0,r_low,y1,y2    ! Copper cold head (Y:11-14")
43 RECTNG,0,r_low,y2,y3    ! Aluminum puck, inner band (over copper)
44 RECTNG,r_low,r_al,y2,y3 ! Aluminum puck, outer band (over foam)
45
46 ! --- Foam parts ---
47 RECTNG,r_low,r_al,yf,y1 ! Foam beside steel, inner radial band (Y:4-11")

```

```

48 RECTNG,r_al,r_foam,yf,y1      ! Foam beside steel, outer radial band (Y:4-11")
49 RECTNG,r_low,r_al,y1,y2      ! Foam beside copper, inner radial band (Y:11-14")
50 RECTNG,r_al,r_foam,y1,y2      ! Foam beside copper, outer radial band (Y:11-14")
51 RECTNG,r_al,r_foam,y2,y3      ! Foam beside aluminum puck (Y:14-14.5")
52 RECTNG,0,r_low,y3,y4         ! Foam right of puck, inner band
53 RECTNG,r_low,r_al,y3,y4       ! Foam right of puck, middle band
54 RECTNG,r_al,r_foam,y3,y4      ! Foam right of puck, outer band
55
56 AGLUE,ALL
57 NUMMRG,ALL
58 ALLSEL,ALL
59
60
61 ! 4. MATERIALS
62
63 ! Foam properties
64 /INPUT,foam_material,mac      ! sets rho_f, k1-k11, cp1-cp11, E20, E293 and a1-a10
65
66 MP,DENS,1,rho_f
67 MP,PRXY,1,0.3
68
69 MPTEMP,,,,,,,,
70 MPTEMP,1,20,30,60,90,120,150
71 MPTEMP,7,180,210,240,270,300
72 MPDATA,KXX,1,1,k1,k2,k3,k4,k5,k6
73 MPDATA,KXX,1,7,k7,k8,k9,k10,k11
74 MPDATA,C,1,1,cp1,cp2,cp3,cp4,cp5,cp6
75 MPDATA,C,1,7,cp7,cp8,cp9,cp10,cp11
76
77 MPTEMP,,,,,,,,
78 MPTEMP,1,0,20,293,300
79 MPDATA,EX,1,1,E20,E20,E293,E293
80
81 MPTEMP,,,,,,,,
82 MPTEMP,1,0,20,77,100,150,200
83 MPTEMP,7,230,260,293,300
84 MPDATA,ALPX,1,1,a1,a2,a3,a4,a5,a6
85 MPDATA,ALPX,1,7,a7,a8,a9,a10
86
87 ! Steel
88
89 MP,DENS,2,7800
90 MP,PRXY,2,0.29
91 MP,KXX,2,15.12
92 MP,C,2,470.5
93
94 MPTEMP,,,,,,,,
95 MPTEMP,1,0,20,40,77,100,150
96 MPTEMP,7,200,250,293,300
97 MPDATA,EX,2,1,2.1010E+11,2.1010E+11,2.1170E+11,2.1440E+11,2.1370E+11,2.1090E+11
98 MPDATA,EX,2,7,2.0720E+11,2.0330E+11,1.9950E+11,1.9950E+11
99
100 MPTEMP,,,,,,,,
101 MPTEMP,1,0,20,40,77,100,150
102 MPTEMP,7,200,250,293,300
103 MPDATA,ALPX,2,1,1.0990E-05,1.0990E-05,1.1770E-05,1.2960E-05,1.3540E-05,1.4440E-05
104 MPDATA,ALPX,2,7,1.4960E-05,1.5230E-05,1.5510E-05,1.5510E-05
105

```

```

06 ! MATERIAL 3: OFHC COPPER, RRR = 100
07 MP,DENS,3,8940
08 MP,PRXY,3,0.343
09 MP,KXX,3,396.9
10 MP,C,3,389.1
11 MPTEMP,,,,,,,,
12 MPTEMP,1,20,293,300
13 MPDATA,EX,3,1,1.36949E11,1.26097E11,1.26097E11
14 MPDATA,ALPX,3,1,1.192E-5,1.65703E-5,1.65703E-5
15
16 ! MATERIAL 4: ALUMINIUM 6061-T6
17 MP,DENS,4,2700
18 MP,PRXY,4,0.33
19 MP,KXX,4,154.3
20 MP,C,4,942.9
21 MPTEMP,,,,,,,,
22 MPTEMP,1,20,293,300
23 MPDATA,EX,4,1,7.78084E10,7.03586E10,7.03586E10
24 MPDATA,ALPX,4,1,1.522E-5,2.26424E-5,2.26424E-5
25
26 TREF,293
27
28 ! 5. MESH - mapped
29
30 MSHKEY,1
31 MSHAPE,0,2D
32 ESHAPE,2
33
34 ALLSEL,ALL
35 LESIZE,ALL,0.25*inch
36 ALLSEL,ALL
37
38 ! Stainless steel
39 ASEL,S,LOC,X,0,r_low+tol
40 ASEL,R,LOC,Y,y0,y1-tol
41 AATT,2,1,1
42 AMESH,ALL
43 ALLSEL,ALL
44
45 ! Copper
46 ASEL,S,LOC,X,0,r_low+tol
47 ASEL,R,LOC,Y,y1+tol,y2-tol
48 AATT,3,1,1
49 AMESH,ALL
50 ALLSEL,ALL
51
52 ! Aluminum puck
53 ASEL,S,LOC,X,0,r_al+tol
54 ASEL,R,LOC,Y,y2+tol,y3-tol
55 AATT,4,1,1
56 AMESH,ALL
57 ALLSEL,ALL
58
59 ! Foam outside steel/copper
60 ASEL,S,LOC,X,r_low+tol,r_foam-tol
61 ASEL,R,LOC,Y,yf+tol,y2-tol
62 AATT,1,1,1
63 AMESH,ALL

```

```

64 ALLSEL,ALL
65
66 ! Foam outside aluminum puck
67 ASEL,S,LOC,X,r_al+tol,r_foam-tol
68 ASEL,R,LOC,Y,y2+tol,y3-tol
69 AATT,1,1,1
70 AMESH,ALL
71 ALLSEL,ALL
72
73 ! Foam right of puck
74 ASEL,S,LOC,X,0,r_foam-tol
75 ASEL,R,LOC,Y,y3+tol,y4-tol
76 AATT,1,1,1
77 AMESH,ALL
78 ALLSEL,ALL
79
80 ALLSEL,ALL
81
82 ! 6. INITIAL CONDITION
83
84 IC,ALL,TEMP,293
85 FINISH
86
87 ! 7. BOUNDARY CONDITIONS
88
89 /PREP7
90
91 ! Left face of the stainless steel (Y=0) held at room temperature
92 NSEL,S,LOC,Y,y0-tol,y0+tol
93 NSEL,R,LOC,X,0,r_low+tol
94 D,ALL,TEMP,293
95 ALLSEL,ALL
96
97 ! Outside surface of the foam held at room temperature
98 ! Outer cylindrical foam surface
99 NSEL,S,LOC,X,r_foam-tol,r_foam+tol
100 NSEL,R,LOC,Y,yf-tol,y4+tol
101 D,ALL,TEMP,293
102 ALLSEL,ALL
103
104 ! Right/end face of the foam held at room temperature
105 NSEL,S,LOC,Y,y4-tol,y4+tol
106 NSEL,R,LOC,X,0,r_foam+tol
107 D,ALL,TEMP,293
108 ALLSEL,ALL
109
110 ! Mechanical boundary condition for axisymmetric cylinder
111 ! Whole left side / symmetry axis of steel, copper, aluminum and foam
112 NSEL,S,LOC,X,0,tol
113 NSEL,R,LOC,Y,y0-tol,y4+tol
114 D,ALL,UX,0
115 ALLSEL,ALL
116
117 ! Bottom face of steel (Y=0) fully constrained in axial direction
118 ! per request: entire steel bottom face, not just one point
119 NSEL,S,LOC,Y,y0-tol,y0+tol
120 NSEL,R,LOC,X,0,r_low+tol
121 D,ALL,UY,0

```

```

222 ALLSEL,ALL
223
224 ! Bare steel outer surface (X=r_low, Y:0-4") held at 293K
225 ! per request: all exposed steel surfaces at room temperature
226 NSEL,S,LOC,X,r_low-tol,r_low+tol
227 NSEL,R,LOC,Y,y0-tol,yf+tol
228 D,ALL,TEMP,293
229 ALLSEL,ALL
230
231 ! Bottom face of foam (Y=yf=4", X:r_low to r_foam) held at 293K
232 ! per request: all exposed surfaces at room temperature
233 NSEL,S,LOC,Y,yf-tol,yf+tol
234 NSEL,R,LOC,X,r_low-tol,r_foam+tol
235 D,ALL,TEMP,293
236 ALLSEL,ALL
237
238 FINISH
239
240
241 ! 8. SINE-SQUARED COOL-DOWN CURVE FOR COPPER COLD HEAD ONLY
242
243 *SET,T_start,293.0
244 *SET,dT,273.0
245 *SET,t_cool,2100.0
246 *SET,N,50
247 *SET,pi,ACOS(-1.0)
248 *SET,omega,pi/2.0/t_cool
249
250 *DIM,TCOOL,TABLE,N,1,1,TIME
251
252 *DO,i,1,N
253     TCOOL(i,0,1) = (i-1.0)/(N-1.0)*t_cool
254 *ENDDO
255
256 *DO,i,1,N
257     *SET,t_i,(i-1.0)/(N-1.0)*t_cool
258     TCOOL(i,1,1) = T_start - dT*(SIN(omega*t_i))**2
259 *ENDDO
260
261
262 ! 9. SOLUTION
263
264 /SOLU
265 ANTYPE,4
266 TRNOPT,FULL
267 AUTOTS,ON
268 NROPT,FULL
269 NEQIT,80
270 KBC,0
271 CNVTOL,HEAT,1,0.001,2,1e-3
272
273 ! Load Step 1a: cool-down curve on copper only (0 -> 600 s)
274
275 TIME,600
276 DELTIM,5,1,10
277
278 ESEL,S,MAT,,3
279 NSLE,S

```



```

280 D,ALL,TEMP,%TCOOL%
281 ALLSEL,ALL
282
283 OUTRES,ALL,ALL
284 SOLVE
285
286 ! Load Step 1b: continue same cool-down curve (600 -> 1500 s)
287
288 TIME,1500
289 DELTIM,2,0.5,5
290
291 ! Copper temperature table remains active
292 OUTRES,ALL,ALL
293 SOLVE
294
295 ! Load Step 1c: continue same cool-down curve (1500 -> 2100 s)
296
297 TIME,2100
298 DELTIM,5,1,10
299
300 OUTRES,ALL,ALL
301 SOLVE
302
303 ! Load Step 2: hold copper at 20 K (2100 -> 5700 s)
304
305 TIME,5700
306
307 ESEL,S,MAT,,3
308 NSLE,S
309 D,ALL,TEMP,20
310 ALLSEL,ALL
311
312 DELTIM,15,5,40
313 OUTRES,ALL,ALL
314 SOLVE
315
316 ! SINE-SQUARED HEATING CURVE
317
318 *SET,T_start_warm,20.0
319 *SET,dT_warm,80.0
320 *SET,t_warm,720.0
321 *SET,t_warm_begin,5700.0
322 *SET,Nw,50
323 *SET,omega_warm,pi/2.0/t_warm
324 *DIM,TWARM,TABLE,Nw,1,1,TIME
325 *DO,i,1,Nw
326     TWARM(i,0,1) = t_warm_begin + (i-1.0)/(Nw-1.0)*t_warm
327 *ENDDO
328 *DO,i,1,Nw
329     *SET,t_i,(i-1.0)/(Nw-1.0)*t_warm
330     TWARM(i,1,1) = T_start_warm + dT_warm*(sin(omega_warm*t_i))**2
331 *ENDDO
332
333 ! Load Step 3: Fast warm-up
334
335 TIME,6420
336 DELTIM,5,1,10
337

```

```

$38 ESEL,S,MAT,,3
$39 NSLE,S
$40 D,ALL,TEMP,%TARM%
$41 ALLSEL,ALL
$42
$43 OUTRES,ALL,ALL
$44 SOLVE
$45
$46 SAVE
$47 FINISH
$48
$49 ! POST
$50
$51 /POST1
$52 SET, LAST
$53 PLNSOL,TEMP
$54 /DSCALE,1,1
$55 PLNSOL,S,1
$56 FINISH

```

C.4 Spherical Tank Model

Because the geometry is annular, radial node selection is carried out in a cylindrical coordinate system (CSYS,1), with the selection tolerance tightened accordingly.

Listing C.3: Complete input file for the spherical tank model.

```

1  ! =====
2  ! Thesis   : Transient Thermo-Mechanical Modelling of Polyurethane Foam
3  !          : Insulation for Non-Vacuum Liquid Hydrogen Storage
4  ! Author   : Gijs Schreuder, TU Delft
5  ! Model    : Spherical tank model (foam and perlite, axisymmetric)
6  ! Case     : Baseline cool-down, 293 K -> 20 K in 2100 s, followed by cold hold
7  ! Solver   : ANSYS Mechanical APDL, coupled-field transient
8  !          : PLANE223 for steel and foam, PLANE55 for the perlite annulus
9  ! Units    : SI (m, kg, s, K, Pa)
10 ! =====
11
12 /CLEAR
13 FINISH
14 /PREP7
15
16 ! 1. PARAMETERS
17 ! Geometry parameters for the spherical tank configuration.
18 *SET,rr1,<inner_steel_radius> ! inner steel radius [m]
19 *SET,steel_t,<steel_t_model> ! ARTIFICIAL steel wall thickness [m] (see Section 3.7)
20 *SET,tfoam,<foam_thickness> ! foam thickness [m]
21 *SET,tperlite,<perlite_thickness> ! perlite thickness [m]
22
23 ! Mesh settings
24 *SET,ntheta,240 ! circumferential divisions over half circle
25 *SET,nsteel,12 ! radial divisions through steel wall
26 *SET,nfoam,36 ! radial divisions through foam
27 *SET,nperlite,24 ! radial divisions through perlite
28

```

```

29 ! 2. ELEMENT TYPES
30
31 ET,1,PLANE223
32 KEYOPT,1,1,11
33 KEYOPT,1,3,1
34
35 ET,2,PLANE55          ! Thermal-only element for the perlite: conducts heat, carries
    no load
36 KEYOPT,2,3,1
37
38 ! 3. KEYPOINTS
39
40 K,100,0,0,0
41 ! Inner steel radius rr1
42 K,11,0, rr1,0
43 K,12,rr1,0,0
44 K,13,0,-rr1,0
45 ! Outer steel radius / steel-foam interface rr2
46 K,21,0, rr2,0
47 K,22,rr2,0,0
48 K,23,0,-rr2,0
49 ! Outer foam radius / foam-perlite interface rr3
50 K,31,0, rr3,0
51 K,32,rr3,0,0
52 K,33,0,-rr3,0
53 ! Outer perlite radius rr4
54 K,41,0, rr4,0
55 K,42,rr4,0,0
56 K,43,0,-rr4,0
57
58 ! 4. LINES
59
60 ! Inner steel boundary
61 LARC,11,12,100,rr1
62 *GET,L_IN1,LINE,0,NUM,MAX
63 LARC,12,13,100,rr1
64 *GET,L_IN2,LINE,0,NUM,MAX
65
66 ! Steel-foam interface
67 LARC,21,22,100,rr2
68 *GET,L_IF1,LINE,0,NUM,MAX
69 LARC,22,23,100,rr2
70 *GET,L_IF2,LINE,0,NUM,MAX
71
72 ! Foam-perlite interface
73 LARC,31,32,100,rr3
74 *GET,L_FP1,LINE,0,NUM,MAX
75 LARC,32,33,100,rr3
76 *GET,L_FP2,LINE,0,NUM,MAX
77
78 ! Outer perlite boundary
79 LARC,41,42,100,rr4
80 *GET,L_OUT1,LINE,0,NUM,MAX
81 LARC,42,43,100,rr4
82 *GET,L_OUT2,LINE,0,NUM,MAX
83
84 ! Radial closing lines for steel
85 L,11,21

```

```

86 *GET,L_TOP_STEEL,LINE,0,NUM,MAX
87 L,13,23
88 *GET,L_BOT_STEEL,LINE,0,NUM,MAX
89
90 ! Radial closing lines for foam
91 L,21,31
92 *GET,L_TOP_FOAM,LINE,0,NUM,MAX
93 L,23,33
94 *GET,L_BOT_FOAM,LINE,0,NUM,MAX
95
96 ! Radial closing lines for perlite
97 L,31,41
98 *GET,L_TOP_PERLITE,LINE,0,NUM,MAX
99 L,33,43
100 *GET,L_BOT_PERLITE,LINE,0,NUM,MAX
101
102 ! 5. AREAS
103
104 ! Steel half-annulus
105 AL,L_TOP_STEEL,L_IF1,L_IF2,-L_BOT_STEEL,-L_IN2,-L_IN1
106 *GET,A_STEEL,AREA,0,NUM,MAX
107
108 ! Foam half-annulus
109 AL,L_TOP_FOAM,L_FP1,L_FP2,-L_BOT_FOAM,-L_IF2,-L_IF1
110 *GET,A_FOAM,AREA,0,NUM,MAX
111
112 ! Perlite half-annulus
113 AL,L_TOP_PERLITE,L_OUT1,L_OUT2,-L_BOT_PERLITE,-L_FP2,-L_FP1
114 *GET,A_PERLITE,AREA,0,NUM,MAX
115 NUMMRG,ALL
116 ALLSEL,ALL
117
118 ! 6. MATERIALS
119
120 ! Foam properties
121 /INPUT,foam_material,mac ! sets rho_f, k1-k11, cp1-cp11, E20, E293 and a1-a10
122
123 MP,DENS,1,rho_f
124 MP,PRXY,1,0.3
125
126 MPTEMP,,,,,,,,
127 MPTEMP,1,20,30,60,90,120,150
128 MPTEMP,7,180,210,240,270,300
129 MPDATA,KXX,1,1,k1,k2,k3,k4,k5,k6
130 MPDATA,KXX,1,7,k7,k8,k9,k10,k11
131 MPDATA,C,1,1,cp1,cp2,cp3,cp4,cp5,cp6
132 MPDATA,C,1,7,cp7,cp8,cp9,cp10,cp11
133
134 MPTEMP,,,,,,,,
135 MPTEMP,1,0,20,293,300
136 MPDATA,EX,1,1,E20,E20,E293,E293
137
138 MPTEMP,,,,,,,,
139 MPTEMP,1,0,20,77,100,150,200
140 MPTEMP,7,230,260,293,300
141 MPDATA,ALPX,1,1,a1,a2,a3,a4,a5,a6
142 MPDATA,ALPX,1,7,a7,a8,a9,a10
143

```

```

44 ! Steel
45
46 MP,DENS,2,7800
47 MP,PRXY,2,0.29
48 MP,KXX,2,15.12
49 MP,C,2,470.5
50
51 MPTEMP,,,,,,,,
52 MPTEMP,1,0,20,40,77,100,150
53 MPTEMP,7,200,250,293,300
54 MPDATA,EX,2,1,2.1010E+11,2.1010E+11,2.1170E+11,2.1440E+11,2.1370E+11,2.1090E+11
55 MPDATA,EX,2,7,2.0720E+11,2.0330E+11,1.9950E+11,1.9950E+11
56
57 MPTEMP,,,,,,,,
58 MPTEMP,1,0,20,40,77,100,150
59 MPTEMP,7,200,250,293,300
60 MPDATA,ALPX,2,1,1.0990E-05,1.0990E-05,1.1770E-05,1.2960E-05,1.3540E-05,1.4440E-05
61 MPDATA,ALPX,2,7,1.4960E-05,1.5230E-05,1.5510E-05,1.5510E-05
62
63 TREF,293
64
65 ! Material 3: Perlite (no EX, PRXY or ALPX: perlite is modelled as a
66 ! mechanically passive fill)
67 MP,DENS,3,rho_p
68 MP,KXX,3,k_p
69 MP,C,3,cp_p
70
71 ! 7. MATERIAL ASSIGNMENT
72
73 ASEL,S,AREA,,A_STEEL
74 AATT,2,,1
75 ALLSEL,ALL
76 ASEL,S,AREA,,A_FOAM
77 AATT,1,,1
78 ALLSEL,ALL
79 ASEL,S,AREA,,A_PERLITE
80 AATT,3,,2 ! Perlite assigned element type 2 (PLANE55), no
      structural DOF
81 ALLSEL,ALL
82
83 ! 8. LINE CONCATENATION FOR MAPPED MESH
84
85 ! Inner steel arc
86 LSEL,S,LINE,,L_IN1
87 LSEL,A,LINE,,L_IN2
88 LCCAT,ALL
89 ALLSEL,ALL
90 ! Steel-foam interface arc
91 LSEL,S,LINE,,L_IF1
92 LSEL,A,LINE,,L_IF2
93 LCCAT,ALL
94 ALLSEL,ALL
95
96 ! Foam-perlite interface arc
97 LSEL,S,LINE,,L_FP1
98 LSEL,A,LINE,,L_FP2
99 LCCAT,ALL
200 ALLSEL,ALL

```

```
201
202 ! Outer perlite arc
203 LSEL,S,LINE,,L_OUT1
204 LSEL,A,LINE,,L_OUT2
205 LCCAT,ALL
206 ALLSEL,ALL
207
208 ! 9. LINE DIVISIONS
209
210 *SET,nhalf,ntheta/2
211 ! Circumferential divisions
212 LESIZE,L_IN1,,nhalf
213 LESIZE,L_IN2,,nhalf
214 LESIZE,L_IF1,,nhalf
215 LESIZE,L_IF2,,nhalf
216 LESIZE,L_FP1,,nhalf
217 LESIZE,L_FP2,,nhalf
218 LESIZE,L_OUT1,,nhalf
219 LESIZE,L_OUT2,,nhalf
220 ! Radial divisions through steel
221 LESIZE,L_TOP_STEEL,,nsteel
222 LESIZE,L_BOT_STEEL,,nsteel
223 ! Radial divisions through foam
224 LESIZE,L_TOP_FOAM,,nfoam
225 LESIZE,L_BOT_FOAM,,nfoam
226 ! Radial divisions through perlite
227 LESIZE,L_TOP_PERLITE,,nperlite
228 LESIZE,L_BOT_PERLITE,,nperlite
229 ALLSEL,ALL
230
231 ! 10. MAPPED MESH
232
233 MSHAPE,0,2D
234 MSHKEY,1
235 ESHAPE,2
236 ! Mesh steel
237 ASEL,S,AREA,,A_STEEL
238 AMESH,ALL
239 ALLSEL,ALL
240 ! Mesh foam
241 ASEL,S,AREA,,A_FOAM
242 AMESH,ALL
243 ALLSEL,ALL
244 ! Mesh perlite
245 ASEL,S,AREA,,A_PERLITE
246 AMESH,ALL
247 ALLSEL,ALL
248 NUMMRG,NODE
249 NUMMRG,ELEM
250 ALLSEL,ALL
251
252 ! 11. INITIAL CONDITION
253
254 IC,ALL,TEMP,293
255
256 ! 12. MECHANICAL BOUNDARY CONDITIONS (SYMMETRY)
257
258 NSEL,S,LOC,X,0
```

```

259 D,ALL,UX,0
260 ALLSEL,ALL
261
262 NSEL,S,LOC,X,0
263 NSEL,R,LOC,Y,-rr4
264 D,ALL,UY,0
265 ALLSEL,ALL
266
267
268 FINISH
269
270 ! 13. SINE-SQUARED COOLING CURVE
271
272 *SET,T_start,293.0
273 *SET,dT,273.0
274 *SET,t_cool,2100.0
275 *SET,N,50
276 *SET,pi,acos(-1.0)
277 *SET,omega,pi/2.0/t_cool
278 *DIM,TCOOL,TABLE,N,1,1,TIME
279 *DO,i,1,N
280     TCOOL(i,0,1) = (i-1.0)/(N-1.0)*t_cool
281 *ENDDO
282 *DO,i,1,N
283     *SET,t_i,(i-1.0)/(N-1.0)*t_cool
284     TCOOL(i,1,1) = T_start - dT*(sin(omega*t_i))**2
285 *ENDDO
286
287 ! 14. COUPLED THERMO-MECHANICAL TRANSIENT SOLUTION
288
289 /SOLU
290 ANTYPE,4
291 TRNOPT,FULL
292 AUTOTS,ON
293 NROPT,FULL
294 NEQIT,80
295 KBC,0
296 CNVTOL,HEAT,1,0.001,2,1e-3
297 SELTOL,1E-6                                ! Selection tolerance for the radial node
      selections below
298
299 ! --- Load Step 1a: 0 -> 600 s ---
300 TIME,600
301 DELTIM,5,1,10
302 CSYS,1                                ! Cylindrical system: X becomes the
      radial coordinate
303 NSEL,S,LOC,X,0,rr2
304 D,ALL,TEMP,%TCOOL%
305 ALLSEL,ALL
306
307 CSYS,1
308 NSEL,S,LOC,X,rr4
309 D,ALL,TEMP,293
310 ALLSEL,ALL
311 CSYS,0
312 OUTRES,ALL,ALL
313 SOLVE
314

```

```

$15 ! --- Load Step 1b: 600 -> 1500 s
$16 TIME,1500
$17 DELTIM,2,0.5,5
$18 OUTRES,ALL,ALL
$19 SOLVE
$20
$21 ! --- Load Step 1c: 1500 -> 2100 s
$22 TIME,2100
$23 DELTIM,5,1,10
$24 OUTRES,ALL,ALL
$25 SOLVE
$26
$27 ! --- Load Step 2: 2100 -> 5700 s
$28 TIME,5700
$29 CSYS,1
$30 NSEL,S,LOC,X,0,rr2
$31 D,ALL,TEMP,20
$32 ALLSEL,ALL
$33 CSYS,1
$34 NSEL,S,LOC,X,rr4
$35 D,ALL,TEMP,293
$36 ALLSEL,ALL
$37 CSYS,0
$38 DELTIM,15,5,40
$39 OUTRES,ALL,ALL
$40 SOLVE
$41 SAVE
$42 FINISH
$43
$44 ! 15. POST
$45
$46 /POST1
$47 SET, LAST
$48 PLNSOL, TEMP
$49 /DSCALE, 1, 1
$50 PLNSOL, S, 1
$51 PLNSOL, EPEL, 1
$52 FINISH

```

C.5 Thermal Load Variations

The cases in this section differ from the baseline only in the imposed temperature history and the associated load-step structure. Geometry, mesh, material data and solver settings are unchanged.

Cool-Down Rate

Three cool-down durations are compared: a fast cool-down of 1050 s, the 2100 s baseline and a slow cool-down of 4200 s. Only the duration of the sine-squared curve and the load-step boundaries change, the temperature range, the material data and the mechanical constraints are identical in all three cases. These cases were solved on mesh Level 4, which is appropriate because the reported quantities are path-based and are extracted away from the corner region.

Listing C.4: Thermal load definition and cool-down load steps for the cooling-rate cases.

```

1  ! =====
2  ! Variation : cool-down rate
3  ! Replaces : the thermal load curve and cool-down load steps in
4  !           mechanism_baseline.mac (Section C.2)
5  ! Cases    : fast (1050 s), baseline (2100 s), slow (4200 s)
6  ! Note     : the load-step boundaries scale proportionally with t_cool, so that
7  !           the finest stepping always covers the steepest part of the curve
8  ! =====
9
10 *SET,t_cool,2100.0      ! 1050.0 (fast) or 4200.0 (slow)
11
12 *SET,T_start,293.0
13 *SET,dT,273.0
14 *SET,N,50
15 *SET,pi,acos(-1.0)
16 *SET,omega,pi/2.0/t_cool
17 *DIM,TCOOL,TABLE,N,1,1,TIME
18 *DO,i,1,N
19     TCOOL(i,0,1) = (i-1.0)/(N-1.0)*t_cool
20 *ENDDO
21 *DO,i,1,N
22     *SET,t_i,(i-1.0)/(N-1.0)*t_cool
23     TCOOL(i,1,1) = T_start - dT*(sin(omega*t_i))**2
24 *ENDDO
25
26 ! --- cool-down load steps, boundaries at 2/7, 5/7 and 7/7 of t_cool ---
27 TIME,2.0*t_cool/7.0
28 DELTIM,5,1,10
29 TIME,5.0*t_cool/7.0
30 DELTIM,2,0.5,5
31 TIME,t_cool
32 DELTIM,5,1,10

```

Table C.2: Cool-down durations and load-step boundaries for the cooling-rate cases.

Case	t_cool [s]	Load-step boundaries [s]
Fast	1050	300 / 750 / 1050
Baseline	2100	600 / 1500 / 2100
Slow	4200	1200 / 3000 / 4200

Stepwise Cool-Down

The stepwise protocol reaches 20 K through three sine-squared cooling segments of 700 s each, separated by two isothermal holds of 120 s at 224.75 K and 88.25 K, followed by the final hold at 20 K. The two intermediate hold temperatures are not chosen arbitrarily: they are the smooth baseline curve evaluated at $t = 700$ s and $t = 1400$ s, so that the stepwise and smooth protocols pass through the same temperatures after the same cumulative cooling time. This is what makes the comparison at matched steel temperatures a controlled one. The case was solved on mesh Level 2.

Each cooling segment uses its own table with its own start and end temperature and an absolute

time offset. The six load steps are: cooling segment 1 ($0 \rightarrow 700$ s), hold 1 ($700 \rightarrow 820$ s), cooling segment 2 ($820 \rightarrow 1520$ s), hold 2 ($1520 \rightarrow 1640$ s), cooling segment 3 ($1640 \rightarrow 2340$ s) and the final hold at 20 K ($2340 \rightarrow 5240$ s).

Listing C.5: Thermal load definition and load-step sequence for the stepwise cool-down protocol.

```

1  ! =====
2  ! Variation : stepwise cool-down protocol
3  ! Replaces  : the thermal load curve and the cool-down load steps in
4  !             mechanism_baseline.mac (Section C.2)
5  ! Case      : three sine-squared cooling segments of 700 s, separated by two
6  !             isothermal holds of 120 s, followed by the final hold at 20 K
7  ! Note      : the intermediate hold temperatures, 224.75 K and 88.25 K, are the
8  !             smooth baseline curve evaluated at  $t = 700$  s and  $t = 1400$  s, so
9  !             both protocols pass through the same temperatures after the same
10 !             cumulative cooling time
11 ! Mesh      : Level 2
12 ! =====
13
14 *SET,pi,acos(-1.0)
15 *SET,N,50
16
17 ! --- Segment 1: 293 K -> 224.75 K over 700 s ( $t = 0$  to 700) ---
18 *SET,T_start1,293.0
19 *SET,T_end1,224.75
20 *SET,dT1,T_start1-T_end1
21 *SET,t_seg1,700.0
22 *SET,omega1,pi/2.0/t_seg1
23 *DIM,TCOOL1,TABLE,N,1,1,TIME
24 *DO,i,1,N
25     TCOOL1(i,0,1) = (i-1.0)/(N-1.0)*t_seg1
26 *ENDDO
27 *DO,i,1,N
28     *SET,t_i,(i-1.0)/(N-1.0)*t_seg1
29     TCOOL1(i,1,1) = T_start1 - dT1*(sin(omega1*t_i))**2
30 *ENDDO
31
32 ! --- Segment 2: 224.75 K -> 88.25 K over 700 s ( $t = 820$  to 1520) ---
33 ! --- Segment 3: 88.25 K -> 20 K over 700 s ( $t = 1640$  to 2340) ---
34 ! (identical construction, with t_seg2_begin = 820.0 and t_seg3_begin = 1640.0)
35
36 ! --- Load-step sequence ---
37 TIME,700      ! LS1: cooling segment 1          DELTIM,5,1,10
38 TIME,820      ! LS2: hold at 224.75 K          DELTIM,2,0.5,5
39 TIME,1520     ! LS3: cooling segment 2          DELTIM,5,1,10
40 TIME,1640     ! LS4: hold at 88.25 K           DELTIM,2,0.5,5
41 TIME,2340     ! LS5: cooling segment 3          DELTIM,5,1,10
42 TIME,5240     ! LS6: final hold at 20 K        DELTIM,15,5,40

```

Warm-Up

Each warm-up case begins from the cooled state after a one-hour cold hold at 20 K. The warm-up itself is a sine-squared increase from 20 K to 100 K, applied over either 3420 s (slow, passive) or 720 s (fast). The block below is appended after the cold-hold load step of the mechanism model.

Listing C.6: Warm-up curve definition and load step for the mechanism model.

```

1  ! =====
2  ! Variation : warm-up rate
3  ! Replaces : nothing; appended after the cold hold in mechanism_baseline.mac
4  !           (Section C.2), which is extended to t = 5700 s
5  ! Cases    : slow passive warm-up (3420 s) and fast warm-up (720 s),
6  !           both from 20 K to 100 K
7  ! Mesh     : Level 2
8  ! =====
9
10 *SET,t_warm,3420.0      ! 720.0 for the fast warm-up
11 *SET,t_warm_begin,5700.0 ! end of the one-hour cold hold
12 *SET,T_start_warm,20.0
13 *SET,dT_warm,80.0
14 *SET,Nw,50
15 *SET,omega_warm,pi/2.0/t_warm
16
17 *DIM,TWARM,TABLE,Nw,1,1,TIME
18 *DO,i,1,Nw
19     TWARM(i,0,1) = t_warm_begin + (i-1.0)/(Nw-1.0)*t_warm
20 *ENDDO
21 *DO,i,1,Nw
22     *SET,t_i,(i-1.0)/(Nw-1.0)*t_warm
23     TWARM(i,1,1) = T_start_warm + dT_warm*(sin(omega_warm*t_i))**2
24 *ENDDO
25
26 ! --- Load Step 3: warm-up ---
27 TIME,t_warm_begin+t_warm      ! 9120 s (slow) or 6420 s (fast)
28 DELTIM,20,5,40                ! 5,1,10 for the fast warm-up
29 ASEL,S,AREA,,1
30 NSLA,S,1
31 D,ALL,TEMP,%TWARM%
32 ALLSEL,ALL
33 NSEL,S,LOC,X,0.16
34 D,ALL,TEMP,293
35 ALLSEL,ALL
36 NSEL,S,LOC,Y,0.15
37 D,ALL,TEMP,293
38 ALLSEL,ALL
39 OUTRES,ALL,ALL
40 SOLVE

```

Table C.3: Warm-up durations, end times and time-step settings.

Case	t_warm [s]	End time [s]	DELTIM
Slow (passive)	3420	9120	20 / 5 / 40
Fast	720	6420	5 / 1 / 10

In the experiment-specific model the warm-up is identical in construction, but the cold region is selected by material number rather than by area, and the ambient boundary conditions applied during preprocessing remain active and are not reapplied.

Listing C.7: Warm-up load step for the experiment-specific model. The warm-up curve definition is identical to Listing C.6.

```

1  ! =====

```

```

2 ! Variation : warm-up rate, experiment-specific model
3 ! Replaces : Load Step 3 in experiment_model.mac (Section C.3)
4 ! Cases : fast warm-up (720 s) and slow passive warm-up (3420 s)
5 ! Note : the cold region is selected by material (copper cold head) rather
6 !       than by area; the ambient boundary conditions applied in PREP7
7 !       remain active and are not reapplied
8 ! =====
9
10 *SET,t_warm,3420.0 ! 720.0 for the fast warm-up
11 *SET,t_warm_begin,5700.0
12
13 ! --- Load Step 3: warm-up ---
14 TIME,t_warm_begin+t_warm ! 9120 s (slow) or 6420 s (fast)
15 DELTIM,20,5,40 ! 5,1,10 for the fast warm-up
16
17 ESEL,S,MAT,,3 ! copper cold head
18 NSLE,S
19 D,ALL,TEMP,%TWARM%
20 ALLSEL,ALL
21
22 OUTRES,ALL,ALL
23 SOLVE

```

Long Cold Hold

To separate the transient response from the residual cold state, the baseline cool-down is followed by an extended hold at 20 K. The solution is run to $t = 15000$ s. The profiles reported in Section 3.11 are extracted up to $t = 11000$ s. This case was solved on mesh Level 4.

Listing C.8: Extended cold-hold load step for the near-steady-state check.

```

1 ! =====
2 ! Variation : long cold hold (near-steady-state check)
3 ! Replaces : the cold-hold load step in mechanism_baseline.mac (Section C.2)
4 ! Purpose : verify that the through-thickness gradient and the transient
5 !           peaks decay towards a lower, near-uniform steady state
6 ! Mesh : Level 4
7 ! Note : the solution is run to t = 15000 s; profiles are reported up to
8 !       t = 11000 s in Section 3.11.3
9 ! =====
10
11 ! --- Load Step 2: extended cold hold (2100 -> 15000 s) ---
12 TIME,15000
13 ASEL,S,AREA,,1
14 NSLA,S,1
15 D,ALL,TEMP,20
16 ALLSEL,ALL
17 NSEL,S,LOC,X,0.16
18 D,ALL,TEMP,293
19 ALLSEL,ALL
20 NSEL,S,LOC,Y,0.15
21 D,ALL,TEMP,293
22 ALLSEL,ALL
23 DELTIM,20,5,60
24 OUTRES,ALL,ALL
25 SOLVE

```

Time-Step Schemes

Three time-stepping schemes are compared: the reference scheme, in which the cool-down is divided into three load steps with the finest stepping over the steepest temperature interval, a coarser scheme with the same load-step structure but larger steps throughout and a single-step scheme in which the entire cool-down is applied as one load step. The cold hold that follows is identical in all three cases. These cases were solved on mesh Level 4.

Listing C.9: Load-step structure and time-step settings for the three time-stepping schemes.

```

1  ! =====
2  ! Variation : time-step sensitivity
3  ! Replaces  : the DELTIM settings and load-step structure of the cool-down in
4  !             mechanism_baseline.mac (Section C.2)
5  ! Cases     : reference, coarser stepping, single load step
6  ! Mesh      : Level 4
7  ! Note      : the cold hold (2100 -> 5000 s, DELTIM,15,5,40) is unchanged in all
8  !             three cases
9  ! -----
10 ! Reference scheme: three load steps, finest stepping over the steep interval
11 ! -----
12 TIME,600
13 DELTIM,5,1,10
14 TIME,1500
15 DELTIM,2,0.5,5
16 TIME,2100
17 DELTIM,5,1,10
18
19 ! -----
20 ! Coarser scheme: same load-step structure, larger steps throughout
21 ! -----
22 TIME,600
23 DELTIM,20,5,40
24 TIME,1500
25 DELTIM,10,2,30
26 TIME,2100
27 DELTIM,20,5,40
28
29 ! -----
30 ! Single-step scheme: the entire cool-down applied as one load step
31 ! -----
32 TIME,2100
33 DELTIM,20,5,60

```

C.6 Material Property Variations

The cases in this section vary a single foam material property while leaving the geometry, mesh, boundary conditions, thermal load and solver settings unchanged. All were solved on mesh Level 4.

Coefficient of Thermal Expansion

The dependence on the foam-steel expansion mismatch is examined by scaling the tabulated foam coefficient of thermal expansion to $0.5\times$, $1.5\times$ and $2.0\times$ its reference value. Every tabulated value is multiplied by the same factor, so the shape of the temperature dependence is preserved and only its magnitude changes.

Listing C.10: Foam thermal expansion block for the CTE scaling cases.

```

1  ! =====
2  ! Variation : foam coefficient of thermal expansion (CTE) scaling
3  ! Replaces  : the foam ALPX block in mechanism_baseline.mac (Section C.2)
4  ! Cases     : CTE 0.5x, 1.5x, 2.0x
5  ! Mesh      : Level 4 (path-based outputs away from the corner, Section 3.11.1)
6  ! =====
7
8  ! Replaces : the foam ALPX block in foam_material.mac (Section C.2)
9  ...
10 *SET,CTEFAC,2.0    ! 0.5, 1.5 or 2.0 for the three scaling cases
11
12 MPTEMP,,,,,,,,
13 MPTEMP,1,0,20,77,100,150,200
14 MPTEMP,7,230,260,293,300
15 MPDATA,ALPX,1,1,CTEFAC*a1,CTEFAC*a2,CTEFAC*a3,CTEFAC*a4,CTEFAC*a5,CTEFAC*a6
16 MPDATA,ALPX,1,7,CTEFAC*a7,CTEFAC*a8,CTEFAC*a9,CTEFAC*a10

```

A separate diagnostic case removes the mismatch entirely by assigning the foam the thermal expansion curve of the steel. This is not a scaling of the reference table but a substitution, and is therefore listed separately.

Listing C.11: Foam thermal expansion block for the equal-CTE diagnostic case, in which the foam is assigned the steel expansion curve.

```

1  ! =====
2  ! Variation : equal-CTE diagnostic case
3  ! Replaces  : the foam ALPX block in mechanism_baseline.mac (Section C.2)
4  ! Purpose   : removes the foam-steel expansion mismatch entirely by assigning
5  !             the foam the CTE curve of the steel
6  ! =====
7
8  ! Replaces : the foam ALPX block in foam_material.mac (Section C.2)
9  ...
10 *SET,CTEFAC,2.0    ! 0.5, 1.5 or 2.0 for the three scaling cases
11
12 MPTEMP,,,,,,,,
13 MPTEMP,1,0,20,77,100,150,200
14 MPTEMP,7,230,260,293,300
15 MPDATA,ALPX,1,1,CTEFAC*a1,CTEFAC*a2,CTEFAC*a3,CTEFAC*a4,CTEFAC*a5,CTEFAC*a6
16 MPDATA,ALPX,1,7,CTEFAC*a7,CTEFAC*a8,CTEFAC*a9,CTEFAC*a10

```

Young's Modulus

The role of the foam stiffness is examined by scaling the tabulated foam Young's modulus to $0.5\times$, $1.5\times$ and $2.0\times$ its reference value, in the same way as for the expansion coefficient. The thermal expansion coefficient is held at its reference values in these cases.

Listing C.12: Foam Young's modulus block for the stiffness scaling cases.

```

1  ! =====
2  ! Variation : foam Young's modulus scaling
3  ! Replaces : the foam EX block in mechanism_baseline.mac (Section C.2)
4  ! Cases      : E 0.5x, 1.5x, 2.0x
5  ! Mesh       : Level 4 (path-based outputs away from the corner, Section 3.11.1)
6  ! =====
7
8  ! Replaces : the foam EX block in foam_material.mac (Section C.2)
9  ...
10 *SET,EFAC,2.0      ! 0.5, 1.5 or 2.0 for the three scaling cases
11
12 MPTEMP,,,,,,,,
13 MPTEMP,1,0,20,293,300
14 MPDATA,EX,1,1,EFAC*E20,EFAC*E20,EFAC*E293,EFAC*E293

```

C.7 Mechanical Boundary Conditions

The influence of the mechanical constraints was assessed by comparing three support conditions. This comparison was performed on a simplified variant of the mechanism geometry with a sharp foam-steel corner and a uniform mapped mesh, rather than on the production model. This is appropriate because the assessment concerns the persistence and location of the internal strain feature along the lower radial path, which runs away from the corner, rather than the corner value itself. All three cases use the same geometry and mesh, so that only the mechanical constraints differ between them.

Listing C.13: Mechanical constraint definitions for the three boundary-condition cases.

```

1  ! =====
2  ! Variation : mechanical boundary-condition comparison
3  ! Replaces : the BOUNDARY CONDITIONS block in mechanism_baseline.mac (C.2)
4  ! Cases      : reference, No Bottom, No Boundary
5  ! Note       : run on a sharp-cornered variant of the mechanism geometry with a
6  !              uniform mapped mesh; see the note below
7  ! -----
8  ! Reference: foam and steel both constrained on the symmetry axis and the base
9  ! -----
10 NSEL,S,LOC,X,0      ! full left edge: steel and foam
11 D,ALL,UX,0
12 ALLSEL,ALL
13 NSEL,S,LOC,Y,0      ! full base: steel and foam
14 D,ALL,UY,0
15 ALLSEL,ALL
16
17 ! -----
18 ! No Bottom: axial constraint on the foam base removed, left edge retained
19 ! -----
20 NSEL,S,LOC,X,0      ! full left edge: steel and foam
21 D,ALL,UX,0
22 ALLSEL,ALL
23 NSEL,S,LOC,Y,0
24 NSEL,R,LOC,X,0,0.04 ! steel base only
25 D,ALL,UY,0
26 ALLSEL,ALL

```

```

27
28 ! -----
29 ! No Boundary: both foam constraints removed, minimal supports on the steel only
30 ! -----
31 NSEL,S,LOC,X,0
32 NSEL,R,LOC,Y,0,0.06           ! steel left edge only
33 D,ALL,UX,0
34 ALLSEL,ALL
35 NSEL,S,LOC,Y,0
36 NSEL,R,LOC,X,0,0.04           ! steel base only
37 D,ALL,UY,0
38 ALLSEL,ALL

```

C.8 Mesh Definitions

Mapped Mesh Levels

The mapped foam mesh is defined by three division counts: the number of divisions on the vertical interface line, the number on each half of the corner arc, and the number on the horizontal interface line. Their sum gives the number of divisions on each outer foam boundary, so that the concatenated interface and the opposite outer boundary always match and the foam region can be mapped as a single four-sided area. The seven refinement levels are obtained by varying these three numbers. The resulting element counts are those listed in Table C.4, and correspond to the foam region only.

Listing C.14: Parameterised line divisions and steel mesh settings for the mapped mesh levels.

```

1 ! =====
2 ! Variation : mapped foam mesh refinement levels
3 ! Replaces  : the LINE DIVISIONS and STEEL MESH blocks in
4 !             mechanism_baseline.mac (Section C.2)
5 ! Cases     : mesh Levels 1 to 7 (Level 1 finest, Level 7 coarsest)
6 ! Note      : the level is set through NI, NA and NT below; see Table 3.8
7 ! =====
8
9 *SET,NI,48           ! divisions on the vertical interface line
10 *SET,NA,4            ! divisions on each half of the corner arc
11 *SET,NT,28           ! divisions on the horizontal interface line
12 *SET,NF,NI+2*NA+NT   ! resulting divisions on each foam outer boundary
13
14 ! Inner interface (concatenated into a single logical line)
15 LESIZE,2,,,NI,4
16 LESIZE,5,,,NA
17 LESIZE,6,,,NA
18 LESIZE,3,,,NT,4
19
20 ! Foam outer boundaries
21 LESIZE,7,,,NF,4
22 LESIZE,8,,,NF/2
23 LESIZE,9,,,NF/2
24 LESIZE,10,,,NF,4
25
26 ! Steel boundaries
27 LESIZE,1,,,NT

```



```

28 LESIZE,4,,NI
29
30 ! Steel region, free-meshed (see table for the element size per level)
31 ESIZE,0.00005
32 MSHKEY,0
33 ASEL,S,AREA,,1
34 AMESH,ALL
35 ALLSEL,ALL

```

Table C.4: Division counts and foam element counts for the seven mapped mesh levels. NI is the number of divisions on the vertical interface line, NA the number on each half of the corner arc, NT the number on the horizontal interface line, and NF the resulting number on each outer foam boundary.

Level	NI	NA	NT	NF	Foam elements
1	60	4	36	104	10 816
2	48	4	28	84	7 056
3	36	4	22	66	4 356
4	24	2	14	42	1 764
5	16	2	10	30	900
6	10	2	6	20	400
7	6	2	4	14	196

The steel region is free-meshed separately and does not form part of the convergence ladder. Its element size was varied between runs and the values used are given in Table C.5.

Table C.5: Steel element size used with each mapped mesh level.

Level	Steel ESIZE [m]
1	0.00005
2	0.00005
3	0.0005
4	0.0005
5	0.0005
6	0.0005
7	0.0005

Free-Mesh Variants

In addition to the mapped mesh ladder, two free-meshing strategies were included in the convergence comparison. The first uses the ANSYS smart-size algorithm alone, at levels 5 (coarsest) through 1 (finest). The second combines the finest smart-size setting with a prescribed global element size on the foam, giving two further variants. In all free-mesh cases both regions are free-meshed, no line concatenation is applied, and the element edge length on each half of the corner arc is prescribed directly.

These smart-size levels are a separate refinement family and are independent of the mapped mesh Levels 1–7 of Table C.4.

Listing C.15: Meshing block for the free-mesh variants used in the convergence comparison.

```

1  ! =====
2  ! Variation : free-mesh strategies for the mesh-convergence comparison
3  ! Replaces  : the LINE DIVISIONS, FOAM CONCATENATION and MAPPED MESH blocks in
4  !             mechanism_baseline.mac (Section C.2)
5  ! Cases     : SMRTSIZE 5 to 1 (five variants), and SMRTSIZE,1 combined with a
6  !             prescribed global element size (two variants)
7  ! Note      : both regions are free-meshed (MSHKEY,0); no line concatenation is
8  !             applied. These are the smart-size levels of Figure 3.11 and are
9  !             independent of the mapped mesh Levels 1-7 of Table 3.8.
10 ! =====
11
12 *SET,SS,1                ! smart-size level: 5 (coarsest) to 1 (finest)
13
14 ! --- Steel ---
15 MSHKEY,0
16 MSHAPE,0,2D
17 SMRTSIZE,SS
18 LESIZE,5,R/4             ! element edge length on the lower arc half
19 LESIZE,6,R/4             ! element edge length on the upper arc half
20 ASEL,S,AREA,,1
21 AMESH,ALL
22 ALLSEL,ALL
23
24 ! --- Foam ---
25 MSHKEY,0
26 SMRTSIZE,SS
27 ! ESIZE,0.0015           ! ESIZE variant 1: smart-size 1 plus prescribed size
28 ! ESIZE,0.002            ! ESIZE variant 2
29 ASEL,S,AREA,,2
30 AMESH,ALL
31 ALLSEL,ALL

```

Table C.6: Free-mesh variants used in the convergence comparison.

Variant	SMRTSIZE	Foam ESIZE [m]
Smart-size only	5, 4, 3, 2, 1	—
Smart-size with prescribed size, variant 1	1	0.0015
Smart-size with prescribed size, variant 2	1	0.002

C.9 Post-Processing Procedure

The post-processing was carried out interactively in the general postprocessor rather than through a batch script. The command sequences below are reproduced from the ANSYS session logs and show the operations that were applied to each solved case. Between cases only the result file, the time instants and the path change. Comments have been added for readability.

Path Definition and Extraction

Before each path is defined, the steel is deselected so that only foam elements and their nodes remain active. This prevents values from being mapped across the material discontinuity at the interface. The path itself is defined between two picked nodes and divided into 60 intervals. Because node numbers differ between meshes, the reproducible specification of each path is the pair of coordinates given in Table 3.9, not the node numbers.

Each quantity of interest is then mapped onto the path with PDEF at a given result time, using averaged nodal values. PLPATH plots the mapped quantities along the path, and PRPATH prints their values, which were transferred to excel for plotting and comparison.

Listing C.16: Path definition and quantity extraction, reproduced from the ANSYS session log.

```

1  /POST1
2  SET, LAST
3
4  ! --- select the foam only, so no values are mapped across the interface ---
5  ESEL, S, MAT, , 1
6  NSLE, S
7  EPLOT
8  NPLOT
9  NLIST                      ! identify the node numbers at the path end points
10
11 ! --- path between two picked nodes, 60 divisions ---
12 FLST, 2, 2, 1
13 FITEM, 2, 102              ! start node
14 FITEM, 2, 4979            ! end node
15 PATH, test1, 2, 60, 60,
16 PPATH, P51X, 1
17 PATH, STAT
18
19 ! --- map quantities onto the path at a given time instant ---
20 SET, , , 1, , 1000, ,      ! repeated for 100, 500, 1000, 2000, 4000, 5000 s
21 AVPRIN, 0, ,
22 PDEF, EPEL11, EPEL, 1, AVG  ! first principal elastic strain
23 PDEF, EPELX1, EPEL, X, AVG  ! radial component
24 PDEF, EPELY1, EPEL, Y, AVG  ! axial component
25 PDEF, EPELZ1, EPEL, Z, AVG  ! hoop component
26 PDEF, EPELXY1, EPEL, XY, AVG ! in-plane shear
27 PDEF, EPTHX1, EPTH, X, AVG  ! thermal strain, radial
28 PDEF, EPTHY1, EPTH, Y, AVG  ! thermal strain, axial
29 PDEF, ss1000, S, 1, AVG     ! first principal stress
30
31 PLPATH, EPEL11              ! plot the mapped quantities along the path
32 PRPATH, EPEL11, EPELX1, EPELY1, EPELZ1, EPELXY1 ! print the values

```

Nodal Maxima and Mesh Statistics

The maximum foam stress and elastic strain were obtained by listing the nodal principal values over the selected foam region at successive time instants and reading the maximum from the resulting list. The element and node counts reported for the mesh convergence study were obtained in the same way, by selecting the foam elements and their attached nodes and retrieving the counts directly.

Listing C.17: Nodal maximum listing and foam mesh statistics, reproduced from the ANSYS session log.

```

1  ! --- foam nodal maximum at a given time instant ---
2  ESEL,S,MAT,,1
3  NSLE,S
4  SET, , ,1, ,2000, ,
5  PRNSOL,S,PRIN                ! lists nodal principal stresses; maximum read from list
6  PRNSOL,EPEL,PRIN            ! lists nodal principal elastic strains
7
8  ! --- element and node counts for the mesh-convergence table ---
9  ESEL,S,MAT,,1
10 *GET,N_FOAM,ELEM,,COUNT
11 *STATUS,N_FOAM
12 NSLE,S
13 *GET,N_FOAM_NODES,NODE,,COUNT
14 *STATUS,N_FOAM_NODES

```

Principal-Direction Vector Plots

The principal stress and principal elastic strain directions were visualised with element-based vector plots at successive time instants. Because a dense mesh produces overlapping and unreadable arrows, these plots were generated on a separate coarse model with a sharp foam-steel corner. They are therefore interpreted for direction only and not for magnitude. The same arrow scaling was applied at every time instant so that the plots can be compared directly with one another.

Listing C.18: Principal-direction vector plotting, reproduced from the ANSYS session log.

```

1  /POST1
2  ESEL,S,MAT,,1
3  NSLE,S
4  SET, , ,1, ,200, ,          ! also 1000 s and 2000 s
5  /VSCALE,1,1.5,0             ! uniform arrow scaling across all time instants
6  PLVECT,S, , , ,VECT,ELEM,ON,0 ! first principal stress directions
7  PLVECT,EPEL, , , ,VECT,ELEM,ON,0 ! first principal elastic strain directions

```

The coarse model used for these plots is listed below. It uses the same element type, material data, boundary conditions and thermal load as the mechanism model, but the geometry is built from four rectangular areas without a corner fillet, and the mapped mesh is deliberately coarse.

Listing C.19: Coarse sharp-corner model used to generate the principal-direction vector plots.

```

1
2  ! =====
3  ! Thesis   : Transient Thermo-Mechanical Modelling of Polyurethane Foam
4  !          : Insulation for Non-Vacuum Liquid Hydrogen Storage
5  ! Author   : Gijs Schreuder, TU Delft
6  ! Model    : Coarse sharp-corner variant of the mechanism model, axisymmetric
7  ! Case     : Baseline cool-down, 293 K -> 20 K in 2100 s, followed by cold hold
8  ! Solver   : ANSYS Mechanical APDL, coupled-field transient (PLANE223)
9  ! Units    : SI (m, kg, s, K, Pa)
10 ! =====
11
12 /CLEAR
13 FINISH
14 /PREP7

```

```
15
16 ! 1. ELEMENT TYPE
17
18 ET,1,PLANE223
19 KEYOPT,1,1,11
20 KEYOPT,1,3,1
21
22 ! 2. GEOMETRY
23
24 RECTNG,0,0.04,0,0.06
25 RECTNG,0.04,0.16,0,0.06
26 RECTNG,0.04,0.16,0.06,0.15
27 RECTNG,0,0.04,0.06,0.15
28 AGLUE,ALL
29 NUMMRG,ALL
30
31 ! 3. MATERIALS
32
33
34 ! Foam properties
35 MP,DENS,1,rho_f
36 MPTEMP,1,20,30,60,90,120,150
37 MPTEMP,7,180,210,240,270,300
38 MPDATA,KXX,1,1,k1,k2,k3,k4,k5,k6
39 MPDATA,KXX,1,7,k7,k8,k9,k10,k11
40 MPDATA,C,1,1,cp1,cp2,cp3,cp4,cp5,cp6
41 MPDATA,C,1,7,cp7,cp8,cp9,cp10,cp11
42
43 ! Steel
44 MP,DENS,2,7800
45 MP,PRXY,2,0.3
46 MP,KXX,2,15.12
47 MP,C,2,470.5
48 MPTEMP,,,,,,,,
49 MPTEMP,1,0,293,300
50 MPDATA,EX,2,1,210.25E9,199.5E9,198.9E9
51 MPDATA,ALPX,2,1,1.031E-5,1.551E-5,1.551E-5
52 TREF,293
53
54 ! 4. MATERIAL ASSIGNMENT
55
56 ! steel
57 ASEL,S,LOC,X,0.001,0.039
58 ASEL,R,LOC,Y,0.001,0.059
59 AATT,2,1,1
60 ALLSEL,ALL
61
62 ! foam
63 ASEL,S,LOC,X,0.001,0.159
64 ASEL,U,LOC,X,0.001,0.039
65 AATT,1,1,1
66 ALLSEL,ALL
67
68 ! 5. MAPPED MESH
69 MSHKEY,1
70 MSHAPE,0,2D
71 ESHAPE,2
72
```

```
73 ! Bottom edges, X:[0,0.04] (steel/foam-left block width)
74 LSEL,S,LOC,Y,0
75 LSEL,R,LOC,X,0,0.04
76 LESIZE,ALL,,,3
77 ALLSEL,ALL
78 LSEL,S,LOC,Y,0.06
79 LSEL,R,LOC,X,0,0.04
80 LESIZE,ALL,,,3
81 ALLSEL,ALL
82 LSEL,S,LOC,Y,0.15
83 LSEL,R,LOC,X,0,0.04
84 LESIZE,ALL,,,3
85 ALLSEL,ALL
86
87 ! Bottom edges, X:[0.04,0.16] (right block width)
88 LSEL,S,LOC,Y,0
89 LSEL,R,LOC,X,0.04,0.16
90 LESIZE,ALL,,,10
91 ALLSEL,ALL
92 LSEL,S,LOC,Y,0.06
93 LSEL,R,LOC,X,0.04,0.16
94 LESIZE,ALL,,,10
95 ALLSEL,ALL
96 LSEL,S,LOC,Y,0.15
97 LSEL,R,LOC,X,0.04,0.16
98 LESIZE,ALL,,,10
99 ALLSEL,ALL
100
101 ! Vertical edges, Y:[0,0.06] (lower block height)
102 LSEL,S,LOC,X,0
103 LSEL,R,LOC,Y,0,0.06
104 LESIZE,ALL,,,5
105 ALLSEL,ALL
106 LSEL,S,LOC,X,0.04
107 LSEL,R,LOC,Y,0,0.06
108 LESIZE,ALL,,,5
109 ALLSEL,ALL
110 LSEL,S,LOC,X,0.16
111 LSEL,R,LOC,Y,0,0.06
112 LESIZE,ALL,,,5
113 ALLSEL,ALL
114
115 ! Vertical edges, Y:[0.06,0.15] (upper block height)
116 LSEL,S,LOC,X,0
117 LSEL,R,LOC,Y,0.06,0.15
118 LESIZE,ALL,,,7
119 ALLSEL,ALL
120 LSEL,S,LOC,X,0.04
121 LSEL,R,LOC,Y,0.06,0.15
122 LESIZE,ALL,,,7
123 ALLSEL,ALL
124 LSEL,S,LOC,X,0.16
125 LSEL,R,LOC,Y,0.06,0.15
126 LESIZE,ALL,,,7
127 ALLSEL,ALL
128 AMESH,ALL
129 ALLSEL,ALL
130
```

```

31 ! 6. INITIAL CONDITION
32 IC,ALL,TEMP,293
33 FINISH
34
35
36 ! 7. BOUNDARY CONDITIONS
37
38 /PREP7
39 NSEL,S,LOC,X,0
40 D,ALL,UX,0
41 ALLSEL,ALL
42 NSEL,S,LOC,Y,0
43 D,ALL,UY,0
44 ALLSEL,ALL
45 ALLSEL,ALL
46 FINISH
47
48 ! SINE-SQUARED AFKOELCURVE
49
50 *SET,T_start,293.0
51 *SET,dT,273.0
52 *SET,t_cool,2100.0
53 *SET,N,50
54 *SET,pi,acos(-1.0)
55 *SET,omega,pi/2.0/t_cool
56 *DIM,TCOOL,TABLE,N,1,1,TIME
57 *DO,i,1,N
58     TCOOL(i,0,1) = (i-1.0)/(N-1.0)*t_cool
59 *ENDDO
60 *DO,i,1,N
61     *SET,t_i,(i-1.0)/(N-1.0)*t_cool
62     TCOOL(i,1,1) = T_start - dT*(sin(omega*t_i))**2
63 *ENDDO
64
65 ! SOLUTION
66
67 /SOLU
68 ANTYPE,4
69 TRNOPT,FULL
70 AUTOTS,ON
71 NROPT,FULL
72 NEQIT,80
73 KBC,0
74 CNVTOL,HEAT,1,0.001,2,1e-3
75
76 ! --- Load Step 1a:(0 -> 600s) ---
77 TIME,600
78 DELTIM,5,1,10
79 NSEL,S,LOC,X,0,0.04
80 NSEL,R,LOC,Y,0,0.06
81 D,ALL,TEMP,%TCOOL%
82 ALLSEL,ALL
83 NSEL,S,LOC,X,0.16
84 D,ALL,TEMP,293
85 ALLSEL,ALL
86 NSEL,S,LOC,Y,0.15
87 D,ALL,TEMP,293
88 ALLSEL,ALL

```

```

189 OUTRES,ALL,ALL
190 SOLVE
191
192 ! --- Load Step 1b:(600 -> 1500s) ---
193 TIME,1500
194 DELTIM,2,0.5,5
195 OUTRES,ALL,ALL
196 SOLVE
197
198 ! --- Load Step 1c:(1500 -> 2100s) ---
199 TIME,2100
200 DELTIM,5,1,10
201 OUTRES,ALL,ALL
202 SOLVE
203
204 ! --- Load Step 2:(2100 -> 5000s) ---
205 TIME,5000
206 NSEL,S,LOC,X,0,0.04
207 NSEL,R,LOC,Y,0,0.06
208 D,ALL,TEMP,20
209 ALLSEL,ALL
210 NSEL,S,LOC,X,0.16
211 D,ALL,TEMP,293
212 ALLSEL,ALL
213 NSEL,S,LOC,Y,0.15
214 D,ALL,TEMP,293
215 ALLSEL,ALL
216 DELTIM,15,5,40
217 OUTRES,ALL,ALL
218 SOLVE
219 SAVE
220 FINISH
221
222 ! POST
223
224 /POST1
225 SET, LAST
226 PLNSOL, TEMP
227 /DSCALE, 1, 1
228 PLNSOL, S, 1
229 PLNSOL, S, EQV
230 FINISH

```

Contour Plots and Animations

Contour plots of temperature, first principal stress and first principal elastic strain were generated for orientation and for the field figures in Chapter 4. Where fields are compared between time instants or between cases, the contour range was fixed manually through the uniform-contour setting of the plot controls, so that the same colour scale applies to every frame of a comparison.

Animations over the full transient were produced in two ways. The built-in time-based animation steps through the stored result sets between two times. Alternatively, an explicit loop steps through a prescribed number of time instants, which fixes the step size in seconds directly and makes it independent of how many substeps were actually written by the solver. ANSYS

then selects the nearest stored result at or before each requested time.

Listing C.20: Animation commands, reproduced from the ANSYS session log.

```

1  PLNS,S,1
2  ANTIME,40,0.5, ,1,2,0,5000      ! 40 frames, 0.5 s per frame, t = 0 to 5000 s
3
4  ! --- alternative: explicit time stepping, independent of stored substeps ---
5  *SET,t_anim_start,0
6  *SET,t_anim_end,4000
7  *SET,n_anim_steps,60
8  *SET,dt_anim,(t_anim_end-t_anim_start)/n_anim_steps
9  *DO,jj,1,n_anim_steps
10     *SET,t_now,t_anim_start+jj*dt_anim
11     SET,,,,,t_now
12     /REPLOTT
13     /WAIT,0.5
14 *ENDDO

```

Steel Temperature Time History

The temperature history of the cooled metal region was extracted with the time-history post-processor at a single representative node in the steel. A single node is adequate because the entire steel region is driven by a prescribed uniform temperature and therefore cools uniformly. This history was used to determine the times at which each case reaches a given steel temperature, which is the basis for all comparisons made at matched steel temperature rather than at matched elapsed time.

C.10 Analytical Thermal Benchmark

This section lists the input file for the one-dimensional foam-slab benchmark used for the analytical thermal check of Section 3.11. The model differs deliberately from the three production models in three respects. The coefficient of thermal expansion is set to zero for both materials, since this benchmark verifies the transient heat-conduction solution alone and no mechanical response is required. The steel block acts solely as the imposed 20 K source, and the comparison with the closed-form solution is made across the 0.12 m foam layer to its right. The closed-form solution against which these profiles are compared was evaluated separately and does not form part of this script.

Listing C.21: Input file for the one-dimensional foam-slab benchmark used in the analytical thermal check.

```

1
2  ! =====
3  ! Thesis   : Transient Thermo-Mechanical Modelling of Polyurethane Foam
4  !          : Insulation for Non-Vacuum Liquid Hydrogen Storage
5  ! Author   : Gijs Schreuder, TU Delft
6  ! Model    : One-dimensional foam-slab benchmark, foam thickness L = 0.12 m
7  ! Case     : Step change to 20 K at the left boundary, 293 K at the outer
8  !          : boundary, initial temperature 293 K throughout
9  ! Solver    : ANSYS Mechanical APDL, transient (PLANE223)
10 ! Units     : SI (m, kg, s, K, Pa)
11 ! =====

```

```

12
13 /CLEAR
14 FINISH
15 /PREP7
16
17 ET,1,PLANE223
18 KEYOPT,1,1,11
19 KEYOPT,1,3,0                                ! Plane 2-D behaviour, NOT axisymmetric: this
        is a 1-D slab benchmark
20
21 K,1,0,0,0
22 K,2,0.04,0,0
23 K,3,0.04,0.15,0
24 K,4,0,0.15,0
25 K,5,0.16,0,0
26 K,6,0.16,0.15,0
27
28 L,1,2      ! L1 bottom steel
29 L,2,3      ! L2 right steel (interface)
30 L,3,4      ! L3 top steel
31 L,4,1      ! L4 left steel
32 L,2,5      ! L5 bottom foam
33 L,5,6      ! L6 right foam
34 L,6,3      ! L7 top foam
35
36 AL,1,2,3,4
37 AL,5,6,7,2
38
39 NUMMRG,ALL
40 ALLSEL,ALL
41
42 ! MATERIALEN
43
44 ! Foam properties
45 MP,DENS,1,rho_f
46 MPTEMP,1,20,30,60,90,120,150
47 MPTEMP,7,180,210,240,270,300
48 MPDATA,KXX,1,1,k1,k2,k3,k4,k5,k6
49 MPDATA,KXX,1,7,k7,k8,k9,k10,k11
50 MPDATA,C,1,1,cp1,cp2,cp3,cp4,cp5,cp6
51 MPDATA,C,1,7,cp7,cp8,cp9,cp10,cp11
52
53 ! Steel
54 MP,DENS,2,7800
55 MP,KXX,2,15.12
56 MP,C,2,470.5
57 MP,EX,2,2.0E+11
58 MP,PRXY,2,0.29
59 MP,ALPX,2,0                                ! CTE set to zero: this check verifies the thermal
        solution only
60
61 TREF,293
62
63 ASEL,S,AREA,,1
64 AATT,2,1,1
65 ALLSEL,ALL
66
67 ASEL,S,AREA,,2

```

```
68 AATT,1,1,1
69 ALLSEL,ALL
70
71 ! Mesh
72 MSHKEY,1
73 MSHAPE,0,2D
74 ESHAPE,2
75
76 LESIZE,1,,16
77 LESIZE,2,,60
78 LESIZE,3,,16
79 LESIZE,4,,60
80 LESIZE,5,,48
81 LESIZE,6,,60
82 LESIZE,7,,48
83
84 AMESH,ALL
85 ALLSEL,ALL
86 NUMMRG,NODE
87 NUMMRG,ELEM
88
89 IC,ALL,TEMP,293
90 FINISH
91
92 /PREP7
93 NSEL,S,LOC,X,0
94 D,ALL,UX,0
95 ALLSEL,ALL
96 NSEL,S,LOC,Y,0
97 D,ALL,UY,0
98 ALLSEL,ALL
99 FINISH
100
101 /SOLU
102 ANTYPE,4
103 TRNOPT,FULL
104 AUTOTS,ON
105 NROPT,FULL
106 NEQIT,80
107
108 KBC,1                                ! Stepped boundary conditions, matching the
    analytical step-change problem
109
110 CNVTOL,HEAT,1,0.001,2,1e-3
111
112 NSEL,S,LOC,X,0,0.04
113 D,ALL,TEMP,20
114 ALLSEL,ALL
115
116 NSEL,S,LOC,X,0.16
117 D,ALL,TEMP,293
118 ALLSEL,ALL
119
120 OUTRES,ALL,ALL
121
122 TIME,100
123 DELTIM,2,0.5,5
124 SOLVE
```

```
25  
26 TIME,500  
27 DELTIM,5,1,10  
28 SOLVE  
29  
30 TIME,1000  
31 DELTIM,5,1,20  
32 SOLVE  
33  
34 TIME,2100  
35 DELTIM,10,2,30  
36 SOLVE  
37  
38 TIME,5000  
39 DELTIM,20,5,60  
40 SOLVE  
41  
42 SAVE  
43 FINISH  
44  
45 /POST1  
46 SET, LAST  
47 PLNSOL, TEMP  
48 FINISH
```