ON THE STEERING QUALITIES OF SHIPS

by

K. NOMOTO, T. TAGUCHI, K. HONDA and S. HIRANO
Department of Naval Architecture, Osaka University, Japan

Preface

It is a desirable quality for a ship to be well-behaved in steering; to keep her course without difficulty and to change her direction quickly when desired.

In consequence, there have been a number of scholarly studies on steering and turning [1], [2], [3], [4], [5], the standard work probably being that of Davidson and Schiff in 1946. They employ equations of motion with several coefficients: some of them refer to the inertia and some to the moment of inertia of the ship; some, called "resistance devatives", indicate resistance of water; and others indicate forces acting on her rudder. The solution of the equations determines the motion of a ship for a given helm angle. Thus the dynamic character of a ship in steering is described by the equations and consequently by a set of these coefficients.

The method has, however, a practical difficulty. Determination of the coefficients for a given ship requires lengthy experimental procedures employing somewhat intricate instrumentation such as a three component dynamometer, a rotating-arm apparatus or curved models. This is probably the reason why studies on steering have largely been limited to fundamental researches in experimental tanks.

The authors attempted to determine those indices which describe the steering quality just as do the equations of motion, and which may be determined from observations of a given amount of helm angle and the resulting motion of a ship or a free-running, self-propelled model. This is quite possible in principle; because the amount of helm angle is connected to the motion of a ship through her dynamic character, it is natural that the simultaneous observations of the two should provide information on her dynamic character in steering. This point of view has a close relation to the concept of transfer function which has been used successfully in communication and control engineering. In the analytical sense, the determination of the indices is equivalent to the determination of the coefficients of the equation of motion. Both of them define the governing equation of motion and consequently provide a complete expression of steering quality of a ship. The former procedure has, however, the advantage that it does not require experimental procedures as lengthy as the latter.

The first section of this paper deals with the description of the steering quality in terms of the indices, associated with the transfer function expression. The second section relates to the methods of determining the steering quality indices according to the above procedure.

Although the method of variation of the helm angle for this purpose may be arbitrary in principle, actually there are several preferable methods; for instance, sinusoidal steering with various frequencies, impulsive steering, that first setting a certain helm then putting it back right away, and zig-zag steering as proposed by Kempf in 1932 [6]. The zig-zag test is useful for obtaining a brief description of the indices for an actual ship from a simple measurement with her compass and a conventional watch. The first two types of steering are utilized for those tests which provide complete determination of the indices through some more refined measurement, for instance, rate-gyro measurement.

Another and probably more substantial aim of the present treatment of steering quality relates to a more generalized investigation of the steering problem considering not only the steering quality of a ship, but also the dynamic qualities of her helmsman or auto-pilot and steering gear. As a matter of fact, the steering is conducted by a helmsman or an auto-pilot that detects continually the motions of the ship and regulates the helm angle so that the motion realized may approach the pre-arranged motion, say, running along a straight course or changing the heading to a new course. In other words, a ship, a helmsman or an auto-pilot and a steering gear constitute a feed-back system as shown in Fig. 1. In such a system, the motion of its last element is continually fed back to the first element to drive the system jointly with external forces, hence the dynamic character of the system is compounded of the dynamic character of each element. In consequence, considerations of a closed system composed of a ship, a helmsman and a steering gear should yield a more realistic treatment on steering of ships. This is particularly true in the problem of course-keeping in a sea way, because it depends on not only the passive action of a hull and appendages but also the active action of a rudder, which is just the result of the feed-back.

Analytically, the considerations of a feed-back system mean treatments of equations of motion of all elements simultaneously. Obviously the more complicated the details of a feed-back system, the more difficult is an orthodox procedure dealing with simultaneous differential equations, especially do some nonlinearities in auto-pilots which often

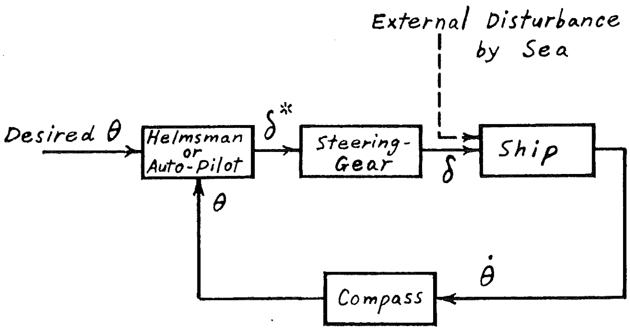

Fig. 1. *Scheme of steering of a ship*

have dominant effects on the course-keeping performances introduce troublesome problems. In such circumstances, the recent approach to a feed-back system is much more advantageous. In this method, the dynamic character of an element is described using a transfer function defined from a relation between an external excitation applied to it and its resulting motion. Then through proper operations of the transfer function on all elements of the system, the results as obtained through the solution of the simultaneous equations are approached. The present description of steering qualities in terms of the steering quality indices or transfer function is the one which just follows this approach to suit the treatment on the generalized steering problem. A synopsis of such a treatment on the steering problem is found in the appendix.

1. DESCRIPTION OF STEERING QUALITIES OF SHIPS

1.1. *The Steering Quality Indices*

The present work starts from the modern studies on steering of ships employing the equations of motion and resistance derivatives, the general procedure for which is as follows:

1. to follow the method of hydrodynamics dealing with motion of a solid through a perfect liquid, selecting a coordinate system moving with the ship, the origin of which is at the C.G. of the ship, x-axis along her longitudinal direction and z-axis vertical;
2. to consider the coupled motion of rotation about the vertical axis (turning angular motion) and translation along the lateral axis (drifting motion), neglecting their dependence upon the other motions involving the deceleration caused by steering;
3. to express the water resistance caused by the ship motion as a linear function of drifting velocity and turning angular velocity of the ship.

According to Davidson [4], the equations of motion are:

$$m_2\,\psi' + C_l\,\psi - (m_1 - C_f)\,\Omega = C_\lambda\,\delta$$
$$n\,\Omega' + C_k\,\Omega - C_m\,\psi = C_\mu\,\delta \qquad \ldots\ldots\ldots (1)$$

where:

$$\Omega_{(S)} = \left(\frac{l}{V}\right)\dot\theta;$$

$\dot\theta$ = turning angular velocity;
V = ship speed;
ψ = drift angle;
δ = helm angle;
l = ship length;

$'$ denotes $\dfrac{d}{ds} = \left(\dfrac{l}{V}\right)\dfrac{d}{dt}$, $\quad S = \dfrac{V}{l}\,t$

m_1, m_2, n = coefficient of inertia;
C_l, C_k, C_m, C_f = coefficient of resistance;
C_μ, C_λ = coefficient of rudder force.

The former equation is the equation of lateral translation and the latter is the equation of turning angular motion.

Although this nondimensional expression is advantageous, particularly with regard to the hydrodynamic aspect, another expression in terms of time is often preferable for practical purposes such as design and treatments of the system involving other elements besides a ship. Then rewriting the equations of motion in terms of time:

$$\left(\frac{l}{V}\right) m_2\,\frac{d\psi}{dt} + C_l\,\psi - \left(\frac{l}{V}\right)(m_1 - C_f)\,\dot\theta = C_\lambda\,\delta$$
$$\left(\frac{l}{V}\right)^2 n\,\frac{d\dot\theta}{dt} + \left(\frac{l}{V}\right) C_k\,\dot\theta - C_m\,\psi = C_\mu\,\delta \qquad \ldots\ldots\ldots (2)$$

Eliminating the drift angle ψ from the equations, we obtain:

$$T_1 T_2\,\frac{d^2\dot\theta}{dt^2} + (T_1 + T_2)\,\frac{d\dot\theta}{dt} + \dot\theta = K\,\delta + K\,T_3\,\frac{d\delta}{dt}$$

where:

$$K = \left(\frac{V}{l}\right)\frac{C_m\,C_\lambda + C_l\,C_\mu}{C_l\,C_k - m\,C_m},$$

$$T_1\,T_2 = \left(\frac{l}{V}\right)^2 \frac{m_2\,n}{C_l\,C_k - m\,C_m},$$

$$(T_1 + T_2) = \left(\frac{l}{V}\right)\frac{m_2\,C_k + n\,C_l}{C_l\,C_k - m\,C_m},$$

$$T_3 = \left(\frac{l}{V}\right)\frac{m_2\,C_\mu}{C_m\,C_\lambda + C_l\,C_\mu}$$

and where $m = m_1 - C_f$.

Then a set of values of K, T_1, T_2 and T_3 describes the dynamic character of a ship just as do the equations of motion (2). K, T_1, T_2 and T_3 may be defined as the steering quality indices.

In the same manner it is possible to define corresponding indices for the nondimensional expression as follows:

$$K' = \left(\frac{l}{V}\right) K, \quad T_1' = \left(\frac{V}{l}\right) T_1, \quad T_2' = \left(\frac{V}{l}\right) T_2$$

and $T_3' = \left(\frac{V}{l}\right) T_3$.

These indices are composed of the nondimensional coefficients of equation (1), excluding the dimensional term $\left(\frac{V}{l}\right)$.

It is not necessary that the steering quality indices be deduced from the equations of motion in the present form; equations of motion in every other form based on the general procedure suggested at the opening of the discussion may be reduced to the above form of equation of motion in respect to $\dot{\theta}$, and accordingly may introduce just the same coefficients K, T_1, T_2 and T_3.

1.2. *Meaning of the Steering Quality Indices*

Meanings of T_1 and T_2. The general solution of the equation of motion which determines the general character of all possible motions of a ship has the following form:

$$\dot{\theta}\ (t) = C_1\ e^{-t/T_1} + C_2\ e^{-t/T_2}$$

or:

$$\Omega_{(S)} = C_1'\ e^{-S/T_1'} + C_2'\ e^{-S/T_2'}$$

Then it may be said that with respect to T_1 and T_2 (or T_1' and T_2'):

1. After a small disturbance, the disturbed motion of a ship with her rudder amidships is damped exponentially with a rapidity dependent on the exponents T_1 and T_2. The smaller T_1 and T_2 the quicker the decay of the motion.
2. When a certain helm angle is set, a ship enters into a turn exponentially with a rapidity dependent on the exponent T_1 and T_2. The smaller T_1 and T_2 the quicker the build-up of turning.
3. Positive values of both of them assure a stable dynamic character and a negative value of either or both indicates an unstable dynamic character. Any imaginary value of them indicates the presence of an inherent periodic yawing, which is never the case except in some curious hypothetical ship outside normal practice.

In a word, T_1 and T_2 are indices of stability on course and quick response in steering. These two abilities are not quite the same thing, because the former means quick response to zero helm angle. Since the larger one of them, say T_1 is considerably larger than the other for ordinary ships, it may be reasonable to concentrate on T_1 as does Davidson. His stability index P_1 is equivalent to $-\frac{1}{T'_1}$. Another way to simplify the problem in a practical manner is to unify T_1, T_2 and T_3 to give a new effective time constant T. This will be discussed later.

Meaning of K. It should be clear that, from the equation of motion, K (or K') has the dimensions of an angular velocity $\dot{\theta}$ (or Ω) in steady turning with a corresponding helm angle. In turnings with large helm angles, however, the realized steady turning angular velocity is generally different from that steady turning velocity which the ship approaches during the initial phase of turning, because considerable speed reduction and gradual variations of the resistance-derivatives arise to nonlinearize the motion. Since a usual steering motion is regarded as succeeding the initial phases of turning, it is more reasonable to define K with respect to the steady turning which a ship approaches than to define it with respect to the realized steady turning.

It should be noted that the circumstances are completely different for an unstable ship; she tends to turn and to approach an infinite turning velocity even with an infinitesimally small helm angle, while her wild motion is eased soon after through nonlinearity as shown by Davidson. Consequently, the definition of K for an unstable ship is not so clear as for a stable ship. A certain "effective" K may be defined for an unstable ship with respect to the motion realized and it has a meaning similar to K for a stable ship in the practical sense. Such an effective K is, however, far from the K in the formal sense, the coefficient of the equation of motion; this formal K has generally a negative value for an unstable ship. Fortunately, it seems that this difficulty is one of rather theoretical interest, because instability is a result of bad design and is corrected in most cases by an additive fin or by increasing the rudder area.

To summarise, K is an index of turning ability, related closely to Davidson's index $\left(\frac{D}{l}\right)_{min}$.

Meaning of T_3. T_3 represents the contribution of steering speed in initiating a turning motion. As a matter of fact, however, no ship has such great steering speed or such large T_3 that the effect may become dominant, so far as is known. In addition, T_3 and T_2 affect a ship's dynamic character in such a manner that their effects cancel each other, as is evident in the transfer function expression discusssed later. To unify T_1, T_2 and T_3 to an effective time constant T is reasonable in these circumstances.

In regard to the effect of steering speed, it should be noted that the effect expected from the equation of motion results from coupling between turning angular motion and lateral drifting motion, excluding a possible similar effect arising from an additional rudder force dependent on the time rate of the helm angle. Considering that the recent measurements of rudder forces in actual ships and on the model scale suggest the presence of the latter effect, the actual value of T_3 should be somewhat larger than T_3 calculated from the coefficients obtained from the equations. In fact, it would seem that T_3 determined from observations of motion of a free model has a little higher value than the one expected from her equations of motion.

1.3. *Responses of Ships to Some Typical Steering Procedures in Terms of the Indices*

To examine the motions of the ship excited by some typical steeringprocedures in terms of the steering quality indices is a reasonable way for visualizing the dynamics of steering.

Fundamental Type of Steering and First-Order Simulation

At first, let us consider such steering in which the helm angle increases uniformly up to a certain angle δ_0, then remains at this value. Every steering process with common steering gears may be considered as a sequence of this type to a fair approximation. The response of a ship is easily obtained from the equation of motion, as follows, where t_1 is the time when δ_0 is reached. That is, during $t \leqq t_1$:

$$\dot{\theta}_{(t)} = \frac{K\,\delta_0}{t_1}\{t - (T_1 + T_2 - T_3)\} + \frac{K\,\delta_0}{t_1\,(T_1 - T_2)} \{T_1\,(T_1 - T_3)\,e^{-t/T_1} - T_2\,(T_2 - T_3)\,e^{-t/T_2}\}$$

$$\theta_{(t)} = \frac{K\,\delta_0}{t_1}\left\{\frac{t^2}{2} - (T_1 + T_2 - T_3)\,t\right\} + \frac{K\,\delta_0}{t_1\,(T_1 - T_2)}\{T_1^2\,(T_1 - T_3)\,(1 - e^{-t/T_1}) - T_2^2\,(T_2 - T_3)\,(1 - e^{-t/T_2})\}$$

and after the time t_1:

$$\dot{\theta}_{(t)} = K\,\delta_0 - \frac{K\,\delta_0}{t_1\,(T_1 - T_2)}\{T_1\,(T_1 - T_3)\,(e^{-t_1/T_1} - 1)\,e^{-t/T_1} - T_2\,(T_2 - T_3)\,(e^{-t_1/T_2} - 1)\,e^{-t/T_2}\}$$

$$\theta_{(t)} = K\,\delta_0\,t - K\,\delta_0\,\{(T_1 + T_2 - T_3) + \frac{t_1}{2}\} + \frac{K\,\delta_0}{t_1\,(T_1 - T_2)}\{T_1^2\,(T_1 - T_3)\,(e^{-t_1/T_1} - 1)\,e^{-t/T_1} - T_2^2\,(T_2 - T_3)\,(e^{-t_1/T_2} - 1)\,e^{-t/T_2}\}$$

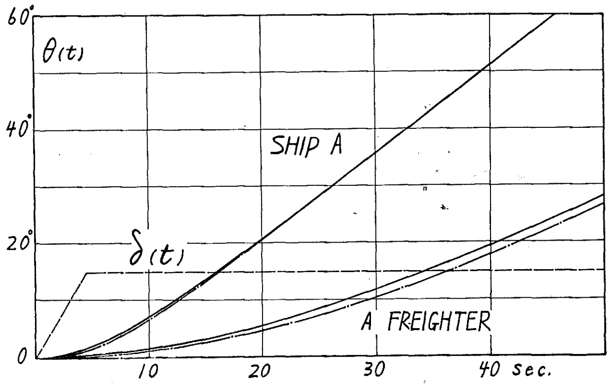

Fig. 2. *Transient phase of turning*

	K	T_1	T_2	T_3	V_{kt}	l
Ship a	0.10	5.9	1.1	2.9	32	115 m
A freighter	0.090	45	6.0	10	16	138 m

———·———: *First-order simulation with* $T = \dfrac{(T_1 - T_3)T_1 - (T_2 - T_3)T_2}{T_1 - T_2}$

The responses of two representative ships are calculated as is shown in Fig. 2. One of them is produced from the "ship A" in the paper of Schiff and Gimprich [9] by estimating probable values of her speed and length. The other is a full-loaded, ocean-going freighter. The results suggest that a ship may be simulated by a first-order system with the same K and a proper time constant T, as has already been suggested from the dominance of T_1 over T_2 and T_3. A first-order system generally means a system with a governing equation of motion of the first-order.

A possible way of defining the "effective" time constant T, that means the T of the first-order system simulating the ship, is to select T so that the motions of a ship and the first-order simulating system may coinside in the ultimate phase. This definition gives:

$$T = T_1 + T_2 - T_3$$

thus showing directly the cancelling effect between T_2 and T_3, and the resulting simulation is satisfactory for a ship with relatively small T_1, as is shown in Fig. 2. Another definition of T is provided by selecting T so that a certain realized motion may be simulated most closely on the whole. Considering that every actual process of steering is a sequence of transient phases, such definition of T may be more reasonable, when it is applied to some typical steering, for instance, the zig-zag steering proposed by Kempf. The analyses of a number of zig-zag tests for actual ships seem to confirm this idea.

After all, it may be said that steering motions of ships are substantially first-order phenomena, and consequently the steering qualities of ships are described in brief by the two fundamental indices, namely K, an index of turning ability, and T, an index of quick-response in steering and the dynamic stability on course. In other words, the actual steering motions of ships may be well described by the first-order equation of motion:

$$T \frac{d\dot{\theta}}{dt} + \dot{\theta} = K\delta \quad \ldots\ldots \quad (3)$$

This conclusion is of fundamental importance in researches on steering problems.

The meaning of steering quality in terms of the indices K and T may be visualized by comparing the steering motions of several ships with different K and T. Fig. 3 illustrates such a result for a supertanker and her imaginary sister ships introduced by some variation of K and T. As is evident from the figure, a combination of large K and small T means a quick build-up of a large turning moment and this is just the feature of superior steering quality, because a ship with such character may outdo other ships in all phases of steering. While a ship with both smaller K and T is distinguished in the earlier phases of steering by her quick build-up of turning, after a while her rival with both larger K and T overtakes and finally leaves her behind displaying great turning ability. Considering these circumstances, it may be concluded that steering quality ought to be expressed by both K and T because it depends on both great turning ability and the quick build-up of the turning.

Trapezoidal Steering. Now consider a different type of steering, in which $\delta_{(t)}$ increases uniformly to δ_0, then remains at δ_0 up to t_2 and decreases to zero at the same rate as the first step. Because this steering may be resolved into two steerings of the former type, one of which has the reverse sign and lags by t_2 behind the other, the response of a ship to the steering may be obtained by superposition of the former results. Fig. 4 illustrates these responses for the preceding four ships.

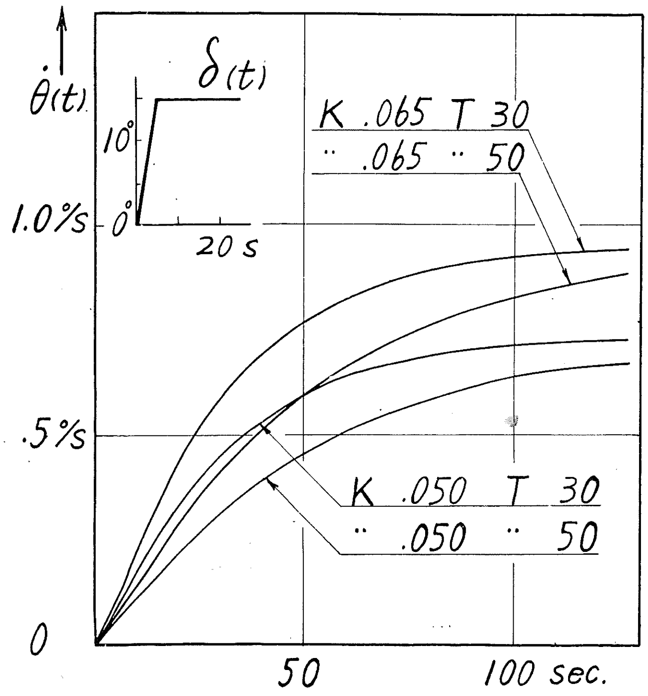

Fig. 3. *Comparative illustration of build-up of turning*

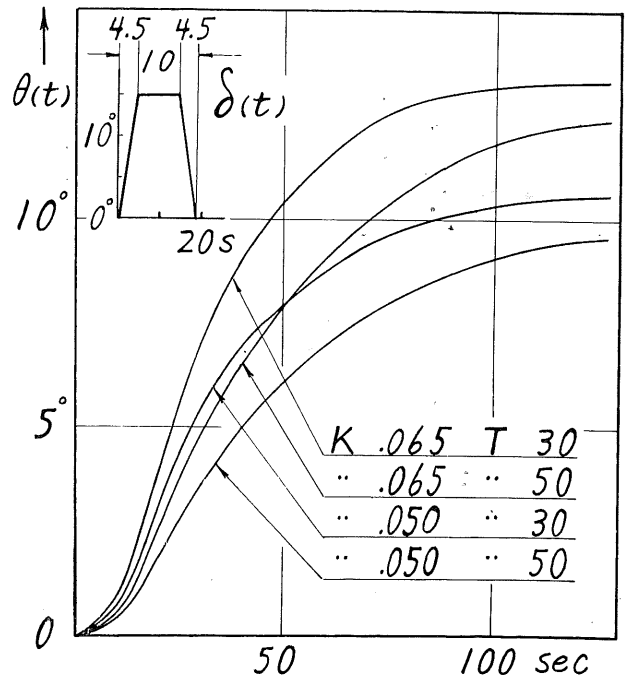

Fig. 4. *Comparative illustration of course change by trapezoidal steering*

A total change of ship's heading by a steering of this type, $\theta_{(\infty)}$, that is regarded as the entire effect of the steering, may be obtained directly by integrating the equation of motion from 0 to ∞, that is:

$$T_1 T_2 \int_{0-}^{\infty} \dddot{\theta}_{(t)}\, dt + (T_1 + T_2) \int_{0-}^{\infty} \ddot{\theta}_{(t)}\, dt + \int_{0-}^{\infty} \dot{\theta}_{(t)}\, dt =$$

$$= K \int_{0-}^{\infty} \delta_{(t)}\, dt + K T_3 \int_{0-}^{\infty} \dot{\delta}_{(t)}\, dt$$

therefore:

$$\theta_{(\infty)} = K \int_{0-}^{t_1 + t_2} \delta_{(t)}\, dt$$

Obviously this result applies to all steerings of finite durations, thus providing another definition of K; the total change of ship's heading by such a steering which ceases in a finite time, is proportional to the time integral of the helm angle curve $\delta_{(t)}$ regardless of its shape, and K is simply the proportionality constant. This is an important feature of steering motions of ships, which is sometimes called "integral character".

The meaning of T is also evident in this case, that is, the smaller the value of T the quicker the display of the entire effect of a steering. Since one steering is generally followed by another in actual steering processes, the quick display of the effect of a steering is important as well as the intensity of the effect.

Sinusoidal Steering. Finally let us turn to another type of steering, in which the helm angle varies sinusoidally with time. Although this steering seems less realistic, it should be remembered that every actual steering may be resolved at least in principle into many sinusoidal steerings with various amplitudes and frequencies. The full significance of the sinusoidal steering will be clear in relation to the transfer function in the next section.

The response of a stable ship to a sinusoidal steering is obtained from her equations of motion, as follows:

$$\dot{\theta}_{(t)} = A_{(\omega)}\ \delta_0 \sin \{\omega\ t + \varphi_{(\omega)}\}$$

$$A_{(\omega)} = K \sqrt{\frac{1 + T_3^2\ \omega^2}{1 + (T_1^2 + T_2^2)\ \omega^2 + T_1^2\ T_2^2\ \omega^4}}$$

$$\varphi_{(\omega)} = -\tan^{-1} \frac{(T_1 + T_2 - T_3)\ \omega + T_1\ T_2\ T_3\ \omega^3}{1 - (T_1\ T_2 - T_2\ T_3 - T_3\ T_1)\ \omega^2}$$

where $2\ \delta_0$ and ω are the amplitude and the frequency of the helm angle respectively. Since the response of a ship is also a sinusoidal function with the same frequency, it is defined completely by the amplitude ratio $A_{(\omega)}$ and the phase difference $\varphi_{(\omega)}$. $A_{(\omega)}$ and $\varphi_{(\omega)}$ for the two representative ships formerly shown are illustrated in Fig. 5, in the form of 20 log $A_{(\omega)}$ and $\varphi_{(\omega)}$ in degrees against log ω. The most remarkable features of the results are the decreasing amplitude and the growing phase lag of ship motions with increase of frequency ω, which are much accelerated after $\omega = \frac{1}{T_1}$ is reached. Namely, ships are more insensible to steerings with higher frequencies. This quality is called "low-pass character", and it is closely related to integral character already mentioned.

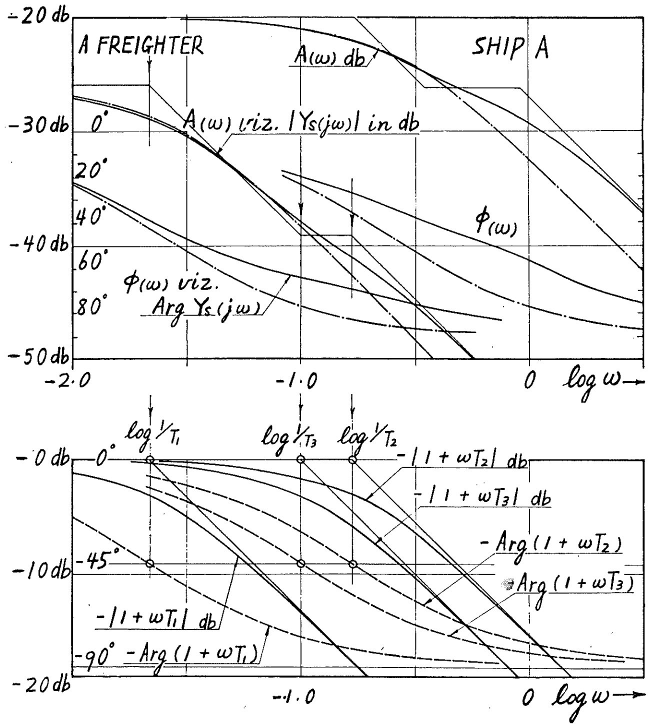

Fig. 5. *Above: Responses of two ships to sinusoidal steering. This figure is also a representation of transfer functions of the ships.* ———·———*: First-order simulation with* $T = \frac{(T_1 - T_3)T_1 - (T_2 - T_3)T_2}{T_1 - T_2}$

Below: Resolution of the transfer function of the freighter

Reconsidering the meaning of steering quality in the light of sinusoidal steering, a combination of large K and small T is again the indication of a superior quality, because such a ship has a great sensitivity to sinusoidal steerings of all frequencies. On the other hand, a ship with both smaller K and T has a relatively greater sensitivity to the higher frequencies, covering her poor turning ability by quick-response while another ship with both larger K and T is superior for the lower frequencies, in which her inherent great turning ability may be displayed sufficiently in spite of her poor quick-response. These circumstances suggest another significant phase of steering quality.

1.4. *Transfer Function Expression of the Steering Quality*

The theoretical background of the steering quality indices is found in the transfer function expression of steering quality. On the other hand, this expression displays its unique utility in treatments of the generalized steering problem as is stated in the preface.

The transfer function of a dynamic system is introduced by taking the Laplace transform of the governing equation of motion. Let us apply the operation to the equations of motion of a ship in steering, equation (2), associating the procedure which has introduced the steering quality indices. Taking the Laplace transforms of both sides of the equations:

$$-\left(\frac{l}{V}\right) m_2\ \psi_{(0-)} + \left\{\left(\frac{l}{V}\right) m_2\ p + C_l\right\} \psi_{(p)} - \left(\frac{l}{V}\right) m\ \dot{\theta}_{(p)} = C_\lambda\ \delta_{(p)}$$

$$-\left(\frac{l}{V}\right)^2 n\ \dot{\theta}_{(0-)} + \left\{\left(\frac{l}{V}\right)^2 n\ p + \left(\frac{l}{V}\right) C_k\right\} \dot{\theta}_{(p)} - C_m\ \psi_{(p)} = C_\mu\ \delta_{(p)}$$

where:

$$\psi_{(p)} \equiv \int_{0-}^{\infty} \psi_{(t)}\ e^{-pt}\ dt,\ \dot{\theta}_{(p)} \equiv \int_{0-}^{\infty} \dot{\theta}_{(t)}\ e^{-pt}\ dt,\ \delta_{(p)} \equiv \int_{0-}^{\infty} \delta_{(t)}\ e^{-pt}\ dt.$$

The time origin is selected at the beginning of steering. Eliminating $\psi_{(p)}$ from the simultaneous equations, and also eliminating $\psi_{(0-)}$ by the condition at the beginning of steering, that is:

$$\left(\frac{l}{V}\right)^2 \ddot{\theta}_{(0-)} + \left(\frac{l}{V}\right) \dot{\theta}_{(0-)} - C_m\ \psi_{(0-)} = 0,$$

we obtain:

$$\dot{\theta}_{(p)} = \left(\frac{V}{l}\right)\frac{C_m C_\lambda + C_l C_\mu}{C_l C_k - m C_m} \frac{1 + \left(\frac{l}{V}\right)\frac{m_2 C_\mu}{C_m C_\lambda + C_l C_\mu}}{1 + \left(\frac{l}{V}\right)\frac{m_2 C_k + n C_l}{C_l C_k - m C_m} p + \left(\frac{l}{V}\right)^2 \frac{m_2 n}{C_l C_k - m C_m} p^2} \cdot \delta_{(p)} +$$

$$+ \frac{\left\{\left(\frac{l}{V}\right)^2 \frac{m_2 n}{C_l C_k - m C_m} p + \left(\frac{l}{V}\right)\frac{m_2 C_k + n C_l}{C_l C_k - m C_m}\right\} \dot{\theta}_{(0-)} + \left(\frac{l}{V}\right)^2 \frac{m_2 n}{C_l C_k - m C_m} \ddot{\theta}_{(0-)}}{1 + \left(\frac{l}{V}\right)\frac{m_2 C_k + n C_l}{C_l C_k - m C_m} p + \left(\frac{l}{V}\right)^2 \frac{m_2 n}{C_l C_k - m C_m} p^2}$$

Unifying the coefficients, and factorising the denominator:

$$\dot{\theta}_{(p)} = \frac{K(1 + T_3 p)}{(1 + T_1 p)(1 + T_2 p)} \delta_{(p)} + \frac{\{T_1 T_2 p + (T_1 + T_2)\} \dot{\theta}_{(0-)} + T_1 T_2 \ddot{\theta}_{(0-)}}{(1 + T_1 p)(1 + T_2 p)} \quad \ldots\ldots\ldots (4)$$

The first term corresponds to the ship motion excited by the steering, and the second corresponds to the memory of her motion at the beginning of the steering. Therefore, a rational function:

$$\frac{K(1 + T_3 p)}{(1 + T_1 p)(1 + T_2 p)} \equiv Y_{S(p)}$$

describes the response character of the ship to steering, which may be called the transfer function of the ship in steering. Now the close relation between the transfer function and the steering quality indices is quite evident, that is, the indices are the coefficients of the transfer function.

1.5. *Meaning of the Transfer Function and Sinusoidal Steering*

Defining the transfer function in analytical terms, the transfer function of a system is the ratio of the Laplace transform of a motion of the system excited from rest, to the Laplace transform of that excitation. Because of the unique correspondence between a function and its Laplace transform, the transfer function, which transfers the Laplace transform of an external excitation to the Laplace transform of the resulting motion, must be something connecting the excitation to the resulting motion. In consequence, the transfer function describes completely a dynamic character of a system. Leaving the more detailed formulation of the application of the Laplace transform for the solution of linear differential equations, let us turn to visualize the meaning of the transfer function, relating it to the concept of harmonic analysis which suggests to us that every actual motion may be regarded as a sum of sinusoidal motions with various frequencies and amplitudes.

The Laplace transform of such a function which is identically zero for all negative values of t, may be considered as the Fourier transform of the product of the function and $e^{-\beta t}$ where β is the real part of p, the parameter of the Laplace transform. For instance, a process of applying the helm angle $\delta_{(t)}$ is such a function if the time t is measured from the beginning of the steering. Then the Laplace transform of $\delta_{(t)}$ is:

$$\int_{0-}^{\infty} \delta_{(t)} e^{-pt} dt = \int_{0-}^{\infty} e^{-\beta t} \delta_{(t)} e^{-j\omega t} dt =$$

$$= \int_{-\infty}^{\infty} e^{-\beta t} \delta_{(t)} e^{-j\omega t} dt$$

where ω is the imaginary part of p, and j denotes $\sqrt{-1}$. This is the Fourier transform of $e^{-\beta t} \delta_{(t)}$, which may be considered as the steering described to some modified scale. Since the Fourier transform may be considered as a distribution of Fourier coefficients as a function of frequency, the magnitude and argument of the Laplace transform of $\delta_{(t)}$ is respectively an amplitude — exactly, a density of amplitude as a function of frequency — and phase angle of a sinusoidal element of the frequency ω involved in $e^{-\beta t} \delta_{(t)}$, into which elements $e^{-\beta t} \delta_{(t)}$ may be resolved according to the concept of harmonic analysis. The same interpretation applies to the Laplace transform of the ship motion $\dot{\theta}_{(t)}$ excited by $\delta_{(t)}$, because the motion starts with the same time origin.

The modification factor $e^{-\beta t}$ is introduced in order to enable the integral to express the function concerned as the summation of numerous sinusoidal elements. Accordingly, the least necessary value of β depends on the inherent irregularity of the function, say in the case of $\delta(t)$ and $\dot{\theta}(t)$. For all values of $\delta(t)$ with finite helm angles and $\dot{\theta}(t)$ of stable ships (ships with positive T_1 and T_2) the least necessary value of β is zero or at the largest case tends to zero, while a finite β is required for $\dot{\theta}(t)$ of an unstable ship. In other words, her irregular motion cannot be resolved into sinusoidal elements without modification by the multiplication of $e^{-\beta t}$.

Leaving the unstable ships for later discussion, it may be said that for stable ships, every manner of steering, $\delta_{(t)}$, and the response of a ship to it, $\dot{\theta}_{(t)}$, may be resolved individually into generally infinite numbers of sinusoidal elements, and the amplitudes and phases of these elements as functions of ω are given as the magnitudes and arguments of the Laplace transform of $\delta_{(t)}$ and $\dot{\theta}_{(t)}$ respectively, when the parameter p is selected as $j\omega$. Furthermore, a sinusoidal element of $\dot{\theta}_{(t)}$ with a certain frequency must have been excited by a sinusoidal element of $\delta_{(t)}$ with the same frequency. This is evident from the fundamental qualities of linear stable systems that a sinusoidal excitation generates a sinusoidal motion with the same frequency, and the summation of responses to certain elemental excitations equals the response to the summation of the elemental excitations. Now obviously, the response of a ship to arbitrary steering is completely described in terms of the response to sinusoidal steering; every $\delta_{(t)}$ may be resolved into sinusoidal steerings and the response of a ship to the $\delta_{(t)}$ may be constructed from the sinusoidal ship motions for the elemental sinusoidal steerings. The transfer function of a ship is the ratio of the Laplace transform of her motion $\dot{\theta}_{(t)}$ to the Laplace transform of that $\delta_{(t)}$ which excited the $\dot{\theta}_{(t)}$, and when p is selected as $j\omega$, the magnitude and argument of the Laplace transform of $\dot{\theta}_{(t)}$ or $\delta_{(t)}$ equal the amplitude and phase of a sinusoidal element of $\dot{\theta}_{(t)}$ or $\delta_{(t)}$ respectively with the frequency ω. Accordingly, the magnitude and argument of the transfer function for the parameter $j\omega$ gives respectively amplitude ratio and phase difference of a sinusoidal element of $\dot{\theta}_{(t)}$ with a frequency ω to a sinusoidal element of $\delta_{(t)}$ with the same frequency. Thus the transfer function defines response character of a ship in steering by defining directly her response to the sinusoidal steering.

This interpretation of the transfer function is useful in actual applications. It is also the origin of the nonlinear transfer function which plays an important role in researches on generalized steering problems as stated in the preface.

For unstable ships, this expression of the response character cannot be applied without some modification, because her inherent motions are so wild that they cannot be resolved into sinusoidal elements. The utility of the modification factor $e^{-\beta t}$ is to measure $\dot{\theta}_{(t)}$ and $\delta_{(t)}$ on the same distorted scale as to be able to apply the similar expression of the response character in terms of the sinusoidal response to unstable ships. Then the transfer function for a complex parameter with a proper real part β may define even the response character of unstable ships, at least in the logical sense.

As is evident from the present discussion, the sinusoidal steering in the last section has a fundamental significance in the aspect of the transfer function treatment of steering of ships. Noting the meaning of the transfer function for an imaginary parameter $j\omega$, we obtain easily:

$$A_{(\omega)} = |Y_{S(j\omega)}| = K \left| \frac{1 + j\omega T_3}{(1 + j\omega T_1)\,(1 + j\omega T_2)} \right|$$

$$\varphi_{(\omega)} = Arg\, Y_{S(j\omega)} = \\ = Arg \frac{1 + j\omega T_3}{(1 + j\omega T_1)\,(1 + j\omega T_2)} \qquad \ldots\ldots\ldots (5)$$

Then Fig. 5 may be considered a representation of the transfer function of the two ships. In fact, such figures, so-called Bode's diagrams, are often employed to represent transfer functions in control and communication engineering. A distinct advantage of Bode's expression is that the construction of the transfer function may be clearly inspected in the expression. Because:

$$\log |Y_{S(j\omega)}| = \log K + \log |1 + j\omega T_3| - \\ - \log |1 + j\omega T_1| - \log |1 + j\omega T_2|,$$

$$Arg\, Y_{S(j\omega)} = Arg\,(1 + j\omega T_3) - \\ - Arg\,(1 + j\omega T_1) - Arg\,(1 + j\omega T_2),$$

then in the expression the transfer function may be resolved into these simple elemental functions, as is shown in Fig. 5. In this connection, Bode's straight-line approximation of $20 \log |Y_{S(j\omega)}|$ is useful for describing the transfer function of a ship briefly. It is shown in the figure with fine straight lines.

Inspecting the transfer function in this manner, there reappears the important conclusion of the last section, viz. that steering motions of ships are substantially first-order phenomena. Since T_2 and T_3 have values which are of the same order and are much smaller than T_1 for actual ships so far as is known, $(1 + T_3\,p)$ in the numerator and $(1 + T_2\,p)$ in the denominator tend to cancel each other. Then the transfer function of a ship in steering may be simulated by a first-order transfer function $\frac{K}{1 + T\,p}$. The simulating first-order transfer function is shown in the figure with chain lines. As is obvious in this case, the first-order simulation of a transfer function of a ship in steering is satisfactory in the lower and intermediate frequency ranges, but not so satisfactory in the higher frequency range. However, the disagreement for the higher frequency is not an important matter, because a ship is more insensible to the higher frequency element of steering, and in addition, actual steering does not involve such high frequency elements.

TABLE I

	TANKER		TANKER	TANKER	FREIGHTER	
$L_{pp} \times B \times D$	$201 \times 28.2 \times 14.6$		$181 \times 25.4 \times 13.5$	$167 \times 22.0 \times 12.2$	$145 \times 19.5 \times 12.2$	
Condition	Full-load	Ballast	Full-load	Full-load	Full-load	Light-load
$\triangle$	50,618	21,114	36,921	27,137	15,780	—
d_a	10.814	7.118	10.150	9.31	8.78	—
d_f	10.814	2.667	10.140	9.29	7.26	—
d_m	10.814	4.893	10.145	9.30	8.02	—
$L_{WL} = l$	205.4	196.3	186.4	171.5	148	—
V_{kt}	17.43	18.9	17.04	15.86	14.8	18
C_b/C_p	.80/.81	.75/.77	.777/.784	.767/.776	—	—
$\nabla / \left(\frac{L_{WL}}{10}\right)^3$	5.70	2.72	5.58	5.24	4.75	—
l/B	7.28	6.96	7.34	7.79	7.59	—
B/d_m	2.61	5.76	2.50	2.37	2.43	—
A_R	28.67	26.7	25.67	23.78	18.0	—
$A_R/L_{pp} \times d_m$	1/77.4	1/36.0	1/72.2	1/65.3	1/64.6	—
K	.0527	.0380	.0553	.0521	.0516	.043
T	46.0	10.6	41.9	33.6	24.7	7.00
$\left(\frac{V}{l}\right)$	.0437	.0495	.0484	.0488	.0514	—
K'	1.20	.768	1.15	1.07	1.00	—
T'	2.01	.524	2.01	1.64	1.27	—

Note: = Displacement in K.T. *L*'s and *d*'s = in metres *A* = Rudder area in square metres

1.6. *Design Problems in Terms of the Indices*

The preceding comparative considerations of steering qualities of ships in terms of K and T may be summarised as follows:

1. a ship with larger K and smaller T is superior in all phases of steering;
2. considering simultaneous variations of K and T, a ship with both small K and T is superior to a ship with both large K and T in the earlier phases of steering and in more frequent steering, and vice versa.
   Furthermore, Davidson and Schiff [4] have shown that;
3. smaller T is absolutely effective for course-keeping in a seaway.
   These conclusions provide a fundamental principle for design work concerned with steering, associated with the following circumstances wellknown to ship designers;
4. better turning ability with better course stability (larger K with smaller T) is realized generally only by a larger rudder for a given hull form which must be designed usually without regard to steering quality;
5. "cut-up" arrangement of bow or stern, designed trim and equipment of a fin or skeg are all effective in producing better turning ability at the expense of course stability or vice versa (to increase or decrease both K and T simultaneously).

In other words, the rudder size determines the amount of steering quality, while "cut-up" and other similar arrangements contribute to a distribution of it between turning ability and quick-response in steering.

Since it is desirable to avoid a large rudder if possible, the first process of design is to select the least necessary rudder size according to the requirements as to steering quality in the design; for example, whalers and harbour-tugs should have relatively larger rudders than freighters. The next process is to design "cut-up" and other similar arrangements from the consideration of the optimum shares of the two fundamental abilities of a ship in steering, that is, great turning power and

FREIGHTER	FREIGHTER	BULK-CARRIER	WHALER	WHALER	COASTER	REFRIGERATING FISH-CARRIER
132 × 18.2 × 11.7	114 × 16.4 × 9.3	152 × 20.6 × 12.7	46 × 8.2 × 4.4	42 × 7.8 × 4.45	44 × 7.6 × 3.7	67 × 10.8 × 5.7
Ballast	Ballast	Ballast	Arrival	Arrival	Ballast	Ballast
5,890	4,180	8,828	—	—	303	943
5.100	4.58	5.614	—	4.70	2.750	3.280
2.010	1.96	2.438	—	4.00	.590	.990
3.555	3.27	4.020	3.95	4.35	1.670	2.135
126.8	109.6	147.3	47	43	43.00	63.52
17.09	15.68	17.22	15.5	14.5	12.37	13.02
.685/.705	.687/.703	.698/.720	—	—	.535/.565	—
2.83	3.12	2.69	—	—	3.72	3.59
6.96	6.68	7.15	5.73	5.51	5.66	5.88
5.12	5.01	5.12	2.08	1.79	4.55	5.06
14.29	11.84	16.83	—	5.0	2.95	5.82
1/32.8	1/30.2	1/35.2	—	1/37.4	1/41.1	1/23.3
.0570	.0544	.0430	.20	.20	.12	.075
8.22	7.26	9.99	7.6	5.0	7.2	5.7
.0666	.0735	.0601	.169	.178	.148	.086
.860	.740	.716	1.18	1.12	.808	.87
.550	.533	.600	1.28	.89	1.06	.49

quick-response. This optimizing problem means a selection between both small and large values of *K* and *T*. Considering the superiority of small *K* and *T* in the earlier phases of, and more frequent steering, the smaller *K* and *T* are suitable to ships engaged in more confused operations. It should be noted that optimization is important not only by itself, but also for the least necessary rudder size.

In order to conduct design work in this manner, systematic plottings of *K* and *T* values for various kinds of ships with different dimensions and forms would be useful. One of their uses is for selecting suitable *K* and *T* for a certain design by referring to the data of ships in similar service, and another is for estimating the effects of hull form, relative rudder size and other design particulars on the *K* and *T* values. Table I illustrates the *K* and *T* for a number of actual ships obtained through analyses of zig-zag tests. While the data give some idea of *K* and *T* for normal kinds of ships, many more such data are desirable to construct the systematic diagrams of *K* and *T* values mentioned above. Systematic researches employing free self-propelled models should also be utilized for the purpose.

Another and more logical way to estimate suitable *K* and *T* values for a certain design may be provided from the analysis of the steering operations in which the ship concerned would be engaged. For example, the necessary *K* and *T* for a whaler in chasing operation may be determined with reference to the representative (or significant) frequency and amplitude of ceaseless variation of her desired course depending upon the unpredictable behaviour of a chased whale. It is probable that this kind of analysis would often require some statistical approach such as the harmonic analysis of random phenomena, and such a treatment should be an important item in the steering problem.

2. EXPERIMENTAL DETERMINATION OF THE STEERING QUALITY INDICES

2.1. *General Considerations*

The present manner of describing the steering quality leads us naturally to an experimental procedure for determining the description for a given ship by observation of her actual steering motions.

In fact, as is shown in the preceding section, the transfer function for a stable ship is:

$$Y_{S(j\omega)} = \frac{\dot{\theta}_{(j\omega)}}{\delta_{(j\omega)}} = \frac{\int_{0-}^{\infty} \dot{\theta}_{(t)}\, e^{-j\omega t}\, dt}{\int_{0-}^{\infty} \delta_{(t)}\, e^{-j\omega t}\, dt} \quad \ldots \ (6)$$

selecting the parameter p as $j\,\omega$. $\dot{\theta}_{(t)}$ is a ship motion excited by $\delta_{(t)}$. Then, simultaneous measurements of an arbitrary $\delta_{(t)}$ and $\theta_{(t)}$ excited by that $\delta_{(t)}$ give us $Y_{S(j\omega)}$ generally through the numerical integrations of $\dot{\theta}_{(j\omega)}$ and $\delta_{(j\omega)}$. These are synopses of the frequency response test and the transient response test. Those procedures suit the relatively fundamental researches with free self-propelled model tests and some specially arranged experiments on the full size, because they introduce somewhat exaggerated instrumentations for actual ships with precise description of steering quality. Since the frequency response and transient response procedures require accurate measurements of angular velocities, a rate-gyro with an electric pick-up may be the most preferable means of measurement. Fig. 6 shows the principal parts of such a rate-gyro used in the Osaka University Tank. While a VHF-wireless control of a free self-propelled model may be an excellent means for those tests, satisfactory results are also given by a "fishing-rod" arrangement shown in Fig. 7, in which a self-propelled model is connected to a control-and-measuring stand on land through a bundle of fine vinyl-shielded cords carried on a fishing-rod.

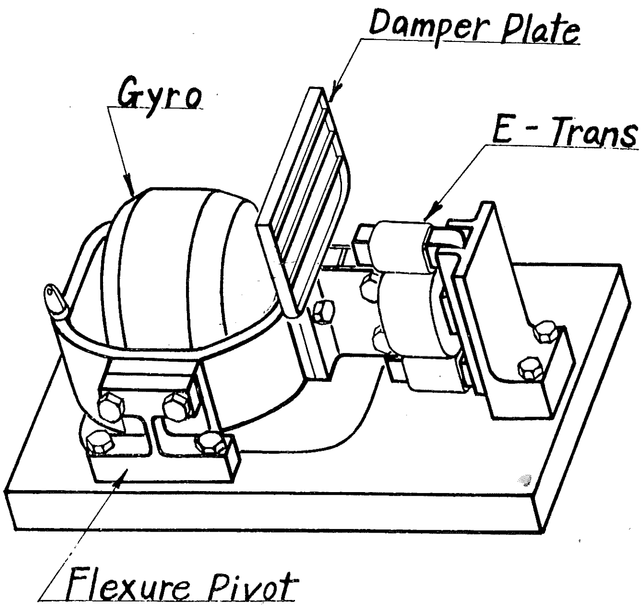

Fig. 6. *Principal parts of rate-gyro, showing upside down*

An electric-driven gyro is hung from a bed plate through a pair of flexure pivots. The gyroscopic moment proportional to turning angular velocity indices a slight deflection of the pivots, which may be measured by a differential transformer with an E-shaped core. The damper plate is immersed in an oil-pot for filtering out the mechanical vibrations and other undesirable "noise".

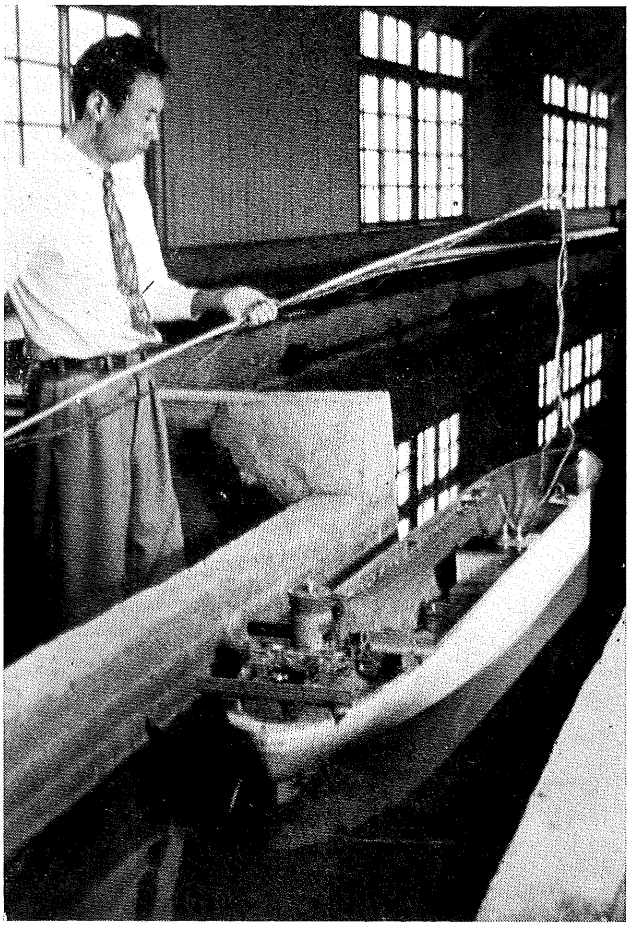

Fig. 7. *A free self-propelled model prepared for steering test*

On the other hand, it may be advantageous for practical researches, if some test means for determining a brief description of steering quality of an actual ship can be devised with no special instrumentation. It is an adequate procedure for the purpose to determine K and T from the Kempf's zig-zag test so that a first-order system defined by that K and T simulates the test results most closely. The necessary observations of ship's heading and helm angle may be performed by a conventional compass and some other simple apparatus.

2.2. *Frequency Response Method — Sinusoidal Steering*

Selecting sinusoidal steering as $\delta_{(t)}$ in equation (6) of the above procedure, it is not necessary to perform the numerical integrations of $\dot{\theta}_{(j\omega)}$ and $\delta_{(j\omega)}$, which means harmonic analysis of $\dot{\theta}_{(t)}$ and $\delta_{(t)}$, because the steering involves only a single frequency. Namely, as is shown in the preceding section, the response of a stable ship to sinusoidal steering is:

$$\dot{\theta}_{(t)} = A_{(\omega)}\, \delta_0 \sin \{\omega t + \varphi_{(\omega)}\}$$

The amplitude ratio $A_{(\omega)}$ and phase difference $\varphi_{(\omega)}$ are:

$$A_{(\omega)} = |\, Y_{S(j\omega)}\, | \quad \text{and} \quad \varphi_{(\omega)} = Arg\ Y_{S(j\omega)}$$

Then, measuring simultaneously the ship motion and her helm angle for various frequencies, $|Y_{S(j\omega)}|$ and $Arg\ Y_{S(j\omega)}$ may be obtained directly. This information is sufficient in principle for the determination of the steering quality indices, K, T_1, T_2 and T_3. Actually the indices may be determined by solving the algebraic simultaneous equations from the values of $|Y_{S(j\omega)}|$ at four frequencies, which are distributed over the frequency range from $\frac{1}{T_1}$ to $\frac{1}{T_2}$ through $\frac{1}{T_3}$.

The frequency response method is the most precise procedure for determining T_2 and T_3, because of the reliability of the method in the higher frequencies. On the other hand, the method has some difficulties in the lengthy test procedure needed for application to actual ships, and in the considerable extent of the test basin required for experiments on the model scale in the low frequency range.

Such tests have been carried out at the Osaka University Tank, a result of which is illustrated in Fig. 12, in a nondimensional form. Sinusoidal steering is performed by an eccentric-mechanism driven by a small DC-motor through a reduction gear, shown in Figs. 8, 9. In order to determine all of the indices with sufficient accuracy the test must be extended to much lower frequencies, which is impossible with a test basin as small as the one used. Since the circumstances are more or less similar in all cases, some other procedure must be devised.

2.3. *Transient Response Method — Impulsive Steering*

From this point of view, another type of steering in which the whole process is finished in a relatively short time is convenient, because such steering does not require so large a test basin. Furthermore, since the impulsive $\delta_{(t)}$ involves sinusoidal elements with the higher frequencies, it is able to provide the information of $Y_{S(j\omega)}$ for relatively higher ω with a fair accuracy. On the other hand, over steep $\delta_{(t)}$ is not desirable because its large amplitude and convulsive manner would disturb the motion beyond the usual linear range. Then the optimum width of the impulsive steering may be about the reciprocal of the highest frequency up to which the determination of $Y_{S(j\omega)}$ is desired.

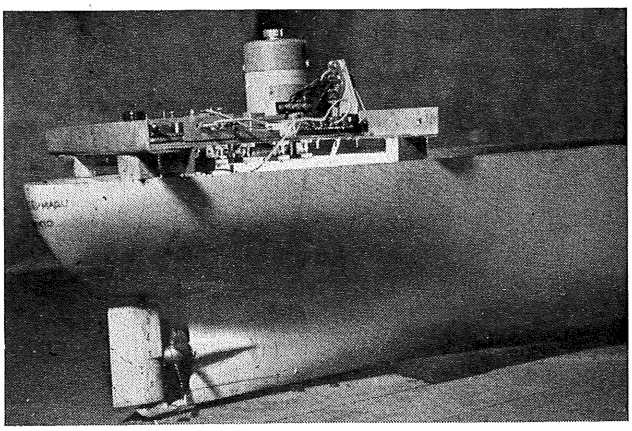

Fig. 8. *Stern arrangement*

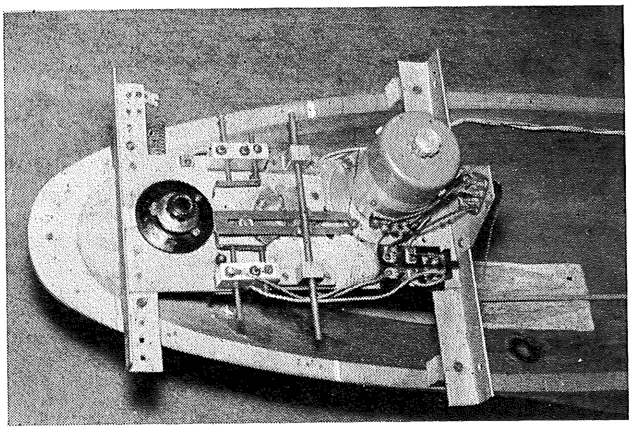

Fig. 9. *Steerling gear*

A pair of position-adjustable electric contacts is utilized to indicate a phase of and to actuate a relay circuit producing a "cosine impulsive steerling". The black disk is a helm angle indicator with fine adjustment of the neutral position of the rudder.

The selection of a detailed form of the impulsive $\delta_{(t)}$ may be arbitrary, that is trapezoidal, trigonometric or parabolic. For instance, the "cosine impulse" that is in practice at the Osaka University Tank is defined as follows (see Fig. 10):

$$\delta_{(t)} = \delta_0 \cos \frac{\pi}{t_1} t \quad \text{for} \quad -t_1/2 \leqq t \leqq t_1/2$$

$$\delta_{(t)} = 0 \quad \text{for all other times}$$

Cosine impulsive steering may be performed by a sinusoidal steering mechanism with a proper relay circuit attached to it. The Laplace transform of the cosine impulse with a parameter $j\omega$ is:

$$\delta_{(j\omega)} = \int_{-t_1/2}^{\infty} \delta_0 \cos \frac{\pi}{t_1} t \cdot e^{-j\omega t}\, dt =$$

$$= \frac{2\ \delta_0\ t_1}{\pi} \frac{\cos \frac{\omega t_1}{2}}{1-(\omega t_1/\pi)^2} \quad \ldots\ldots \quad (7)$$

where the time is measured from $-t_1/2$ in this case, as it is the time from when $\delta_{(t)}$ has started. If the cosine form of $\delta_{(t)}$ is assumed as a whole, the above numerical integration of $\delta_{(j\omega)}$ may be substituted by this formal integration, because any local deviation of $\delta_{(t)}$ has no effect on $\delta_{(j\omega)}$ except at much higher frequencies.

The integration of $\dot\theta_{(j\omega)}$ for a measured $\dot\theta_{(t)}$ may be carried out in two steps, into which the integration is divided at a time t_b. t_b is selected so large that the motion $\dot\theta_{(t)}$ has already become a single exponential decay defined by T_1 at that time. It

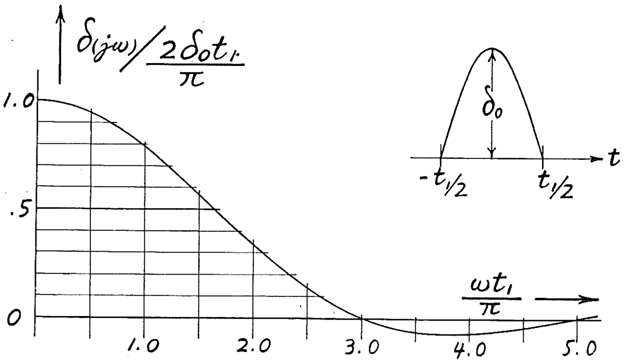

Fig. 10. *Fourier transform of cosine impulse*

must become so, because $\dot{\theta}_{(t)}$ after the finish of $\delta_{(t)}$ is an inherent motion of the form:

$$C_1\, e^{-t/T_1} + C_2\, e^{-t/T_2}$$

and $T_1 \rangle\rangle\, T_2$. Experience shows that t_b equal to twice T_1 seems sufficient for satisfactory results.

The first step is a numerical integration over the earlier and intermediate phases of $\theta_{(t)}$, applying Simpson's rule for each of the real and imaginary parts of the integral, that is:

$$\int_{-t_1/2}^{t_b} \dot{\theta}_{(t)}\, e^{-j\omega t}\, dt = \int_{-t_1/2}^{t_b} \dot{\theta}_{(t)} \cos \omega t\, dt - $$

$$- j \int_{-t_1/2}^{t_b} \dot{\theta}_{(t)} \sin \omega t\, dt \quad \ldots\ldots\ldots \quad (8)$$

where the time is measured from $-\ t_1/2$ again. The second step is the formal integration of $C_1\, e^{-\left(\frac{1}{T_1} + j\omega\right)t}$ since $C_2\, e^{-t/T_2}$ has already vanished at t_b selected reasonably large, hence:

$$\int_{t_b}^{\infty} \dot{\theta}_{(t)}\, e^{-j\omega t}\, dt = \frac{C_1\, T_1\, e^{-t_b/T_1}}{1 + (\omega\, T_1)^2} [(\cos \omega t_b -$$

$$- \omega T_1 \sin \omega t_b) - j\, (\sin \omega t_b + \omega T_1 \cos \omega t_b)]$$

$$\ldots\ldots\ldots \quad (9)$$

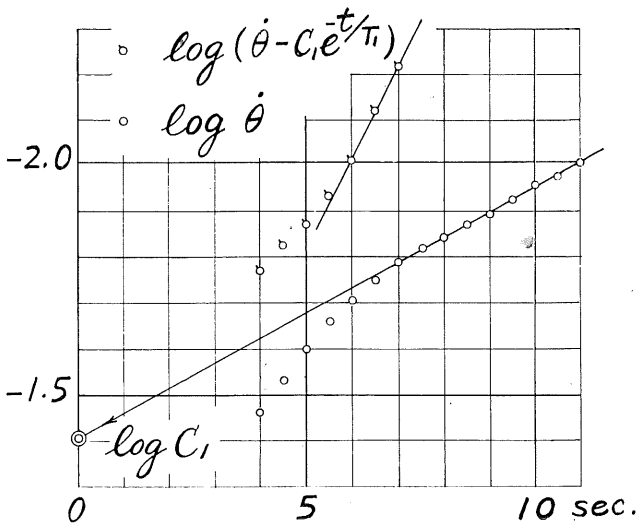

Fig. 11. *Logarithmic plot of measured* $\Theta_{(t)}$

C_1 and a first approximate value of T_1 for this calculation are determined directly from the measured $\dot{\theta}_{(t)}$ by means of a logarithmic plot as illustrated in Fig. 11. Plotting log $\dot{\theta}_{(t)}$ against t, it approaches asymptotically a straight line with a gradient equal to $-\ \frac{1}{T_1}$ log e, because:

$$\log C_1\, e^{-t/T_1} = \log C_1 - \frac{t}{T_1} \log e$$

C_1 may be determined by extending the line to $t = 0$. Now summing up the two integrals, $\dot{\theta}_{(j\omega)}$ may be obtained.

In practice some skill is required to operate a self-propelled model so that the disturbed motion completely vanishes before a transient response test, particularly in a small test basin. Then occasionally the above procedure must be followed by a correction to separate out the residuals of the disturbed motion. According to equation (4), the correction term to be subtracted from $\dot{\theta}_{(j\omega)}$ obtained from the measured $\dot{\theta}_{(t)}$ in the above procedure is:

$$\frac{\{(T_1 + T_2) + j\, \omega\, T_1\, T_2\}\, \dot{\theta}\, (-\ t_1/2) + + T_1\, T_2\, \ddot{\theta}\, (-t_1/2)}{(1 + j\, \omega\, T_1)\, (1 + j\, \omega\, T_2)}\; e^{j\frac{\omega t_1}{2}}$$

The term $e^{j\frac{\omega t_1}{2}}$ is necessary in this case because the origin $-\ t_1/2$, the beginning of steering, is not infinitesimal small. $\dot{\theta}\, (-\ t_1/2)$ and $\ddot{\theta}\, (-\ t_1/2)$ are an angular velocity and its time rate measured at $-\ t_1/2$. T_2 for the correction may be determined as is T_1; plotting log $[\dot{\theta}_{(t)} - C_1\, e^{-t\, T_1}]$ against t, the gradient of its asymptote equals $-\ \frac{1}{T_2}$ log e.

Now $Y_{S(j\omega)}$ may be determined by dividing $\dot{\theta}_{(j\omega)}$ by $\delta_{(j\omega)}$. A result of this procedure is shown in Fig. 12 in the nondimensional form, with the frequency response result. The transient response test may be considered as the one which excites a ship by such $\delta_{(t)}$ as is composed of numerous sinusoidal steerings, and her frequency response for each sinusoidal steering is obtained at one stroke, by resolving the excited motion into sinusoidal elements. Therefore the reliability of the method must decrease rapidly with increase of frequency, because the amplitude ratio $|\, Y_{S(j\omega)}\, |$ decreases with increase of ω, and in addition, actual impulsive steering involves a smaller portion of the higher frequency elements. The superiority of the frequency response method in the higher frequency range results from these circumstances.

In conclusion then, the most reasonable procedure for determination of the steering quality indices from a free self-propelled model test is as follows:

1. to obtain $|\, Y_{S(j\omega)}\, |$ for the relatively lower frequencies from the transient response tests;

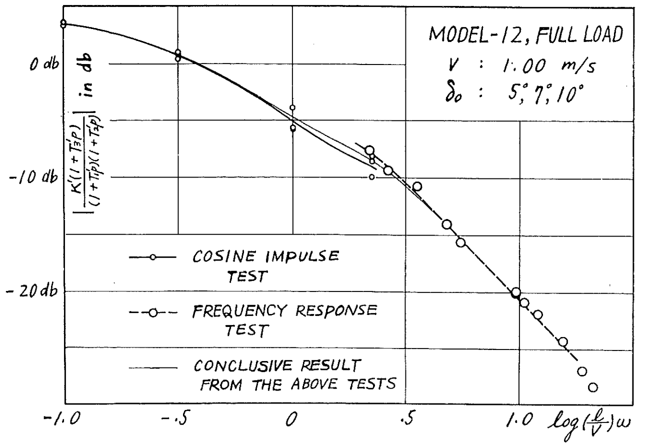

Fig. 12 *Steering test results of a free self-propelled model*

2. to obtain $|Y_{S(j\omega)}|$ for the higher frequency range from the frequency response test;
3. then to determine K, T_1, T_2 and T_3 by solving the algebraic simultaneous equations from four values of $|Y_{S(j\omega)}|$ reasonably distributed over the frequencies. Other values of $|Y_{S(j\omega)}|$ may be used as a check on the results.

A result of this general procedure is also shown in Fig. 12 with a fine line. These researches using self-propelled models may be carried out in a towing tank of medium size or a conventional swimming pool. They are useful for assembling systematic information on the contributions of hull forms, relative rudder areas, stern arrangements and other factors affecting the steering quality. In this connection, it seems that scale effect on steering quality is worth examination, particularly for the usual stern arrangement of most merchant ships. This is the reason why some special researches on steering quality using full-size ships are desirable.

2.4. *Kempf's zig-zag test*

The zig-zag test proposed by Kempf is carried out in the following manner (see Fig. 13):

1. set a certain helm angle (e.g. 15°) to starboard;
2. when ship's course deviation has reached this angle to starboard, reverse the helm to the same angle to port;
3. when the course deviation has reached the same angle to port, reverse the helm again to the same angle to starboard;
   and so on.

This zig-zag steering may be well simulated by a sequence of trapezoidal steerings, as is obvious in the result. Employing the first-order equation of motion (3), the following equations are deduced by calculating the ship motion for the zig-zag steering, where θ_e, θ'_e and θ''_e are succeeding extreme course deviations which appear respectively at times t_e, t'_e and t''_e, and t_2, t_4 and t_6 are the times at which the helm is reversed. That is, equating the calculated $\dot{\theta}_{(t)}$ to zero at t_e, t'_e and t''_e and ignoring some of the exponential terms that become negligible through damping, we obtain for the case where $t_e \geqq t_2 + 2\,t_1$, as is so for most ships:

$$T\,(e^{-t_e/T} - e^{-(t_e-t_1)/T} - e^{-(t_e-t_2)/T} + e^{-(t_e-t_2-2t_1)/T}) = t_1$$
$$T\,(e^{-(t'_e-t_2)/T} - e^{-(t'_e-t_2-2t_1)/T} - e^{-(t'_e-t_4)/T} + e^{-(t'_e-t_4-2t_1)/T}) = t_1$$
$$T\,(e^{-(t''_e-t_4)/T} - e^{-(t''_e-t_4-2t_1)/T} - e^{-(t''_e-t_6)/T} + e^{-(t''_e-t_6-2t_1)/T}) = t_1 \quad \ldots\ldots\ldots (10)$$

For the case where $t_e \leqq t_2 + 2\,t_1$, as is possible in some cases:

$$T\,(1 + e^{-t_e/T} - e^{-(t_e-t_1)/T} - e^{-(t_e-t_2)/T}) = t_e - (t_1 + t_2)$$
$$T\,(1 + e^{-(t'_e-t_2)/T} - e^{-(t'_e-t_2-2t_1)/T} - e^{-(t'_e-t_4)/T}) = t'_e - (t_1 + t_4)$$
$$T\,(1 + e^{-(t''_e-t_4)/T} - e^{-(t''_e-t_4-2t_1)/T} - e^{-(t''_e-t_6)/T}) = t''_e - (t_1 + t_6) \quad \ldots\ldots\ldots (11)$$

Furthermore, substituting t_e, t'_e and t''_e into the calculated $\theta_{(t)}$, for the case where $t_e \geqq t_2 + 2\,t_1$:

$$\frac{1}{K} = \frac{\delta_0}{\theta_e}\,(-\,t_e + 2\,t_2 + 3/2\,t_1)$$
$$\frac{1}{K} = \frac{\delta_0}{\theta'_e}\,(t'_e + 2\,t_2 - 2\,t_4 - 1/2\,t_1)$$
$$\frac{1}{K} = \frac{\delta_0}{\theta''_e}\,(-\,t''_e + 2\,t_2 - 2\,t_4 + 2\,t_6 + 3/2\,t_1) \quad \ldots\ldots\ldots (12)$$

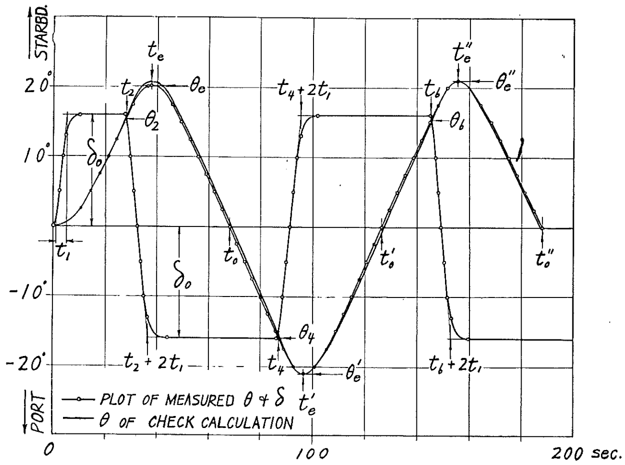

Fig. 13. *Results of zig-zag test for a freighter in ballast condition*

and for the case where $t_e \leqq t_2 + 2\,t_1$:

$$\frac{1}{K} = \frac{\delta_0}{\theta_e\, t_1}\{(t_1 + t_2)\, t_e - 1/2\,(t_e^2 + t_1^2 + t_2^2)\}$$

$$\frac{1}{K} = \frac{\delta_0}{\theta'_e\, t_1}\{-(t_1 + t_4)\, t'_e + 1/2\,(t'^2_e + t_1^2 + t_4^2) + t_1\,(t_1 + 2\,t_2)\}$$

$$\frac{1}{K} = \frac{\delta_0}{\theta''_e\, t_1}\{(t_1 + t_6)\, t''_e - 1/2\,(t''^2_e + t_1^2 + t_6^2) - 2\,t_1\,(t_4 - t_2)\} \qquad \cdots\cdots\cdots (13)$$

These equations may be utilized in the determination of K and T substituting the measured values of θ_e, θ'_e, θ''_e, t_e, t'_e, t''_e, t_2, t_4, t_6, δ_0 and t_1 in them. Actually, since the equations (10), (11) cannot be solved formally, "step by step" or trial and error methods must be employed, starting from a first approximation for T. A convenient way of estimating this approximate T is to equate it to the measured value of $(t_e - t_2)$.

Similar procedures may also be applied to t_o, t'_o and t''_o, the times when the ship's heading has passed the initial course in sequence and to t_2, t_4 and t_6. Equating the calculated $\theta_{(t)}$ to zero at t_o, t'_o and t''_o and to θ_2, θ_4 and θ_6 at t_2, t_4 and t_6 respectively, we obtain for the case where $t_o \geqq t_2 + 2\,t_1$, as is always so for actual ships, so far as is known:

$$T = (t_o - 2\,t_2 - 3/2\,t_1) + \frac{T^2}{t_1}\left(e^{-t_o/T} - e^{-(t_o - t_1)/T} - e^{-(t_o - t_2)/T} + e^{-(t_o - t_2 - 2t_1)/T}\right)$$

$$T = (t'_o + 2\,t_2 - 2\,t_4 - t_1/2) + \frac{T^2}{t_1}\left(e^{-(t'_o - t_2)/T} - e^{-(t'_o - t_2 - 2t_1)/T} - e^{-(t'_o - t_4)/T} + e^{-(t'_o - t_4 - 2t_1)/T}\right)$$

$$T = (t''_o - 2\,t_2 + 2\,t_4 - 2\,t_6 - 3/2\,t_1) + \frac{T^2}{t_1}\left(e^{-(t''_o - t_4)/T} - e^{-(t''_o - t_4 - 2t_1)/T} - e^{-(t''_o - t_6)/T} + e^{-(t''_o - t_6 - 2t_1)/T}\right)$$

and unconditionally: $\cdots\cdots\cdots (14)$

$$\frac{1}{K} = \frac{\delta_0}{\theta_2}\left\{(t_2 - t_1/2) - T + \frac{T^2}{t_1}\left(-e^{-t_2/T} + e^{-(t_2 - t_1)/T}\right)\right\}$$

$$\frac{1}{K} = \frac{\delta_0}{\theta_4}\left\{(2\,t_2 - t_4 + 3/2\,t_1) + T + \frac{T^2}{t_1}\left(-e^{-t_4/T} + e^{-(t_4 - t_1)/T} + e^{-(t_4 - t_2)/T} - e^{-(t_4 - t_2 - 2t_1)/T}\right)\right\}$$

$$\frac{1}{K} = \frac{\delta_0}{\theta_6}\left\{(2\,t_2 - 2\,t_4 + t_6 - t_1/2) - T + \frac{T^2}{t_1}\left(e^{-(t_6 - t_2)/T} - e^{-(t_6 - t_2 - 2t_1)/T} - e^{-(t_6 - t_4)/T} + e^{-(t_6 - t_4 - 2t_1)/T}\right)\right\} \qquad \cdots\cdots\cdots (15)$$

In actual applications of this approach, mean values of somewhat different K and T resulting from the equations (10), (12), (14), (15) or (11), (13), (14), (15) may be defined as the K and T describing the steering quality of the ship if the differences are not large.

Since the equations are calculated for zig-zag steering with equal helm angles on both sides, it is often necessary to apply a correction for "residual helm", i.e. an error of the neutral position of the rudder owing to an incomplete adjustment of the steering gear or the telemotor link, turning effect of the propeller race in a single-screw ship and other miscellaneous factors. Even if the residual helm be almost negligible, its effect on $\theta_{(t)}$ is often appreciable because the effect is cumulative as the test progresses; it is as if a slow turning is added to the ship's motion. In fact, analyses of actual zig-zag tests often show some difference between the maximum turning angular velocities to starboard and to port in spite of the same helm angle on both sides. Assuming that this hidden turning is present constantly during a test, its rate may be estimated as:

$$\frac{1}{2}\left|\frac{\theta_6}{t_6 - t'_o} - \frac{\theta_4}{t_4 - t_o}\right|_{measured}$$

That is, because the measured values of $\dfrac{\theta_6}{t_6 - t'_o}$ and $\dfrac{\theta_4}{t_4 - t_o}$ nearly equal the ultimate rate of the course deviation to starboard and to port respectively including the hidden turning, hence the difference of the two is twice the hidden turning rate. Estimating the rate in this manner, the residual helm correction may be applied graphically, as is shown in Fig. 14. In some cases, the correction affects considerably θ'_e s, t'_o s, and θ_2, θ_4, θ_6, while it has much less effect on t'_e s even for those cases. This is natural because the effect of the residual helm on $\theta_{(t)}$ does not accumulate and consequently

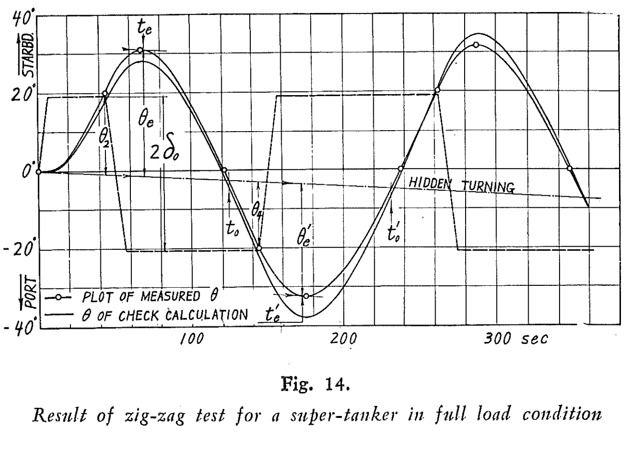

Fig. 14.

Result of zig-zag test for a super-tanker in full load condition does not grow to so considerable an extent. Considering these circumstances, the above approach to T from t'_e s seems more reliable than a similar approach from t'_o s.

Figs. 13 and 14 show two typical test results comparing the calculated motion for mean K and T determined by the present procedure. The disagreement for the super-tanker is that kind which is often found for those ships with large T. It may depend upon speed reduction caused by steering and other nonlinear effects, because it can not be reduced by advancing the order of linear treatment by recalling T_2 and T_3. In other words, it suggests a limitation of the linear treatment. However, it should be remembered that the helm angle of 20° employed in the test is rather large for usual steering. Considering that there are fewer nonlinear effects in the easy motions of usual operation, it is reasonable to carry out a zig-zag test employing a smaller helm angle as in usual steering, say 10°. The use of small helm angle is recommended particularly for the tests in the full-load condition, in which a ship has necessarily a larger value of T. When a steering quality for a larger helm angle is specially required, it may be reasonable to apply a similar procedure to the earlier period of the test up to t_o.

In carrying out the zig-zag test, it is preferable to record the course deviation and the helm angle continually if possible, since such a continuous record provides more reliable results than intermittent measurements at t'_e s, t'_o s and at some other times as is usually done. For this purpose, it is convenient to use an electric-pen-recorder having several pens. Each observer, who is located before a compass and a helm-indicator, presses a push-button actuating individually one of the pens, at intervals of some degrees of his reading — e.g. each 1° or 2.5° of θ and 5° of δ. The time marks are supplied to another pen by an electric-contact watch every second. The continuous measurement may be recommended particularly for a zig-zag test in a sea-trial conducted by the staff of a dockyard. On the other hand, this measurement would be somewhat troublesome for a zig-zag test in a sea way conducted by an officer an board. In such a case, when the usual intermittent observation is employed, attention must be paid to the observation of the helm angle. Since an electro-hydraulic steering gear considerably decelerates when approaching the desired helm angle, it is not reasonable to estimate t_1 from the time when steering is finished. A satisfactory way to estimate t_1 is to measure the period between the start of the reversal of the helm and the time when it passes amidships. Of course, this helm is the actual helm and is not the helm on the steering wheel; the former follows the latter with some time lag. This measurement may be performed by a stopwatch.

Present Conclusion

Although investigations on steering quality according to the present scheme are still under way, it may be concluded that:

1. The steering quality of a ship may be described by a transfer function of the form $\frac{K(1+T_3 p)}{(1+T_1 p)(1+T_2 p)}$ or a set of the steering quality indices K, T_1, T_2 and T_3. These indices, coefficients of the transfer function, are functions of hull form, rudder size and other particulars of a ship. Analytically, they are composed of coefficients of the equations governing the motion.
2. In a more practical sense, steering motions of ships are substantially first-order phenomena, and consequently they are described in brief by a first-order equation of motion:
$$T\frac{d\dot{\theta}}{dt}+\dot{\theta}=K\delta$$
Accordingly, steering quality may be described briefly by two fundamental indices K and T.
3. It may be expected that this expression of steering quality would provide a reasonable basis for design work relating to steering and turning. In this connection, consideration of a feed-back system composed of a ship, a steering-gear and a helmsman or an auto-pilot may be an important aid.
4. Use of the transient response test and frequency response test of a free-running, self-propelled model is an effective means of approaching the steering quality of a given ship. It provides a full description of steering quality in terms of K, T_1, T_2 and T_3 through a procedure not so lengthy as the usual approach of resistance derivatives.
5. Kempf's zig-zag test followed by an analysis employing the first-order equation of motion provides a reasonable figure for steering qualities of an actual ship in terms of K and T. The zig-zag test would bring out many things about steering quality if it were carried out for many ships.

Acknowledgment

Acknowledgment is made of co-operation by the staff of the Kawasaki Dockyard Co., Osaka Shipbuilding Co., Hitachi Shipbuilding Co., Sanoyasu Dockyard Co. and Shioyama Dockyard Co., on sea trials.

Acknowledgment is also made of the help given by the staff of the Osaka University Tank.

APPENDIX

It seems that the transfer functions of ships are linear, at least within usual steering; this means in fact that the governing equations of motion are linear, as has been generally supposed. The electro-hydraulic steering gear that is the most usual type for present sea-going ships, has also a linear transfer function up to a moderate helm angle, while the steering gear direct-driven by an electric motor or a steam engine cannot be regarded as a linear element. Various auto-pilots are also non-linear elements because of their artificial dead-bands for "the weather adjustment", on-off elements as a contactor and a drive motor and in some cases, back-lash in feed-back links for stabilizing the systems. Consideration of these non-linearities is necessary to advance the research on automatic steering of ships beyond the earlier works by Minorski [8] and Schiff and Gimprich [9].

Although the transfer function was defined originally for linear systems, present control engineers such as Kochenburger [10] and Johnson [11] have generalized the concept in a practical sense to the descriptions of non-linear systems. The non-linearities of auto-pilots and direct-driven steering gears are adequately described by this non-linear transfer function or the so-called describing function.

Describing an open character of each element in this manner, the stability of the whole system is characterized by means of the generalized Nyquist criterion [10], [11]. According to that method, the wellknown eratic performances of most auto-pilots in heavy weather may be considered as a sort of self-exciting oscillation of the system, and fair improvements are expected by putting a dead-band with a lowpass character instead of a mere dead-band as is usual, as suggested by Motora [13]. With regard to auto-piloting in a sea way, it is suggested that the principle of "minimizing the root-mean-square error", which in the present case means to minimize the root-mean-square value of the course deviations, may provide a reasonable basis for auto-pilot design. While the method is attractive with its fine logic and is honoured by success in control and communication engineering, there is still some question of its validity with respect to the auto-pilot design. An auto-pilot designed following this principle would require ceaseless steering in heavy sea in response to each wave encountered, while it would surely minimize the yawing as a whole. Considering the maintenance of an auto-pilot and a steering gear, is its performance really so desirable? Probably the method would require a certain modification for practical applications. At any rate, it seems that auto-piloting in a sea way is now the most important subject in the steering problem of ships.

Another approach is the use of analogue-computers or simulators; particularly a single-purposed anglogue-computer with merely several integral units, which simulates a ship and a steering gear in actual time scale may be a simple and effective instrument. Considering the large time constants of ships and easy connection with an actual auto-pilot or its model, the electro-mechanical integral unit employing a two-phase motor and an induction tacho-generator may be superior to the usual electronic integral units. A decided advantage of the method is that it easily determines the response of the whole system to various external forces, even if the system involves some non-linear elements. The simulator also may be useful for determining the character of an actual helmsman by connecting it to him, as if he is on duty behind a steering wheel.

While the dynamic character of a helmsman is naturally a most complicated subject, the daring attempts to describe human operations by transfer functions are having a growing success in control engineering [14], [15]. In the problem of steering of ships, however, the subject is not so severe as in the case of a modern high-speed air-plane, in which the limitation of quick-response of human nervous systems induces a vital difficulty. The studies on the open character of a helmsman may be useful rather in the meaning that his character provides a fair standard for auto-pilot design, since no present auto-pilot is as well-behaved as a skilled helmsman particularly in heavy seas.

References

1. *Akazaki, S.*: "Experimental Investigation on the Turning of Ships". Journal of the Society of Naval Architects in Japan (J.S.N.A.J.), Vol. 61, 1937.
   And the successive papers by the author up to J.S.N.A.J., Vol. 73, 1951.
2. *Weinblum, G.*: "Über die Kursstabilität von Schiffen". Schiffbau, 1938.
3. *Motora, S.*: "Course Stability of Ships". J.S.N.A.J., Vol. 77, 1946.
4. *Davidson, K.* and *Schiff, L.*: "Turning and Course-Keeping Qualities of Ships". T.S.N.A.M.E., 1946.
5. *Inoue, S.*: "Course Stability and Steady Turning of a Ship". J.S.N.A.J., Vol. 90, 1953.
   And the preceding papers by the author from J.S.N.A.J., Vol. 90, 1951.
6. *Kempf, G.*: "Measurements of the Propulsive and Structural Characteristics of Ships". T.S.N.A.M.E., 1932.
7. *Smith, G. A.* and *Triplett, W. C.*: "Experimental Flight Methods for Evaluating Frequency-Response Characteristics of Air-craft". IRD Frequency-Response Symposium. Trans. of A.S.M.E., 1954.
8. *Minorski, N.*: "Directional Stability of Automatically Steered Bodies". J.A.S.N.E., 1922.
9. *Schiff, L.* and *Gimprich, M.*: "Automatic Steering of Ships by Proportional Control", T.S.N.A.M.E., 1949.
10. *Kochenburger, R. J.*: "Frequency Response Method of Analyzing and Synthesizing Contactor Servomechanisms". Trans. American Society of Electrical Engineers (T.A.I.E.E.), 1950, Part I.
11. *Johnson, E. C.*: "Sinusoidal Analysis of Feedback Control System Containing Nonlinear Elements". T.A.I.E.E., 1952, Part II.
12. *Seacord, C. L.*: "Application of Frequency Response Analysis to Aircraft Autopilot Stability". J. Aeronautical Science, Aug., 1950.
13. *Motora, S.*: "On the Automatic Steering and Yawing of Ships in Rough Seas". J.S.N.A.J., Vol. 94, 1953.
14. *North, J. D.*: "The Human Transfer Function in Servo Systems". A paper contributed to the conference at Cranfield, 1951 and publicated in "Automatic & Manual Control" by Butterworths Publications.
15. *Decker, J. L.*: "The Human Pilot and the High-Speed Airplane". J. Aeronautical Sciences, Aug. 1956.