Josephson Current through Charge Density Waves

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The effect of the collective charge density wave (CDW) motion on the Josephson current in a superconductor/charge density wave/superconductor junction is studied theoretically. By deriving the kinetic equations for the coupled superconductor-CDW system, it is shown that below the critical current the CDW does not move. Biased above this value, the Josephson current oscillates as a function of the velocity of the sliding CDW and the collective mode acts as a nonlinear shunting resistor parallel to the Josephson channel. Internal mode locking of the Josephson and CDW frequencies causes oscillations in the current-voltage characteristics and plateaus in the CDW conductance. [S0031-9007(97)04665-6]

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The Josephson effect [1] is known to exist in superconductor hybrid structures, where two superconductors are separated by insulating barriers or normal metals [2]. Recently, the theory of the Josephson effect through systems which support a non-Fermi (Tomonaga-Luttinger) liquid ground state has received much attention due to rapid developments in fabrication technology [3]. In this Letter we investigate the Josephson current through a different yet related system, namely, a strongly anisotropic metal with a charge density wave (CDW) instability.

The ground state of CDW's consist of a lattice distortion coupled to an electron density modulation $n_{\mathrm{CDW}} \propto |\Delta_c(x,t)| \cos[2k_F x - \chi(x,t)]$. The amplitude of the complex CDW order parameter Δ_c is half the Peierls energy gap at the Fermi wave vectors $\pm k_F$, and its phase χ denotes the position of the density wave relative to the crystal lattice. Despite the insulating quasiparticle spectrum, incommensurate CDW's allow for a unique collective mode of transport, which distinguishes them from ordinary insulators. The collective current, which results from the sliding motion of the CDW, is proportional to $\dot{\chi} \equiv \partial_t \chi$, and leads to remarkable electric behavior, such as non-Ohmic conductivity and narrow-band noise [4].

It is interesting to investigate whether the Josephson effect in a superconductor/charge density wave/superconductor (S/C/S) junction will be affected by the sliding CDW motion. Therefore, we consider a current biased S/C/S junction, which consists of parallel one-dimensional CDW chains of length L, sandwiched between two large superconductors characterized by the paring potential $|\Delta_s|$ and phases φ_R and φ_L. Recent progress in controlled deposition of thin films of CDW's may lead to the fabrication of such mesoscopic-scale heterostructures in the near future [5].

Using the Keldysh formalism for superconductors [6] and CDW's [7] we have formulated a consistent framework for the quasiclassical dynamics of the coupled superconductor-CDW system. It is shown that the CDW is immobile under a certain critical current. Biased above this value, the CDW starts to slide and the Josephson current has an oscillatory behavior as a function of the phase $\theta = \varphi_R - \varphi_L + \dot{\chi} L/v_F$, where v_F is the Fermi velocity. Apparently CDW motion induces a dynamical phase which is added to the conventional superconductor phase difference. Finally, we show that in the presence of pinning the collective mode causes a nonlinear shunting resistance parallel to the Josephson channel. Typical current-voltage characteristics show sharp oscillations caused by internal mode locking of the CDW and Josephson frequencies.

The dynamics of superconductor and CDW systems can be described simultaneously by the semiclassical Green functions $g^i_{\alpha\beta}(x;t,t')$ where $i = \{R, A, K\}$ and $\alpha, \beta = \{1,2,3,4\}$. The retarded $\mathbf{g}^R$ and advanced functions $\mathbf{g}^A$ determine the excitation spectrum, and the Keldysh function $\mathbf{g}^K$ describes the kinetics of the system. The subscripts 1, 3 refer to right and left moving electrons with spin up and 2, 4 to left and right moving holes with spin down. Throughout the paper "caret" denotes (2×2) matrices and boldface (4×4) matrices. The Green functions satisfy the equation of motion

$$i\hbar v_F \partial_x \mathbf{g}^i + \mathbf{H} \circ \mathbf{g}^i - \mathbf{g}^i \circ \mathbf{H} = 0 \tag{1}$$

where

$$\begin{aligned}
&\mathbf{H} = i\hbar \partial_t \boldsymbol{\sigma}_3 \boldsymbol{\Sigma}_3 - \Phi \boldsymbol{\Sigma}_3 + \boldsymbol{\Delta},\\
&\boldsymbol{\sigma}_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix}, \qquad \boldsymbol{\Delta} = \begin{pmatrix} \hat{\Delta}_s & -\hat{\Delta}_c \\ \hat{\Delta}_c^\dagger & -\hat{\Delta}_s \end{pmatrix},\\
&\boldsymbol{\Sigma}_1 = \begin{pmatrix} 0 & \hat{1} \\ \hat{1} & 0 \end{pmatrix}, \qquad \boldsymbol{\Sigma}_2 = i\begin{pmatrix} 0 & -\hat{1} \\ \hat{1} & 0 \end{pmatrix},\\
&\boldsymbol{\Sigma}_3 = \begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix}.
\end{aligned} \tag{2}$$

Here Φ is the quasiparticle potential, and σ_k with $k = \{1,2,3\}$ are the three Pauli matrices. The dot operation $\circ$ denotes internal time integrations as well as matrix multiplications. The self-energy term for impurity scattering is neglected throughout this paper. The matrix $\hat{\Delta}_s$ is given by $\hat{\Delta}_{s,11} = \hat{\Delta}_{s,22} = 0, \hat{\Delta}_{s,12} = -\hat{\Delta}^*_{s,21} = |\Delta_s| \exp(i\varphi)$, and $\hat{\Delta}_c = \hat{1}|\Delta_c| \exp(i\chi)$.

It is convenient to gauge away both phases φ and χ by applying the unitary transformation

$$\tilde{\mathbf{g}}^i = \mathbf{U}^\dagger(x,t) \circ \mathbf{g}^i \circ \mathbf{U}(x,t'), \tag{3}$$

where $\mathbf{U} = \exp(\frac{i}{2}\mathbf{\Sigma}_3\chi + \frac{i}{2}\boldsymbol{\sigma}_3\varphi)$. Disregarding local variations of $\dot{\chi}$, we look for a stationary state solution of the form $\mathbf{g}(x; t - t')$, which can be treated by the Fourier transformation

$$\tilde{\mathbf{g}}^i(x; t - t') = \int \frac{d\epsilon}{2\pi} \tilde{\mathbf{g}}^i(x,\epsilon) e^{-i\epsilon(t-t')/\hbar}. \tag{4}$$

The stationary-state equation of motion for the Fourier transformed function is

$$i v_F \partial_x \tilde{\mathbf{g}}^i + [\epsilon \boldsymbol{\sigma}_3 \mathbf{\Sigma}_3 - \tilde{\Phi}\mathbf{\Sigma}_3 - e v_F \tilde{A} \boldsymbol{\sigma}_3 + i|\mathbf{\Delta}|, \tilde{\mathbf{g}}^i]_- = 0, \tag{5}$$

with $\tilde{\Phi} = \Phi + \frac{1}{2}\hbar v_F \partial_x \chi + \frac{1}{2}\hbar\dot{\varphi}$, $\tilde{A} = \hbar\dot{\chi}/2ev_F + \hbar\partial_x\varphi/2e$, and $|\mathbf{\Delta}| = |\Delta_s|\boldsymbol{\sigma}_2\mathbf{\Sigma}_3 - |\Delta_c|\mathbf{\Sigma}_2$, and the brackets $[\,]_-$ denote commutation. From the structure of this equation the well-known duality between the superconductor and CDW phases $v\partial_x\chi \Leftrightarrow \dot{\varphi}$ and $\dot{\chi} \Leftrightarrow v_F\partial_x\varphi$ is observed. The gradient of the superconductor phase and the time derivative of the CDW phase correspond to an electrical current, whereas the gradient of the CDW phase and the time derivative of the superconductor phase correspond to an electronic potential.

The retarded and advanced Green functions in the superconductors are determined from the stationary-state equation of motion Eq. (5) with $\tilde{\Phi} = |\Delta_c| = 0$. Assuming Δ_s to be constant in the superconductors, it is convenient to apply the Bogoliubov transformation to diagonalize $\mathbf{H}$

$$\mathcal{G} = \boldsymbol{\vartheta}^{-1}\tilde{\mathbf{g}}\boldsymbol{\vartheta}, \qquad \boldsymbol{\vartheta} = \begin{pmatrix} \hat{\vartheta}_+ & 0 \\ 0 & \hat{\vartheta}_- \end{pmatrix}, \qquad \hat{\vartheta}_\pm = \begin{pmatrix} u_\pm & -v_\pm \\ v_\pm & -u_\pm \end{pmatrix}, \tag{6}$$

where $u_\pm$ and $v_\pm$ are the gauge transformed BCS coherence factors given by

$$u_\pm = \sqrt{\frac{1}{2}\left(1 + \frac{\lambda_\pm}{\varepsilon_\pm}\right)}, \qquad v_\pm = -\sqrt{\frac{1}{2}\left(1 - \frac{\lambda_\pm}{\varepsilon_\pm}\right)},$$
$$\lambda_\pm = \mathrm{sgn}(\varepsilon_\pm)\sqrt{\varepsilon_\pm^2 - |\Delta_s|^2}\,\Theta(|\varepsilon_\pm| - |\Delta_s|) + i\sqrt{|\Delta_s|^2 - \varepsilon_\pm^2}\,\Theta(|\Delta_s| - |\varepsilon_\pm|). \tag{7}$$

Here $\varepsilon_\pm = \epsilon \mp \hbar\dot{\chi}/2$, and Θ is the Heaviside step function.

For the inhomogeneous S/C/S system Eq. (5) has to be supplemented by boundary conditions, which adequately describe the two superconductor interfaces. Here we restrict ourselves to the ideal case where no defects or potential barriers are present at the interfaces. In order to derive the boundary conditions it will be convenient to decompose the (4×4) Green functions into four (2×2) blocks as follows:

$$\mathcal{G} = \begin{pmatrix} \hat{\mathcal{G}} & \hat{\mathcal{F}} \\ -\hat{\bar{\mathcal{F}}} & -\hat{\bar{\mathcal{G}}} \end{pmatrix}. \tag{8}$$

Following Zaitsev [8], we require that the diagonal blocks are normalized as usual, and the nondiagonal blocks must satisfy the following relations on the left $(x = 0)$ and right $(x = L)$ interfaces:

$$\hat{\mathcal{G}}^2 = \hat{\bar{\mathcal{G}}}^2 = \hat{1}, \tag{9a}$$
$$\hat{\mathcal{G}}\hat{\mathcal{F}} = -\hat{\mathcal{F}}, \qquad \hat{\bar{\mathcal{G}}}\hat{\bar{\mathcal{F}}} = \hat{\bar{\mathcal{F}}} \qquad (x = 0), \tag{9b}$$
$$\hat{\mathcal{G}}\hat{\mathcal{F}} = \hat{\mathcal{F}}, \qquad \hat{\bar{\mathcal{G}}}\hat{\bar{\mathcal{G}}} = -\hat{\bar{\mathcal{F}}} \qquad (x = L). \tag{9c}$$

Together with Eq. (5), it follows that the components of the retarded Green function at the interfaces are

$$\begin{aligned} &\hat{\mathcal{G}}_{11} = \hat{\bar{\mathcal{G}}}_{11} = -\hat{\mathcal{G}}_{22} = -\hat{\bar{\mathcal{G}}}_{22} = 1 \qquad (x = 0, L), \\ &\hat{\mathcal{G}}_{12} = \hat{\bar{\mathcal{G}}}_{21} = \hat{\mathcal{F}}_{11} = \hat{\mathcal{F}}_{12} = \hat{\bar{\mathcal{F}}}_{21} = \hat{\bar{\mathcal{F}}}_{22} = 0 \qquad (x = 0), \\ &\hat{\mathcal{G}}_{21} = \hat{\bar{\mathcal{G}}}_{12} = \hat{\mathcal{F}}_{21} = \hat{\mathcal{F}}_{22} = \hat{\bar{\mathcal{F}}}_{11} = \hat{\bar{\mathcal{F}}}_{12} = 0 \qquad (x = L). \end{aligned} \tag{10}$$

The boundary conditions (10) state that both quasielectrons and quasiholes which move away from the CDW region into the ideal superconducting leads will never be reflected into quasiparticles moving in the opposite direction [9]. The off-diagonal components of the diagonal blocks express Andreev scattering, and the diagonal components of the nondiagonal blocks contain the normal backscattering. The boundary conditions for the advanced Green functions are obtained by the relation $\mathbf{g}^A = -\boldsymbol{\sigma}_3(\mathbf{g}^R)^\dagger\boldsymbol{\sigma}_3$.

In principle Eqs. (5) and (10) are sufficient to calculate the total current I through the system

$$I = \frac{ev_F N(0)}{8}\int d\epsilon\, \mathrm{Tr}\, \boldsymbol{\sigma}_3 \tilde{\mathbf{g}}^K + ev_F N(0)\hbar\dot{\chi}, \tag{11}$$

with e the electron charge and $N(0) = (\pi\hbar v_F)^{-1}$ the density of states at the Fermi level for one spin direction. Details of the calculation will be given elsewhere [10]. Here we present only the final result for the current through the S/C/S junction,

$$I = \frac{e}{\pi}\dot{\chi} + \frac{e}{h}\mathrm{Im}\int d\epsilon \frac{\sin\eta}{\cos\eta - \cos\zeta} h_0^+,$$
$$h_0^+ = \tanh\left(\frac{\epsilon - \hbar\dot{\chi}/2}{2k_BT}\right) + \tanh\left(\frac{\epsilon + \hbar\dot{\chi}/2}{2k_BT}\right). \tag{12}$$

In this expression we have defined

$$\eta = \theta + \xi_+ - \xi_-, \qquad \xi_\pm = \frac{i}{2}\ln\frac{\varepsilon_\pm - \lambda_\pm}{\varepsilon_\pm + \lambda_\pm},$$

$$\cos\zeta = 1 - 2\Big(\cos\frac{\lambda_c L}{\hbar v_F}\sin\frac{\xi_+ + \xi_-}{2} - \frac{\epsilon}{\lambda_c}\sin\frac{\lambda_c L}{\hbar v_F}\cos\frac{\xi_+ + \xi_-}{2}\Big)^2, \qquad (13)$$

$$\lambda_c = \sqrt{\epsilon^2 - |\Delta_c|^2}\,\Theta(|\epsilon| - |\Delta_c|) + i\sqrt{|\Delta_c|^2 - \epsilon^2}\,\Theta(|\Delta_c| - |\epsilon|).$$

The angle $\theta = \varphi_R - \varphi_L + \dot\chi L/v_F$ governs the oscillating behavior of the Josephson current. Evidently, the extra phase factor arises from the line integral of the vector potential $\tilde A$ along the junction $\frac{2e}{\hbar}\int_0^L dx\,\tilde A = \dot\chi L/v_F$. Because the CDW velocity is restricted to $\hbar\dot\chi < |\Delta_s| < |\Delta_c|$ and the maximum supercurrent decreases exponentially $\propto \exp(-L/\xi_c)$, oscillations are expected in the range where L is of the order of the CDW coherence length $\xi_c = \hbar v_F/|\Delta_c|$. In the static limit where $\dot\chi = 0$, Eq. (12) corresponds to the Josephson current through a band insulator with an energy gap $2|\Delta_c|$. Taking the limit $|\Delta_c| \to 0$ reproduces the well-known expressions for an ideal S/N/S junction [11,12].

Equation (12) expresses the total current through the S/C/S junction as a function of the superconductor phases $\varphi_{R,L}$ and the sliding velocity $\dot\chi$. In order to obtain a closed set of equations, however, it is necessary to derive an additional relationship between $\dot\chi$ and the superconductor phase difference. The additional closing relation can be derived microscopically from the self-consistency relation for the phase of the CDW order parameter

$$\int d\epsilon \ \mathrm{Tr}\ \boldsymbol{\Sigma}_1 \tilde{\mathbf{g}}^K = 0, \qquad (14)$$

and the Keldysh functions in the reservoirs. The self-consistent solution for the clean junction requires an electrochemical potential difference $\delta\mu = \mu_R - \mu_L$ in the sliding state $\delta\mu = \hbar\dot\chi$, as in Ref. [9]. This implies that below the critical Josephson current the CDW will not move. Biased above the critical current the CDW slides, but since the collective CDW motion is dissipative due to the contact reservoirs, a small potential difference is induced on the superconducting leads and the superconductor phase difference will evolve slowly in time. As a consequence we obtain the relation

$$\delta\mu = \hbar\dot\varphi/2 = \hbar\dot\chi. \qquad (15)$$

The sliding CDW mode thus acts as a shunting resistor parallel to the Joshephson channel. The above quasistationary approximation, where the dynamics of the superconductor phase is neglected on the Josephson term, is therefore justified *a posteriori* [13].

The assumption of an ideal junction is rather restrictive in realistic systems where interface effects and impurities tend to pin the CDW. Although a fully microscopic treatment is beyond the present work, we will treat pinning effects at a phenomenological level. In the short junction limit $L \ll \hbar v_F/|\Delta_s|$, the dynamical phase $\dot\chi L/v_F$ can be neglected and if additionally $|\Delta_c| \gg |\Delta_s|$ the Josephson current can be written as $I_c \sin\varphi$, where I_c is the critical current. We now can model the S/C/S junction as an electronic circuit shown in the inset of Fig. 1. We have added a normal shunting resistance R_N, which may include the additional conductance due to uncondensed quasiparticles in the system. This circuit is equivalent to a conventional overdamped Josephson junction shunted by a normal resistor and a CDW conductor. The dynamics of the superconductor and CDW phases is governed by two coupled nonlinear differential equations

$$I = I_c\sin\varphi + \frac{eN}{\pi}\dot\chi + \frac{\hbar}{2eR_N}\dot\varphi, \qquad (16a)$$

$$\frac{\hbar\dot\varphi}{2e} = V_T\sin\chi + \frac{e}{\pi}NR_c\dot\chi, \qquad (16b)$$

where the second term in Eq. (16a) is the sliding current through a CDW material consisting of N chains. The second equation describes the dynamics of the CDW with a voltage source $\hbar\dot\varphi/2e$ in the single particle model [14], where V_T represents the threshold value required to overcome the pinning potential and R_c denotes the dissipation.

In the limit $R_N \ll R_c, R_T = V_T/I_c$ the solution is the standard expression of a normal shunted Josephson junction $V = R_N\sqrt{I^2 - I_c^2}$ [13]. In the range $R_N \approx R_c \approx R_T$ the dynamics of the CDW becomes important, however. We have solved Eqs. (16) numerically. Figure 1 shows a typical I-V curve of the circuit. We can distinguish three regions. In the region just above I_c the

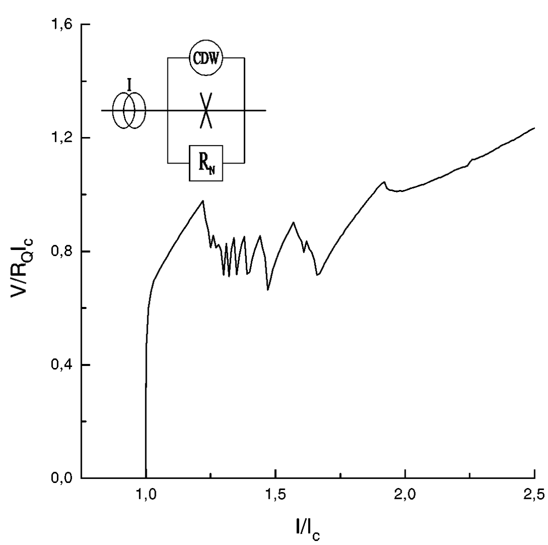

FIG. 1. Typical I-V characteristic of the circuit for R_N, $R_c, R_T = R_Q$. At the onset of depinning sharp oscillations appear, caused by mode locking of the CDW and Josephson frequencies.

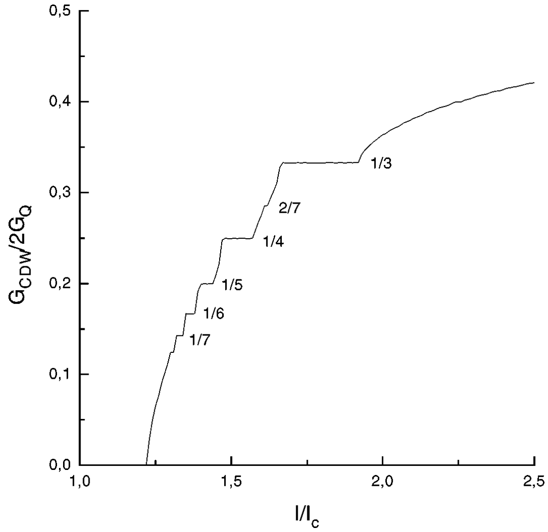

FIG. 2. Plateaus in the CDW conductance at different fractions of the CDW and Josephson frequencies m/n.

Josephson junction is shunted by the normal resistor until the onset of CDW motion. In the large bias regime the CDW slides uniformly and the Josephson junction is shunted by two parallel resistors $R_N R_c/(R_N + R_c)$. In the intermediate region a sharply peaked oscillating structure is seen. This behavior is caused by internal "mode locking" of both the Josephson and CDW frequency. Figure 2 shows the ratio of the CDW and Josephson frequency $\langle\dot{\chi}\rangle/\langle\dot{\varphi}\rangle$ or, equivalently, the CDW conductance G_{CDW} in units of the N-mode quantum conductance $4e^2N/h = 2G_Q$ versus bias current. Harmonic and subharmonic plateaus are observed at different ratios m/n, with m, n integers. In these mode-locked regions the CDW resistance is constant. The voltage across the junction increases with increasing bias current as if effectively shunted by two parallel resistors. Between the plateaus, as the CDW adjusts its resistance to the next (smaller) fraction, the total effective resistance drops, resulting in a decreasing voltage over the junction. Far above the critical current $I \gg I_c$ the Josephson frequency becomes 2 times the fundamental frequency of the narrow-band noise $m/n = 1/2$. This effect is different from conventional mode locking in Josephson junctions and CDW's, where an external oscillating drive causes voltage, respectively, current plateaus in the I-V itself [4,13]. When the resistances R_N, R_c, R_T are of the same order, the oscillations in the I-V should be experimentally observable. Using typical values of a commercial Josephson junction $I_c = 10-100\ \mu$A, $R_N = 10-100\ \Omega$ [15], this condition may be satisfied for several micrometers long CDW samples with threshold fields $E_T = 1-10$ V/cm and damping resistances $R_c = 10-100\ \Omega$.

We conclude by summarizing our results. We have formulated the kinetic equations for a coupled superconductor-CDW system and calculated the current through an ideal superconductor/charge density wave/superconductor junction. In the dc limit the CDW does not move. Biased above the critical current the CDW slides, and the Josephson current oscillates as a function of the CDW velocity. The collective mode acts as a nonlinear shunting resistance parallel to the Josephson channel. Internal mode locking of the Josephson and CDW frequencies causes sharp oscillations in the I-V and plateaus in the CDW conductance.

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