A short review (\*) on

#### RANDOM WAVE BREAKING AND INDUCED CURRENTS

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#### 1. Introduction and summary

The radiation stress concept is basic to the description of currents induced by waves. Gradients of the excess momentum flux due to the presence of waves (or radiation stress) appear as volume forces in the time- and depth-integrated horizontal momentum balance equations. Combined with the continuity equation these three equations yield the mean water surface elevation and both depth-averaged, horizontal velocity components. Since the introduction in 1969/1970 considerable progress has been made with the modelling of depthaveraged currents in the nearshore zone.

In recent years, the modelling of wave-induced currents in the vertical crossshore plane has also made substantial progress. Here, the local time-averaged horizontal momentum equation describes the imbalance between the vertically nonuniform radiation stress and the vertically uniform pressure gradient. This imbalance induces a seaward directed undertow in the cross-shore direction, compensating for a shoreward mass flux above wave trough level.

Latest developments are to combine the latter cross-shore flow field with the aforementioned horizontally two-dimensional flow field to a three-dimensional flow formulation.

In all the above cases either depth-integrated or local radiation stress gradients are important driving terms. Since the magnitudes of the radiation stresses depend on the local wave directions and energy (or more generally the two-dimensional wave energy spectrum), the wave field must be specified a

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priori. This requires a model for nearshore wave propagation with source terms in the energy balance equation of which breaking dissipation is the most important. The classical approach is to adopt monochromatic waves with a constant breaker ratio in the surf zone. With regard to applications, the limitation to monochromatic waves is very restrictive and more or less unnecessary because of the availability of suitable random wave decay formulations, as described in Section 2. A discussion of the several wave-induced flow formulations against the background of the random wave formulation is given in Section 3. Finally, some suggestions for further research are given.

# 2. Energy dissipation, decay and set-up in random, breaking waves

The randomness of wind-generated waves can for many practical purposes be handled with sufficient accuracy on the basis of linear, spectral propagation models with appropriate source terms. However, this approach fails in the nearshore zone because of the highly nonlinear character of the process of depth-induced wave breaking. Yet, modelling of the energy dissipation and attendant wave decay in the coastal zone is important for the prediction of wave-induced currents. This need has prompted the development of non-spectral models in which the local wave field is represented by only three parameters: the energy density (or, equivalently, a characteristic waveheight such as  $H_{\rm rms}$ ), a characteristic frequency (such as the peak frequency  $f_{\rm p}$ ) and a characteristic direction.

All available models for the decay of random, breaking waves known to the authors have in common that they use empirical and/or theoretical knowledge concerning the maximum height  $(H_m)$  which a monochromatic wave field of given frequency (f) can attain in water of a given mean depth (h). This value is in some sense used as an upper limit to the distribution of the local waveheights.

The earlier models (Collins, 1970; Battjes, 1972; Goda, 1975) estimate the local wave energy directly in terms of deep-water wave parameters and the local value of  $H_m$ , which in turn is mainly determined by the depth. These models are not based on an energy balance in which the breaking-induced dissipation is considered explicitly.

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A fundamental drawback of these models is that they are not well founded physically. Practical limitations are that they do not lend themselves well to inclusion of other energy sources or sinks, and that they are basically restricted to areas where the depth decreases monotonically in the propagation direction. The important cases of waves passing over a shoal or bar into deeper water require additional (ad hoc) assumptions.

Battjes and Janssen (1978), hereafter referred to as BJ, presented a model in which the local energy dissipation rate  $(\overline{D})$  in random, breaking waves is estimated explicitly. This is used subsequently in a wave propagation model based on the energy balance in which other sources or sinks can be accommodated without any difficulty. A forward integration procedure is used in which all processes considered, including the breaking-induced dissipation, are adapted to the local depth, so that a uniform formulation suffices whether or not the depth varies monotonically in the propagation direction.

The key element of BJ's model is the consideration of the fraction  $(Q_b)$  of waves breaking at a point as a dependent variable, which is expressed as a function of  $H_{rms}/H_m$ . This function is determined on the basis of a cut off Rayleigh distribution of the waveheights. The value of  $H_m$  in turn depends mainly on the depth h ( $H_m$  = Yh in shallow water, Y being some constant in a given situation).

A second ingredient in BJ's model is fairly classical. It is the estimation of the energy dissipation rate in breaking waves on the basis of the well known analogy to a bore. Combining this with the probability of occurrence of breaking  $(Q_b)$  allows an estimate of the local, mean dissipation rate  $\overline{D}$  in terms of the local energy density (among others).

A calibration and verification of BJ's model on the basis of laboratory and field data (Battjes and Stive, 1985) has shown that BJ's model gives very realistic predictions of the rms-waveheight decay due to breaking, not only on more or less plane beaches, but also on barred beaches and over a shoal. Two examples are given in Fig. 1 and 2, which also show a comparison between predicted and measured set-up of the mean water level. The optimal values of the parameter  $\gamma$ , which occurs in the expression for  $H_m$ , were found to vary slightly but systematically with the incident wave steepness. Using a

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Fig. 1 Comparison of computed results with laboratory data. The figures show profile of bottom elevation below SWL, (d), mean water level above SWL,  $(\bar{n})$ , and rms wave height  $(H_{rms})$  versus distance normal to shore (x). Data points: measured values with offshore reference value encircled. Solid curves: computed values of  $\bar{n}$  and  $H_{rms}$  based on a parameterization of  $\gamma$  (after Battjes and Stive, 1985)

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Fig. 2 Comparison of computed results with field data (Egmond beach). For legend, see Figure 1. The vertical line segments indicate one standard deviation on either side of the plotted data point (after Battjes and Stive, 1985)

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parameterization of this dependence, the values of  $H_{\rm rms}$  in the decay-zone could be predicted with rms relative error of about 6% only.

Thornton and Guza (1983) have presented empirical data on the distribution of the heights of breaking and non-breaking waves in the surf zone. They used these data for a modification of BJ's model, consisting of taking the detailed (parameterized) distribution of the heights of breaking waves into account for the calculation of the expected value of D, whereas BJ's model only uses the estimated rms-value of those heights. However, this modification does not give significantly different results for the prediction of the dissipation rate or the resulting waveheight decay.

In applications of the dissipation model described above for the prediction of nearshore currents, the propagation distances are usually relatively short, and the breaking-induced dissipation is strongly predominant compared to other sources or sinks. Moreover, the peak frequency can then be considered as a constant. However, the model can also be incorporated in wave prediction schemes for larger areas, where wind input and bottom resistance should be taken into account, and the peak frequency is variable. The inclusion in such schemes of an explicit sink representing breaking-induced dissipation is of course particularly important in areas with shallow water and/or strong currents, where it has been found to function satisfactorily (see e.g. Dingemans et al., 1984).

## 3. Wave-induced currents

The first models for wave-induced currents based on the radiation stress concept, started in 1969/1970, dealt with alongshore uniform flow on a plane beach. These were later extended to more arbitrary, horizontally two-dimensional beaches. A review of these is given by Basco (1983; also note the discussion by Kirby and Dalrymple, 1984). In the next two subsections we highlight some aspects of these cases when dealing with random waves. The restriction to depth-averaged current motions is released in the third and fourth subsection where recent formulations are described for the breaking wave-induced return flow or undertow, and the combination of this vertically distributed flow with a vertically distributed nearshore current field.

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#### the one-dimensional case

The case considered here is characterized by uniformity alongshore. The first presentations on wave-induced longshore currents (Bowen, 1969, Thornton, 1969, Iwata, 1970 and Longuet-Higgins, 1970) consider monochromatic waves with a constant breaker ratio in the surf zone. If in these models no lateral mixing is taken into account, the resulting longshore current distribution is discontinuous, with zero velocity outside the surf zone, a maximum velocity at the break point and then a gradual decrease to zero at the waterline. To smooth the unrealistic discontinuity in the current profile, it is necessary to introduce lateral mixing. This need to smooth the current profile is reduced or even eliminated if a random wave formulation is considered (Collins, 1970; Battjes, 1972). The spatial spreading of wave breaking intensity smoothes the rate of energy dissipation and thus radiation stress gradients, resulting in smooth longshore current profiles.

The latter aspect is well illustrated in the paper of Thornton and Guza (1985), where a comparison is made between field experiments and theory for both a linear and a nonlinear bottom shear stress formulation and with and without lateral mixing. They use their narrow band wave transformation model (Thornton and Guza, 1983, which is in fact an extension of Battjes and Janssen, 1978, see also the foregoing section). The results obtained (see Figure 3) indicate that there is only a marginal need to include a nonlinear bottom shear stress or lateral mixing to obtain the best realizable agreement with the measurements.

#### the horizontally two-dimensional case

The more complex horizontally two-dimensional case requires the important extension of the horizontal momentum balance equations with the nonlinear advective terms. The ten numerical models that attempt to describe the twodimensional current system, examined and rejected for engineering purposes by Basco (1983), either lack the advective terms or are limited to unidirectional, monochromatic waves. The importance of the advective terms is well illustrated by Wu and Liu (1984) and by Wind and Vreugdenhil (1986). The latter compare measured and computed rip currents near a structure. With regard to applications, the limitation to monochromatic waves is particularly

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- Fig. 3 Upper: rms wave height (H<sub>rms</sub>) versus distance normal to shore; computations (solid lines) and measurements (data points)
  - Center:longshore current (v) versus distance normal to shore; computations for linear bottom shear stress with mixing (solid lines) and without mixing (broken lines) and measurements (data points)
  - Lower: longshore current (v) versus distance normal to shore; computations for non-linear bottom shear stress (solid lines) and linear bottom shear stress (broken lines) and measurements (data points)

(After Thornton and Guza, 1985)

restrictive. Inclusion of the random wave decay formulation, e.g. in a unidirectional, parabolic refraction-diffraction wave model (Dingemans et al, 1984) or a multidirectional refraction wave model (Booij et al, 1985) should provide the improvement needed. This is illustrated in Figure 4 where a comparison is made between laboratory measurements in a directional wave basin and computations with a current model including radiation stress terms calculated from the above multidirectional refraction wave model (after Dingemans et al, 1986).

A numerical aspect that requires specific attention is that of the estimation of the spatial derivatives of wave energy, which determine the radiation stress gradients. It appears that careful physical or numerical treatment of the wave field is required to prevent spurious current patterns (see De Vriend, 1984, for an example of a physical trap). An elegant method to overcome this problem is to make use of the property (Longuet-Higgins, 1970; Battjes, 1974) that within the refraction or geometric optics approximation a part of the radiation stress gradient is irrotational; it can be balanced by pressure gradients, so that it does not induce currents. The remaining part is proportional to the energy dissipation rate. If this quantity is known explicity, as in the BJ model, the calculation of spatial energy gradients through numerical differentiation is no longer necessary. To what extent this property is also valid for the general refraction-diffraction situation is still under investigation (Dingemans, 1986). However, the authors expect that it holds for the more general case in view of the conservation of circulation in an ideal fluid driven by conservative forces.

# the vertically two-dimensional case

Experiments confirm that the seaward directed returnflow or undertow in the surf zone -which compensates for the shoreward directed mass flux above wave trough level- is driven by the imbalance between the vertically non-uniform wave momentum flux on the one hand and the vertically uniform pressure gradient on the other hand, in qualitative agreement with the hypothesis of Dyhr-Nielsen and Sørensen (1970). Quantitative evaluations of these ideas leading to models for the circulation have been presented by Dally (1980), Börecki (1982), Svendsen (1984), and very recently by Stive and Wind (1986). The latter study emphasizes the importance of the boundary condition at wave trough level due to the momentum decay above this level, which is the result of wave breaking.

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Fig. 4 Comparison between measurements (upper diagram) in a directional wave basin and computations (lower diagram) with a current model driven by radiation stresses from a random, multi-directional refraction wave model (after Dingemans et al, 1986).

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The above cross-shore flow models are all based on a periodic wave formulation. To apply these models in the case of random waves Stive and Battjes (1984) have shown that satisfactory results are obtained, by simply applying the periodic formulation to that fraction of the waves that are breaking (e.g. as modelled through  $Q_b$ , see Section 2). This is illustrated in Figure 5, which concerns the same Egmond field measurements as shown in Figure 2. The randomwave cross-shore flow model thus obtained has been used to formulate a crossshore sediment transport model. This model was recently extended to accomodate effects of wave asymmetry on the sediment transport rate (Stive, 1985). It has been found to give realistic predictions of the evolution of beach profiles including erosion of a duneface under storm surge conditions.

### the three-dimensional case

So far, the modelling of wave-induced currents in the vertical cross-shore plane has not been integrated in the horizontally two-dimensional models mentioned earlier. A first effort in combining the two aspects of nearshore water motions to a three-dimensional formulation is described by De Vriend and Stive (1986), who apply this formulation in the modelling of nearshore morphology.

Assuming that the wave field in the nearshore region considered is known, they distinguish four consecutive steps in the modelling of the mean currents, viz.

- (a) derivation of a shape function for the vertical distribution of the velocity of the 'primary' current, which is defined as the mean current driven by the depth-averaged wave- and tide-induced forces,
- (b) derivation of the horizontally two-dimensional depth-averaged primary current velocity field,
- (c) derivation of a shape function for the vertical distribution of the waveinduced secondary current velocity,
- (d) derivation of the wave-induced secondary current intensity.

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Fig. 5 Results for the same case as shown in Figure 2 (field, Egmond beach). For legend upper and lower diagram see Figure 1. The center diagrams give the fraction of breaking waves  $(Q_b)$  and the near-bottom return flow velocity (u).

## Ref (a):

It is assumed that the horizontal components of the primary velocity can be described as the product of their depth-average and a normalized vertical distribution function, that is invariant or at most weakly variable in the horizontal plane. With this assumption and a similarity hypothesis for the turbulence viscosity, the horizontal momentum equations can be reduced to an ordinary differential equation describing this vertical distribution function.

The solution also establishes the relation between the primary component of the mean bottom shear stress and the depth-averaged velocity.

## Ref (b):

The depth-mean current field is solved from the two-dimensional, depthaveraged continuity and momentum equations, including the wave-induced forces and the effect of waves on the bottom shear stress (also see a).

## Ref (c):

The wave-induced undertow turns out to be directed opposite to the direction of the wave energy flux. It is assumed to consist of two parts, each of which is written as the product of an intensity that is variable in the horizontal plane and a normalized vertical distribution function that is horizontally invariant or at most weakly variable. This vertical distribution function is computed in a similar way as the one for the primary current. An important boundary condition here is formed by the wave momentum decay above the wave trough level.

## Ref (d):

The wave-induced secondary current intensity or mean undertow is derived from the effects induced by the in-phase relationship between surface elevation and horizontal wave velocity and by the breaking wave surface roller.

The above approach should be considered as a first step towards the integration of the historically separated treatment of longshore and cross-shore water motions (and sediment transports).

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# 4. Suggestions for further research

Against the background of wave-induced currents we suggest the following research efforts.

On the theoretical side the main topic is the modelling of turbulence (eddy viscosity?) and the bottom shear stress. More specifically this requires insight in the detailed interaction between the bottom boundary layer and the fluid right above this layer in the general situation of waves and currents under arbitrary angles. On the experimental side there is still a lack of high quality nearshore current data, especially under well defined conditions. These topics pertain not only to the modelling of nearshore currents. They are also relevant to the modelling of sediment motion (stirring, convection and dispersion under the combined action of waves and currents), and dispersion of pollutants.

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