Noise Amplification and Ill-convergence of Richardson-Lucy Deconvolution

Liu, Y.; Panezai, S.; Wang, Y.; Stallinga, S.

DOI
10.1364/ISA.2024.IW3D.2

Publication date
2024

Document Version
Final published version

Citation (APA)
Liu, Y., Panezai, S., Wang, Y., & Stallinga, S. (2024). *Noise Amplification and Ill-convergence of Richardson-Lucy Deconvolution*. Paper presented at Imaging Systems and Applications 2024, Toulouse, France. https://doi.org/10.1364/ISA.2024.IW3D.2

Optica Imaging Congress (3D, AO, COSI, IS, pcAOP) © Optica Publishing Group 2024

Noise Amplification and Ill-convergence of Richardson-Lucy Deconvolution

Yiming Liu, Spozmai Panezai, Yutong Wang, and Sjoerd Stallinga
Department of Imaging Physics, Delft University of Technology, The Netherlands
s.stallinga@tudelft.nl

Abstract: Richardson-Lucy deconvolution can offer an increase in contrast, but converges poorly, and is sensitive to noise. We show that the Cramér Rao Lower Bound (CRLB) diverges, which explains the problematic behaviour.

An important quest in the field of imaging is to devise instruments and methods to deliver the sharpest and most contrast rich images possible. Computational enhancement of raw images acquisitions are an important and broadly applied inroad to do so. This enhancement can be achieved via image processing steps such as filtering operations that are applied ad hoc, agnostic to the underlying physics of the image formation, or via learning based data driven approaches. Deconvolution, on the other hand, provides an estimate of the underlying object using statistical inference and a model of the image formation. The archetypical algorithm in this field is Richardson-Lucy (RL) deconvolution [1,2]. An important advantage is that it enables reconstruction of out-of-band information, depending on the type of object that is imaged [3].
The application of RL deconvolution to practical imaging settings in astronomy or microscopy has brought to light that the algorithm converges slowly, if at all. Moreover, with increasing number of iteration steps an apparent noise structure builds up, originating from small perturbations of the input due to physical and/or numerical noise (see Figure 1).

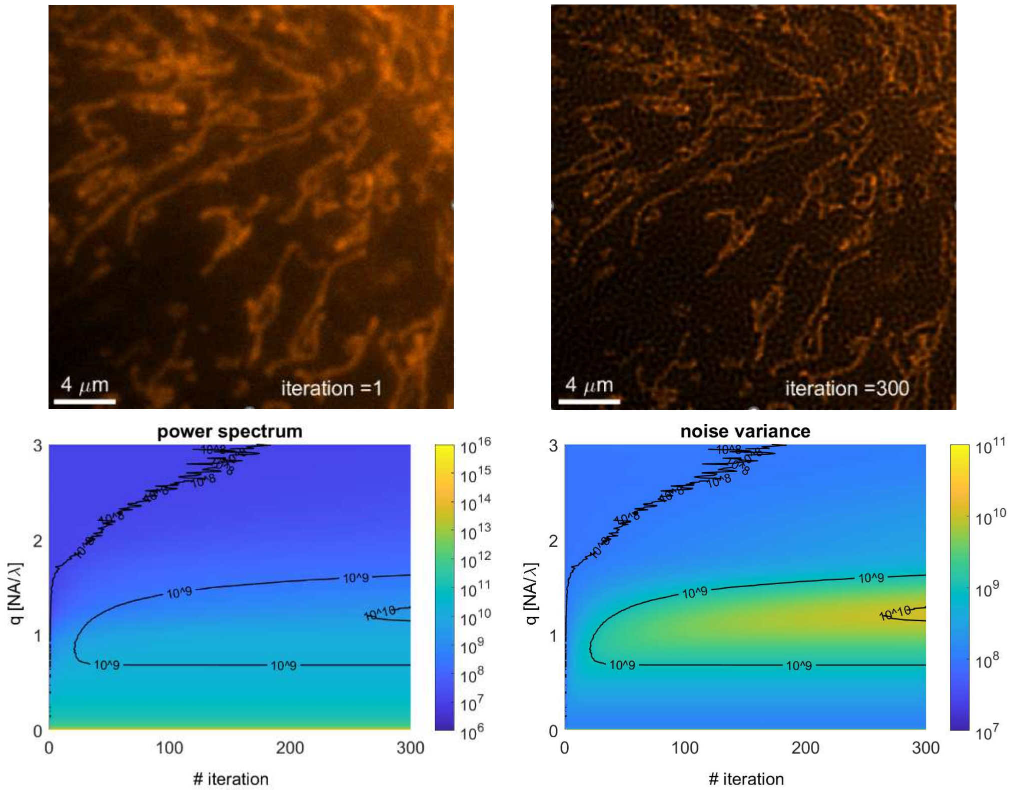

Fig. 1. Top: Experimental acquisition of a sample of stained mitochondria, and RL deconvolution result, showing increased contrast as well as enhanced noise. Bottom: Power spectrum and noise variance in Fourier space as a function of the number of iteration steps, derived from 10 noise independent acquisitions, indicating ill-convergence and noise blowup.

In the presentation, we address the questions why RL deconvolution has a problematic convergence or why the procedure is so sensitive to noise, using a Cramér Rao Lower Bound (CRLB) analysis. As the RL algorithm is a form of Maximum Likelihood Estimation for the ground truth object, a hypothetical well behaved optimum must have a lower bound on the precision of the estimate of the object, and this lower bound is the CRLB. We will show that the CRLB is given by:

$$\langle|\Delta\hat{x}_k|^2\rangle \geq K\frac{\mu_{av}}{|\hat{g}_k|^2} \tag{1}$$

Here, the left hand side of the inequality is the variance of the Fourier Transform of the estimate of the fluorescent object. The lower bound on the right hand side contains the total number of pixels K, the average number of expected photons per pixel μ_{av} and the Optical Transfer Function (OTF) of the microscope $\hat{g}_k$. The index k denotes the spatial frequency $\vec{q}_k$, and the inequality holds for all $|\vec{q}_k| < 2NA/\lambda$, where λ is the wavelength and NA is the Numerical Aperture. As the OTF goes to zero as the spatial frequency approaches the cut-off spatial frequency, the CRLB will diverge. It follow that the original assumption of a regular, well-behaved optimum must be false, and that noise blow-up is an inevitable consequence of doing RL-deconvolution. It also intimately connects noise sensitivity and noise amplification to a lack of convergence of the iterative procedure.In the presentation we will also provide a review of existing and an outlook on new mitigation strategies for this problematic behaviour.

3. References

[1] W. Richardson, "Bayesian-based iterative method of image restoration," J. Opt. Soc. Am. **62**, 55-59 (1972).
[2] L.B. Lucy, "An iterative technique for the rectification of observed distributions," Astron. J. **79**, 745-754 (1974).
[3] R. Heintzmann, "Estimating missing information by maximum likelihood deconvolution," Micron **38**, 136-144 (2007).