DIRECT NUMERICAL SIMULATION OF WEAKLY SPANWISE-ROTATING TURBULENT PLANE COUETTE FLOW

Jie Gai, Zhenhua Xia & Qingdong Cai
State of Key Laboratory for Turbulence and Complex Systems, College of Engineering, Peking University, Beijing, P.R. China

Abstract In this report, we conduct direct numerical simulations (DNS) of weakly spanwise-rotating plane Couette flows at Reynolds number $Re_w = U_w h/\nu = 1300$ (here, U_w is the half the wall velocity difference, and h is half-channel height). A series of simulations with different rotation numbers $Ro = 2\Omega h/U_w$ (Ω is constant angular velocity component in the spanwise direction) is carried out to investigate the effect of Ro on the flow statistics. Our results show that the flow statistics are affected by the Ro, and a "critical" rotation number Ro^* (between $Ro = 0.01$ and $Ro = 0.05$) is observed, where the kinetic energy of secondary flow contributes about a half of the turbulent kinetic energy, and the mean shear rate at the center line reaches a minimum value. We conjecture that different mechanisms should exist around Ro^*, and will be investigated further.

INTRODUCTION

Owing to the effect of the Coriolis force, the shear flow may be either stabilizing or destabilizing depending on the direction of rotation. For a spanwise-rotating plane Couette flow (RPCF), if the system rotation has the opposite sign compared to the mean flow vorticity then the flow becomes destabilized (anticyclonic rotation), whereas the flow becomes stabilized (cyclonic rotation) if they have the same sign.

System rotation substantially affects the mean flow pattern as well as the turbulence structure. Tsukahara et al. [1] reported the results of a systematic experimental investigation into the RPCF, and 17 different flow regimes have been identified. The flow-state diagram showed that, with increasing anticyclonic rotation, the turbulent flow undergoes a transition from a secondary flow with two-dimensional roll cells to a flow in the form of three-dimensional meandering roll-cells. Bech and Andersson [2, 3] applied the DNS to investigate the RPCF with the rotation numbers $Ro = \pm 0.01,\ 0.10,\ 0.20$ and 0.50. The weak but yet obvious roll cells observed already at $Ro = 0.01$ become more regular and energetic at $Ro = 0.10$ and 0.20. At the higher rotation rate $Ro = 0.50$, however, a disordering of the counter rotating vortices appears. The mean velocity at $Ro = 0.01$ exhibits a lower shear around the center line than that at non-rotating case, because the rather steady and persistent secondary flow (or roll cells) causes strong mixing in the wall-normal direction. Besides, the mean shear rate increases when the rotation number Ro varies from 0.1 to 0.5. The changing distribution of the mean velocity is accompanied by substantial changes in the structure of the fluctuating flow field, both in the sense of secondary motion and the residual motions. Based on the study of Bech and Andersson, there must be a minimum value of the mean shear rate in the core region between $Ro = 0.01$ and $Ro = 0.1$. In the present work, we will conduct more DNSs at low Ro cases around this region, and hope to find a more accurate "critical" rotation number Ro^*. Also, the underlying different mechanisms will be investigated.

RESULTS AND DISCUSSION

Ro	Re_τ	T_s	$\frac{\partial \langle u \rangle}{\partial y}\|_{y=0}$	k	k''	k^s
0.00	82.15	23.7	0.1782	3.60	3.46	0.14
0.005	83.46	6.42	0.0675	4.12	2.90	1.22
0.01	84.78	6.52	0.0455	4.28	2.78	1.50
0.02	-	-	-	-	-	-
0.03	-	-	-	-	-	-
0.04	-	-	-	-	-	-
0.05	100.63	7.74	0.0507	6.35	1.65	4.70
0.10	105.87	8.15	0.1102	7.59	1.30	6.29
0.15	107.68	8.28	0.1550	8.75	1.24	7.50
0.20	106.16	8.17	0.2031	9.01	1.67	7.34
0.30	101.33	7.79	0.2864	8.52	2.55	5.97
0.50	89.63	6.89	0.4708	5.31	4.87	0.44

Table 1. Statistics of our simulations. The Reynolds number $Re_\tau = u_\tau h/\nu$, and u_τ is friction velocity. The mean shear rate at the center line are scaled with U_w/h. The sampling time for the statistics T_s are given in h/u_τ. Last three columns are wall-normal averaged kinetic energy scaled with u_τ^2 for $Ro = 0.0$.

In our simulations, a pseudo-spectral method is used to solve the incompressible Navier–Stokes equations on a box with size of $10\pi h \times 2h \times 4\pi h$. The number of grid points is $256 \times 70 \times 256$, which is the same as that used by Bech and Andersson [2, 3]. The flow parameters and the basic flow statistics are listed in Table 1.

In our work, the instantaneous velocity u_i is decomposed into three parts using a triple-decomposition approach [4],

$$u_i(x, y, z, t) = \langle u_i \rangle(y) + u_i'(x, y, z, t) = \langle u_i \rangle(y) + u_i^s(y, z) + u_i''(x, y, z, t)$$

Here, $\langle \cdot \rangle$ denotes averaging over time, streamwise (x-) and spanwise (z-) directions, and u_i^s is the velocity field corresponding to the secondary flow, which is obtained by averaging the velocity over time and x-direction and subtracting its mean value $\langle u_i \rangle$. For clarity, we call u_i' the fluctuation velocity and u_i'' the velocity of the residual field, respectively. Under the same decomposition, the instantaneous kinetic energy $K \equiv \langle u_i u_i \rangle / 2$ (the repeated subscripts imply summation) can be decomposed into three parts, i.e., $K = \frac{1}{2}\langle u_i \rangle \langle u_i \rangle + \frac{1}{2}\langle u_i^s u_i^s \rangle + \frac{1}{2}\langle u_i'' u_i'' \rangle$. Here, $\langle u_i^s u_i^s \rangle / 2 \equiv k^s$ is the kinetic energy of the secondary flow, and $\langle u_i'' u_i'' \rangle / 2 \equiv k''$ is the residual kinetic energy. The turbulent kinetic energy is $k \equiv \frac{1}{2}\langle u_i' u_i' \rangle = k^s + k''$.

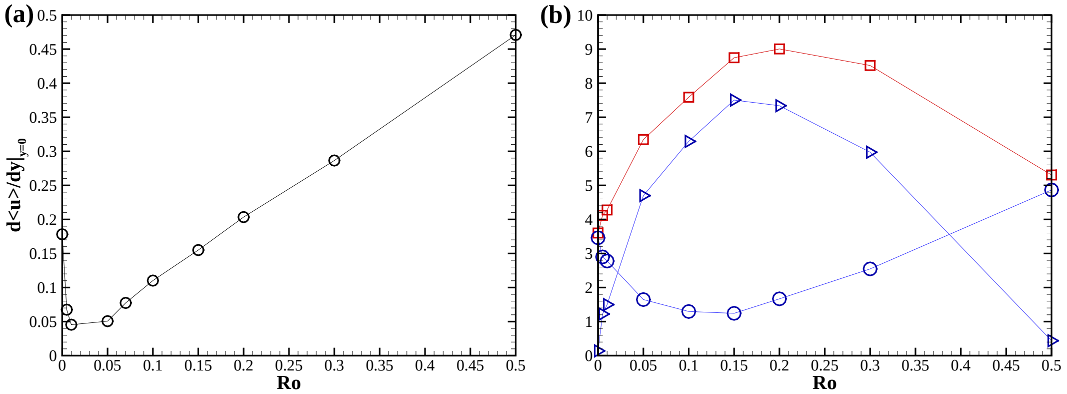

Figure 1. (a) The mean shear rate at the center line. (b) The time- and volume- averaged kinetic energy k_{all} (□), k_{all}'' (○) and k_{all}^s (▷) scaled with u_τ^2 for $Ro = 0.0$.

As shown in Figure 1(a), the mean shear rate at the center line decreases for $0.0 < Ro < 0.01$, and increases almost linearly when the Ro is greater than 0.05. A minimum value can be expected between $Ro = 0.01$ and $Ro = 0.05$. The turbulent kinetic energy (k_{all}) and its two parts (k_{all}^s and k_{all}'') in the whole field are shown in Figure 1(b). It is clearly seen that k_{all} and k_{all}^s increase with Ro when $Ro \lesssim 0.15$ and decrease when $Ro \gtrsim 0.2$, while k_{all}'' shows an opposite behavior to k_{all}^s. An intersection of the k_{all}^s and the k_{all}'' can be clearly identified between $Ro = 0.01$ and $Ro = 0.05$, which is very close to the value when the mean shear rate at the center line reaches its minimum. We denote this value as the "critical" rotation number $Ro*$.

The flow structures from our simulation at $Ro = 0.01$ and $Ro = 0.05$ are the two-dimensional roll cells and the three-dimensional meandering roll cells, respectively. We conjecture that these different flow patterns should be closely related to the changes of the flow statistics, and the underlying mechanism will be studied further in details.

References

[1] T.Tsukahara, N.Tillmark, and P. H. Alfredsson. Flow regimes in a plane Couette flow with system rotation. *Journal of Fluid Mechanics* **648**: 5 -33, 2010.

[2] K.H.Bech and H.I.Andersson. Secondary flow in weakly rotating turbulent plane Couette flow. *Journal of Fluid Mechanics* **317**:195 - 214, 1996.

[3] K.H.Bech and H.I.Andersson. Turbulent plane Couette flow subject to strong system rotation. *Journal of Fluid Mechanics* **347**:289 - 314, 1997.

[4] M. J. Lee and J. Kim. The structure of turbulence in a simulated plane Couette flow. *8th Symposium on Turbulent Shear Flows (Munich)* :5-3-1-5-3-6, 1991.

[5] M.Barri and H.I.Andersson. Computation experiments on rapidly rotating plane Couette flow. *Commun. Comput. Phys.* **7**:683-717, 2010.

[6] K.H.Bech and H.I.Andersson. Growth and decay of longitudinal roll cells in rotating turbulent plane Couette flow. *ADVANCES IN TURBULENCES VI* **36**:91-94, 1996.

[7] Tsukahara Takahiro. Structures and turbulent statistics in a rotating plane Couette flow. *Journal of Physics: Conference Series* **318**: 022024, 2011.