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NON-LINEAR DYNAMICS OF FLEXIBLE RISERS BY THE FINITE ELEMENT METHOD

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ABSTRACT

A finite element solution for the analysis of the non-linear dynamic axial and lateral motions of a flexible riser is presented. The governing equations are derived including non-linear stiffness, excitation due to surface vessel surge, pitch and heave motions, and the effects of waves and currents. The technique accounts for non-linearities such as hydrodynamic damping and large angle deflections in the flexible riser. Results from the simulation model are obtained in the time domain and make use of a general purpose finite element computer program and software developed by the authors. The static analysis under current loading only as well as normal mode analyses are presented and results are included for risers having relatively low bending stiffness.

The results from the dynamic simulation of the flexible riser, using the finite element approach, are compared with those obtained in scale model experiments, where riser motions are monitored using a purpose-designed underwater TV viewing system. The correlations obtained are used to improve the computer simulation model particularly through a better definition of the relevant boundary conditions. These correlations also provide a check on the relative accuracy of results obtained by the mathematical and physical models.

NOMENCLATURE

C_d	fluid drag coefficient
C_m	added mass coefficient
D	outer diameter of the riser pipe
EA	axial rigidity of the riser pipe
EI	bending rigidity of the riser pipe
F_{EA}	external force along the riser element
F_{GJ}	external torsion along the riser element
F_v, F_w	external forces in two v, w orthogonal direction to axis of riser element respectively
$f(x)$	polynomial functions defined shape
$F(x)$	external loading function (on the riser element)
GJ	torsional rigidity of the riser pipe
K_{ij}	stiffness value in i,j row and column o[f] elemental stiffness matrix
$K_{T_{ij}}$	stiffness value in i,j row and column o[f] elemental geometric stiffness matrix
KE	kinetic energy of differential element
m	mass per unit length, includes rise[r] mass, mass of internal fluid and adde[d] mass term
m_u	effective mass per unit length along th[e] riser element
m_x	effective moment of the riser element
M_{ij}	mass value in i,j row and column o[f] elemental mass matrix
PE	potential strain energy of differen[tial] element
q_i	ith amplitude function
s	length of riser
$s(x)$	shape function
T	effective tension
T_o	constant effective tension in element
T'	varying effective tension, defined i[n] the text
t	time
u, v, w	deformations of differential elemen[t] along and orthogonal to axis of riser
$\bar{V}, \dot{\bar{V}}$	relative velocity and acceleratio[n] orthogonal to axis of riser
$\bar{W}, \dot{\bar{W}}$	relative velocity and acceleratio[n] orthogonal to axis of riser
$\bar{W}_v, \bar{W}_w$	components of riser apparent weigh[t] orthogonal to axis riser

Dynamic Matrix Equation

$[M]$	mass matrix
$[C]$	damping matrix
$[K]$	stiffness matrix
$[F]$	force matrix
$[K]_o$	elemental flexural stiffness matrix
$[K]_T$	elemental geometric stiffness matrix
$[K]_{To}, [K]_{T'}$	elemental stiffness matrix defined i[n] equation (8)
$\{q\}, \{\dot{q}\}, \{\ddot{q}\}$	displacement, velocity and acceleratio[n] nodal amplitude matrices

Superscripts in Matrix Equation

A	assembled matrix
e	element matrix

Subscripts in Matrix Equation

G	global coordinate system
L	local coordinate system

Greek Symbols

α, β, γ	angle deformations of differential element
ρ	density of sea water
λ_i	eigenvalue
ω_i	eigen-frequency (radians per unit time)

Miscellaneous

$(\dot{\ }), (\ddot{\ })$	first and second derivatives with respect to time
$(\ )', (\ )''$	first and second derivatives with respect to axial length
$\partial u/\partial s, \partial^2 u/\partial s^2$	first and second partial derivatives of u with respect to s
$\partial^2 u/\partial q_i \partial q_j$	second cross partial derivatives of u with respect to q_i and q_j
$\sum$	summation sign
$\int$	integral sign

INTRODUCTION

The dynamic behaviour of an offshore structure can be of considerable importance in relation to its design since motions and loads under the prevailing environmental conditions may be dominated by dynamic components. This can apply to both normal operating conditions, which have a bearing on fatigue assessment, and also maximum design sea conditions when peak values are of primary interest. In recent years, the flexible riser has received considerable attention as it has been regarded as a viable alternative to rigid marine risers in offshore floating production systems. A number of theoretical investigations for flexible risers have been carried out. The catenary solution and stiffened catenary solution were adopted in static analysis (1,2). These approaches have a particular value in providing initial conditions for dynamic analysis with a simple form. The finite element method can be conveniently applied in both static and dynamic analysis (2,3,4,5,6). This approach allows one to handle the numerous problems related to the non-linear geometry, excitation due to surface vessel vibration, environmental loadings from waves, currents and the boundary conditions from the seabed and the subsurface buoy. Investigations using tests at full scale and model scale have also been carried out (7,8).

This paper is an extension of previous work on flexible risers. The theoretical approach is based on nonlinear finite element methods. The technique accounts for nonlinearities due to geometry, hydrodynamic damping and complex boundary conditions. The dynamic analysis has been carried out in the time domain and use is made of a general purpose finite element computer program. Scale model experiments were designed to complement the theoretical studies and to provide suitable validation of the program.

GOVERNING EQUATIONS

A flexible riser can be represented by a flexible structural member. It has relatively low bending stiffness and the axial forces may play a significant part in dynamic responses. The usual beam/column equations will not be suitable for solving the problem. In the theoretical development, the basic assumptions are that the strains are small, although the displacements of the riser itself will be large due to the overall flexibility of the system. Furthermore, it is assumed that motions in the direction of one principal axis (see Figure 1) only affect motions in the other through the coupling effect due to the non-linear axial forces. The governing equations can be conveniently handled by use of a local element oriented coordinate system. Figure 1 shows that a differential element oriented arbitrarily in three dimensional space is defined in terms of the local element coordinate system. The governing equations in three dimensions can be written

$$m\frac{\partial^2 v}{\partial t^2} + \frac{\partial^2}{\partial s^2}\left[EI\frac{\partial^2 v}{\partial s^2}\right] - \frac{\partial}{\partial s}\left[T(s)\frac{\partial v}{\partial s}\right] = F_v - \bar{W}_v$$

and

$$m\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial s}\left[EI\frac{\partial^2 w}{\partial s^2}\right] - \frac{\partial}{\partial s}\left[T(s)\frac{\partial w}{\partial s}\right] = F_w - \bar{W}_w \qquad (1)$$

where

$$F_v = \tfrac{1}{2}\,\rho.C_d.D\,\bar{V}|\bar{V}| + \frac{\pi}{4}D^2.\rho.C_m.\dot{\bar{V}}$$

$$F_w = \tfrac{1}{2}\,\rho.C_d.D.\bar{W}|\bar{W}| + \frac{\pi}{4}D^2.\rho.C_m.\dot{\bar{W}}$$

and $\bar{V}$, $\bar{W}$ are relative velocities in the two principal axes of the element and $\dot{\bar{V}}$, $\dot{\bar{W}}$ are fluid accelerations. $\bar{W}_v$, $\bar{W}_w$ are components of apparent weight in the two principal axes of the element.

If the effects of torsion and elongation are also included we obtain

$$m_u\frac{\partial^2 u}{\partial t^2} - EA\frac{\partial^2 u}{\partial s^2} = F_{EA} \qquad (2)$$

and

$$m_\alpha\frac{\partial^2 \alpha}{\partial t^2} - GJ\frac{\partial^2 \alpha}{\partial s^2} = F_{GJ}$$

where m_u is the effective mass per unit length along the element
m_α is the effective moment
F_{EA} is the applied forces along the element
F_{GJ} is the applied torsion in the element

The finite element method is a convenient way to solve these nonlinear equations. The method has the advantage that it is possible to model different parts of the riser system with different levels of sophistication. Another advantage is the capability of modelling the complex boundary conditions including all the moving supports (viz surface vessel motions, subsurface buoy motions) with six degrees of freedom, as well as representing any riser/seabed interaction effects.

FINITE ELEMENT FORMULATION

The formulation of the relevant equations in developing the finite element model is based on energy principles. The potential and kinetic energies of a differential element of length l are given by

$$\text{Total} = PE + KE$$

$$= \frac{1}{2}\int_0^l \{[EI(v''^2 + w''^2) + T(v'^2 + w'^2)$$

$$+ EA\,u'^2 + GJ\,\alpha'^2]$$

$$+ [m(\dot{v}^2 + \dot{w}^2) + m_u\,\dot{u}^2 + m_\alpha\dot{\alpha}^2]\}dx \qquad (3)$$

The shape functions are chosen to represent the element shape using each of the 12 unit deformations at the two ends of the element. These functions, $f(x)_i$, $i = 1,12$ are generally polynomials in terms of the length, l, along the element and may be defined by the algebraic sum of each function times its corresponding amplitude, q_i, $i = 1,12$. The shape functions of an element are then given as

$$g(x) = \sum_{i=1}^{12} f(\alpha)_i q_i \qquad (4)$$

By applying the energy minimisation principle, the dynamic matrix equation in the local coordinate system becomes

$$[M]^e_L\{\ddot{q}\}^e_L + [C]^e_L\{\dot{q}\}^e_L + [K]^e_L\{q\}^e_L$$

$$= \{\bar{F}\}^e_L \qquad (5)$$

where

$$M_{ij} = \partial^2 KE/(\partial\dot{q}_i\,\partial\dot{q}_j)$$

$$K_{ij} = \partial^2 PE/(\partial q_i\,\partial q_j)$$

$\{F\}^e_L$ is the generalised force vector and is calculated from

$$F_i = \int_0^l f(x)_i F(x)_i dx$$

where $F(x)_i$ is the distributed applied loadings on the riser element due to weight, buoyancy and hydrodynamic forces, $[C]^e_L$ is the damping matrix accounting for structural damping and is included for completeness, but generally may be neglected because of the much larger hydrodynamic damping components in $[F]^e_L$.

ELEMENTAL STIFFNESS MATRIX

The elemental stiffness matrix is given by

$$K_{ij} = \partial^2 PE/\partial q_i \partial q_j$$

Since the axial force, T, appears in the total expression for the potential energy, the elemental matrix will now take the form

$$[K] = [K]_o + [K]_T \qquad (6)$$

where $[K]_o$ is a general flexural stiffness matrix of a beam element, which can be found in standard text books. $[K]_T$ is called the geometric stiffness matrix.

This geometric stiffness matrix, $[K]_T$, can be obtained by differentiation of the effective tension in equation (3):

$$K_{T_{ij}} = \frac{\partial^2 (PE)_T}{\partial q_i \partial q_j} = \int_0^l T f'_i f'_j dx$$

where

$$(PE)_T = \frac{1}{2}\int_0^l T(v'^2 + w'^2)dx$$

If the axial force, T, takes a linear variation $T = T_o + T'x$, then

$$K_{T_{ij}} = T_o \int_0^l f_i' f_j' dx + T' \int_0^l f_i' f_j' x dx \qquad (7)$$

or

$$[K]_T = [K]_{To} + [K]_{T'} \qquad (8)$$

where $[K]_{To}$ is the stiffness matrix for constant axial force in the element and where $[K]_{T'}$, is the matrix attributable to axial force variation in the element.

The effective axial forces play an important role in the dynamic responses of the riser. In the case of the flexible riser with low bending stiffness, the geometric stiffness matrix, $[K]_T$, may be very significant particularly in the region of the riser where the tensions are large (viz near the upper end).

DYNAMIC MATRIX EQUATION

The elemental mass, stiffness, damping and force matrices must be assembled into a total set of equations in order to arrive at a system solution. This is achieved by using a transformation matrix. The assembled global system of equations is then

$$[M]^{A}_{G}\{\ddot{q}\}^{A}_{G} + [C]^{A}_{G}\{\dot{q}\}^{A}_{G} + [K]^{A}_{G}\{q\}^{A}_{G} = \{F\}^{A}_{G} \qquad (9)$$

The solution for the nonlinear equation (9) usually makes use of time domain solutions.

If the external loadings and floating facility motions are defined the dynamic solutions in the time domain can be carried out. It should perhaps be pointed out that the geometric stiffness matrix is mainly due to the static axial forces. In practice, however, the time varying axial forces have less influence, and are neglected. In practice hydrodynamic forces can be calculated with reasonable accuracy on the assumption that the inclination of a particular element does not change as a result of the dynamic response of the system.

NUMERICAL EXAMPLE

The numerical work is based on the use of a general purpose finite element program together with additional software developed by the authors. This powerful program allows one to take advantage of modelling more elements. This means that one can use small length elements instead of long ones. The description of the structural data and details of a double catenary flexible riser system are given in Table 1.

A quasi-static case was first run to check out the influence of current in the riser assuming a particular offset for the vessel. Figure 2 shows displacements and Figure 3 shows the displacement vectors for this situation neglecting hydrodynamic forces on the subsurface buoy. The evaluation of mode shapes is obviously the first phase in the dynamic analysis. Figure 4 shows the first two mode shapes and Figure 5 shows those for modes 3, 5, 7 and 8. These results show that a number of modes have natural periods in the range of predominant wave energy, i.e. 10 < T < 30 sec, which indicates the possibility of significant dynamic response of the flexible riser. The solution in the time domain was carried out without surface vessel motions. This means that the upper end of the riser was maintained in a fixed position. In practice, this will be the case for fixed platforms or non-compliant floating structures, e.g. DP vessels under light seas and stiffly moored systems. These results can also be used to demonstrate the influence of vessel motions on the riser dynamic response. Figure 6 shows the dynamic responses of the flexible riser under wave action only in true scale. Figure 7 gives an amplified view with displacements times five to illustrate more clearly the response shape for the riser. Figure 8 shows dynamic responses together with the mode shapes. In this particular case, it can be seen that the dynamic response is dominated by the third mode component. This is because the natural period for this mode (8.9 secs) is relatively close to that of the wave period for this particular case (T = 12 secs).

COMPARISON WITH MODEL TEST RESULTS

The computer model was validated by comparison with model test results at 1/50th scale. More detailed discussion of the model tests is given in a previous publication of the authors (2). The model riser was constructed using PVC tubes. It was found that the flexural stiffness was not easy to model correctly at 1/50th scale due to potential buckling problems in the model riser. For this reason, two distorted models were used. They had the same outer diameter but different bending stiffness. In both cases the riser weight was correctly modelled. This meant that both models would be subjected to the same nominal environmental conditions, i.e. the same wave forces. The responses of both models could thus demonstrate the influence of the bending stiffness. The riser motions were monitored using a purpose-designed underwater TV viewing system (Figure 9). The six cameras were used to simultaneously view six points along the riser model. The displacements were continuously recorded and subsequently analysed to provide information regarding the dynamic behaviour of the riser. The bending moments and tensions were measured by means of purpose designed strain gauges. Figure 10 shows typical dynamic responses of the flexible riser from both the finite element solution and scale model tests with a fixed upper end. Figure 11 shows the bending moments at the upper end. It should be noted that the model test results for bending moment are much less than the corresponding calculated values. This is probably because the damping in the experimental model is greater than that assumed for the simulation model. In this connection it is important to note that due to scaling difficulties it was found impossible to correctly model the bending stiffness of the prototype riser system, as has been previously pointed out. The tests with a surface vessel were also carried out. Figures 12, 13 and 14 show the motion of the vessel. These motions were used to give the appropriate support conditions for input to the finite element simulation. Figure 15 shows the dynamic responses with surface vessel excitations from both the computer simulation and the corresponding model tests.

CONCLUSION

A finite element solution for the non-linear dynamic axial and lateral motions of a flexible riser have been presented. The method proposed takes account of such non-linearities as hydrodynamic damping and large angle deflections in the riser. Numerical results have been given for a typical flexible riser. These included a quasi-static solution to demonstrate the effect of current and offset of the surface vessel. The first few mode shapes are then given to show that certain modes can be excited under particular wave conditions for the riser system under consideration. Time domain solutions are given to show the influence of dynamic effects on riser configuration under the action of surface waves.

Some typical results from a programme of scale model tests of a flexible riser system are presented for comparison with the results of corresponding solutions from the numerical model. Although the physical model had of necessity to be distorted in some respects, these comparisons showed that the finite element numerical model would be likely to produce useful and valid information for the purposes of the design of the prototype riser system.

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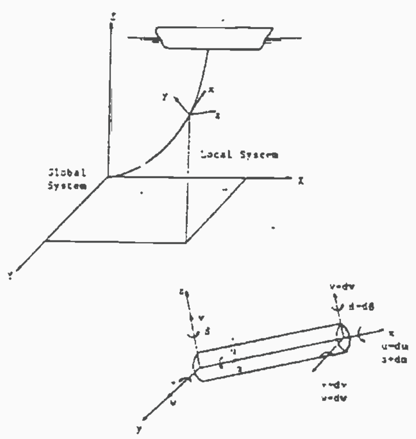


FIGURE 1

Element Deformations in Local System

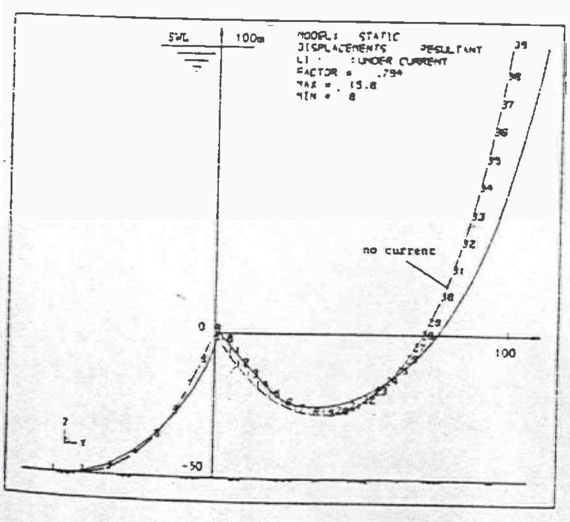


FIGURE 2

Deflection Shape Under Current

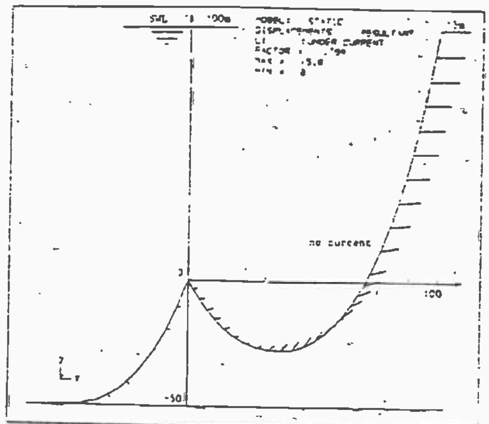


FIGURE 3

Deflection Vectors in Nodal Points

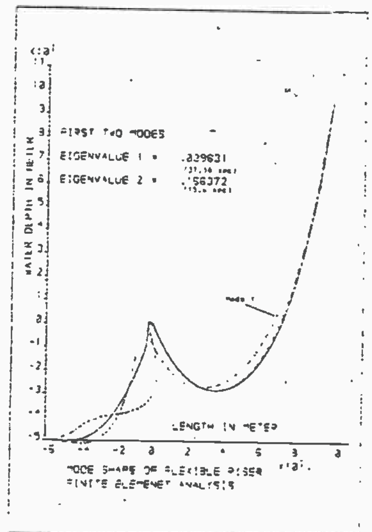


FIGURE 4

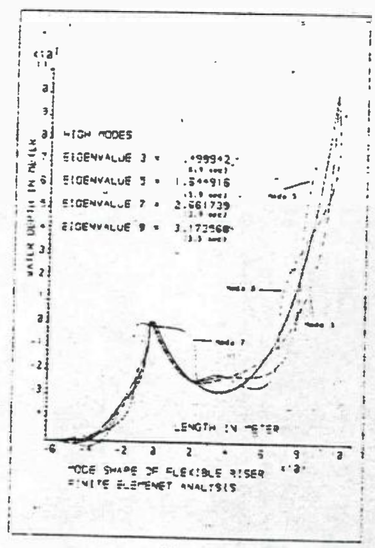


FIGURE 5

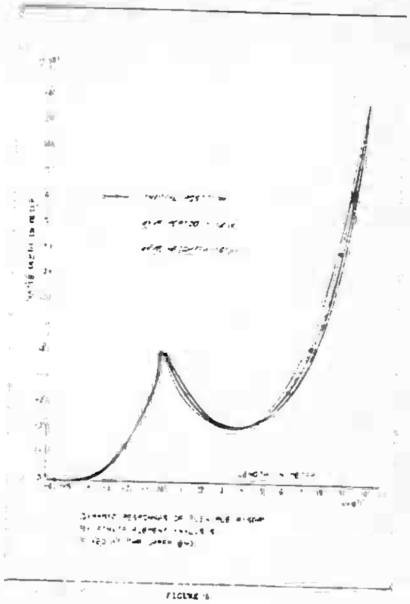


FIGURE 6

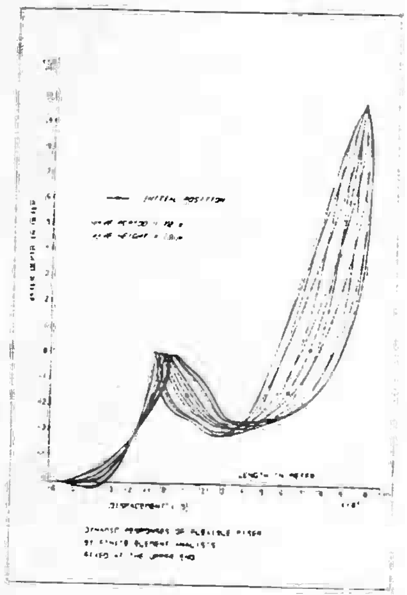


FIGURE 7

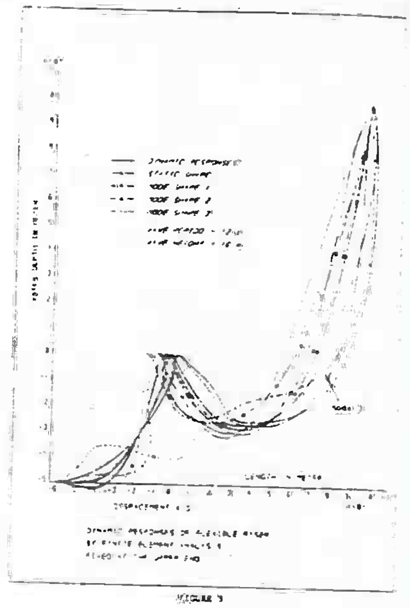


FIGURE 8

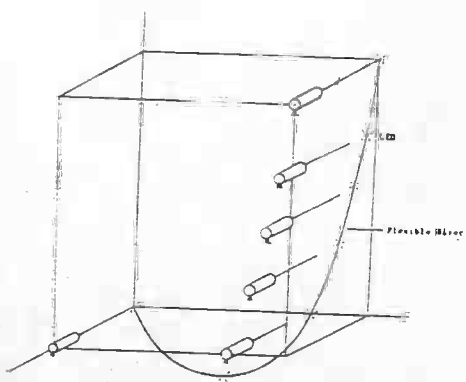


FIGURE 9

Underwater TV Viewing System Monitoring Flexible Riser Motions.

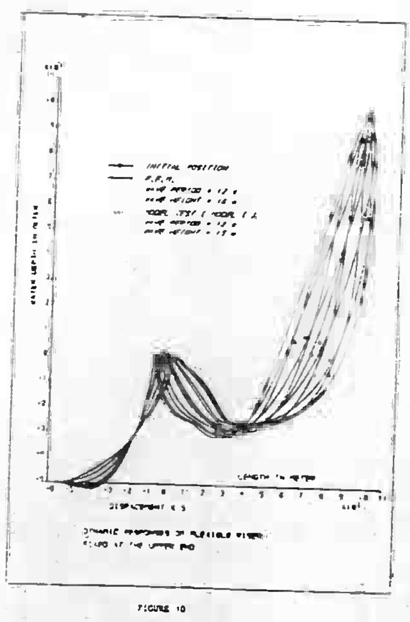


FIGURE 10

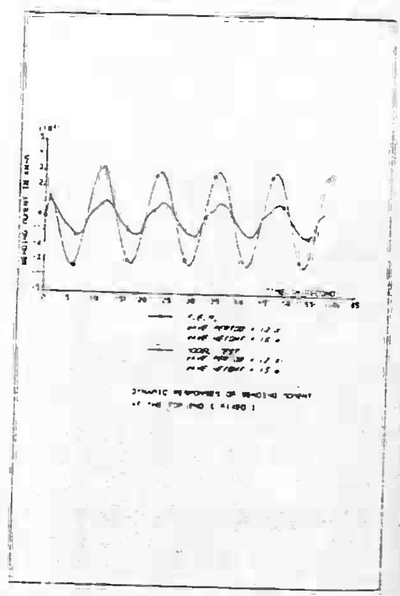


FIGURE 11

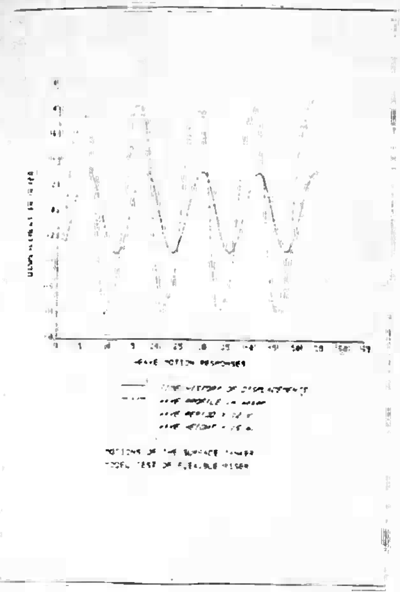


FIGURE 12

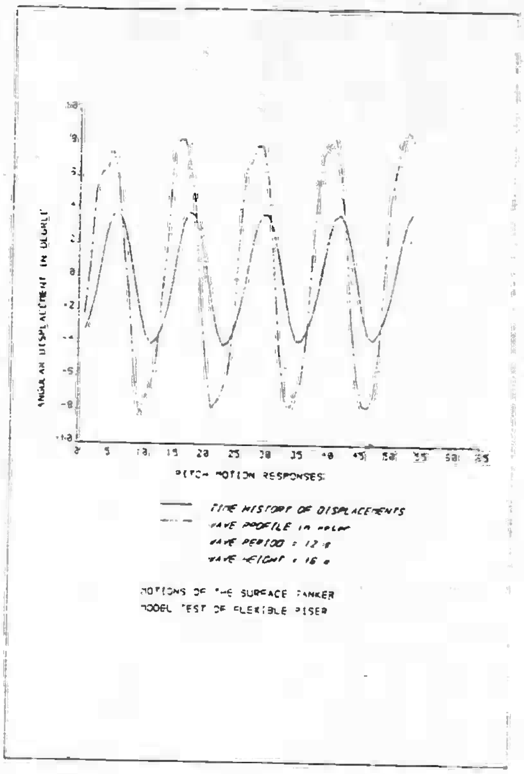


FIGURE 14

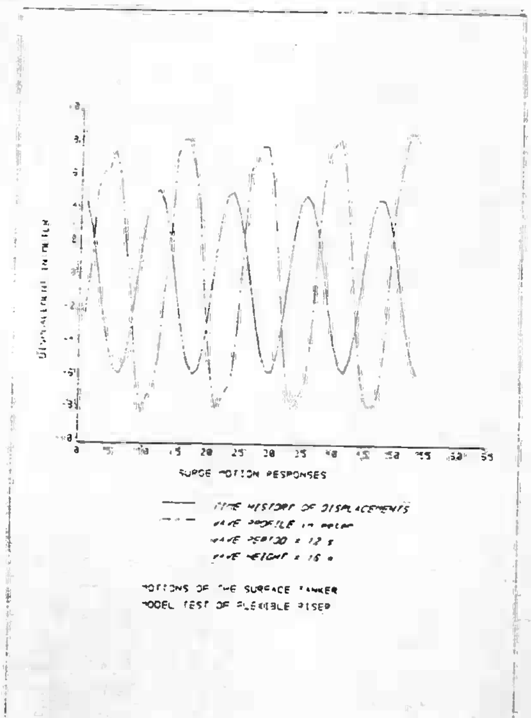


FIGURE 13

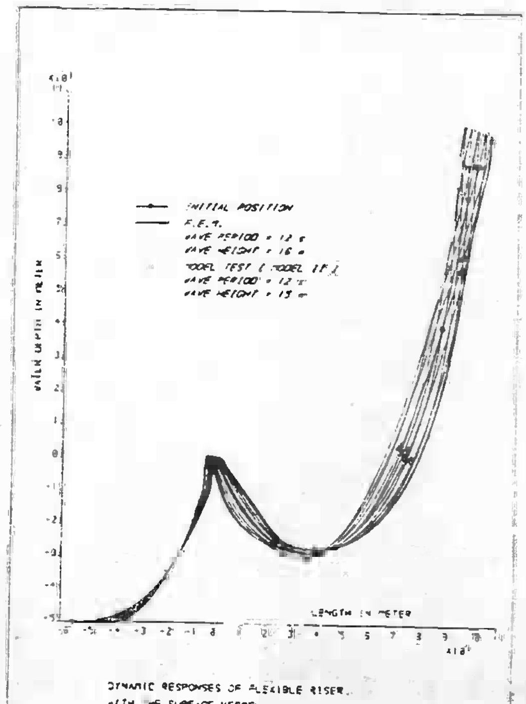


FIGURE 15

TABLE [illegible]

Data for Numerical Case Study of Flexible Riser

Field

Water depth	150m
Subsurface buoy above seabed	50m
Horizontal distance from vessel to subsurface buoy	100m
Well base to subsurface buoy base	55m

Flexible Riser

Outer diameter	0.4m
Inner diameter	0.3m
Bending stiffness EI (or E = 800N/mm^2)	$6.867 \times 10^5 Nm^2$
Axial stiffness EA	$11.529 \times 10^9 N$
Weight in air	160 kg/m
Length from subsurface buoy to deck of surface vessel	200m
Total length	281m
Buoyancy force of subsurface buoy	400,000N

Element Distribution

8 elements between seabed and subsurface buoy
30 elements from subsurface buoy to surface vessel
(elements distributed in relation to initial curvature)
Total elements 38

Quasi-Static

Current velocity	
Surface	1.5m/s
Seabed	0.5m/s

and profile is linearly decreasing from SWL to 50m water depth and constant under 50m depth.

Environment (Dynamic)

Wave height	16m
Wave period	12 sec
Drag coefficient	0.6
Inertia coefficient	2.0
Density of water	1020kg/m^3
Soil support is defined as a spring	$K_s = 5 \times 10^3 N/m$
Time increment	0.25 sec

Surface Vessel Motions

Heave amplitude	± 3.1m
Heave phase angle	- 45°
Surge amplitude	± 5.6m
Surge phase angle	- 270°
Pitch amplitude angle	± 4.1°
Pitch phase angle	- 45°

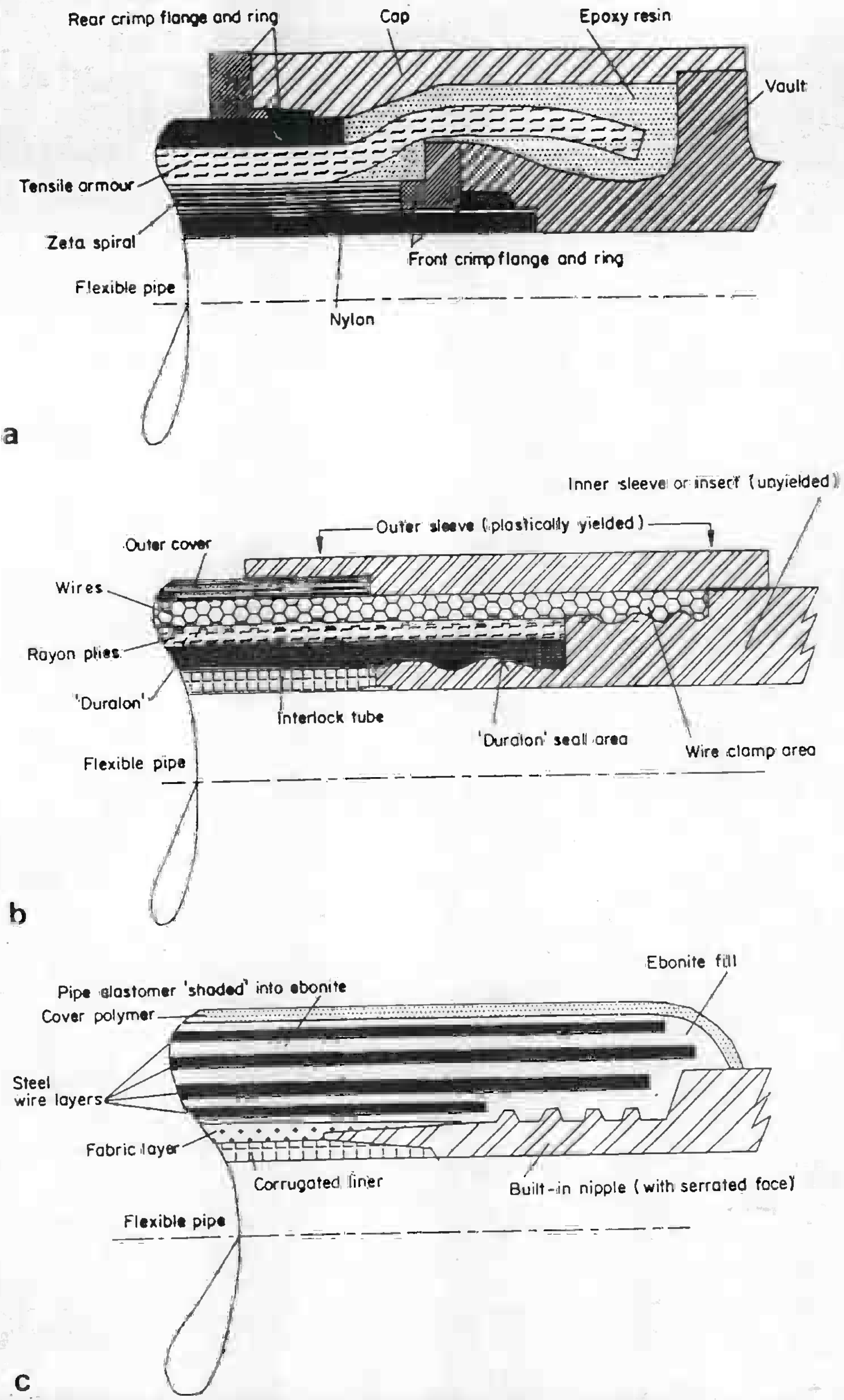
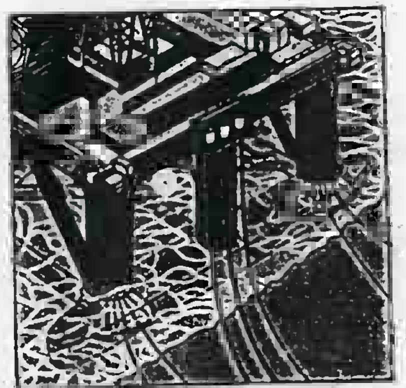


Figure 2 (a) Coflexip termination; (b) Dunlop termination; (c) Pag-O-Flex termination

produce both internal and external resistance to flow. The inner (front) seal which holds the product is in effect axial and, if passed, there is no further resistance to flow radially through the termination unless the 'outer' surface of the crimp ring is designed as a further seal; this is not clear. The manufacturing process would appear to be: (1) to strip the pipe to expose sequential layers, fitting the spiral locking ring; (2) to make up the first crimping flange; (3) to prepare the tensile armours around the termination body; (4) to slide up the cap and, presumably, connect it to the vault; (5) to inject and set the resin; and (6) to make up the final crimping flange. It may be that these last two are reversed, but I know of no published information on the construction of these terminations beyond the brief functional description of Reference 3.

Difficulties with this form of termination arise from the



Flexible Riser Connector System

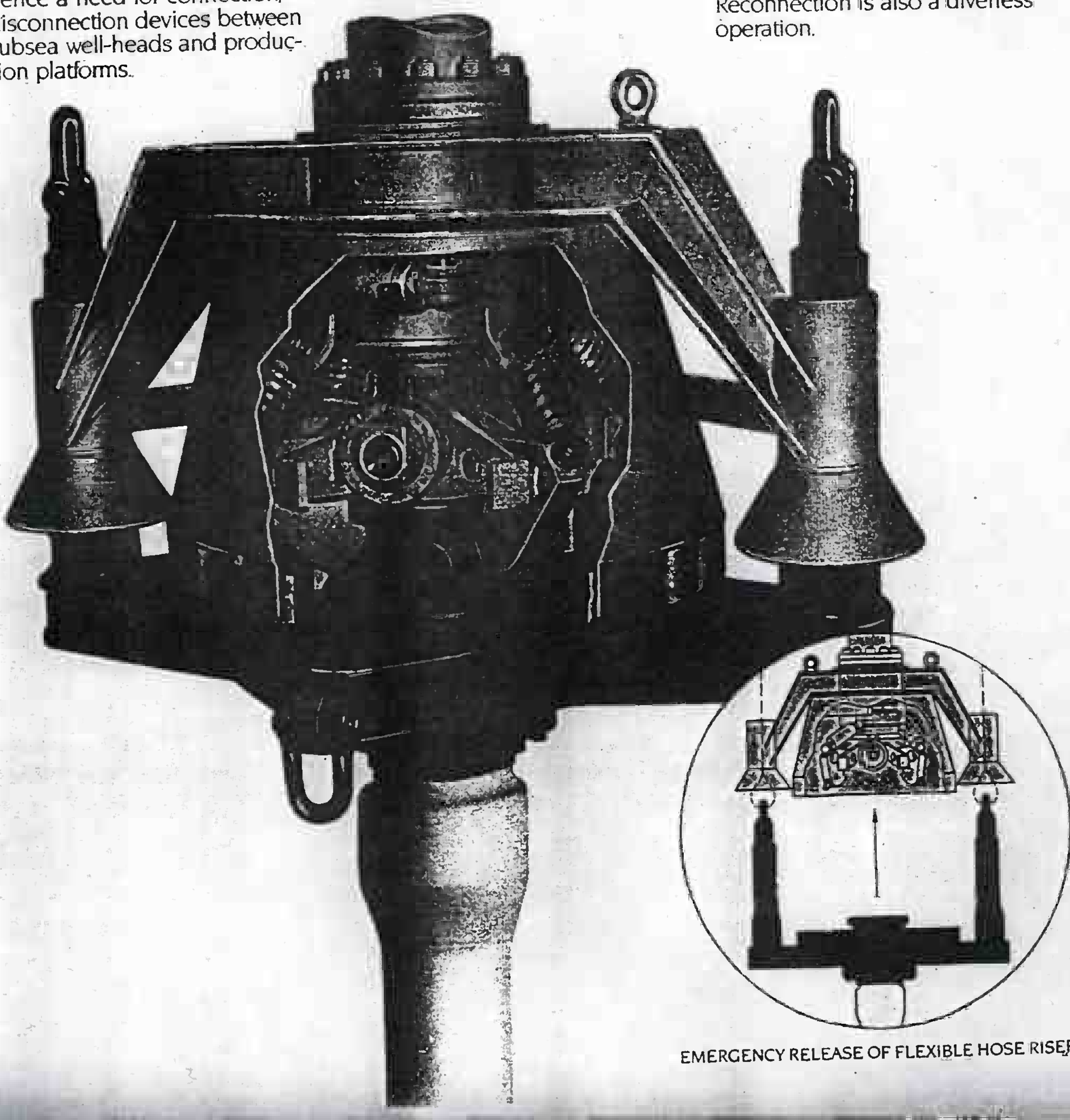
The increasing desire to develop marginal oil fields located in areas where extreme weather conditions prevail has led to the use of floating and temporary production systems.

This, in turn, has led to the use of high pressure flexible risers and umbilical control lines — hence a need for connection/disconnection devices between subsea well-heads and production platforms.

MIB has supplied many custom-designed, high pressure flexible riser coupler systems for production, injection and control umbilical lines to Petrobras, Offshore Brazil and, more recently, to Hamilton Brothers and Sun Oil for their developments on the Duncan and Balmoral Oil Fields.

Based on the well-proven MIB integrated coupler and ball valve design, these systems offer minimal spillage, high angular release, extended maintenance-free periods and long service life.

Installed on the FPV at either pontoon or deck level, shut-off and disconnection of the flexible risers is provided under normal or emergency conditions. Reconnection is also a diverless operation.



EMERGENCY RELEASE OF FLEXIBLE HOSE RISER

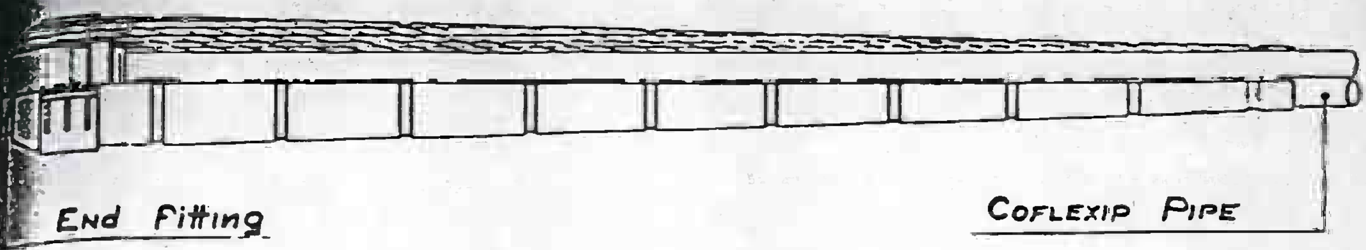


Figure 2.11

Plastic bending restrictor

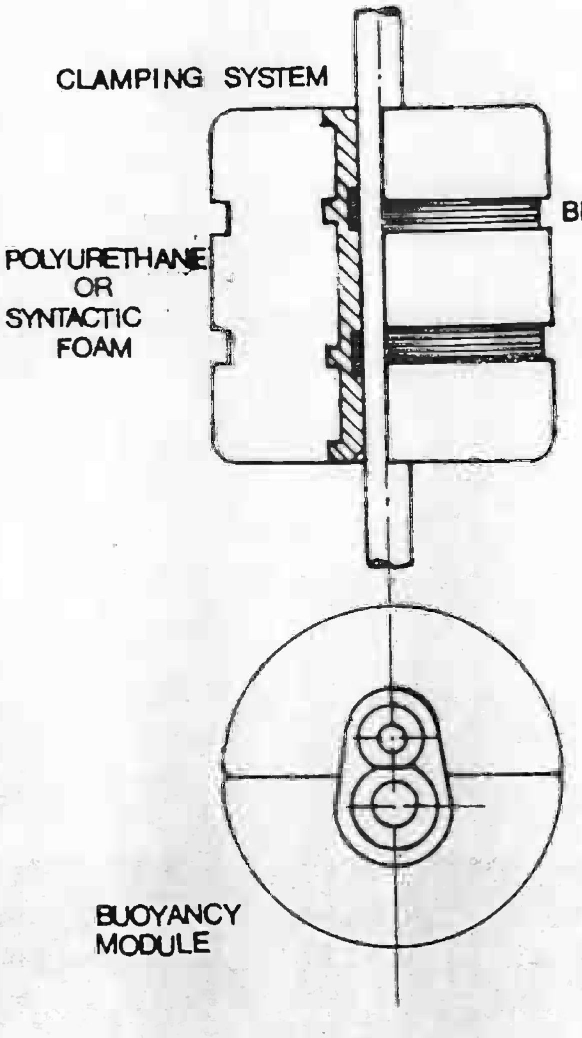


Figure 2.12

Typical buoyancy module

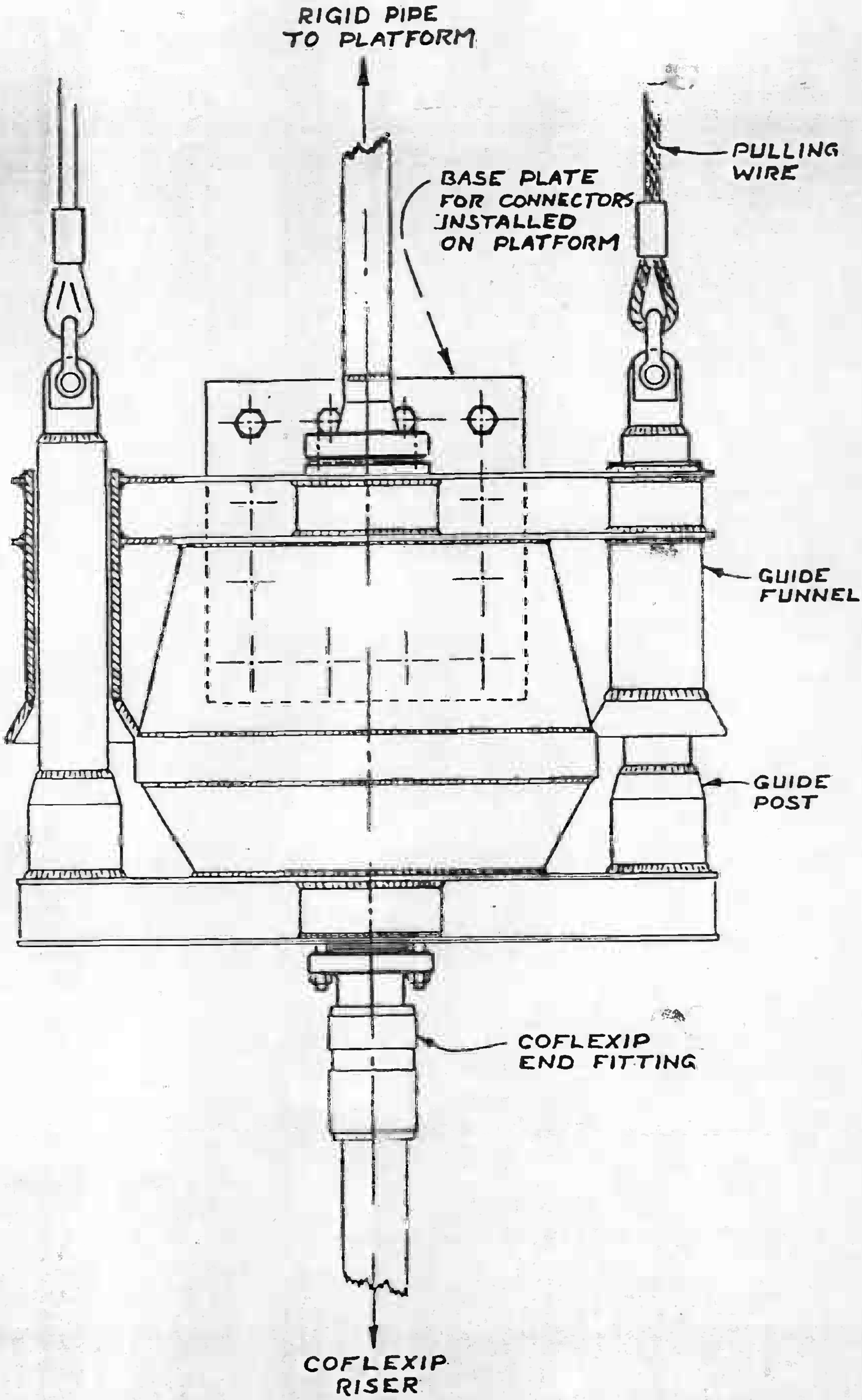


Figure 2.10

A quick-release hydraulic coupling (Coflexip)