The Proposal of a Novel Pattern Matching Technique for Mirror Anamorphosis

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Abstract

Locality mapping captures the displacement imposed by a mirroring surface in anamorphic art. Establishing this mapping for a physical mirror, however, is a difficult task. Applying pattern matching on the distorted reflection is a promising technique, though existing pattern matching strategies are often ill equipped for this particular task. This paper proposes a novel pattern- and pattern matching strategy designed to provide robustness against the artifacts introduced by mirror anamorphosis. The resulting approach, though computationally intense, is able to provide an arbitrary quantity of anchor points given the necessary image quality.

Introduction

Mirror anamorphosis allows for great artistic expression through the use of a reflective surface, morphing a source image into a desired projection. It is, however, not trivial to establish the behaviour of this anamorphosis, and typical approaches come with numerous limitations. This paper will propose a novel solution in the form of pattern matching.

Existing approaches to facilitate anamorphic artistry require object-specific calculations[1] to apply grid formation or mapping functions. This relies on highly regular and well-defined mirroring objects. Manual sampling may be used as an alternative, though this is a very labour-intensive process and is ill suited for use in digital image editors. Ray tracing offers a solution in digital space, but translating physical mirrors to a digital space is not always feasible or even possible. For a physical mirror it is, however, trivially easy to prepare a canvas and take a picture of the resulting mirror anamorphosis.

Digital image processing in the form of pattern matching is widely used in relation to distortion correction, for instance to calibrate cameras [2] and for digitisation of deformed materials[3]. As such, by defining a pattern and analysing the subsequent distortion created through anamorphosis, we may extract the information needed to reproduce this distortion on an arbitrary image. This would be of immense value when creating anamorphic art, and could possibly enable novel optic techniques using mirroring surfaces.

The aim of this research is to deploy pattern matching to establish a translation of locality between a given source image and its reflection to a reasonable degree. To this end, a novel approach to pattern creation- and matching will be laid out as well as its strengths and drawbacks.

The question this paper aims to answer is whether and how we can create a globally unique pattern in such a way that correspondence can be established with its reflection, regardless of the distortion. In proposing a technique to resolve this, we shall provide an analysis on its performance given a resolution and an expected distortion.

To setup a pattern matching approach for mirror anamorphosis, a few steps must be taken. To begin, a pattern must be defined and constructed. This pattern is then applied in the mirror anamorphic scene, resulting in an anamorphic image. Such images must then be clustered based on the colours of the source pattern. This should yield a set of adjacencies between various coloured clusters. A matching algorithm can then use these adjacencies to find correspondence between the source pattern and its anamorphised depiction.

We show that the resulting locality mapping correctly captures the super majority of points in clear images, and only discards a few markers in case of significant visual obstruction. It is further shown that the pattern used can be defined for an arbitrary resolution and can be configured to use any number of colours.

Related Work

Pattern matching is a well-established field with a broad range of approaches. As such, this Related Works section will not be exhaustive. Most intuitive and popular among patterns for pattern matching is the checkerboard pattern. It is widely used in fields such as camera calibration[2] and tracking fabrics in video footage [4]. This approach typically uses the ubiquitous black-and-white checkerboard pattern to establish locality mapping. By detecting the boundaries between black and white tiles, the distorted pattern can be mapped. The greatest advantage of this strategy is its computational simplicity and robustness against colour distortion. In addition, this initial computational simplicity enables the use of corner- and vertex detection algorithms [5] without any ambiguity or complexities introduced through colour distortion. However, finding the global position of any given tile requires a direct or indirect adjacency to both a horizontal and vertical edge of the full pattern. For many applications this relative position can be enriched through inference, but for mirror anamorphosis this is not possible to nearly the same extent.

Two requirements for a pattern matching solution in mirror anamorphosis particularly distinguish this from typical applications: full locality on a partial reflection, and possible multiplicity in the reflection. Firstly, it is the goal of pattern matching in mirror anamorphosis to establish the locality mapping between the source image and its reflection. However, there is no guarantee that the full pattern is reflected. This strongly limits the utility of inference in determining global positioning. Furthermore, mirrors may reflect the same part of the source pattern multiple times. This reduces the applicability of inference even further.

The Hilbert curve is a notable instance of a space-filling curve and already applied in other computer science fields where dimensionality reduction is needed. A particularly valuable property of the Hilbert curve is the interchangability of its fractal pattern, which allows for a localised increase or decrease of curvature [6].

Pattern Generation

The main objective of the pattern we aim to create is to provide global uniqueness. This means that any sufficiently large continuous sample of the pattern must occur only once. This uniqueness must be robust against the kinds of distortion observed in mirror anamorphosis. For this reason adjacency is the attribute used to guarantee uniqueness.

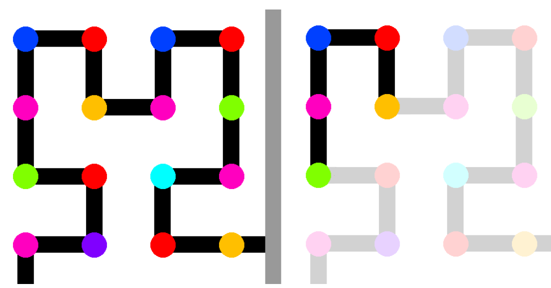

Figure 1: *No singular colour is unique. However, through adjacency, we can establish unique sequences.*

As a way of reducing the dimensionality of the problem, a line will be used to demarcate the space. Thus, uniqueness must be established along a line segment rather than across a two dimensional surface. The shape of the line is not considered, only its continuity and consequent adjacency.
The shape of the line is largely arbitrary, since its shape is not instrumental to establishing uniqueness. The Hilbert curve was chosen to somewhat evenly and unilaterally traverse the plane with the line segment, while avoiding straight segments as much as possible. The Hilbert curve is a space-filling fractal curve. In its lowest dimension, it consists of a "u" shape. Each subsequent dimension consists of four of the lower dimension instances connected together in a particular orientation.

Coloured dots will be used to uniquely identify points along the line, since these are robust against distortion. To establish uniqueness through these colours it must therefore hold that any given sequence of sufficient length must be unique. The dots may be placed arbitrarily along the line. However, placement along the grid used by the Hilbert curve allows for uniform spacing along it. Additionally, this means all connective line segments are straight. Though not used within this paper, some amount of information could be extracted from the resulting distortion.

Let $C_n = \{c_0, c_1, ..., c_{n-1}\}$ be a set of n colours, and C_n^* denote the set of all possible strings of colours from C_n. Let us define $k > 0$ as the minimum length substring that we desire to be uniquely identifiable. It is important to note that this uniqueness must be robust against mirroring, since we may not necessarily be able to maintain the same direction of traversal.

Let us further define $s_n(S, i, k)$ as a function returning a substring of S starting at index i of length k in the form $(s_i, s_{i+1}, \ldots, s_{i+k-1}) \subseteq S \in C_n^*$ with $s_n^{mirror}(S, i, k) = \{s_{i+k-1}, \ldots, s_{i+1}, s_i\} \subseteq S \in C_n^*$

We can thus define a string $U_{k,n} \in C_n^*$ to be uniquely identifiable on k-length substrings when:

$$\forall x, y \left(\begin{array}{c} s_n(U, x, k) = s_n(U, y, k) \iff x = y \\ \wedge \\ s_n(U, x, k) = s_n^{mirror}(U, y, k) \iff x = y \end{array} \right)$$

Though we cannot trivially compute the longest such sequence, we can calculate its upper limit. The longest theoretically possible sequence would contain every unique k-length colour sequence in C_n^*, excluding mirrored instances. We can combine all non-palindromic sequences in mirroring pairs. By taking half of this set, combined with the set of palindromes, we would acquire the maximum number of substrings. Since we know each palindrome corresponds to $\lceil \frac{k}{2} \rceil$-length string in C_n^* we know that there are $|P_{n,k}| = n^{\lceil \frac{k}{2} \rceil}$ k-length palindromes in C_n^*. Considering that a string with t k-length substrings will have a length of $k + t - 1$, we can define an upper limit for the length at

$$|U_{k,n}| \leq k - 1 + \frac{n^k - |P_{n,k}|}{2} + |P_{n,k}|$$

Upper bound for length of $|U_{k,n}|$

	k=3	k=4	k=5	k=6	k=8
n=3	20	48	139	383	3328
n=4	42	139	548	2085	32903
n=5	77	328	1629	7880	195632
n=6	128	669	4000	23441	840463
n=8	290	2083	16644	131333	8390663

Notes

The choice of a Hilbert curve is somewhat arbitrary: it is simply that its properties are quite desirable. Namely, proximity of points on the curve correlates well with proximity in two dimensional space. In addition, no singular property of the mirroring surface would unilaterally obstruct the pattern as it is highly curved compared to for instance a snake-like pattern. Due to the fractal nature of the Hilbert curve, additional techniques may be possible but are not explored here.

Clustering

In order to extract any information out of an image, it must first be subdivided into its constituent regions. For the technique outlined in this paper, this subdivision is based on clusters of a predefined set of colours: background clusters (white), connective clusters (black) and marking clusters (the colours from C_n).

Let P represent the two dimensional array containing the colour values for each pixel, and a decider algorithm $d(a, b)$ be defined such that

$$d(a, b) = \begin{cases} 1, & \text{if } a \text{ and } b \text{ should be clustered} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

i.

We create two-dimensional integer array matching the size of P, with each value starting at -1. This array will track the cluster indices for each pixel. This array we shall call P'. We further create an empty list C which will track the colour of each cluster, and an empty list of pairs A which will track adjacency of clusters.

ii.

To begin the algorithm, we read the colour of the first pixel and append it to C. We set the value $P'_{0,0}$ to zero. After this, we iterate over the elements of P row-by-row.
Note that $P_{0,0}$ refers to the top left pixel.

iii.

For each pixel $P_{x,y}$ we first check the colour of its neighbour to the top, $P_{x,y-1}$. If $d(P_{x,y}, P_{x,y-1}) = 1$ we set $P'_{x,y}$ to equal $P'_{x,y-1}$. If not, we check its left neighbour $P_{x-1,y}$. If $d(P_{x,y}, P_{x-1,y}) = 1$ we set $P'_{x,y}$ to equal $P'_{x-1,y}$.

iv.

In the case no direct left or above neighbours match colours, we iterate over its right neighbours so long as it holds that

$$\forall x_i \in [x, x_n] \; d(P_{x,y}, P_{x_i,y}) = 1$$

. We stop on the lowest value x_n for which $d(P_{x_n,y}, P_{x_n,y-1}) = 1$. If such a match is found, we will assign the value of $P'_{x_n,y-1}$ to all $P'_{x,y}, \ldots, P'_{x_n,y}$.

v.

If no match was found, we assign a new cluster index equal to the length of C to all $P'_{x,y}, \ldots, P'_{x_n,y}$, and afterwards append the value $P_{x,y}$ to C.

vi.

After a cluster has been assigned to a pixel $P_{x,y}$, we check its remaining neighbours in P' for different cluster numbers. We then add a pair of the cluster numbers (c_1, c_2) to A where c_1 and c_2 are the lower and higher cluster numbers respectively.

vii.

Once we have iterated through the entirety of P, we iterate through all cluster adjacency pairs (c_1, c_2) in A and evaluate $d\big(C(c_1), C(c_2)\big)$. If it is 1, we merge the clusters c_1 and c_2.

Notes

Considering the proposed method only needs to categorise a set number of colours, we may introduce a colour conversion at step six. This way, there is significantly lower ambiguity in further grouping of adjacent clusters.
Furthermore, it must be stated that this method is ill suited for scenario's with a high presence of colour gradients.
Finally, this technique in its current description does not support multithreading. Partitioning the image into various smaller pixel sets would enable this, though this would require an additional algorithm to merge the resulting multitude of cluster sets. Given the extensive benefits of parallelism, however, this could prove very much worthwhile.

Matching Algorithm

In order to establish correspondence between the coloured markers in the pattern and the zones detected on the mirrored surface, we must construct line segments from the mirrored surface and resolve any contradictions that may arise due to fractions. Fractions, in this context, refer to discontinuous sections in the locality mapping between the source image and the anamorphised image.
Various effects may cause non-linearity or fractions in the locality mapping. Namely, the surface may itself be discontinuous or it may curve out of view of the camera. Additionally, the mirror may reflect the same section of the source pattern multiple times onto the camera, or reverse the order of traversal. All these phenomena combined make the establishing of the locality mapping a non-trivial task.

To solve this problem, we begin with a rudimentary reconstruction by finding adjacency sequences that match the characteristics of the pattern. This refers to any chain of sequential adjacencies that alternates between black and a sequence color.
Let $C = \{c_0, c_1, \ldots, c_n\}$ be the set of n colours constituting the pattern. In addition let b be the color black and w the color white. Let $K = \{0, 1, \ldots, k-1\}$ be the set of all k indices of the clusters. The function $color(x)$ will return the color of a cluster $x \in K$. i.

We also define the function $Adjacency(x, y)$ which returns true if clusters x and y are adjacent and false otherwise.

ii. We begin the algorithm by constructing the sets $K_c = \{i \mid i \in K \wedge color(i) \in C\}$ and $K_b = \{j \mid j \in K \wedge color(j) = b\}$. We then construct a (possibly disjoint) graph from these points following adjacency. Mathematically, this will be represented as a function $neighbours : K \mapsto K^*$. In implementation, a pythonic dictionary provides the necessary functionality.
For each value in

$$neighbours(a) = \begin{cases} \{i \mid i \in K_c \wedge A(a,i)\}, & \text{if } a \in K_b \\ \{i \mid i \in K_b \wedge A(a,i)\}, & \text{if } a \in K_c \\ \emptyset, & \text{otherwise} \end{cases} \quad (2)$$

iii.

We construct trees from this set. In particular, we select an arbitrary instance $k_i / in K_c$ which has not yet appeared in any tree to create a new instance T. We then find and append all k_j for which holds that $\exists x \big(x \in neighbours(k_i) \ \wedge \ k_j \in neighbours(x)\big)$ which are not yet in T. We repeat this process for each newly appended element, until no new elements can be added to T. In case there are still elements in K_c which have yet to be assigned to a tree, we select one of these as the start of a new instance until eventually all elements of K have been categorised.

iv.

To initiate the mapping process for any given tree, we select the deepest leaf as an initial head and grow from there. Each newly selected head is appended to a substring, so long as the total corresponds to a valid substring of the color sequence defined in the pattern. Candidates for a new head are the parent of the current head, or a child of the current head which was not yet added to the sequence. Only sequences exceeding the minimum identifiable substring length are considered.

v.

After finding the longest valid sequence from the given leaf, the sequence is stored and its corresponding nodes are removed. If this causes the tree to become disjoint, we split the tree up. From the remaining tree or trees, we repeat the process outlined in iv. until no valid sequence exceeding the minimum identifiable substring length can be found.

vi.

The sequences that have been identified now allow for establishing correspondence between the anamorphised image and the source pattern. Interpolation methods may now be applied to create a more complete locality mapping.

Notes

There exist a number of signs within the cluster adjacency which indicate and point towards areas where surface distortion is particularly severe. Namely, any direct adjacency between two coloured clusters indicates a discontinuous zone removing the space between them. Additionally, having more or less than two black clusters neighbouring a given colored cluster shows that some degree of distortion or discontinuity has shifted the pattern. The same holds for any coloured cluster which indirectly (through black clusters) neighbours more than two coloured clusters. Another option not explored in the above method is analysing the shape of the clusters. Since all source shapes are either circular or rectangular and are all small relative to the overall pattern, simple approximation methods to the local distortion may prove to greatly amplify the overall accuracy of the locality mapping.

Results

Pattern Generation

Firstly, the pattern generation will be assessed. In particular, the length of the sequences used to provide uniqueness will be computed and compared to the theoretical upper bound. Since the search is depth-first and the search space grows exponentially with the number of colours n and the minimal identifiable substring length k, the process was cut off after no new longest candidate was found for some time.

The values of n and k were set to all permutations in the set $(3, 4, 5, 6, 8)$. Each coloured line represents a different value of n, with the x-axis representing the chosen k value and the y-axis the corresponding length achieved in log_{10}. We observe as we expected that the length of the sequence grows exponentially with both n and k.

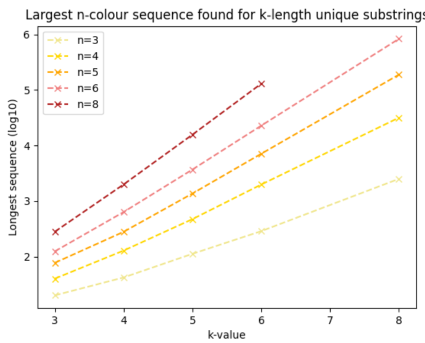

Figure 2: *Plot of the sequence lengths given values for n and k*

Using the same values as displayed above, for each combination of n and k the sequence length was compared to the maximum theoretical value. We observe that for higher values of k we come closer to this maximum. This goes against our expectation for the result of the algorithm, since for these larger sequences we have explored a smaller percentage of the search space.

Pattern Matching

A pattern was chosen for the pattern matching experiments with $n = 8$ colours and a minimum identifiable substring length $k = 5$. This will be used within all tests described in this section.

In order to properly test the algorithm, three scenarios will be tested. Firstly, the pattern will be tested as-is without any modification. This should yield a fully accurate mapping of the pattern, given no distortion will be present. Secondly, a rudimentary spherical distortion will be showcased to show

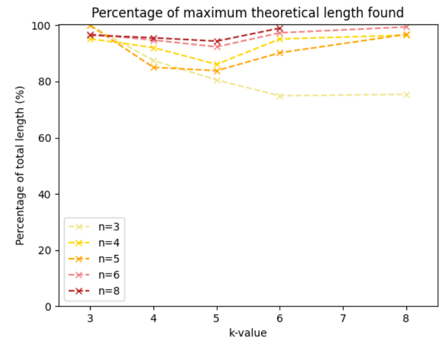

Figure 3: *Plot of the ratio between the longest found and the theoretical upper bound, as a percentage*

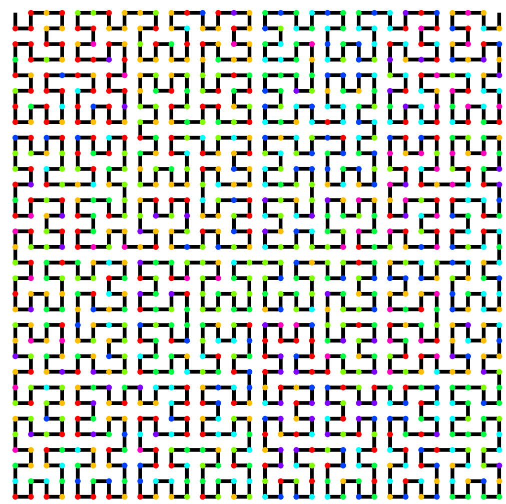

Figure 4: *A generated instance of the pattern described in the paper, with $n = 8$ and $k = 5$*

resilience against light distortion. Further, a section of the scene will be obstructed to showcase that global position can be derived only from partial connectivity.

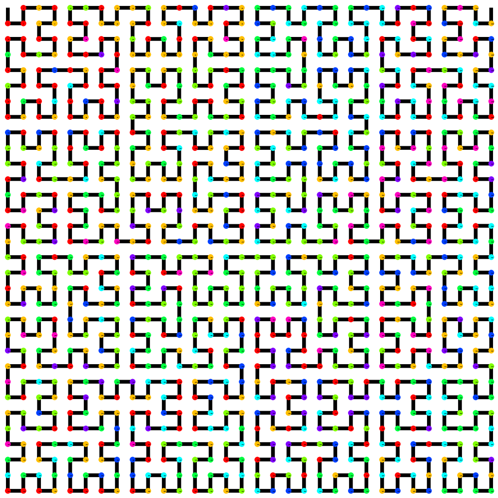

Figure 5: *The predefined pattern, with each colour within the sequence enumerated according to the found values*

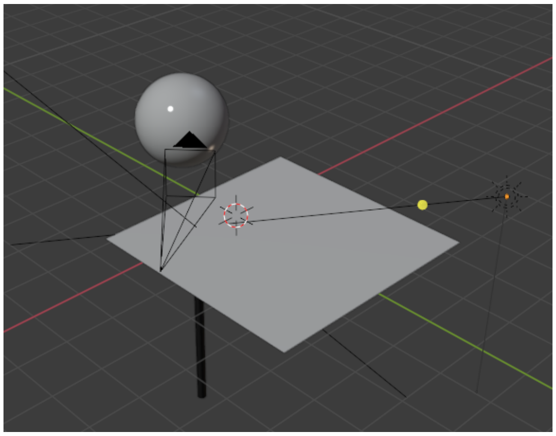

Figure 6: *Scene 1: a textured plane with a reflective sphere*

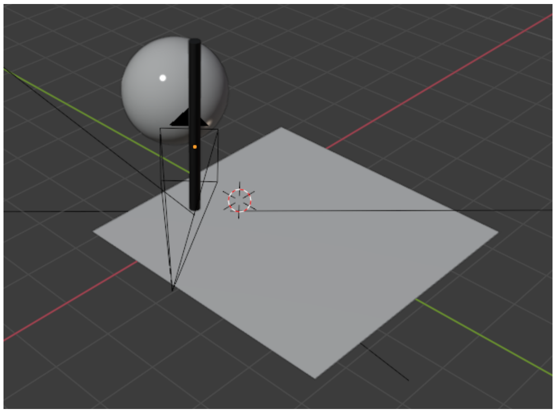

Figure 7: *Scene 2: a textured plane with a reflective sphere and an obstruction*

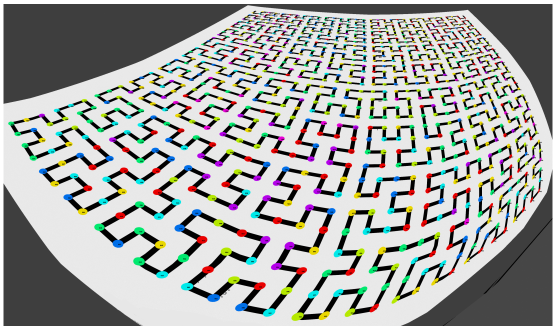

Figure 8: *Scene 1, enumerated*

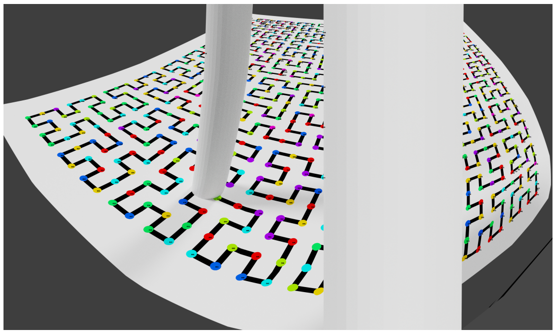

Figure 9: *Scene 2, enumerated*

Discussion

To maintain the scope of the research within the allotted time, many decisions had to be made to narrow down the search space. In particular, it was chosen to not pursue research into colour gradients since this avenue was far more complex and novel than the given time allowed. Additionally, the research presumed that the given count of colours could be adequately distinguished in real-world scenarios.

Within the original scope definition, it was outlined to strive towards the integration of multiple patterns to establish the locality mapping with higher precision and accuracy. It proved, however, that it was much more beneficial for the overall quality of the research to focus on the establishment of a singular pattern.

A notable roadblock and discovery within the exploratory phase of research was around the establishment of the colour sequence. This proved to be much more difficult to algorithmically solve than initially thought, which required significant experimentation on creating a more efficient way to determine these strings.

Conclusion

Succinctly put, this paper sought to propose a novel solution to pattern matching within the context of mirror anamorphosis. A technique would be outlined, and further substantiated through tests. In addition, the benefits and drawbacks would be explored.

An interesting result from the pattern definition is the apparent contradiction in the relation between the length of the minimum identifiable substring length and the permitted resolution: the larger this minimum is set to be, the higher the overall resolution. This can be intuited quite well, as linear growth of the length of the substring will cause an exponential growth in the number of permutations. Though the process of appending these strings is not trivial and not all possible permutations may be added, current data suggests that this reduction does not exceed the exponential growth of the substring space. As such, the algorithm benefits greatly from higher resolution imagery.

1 Future Work

With correspondence between coloured markers established, the relation between the distortion and its original could be exploited to increase the resolution of the locality mapping. Further, the shape and distribution of the markers may be further refined to this end.
The approach as described in this paper does not consider the false adjacency that is detected; in particular, any directly neighbouring coloured clusters are incorrectly adjacent, as are black clusters which neighbour more than two coloured markers. Finding such points within the pattern allows one to establish regions of discontinuity in the locality mapping.
Another avenue not explored is the enrichment of the coloured pattern. for instance, defining sequences which may not occur in the pattern would allow for an improved detection of false positives. Additionally, matching speed could be improved by restricting small marker substrings to regions of the pattern.
This paper set out to provide the conceptual foundation and validation of the described pattern matching approach. In this, the choice of colour values was done largely arbitrary. Further research could be done to find a set of colours optimised against colour distortion.
In a rather tangental approach to that explored in this paper, it may prove fruitfull to use a digital screen in order to establish locality mapping.. This would allow for much more dynamic and accurate mapping, at the cost of a loss in flexibility when it comes to surface size and shape.

References

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Appendix

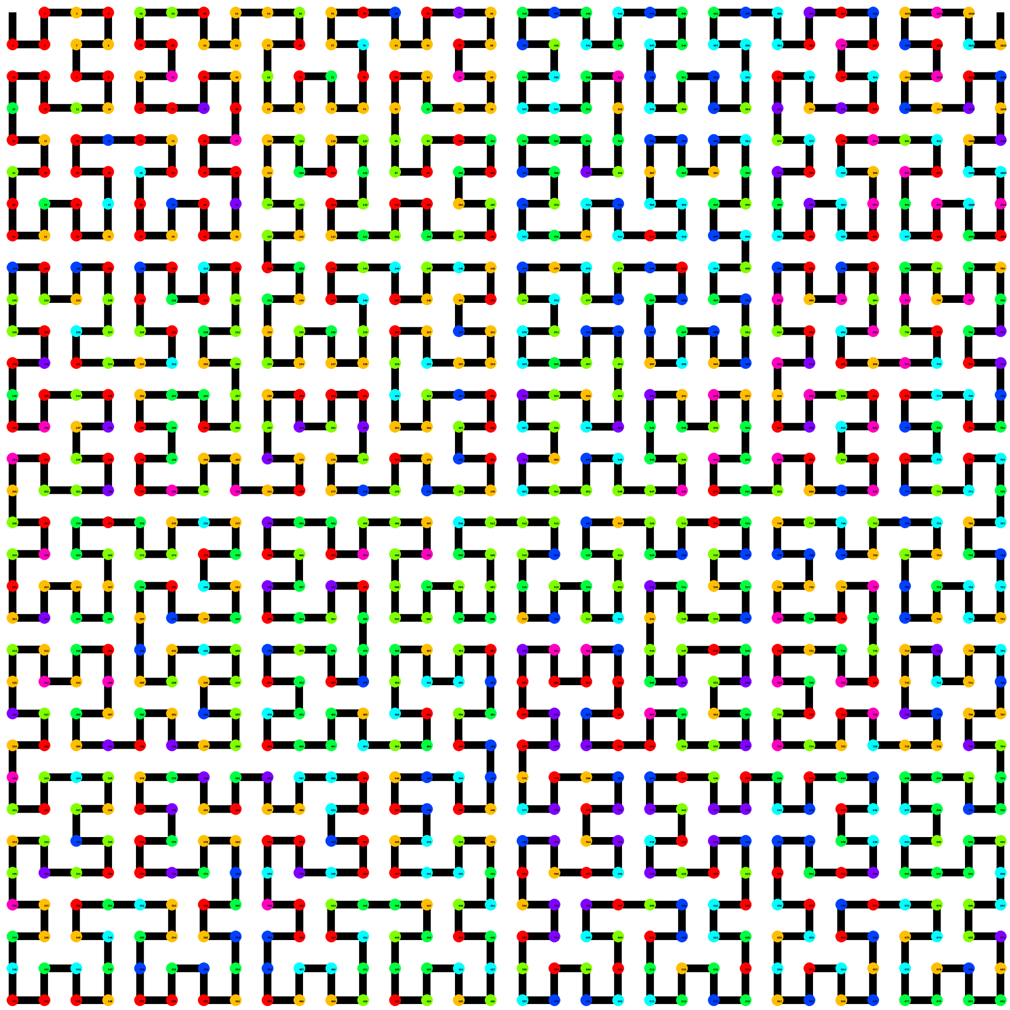

Figure 10: *The predefined pattern, with each colour within the sequence enumerated according to the found values*

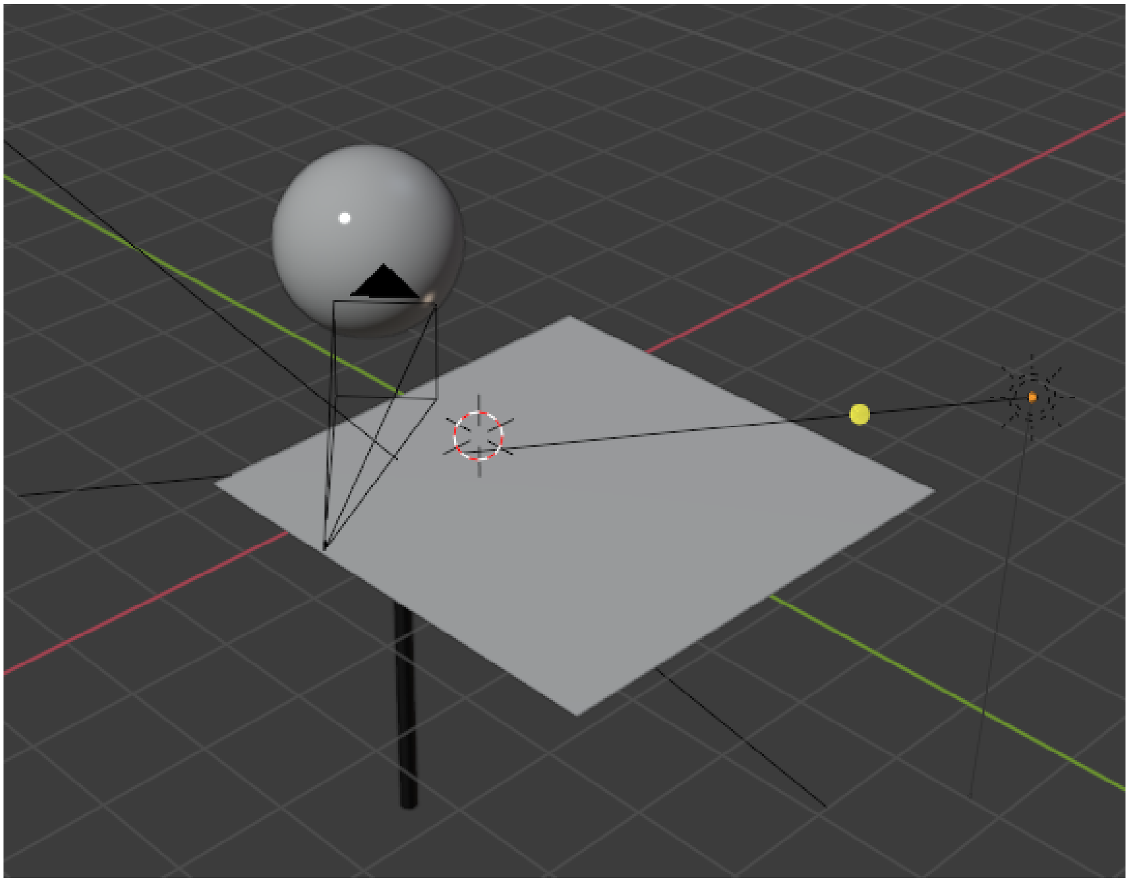

Figure 11: *Scene 1: a textured plane with a reflective sphere*

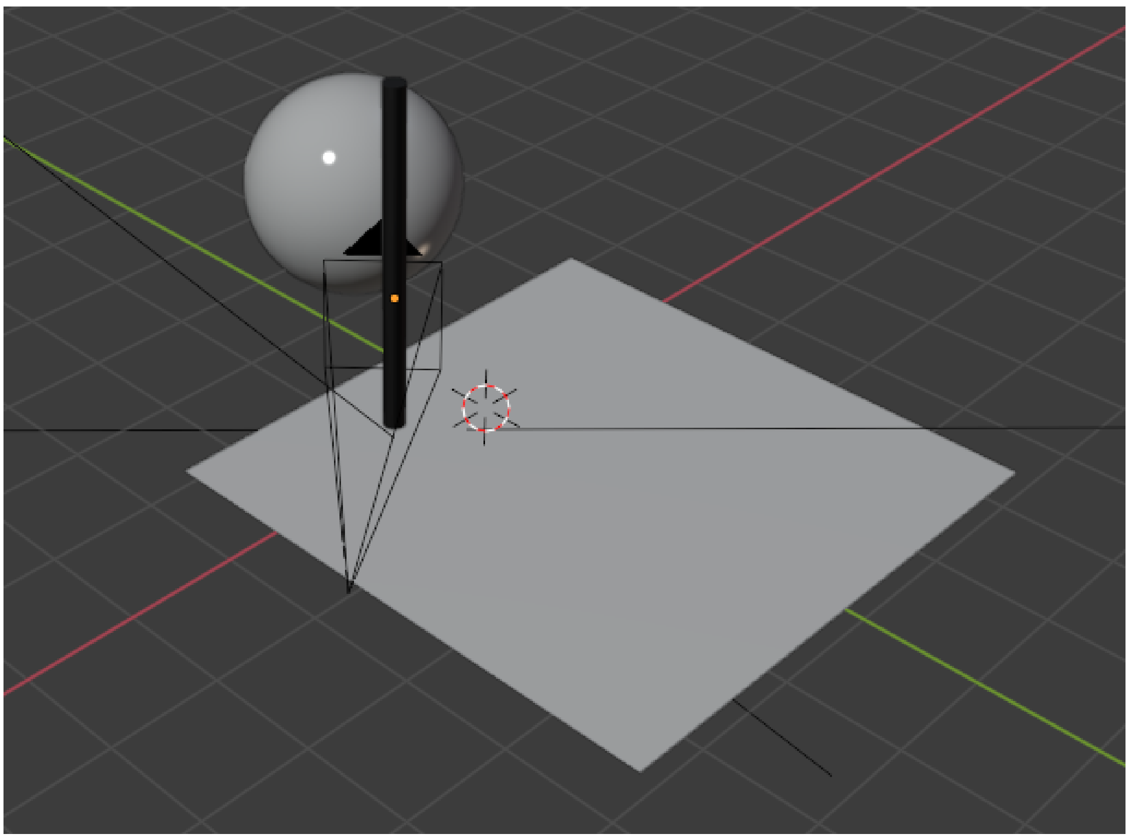

Figure 12: *Scene 2: a textured plane with a reflective sphere and an obstruction*

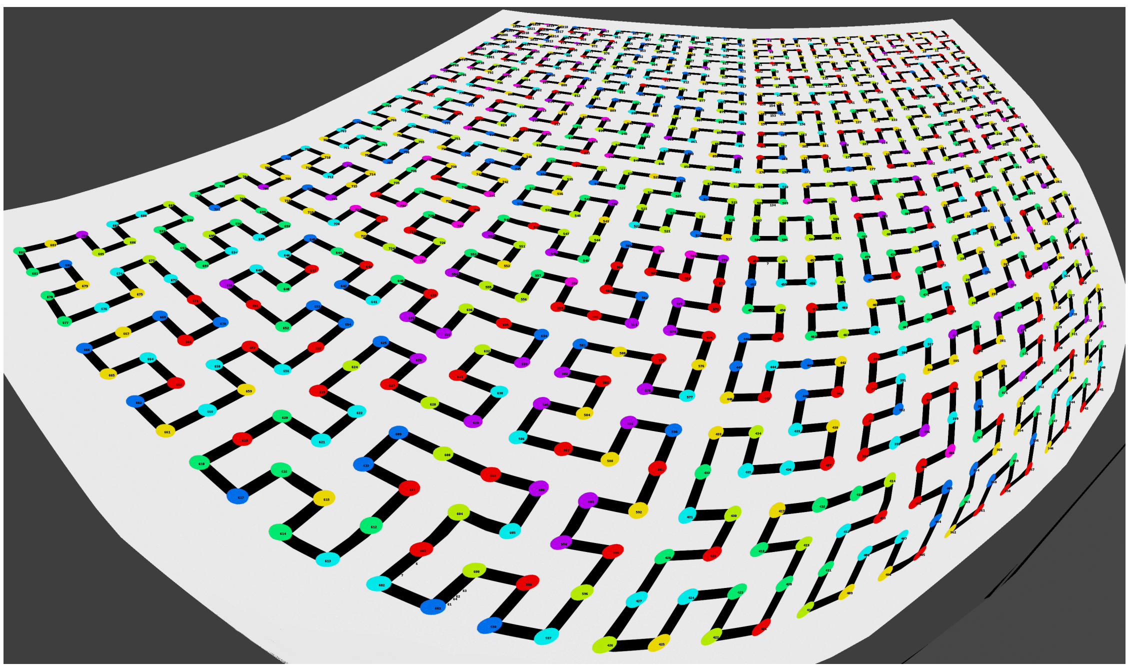

Figure 13: *Scene 1, enumerated*

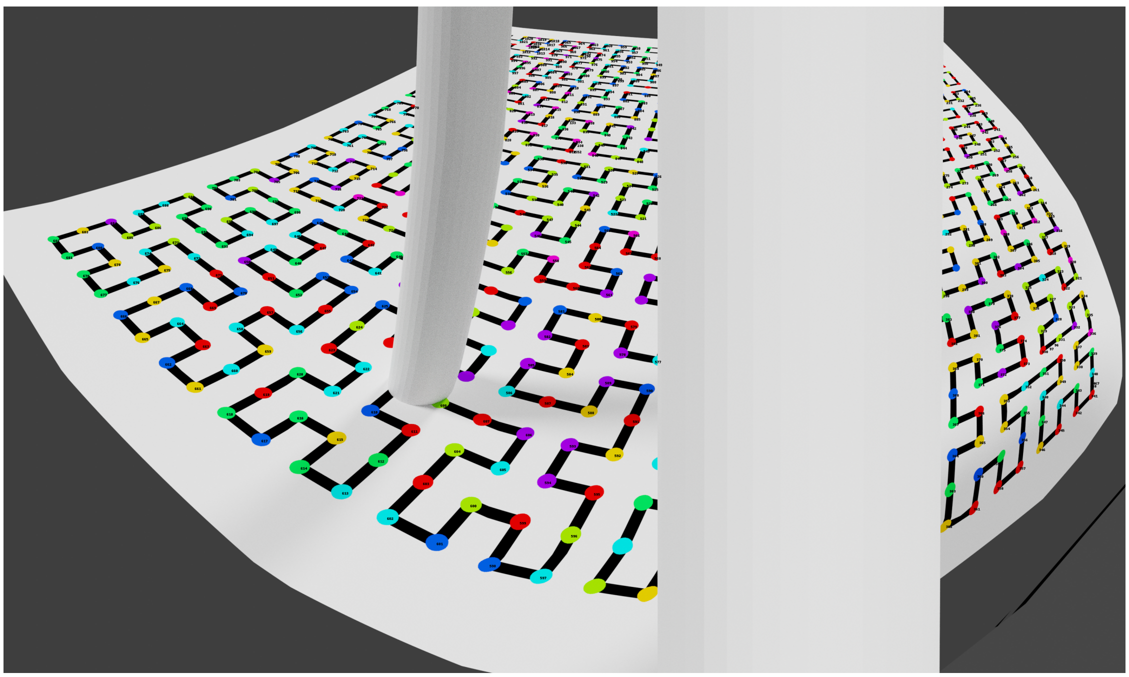

Figure 14: *Scene 2, enumerated*