Reliability analysis of flood defence systems

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In recent years an advanced program for the reliability analysis of flood defence systems has been under development. This paper describes the global data requirements for the application and the set-up of the models. The analysis generates the probability of system failure and the contribution of each element and parameter of the system to this probability. Thus weak links in the flood defence system can be identified, as well as the mechanisms and parameters that contributes most to the probability of failure.

Key words: PC-Ring, dike failure, flood defence system, probability of failure, correlation in time and space, system reliability

1 Introduction

After the flood disaster in the southwest of the Netherlands in 1953, the Delta Committee was installed to advise on short and long-term measures. In 1960 this committee presented the foundations for the current Dutch safety approach against flooding. They also noticed that a safety approach should preferably be based on flood risks. In doing so, probabilities and consequences of flooding would have to be considered together and in coherence. Until 1992 the knowledge necessary for further interpretation was not available.

In recent years an advanced program for the reliability analysis of flood defence systems has been under development, called PC-Ring. It implements the reliability analysis of the elements in a flood defence system, considering all principal failure modes and correlations.

An analysis with PC-Ring calculates the probability of system failure and the contribution of each element and parameter of the system to this probability. Thus weak links in the flood defence can be identified, as well as the mechanisms and variables that contributes most to the probability of failure.

2 Probabilistic framework

A flood defence system is a distinct area surrounded and protected by dikes, dunes, retaining walls, higher grounds, storm surge barriers, sluices and locks. The first step in calculating the failure probability of a flood defence system comprises of subdividing the flood defence system into several parts. A section is defined, as a part of the water defence system in which the expectation of main characteristics in lateral direction may be seen as constants. This refers to characteristics such as crest height, slopes, verges but also on loading conditions, revetment type, soil layers, etc. It could therefore be necessary to divide a specific section for a specific mechanism into two parts, which is not necessary for other mechanisms.
When the flood defence system is divided into sections, the probability of failure of each section and each mechanism can be calculated. In principle, this calculation requires three ingredients:

1. A model describing the mechanism.
2. Data for the deterministic and stochastic variables on the section strength.
3. Data for the deterministic and stochastic variables on the section loading.

In some cases one can choose between several different models. After the calculations for each section the results should be combined to one probability of failure for the complete flood defence system.
Until recent years these calculations were made only for cross sections of flood defence systems with a small number of stochastic variables to describe the water level. Most parts of the calculations (strength model) were based on a deterministic way of thinking. This was however not completely in accordance with the preferred safety approach of the Delta Committee. Therefore the PC-Ring program package is developed.

2.1 ***The PC-Ring program package***

The PC-Ring program package has been designed specially to calculate the reliability of a flood defence system as a composition of sections within a reference period. In order to perform this task in a reliable and flexible way the program package is build up of two programs. The main program calculates the probability of failure per section and per mechanism. The second program combines the probability of failure per section and per mechanism to the probability of failure of the complete flood defence system. In this way it is possible to combine the results of PC-Ring with other failure types (i.e. non-structural failure) as well.
PC-Ring uses a set of approximately 50 variables per mechanism that can be regarded as stochastic or deterministic parameters. In principle, data should be entered as:

- Geometric data (slope, orientation, fetch, etc.)
- Material properties (weight, sliding strength, etc.)
- Hydraulic loading (waves, water level, duration, etc.)

For a deterministic variable a single value has to be entered. For a stochastic variable the following values have to be entered:

- Variation around the average (mean value and standard deviation).
- Spatial correlation within a dike section and between sections
- Correlation in time

With these parameters the influence of the stochastic or deterministic parameters on the failure probability of the flood defence system can be studied in detail.

2.2 ***Failure probability of a section per mechanism***

The main program of PC-Ring calculates the failure probability of a section per mechanism. Failure is the result of the occurrence of one or more of the following mechanisms:

- Overtopping and overflow
- Uplifting and piping
- Inner slope failure
- Damage of revetment and erosion of the dike body
- Erosion of dunes
- Failure of hydraulic structures

Note that some of the above mentioned failure mechanisms do not directly result in flooding. i.e. there is still some but not quantified reserve strength present.
The latter two are not further described in this paper. Figure 1 shows the total set-up of the analysis.

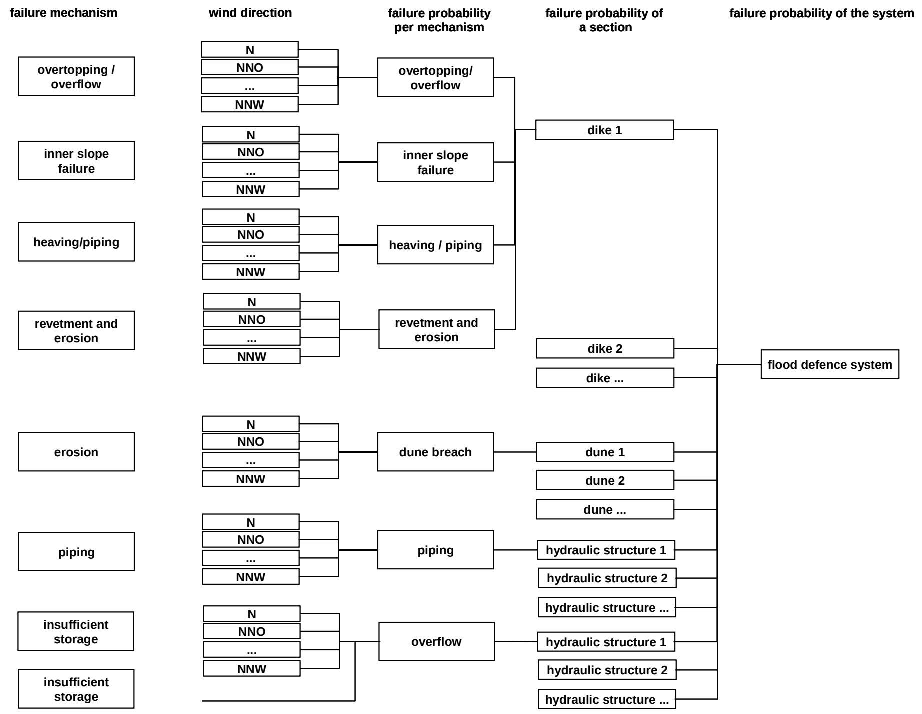


Figure 1: Flow chart for the failure of a flood defence system

The procedure to calculate the failure probability of a section per mechanism is as follows:

1. Calculation of the failure probability of one flood defence cross-section for one tide, one partial failure mode (for instance failure mode overtopping, partial failure mode saturation), given the wind direction.
2. Combination of the partial failure modes resulting in the failure probability of one total failure mode, given the wind direction.
3. Calculation of the failure probability taking into account the probability of the wind direction.
4. Determining the failure probability due to one failure mode for the flood defence section for which the under step 1 mentioned flood defence cross-section is representative taking into account the spatial correlation in one direction.
5. Combining the failure probabilities of all the wind directions.
6. Determining the failure probability for the total regarded period.
7. If needed, taking into account the influence of storm barriers and their closing regime.

2.3 *Failure probability of the total flood defence system*

After calculating all the failure probabilities of the sections for all the mechanisms, the system probability can be analysed. The combination program within the PC-Ring program package does this in the following way:

1. Combining the probabilities of the different failure modes for each flood defence section.
2. Combining all the sections to find the failure probability of total flood defence system.

3 Calculation methods

3.1 *Reliability of a single element*

In PC-Ring the following methods are available to calculate the probability of failure of a single element:

- Numerical integration (NI)
- Crude Monte Carlo (MC)
- Directional sampling (DS)
- Second order reliability method (SORM)
- First order reliability method (FORM)

For probabilistic calculations, the failure mechanism model is usually determined by means of a so-called limit state function, Z=g($\underline{x}$), whereby $\underline{x}$ is the vector of stochastic variables. Per definition, negative values of Z correspond to "*failure*" and positive values of Z, to "*non failure*". The equality g($\underline{x}$) = 0 is called the limit state, the corresponding surface is called the failure surface. The failure probability can formally be expressed as:

$$P_f = P[g(\underline{x}) \leq 0] = \int_{g(\underline{x})<0} f_{\underline{x}}(\underline{\xi})d\xi \tag{1}$$

where $f_{\underline{x}}(\underline{\xi})$ is the joint probability density function of $\underline{x}$. The primary purpose of reliability methods is to evaluate this equation. Usually the reliability index ß is used instead of the probability of failure. The reliability index is related to the probability of failure by:

$$\beta = \Phi^{-1}(P_f) \tag{2}$$

in which Φ is standard normal distribution. The reliability index has the advantage that it is easier in use and related to the safety level, i.e. the safety increases as the index increases.

3.1.1 *Numerical integration*

For the basic reliability problem where all variables are standard normal and independent, equation for the failure probability can be rewritten as:

$$P_f = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \ldots\ldots\ldots \sum_{k=-\infty}^{\infty} I(g(\underline{x}))f(\underline{x})\Delta x \tag{3}$$

where $I(g(\underline{x})) = 1$ if $g(\underline{x}) \leq 0$ and $I(g(\underline{x})) = 0$ if $g(\underline{x}) > 0$

3.1.2 Crude Monte Carlo simulation

The Monte Carlo simulation consists of sampling random x-values from their distributions $f(\underline{x})$ and calculating the relative number of simulations for which $g(\underline{x}) < 0$:

$$P_f = \frac{N_f}{N} \text{ or } P_f = \frac{\sum_i^N I(g(\underline{x}))}{N} \tag{4}$$

where N is the total number of simulations and N_f the number of simulations in the failed state ($g(\underline{x}) \leq 0$).

3.1.3 Crude directional sampling

The basic idea of directional sampling is to remodel the basic variables u into polar coordinates (λ, $\underline{\theta}$). The unit vector $\underline{\theta}$ defines the direction and a scalar quantity λ defines the length of the vector in u-space. The equation for the failure probability (1) is altered into:

$$P_f = \int_{g(\underline{x})\leq 0} dF(x) = \int P(g(\lambda\underline{\theta}) \leq 0) f(\underline{\theta}) d\underline{\theta} \tag{5}$$

where $f(\underline{\theta})$ is the (constant) density on the unit sphere.

For each direction $\underline{\theta}_i$ the value of λ_i is determined for which the limit state function equals zero:

$$g_i = g(\lambda_i \theta_i) = 0 \tag{6}$$

In other words λ_i is a measure of the distance to the limit state in the direction defined by the vector $\underline{\theta}$. The factor λ_i is found via an iteration procedure and requires several limit state function evaluations.

An estimate of the probability of failure P_f can be obtained by performing N directional Monte Carlo simulations of the $\underline{\theta}$-vector. Every simulation results in a sample value P_i:

$$P_i = P(g(\lambda_i \theta_i) < 0) = 1 - \chi_n^2(\lambda_i^2) \tag{7}$$

in which χ^2 is the chi-squared distribution function and n the number of random variables in the limit state function.

An estimate for the probability of failure is calculated as the mean value of the sample values P_i:

$$E(P_f) = \frac{1}{N} \sum_{i=1}^{N} P_i \tag{8}$$

3.1.4 First order reliability method

The present standard in structural reliability methods is the first order reliability method (Hasofer and Lind, 1974). FORM has proven to be highly efficient in the case of smooth limit state functions. Figure 2 shows a two-dimensional *u*-space with an arbitrarily limit state function. The point of the limit state function closest to the origin is called the design point. FORM linearises the limit state function in this point. Since the joint probability density function rapidly decreases with the distance $|u|$ from the origin, the main contribution to the probability integral (1) originates from the region closest to the origin. The probability of failure P_f can be computed from Eq. 2.

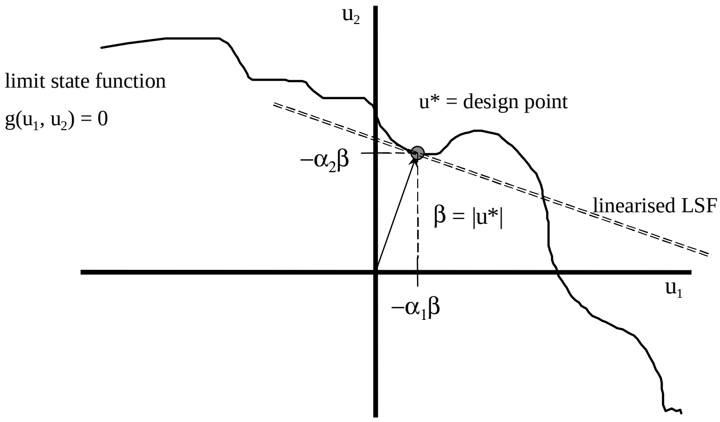


Figure 2: Two-dimensional illustration of u-space, limit state function and design point

The design point is to be found with a minimisation procedure that iterates to the vector $-\alpha\beta$ with the smallest distance from the origin to the failure surface. The length of the vector α is equal to 1. The components of the vector are called influence factors because they show the influence of variables on the limit state function.

3.1.5 Second order reliability method

A linear approximation in the design point will be accurate in cases of linear limit state functions. For strongly curved, but smooth limit state functions the second order reliability method can be used. Instead of a linear approximation of the limit state function, a second order function (parabola) is fitted through the limit state function in the design point.

3.1.6 Combinations of methods

All these reliability methods have there advantages and disadvantages, see for example Waarts (2000). For instance FORM is fast and gives a good definition of the design point. However the probability of failure is sometimes inaccurate due to the linearisation and the computation can have convergence problems. On the other hand the DS method is accurate and stable concerning the probability of failure, but inaccurate for the design point and slow. PC-Ring requires a stable method with an accurate design point because the design point is used in the system calculations. To overcome the disadvantages also a number of combinations of the above mentioned methods are implemented. Optionally PC-Ring can automatically switch to DS if convergence does not occur in a calculation with FORM or SORM. Other options involve the combination of DS and FORM, the first method is then used to find the probability of failure and the second method to find the design point.

3.2 *Reliability of a system*

In PC-Ring the reliability methods are performed on component level or on single failure modes. Reliability analysis covers the complete failure domain in space and time. Consequently, an additional combination procedure of all failure modes has to be performed. The difficulty is to identify the system of failure modes and to compute the reliability index of the system. In combination with FORM, it is customary to analyse the reliability of the system either as parallel or series system or as a combination of both.

3.2.1 Serial and parallel systems

A series system (union) of *m* components is considered to have failed if any of the components fails. When the event that component *j* has failed is denoted by E_{fj} the probability of the union can be determined as:

$$P_f = P\left(\bigcup_{j=1}^{m} E_{fj}\right) \tag{9}$$

A parallel system (intersection) of *m* components is considered to have failed when all of the components fail. When the event that component *j* has failed is denoted E_{fj}, the probability of failure of the intersection can be determined as:

$$P_f = P\left(\bigcap_{j=1}^{m} E_{fj}\right) \tag{10}$$

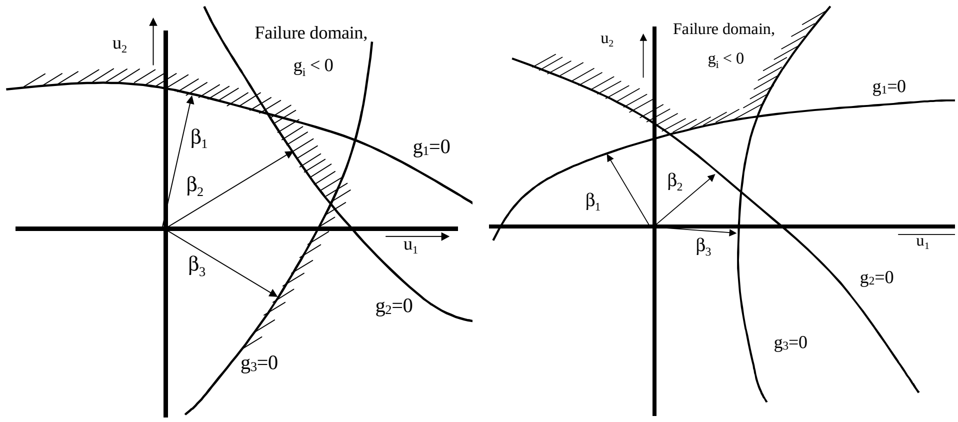


Figure 3: Union (series system) defined by 3 components *Intersection (parallel system) defined by 3 components*

The Hohenbichler and Rackwitz (1983) approximation replaces the probability $P_f = P(g(\underline{x})_1 < 0$ and $g(\underline{x})_2 < 0)$ by the probability:

$$P_f = P(\beta_2 - \rho u' - w\sqrt{1-\rho^2} < 0)P(\beta_1 - u < 0) \quad (11)$$

The approximation is based on the transformation of $g(\underline{x})_1$ into $g(\underline{x})_1 = \beta_1 - u$ and $g(\underline{x})_2$ into $g(\underline{x})_2 = \beta_2 - v$. The parameters u and v are (dependent) standard normal variables. The dependent variables u and v can be replaced by the independent standard normal variables u and w according to:

$$g(\underline{x})_1 = \beta_1 - u\ ,\ \ g(\underline{x})_2 = \beta_2 - \rho u - w\sqrt{1-\rho^2} \quad (12)$$

The probability P_f according to Eq. (11) can be found by introducing the truncated standard normal variable u':

$$u' = \Phi^{-1}\{1 - P(u > \beta_1)\Phi(u)\} \quad (13)$$

The failure probability of the combination of limit state functions $g(\underline{x})_1$ and $g(\underline{x})_2$ can be analysed within the FORM procedure using the following limit state function:

$$g(\underline{x}) = \beta_2 - \rho\Phi^{-1}\{1 - P(u > \beta_1)\Phi(u)\} - w\sqrt{1-\rho^2} \quad (14)$$

Subsequently, the intersection probability (system reliability) is computed by FORM.

3.2.2 Spatial and time correlation

The global form for spatial correlation in one direction in PC-Ring is:

$$\rho(\Delta x) = \rho_x + (1 - \rho_x)exp\left\{-\frac{\Delta x^2}{d_x^2}\right\} \quad (15)$$

where ρ_x is a constant correlation and d_x the correlation distance.

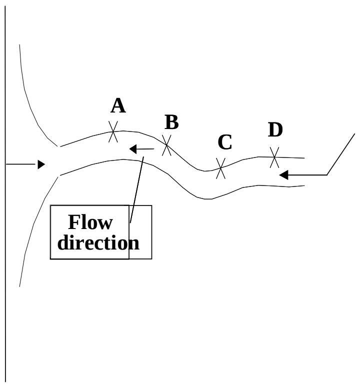
The following auto-correlation function is used when a spatial deviation in two directions has to be taken into account:

$$\rho(r_x, r_y) = exp\left\{-\frac{r_x^2}{d_x^2}\right\}\{(1-\alpha) + \alpha exp\left\{-\frac{r_y^2}{d_y^2}\right\}\} \tag{16}$$

where r_x and r_y are the horizontal respectively vertical distance between two points, d_x and d_y the horizontal respectively vertical correlation distance and α the variance ratio factor.
The correlation in time is taken into account according to the approach of Borges and Castanheta (1971). In this approach time intervals are used. Inside the intervals there is total correlation and between the intervals there is a constant correlation ρ.

Data requirements at the mouth of the river

1. Statistics of water levels, given the wind direction ϕ $F(h_{sea} < h_{sea}|\phi)$
2. Statistics of wind speeds, given the wind direction ϕ $F(\underline{U} < U|\phi)$
3. Probabilities of the wind directions $P(\phi)$
4. Correlation between wind speed U and water level h_{sea}



Data requirements upstream of the river

1. Statistics of discharge Q
2. The magnitude of Δt according to the Borges-Castagneta model

Figure 4: Data requirements of the basic random variables and the statistics that are used to determine the local hydraulic boundary conditions at for instance locations A, B, C and D

4 Limit state functions

One of the main failure mechanisms is overtopping and overflow. In this section limit state functions of these failure mechanisms related to dikes are given. Limit state functions of other failure mechanisms are given in the appendix.
Water discharges passing the crest of a dike either due to overtopping or overflow is the cause of erosion loading on the inside slope. Water discharges due to overflow are in PC-Ring only assumed to be relevant in case of offshore wind or wave heights smaller than 1 mm. In other situations the water discharges are assumed to occur due to wave overtopping.

4.1 ***Overtopping***

The mechanism failure due to overtopping is supposed to occur when at a certain location the quantity of water overtopping as a result of waves and the water level is larger than the crest and inner slope can handle (see Figure 5). This leads to erosion, after which a breach can start to grow and flooding may occur. In case of discharges due to overtopping the limit state function is:

$$Z = m_{qc} q_c - m_{qo} q_o / P_t \tag{17}$$

where q_c is the critical discharge expressing the limit discharge for which damage of the inner slope occurs, q_o is the actual occurring overtopping discharge due to the hydraulic boundary conditions, m_{qc} and m_{qo} are model factors describing the uncertainty of the models for calculating q_c and q_o. Furthermore the occurring discharge is divided by a percentage of the time P_t that overtopping takes place. Hereby the pulsar character of wave overtopping is accounted for.

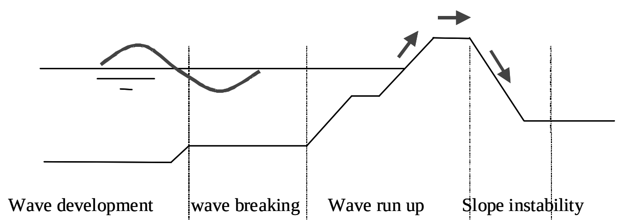


Figure 5: Dike failure mechanism: overtopping

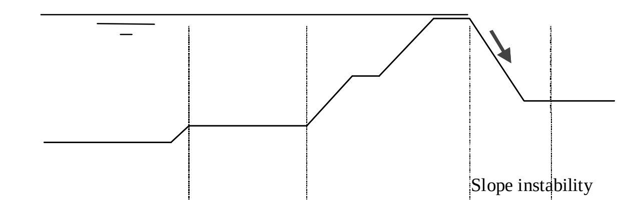


Figure 6: Dike failure mechanism: overflow

4.2 ***Overflow***

The mechanism failure due to overflow is supposed to occur when at a certain location the water level h rises above the critical level of the dike h_{kd} (see Figure 6). In case of discharges due to overflow the limit state function is (Vrouwenvelder et al. 2001):

$$Z = h_{kd} - h = h_d + \Delta h_c - h = h_d + \sqrt[3]{\frac{q_c^2}{0.36g}} - h \tag{18}$$

in which h_d is the crest level of the dike, h_c expresses the critical height for which damage of the inner slope occurs, h the actual occurring water level, and g the gravity.

5 Hydraulic loading models

The limit state functions need hydraulic loading models. A loading model is formed by the following elements: statistics of the stochastic variables, correlation between the variables, physical models and load parameters. The loading model results in the local water level and wave conditions on the sections of the flood defence system. Rivers, lakes and seas threaten flood defences. So one or more correlated loading models may apply to a flood defence system.

*5.1 **Coast and estuaries***

Most coastal load models have three global stochastic variables:

- Water level
- Wind direction
- Wind speed

These stochastic variables are valid for a larger area. To calculate local water levels the marginal statistics of the water level at some measuring stations or the water levels based on model calculations and triangular interpolation (Dekker, 2002) are used. For waters behind storm surge barriers the closure regime, the accuracy of the models that predict the water levels and the probability of failure of the closure is taken into account. To calculate the local wave load the physical relation between wind and water level on the one hand and wave load on the other hand is based on SWAN (1999).

*5.2 **River***

Important processes and variables for the load on rivers are:

- River discharge
- Duration and form of a high discharge
- Wind speed and direction
- Water level at the river mouth
- Duration of the storm onset

For waters behind storm surge barriers the closure regime, the accuracy of the models that predict the water levels and the probability of failure of the closure are taken into account.

The statistics of the discharge has two parts:

1. The distribution of the yearly extremes, which gives the return times of the top discharges,
2. The arbitrary point in time distribution, which gives the average number of days per year that a certain level of discharge is exceeded. The magnitude of correlation in time Δt is calculated according to the Borges-Castagneta model.

A fit through these lines is programmed in PC-Ring. The models SOBEK and WAQUA are used to establish a physical relation between local water level, river discharges, wind speed, wind direction and sea water level.
There may be more than one river that threatens a flood defence. The correlation between the discharges is used to account for the probability that an extreme discharge occurs on both rivers at the same time.
Bretschneider (TAW 1989) give the physical relation between local wind speed, wind direction, local water level and fetch on the one hand and the wave load on the other hand.
The derivation of the combined wind – water level statistics is based on the statistics of the sea or lake level at the river mouth, the statistics of the wind and the correlation between wind and water level .

5.3 ***Lakes***

For lakes there are also three global stochastic variables:

- Water level
- Wind direction
- Wind speed

The statistics of the lake level are taken from HYDRA-M (1999). The statistics of the lake level has two parts:

1. The distribution of the yearly extremes, which gives the return times of the top lake levels,
2. The arbitrary point in time distribution, which gives the average number of days per year that a certain level is exceeded. The magnitude of correlation in time Δt is calculated according to the Borges-Castagneta model.

To relate the local hydraulic loading to the stochastic functions the WAQUA water movement model and the HISWA wave growth model are used.

6 Example

As an example of a PC-Ring calculation the Dutch flood defence system ‘Hoeksche Waard’, is analysed. The flood defence system is shown in Figure 7. The main result of a run using PC-Ring is presented for every section and every mechanism the reliability index β. The PC-Ring result of the flood defence system is given in Table 1. In total the system consists of 130 sections. In this case only the results of 28 sections are given. If no index is given for a particular failure mechanism, a reliability analysis was not considered to be necessary because in a prior assessment the reliability was considered to be high. In the last column of the table the reliability index is given for each section, integrated over all mechanisms. This index is always equal to or lower than the lowest reliability index of each mechanism. In the last row the reliability index is given for each mechanism of the system. Finally, in the lower right corner the reliability index of the whole flood defence system is given for all mechanisms and sections.

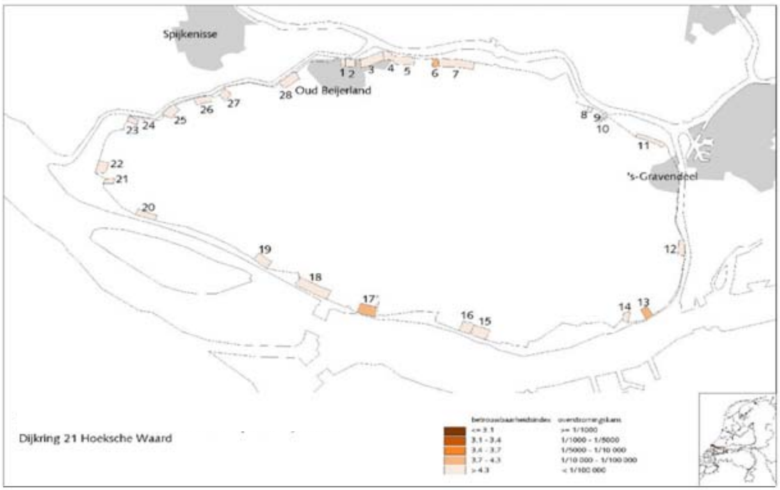


Figure 7: Overview of the flood defence system ‘Hoeksche Waard’. The numbers refer to the section which are analysed

Table 1: Example of a PC-Ring result: reliability indices for each failure mechanism per section and for the total system for a period of 1 year

section	O	GI	P	R	P-HS	N-HS	all
1	5.5						5.5
2	5.7						5.7
3	6.9						6.9
4	4.7						4.7
5	4.8						4.8
6	5.2			5.9	4.4	4.4	4.3
7	5.8	5.7	8.6				5.6
8	6.1				4.4		4.4
9	5.6						5.6
10	5.1						5.1
11	4.7						4.7
12	5.0	4.9	6.9				4.8
13	5.4				4.4	4.4	4.3
14	4.9						4.9
15	5.6	5.3	4.8	6.4			4.8
16	5.1						5.1
17	4.5			6.0	4.4	4.4	4.2
18	4.8						4.8
19	5.0						5.0
20	5.7	6.7	10.5				5.7
21	5.4			6.6			5.4
22	4.9			6.4			4.9
23	5.8						5.8
24	5.4						5.4
25	5.4						5.4
26	5.2					4.4	4.4
27	5.0	5.1	7.3				4.9
28	5.6						5.6
system	4.5	4.8	4.8	5.8	4.1	4.1	3.9

O: overtopping/overflow, GI: geotechnical instability

P: heave and piping, R: revetment

P-HS: piping of hydraulic structure, N-HS: non closure of hydraulic structure

In this case the reliability index of the flood defence system is 3.9 per year. This means that the probability of flooding for the whole defence system is about 0.001 per year, or a return period of about 1 in 1000 years. The reliability is mainly determined by section 17 which has an index of 4.2. The failure probability of section 17 is determined by the mechanism overtopping and overflow. We may consider the spatial correlation between the sections. The flood defence system can be considered as a series system. The lower bound of the failure probability of the system is equal to the maximum failure probability of the sections or to the lowest reliability index which is in this example 4.2. The upper bound of the failure probability of the system is the sum of all individual failure probabilities which can easily be calculated. In this case the upper bound of the failure probability is $4.8 \cdot 10^{-5}$ which is equivalent to a reliability index of 3.89.

7 Closure

With PC-Ring it is possible to calculate the failure probability of sections and the combined failure probability of a flood defence system in an efficient and accurate way. The program uses a range of techniques and methods to calculate the failure probability including the effect of spatial and time correlation in the stochastic variables. The output files give reliability indices β and a list of influence factors α indicating the importance of a variable's uncertainty. These values can be used to identify weak links. Moreover they can be used to prioritise research or dike reinforcement. Combined with a flood model and a model to calculate damage and people at risk, the results of PC-Ring can be used to calculate flood risks.

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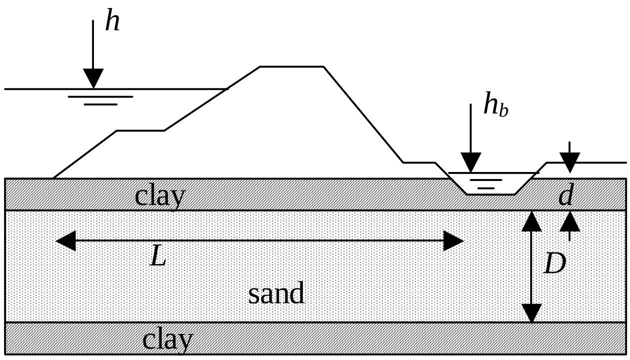
Appendix Limit state functions

In addition to the overtopping and overflow failure mechanisms also the following main failure mechanisms are programmed in PC-Ring.

A.1 Uplifting and piping

A dike fails due to uplifting and piping when two subsequent failure mechanisms occur $P(Z_1 < 0$ and $Z_2 < 0)$. The process leading to failure can be described as follows. First uplifting causes openings in the impervious clay layer covering the sand layer. Secondly, a flow of water through these openings initialises an erosion process. This process can progress from the point of the openings caused by the previous uplifting behind the dike towards the water outside. The erosion process takes the form of pipes undermining the foundation of the dike. These pipes can eventually cause failure. The definition of some variables is shown in Figure A1.



d = thickness of cover layer,

D = thickness of permeable sand layer,

h = local water level,

h_b = water level in the polder,

L = length of the seepage

Figure A1: Dike failure mechanism: uplifting and piping

A.1.1 Uplifting

Uplifting occurs if the difference between the local water level *h*, and the water level "inside", h_b is larger than the critical water level h_c. This is expressed in the limit state function as:

$$Z_1 = m_o h_c - m_h(h - h_b) \tag{A.1}$$

in which m_o is the model uncertainty of the critical water level h_c and m_h the uncertainty of the damping. The critical water level expresses the water level at which uplifting almost occurs. This water level is based on the properties of the impervious layer.

A.1.2 Piping

Piping is the sub mechanism that describes the growth of water bearing 'pipes' due to increasing pressure of the water on the outside. The dike fails as a consequence of piping if the difference between the local water level *h* and the inside water level h_b, reduced with a part of the vertical seepage length *d*, exceeds the critical water level h_p.

$$Z_2 = m_p h_p - (h - 0.3d - h_b) \tag{A.2}$$

in which m_p is the model uncertainty of the model with which h_p is described. Sellmeijer (1988) defines the critical water level h_p as a function of a factor α reflecting the effect of the finite thickness of the water bearing layer, a coefficient *c* for the characteristics of the sand in the eroding water bearing layer, the seepage length *L*, the roll resistance angle θ of the sand, the unit weight of water γ_w and the unit weight of the sand grains γ_k.

A.2 Inner slope failure

The dike fails due to sliding / uplifting when a part of the dike is instable and slides. This phenomenon can occur on the inner or outer slope. Mostly only sliding of the inner slope is taken into account, see Figure A2. Instability of a dike slope occurs if part of the dike slides away along a slip plane. According to Bishop's approach this slip plane is circular and the stability is expressed in the form of a stability factor. To determine this stability factor Γ the dike is divided into vertical slices. The stability factor consists of a ratio between two moments that are taken with respect to the centre of this slip circle.

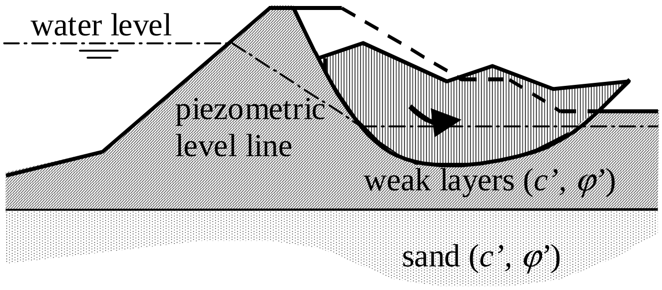


Figure A2: Dike failure mechanism: sliding of the inner slope

For the strength model of the failure mechanism 'inner slope failure' the program MPROSTAB is used. In PC-Ring the results of this strength model are combined with the loading model (hydraulic boundary conditions). This is done by using the results of the MPROSTAB / MPROLIFT models (Calle, 1994, Van, 2002) for a given actual water level *h*.

A.2.1 Influence of the strength model

For the probabilistic stability analysis it is assumed that the sliding surface is finite and the resisting moment of the ends is taken into account. These features are stochastic functions. The calculation of the shear strength is based on "drained" parameters c' (cohesion) and φ' (angle of internal friction) with the following equation:

$$\tau = c' + (\sigma - u)\tan\varphi' \qquad \text{(A.3)}$$

where τ is the shear strength along the bottom of the slice, σ the total normal stress on the bottom of the slice and u the (excess) pore pressure. In an effective stress approach the water tensions in the soil have to be given as input in the form of piezometric level lines.

Based on the modelling of the shear strength and the water (over) tension the local stability factor (i.e. the stability factor for one cross-section) is calculated. The 'deterministic' stability factor Γ for a slip circle is calculated from the following limit state:

$$M_R = \frac{R \sum c_i' + (\sigma - u) \tan \varphi_i' \Delta b_i}{\Gamma} = M_A \qquad \text{(A.4)}$$

where M_A is the driving moment for the slip circle in question (radius R), M_R is the resisting moment at limit equilibrium, Δb_i is the width of slice i at the bottom, u_i is the water tension perpendicular to the bottom of the slice and σ_i is the soil stress perpendicular to the bottom of the slice. The critical slip circle is the slip circle with the smallest stability factor. It is found iteratively. The stability factor of the cross-section is equal to the stability of the critical slip circle.
The limit state function for the probabilistic stability analysis for the strength model is:

$$Z = \Gamma - q \qquad \text{(A.5)}$$

where the limit value q equals 1.0 if there were no model uncertainty in the calculation method. In the probabilistic stability analysis q is a stochastic function. The value depends on the type of tests done to determine the strength parameters.

A.2.2 Influence of the stochastic water level

The program MPROSTAB / MPROLIFT is used to calculate the probability of sliding / uplifting of a cross section at a given water level in three different situations: for instance an average water level, an extreme water level with a return period of 1 in 1000 years and the latter water level minus one meter. So the probability calculated is at first a conditional probability of instability given the water tensions in the earth body at this water level. In PC-Ring the output of MPROSTAB / MPROLIFT is used to calculate the total probability of failure due to instability of the inside slope. To this end, the reliability indices β and the influence factors α are used in the following reliability function:

$$Z = \beta(h) + \sum_{i=1}^{n_{MPROSTAB}} \alpha_i(h)\, u_i \qquad \text{(A.6)}$$

where α_i and β are extracted from MPROSTAB / MPROLIFT at a given water level, and u_i are standard normally distributed variables.

A.3 Damage to the revetment and erosion of the dike body

When a dike fails due to damage of the revetment the failure mechanism consists of two parts. First the revetment gets damaged by wave attack and next the cross section of the dike body decreases by erosion. For most types of revetments this cannot be combined in one reliability function. So the mechanism consists of two sub mechanisms: damage of the revetment and erosion of the dike body. Failure by erosion depends on the duration of the storm t_s, and in fact on the whole course of the water level and wave height.

A.3.1 Grass revetment

A dike with a grass revetment fails when the time that a certain storm needs to damage the grass revetment t_{RT} and to erode the rest of the dike body (t_{RK} + t_{RB}) is shorter than the duration of the storm t_s. The limit state function is given by (Verheij 1997):

$$Z = t_{RT} + t_{RK} + t_{RB} - t_s \tag{A.7}$$

The strength of the grass revetment t_{RT}, is given by:

$$t_{RT} = d_w / E_g \tag{A.8}$$

where d_w is the depth of the grass roots and E_g is the velocity of erosion of the grass revetment which depends on the erodibility of the grass revetment c_g, on the angle of wave attack and the significant wave height H_s, according to:

$$E_g = \frac{r^2 H_s^2}{c_g} \tag{A.9}$$

Erosion strength of the clay cover layer t_{RK} is given by:

$$t_{RK} = \frac{0.4 L_K c_{RK}}{r^2 H_s^2} \tag{A.10}$$

where L_K is the width of the clay cover layer, see Figure A3, c_{RK} a coefficient related to the erodibility of the cover layer, r a reduction factor depending on the angle of wave attack and H_s the significant wave height.

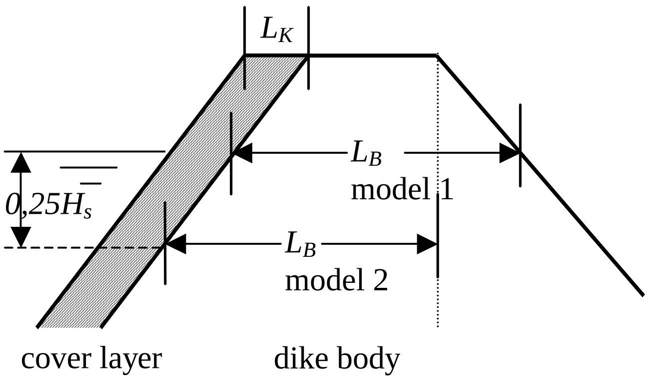


Figure A3: Dike failure mechanism: erosion of dike body
Definitions of width of cover layer and dike body

There are two models in PC-Ring to calculate the erosion strength of the dike body t_{RB}. Model 1 assumes no mixing of material from the cover layer and the body. Model 2 assumes there is some delay in the erosion due to mixing. The erosion strength of the dike body t_{RB} is given by:

$$t_{RB} = \frac{0.4 L_B c_{RB}}{r^2 H_s^2} \tag{A.11}$$

where L_B is the width of the dike body (the definition of L_B depends on the erosion strength model chosen) and c_{RB} a coefficient related to the erodibility of the dike body.

A.3.2 Stone pitching

In PC-Ring there are two types of stone revetment: stone pitching directly on clay and stone pitching on a granular filter. Especially the last type covers most Dutch stone revetments.
The limit state function for the sub mechanism “damage of stone pitching directly on clay” is:

$$Z = c_k \Delta D - rH_s \tag{A.12}$$

where c_k is a coefficient of the strength of the stone pitching, Δ the relative density and D the thickness of the stone pitching.
The limit state function for the sub mechanism erosion of the dike body after damage of the revetment is given by:

$$Z = t_{RK} + t_{RB} - t_s \tag{A.13}$$

Erosion of the dike body is the same for a grass and stone revetment. Thus the calculation of t_{RK} en t_{RB} is equal to Eqs. A.10 and A.11.
The sub mechanism “damage to the stone pitching on granular filter” consists of two sub mechanisms. Therefore the stone revetment has to comply to one of the following two demands: ($Z_{b1} < 0$) or ($Z_{b2} < 0$).

$$Z_{b1} = \frac{c_f (\Delta D \Gamma)^{5/3}}{(\Lambda \tan \alpha_u)^{2/3}} - \frac{r H_s}{(r S_{op})^{1/3}} \tag{A.14}$$

where c_f is a coefficient for the strength of the stone pitching, Δ the relative density, D the thickness of the stone pitching, Γ an influence factor for the friction between stones, towards flow and inertia, Λ a seep-age length, α_u the angle of the outer slope and S_{op} the wave gradient.

$$Z_{b2} = c_{gf} \left(\frac{\sqrt{S_{op}}}{\tan \alpha_u} \right)^{2/3} - \frac{H_s}{\Delta D} \tag{A.15}$$

where c_{gf} is a coefficient (stochastic function) for the strength of the stone pitching.
The limit state function of the sub-mechanism erosion of dike body is given by:

$$Z = t_{RS} + t_{RK} + t_{RB} - t_s \tag{A.16}$$

This limit state function is nearly identical to that of grass with the exception that the rest strength of grass is replaced by that of the stone pitching. The expressions for t_{RK} and t_{RB} are given in Eqs. A.10 and A.11.
The erosion strength of the stone pitching on a granular filter t_{RS} is given by:

$$t_{RS} = 57 \cdot 10^3 \, T_p \, e^{-\sqrt{H_s L_{op}} / c} \tag{A.17}$$

where T_p is the peak period, H_s the significant wave height, L_{op} the wave length and c is a coefficient depending on the angel of wave attack (Vroeg, 1992).

A.3.3 Asphalt cover

The failure of the asphalt cover consists of two sub mechanisms: “failure by water overpressures” and “failure by wave impact”. The sub-mechanism failure due to uplifting by water overpressures occurs when the difference in pressure over the revetment at the water level line exceeds the weight of the asphalt layer. This leads to the following limit state function:

$$Z = \Delta D - 0.21 Q_n (a + v) R_w \tag{A.18}$$

where Δ is the relative density of the asphalt, D the thickness of the asphalt layer, Q_n a factor depending on the angle of the outer slope, a the distance measured vertically from the (fictive) bottom of the closed revetment to the ruling water level, v the distance measured vertically of the ruling water level to the ruling ground water level and R_w the reduction factor for the location of the water level (TAW 2000).

For the sub mechanism failure by wave impact there are two types of asphalt covers:

1. Hydraulic engineering asphalt covers and open stone asphalt covers,
2. Partly penetrated rubble layer.

The limit state function for type 1 is given by:

$$Z = D - D_{needed} \tag{A.19}$$

where D_{needed} depends on the significant wave height H_s and can be derived from TAW (2000).

The limit state function for type 2 is given by:

$$Z = D_{n50} - \frac{H_s \xi_{op}^b}{\Delta_m \psi_u \Phi_{sw} \cos\alpha} \tag{A.20}$$

where D_{n50} is the nominal mean diameter of the revetment material, ξ_{op}^b a wave parameter, Δ_m the relative density, ψ_u a parameter for the penetration of asphalt, Φ_{sw} the stability factor and α the angle of the slope.

For two types of asphalt cover the limit state function of the sub mechanism erosion of dike body after damage of the revetment is given by:

$$Z = t_{RB} - t_s \tag{A.21}$$

The expression for t_{RB} is given in Eq. A.8, though for dikes with an asphalt cover it is assumed that there is no mixing of material from the cover layer and the body.