Near–Far Effects in Land Mobile Random Access Networks with Narrow-Band Rayleigh Fading Channels

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Abstract—**The near–far effect of random access protocols in mobile radio channels with receiver capture is investigated. To this end, the probability of successful reception of a packet from a terminal at a known distance from the central receiver is obtained taking into account Rayleigh fading, UHF propagation attenuation, and the statistics of contending packet traffic in radio nets employing slotted ALOHA, carrier sense multiple access (CSMA) or inhibit sense multiple access (ISMA) protocols. Various models of receiver capture are compared, namely packet error rates for synchronous detection in slow- and fast-fading channels, and the probability that the signal-to-interference ratio is above a required threshold.**

I. Introduction

Dynamic multiple access to a common receiver via fading radio channels is an important issue to the efficiency of mobile data networks. In this paper the case of mobile transmitters sending data packets to a single (fixed) base station over a common radio channel is studied. Examples of popular protocols to provide random access for a large number of users are slotted ALOHA [1]–[16], carrier sense multiple access (CSMA) [17], [18] and related techniques such as inhibit sense multiple access (ISMA) [19]–[21], idle-signal casting multiple access (ICMA) [22] and busy channel multiple access [23]. According to the ALOHA protocol, any terminal is allowed to transmit a packet of data without considering whether another terminal is already transmitting on the common inbound (mobile-to-base) channel. Overlapping transmissions called "collisions" severely reduce the throughput of a channel if the offered traffic load is high. A refinement of the ALOHA protocol is slotted ALOHA. In the latter protocol all packets are transmitted within time slots defined by the base station. This reduces the probability of interference between terminals.

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In CSMA, each mobile terminal first senses the radio channel, and no packet transmission is initiated when the channel is already busy. As this dramatically reduces the number of collisions, the throughput is significantly higher than in slotted ALOHA networks [17]. However, in mobile radio nets with fading channels, a mobile terminal might not be able to sense a transmission by another (remote) terminal. This effect, known as the hidden terminal problem, is avoided in ISMA, where the base station transmits a "busy" signal to inhibit all other mobile terminals from transmitting as soon as an inbound packet is being received [21]. Murase and Imamura [22] addressed a possible extension of the feedback signaling: in idle-signal casting multiple access with collision detection (ICMA-CD) the base station broadcasts "idle," "busy," or "stop" (i.e., collision) messages. Andrisano *et al.* [23] studied the case that idle/busy feedback information is broadcast by on–off keying of a carrier, and they assessed probabilities of erroneously detecting feedback messages. However, even if signaling messages on this feedback channel are always received correctly by all mobile terminals, collisions can nonetheless occur in ISMA for two reasons: i) (re)transmissions of new or rescheduled packets can start during the delay in reception of the inhibit signal, and ii) packets from two or more persistent terminals, awaiting the channel to become idle, can collide immediately after the termination of the previous packet transmission. Considering these collisions, papers such as [17] determine the successful packet traffic based on the two assumptions that, firstly, a data packet is always received correctly in the absence of collisions and, secondly, all packets involved in a collision are lost. In mobile radio nets both assumptions should be reconsidered. Channel imperfections may cause loss of a data packet even if no interference from other terminals occurs; further, each received signal experiences fading, such that the received power can substantially vary for each terminal position. In the event of a collision, the strongest contending signal may then capture the receiver. Compared to remote terminals, nearby terminals thus experience a higher probability of success in transmitting a packet. This near–far discrimination has been studied previously for slotted ALOHA, e.g., [1], [2], [4], [5], but the performance of CSMA and ISMA protocols, which are also commonly used (see e.g., [25]), received relatively little attention. This has motivated the authors to present results for unslotted nonpersistent and p-persistent ISMA over fading radio channels and to compare these with the performance

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of slotted ALOHA. The results presented here for ISMA also apply for those CSMA nets where the hidden terminal problem is negligible.

Section II analyzes packet traffic over the radio channel. The probability of overlapping transmissions is studied for one particular "test" packet with *a priori* known properties, such as the area mean received power. Our method, presented in brief in [24], differs from other papers on ISMA and CSMA, where the Poisson distributed arrivals of packets at the receiver are studied without any specific knowledge about each packet [17], [20], [21]. Probabilities of a successful transmission, conditional on the position of the transmitter of the test packet, are developed to investigate the near–far effect. Section III summarizes the mechanisms of the mobile fading considered. Various models for receiver capture in a narrowband channel are reviewed and extended in Sections IV and V: in Section IV, block error probabilities are considered to evaluate the probability of capture, and in Section V a packet is considered to be received correctly if the signal-to-interference ratio is above a certain capture threshold. Results are compared and discussed in Section VI, and conclusions are given in Section VII.

II. Packet Traffic

All data packets are assumed to be uniform duration, equal to the (normalized) unit of time and, in the event of slotted ALOHA, equal to the duration of a time slot. If a packet is received correctly, the base station transmits an acknowledgment over the outbound channel. Assuming that these acknowledgments are never lost, the mobile terminal deems its packet to be lost if no acknowledgment is received, and retransmits the packet after waiting a random time. An infinite population of (independent) terminals is assumed, and the probability that one particular terminal attempts a (re)transmission during a certain interval of time is assumed to offer an infinitely small contribution to the total traffic. The total attempted packet traffic is denoted as G_t, expressed in the average number of attempted packets transmissions per unit of time (ppt). This (composite) traffic G_t includes not only the initial attempts to transmit newly arrived packets, but also includes attempts to retransmit previously unsuccessful packets. We assume that the number of (re)transmission attempts during a time interval of duration T is Poisson distributed, with mean TG_t. The probability of n attempts during T is

$$P_n(n) = \frac{(G_tT)^n}{n!} \exp(-G_tT) \tag{1}$$

with $(n = 0, 1, 2, \cdots)$. We address steady-state operation of the channel, i.e., G_t is assumed to be constant with time. Such stable behavior of the net [13] requires random waiting times to be sufficiently long to ensure uncorrelated interference during the initial and any successive transmission attempts.

The average number of attempted packet transmissions originating from a (normalized) distance r $(0 < r < \infty)$ from the central receiver, per (normalized) unit of time and per (normalized) unit of area, is denoted as $G(r)$ [1], [2]. The unit of distance (thus also the unit of area) is normalized to ensure that the major part of the traffic is transmitted in the range $0 < r < 1$ (see Section III). The total attempted packet traffic G_t is found from (polar) integration of $G(r)$ over the service area, viz.

$$G_t = \int_0^\infty 2\pi r G(r)\, dr. \tag{2}$$

The following probabilities of successful reception are defined.

$Q(r)$ is the unconditional probability of the successful reception of a test packet generated in a terminal at a distance r from the central base station, taking account of the probability of permission to transmit and averaged over the number of interfering packets n and over the unknown positions of the interfering terminals.

$q_n(r)$ is the probability of correct reception of a test packet transmitted from a distance r, given the number of interfering packets n, but averaged over the unknown positions from which these interfering signals originate.

q_n is the probability of successful reception given that the test packet is transmitted in the presence of n interfering signals, averaged over the unknown positions of all mobile terminals, including the one transmitting the test packet. Hence

$$q_n = \frac{1}{G_t} \int_0^\infty 2\pi r q_n(r) G(r)\, dr. \tag{3}$$

C_{n+1} is the expectation value of the number of correctly received packets in the event that $n + 1$ packets collide. If receiver capture is mutually exclusive for each of the $n + 1$ packets, as assumed in [1]–[7], [11], [13], [15], [20], and [21], $C_{n+1} = (n + 1)q_n$. In Section IV, this is discussed in more detail.

The total throughput S_t of the network is defined as the expected number of transmitted packets per unit of time that are detected correctly at the base station, i.e.:

$$S_t = \int_0^\infty 2\pi r Q(r) G(r)\, dr. \tag{4}$$

A. Slotted ALOHA

According to the ALOHA protocol, a mobile terminal transmits its packets regardless of other transmissions in the same time slot. The probability $P_n(n)$ that the test packet experiences interference from n other (contending) signals in the same time slot is assumed to be Poisson distributed, with mean G_t, thus with $T = 1$ inserted in (1). The probability $Q(r)$ of a successful transmission is

$$Q(r) = \sum_{n=0}^{\infty} P_n(n) q_n(r). \tag{5}$$

The total packet throughput results from

$$S_t = G_t \sum_{n=0}^{\infty} P_n(n) q_n = \sum_{i=1}^{\infty} P_n(i) C_i \tag{6}$$

where i represents the total number of packets in a time slot ($i = 0, 1, \cdots$ and $C_0 = 0$).

B. Inhibit Sense Multiple Access (ISMA)

In ISMA, the radio system is supplemented by an outbound signaling of the status of the channel: either "busy" or "idle." When the base station receives an inbound packet, a "busy" signal is broadcast to all mobiles after a processing delay d. This delay is normalized to the unity duration of each data packet ($0 \leq d < 1$). After termination of all $n+1$ contending transmissions, the base station starts transmitting an "idle" signal after a delay also of duration d. In CSMA, the delay is mainly caused by the time a mobile terminal takes to switch from reception to transmission mode (powerup), after sensing the radio channel for carriers from other active terminals [25].

The busy period is the period during which the base station broadcasts a busy signal plus the preceding signaling delay d. For memoryless Poisson arrivals, the expected duration I of the idle period between two busy periods equals the average lapse of time until a new packet arrival occurs, thus $I = G_t^{-1}$ [17]. The average duration B of the busy period depends on the signaling delay d and on the persistency p in scheduling inhibited packets [17]. We address an unslotted version of p-persistent CSMA and ISMA. If the channel is idle, the packet is transmitted immediately. If the channel is busy, the terminal performs a binary random experiment: with probability p the terminal transmits the packet as soon as the channel becomes idle. Such an attempt is considered successful unless the packet is destroyed in a collision. Alternatively, with probability $1-p$, the packet arriving at an instant when the receiver is busy is rescheduled with a random delay. Such an inhibited packet is considered to be unsuccessful and a next attempt is to be performed after waiting a random time. For nonpersistent CSMA and ISMA, rescheduling always occurs if the channel is busy at the instant of sensing.

1) Unslotted Nonpersistent ISMA: If a packet arrives at a nonpersistent terminal when the base station transmits a "busy" signal (denoted by event H_B), the attempt is considered to have failed. The packet is rescheduled for later transmission. With probability $I/(I+B)$, a test packet transmitted from a distance r starts at an instant when the channel is idle. This event is denoted as H_I. A collision can occur if one or more other terminals start transmitting during the time delay d of the inhibit signal. The conditional probability of n transmissions overlapping with the test packet that initiated the busy period is

$$R_n(n|H_I) = \frac{(dG_t)^n}{n!} \exp(-dG_t). \tag{7}$$

Alternatively, the test packet itself starts during a period of duration d when the channel is busy because of a transmission by another terminal, but seems idle since the inhibit signal is not yet being broadcast. This event, denoted as H_d, occurs with probability $d/(B+I)$. This packet thus experiences interference from at least one contender. The additional, $n-1$ contending signals arrive independently of this event, so they are Poisson distributed. The conditional probability of n interferers is found from

$$R_n(n|H_d) = \frac{(dG_t)^{n-1}}{(n-1)!} \exp(-dG_t) \tag{8}$$

where ($n = 1, 2, \cdots$). Taking account of the above three possible events H_B, H_I, and H_d, the unconditional probability of successful transmission $Q(r)$ is

$$\begin{aligned} Q(r) &= P(H_I)Q(r|H_I) + P(H_d)Q(r|H_d) \\ &= \frac{I}{B+I} \sum_{n=0}^{\infty} R_n(n|H_I) q_n(r|H_I) \\ &\quad + \frac{d}{B+I} \sum_{n=1}^{\infty} R_n(n|H_d) q_n(r|H_d). \end{aligned} \tag{9}$$

The probability of capture $q_n(r)$ depends, among other things, on the probability to acquire receiver synchronization, which, in general, depends on the channel status (H_d or H_I) at the arrival of the packet. This is elaborated in Section IV for a simplified synchronization model. The busy period was shown in [17] to be of average duration

$$B = 1 + 2d - \frac{1}{G_t}[1 - \exp(-dG_t)]. \tag{10}$$

The total channel throughput S_t is found from the integration

$$S_t = \int_0^{\infty} \frac{2\pi r G(r)}{B+I} \exp(-dG_t) \sum_{n=0}^{\infty} \cdot \left[\frac{d^n G_t^{n-1}}{n!} q_n(r|H_I) + \frac{n d^n G_t^{n-1}}{n!} q_n(r|H_d) \right] dr. \tag{11}$$

After interchanging the order of integration and the summation, we recover the expression used in [20], [21] by considering Poisson arrivals of packets at the receiver in the base station, viz.:

$$S_t = \frac{1}{B+I} \exp(-dG_t) \sum_{n=0}^{\infty} \frac{d^n G_t^n}{n!} C_{n+1} \tag{12}$$

where we assumed $C_{n+1} = q_{n|H_I} + n q_{n|H_d}$. For instantaneous inhibit signaling ($d \to 0$), collisions can never occur in nonpersistent ISMA, and (12) reduces to $S_t \to G_t(1+G_t)^{-1}$, in agreement with [17].

2) Unslotted p-Persistent ISMA: We now consider unslotted p-persistent ISMA ($0 \leq p \leq 1$) without signaling delay ($d = 0$), although in practice only nonzero values of d can be achieved. The case $d = 0$ thus represents an upperbound on throughput performance. A busy period can consist of a number of packet transmissions in succession because a terminal may start transmitting as soon as the previous transmission by another mobile station is terminated. If a packet arrives during an idle period (event H_I), the probability of correct reception of this initial packet is $q_0(r)$. During the transmission of this packet, a random number of k ($k = 0, 1, 2, \cdots$) terminals sense the channel busy with probability $P_n(k)$, with $T = 1$ in (1). When the channel goes idle, each of the k terminals starts transmitting with probability p. For a test packet arriving during a busy period (event H_B), the probability of n interfering packets is thus

$$P_n(n|p) = \sum_{k=n}^{\infty} \binom{k}{n} P_n(k)(1-p)^{k-n} p^n = \frac{(pG_t)^n}{n!} \exp(-pG_t). \tag{13}$$

In particular, the probability that the busy period is terminated, i.e., none of the k terminals starts transmitting, is

$$P_n(0|p) = \exp(-pG_t). \tag{14}$$

The probability $P_m(m)$ of continuing transmissions during m ($m = 0, 1, \cdots$) units of time, concatenated to the initial packet is

$$P_m(m) = \exp(-pG_t)[1 - \exp(-pG_t)]^m. \tag{15}$$

On the average, a busy period thus has the total duration

$$B \triangleq \mathrm{E}_m[1+m] = 1 + \exp(-pG_t) \sum_{m=0}^{\infty} m[1 - \exp(-pG_t)]^m = \exp(+pG_t) \tag{16}$$

where E_m denotes the expectation over m. The probability of a successful transmission $Q(r)$ is

$$Q(r) = P(H_I)Q(r|H_I) + P(H_B)Q(r|H_B) = \frac{I}{B+I} q_0(r) + \frac{B}{B+I} pP_n(n|p)q_n(r). \tag{17}$$

Using $I = G_t^{-1}$, (13), and (16), (17) becomes

$$Q(r) = \frac{q_0(r) + p\,G_t \sum_{n=0}^{\infty} \frac{(pG_t)^n}{n!} q_n(r)}{1 + G_t \exp(pG_t)}. \tag{18}$$

After integration, the total channel throughput S_t is obtained from

$$S_t = G_t \frac{q_0 + \sum_{i=1}^{\infty} \frac{(pG_t)^i}{i!} C_i}{1 + G_t \exp(pG_t)}. \tag{19}$$

In the special case $p = 0$, we recover the result for unslotted nonpersistent ISMA without signaling delay ($d = 0$). The classical case of 1-persistent CSMA on wired channels is found by inserting $q_0 = 1$ and $q_n = 0$ for $n \geq 1$ in (19). The throughput then becomes

$$S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)}.$$

This agrees with expressions derived for Poison arrivals at a receiver without capture [17].

III. Channel Model

The normalized local-mean power $\overline{p}_j$ received from the jth mobile terminal ($j = 0, 1, \cdots$) at a (normalized) distance r_j from the central receiver is taken to have the form [26]:

$$\overline{p}_j = r_j^{-4}. \tag{20}$$

If the position of the terminal is unknown, the probability density function (PDF) of the mean power $\overline{p}_j$ is found from [2]

$$\left| f_{\overline{p_j}}(\overline{p_j})\,d\overline{p_j} \right| = \left| \frac{2\pi r_j G(r_j)}{G_t}\, dr_j \right|. \tag{21}$$

In the following, we assume the quasi-uniform spatial distribution of the offered channel traffic suggested in [2], and also in [8], [9], [11], [15], [16], namely:

$$G(r) = \frac{G_t}{\pi} \exp\left(-\frac{\pi}{4} r^4\right). \tag{22}$$

This is an approximation of the exactly uniform distribution

$$G(r) = \begin{cases} \frac{G_t}{\pi}, & 0 < r < 1 \\ 0, & \text{elsewhere} \end{cases}$$

by a smooth analytical function. The main part of the traffic thus arrives from the normalized distance $0 < r < 1$, whereas beyond $r = 1$ the intensity of the attempted traffic rapidly decreases. The main reason to consider a quasi-uniform, rather than an exactly spatial distribution is the convenient analytical expression found for the PDF of the joint power of n uncorrelated signals, viz. [2]:

$$f_{\overline{P}_t}(\overline{P_t}|n) = \frac{n}{2} \overline{P}_t^{-\frac{3}{2}} \exp\left(-\frac{\pi n^2}{4\overline{P}_t}\right) \tag{23}$$

where $\overline{P_t}$ $\left(\overline{P_t} = \sum \overline{p_j}\right)$ is the local-mean power of the joint interference signal.

Further, Rayleigh fading in a narrow-band channel is assumed [26]. The instantaneous amplitude ρ_j of the jth carrier is Rayleigh distributed, with PDF:

$$f_{\rho_j}(\rho_j|\overline{p_j}) = \frac{\rho_j}{\overline{p}_j} \exp\left(-\frac{\rho_j^2}{2\overline{p_j}}\right) \tag{24}$$

for $0 < \rho_j < \infty$. This Rayleigh distribution is due to Gaussian in-phase and quadrature carrier components ζ_j and ξ_j, respectively, with identical with zero mean and a variance equal to the local-mean power. The corresponding total instantaneous (in-phase plus quadrature) power p_j $\left(p_j = 1/2\rho_j^2 = 1/2\zeta_j^2 + 1/2\xi_j^2\right)$ received from the jth mobile terminal is exponentially distributed about the mean power, viz.:

$$f_{p_j}(p_j|\overline{p_j}) = \frac{1}{\overline{p_j}} \exp\left(-\frac{p_j}{\overline{p_j}}\right) \tag{25}$$

with $p_j \geq 0$. Combining the statistical fluctuations caused by the spatial distribution of the mobile terminals (22) and by Rayleigh fading, the unconditional PDF's of the amplitude ρ_j and the in-phase component ζ_j are

$$f_{\rho_j}(\rho_j) = \frac{\sqrt{2\pi}\,\rho_j}{\left(\frac{\pi}{2} + \rho_j^2\right)^{\frac{3}{2}}} \tag{26}$$

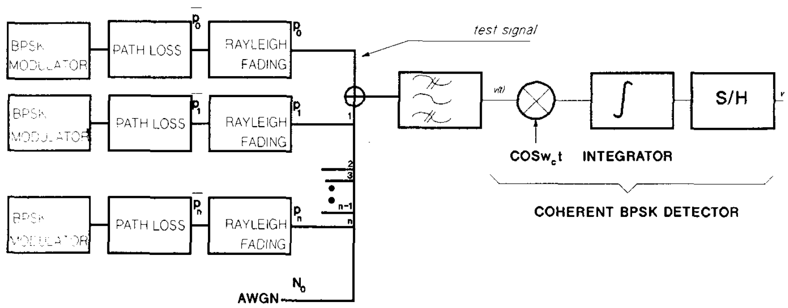


Fig. 1. Correlation receiver for coherent detection of BPSK in a mobile radio channel with multiple interferers and Gaussian noise.

for $0 < \rho_j < \infty$, and

$$f_{\zeta_j}(\zeta_j) = \frac{1}{\sqrt{2\pi}\left(\frac{\pi}{2} + \zeta_j^2\right)} \tag{27}$$

for $-\infty < \zeta_j < \infty$, respectively [8]. Any retransmitted or rescheduled packet is assumed to experience uncorrelated fading and path loss.

IV. Probability of Packet Error

In this section, a test packet of L bits is assumed to have captured the receiver if and only if the bit sequence detected by the receiver entirely matches the bit sequence in the test packet. The assumption to require *all* bits in the test packet to be received correctly is pessimistic. Most practical implementations of packet radio employ some form of forward error correction coding, which allows a test packet to be received correctly as long as the number of errors is within the error correcting capability of the code [7], [10], [14], [15]. The case of fast fading, i.e., with signal amplitudes independent from bit to bit, is studied considering two different models: model **I.A** describes a receiver that perfectly locks to the test packet, whereas in model **I.B** the receiver selects one favorite packet out of the $n+1$ contending signals. In the latter case, the test packet is always assumed to be lost if the receiver happens to lock to another signal.

With slow fading, which will be studied for five cases (model **II.A–II.E**), the amplitude and phase of each signal are assumed constant for the entire duration of the packet. Models **II.A** and **II.B** correspond to the receiver synchronization behavior assume in **I.A** and **I.B**, respectively. The models **II.C–II.E** are introduced to approximate the effect that the carrier recovery in the receiver is impaired by interfering signals.

A. Receiver Model

The classical correlation receiver [27] is considered, which is known to be optimal for time invariant channels with additive white Gaussian noise (AWGN), but free of interference. We study the performance of this receiver with n cochannel signals interfering with the test signal (Fig. 1). At least during one bit, the phase θ_j and the amplitude ρ_j of each signal is assumed to remain constant. For each of the interfering signals, exactly overlapping bit periods are assumed, i.e., no bit synchronization offset is taken into account. The received signal $v(t)$ is on the form

$$v(t) = \rho_0\kappa_0\cos(\omega_c t + \theta_0) + \sum_{j=1}^{n}\rho_j\kappa_j\cos(\omega_c t + \theta_j) + n(t) \tag{28}$$

where κ_j ($\kappa_j = \pm 1$) represents phase reversals due to BPSK modulation of the jth carrier and $n(t)$ is the AWGN, bandpass filtered for the transmission bandwidth B_T in any practical receiver. The test signal is denoted by index 0. Fig. 2 illustrates the phase-quadrature constellation for a test packet in the presence of three interferers and (bandlimited) AWGN. Three idealized cases for the phase of the local oscillator in the coherent detector are compared, namely, a receiver locked to the test signal (Fig. 2(a)), a receiver locked to one of the interferers (Fig. 2(b)), and a receiver with arbitrary (but constant) phase (Fig. 2(c)). The latter two events correspond to extreme cases of carrier phase errors in the receiver, caused by signals competing with the test packet.

In the detector, the received composite signal $v(t)$ is multiplied by a locally generated cosine ($2\cos\omega_c t$) and integrated over the entire (normalized) bit duration T_b ($T_b L = 1$). The received energy per bit is $E_b = p_0 T_b \cos^2\theta_0 = 1/2\rho_0^2 T_b \cos^2\theta_0$. The decision variable for synchronous bit extraction from a test packet in the presence of n interferers is

$$v = \frac{1}{T_b}\int_0^{T_b} 2v(t)\cos(\omega_c t)\,dt = \rho_0\kappa_0\cos(\theta_0) + \sum_{j=1}^{n}\rho_j\kappa_j\cos(\theta_j) + n_i. \tag{29}$$

In a Rayleigh fading channel, both the in-phase components ζ_j ($\zeta_j = \rho_j\cos\theta_j$) of the n interferers with random phase relative to the local oscillator in the receiver, and the noise sample n_i, are independent Gaussian variables. Phase reversals of bit synchronous interference do not change the Gaussian PDF. The rate of fading determines the correlation of amplitude and phase in successive bits. Fast and slow fading are distinguished.

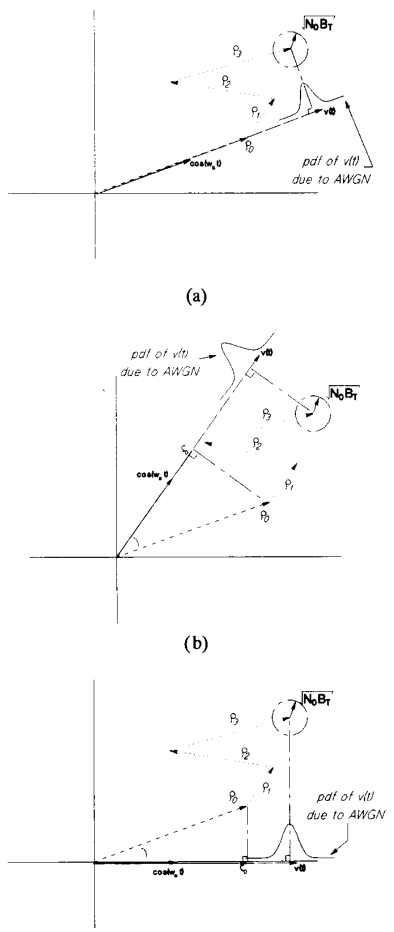


Fig. 2. Phasor diagram for a test signal ρ_0(--), three interferers $\rho_1 \cdot\cdot \rho_3$(..) and AWGN $n(t)$ offered to a coherent detector (—). AWGN produces the Gaussian probability density $N(\sum \rho_j \cos\theta_j, N_0 B_T)$ of the instantaneous signal $v(t)$. (a) Detector locked to the test packet. (b) Detector locked to the first interfering signal. (c) Detector with arbitrary phase.

B. Fast Fading

Fast Rayleigh fading is defined by the duration of a packet being substantially longer than the average nonfade duration of the channel [28]. During reception of a packet, the signal is expected to experience several fades. On the other hand, to ensure proper detection of BPSK signals, at least during a number of successive bits, the carrier phase must remain sufficiently constant. Both conditions are assumed to be fulfilled simultaneously. During each bit, a synchronous detector sees Gaussian interference and noise in the decision variable v with average energy $\overline{P_t}T_b + N_0$, with N_0 the spectral density of the AWGN. The corresponding conditional bit error probability for a receiver locked to the test packet ($\theta_0 = 0$) is

$$P_b(e|\theta_0 = 0, p_0, \overline{P_t}) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{p_0 T_b}{\overline{P_t}T_b + N_0}}\right) \tag{30}$$

where the instantaneous power p_0 of the test signal is exponentially distributed (25). After averaging over the Rayleigh fading of the test packet, the bit error probability is

$$P_b(e|\theta_0 = 0, \overline{p_0}, \overline{P_t}) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p_0}T_b}{\overline{p_0}T_b + \overline{P_t}T_b + N_0}}. \tag{31}$$

Limiting cases agree with [27], [29] for a fading AWGN channel $(\overline{P_t} = 0)$ and with [30] for a noise-free channel $(N_0 = 0)$. The probability of correct reception for BPSK without differential encoding is obtained from

$$q_n(r|\overline{P_t}) = \left[1 - P_b(e|\overline{p_0} = r_0^{-4}, \overline{P_t})\right]^L. \tag{32}$$

Here, it has been assumed that the received amplitude and phase of all signals are statistically independent from bit to bit. After integrating over the mean power of the joint interference signal (23) and substituting $\overline{P_t} = s^{-4}$, the probability of correct reception is found to be

$$q_n(r|\theta_0 = 0) = \int_0^\infty 2ns \exp\left(-\frac{\pi}{4}n^2 s^4\right) \cdot \left[\frac{1}{2} + \frac{1}{2}\sqrt{\frac{T_b r^{-4}}{T_b r^{-4} + T_b s^{-4} + N_0}}\right]^L ds \tag{33}$$

This probability is depicted in Fig. 3.

Two capture models are now defined.

Model I.A: In the event of fast Rayleigh fading, any packet is expected to experience several deep fades. Correct reception of all L bits in the test packet is likely to occur only if its received signal power, averaged over the packet duration, significantly exceeds the power of the interfering signals. We assume that, in the case of correct reception, the signal is also sufficiently strong to maintain perfect receiver synchronization during the entire packet. On the other hand, for a weak signal the probability of correct reception is negligible irrespective of the carrier phase synchronization, provided L is sufficiently large. This leads to the (optimistic) model

$$q_n(r) = q_n(r|\theta_0 = 0) \tag{34a}$$

and

$$C_{n+1} = (n+1)q_{n|\theta_0=0} = (n+1)\int_0^\infty 2\pi r q_n(r|\theta_0 = 0)G(r)\,dr. \tag{34b}$$

Model I.B: The receiver randomly locks to the kth signal $(k = 0, 1, \cdots, n)$, where k does not necessarily represent the test packet or the strongest contending packet. Correct reception of the test packet occurs if and only if the receiver locks to the test packet, i.e., if the receiver selects $k = 0$, and no bit error occurs [8]. This produces the (pessimistic) relations

$$q_n(r) = P(k=0)q_n(r|\theta_0 = 0) = \frac{1}{(n+1)}q_n(r|\theta_0 = 0) \quad \text{and} \quad C_{n+1} = q_{n|\theta_0=0}. \tag{35}$$

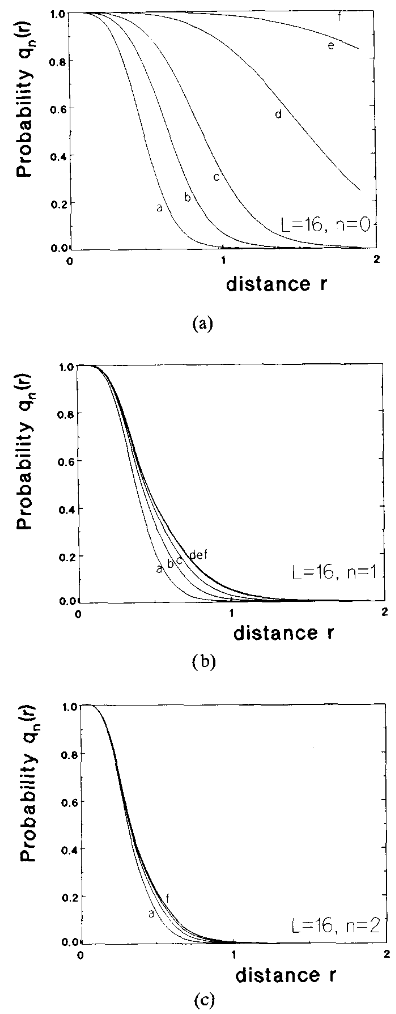


Fig. 3. Capture probability $q_n(r)$ for coherent detection given the presence of n interferers, for a receiver locked to the test packet in a fast-fading channel. The median C/N ratio is 0, 5, 10, 20, 30, and 50 dB for curves $a, \cdots f$, respectively. The packet length is $L = 16$ b. The number of interferers is (a) $n = 0$, (b) $n = 1$, and (c) $n = 2$.

For a comparison of model **I.A** and **I.B** we refer to [9]. The results for the two models appeared essentially different for large offered traffic loads.

C. Slow Fading

To fully exploit the benefits from receiver capture, packets should be short with respect to the average nonfade duration [28]. For packets of sufficiently short duration, the received amplitude and carrier phase may be assumed constant throughout the entire duration of a packet for each of the $n + 1$ signals. This condition is satisfied if the motion of each transmitter during a packet time is negligible compared to the wavelength [26]. Packet error rates for a slow fading (test) signal were reported in [29] for channels with AWGN. For angle modulated signals with constant envelope, a Gaussian approximation of PDF of the interference signal becomes inaccurate: successive interference samples in the decision variable occur with constant in-phase amplitude $\zeta_1, \cdots, \zeta_n$ throughout the entire packet. Zhang and Pahlavan [10] studied correlated interference samples in successive bits. Their model is elaborated here. In particular, the distinction between the *a priori* selected test packet and the packet that acquires receiver synchronization is further developed.

As illustrated by Fig. 2, the components $v_i(t)$ of the test signal plus interference and (bandlimited) AWGN in phase with the oscillator in the receiver have a Gaussian PDF with variance $N_0 B_T$ and mean value

$$E[v_i(t)] = \sum_{j=0}^{n} \rho_j \kappa_j \cos\theta_j. \tag{36}$$

Whenever the test packet contains a "1" ($\kappa_0 = 1$), the detector makes a bit error if the decision variable happens to be less than zero. For reasons of symmetry of the events $\kappa_0 = \pm 1$, the bit error probability, conditional on the set of bits $\{\kappa_j\}$ in the interfering packets and on the envelope and phase of each signal, is

$$\begin{aligned} P_b\left(e|\{\rho_j, \theta_j, \kappa_j\}_{j=0}^n\right) &= P_b\left(e|\kappa_0 = 1, \{\rho_j, \theta_j, \kappa_j\}_{j=0}^n\right) \\ &= \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho_0 \cos\theta_0 + \sum_{j=1}^{n} \kappa_j \rho_j \cos\theta_j}{\sqrt{2N_0/T_b}}\right). \end{aligned} \tag{37}$$

After coherent detection, the integrated noise n_i is uncorrelated during each successive bit. Further, the mean value of the PDF of the interference plus noise sample changes due to phase reversals in the $n+1$ signals. With n interfering signals, there are 2^n different sets $\{\kappa_j\}$ and all are assumed to be equally likely, so

$$\begin{aligned} P_{b_0}\left(e|\{\rho_j, \theta_j\}_{j=0}^n\right) = 2^{-n-1} &\sum_{\kappa_1=\pm1,\cdots,\kappa_n=\pm1} \\ &\cdot\, \mathrm{erfc}\left(\frac{\rho_0 \cos\theta_0 + \sum_{j=1}^{n} \kappa_j \rho_j \cos\theta_j}{\sqrt{2N_0/T_b}}\right). \end{aligned} \tag{38}$$

Each in-phase component ζ_j ($\zeta_j = \rho_j \cos\theta_j$) either has probability density (27) (if θ_j is random) or has the density (26) (if the receiver is locked to the jth signal). For BPSK, the capture probability conditional on the carrier phasors $\{(\rho_j, \theta_j)\}$ received from the interference signals, or, equivalently, conditional on the Cartesian set $\{(\zeta_j, \xi_j)\}$, is

$$q_n\left(r\,\middle|\,\{\rho_j, \theta_j\}_{j=0}^n\right) = \left[1 - P_b\left(e\,\middle|\,\{\rho_j, \theta_j\}_{j=0}^n\right)\right]^L. \tag{39}$$

For differential encoding (DPSK), phase reversal of the carrier of the test packet (and thus inverting all L bits) does not affect the probability of a bit error. Consequently, the conditional packet error probability for coherent detection of

DPSK follows from

$$q_n\left(r\,\middle|\,\{\rho_j,\theta_j\}_{j=0}^n\right) = \left[1 - P_b\left(e\,\middle|\,\{\rho_j,\theta_j\}_{j=0}^n\right)\right]^L + \left[P_b\left(e\,\middle|\,\{\rho_j,\theta_j\}_{j=0}^n\right)\right]^L. \quad (40)$$

The effect that one extra bit is required for differential encoding is ignored here. Three cases of carrier synchronization of the receiver are considered: a receiver locked to the test packet (Fig. 2(a)), a receiver locked to another packet (Fig. 2(b)), and a receiver with a constant but arbitrary phase with respect to each signal (Fig. 2(c)).

a) The capture probability $q_n(r|\theta_0 = 0)$ for receiver locked to the test packet is obtained by integrating over the in-phase amplitudes $\{\rho_0, \zeta_1, \cdots, \zeta_n\}$. So

$$q_n(r|\theta_0 = 0) = \int_0^\infty d\rho_0 \int_{-\infty}^\infty d\zeta_1 \cdots \int_{-\infty}^\infty d\zeta_n \cdot q_n(r|\{\rho_j,\theta_j\},\theta_0 = 0) \cdot f_{\rho_0|r} f_{\zeta_1} \cdots f_{\zeta_n}. \quad (41)$$

b) If the carrier recovery circuit in the receiver is accidentally locked to the kth interference signal ($k \neq 0$), the decision variable, evaluated for a test packet with a random (but constant) phase $\theta_0 \in [0, 2\pi\rangle$ relative to the receiver, goes into

$$v = \kappa_0\zeta_0 + \kappa_k\rho_k + \sum_{j=1, j\neq k}^n \kappa_i\zeta_i + n_i. \quad (42)$$

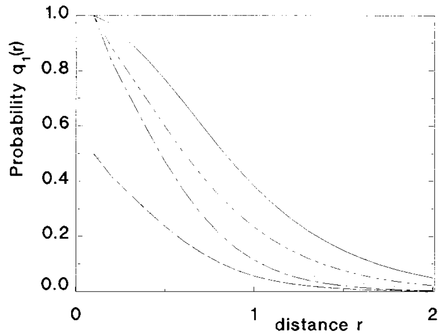
Here, we assumed perfect bit alignment for each of the $n+1$ colliding signals. The capture

$$q_n(r|\theta_k = 0, 1 \le k \le n) = \int_{-\infty}^\infty d\zeta_0 \int_{-\infty}^\infty d\zeta_1 \cdot\cdot \left(\int_0^\infty d\rho_k\right) \cdot\cdot \int_{-\infty}^\infty d\zeta_n \; q_n\left(r|\{\rho_j,\theta_j\}_{j=0}^n, \theta_k = 0, 1 \le k \le n\right) \cdot f_{\zeta_0|r} f_{\zeta_1} \cdot\cdot (f_{\rho_k}) \cdot\cdot f_{\zeta_n}. \quad (43)$$

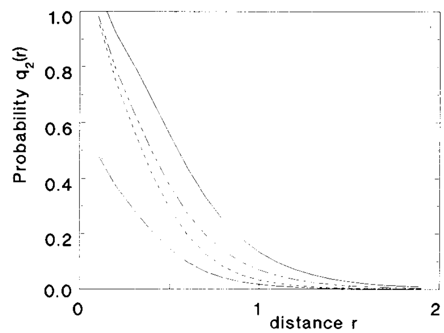
c) The phase reference of the coherent detector is assumed constant but arbitrary, i.e., it is not related to the phase of any of the $n+1$ signals. Analogous to the previous models, perfect bit alignment is considered for ease of analysis. Each signal produces a Gaussian in-phase component. The capture probability of error-free detection turns into

$$q_n(r|\text{no lock}) = \int_{-\infty}^\infty d\zeta_0 \cdots \int_{-\infty}^\infty d\zeta_n q_n\left(r\,\middle|\,\{\rho_j,\theta_j\}_{j=0}^n\right) \cdot f_{\zeta_0|r} f_{\zeta_1} \cdot\cdot f_{\zeta_n}. \quad (44)$$

Numerical results, e.g., in Fig. 4, have been obtained by computation of the above n dimensional integrals. Due to the wide dynamic range of the signals, numerical results obtained from the computations tended to become less accurate for $r < 0.1$. For a coherent receiver locked to the test packet, BPSK and DPSK are seen to give almost equal capture probabilities. If the receiver locks to an interfering signal or if the receiver has an arbitrary phase, differential encoding increases the probability of correct reception by almost a factor of 2.



(a)



(b)

Fig. 4. Capture probability $q_n(r)$ for coherent detection given the presence of one interferer, for a receiver locked to the test packet (—), for a receiver locked to the interferer with BPSK (--) and DPSK (-...-), and for a receiver with arbitrary phase with BPSK (...) and DPSK (-.-) in a slow-fading channel. The median C/N is 20 dB. The packet length is $L = 16$ b. (a) 1 interferers ($n = 1$). (b) 2 interferers ($n = 2$).

We now postulate five capture models, based on the aforementioned cases of idealized carrier synchronization.

*Model **II.A**:* Analogous to model **I.A**, perfect carrier synchronization is always assumed for the test packet. We apply (34): for each of the signal k ($k = 0, 1, \cdots, n$) in the collision, we add the probability of capture if the receiver were perfectly locked to signal k. Particularly if two signals with approximately equal power are competing, this assumption is optimistic.

*Model **II.B**:* The receiver locks to one of the $n+1$ contending signals. Capture occurs if and only if the receiver locks to the test packet *and* the detected bit sequence is identical to the bit sequence of the test packet. The capture probability is found from (35), (39), and (41). In case of ISMA the receiver locks to the first arriving packet, thus to the one that terminates the idle period.

The models **II.C–II.E** address the effect of inaccuracies in receiver synchronization with respect to the phase of the test packet, caused by interfering signals. Extreme cases are considered: the receiver is assumed to lock perfectly to the carrier of one of the $n+1$ signals, but this does not exclude other packets from being received correctly. The probability of correct detection of the test packet is evaluated for events of reception with and without phase offset from (41) and (44).

For CSMA or ISMA, the receiver is assumed to maintain lock to the first arriving packet (with H_I). The capture probability for the initial packet is found from

$$q_r(r|H_I) = q(r|\theta_0 = 0) \tag{45}$$

with (41), whereas for each successive packet, the capture probability is taken to have the form

$$q_n(r|H_d) = q_n(r|\theta_k = 0, 1 \le k \le n) \tag{46}$$

with (43).

*Model **II.C**:* The receiver locks to one of the $n+1$ BPSK signals. The probability that one of the $n+1$ packets is received correctly is $C_{n+1} = q_n(r|k=0) + nq_n(r|k \neq 0)$.

*Model **II.D**:* This model is identical to **II.C**, except that we assume coherent detection of differential encoding (DPSK).

*Model **II.E**:* The phase reference of the coherent detector is constant but arbitrary, i.e., it is not related to the phase of any of the $n+1$ signals. With DPSK, the probability of correct packet reception follows from (44) and $C_{n+1} = (n+1)q_n$.

The models **II.C**, **II.D**, and **II.E** require constant carrier phases that are not affected by mobile fading of different carrier frequencies throughout the busy period. Consequently, these models are limited to slow fading and become inappropriate with fast fading.

V. Capture Ratio

In 1977, Abramson [1] suggested the "vulnerability circle" to study receiver capture: a packet transmitted from a terminal at a distance r is received correctly if no other packet is transmitted within a circle of radius cr, with c a system constant. Arnbak and Van Blitterswijk [2] extended this model taking Rayleigh fading and cumulation of interference power (if $n = 1, 2, \cdots$) into account. A test packet is considered to capture the receiver in the base station if and only if its instantaneous power p_0 exceeds the instantaneous joint interference power P_t by at least a threshold factor z. Received powers are assumed constant during the packet time (slow fading).

This capture model, based on the cochannel rejection ratio of typical receivers, is reasonable for popular analog FM modulation by an FFSK subcarrier [25] or for the commonly used direct frequency modulation of a carrier by a suitably filtered baseband bit sequence: satisfactory reception occurs as long as the signal remains above the FM threshold. Further, the capture ratio model provides a counterpoise to the foregoing computations based on packet error rates for idealized bit and carrier synchronization. The value of $z = 4$ (6 dB), as considered here for numerical examples, is believed to be an optimistic representation of the immunity of practical narrowband receivers against constant-envelope interference ($n = 1$) or Gaussian distributed interference ($n \to \infty$). Practical values of z also depend on synchronization capabilities of the receiver and on the type of error correction coding applied.

In [5] and [13] it was shown that the refinements in [2] are equivalent to replacing the vulnerability circle [1] by spatially weighing the interference traffic $G(x)$ by an analytical weight function $W(x,r)$: For channels without shadowing, the probability of capture $q_n(r)$ was found to be

$$q_n(r) = q_1^n(r) = \left[\frac{1}{G_t}\int_0^\infty W(x,r)\,G(x)\,2\pi x\,dx\right]^n \tag{47}$$

where

$$W(x,r) \triangleq x^4\left(x^4 + zr^4\right)^{-1} \tag{48}$$

and $(n = 0, 1, \cdots)$. The vulnerability circle [1] is recovered by replacing this weight function by a step function. For the quasi-uniform spatial distribution (22), one finds after applying [31, eq. (7.4.1)]

$$q_n(r) = \left[1 - \frac{\pi}{2}\sqrt{z}\,r^2\,\mathrm{erfc}\!\left(\frac{\sqrt{z\pi}}{2}\,r^2\right)\exp\!\left(\frac{zr^4\pi}{4}\right)\right]^n \tag{49}$$

with $\mathrm{erfc}(\cdot)$ the complementary error function [32]. A similar expression for the exactly uniform distribution is reported in [24]. An investigation of various spatial distributions and a further comparison of the vulnerability circle [1] with the capture ratio model [2] is presented by Lau and Leung in [11] for channels without Rayleigh fading. A possible technique to consider the additional effect of a receiver noise floor on $q_n(r)$ will be proposed in [16], [34].

Using the series expansion of the exponential function, the probability of a successful transmission for slotted ALOHA is found on the form of

$$Q(r) = \exp\{-G_t(1 - q_1(r))\}. \tag{50}$$

Using (47) with (9), the probability of correct reception in nonpersistent ISMA becomes

$$Q(r) = \frac{\exp\{-dG_t(1 - q_1(r))\}\{1 + q_1(r)dG_t\}}{G_t(1 + 2d) + e^{-dG_t}}. \tag{51}$$

Finally, the probability $Q(r)$ for unslotted p-persistent ISMA with zero signaling delay is obtained from (47) and (18) as

$$Q(r) = \frac{1 + pG_t\exp\{pG_tq_1(r)\}}{1 + G_t\exp(pG_t)} \tag{52}$$

VI. Computational Results and Discussion

A. Probability of Success and Near–Far Effect

Capture probabilities as obtained from (33), (41), and (47) are depicted in Figs. 5(a)–(c), respectively. For any n, the packet error probability in a fast-fading channel (Fig. 5(a)) is seen to increase rapidly beyond a certain distance, particularly for fast fading. Such a turnover point is less apparent for slow fading (Fig. 5(b)), and distant terminals experience a relatively high probability of capture: with slow Rayleigh fading, a packet form a distant terminal may under certain circumstances even survive a collision with a more nearby terminal. However, if the slow-fading channel is evaluated from a receiver threshold of $z = 4$ (Fig. 5(c)) such high probabilities

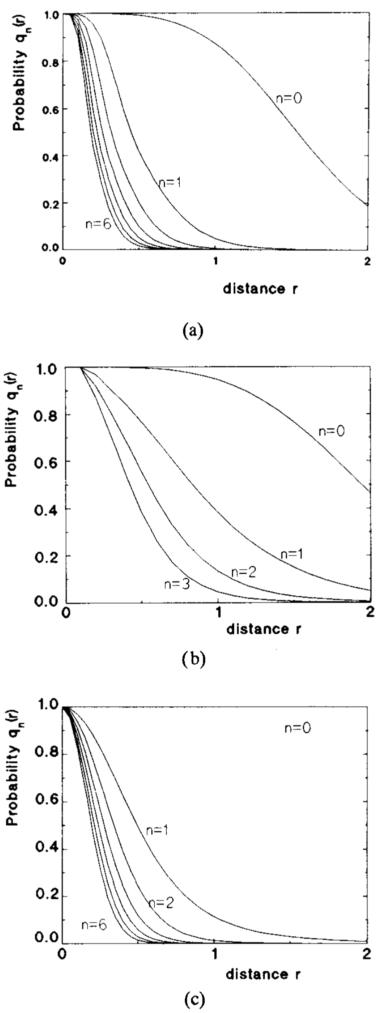


Fig. 5. Capture probability $q_n(r)$ for coherent detection given the presence of n interferers. (a) and (b) Fast fading and slow fading, respectively, for a receiver locked to the test packet, and median C/N of 20 dB, and a packet length of $L = 16$ b. (c) Capture ratio of $z = 4$ (6 dB).

of capture are not confirmed. This indicates that in (41) a substantial portion of the successful packets are considered to capture the receiver at remarkably low C/I ratios. Practical narrow-band receivers appear to require a C/I ratio of at least 6 dB to perform reliable detection and synchronization [25]. This suggests that considering packet error rates for a perfectly synchronized receiver are presumably optimistic.

In Fig. 6, capture probabilities $q_n(r)$ have been used to find the probability $Q(r)$ of a successful transmission for various protocols. The models **I.A**, **II.A**, and **III** are compared for an offered traffic of $G_t = 1$ ppt in Figs. 6(a)–(c), respectively. Slotted ALOHA is seen to result in the most significant near–far unfairness because of prevailing packets from nearby users. In contrast to this, nonpersistent ISMA without delay $(d = 0)$ provides a uniform probability of access for all terminals, although $Q(r)$ degrades for terminals beyond $r = 1$ because of noise. A signaling delay $(d > 0)$ is known to degrade the average network performance for receivers with capture [20], [21] and without [17] capture. Nonetheless, it appears that nearby users benefit from a signaling delay. This is explained by a high probability of capture for strong packets arriving during the inhibit signaling delay. For nonpersistent ISMA in slow-fading channels, the probability of a successful transmission according to the models **II.A–II.E** are compared in Fig. 7. For terminals near the receiver, ignoring capture in the event of H_d, as it occurs in model **II.B**, gives pessimistic estimates for the probability of successful transmission. Even if packets arriving during the delay d fail to seize full carrier synchronization, as e.g., is assumed in models **II.C–II.E**, they nonetheless have a substantial probability of being received correctly. Results for the various models mainly differ for small r. For weak signals arriving from remote terminals, capture is mostly limited to cases with no contenders at all. Hence, probabilities for various models converge for large r, except for model **II.E**.

B. Spatial Average Probability of Success and Total Throughput

Table I summarizes the probability C_{n+1} that one out of $n + 1$ packets captures the receiver for the various models defined. If a data packet arrives without interference $(n = 0)$, a median C/N of 20 dB causes an average outage probability of less than 4% $(q_0 \approx 0.96)$ in a slow-fading channel and 8% $(q_0 \approx 0.92)$ in a fast-fading channel. If interference is present $(n \geq 1)$, the models for packet error rates produce widely different estimates of the probability that one out of $n + 1$ signals is received correctly. It is seen that model **II.A** becomes unacceptable for determining C_{n+1} if n is small: it appears far too optimistic to ignore carrier synchronization errors in the detection of the test packet. The other extreme, model **II.B**, is believed to underestimate capture performance since it ignores the fact that any narrow-band receiver is likely to lock to the *strongest* signal, rather than to a *random* signal. The threshold model **III** with $z = 4$ (6 dB) tends to produce results which represent a good average of estimates for C_{n+1} according to the models **II.A–II.E**. For $n > 1$, particularly the estimates of C_{n+1} by model **II.C** closely agree with a capture ratio of $z = 4$ (6 dB).

The steady-state throughput S_t of the network in its entirety is studied in Fig. 8 for model **II.A** and **III**. Similar curves for the models **II.A–II.E** are not depicted because of the large computational effort required to evaluate $q_n(r)$ for $n > 3$. As seen in Fig. 8, for low offered traffic loads ($G_t < 1$ ppt), slotted ALOHA and low-persistent ISMA, say $p < 0.1$, have almost equal total throughput. Unslotted ISMA with highly persistent terminals $(p \to 1)$ yields higher throughput than low-persistent ISMA. For reasonably high traffic loads ($3 < G_t < 10$ ppt), nonpersistent ISMA with a small signaling delay, say $d < 0.1$, outperforms slotted ALOHA and 1-persistent ISMA. At these traffic loads, collisions occur frequently, particularly in slotted ALOHA. Their effect is, however, significantly less disastrous than in channels without capture. This allows the application of relatively high persistence (Fig. 8). In mobile radio channels, a persistence of 10% $(p = 0.1)$ imposes negligible throughput degradation as compared to nonpersistent ISMA, even for high offered traffic loads ($G_t < 10$ ppt). This is in contrast to the curves presented

TABLE I
EXPECTED NUMBER OF CORRECTLY RECEIVED PACKETS C_{n+1} PER UNIT OF TIME FOR THE MODELS I.A, I.B, II.A–II.E, III. MEDIAN C/N IS 20 dB, EXCEPT FOR THE MODEL III, WHERE NOISE IS IGNORED

n+1	I.A	I.B	II.A	II.B	II.C	II.D	II.E	III
1	0.92	0.92	0.97	0.97	0.97	0.97	0.84	1.00
2	0.55	0.28	1.17	0.59	0.75	0.91	0.87	0.67
3	0.46	0.15	1.12	0.37	0.57	0.77	0.77	0.52
4	0.42	0.11	1.01	0.25	0.46	0.67	0.67	0.42

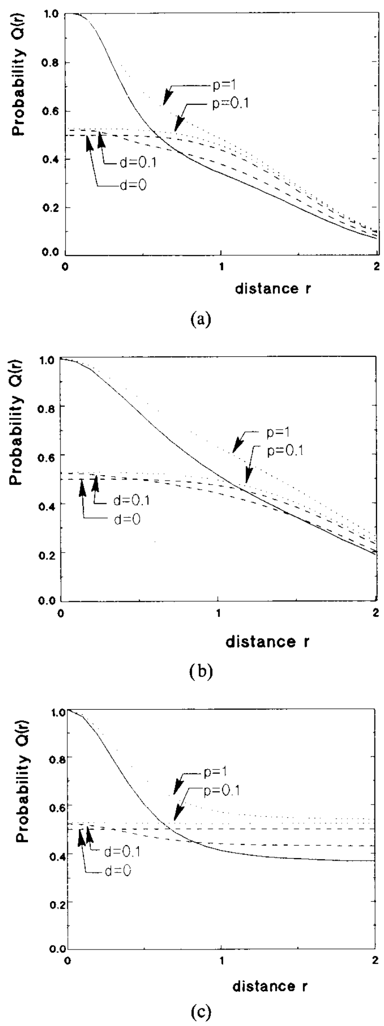


Fig. 6. Access probability $Q(r)$ versus the distance of the mobile terminal, for slotted ALOHA (—), nonpersistent ISMA (- -) and p-persistent ISMA (...). (a) and (b) Fast fading and slow fading, respectively, for a receiver locked to the test packet, with median C/N of 20 dB, and a packet length of L = 16 b. (c) Capture ratio of z = 4 (6 dB).

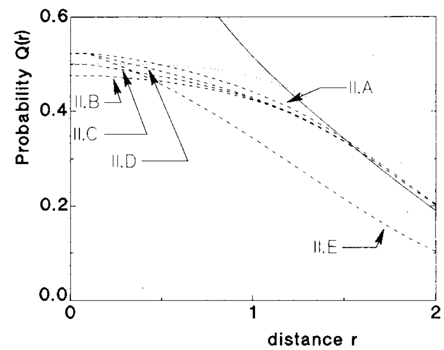


Fig. 7. Access probability $Q(r)$ for packets of 16 b in a slow-fading channel as a function of the distance for nonpersistent ISMA (- -) with delay $d = 0.1$. $Q(r)$ with model **II.A** for slotted ALOHA (—), and for nonpersistent ISMA $(\cdot\cdot\cdot)$ with $d = 0$. The median signal-to-noise is 20 dB.

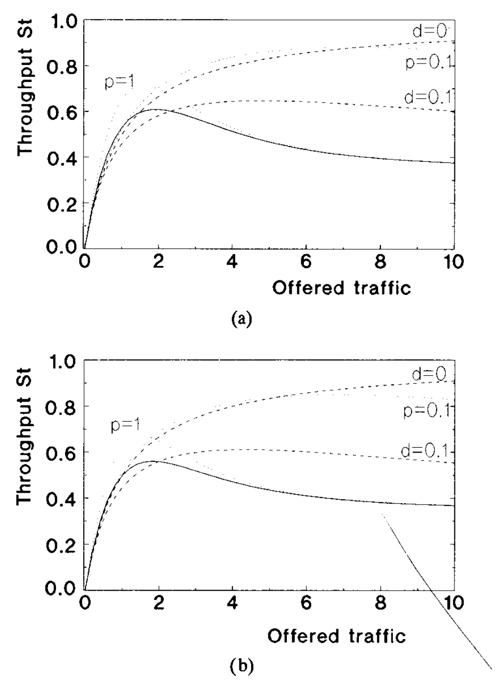


Fig. 8. Total channel throughput S_t versus attempted traffic G_t for slotted ALOHA (—), nonpersistent ISMA (- -) and p-persistent ISMA (...) (a) (Model **II.A**): Fast fading with a median C/N of 20 dB and a packet length of L = 16 b. (b) (Model **III**): Capture ratio z = 4 (6 dB).

in [17] for slotted p-persistent channels without capture, where for $G_t \approx 10$ ppt the throughput is substantially diminished for any $p > 0.1$.

C. High Attempted Traffic Loads

According to model **III**, the limit for high offered traffic $(G_t \to \infty)$ yields nonzero limits, though only packets from the immediate vicinity of the receiver contribute to the throughput.

For slotted ALOHA and p-persistent ISMA ($p > 0$) without signaling delay, the throughput at high traffic loads approaches the limit $S_t \rightarrow C_\infty = 2/(\pi\sqrt{z})$ which was also observed in [11] for channels without Rayleigh fading. Fig. 8(b) shows that this theoretical limit is approached very slowly for small values of p. Similarly, for nonpersistent ISMA with a propagation delay, the asymptotical throughput S_t becomes

$$\lim_{G_t \rightarrow \infty} S_t = \frac{2}{\sqrt{z}\,\pi(1+2d)} \tag{53}$$

except when $d = 0$. Remarkably, this is a factor $1 + 2d$ less than in the case of slotted ALOHA and unslotted p-persistent ISMA ($p > 0$) without signaling delays. This theoretical limit is also approached very slowly: in contrast to the case for extremely high offered traffic, for reasonable high traffic loads ($3 < G_t < 10$ ppt) and for sufficiently small signaling delay, nonpersistent ISMA gives significantly higher throughput than slotted ALOHA. All the above limits critically depend on the assumption of uniform traffic in the vicinity of the central receiver ($r \rightarrow 0$). For spatial distributions that prohibit terminals to be arbitrarily close to the receiver [3], [7], [11], [13], one finds $C_\infty \rightarrow 0$. In this event, S_t is found to reduce to zero for $G_t \rightarrow \infty$, except for the theoretical case of $d = 0$ in nonpersistent ISMA.

D. Packet Length and Coding

Throughout this paper a packet length of $L = 16$ bits has been considered to keep the runtime of software routines for computation of packet success probabilities (especially in the model for slow fading) within acceptable time margin, and to make it possible to compare the computed results with results in [10]. The packet length $L = 16$ corresponds to relatively short packets. Such short packets may be impractical if data files have to be transferred in a random access mode. Moreover, in the case that longer messages are split into many packets each of such duration, the Poisson model for packet arrival may become less appropriate.

Nonetheless, we feel that short packet lengths may make sense in particular practical cases; for instance, if the random access channel is merely used to place requests for communication sessions of longer duration in separate channels. Examples might call the request channel of telephone or closed user groups trunking networks, or the reservation of channel capacity in burst-type transmission of data. In some vehicle control or fleet management, systems require exchange of very short routine messages (e.g., containing vehicle status).

The assumption of a finite packet length is at odds with the assumption of an infinite population of terminals: the simple practical requirement that different terminals should transmit different messages (e.g., containing a unique address of the transmitter) can only be satisfied if $L \geq \log_2 N$, with N the size of the population of terminals. We assumed terminals to transmit "random" patterns of L bits, without avoiding the probability that multiple terminals may offer the same bit sequence. Even if the packet is buried in noise or interference the correct bit sequence is retrieved with probability $1/2^L$. Hence, the probabilities $q_n(r)$ and q_n are always larger than $1/2^L$, and for slotted ALOHA $Q(r) > 1/2^L$ and $S_t > 1/2^{-L} G_t$, except for model **I.B** and **II.B**. With the above considerations, it is clear that the models **I.A**, **II.A**, and **II.C–II.E** are inappropriate to assess the limit for $G_t \rightarrow \infty$: one would find $S_t \rightarrow \infty$, since for $G_t \rightarrow \infty$ each time slot is expected to contain many offered packets with a bit sequence identical to the detected one. A more detailed discussion of the effect of coding and allocation of codewords to different terminals in a multiuser network is contained in [15].

It has been verified that for slow fading, longer packets ($L \gg 16$) do not experience a substantially reduced probability of correct reception, particularly if the (joint) interference signal is peak limited (e.g., if n is small). With fast fading, however, performance was seen to degrade substantially for longer packets; and the application of long packets in a fast-fading channel without error correcting codes may not be very appropriate (see also [14]).

VII. Conclusion

The probability distribution of the number of interfering signals experienced by an *a priori* selected "test" packet has been formulated for CSMA and ISMA. The probability of receiver capture has been derived, taking account of Rayleigh fading and UHF pathloss. Computational results have been presented for the throughput of ISMA (or CSMA) networks with unslotted nonpersistent and p-persistent terminals, and have been compared with slotted ALOHA. In land mobile radio channels, the throughput of random access networks is significantly higher than in channels without receiver capture. Highly persistent terminals may be employed for reasonable offered traffic loads, without any substantial sacrifice of the network throughput.

Mobile nets employing slotted ALOHA or 1-persistent ISMA have been found to substantially favor nearby terminals. For high traffic loads, this is at the cost of the probability of successful transmission by distant terminals. This near–far discrimination is (almost) absent for nonpersistent ISMA with zero (or small) signaling delay. The evenly distributed probability of access for all terminals in the service area is not eroded if unslotted p-persistent ISMA is applied, provided that the persistency is not too large.

The probability of receiver capture has been assessed from the probability of a block error and compared with the probability that the C/I-ratio fails to exceed a certain cochannel rejection ratio. For the case that no error correction coding is employed, it has been confirmed that the probability of correct reception of a data packet in a slow-fading channel is substantially higher than in a fast- fading channel. Idealized synchronization models for a slow-fading channel showed relatively small block error probabilities and a high immunity of a coherent receiver against contending transmissions, particularly if the number of interferers is small. A receiver with a cochannel rejection ratio of 6 dB showed somewhat less optimistic results. For a large number of interfering signals, capture probabilities assessed by considering a cochannel rejection ratio did not divert widely from the results obtained from the block error probabilities. It is our impression that the capture ratio model offers a simple and relatively reliable alternative to support generic evaluation of the performance of mobile random access channels. This impression is in contrast to results from studies of wideband (spread spectrum) systems (e.g., [33]), in which the differences of received power between various signals are assumed to be less pronounced because of the absence of (narrow-band) fading. The tedious numerical computations required to assess packet error rates are in sharp contrast to the convenient closed-form expressions obtained using the capture ratio model.

Assessment of the effects of (combined) error detection and error correction coding has not been included in the analysis. The discussion nonetheless suggested that this topic is of particular interest.

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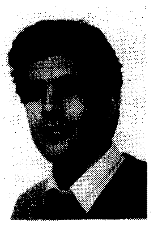

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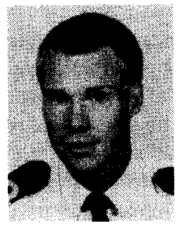

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