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Minimizing cost of empty container repositioning in port hinterlands, while taking repair operations into account

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Abstract

Shipping companies are striving to optimize their empty container repositioning strategies **which also contributes to reduced congestion and environmental improvements**. In this paper we propose a multi-commodity model that makes an explicit distinction between flows of non-damaged containers, on the one hand, and flows of damaged containers, on the other. The model is tailored for the repositioning of these containers in the representative setting of a network of off-dock empty depots, ocean terminals, and inland terminals. In our case study, cost savings of up to 17% are found, depending on the composition of the network, container type, and particular evacuation and repositioning strategy. In particular, directly transporting containers from inland terminals to other inland terminals (direct repositioning) results in cost savings of up to 15% for dry containers and up to 17% for reefer containers. Furthermore, the total costs might be optimized by actually preventing the container failure from occurring possibly leading to considerable additional cost reductions. Finally, exporting damaged containers might seem to be the optimal solution from a regional cost perspective, but, this does **not** necessarily lead to total cost optimization from the global perspective.

Keywords: damaged containers, repositioning, hinterland, optimization

1 Introduction

Shipping companies mainly focus on providing transport between major ports in a global network. Chang et al., (2015) analyzed the minimum transportation cost for the repositioning of empty containers for an entire shipping network. However, also a trend of incorporating the port hinterland into a carrier's supply chain can also be observed (Gadhia et al., 2011). In general, the carriers' customers are not located directly near the terminal, and it is therefore necessary for shipping companies to transport containers between the ocean terminals and provide empty containers at the customer's front door in the hinterland. This is complicated due to the existence of large trade imbalances between the continents (e.g. from Asia to Europe). These imbalances contribute to policy requests to reduce these additional empty transport flows causing congestion and environmental problems on a local and regional level. In an ever-growing volatile container transport market, cost reductions and efficiency improvements are required. For container carriers it is therefore crucial to (re)position empty containers optimally (i.e. at the lowest possible cost). Indirectly this also contributes to reduced congestion and environmental pollution.

1.1 Empty container repositioning in a regional network

Empty container repositioning is performed at various network levels, viz. global, regional, and local scales. The local level covers the repositioning of empty containers between inland terminals or depots and surrounding customers. The regional level focuses on hinterland transport between inland terminals, off-dock empty depots, and ocean terminals. In a research, Mittal et al., (2013) determined optimal inland-empty container depot locations under stochastic demand for the New York/New Jersey port region. The global level focuses on balancing international trade flows between ocean terminals. Inland terminals hereby serve as nodes which connect the regional and local-scale network, while ocean terminals serve as gateways to interconnect the global scale with the regional-scale network. Trade imbalances can be observed leading to regions being either surplus (i.e. import dominated) or deficit (i.e. export dominated) regions, resulting in empty container transport. At a regional level, this results in repositioning flows between the deficit and surplus areas on a regional scale. At a global scale, this results in what are called 'evacuation flows' between continents (e.g. from Europe to Asia). In general, approximately 20% of the exports are empties, but a wide range from 0-90% can be observed. Overall this means that empty flows can be considerable.

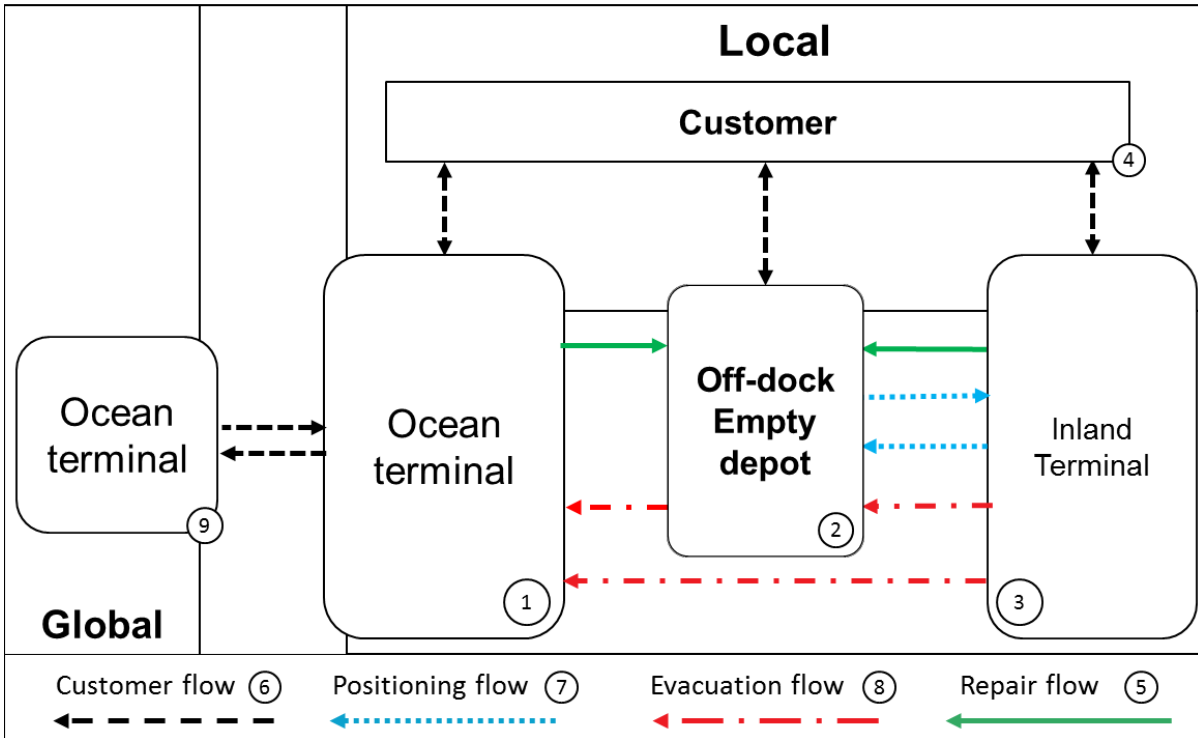


Figure 1 An overview of the regional network surrounding the ocean terminal in relation to the transport super-network. There are various types of empty-container flows between: ocean terminals at a port (1), off-dock empty depots at the port (2), inland terminals in the port hinterland regional network (3), and customers (4). Off-dock empty depots serve as container storage locations, from where containers are picked up, and to where they are returned to serve export demand. As illustrated in Figure 1 below, flow interactions exist between these different locations: repair flows for transporting damaged empty containers to workshops located at depots (5), customer flows (6) to meet local demand, repositioning flows (7) to meet regional demand and evacuation flows (8) towards a global network to serve overseas deficit areas (9).

Meeting customers' demand globally through the repositioning of empty containers follows a hierarchical order from local via regional to global scale, until the costs exceed the price of producing new containers (Theofanis & Boile, 2009). At each location what is called a 'safety stock' in the form of a Target Stock Level (TSL) is maintained to meet demand. The TSL is based on historic data and the carrier's expert knowledge. Hardly any information regarding the actual distribution and availability of empty containers throughout the network is available.

1.2 Non-damaged versus damaged empty containers

Containers are a commodity that is not handled gently. They are built to last, however during the transport process, containers can get damaged, which is often inflicted by careless handling on a terminal or during transport, failure of cooling equipment, regular maintenance, etc.. This can have a significant influence on the available supply of empty containers. Currently, as soon as a container is damaged, it is taken out of service until it has been repaired. A repair activity is a direct reduction of the available supply of empty containers for meeting export demand. Given the high failure rates (20-25%), understanding the cost impact of extra container movement for meeting demand is quite relevant. Current methods for repositioning do not take this reduction of supplies into account, resulting in higher costs due to inefficient transport, handling, storage, and repair operations. A more comprehensive approach that explicitly takes into account the presence of both damaged and non-damaged empty containers is therefore desired. This article aims to fill this research gap and investigates how the total costs of repositioning damaged and non-damaged empty containers can be optimized, while keeping operations in the hinterland and ocean terminals in mind. Section 2 provides an overview of the scientific literature for modeling empty container repositioning. Section 3 gives a detailed explanation of the developed mathematical model. In Section 4 the mathematical model is implemented for a case study in the Port of Rotterdam. Section 5 gives the conclusions and further research opportunities.

2 Scientific state of the art in empty container repositioning modeling

To understand the behavior of cost induced by the interaction between damaged and non-damaged containers while keeping different strategies of repositioning empty containers in mind an optimization model needs to be developed. Several mathematical models have been proposed in the literature for general network optimization problems in container liner shipping (see the overview by Tran & Haasis (2013)), as well as more specifically for empty container repositioning (see Braekers et al. (2011), Song & Dong (2015) for extensive overviews). However, little research has been done that takes damaged containers into account. The goal of this research study is to identify how to model multi-commodity container repositioning problems and how to solve them. We use Braekers et al. (2011) as starting reference for our literature review. Crainic et al. (1993) propose a single commodity model, which has been used by many authors. In this article, we also use this single commodity model as a basis, as its

assumptions and model dynamics closely match our case. Choong et al. (2002) present a model to investigate the impact of the length of the planning horizon. Their paper provides a better understanding of how to implement a container allocation problem to a fixed network, yet no damaged containers are taken into account. Olivo et al. (2005) provide a single commodity model which takes container leasing into account. Their paper mentions the importance of the repair of damaged containers, although it is not included in their model. The implementation of customers and inland terminals as aggregated nodes, henceforth known as ‘macro-nodes’, will also be used for our model. This approach is useful for a tactical model, where the focus is to understand the impact of repositioning strategies between terminal nodes on total cost through network optimization. The approach leaves out the interaction with and decisions by customers in order to make optimization less computationally intensive. Jula et al. (2006) present a single commodity model, which implements the solutions depot direct and street-turn, but leaves out the effect of damaged containers. The methods presented are used to investigate the impact of the solutions on total cost. Wang & Wang (2007) present a model that minimizes the cost of empty container repositioning with a focus on inventory at locations. Empty container stock is managed through a TSL, which is implemented as an equality constraint in their model. Furió et al. (2013) present a Decision Support System tool that reviews street-turn in a model implemented for a case at the Port of Valencia. No evacuation demand to locations outside the scope of their paper is taken into account. Moon et al. (2013) present a model that investigates the influence of foldable containers on total costs. The proposed model has inspired the formulation of our model, yet the implementation of foldable containers is left out of the scope of our paper, since they currently represent only a small portion of existing containers. Olivo et al. (2013) present a deterministic multi-commodity model that takes future requirements into account.

Table 1 provides the review of the model types and solution methods found in the scientific literature. Deciding on the model type depends on a number of choices between alternatives, i.e. nonlinear or linear relationships; explicit or implicit system equations; discrete or continuous states; deterministic or stochastic variables; and static or dynamic models.

The existing literature presents models that allow the addition of multiple container states and solutions to the empty container repositioning problem. However, no papers have been found that take into account damaged containers and the constant evacuation of empty containers. We therefore propose a new Linear Deterministic Discrete Dynamic Mathematical Optimization

model for finding an optimal cost solution for the repositioning of empty containers through off-dock depots, while taking both damaged and non-damaged containers into account. The proposed model is then used to investigate the impact on different network topologies, and the results of applying the following solutions to the network: *street-turn*; *moving containers directly between importers and exporters* (Furió et al., 2013) or *depot-direct*; *empty containers are stored and maintained at off-dock depots next to deep-sea terminals* (Jula et al., 2006). These solutions, when applied correctly, can help to eliminate a full transport leg, as the purpose of the model is to support decision making on empty container repositioning.

Table 1 Existing literature on relevant empty container repositioning models and their relationship to our proposed model

Authors	Model type						Solution methods						Contributions
	Linear	Explicit	Continuous	Deterministic	Dynamic	Multi-commodity	Substitution	Street turn	Leasing	Depot direct	Foldable containers	Container repair	
(Crainic et al., 1993)	X	X	X	X	X	X	X		X				Cost function design
(Choong et al., 2002)	X	X	X	X	X				X				Cost function design
(Olivo et al., 2005)	X	X	X	X	X				X				Aggregated macro-nodes
(Jula et al., 2006)	X	X	X	X	X	X		X		X			Depot direct solution
(Moon et al., 2013)	X	X	X	X	X						X		Cost function design
(Wang & Wang, 2007)	X	X	X	X	X	X							Inventory equality constraint
(Furió et al., 2013)	X	X	X	X	X	X		X					Cost function design and street-turn solution
(Olivo et al., 2013)	X	X	X	X	X		X		X				Influence of future requirements (TSL)
This article	X	X	X	X	X	X		X		X		X	

This article contributes to the literature in three ways: (1) it provides an optimization model for the repositioning of empty containers in the hinterland while incorporating flows of damaged containers; (2) it applies the model to realistic instances based on real data from Maersk Line;

and (3) it provides managerial insights that support empty container repositioning while taking into account failure of containers and repair.

3 Model incorporating ‘damaged’ and ‘non-damaged’ empty containers

The mathematical model proposed optimizes the cost of transporting damaged and non-damaged empty containers in a network. The problem is known as a ‘minimum-cost network flow problem’, which optimizes the objective function restricted by flow conservation constraints.

3.1 Model assumptions

A number of assumptions have been made to be able to model empty container repositioning:

- *A time-step of 1 week with a horizon of 52 weeks is assumed.* The available dataset (provided by Maersk Line) contains weekly data points for 52 weeks, which allows the incorporation of seasonal effects. Also, as container repair takes roughly 1 week, the repair prices can be taken into account by assuming a single period repair.
- *All transport in the model is performed by barge without capacity constraints.* The model proposed is made from the carrier’s point of view, and in this case the barge modality is the most economic option.
- *Depots which offer repair facilities are limited by a repair capacity representing the number of repairable containers per time-step.* A repair facility is limited by the number of staff and available equipment.
- *No capacity constraints exist at the other nodes in the network.* Each inland terminal has sufficient storage space, and is often capable of expanding its capacity.
- *No backlogging is permitted, and no delivery window is included.* Containers are delivered within a single time period to the location with demand. This is in alignment with the carrier’s operational policy.
- *Move costs are a combination of transport costs and two handling costs for the start and end node per arc.* Each arc in the network represents the movement of a container between two nodes, which is subject to a combination of costs.
- *Each node represents a location in a container transport network plus its surrounding customers.* Customers generate demand and supply in a container supply chain with respect

to empty containers. The physical transport to and from customers is outside the scope of our model, which means that the various inland terminals, ocean terminals, and empty depots represent the origin and destinations in the supply chains.

- *Containers can only be repaired once inside the network.* A repaired container is delivered to the customer in the following time-step and as such they leave the model. The total lifecycle of a container is left out of scope.
- *Containers can also be repaired outside of the network.* The container volume to be repaired inside of the considered network (Port of Rotterdam) is assumed to be 25%. The remaining 75% of repairs is carried out outside of the considered network. Evacuation flows represent these external repairs.
- *All move costs are unit costs independent of distance and time travelled.* Carriers make price agreements with inland shippers depending on the distance, travel time, and container frequency between the legs in their network.

3.2 Network representation

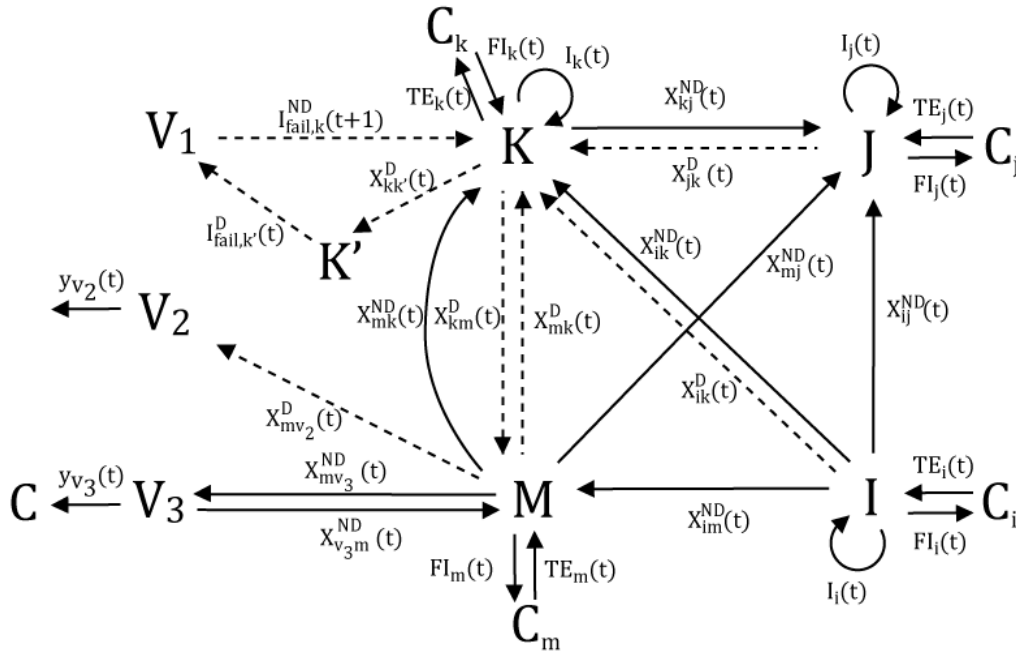


Figure 2 Graphical representation of the scoped network of nodes and arcs, allowing for both damaged and non-damaged container flow.

We define the network $G(N, A)$ with nodes N and arcs A . We consider flows of both damaged containers (on arcs A^D) and non-damaged containers (on arcs A^{ND}), with disjoint union $A = A^D \cup A^{ND}$. As a consequence, the network consists of two separate subgraphs $G^D = G(N, A^D)$ and $G^{ND} = G(N, A^{ND})$. The node set N consists of ocean terminal M , off-dock empty depots K , workshops K' (located at a depot location), surplus inland terminals I , deficit inland terminals J , virtual nodes V_1 and V_2 for damaged flows, virtual node V_3 for non-damaged flow and node specific customers C . Figure 2 illustrates the structure of the network by indicating the flows between the node sets.

The network as proposed is balanced with incoming and outgoing flows. Subgraph G^D (dashed arrows) focuses only on the damaged containers, and provides repair at a workshop either within the region or outside the scope of the network. The G^{ND} subgraph (solid arrows) focuses on the flow of non-damaged containers for the purpose of meeting shipping demand at customers.

The flow volumes between nodes N are decision variables. For example, the flow volume of damaged containers between inland terminal $i \in I$ and workshops $k' \in K'$ is given by the non-negative real number $X_{ik'}^D$.

The flow volumes between customers C and nodes N are input variables, where FI represents flow ‘From Import customers’ and TE represents flow ‘To Export customers’. The damaged container flow leaves the network through the set of virtual nodes $V_2 = \{v_2\}$, and the non-damaged container flow leaves the network through the set of virtual nodes $V_3 = \{v_3\}$.

In network G^D , a depot has two tasks and is represented by two separate nodes: first, serving as a transshipment and source node $k \in K$ and second, as a workshop $k' \in K'$. Containers become available for repositioning or evacuation purposes after being repaired at a workshop. Node $V_1 = \{v_1\}$ represents a sink node with inflow equal to the sum of locally repaired damaged containers. Node $V_2 = \{v_2\}$ represents a sink node with inflow equal to the sum of the evacuated damaged containers. Inland terminal nodes $I \cup J$ serve as a source node for repair flows. Ocean terminal nodes M have two functions, being combined transshipment and source nodes. The arcs between K and K' represent the number of repair activities $X_{kk'}^D$ of damaged containers. $I_{fail,k'}^D(t)$ is the amount of containers originating from workshop $k' \in K'$, that have failed in time-step t , and will re-enter the system at time-step $(t + 1)$ as flow of repaired containers $I_{fail,k}^{ND}(t + 1)$ to an off-dock empty depot $k \in K$.

$X_{ik}^D, X_{jk}^D, X_{mk}^D, X_{km}^D, X_{mv_2}^D$ (dashed arrows) represent the flows between nodes in the network of damaged containers. Flows between the nodes I, J, K, M represent the transport of empty containers to repair facilities. Flows between an ocean terminal M and overseas repair facility V_2 represent the transport of empty containers to repair facilities overseas. Flows from a repair facility $k' \in K'$ via virtual node V_1 and off-dock empty depot $k \in K$ represent the repair of empty containers.

The G^{ND} graph is similarly constructed as the G^D graph with some differences, but only operates non-damaged empty containers for repositioning and evacuation purposes. The main differences are as follows. The off-dock empty depots K are either sink-and-transshipment or source-and-transshipment nodes. Inland terminal nodes are either sink nodes I or source nodes J . An ocean terminal $m \in M$ serves as a sink-and-transshipment or source-and-transshipment node. Node V_3 has been added to serve as the location representing the overseas location, from where containers enter or leave the network depending on the overall state of the network at time-step t .

$X_{mk}^{ND}, X_{ik}^{ND}, X_{kj}^{ND}, X_{mj}^{ND}, X_{im}^{ND}, X_{ij}^{ND}, X_{mv_3}^{ND}, X_{v_3m}^{ND}$ (solid arrows) represent the flows between nodes in the network of non-damaged containers. Flows of non-damaged containers between the nodes I, J, K, M represent the transport of empty containers to fulfill demand. Flows between ocean terminals M and an overseas ocean terminal $V_3 = \{v_3\}$ represent the transport of empty containers to and from overseas locations. Flows between the virtual nodes V_1 and the empty depots K represent repaired containers becoming available for empty container demand. For a detailed description of the optimization model, see Annex 1.

4 Case study: Maersk Line

In this section, we apply the model, as sketched in the previous section and described in Annex 1, to the Maersk Line case study. Simulations are carried out using an implementation of the model in Matlab 2014a, in combination with the linear optimization toolbox (Kay, 2014) for investigating the impact of various scenarios.

The Maersk Line dataset provides gate in (i.e. containers returning from import customers) and gate out (i.e. empty containers sent to export customers) frequencies at inland terminals per week. The implemented model is subject to various scenarios which show the potential of the model and the contribution of container repair in the total repositioning cost. A set of 3 ocean

terminals, 4 off-dock empty depots, and 14 inland terminals are considered, corresponding to the main hinterland of the Port of Rotterdam. The investigated locations are in a surplus area, meaning that sufficient supply exists and therefore delivery time constraints are left out of the scope of the model. The case study had the following characteristics;

Number of periods (weeks):	52
Amount of 20' dry containers (20DC):	37500
Amount of 40' dry containers (40DC):	26500
Amount of 40' highcube containers (40HC):	42400
Amount of 40' highcube reefer containers (40HR):	25900

In order to explore cost-saving opportunities, a number of scenarios have been studied. Based on Maersk Line data, empty containers feature a failure rate of 20 – 25%.

4.1 *Experimental setup*

The model is now used to investigate several operational scenarios. In particular the influence of opening or closing off-dock empty depots and ocean terminals was of interest for this case study. Moreover, some of the existing solutions found in literature that have a direct relationship to managing empty container flow need to be considered, because they change the physical network that drives the model. Each scenario is a combination of four scenario variables, which alter the shape of the matrix that describes the network investigated, specifically:

1. *Ocean terminal topology*: see Table 2. The scenario variables change when an ocean terminal in the network is opened or closed. Opening or closing an ocean terminal has a direct influence on the network and on total cost, as certain network arcs can(not) be selected for transport.
2. *Off-dock empty depot topology*: see Table 3. The scenario variables represent which off-dock empty depots are open in the network.
3. *Forced repositioning*: see Table 4. The scenario variables allow the forced repositioning of empty containers through depots.
4. *Move type scenarios*: see Table 5. The scenario variables allow direct transport between inland terminals (reposition moves) or between inland terminals and ocean terminals (evacuation moves) in the network of arcs and nodes. Indirect refers to if transport went

through an off-dock empty depot to get to its final destination and direct refers to transport that neglects the presence of off-dock empty depots.

Table 2 Overview of the ocean terminal topology scenario variables (based on Maersk Line operations)

<i>Scenario name:</i>	<i>Ocean terminals</i>		
	<i>Terminal 1⁽¹⁾</i>	<i>Terminal 2⁽¹⁾</i>	<i>Terminal 3⁽¹⁾</i>
<i>'Base'</i>	Open	Closed	Open
<i>'Transition 1'</i>	Open	Open	Open
<i>'Transition 2'</i>	Closed	Open	Open
<i>'Future'</i>	Closed	Open	Closed

Note: 1. For confidentiality reasons the actual terminal and depot names are left out.

Table 3 Overview of empty depot topology scenario variables (based on Maersk Line operations)

<i>Depot</i>	<i>Owner</i>	<i>Size</i>	<i>Container types</i>
<i>A</i>	Third party ⁽¹⁾	Large	20' dry, 40' dry & 40' highcube dry ⁽²⁾
<i>B</i>	Third party ⁽¹⁾	Small	20' dry, 40' dry & 40' reefer
<i>C</i>	Third party ⁽¹⁾	Medium	40' reefer
<i>D</i>	Maersk Group	Large	20' dry, 40' dry, 40' highcube dry ⁽²⁾ & 40' reefer

Note: 1. For confidentiality reasons the actual terminal and depot names are left out.

2. Highcube dry containers are 1 foot taller than dry containers

Table 4 Overview of forced repositioning scenario variables (based on Maersk Line operations)

	<i>Equal depot flow distribution</i>	<i>Unequal depot flow distribution 1</i>	<i>Unequal depot flow distribution 2</i>
<i>2 depots open</i>	50/50	40/60	80/20
<i>3 depots open</i>	33.3/33.3/33.3	20/20/60	20/40/40
<i>4 depots open</i>	25/25/25/25	20/20/20/40	None

Table 5 Overview of move type scenario variables (based on Maersk Line operations)

	<i>Repositioning</i>	<i>Evacuation</i>
<i>Move type 1</i>	Indirect	Indirect
<i>Move type 2</i>	Direct	Direct
<i>Move type 3</i>	Indirect	Direct
<i>Move type 4</i>	Direct	Indirect

The initial and current network composition per container type is referred to as the 'base' result. For each scenario, the focus lies on the relative cost savings compared to the 'base' result. The deterministic optimization model is applied to an instance where for each time step, failure rates are drawn uniformly from the interval from 20% to 25%. Each scenario is run five times to reduce variation in these randomly drawn instances. The established scenario variables result in over 36,000 combinations to be determined by the model. To reduce the computational

complexity, a heuristic has been established that works as follows: (1) Determine the optimal solution while varying empty depot and ocean terminal topology scenarios; (2) Use the best solution to further investigate the influence from forced repositioning through depots; and (3) Use the best solution to investigate the influence of direct repositioning and direct evacuation scenarios. The main focus of the case study is to discover the cost impact of opening or closing empty depots in various scenarios with open or closed ocean terminals in the focus area. The application of step 2 and 3 further illustrate other means of reducing cost for the company. This heuristic helps reduce the size of the investigation to 9,200 variants. The heuristic is applied separately to the 4 container types investigated in the case: namely, 20 ft. dry containers (20DC), 40 ft. dry containers (40DC), and 40 ft. highcube dry containers (40HC), 40 ft. highcube reefers (40HR). Finally, a sensitivity analysis identifies the model's cost drivers and provides input to the recommendations.

4.2 Results

Prior to understanding the results presented by the model, it is important to know the dispersion of the containers through the network. Depending on the location of companies with transport demand in the investigated network, the following statement applies; all dry container types seem to be required more at and around inland terminals in comparison to reefer containers, which are required more in the port area. In the Rotterdam port area a large quantity of customers shipping perishables exists in comparison with further inland. Dry containers in comparison to reefers thus require a different approach with respect to optimal repositioning. All numerical values are deformed to preserve confidentiality of the data.

4.2.1 Step 1. Determine the optimal solution while varying the empty depot and ocean terminal topology scenarios

Running the model with respect to Step 1 of the heuristic provides the results found in Figure 3 and Figure 4. A number of insights are obtained when investigating the results further:

1. The relative weight between transport and handling costs seems to fluctuate depending on container type. This is caused by differences in locations where there is container availability and locations where containers are required. Furthermore, the distance

travelled by empty containers influences the size of transport costs. When combined, these aspects result in roughly 75% of the total cost.

2. Storage costs are constant per container type, because the pre-set TSL is constant over time for the different locations. In the current setup, storage costs have minimal influence on total cost, because the costs are only charged at off-dock empty depots and because the unit costs are low.
3. The transport costs of the 40HR container scenario are significantly lower than the transport costs in the other scenarios, as can be explained by the fact that 40HR containers are more required in the Rotterdam port area.
4. The 40HR container cost composition is different compared to the cost composition of the other containers, resulting in transport and handling cost aspects accounting for 50% and repair cost accounting for 45%. The repair of reefers has a larger impact on the total cost.

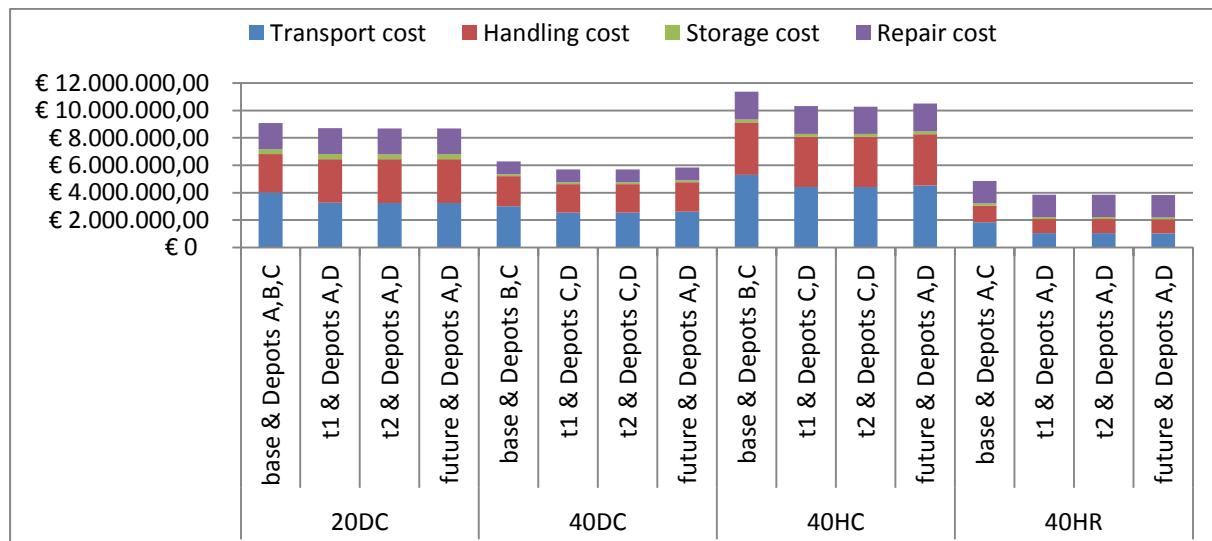


Figure 3 Results of the first step in the heuristic

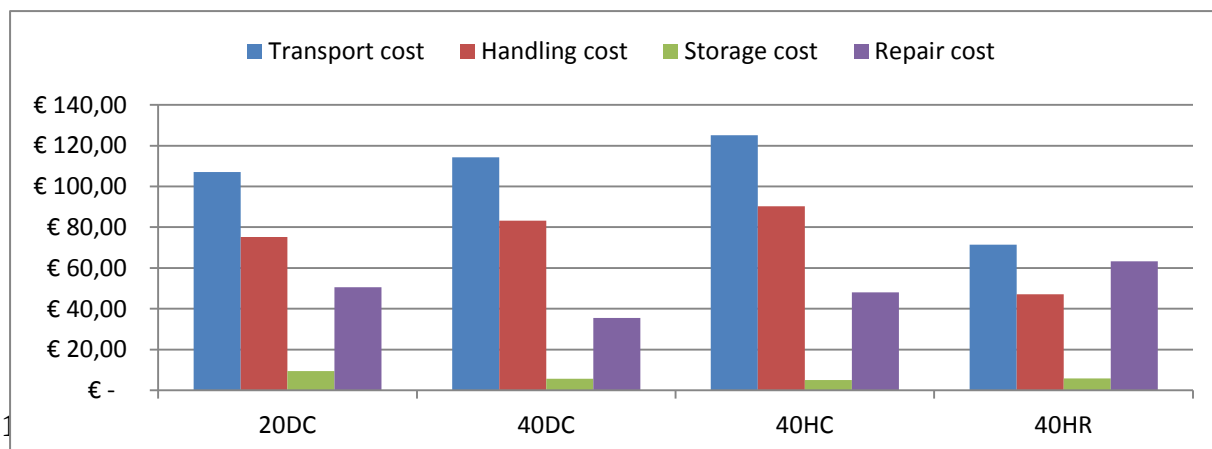


Figure 4 Unit cost per cost category and container type for the 'base' scenario.

Some conclusions can be drawn from the impact of damaged containers on empty container management. Empty container supply is reduced by the repair rate, resulting in extra container moves being required in order to supply the network with empty containers. A damaged container needs to be transported to a workshop for repair, resulting in the possibility of a location changing from a surplus to a deficit of supply. This risk can be mitigated by incorporating a safety factor into the TSL value. Currently, the storage cost at inland terminals is low, resulting in almost no impact of an increased TSL on operational cost. However, a higher TSL does mean a larger container fleet. Finally, the repair costs of containers are not directly related to the other cost aspects, but, nevertheless, they do impact indirectly. The equipment can be managed in a better way when knowledge of the life cycle of the empty container, whether it is a dry or a reefer container, is used; see also (Lam and Lee, 2011). This also applies to the extra costs that are incurred when containers are repaired outside the regional network. Damaged empty container transport only makes sense if it results in lower total costs compared to repair overseas.

4.2.2 Step 2. Use an optimal solution to further investigate the influence from forced repositioning through depots

Table 6 Overview of cost improvements compared to the base scenario (Step 1)

		transition 1	transition 2	future
20 DC	Open depots	A, D	A, D	A, D
	Steering	60/40	60/40	40/60
	Cost change	-3,91%	-3,97%	-3,92%
40DC	Open depots	C, D	C, D	A, D
	Steering	60/40	60/40	20/80
	Cost change	-8,81%	-8,90%	-6,53%
40HC	Open depots	C, D	C, D	A, D
	Steering	60/40	40/60	60/40
	Cost change	-8,23%	-8,12%	-6,27%
40HR	Open depots	A, D	A, D	A, D
	Steering	40/60	40/60	40/60
	Cost change	-17,64%	-17,37%	-17,82%

Running the model with respect to Step 2 of the empirical setup provides the results found in Table 6. An unsteered network, in other words a solely mathematically optimal result, will send all flows over the arc which has the lowest cost, until capacity is reached. The current model is hardly restricted by capacity constraints. In the optimal case, the model will, therefore, send all

containers to a single cheapest depot. In real-life a carrier, such as Maersk Line, is unlikely to consider only the cheapest depot for monopoly reasons and because of the importance of spreading the risk of operations. A difference in cost per container type seems to impact how containers move around in a forced network

Table 7 Cost improvements due to move type scenarios compared to ‘only individual move type’

type	ocean topology	Scenario			
		only indirect move type	direct evacuation and direct repositioning	only direct repositioning	only direct evacuation
20DC	base	0%	-13%	-12%	-5%
	transition 1	0%	-11%	-10%	-4%
	transition 2	0%	-11%	-10%	-4%
	future	0%	-11%	-9%	-4%
40DC	base	0%	-14%	-13%	-5%
	transition 1	0%	-10%	-9%	-3%
	transition 2	0%	-10%	-9%	-3%
	future	0%	-7%	-5%	-3%
40HC	base	0%	-16%	-15%	-5%
	transition 1	0%	-13%	-10%	-4%
	transition 2	0%	-12%	-10%	-4%
	future	0%	-8%	-6%	-3%
40HR	base	0%	-29%	-29%	-15%
	transition 1	0%	-16%	-17%	-1%
	transition 2	0%	-17%	-16%	0%
	future	0%	-17%	-16%	0%

4.2.3 Step 3. Use optimal solutions to investigate influence of direct repositioning and direct evacuation scenarios.

Running the model with respect to Step 3 of the empirical setup provides the results found in Table 7. Move types show an immediate advantage of using direct positioning for serving regional balancing purposes. Even by allowing direct evacuation, a cost reduction can be obtained. These results might give the impression that, by allowing direct connections between export and import customers, an easy and obvious cost reduction can be made. However, it is

important to realize what the model results do not show. The implementation of direct positioning or direct evacuation makes the planning of transport a more complex, and therefore a more costly operation. However, customers tend to concentrate around terminals, and therefore it might be worthwhile to further explore direct repositioning (van den Heuvel et al., 2013). Another aspect that should be considered is the congestion that can occur at the ocean terminal when the off-dock depots are removed from the network. Better communication between all parties is required with more streamlined planning to allow for direct transport to be implemented.

4.3 Sensitivity of the model

4.3.1 Sensitivity to failure rate η

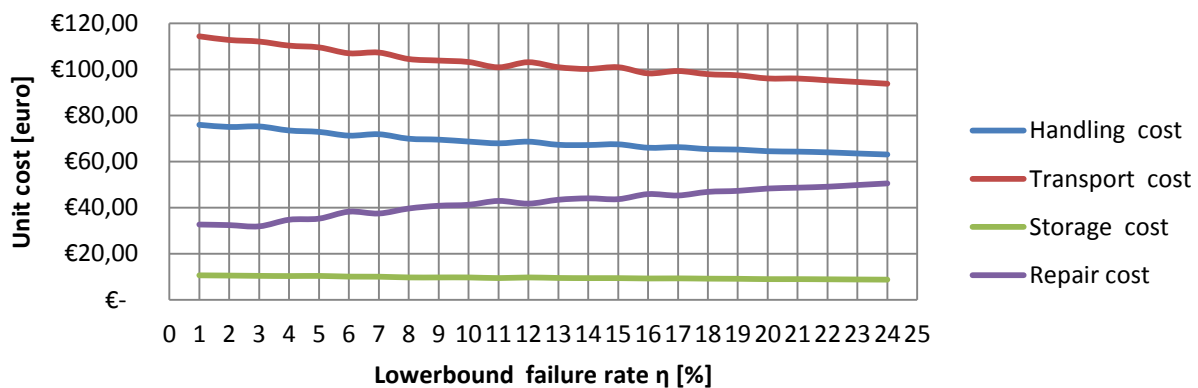


Figure 5 Results from a sensitivity analysis on the failure rate η with a lower-bound from 1 to 24% and an upper-bound of 25%

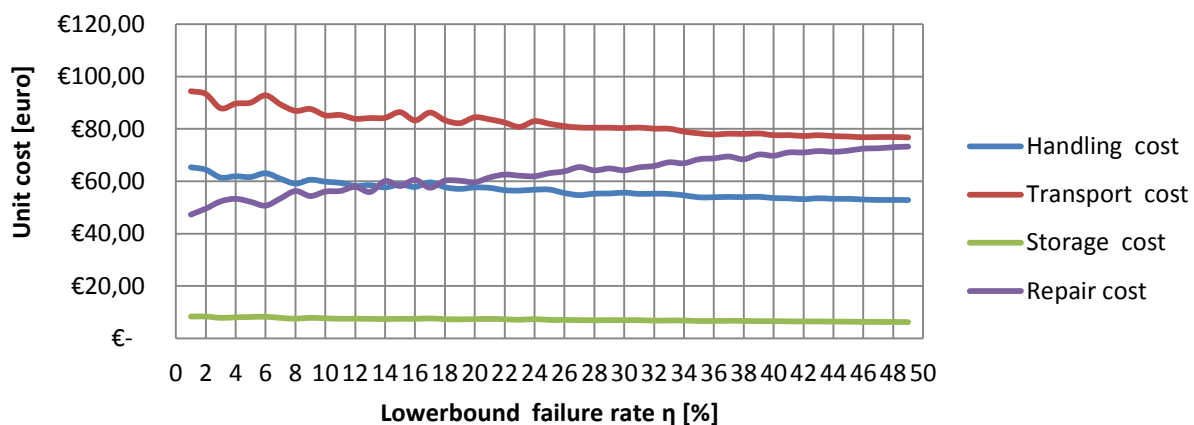


Figure 6 Results from a sensitivity analysis on the failure rate η with a lower-bound from 1 to 49% and an upper-bound of 50%

The container failure rate is limited by a lower-bound and upper-bound value, which, for the scenario evaluation in Section 4.2 were set between 20% (lower bound) and 25% (upper bound). For the sensitivity analysis, the failure rate lower-bound is varied in steps of 1% up to the upper-bound limit. Three upper-bound limits, i.e. 25% (Figure 5), 50% (Figure 6) and 75%, are investigated. The model showed infeasibility when the upper-bound failure rate was set to 75%, which occurs due to the number of damaged containers exceeding the repair capacity of the workshops. The result showed that transport and handling costs decrease as the failure rate increases. It is important to realize that the decrease depends on where containers become damaged in the network. An increase in damaged containers could in the short term mean less evacuation moves from inland locations, but in the long term results in more repositioning moves from other locations to meet the demand, because the network becomes more deficit.

4.3.2 Sensitivity to local repair rate α

The local repair rate is a factor that describes the number of containers which are repaired in Rotterdam instead of overseas. In the model, the local repair rate has been set at 75%. To identify the sensitivity of this factor on the model results, the local repair rate is varied between 0 and 99%. A value of 0% means that no containers are repaired in the model and the only repair cost

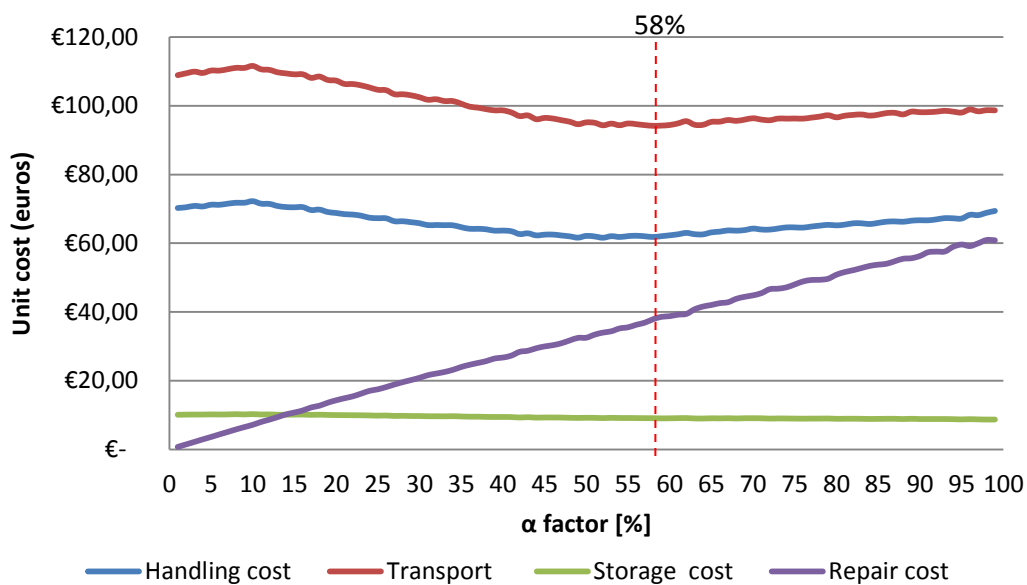


Figure 7 Sensitivity of the local repair rate

spent is to transport the containers to the ocean terminal. A value of 99% results in almost all containers being repaired in the Port of Rotterdam, allowing for a maximum number of containers, which can be used for future demand in the next time-step. Figure 7 shows the results from this test. Notice that at a local repair rate equal to 58%, a critical point is found, after which transport and handling costs start increasing again. The lower the local repair rate is, the more containers are repaired elsewhere, which does not necessarily benefit the company at a global scale.

5 Conclusion and recommendations

This paper focuses on empty container repositioning through off-dock empty depots located in the port area. The main research question in this paper has been: “How can total costs be optimized in the repositioning of empty containers through off-dock empty depots, while taking account of operations in the hinterland and ocean terminals?” Empty container repositioning is a non-revenue generating operation, yet it is an important and costly part of meeting customer export demand. Large container ports where deep-sea foreland and hinterland meet occupy important positions in empty container repositioning. In these areas, a more efficient repositioning system for empty container movement will also contribute to reduced congestion and emissions.

Three contributions can be distinguished. First, the article contributes to the scientific state of the art by developing a multi-commodity model that takes into account container failure and repair. The purpose of the model is to support decision making on empty container repositioning through a network of inland terminals, depots, and ocean terminals. The model takes into account all the described flows in the regional transport network of Rotterdam and its hinterland, including different types of repair flows. The model investigates the opportunities to improve container handling in the Port of Rotterdam and its hinterland from a network perspective, but now for a single carrier (Konings, 2007). The proposed model is then used to investigate the impact on different network topologies and to apply the street-turn (Furió et al., 2013) or depot-direct (Jula et al., 2006) solutions.

Second, the performance of the model is assessed using Maersk Line reference datasets. Third, a number of managerial takeaways can be inferred from the study, which considers a number of scenarios based on: (1) Different terminal combinations in the network; (2) Forced repositioning;

and (3) Direct transport between terminals. These scenarios demonstrate the potential of the model, and result in several important conclusions. First, operational costs related to empty container repositioning are affected by many variables, of which container repair is an important element. Given the high container failure rate, total costs might be optimized by actually preventing the container failure from occurring possibly leading to considerable cost reductions. Secondly, damaged containers account for nearly 20% for dry containers and 45% for reefer containers of the total repositioning costs, depending on where the final customer is located. Dry containers seem to be required more around inland terminals compared with reefer containers, which are required more in the port area. Certain container types might thus require a dedicated approach with respect to total cost optimization. Thirdly, a balance needs to be struck between exporting damaged containers to cheap repair facilities, and repairing them within the region. Exporting damaged containers might seem to be the optimal solution from a regional perspective. However, this does not necessarily lead to total cost optimization from the global perspective. Therefore, empty container management total costs need to be optimized through collaboration on a global scale within the company. Finally, directly transporting containers from inland terminals to other inland terminals (direct repositioning) results in cost savings of up to 15% for dry containers and up to 17% for reefer containers. However, the resulting cost-savings tend to be overstated due to the randomness of the failure rates which makes repositioning a challenging task leading to lower actual cost-savings. Direct repositioning from customer to customer might lead to possibilities for further cost savings. However, direct positioning makes the planning of transport a more complex, and therefore a more costly, operation. But customers tend to concentrate around terminals and direct repositioning might also reduce congestion, thus making it worthwhile to explore this further. In the end, a more efficient reposition will also contribute to a more sustainable port area (it results in reduced congestion and emissions). Numerous further research opportunities exist. First, categorizing container repair in types of repairs results in more accurate determination of the repair cost drivers. Secondly, including multiple hinterland modalities, such as barge and rail, would also improve the model's resemblance to reality. Thirdly, the effect of a time-step of 1 day could be implemented to analyze its impact on cost and model accuracy. Also, the current deterministic model could be extended by taking stochastic input into account. And, finally, the object of study could be enlarged in order to analyze the effects of empty container repositioning in global networks.

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Annex 1. Optimization model

Indices	Description
i	Surplus Inland Terminals, $i \in I = \{1, \dots, ITS\}$ where ITS is the number of Inland Terminals, which are surplus, for time-step t
j	Deficit Inland Terminals, $j \in J = \{1, \dots, ITD\}$ where ITD is the number of Inland Terminals, which are deficit, for time-step t
k	Off-dock empty depots, $k \in K = \{1, \dots, OD\}$ where OD is the number of Off-dock empty depots
k'	Workshops, $k' \in K' = \{1, \dots, WS\}$ where WS is the number of workshops
m	Ocean Terminal, $m \in M = \{1, \dots, OT\}$ where OT is the number of Ocean Terminals
v	Virtual nodes; three virtual nodes exist
t	Periods, $t \in T = \{1, \dots, T\}$ where T is the number of periods in the planning horizon
Decision Variables	Description
X^D	'Damaged' empty container flow (Decision variable)
X^{ND}	'Non-damaged' empty container flow (Decision variable)
Input variables	Description
$FI_n(t)$	On arc (C, n) with $n \in I \cup J \cup K \cup M, c \in C$ Empty containers entering a node from import customers at t
$TE_n(t)$	On arc (n, C) with $n \in I \cup J \cup K \cup M, c \in C$ Empty containers leaving a node to export customers at t
Parameters	Description
α	'local repair' rate, describing containers that are too expensive to repair within the reviewed scope
η	'Failure' rate of containers to separate 'non-damaged' containers from 'damaged' containers
C^A	Arc costs for repositioning and evacuation of containers specific for all arcs
C^{Imp}	Import arc costs of containers when a terminal becomes deficit
C^{Floc}	Repair costs specific for a depot and container type
C^{Fevac}	Repair evacuation costs, representing the cost of performing repair on containers outside scope
C^{Evac}	Evacuation costs
n_{cap}	Physical capacity of an inland terminal and off-dock empty depot
$D_{capk'}$	Repair capacity of off-dock empty depot
$I_{fail}(t)$	Damaged containers requiring repair at the depot at t
$TSL \forall i, j, k, m$	Target stock level at time-step t
$I(t) \forall i, j, k, m$	Target stock level from time-step $(t-1)$ equal to inventory at t
$y_{v1}(t)$	Number of containers repaired at t
$y_{v2}(t)$	Number of damaged containers evacuated at t
$y_{v3}(t)$	Number of containers evacuated/imported at t
Y_m	Binary value used to 'open' or 'close' arcs connected to ocean terminals
Y_k	Binary value used to 'open' or 'close' arcs connected to off-dock empty depot

Cost function

The carrier aims to minimize cost by optimizing flow in the network with limitations occurring due to container failure. The cost function, which sums all costs for moving containers through the network between all nodes $\{i,j,k,m\}$ for each time-step is given by (1).

$$\begin{aligned}
 & \sum_{k=1}^{OD} \sum_{k'=1}^{WS} C_{kk'}^{Floc} X_{kk'}^D(t) + \sum_{k=1}^{OD} \sum_{j=1}^{ITS} C_{kj}^A X_{kj}^D(t) + \sum_{i=1}^{ITS} \sum_{k=1}^{OD} C_{ik}^A X_{ik}^D(t) + \sum_{k=1}^{OD} \sum_{m=1}^{OT} C_{km}^A X_{km}^D(t) + \\
 & \sum_{m=1}^{OT} \sum_{k=1}^{OD} C_{mk}^A X_{mk}^D(t) + \sum_{m=1}^{OT} \sum_{v_2=1}^1 C_{mv_2}^{Fevac} X_{mv_2}^D(t) + \sum_{k=1}^{OD} \sum_{j=1}^{ITD} C_{kj}^A X_{kj}^{ND}(t) + \sum_{i=1}^{ITS} \sum_{k=1}^{OD} C_{ik}^A X_{ik}^{ND}(t) + \\
 & \sum_{i=1}^{ITS} \sum_{j=1}^{ITD} C_{ij}^A X_{ij}^{ND}(t) + \sum_{i=1}^{ITS} \sum_{m=1}^{OT} C_{im}^A X_{im}^{ND}(t) + \sum_{m=1}^{OT} \sum_{j=1}^{ITD} C_{mj}^A X_{mj}^{ND}(t) + \sum_{m=1}^{OT} \sum_{v_3=1}^1 C_{mv_3}^{Imp} X_{mv_3}^{ND}(t) + \\
 & \sum_{v_3=1}^1 \sum_{m=1}^{OT} C_{v_3m}^{Evac} X_{v_3m}^{ND}(t)
 \end{aligned} \tag{1}$$

The cost function contains all decision variables X and their respective unit flow cost C . For each time-step the optimal cost is calculated. Storage and inventory is determined at the end of each time-step. For each time-step the network can change completely, i.e., demand patterns can change due to seasonality resulting in a deficit location becoming surplus, resulting in the size of the set of ITD and ITS to be dependent on time.

Equality constraints

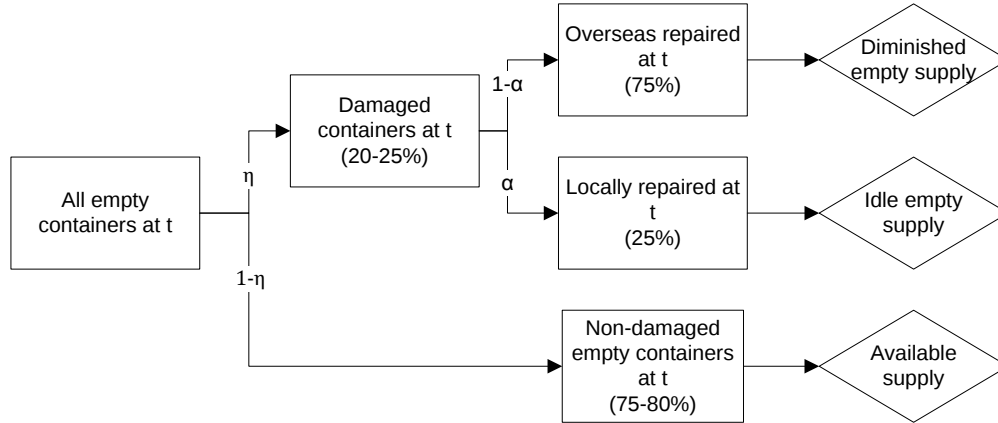


Figure 7 Supply determination process

Equations (2a) until (9) provide the flow balancing per node as indicated in Figure 7. Per index the subgraph G^D and the subgraph G^{ND} equality constraints are given. Node J serves in subgraph G^{ND} as a sink node and in subgraph G^D as a source node, because a node with an empty container requirement can also suffer from damage to containers. Prior to local balancing of

empty containers, available supply at source and sink nodes are divided into ‘damaged’ and ‘non-damaged’ containers per time-step through a ‘failure’ rate η (see (2)-(4)) , which is a randomly generated value subject to a lower bound and an upper bound limit. In each time-step, this results in a random number of damaged containers. Damaged containers are transported to empty depots, where workshops are located, for local repair, or to ocean terminals for overseas repair at cheap locations at the ‘local repair’ rate α . Figure 2 shows both the local repair rate and the failure rate have an influence on the available supply at the various nodes I, J, K, M .

$$i \in I : \begin{cases} \eta FI_i(t) = \sum_i X_{ik}^D(t) \\ (1-\eta) FI_i(t) + \sum_i I_i(t-1) \\ = I_i(t) + TE_i(t) + \sum_i X_{ij}^{ND}(t) + \sum_i X_{ik}^{ND}(t) + \sum_i X_{im}^{ND}(t) \end{cases} \quad \begin{matrix} (2a) \\ (2b) \end{matrix}$$

$$j \in J : \begin{cases} \eta FI_j(t) = \sum_k X_{jk}^D(t) \\ (1-\eta) FI_j(t) + I_j(t-1) + \sum_i X_{ij}^{ND}(t) + \sum_k X_{kj}^{ND}(t) + \sum_m X_{mj}^{ND}(t) \\ = I_j(t) + TE_j(t) \end{cases} \quad \begin{matrix} (3a) \\ (3b) \end{matrix}$$

$$k \in K : \begin{cases} \eta FI_k(t) + \sum_j X_{jk}^D(t) + \sum_i X_{ik}^D(t) + \sum_m X_{mk}^D(t) = \sum_m X_{km}^D(t) + \sum_{k'} X_{kk'}^D(t) \\ (1-\eta) FI_k(t) + I_k(t-1) + I_{fail}(t-1) + \sum_m X_{mk}^{ND}(t) + \sum_i X_{ik}^{ND}(t) \\ = I_k(t) + TE_k(t) + \sum_j X_{kj}^{ND}(t) \end{cases} \quad \begin{matrix} (4a) \\ (4b) \end{matrix}$$

$$k' \in K' : \begin{cases} \sum_{k'} I_{fail,k'}^D(t) = \sum_k X_{kk'}^D(t) \\ \sum_{k'} I_{fail,k'}^D(t) = \sum_k I_{fail,k}^{ND}(t+1) \end{cases} \quad \begin{matrix} (5a) \\ (5b) \end{matrix}$$

$$m \in M : \begin{cases} \eta FI_m(t) + \sum_k X_{km}^D(t) = X_{mv_2}^D(t) + \sum_k X_{mk}^D(t) \\ (1-\eta) FI_m(t) + I_m(t-1) + X_{v_3m}^{ND}(t) + \sum_i X_{im}^{ND}(t) \\ = I_m(t) + TE_m(t) + \sum_k X_{mk}^{ND}(t) + X_{mv_3}^{ND}(t) + \sum_j X_{mj}^{ND}(t) \end{cases} \quad \begin{matrix} (6a) \\ (6b) \end{matrix}$$

Equation (7) through (9) illustrate how the model determines the amount of containers that go to the three virtual nodes. Equation (7) determines the total flow of damaged containers repaired in

time step t by multiplying the total amount of supply by the failure rate and the local repair rate. Equation (8) calculates the total flow of damaged containers repaired overseas in time step t by multiplying the total supply by the failure rate multiplied by the evacuation rate, which is the amount of containers not repaired locally. Equation (9) calculates the total flow of non-damaged containers and repaired containers evacuated in time step t by adding the non damaged supply to the required storage plus the amount of containers repaired in the previous time step minus the previous storage and minus the required demand. Determining these three sink flows the model becomes balanced and therefore solvable.

$$y_{v_1}(t) = \alpha \eta \left(\sum_i FI_i(t) + \sum_j FI_j(t) + \sum_k FI_k(t) + \sum_m FI_m(t) \right) \quad (7)$$

$$y_{v_2}(t) = (1 - \alpha) \eta \left(\sum_i FI_i(t) + \sum_j FI_j(t) + \sum_k FI_k(t) + \sum_m FI_m(t) \right) \quad (8)$$

$$\begin{aligned} y_{v_3}(t) = & (1 - \eta) \left(\sum_i FI_i(t) + \sum_j FI_j(t) + \sum_k FI_k(t) + \sum_m FI_m(t) \right) \\ & - \left(\sum_i TE_i(t) + \sum_j TE_j(t) + \sum_k TE_k(t) + \sum_m TE_m(t) \right) + \left(\sum_i I_i(t) + \sum_j I_j(t) + \sum_k I_k(t) + \sum_m I_m(t) \right) \\ & - \left(\sum_i I_i(t-1) + \sum_j I_j(t-1) + \sum_k I_k(t-1) + \sum_m I_m(t-1) \right) + \sum_k I_{fail,k}^{ND}(t-1) \end{aligned} \quad (9)$$

Capacity constraint

Containers that are repaired at a workshop in the previous time-step become available for repositioning or evacuation in the current time-step. Each node in the G^{ND} subgraph is limited by a capacity constraint as found in equations (10)-(13).

$$0 \leq \sum_k X_{ik}^{ND}(t) + \sum_j X_{ij}^{ND}(t) + \sum_m X_{im}^{ND}(t) \leq n_{cap,i} \quad (10)$$

$$0 \leq \sum_k X_{kj}^{ND}(t) + \sum_m X_{mj}^{ND}(t) + \sum_i X_{ij}^{ND}(t) \leq n_{cap,j} \quad (11)$$

$$0 \leq \sum_j X_{kj}^{ND}(t) + \sum_i X_{ik}^{ND}(t) + \sum_m X_{mk}^{ND}(t) \leq n_{cap,k} \quad (12)$$

$$0 \leq \sum_k X_{km}^{ND}(t) + \sum_k X_{mk}^{ND}(t) + \sum_i X_{im}^{ND}(t) + \sum_j X_{mj}^{ND}(t) + X_{mv_3}^{ND}(t) + X_{v_3m}^{ND}(t) \leq n_{cap,m} \quad (13)$$

Repair constraint

Equation (14) ensures no more is repaired at a repair shop than the facility allows., given by

$$0 \leq \sum_k X_{kk'}^D(t) \leq D_{capk} \quad (14)$$

Topology selection

Equations (15) and (16) are used for the investigation of open/closed corridors in the model, i.e. setting Y_k to 0 a set of arcs are closed and by setting it to 1 that set of arcs are open. M represents the total flow on the affected arcs for time step t .

$$\begin{aligned} \sum_i X_{ik}^D(t) + \sum_j X_{jk}^D(t) + \sum_{k'} X_{kk'}^D(t) + \sum_m X_{km}^D(t) + \sum_{k'} X_{kk'}^D(t) + I_{fail}(t) \\ + \sum_i X_{ik}^{ND}(t) + \sum_j X_{jk}^{ND}(t) + \sum_m X_{km}^{ND}(t) + \sum_m X_{mk}^{ND}(t) + I_{fail}(t-1) \leq Y_k M \end{aligned} \quad (15)$$

$$\begin{aligned} \sum_k X_{km}^D(t) + \sum_k X_{mk}^D(t) + X_{mv2}^D(t) + \sum_i X_{im}^{ND}(t) + \sum_j X_{mj}^{ND}(t) + \sum_k X_{km}^{ND}(t) \\ + \sum_k X_{mk}^{ND}(t) + X_{mv3}^{ND}(t) + X_{v3m}^{ND}(t) \leq Y_m M \end{aligned} \quad (16)$$

Non-negativity constraint

Equation (17) is the non-negativity integer constraint.

$$\begin{aligned} X_{ik}^D(t), X_{jk}^D(t), X_{kk'}^D(t), X_{km}^D(t), X_{mk}^D(t), X_{mv2}^D(t), I_{fail}(t), X_{ik}^{ND}(t), X_{ij}^{ND}(t), X_{kj}^{ND}(t), X_{km}^{ND}(t), \\ X_{mk}^{ND}(t), X_{im}^{ND}(t), X_{mj}^{ND}(t), X_{mv3}^{ND}(t), X_{v3m}^{ND}(t), I_{fail}(t-1) \geq 0 \quad \forall i, j, k, m, v1, v2, v3 \in N \end{aligned} \quad (17)$$

In our case, we consider a flow perspective, and consequently we can relax the integer restraint on the variables representing the arcs. Furthermore by comparing results between two cases any existing error due to considering non-integer values is minimized.