The design of a large distribution system for the allocation of stores and the routing of vehicles

A Dutch case study

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Preface

This thesis represents the end of my master's in Transport, Infrastructure and Logistics at the Delft University of Technology. Without the assistance of several people, this thesis would not have come to its current conclusion.

Firstly, I would like to thank my daily supervisors, Ir. Mark Duinkerken and Dr. Yousef Maknoon, for their invaluable critiques, discussions, and assistance. I have greatly appreciated your approachability and helpfulness throughout our numerous meetings. I also want to thank Dr. Bilge Atasoy for chairing my thesis committee. You have supported me with your knowledge regarding your expertise.

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Mike Rietveld
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Executive Summary

Efficient distribution is crucial in many industries today, especially in retail, where customer demand is growing rapidly. The rise of online shopping and automation has led to a shift towards the use of Central Distribution Centers (CDCs). Companies in the retail industry often follow this trend, which requires them to rethink how they handle their distribution. This research focuses on two main challenges: first, changing the distribution system to adapt to new CDCs, and second, dealing with the computational complexities when optimizing the distribution on a large scale. The aim is to design a large distribution system for the allocation of stores and the routing of vehicles.

An extended analysis is performed to reach this goal. The analysis can be divided into two parts:

1. A computational analysis examines several cases with different computational improvements. This computational plan shows that the adjustments to the Gurobi optimization tool have a positive influence on the run time.
2. An experimental analysis consists of a base case, validation of the base case, and five experimental cases.

New elements are introduced to the mathematical model, which are primarily focused on the routing of vehicles. We also improve the computation by implementing a start solution, adjusting the parameters of the Gurobi optimization tool and including a callback function which bounds the run time. This model will determine the routing decisions for demand from several Distribution Centers (DCs) to the stores and will find the optimal vehicle routes. A case study at a company in the Netherlands provides insights into the changing world of distribution systems. It offers practical solutions for businesses trying to make their distribution processes better.

The results are compared from three perspectives. It must be considered that the clustering of data impacts the results, and sub-optimal outcomes are present because certain gaps in the data are not entirely eliminated in the experiments, affecting result interpretation. This error and uncertainty are based on the level of aggregation of data and the gap size of the solution.

First, they are compared based on the performance metrics. The main KPI is the most important metric to compare the cases on, but the other metrics need to be considered too. Consolidation of demand at the CDCs is proven to be a successful improvement, while an improvement in costs of 5.4% can be achieved. The total costs influence the third and fourth cases, so these cases can not be compared with the others. What can be concluded from these cases is that the metrics of both perform as expected. When lowering the total costs of transportation between the CDCs and Regional Distribution Centers (RDCs), the third case, the percentage of indirect delivery will increase. When increasing the volume, in the fourth case, the direct delivery increases too. And when decreasing the volume, the direct delivery decreases too.

Second, the results are compared based on their routing decisions. In some cases, the indirect delivery is more attractive. In others, the direct delivery, there is a difference in the postal codes with the highest influence on the routing decisions. Besides this fact, two postal codes pop up in all cases as postal codes with a significant influence on the total costs, considering a difference in routing decisions. These postal codes are 48 and 94. Therefore, it is advised to change the routing decisions of these postal codes.

Third, the relationship between routing decisions, distances and volumes is discovered. As a result, there are no convincing relationships between the routing decisions and distances on its own and between the routing decisions and volumes. However, a relationship between all three can be determined using a logistic regression. The coefficients of this logistic regression function (a, b, c and d) are fitted on the data. A probability of the routing decision can be determined where the coefficients fit the data and the distances and volume of the new store can be entered. Here, with an accuracy between 63.1%

for the second case and 78.0% for the third case, advice can be given for the routing decision based on the distance of a store to the CDCs, the distance of a store to an RDC plus the distance of this RDC to the CDCs, and the volume of demand of this store.

All in all, this research aimed to design an effective distribution system for store allocation and vehicle routing, addressing the main research question. Four sub-research questions guided the study, delving into system characteristics, planning model formulation, system performance, and the generalizability of findings. The analysis revealed that consolidating demand at CDCs can lead to significant cost improvements because of the higher truckloads. Changing transport costs between CDCs and RDCs and adjusting order volumes can influence delivery strategies. Moreover, the study identified specific postal codes as crucial in routing decisions. Despite the absence of direct relationships between routing decisions, distances, and volumes, logistic regression models provide guidance. Notably, the findings suggest that this distribution model can be adapted to various scenarios with different network structures, making a general contribution to the literature. Overall, this research contributes valuable insights into the design of large-scale distribution systems, offering a foundation for more efficient and cost-effective store allocation and vehicle routing strategies.

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1 Introduction

Distribution plays a significant role in many industries due to growing demand (Munasinghe & Rupasinghe, 2016). The same is true for the retail industry. Retailers must ensure that their supply chains are effective and able to fulfil consumer needs. To do this, a focus is needed on inventory management and transportation (Lagorio & Pinto, 2021). Several factors influence the need for innovation in inventory management. First, as companies struggle to recruit employees, they are forced to automate their inventory systems (PWC, 2023). Second, customers are increasingly getting used to a flexible market where last-minute changes can be made to their orders and where orders can be delivered at short notice (Tarry, 2022). This plays a significant role in the growth of e-commerce in the retail industry (Morgan Stanley, n.d.). To increase the efficiency of a company's inventory system and transportation, there is a trend of more and more companies returning to automated Central Distribution Centres (CDCs) where they can easily handle a large proportion of orders at one location. Bol.com and Albert Heijn are examples of such companies where this trend has also impacted and which has also brought to life an automated CDC that (partially) replaces several non-automated Distribution Centres (DCs) (Stad, 2022). The building of new CDCs forces companies to adjust their transport schedules. Stores must be reassigned to the Regional Distribution Centre (RDC) they are currently assigned to or the new CDC. The demand can be delivered from the CDC directly or from the CDC via an RDC. Consolidation of demand can take place at an RDC, which can make it attractive to deliver from that DC. Here, financial and environmental aspects need to be taken into account by the company. The financial aspect is important to remain competitive in the market. Higher costs in transport and logistics will eventually be passed on to the customer. Today, the environment is becoming an increasingly important aspect to consider. Society and the government are both encouraging improvements in this area. As a direct result, companies must have this focus too.

1.1. Main challenge

A challenge that arises from the described developments is the need for a change of structure of the distribution system. This challenge can be split into two parts: an industrial challenge and a computational challenge.

Industrial challenge

The industrial challenge arises due to the opening of the new DC. The distribution system will change, and this will influence the optimal allocation of stores to the DCs. Stores will be supplied directly or indirectly from the new DC. When stores are supplied indirectly, demand is first sent to an RDC, where it will be consolidated for delivery to the stores. Here, the challenge arises to adjust the distribution system to reallocate the stores to the DCs. This allocation is based on the aforementioned trade-off between cost and environment. Three sets of vehicles can be used for the transportation of demand, which all can be assigned to one of the routing options. So, the routing of vehicles needs to be revised to meet the new allocation of stores.

Computational challenge
A second challenge arises due to the size of the distribution system. The larger the distribution system, the higher the number of options for allocation and routing, which all need to be evaluated. The complexity influences the computational forces required to solve the model. It is therefore necessary to look at how this complexity can be reduced while searching for new optimal outcomes. It is important to keep evaluating performance while searching for a solution to this challenge. All in all, this ultimately results in a main challenge: the design of a large distribution system for the allocation of stores and the routing of vehicles.

The described challenge reflects a more general challenge in many industries. In today's industry, transport and logistics are going to play an increasingly important role. Costs must be as low as possible to maintain competitive prices and remain competitive in the market. In addition, companies are being judged increasingly harshly for environmentally unfriendly behaviour. Also, companies are automating their DCs to improve the efficiency of their operations. This is often done in new locations, and, as a result, a part of the stores are eventually supplied from a new DC. A trend here is that such an automated DC can take over some or all of the roles of several smaller, non-automated DCs. Therefore, this research on the allocation of stores and the routing of vehicles must be done.

1.2. Contribution

The design of a large distribution system for the allocation of stores and the routing of vehicles will be studied in this research. There have previously been numerous papers written on this subject and its variations. Andersen et al. (2009) formulates a fixed charge capacitated multi-commodity network design. Each of the arcs has a fixed cost and certain capacity. An arc-based formulation is used for this network. The main objective is to minimize the total costs, including fixed and flow costs. The service network design with asset management includes constraints which ensure that the number of vehicles entering and leaving a node must be equal, (Andersen et al., 2009). An upper bound of the fleet size is taken into account, and costs are linked to using an asset. Cheong et al. (2007) introduced a network design which contains multiple suppliers, consolidation hubs, supply hubs and manufacturers. To enable make-to-order production, the suppliers transport components to the warehouses, which regularly restock the makers. Every warehouse has a specific manufacturing plant it serves. Each warehouse must send suppliers replenishment orders according to each manufacturer's daily final assembly schedule. This research aims to determine which of the possible hubs need to be opened and what the minimal total logistic costs are. These costs consist of the transportation costs from the suppliers to the hubs, the transportation costs from the hubs to the warehouses, the expenses associated with keeping goods in the warehouses, and the fixed and variable costs associated with operating the hubs. The distribution network of a consumer goods company is investigated by Cintron et al. (2010). Reducing consumer demand supplied by the RDC and increasing supply from independent distributors or directly from the plants' warehouses may decrease supply chain distribution expenses. The model contains several factories, each of which produces a variety of goods. As long as every item in the container is produced in the designated facility, it is thought that a variety of products can be shipped directly from a factory to a consumer. The objectives are to maximize total profit, minimize lead time, maximize power and maximize credit performance. Integrated distribution network design problems often include several plants, depots, transit points and customers. Ambrosino and Scutella (2005) provides a static model for such a network. They do not take into account inventory management, so there is no difference between a transit point and a regional depot. Alikhani et al. (2021) proposes a supply chain network design which focuses on resilience against disruptions. The network consists of several suppliers, distribution centres and stores. The distribution centres can function as a cross-dock or as a warehouse. This research focuses on determining where to locate facilities and schedule the distribution of demand. The objective is to minimize the total costs of facility location and distribution. Even though many of these transportation problems exist in the literature already, there is still room for research. The gap in the literature that will be explored in this paper relates to:

> The allocation of stores and the routing of vehicles for large-scale distribution systems based on the opening of new DCs.

The contribution can be split into two parts: a scientific contribution and a business contribution.

Scientific contribution
The design of a large distribution system for the allocation of stores and the routing of vehicles will be studied in this research. Since there are no known solutions to this particular case, this is considered a scientific contribution. First, the mathematical model for the distribution system of this research will be built on existing models. New parts will be added to the combination of existing parts. The new parts will involve the use of various modalities that can be deployed on limited parts of the system. All in all, this model will be tailored for this specific type of distribution system, which contains multiple DCs and a large number of stores. Second, finding an optimal solution for these complex systems can be computationally challenging. The optimization problems for such large-scale distribution systems are currently still a topic where much research can be done. A distribution system quickly becomes complex with a number of DCs and stores, and this grows exponentially. The optimization tool will automatically select heuristics that fit the type of optimization, and that will help solve the model more efficiently. The combination of a start solution, adjustments regarding the input parameters of the optimization tool, a callback function and math-based heuristics will be implemented to avoid reaching the computational limits.

Business contribution
Openings of new central DCs are due to the trend of automating DCs, which affects companies' transport and logistics. There is a trade-off for companies to determine how much demand needs to be delivered from the current DCs and how much from the new ones. Whereas previously, the choice of allocation was often based on the smallest distance to a DC, this is different with this new form of distribution system. Here, the choice has to be made whether it is more advantageous to divert demand via RDCs. Given the various delivery options that arise, this choice becomes a lot more complex. This study will provide a solution to this complex issue and help businesses further with the design of their distribution system.

So, this optimisation with the aim of designing a large distribution system for the allocation of stores and the routing of vehicles is an expected contribution to science due to the new parts of the mathematical model, the implementation of a start solution, input parameters, a callback function and math-based heuristics, and the contribution to logistics companies.

1.3. Problem statement

Retailers need to stay innovative to fulfil consumers' needs. The high expectations of consumers and the shortage in the labour market are already two reasons to improve transport and logistics in the retail industry. These reasons complement the already existing trend where companies increase their efficiency by building automated CDCs. It is necessary to reassign stores to either the current RDC or the new CDC. An RDC may see demand consolidation, which could make it more appealing to deliver from that DC. Here, both financial and environmental factors must be taken into account. These developments bring some challenges. Retailers must keep a competitive position in the market, so the costs need to be as low as possible while keeping an eye on the emissions of the transport. Furthermore, solving such an optimization problem for a large-scale distribution system may bring computational challenges. Therefore, the goal of this research is to:

> Design a large distribution system for the allocation of stores and the routing of vehicles.

1.4. Research questions

Several research questions have been formulated to investigate the gap in the literature and the corresponding goal of the study. The main research question will be answered by four sub-research questions, Table 1.1.

How can a large distribution system be designed for the allocation of stores and routing of vehicles?
1. What are the characteristics of this distribution system?
2. Which methods can be used to formulate a planning model regarding this distribution system?
3. How do you evaluate the performance of the system?
4. What is the performance of the system, given the data from the case study?

Table 1.1: Research questions

1.5. Research methods

Table 1.2 indicates the methods that will be used to answer the research questions. These methods are further elaborated on in the following sections.

How can a large distribution system be designed for the allocation of stores and routing of vehicles?	
1. What are the characteristics of this distribution system?	Desk research & expert consulting
2. Which methods can be used to formulate a planning model regarding this distribution system?	Literature & expert consulting
3. How do you evaluate the performance of the system?	Modelling
4. What is the performance of the system, given the data from the case study?	Case study

Table 1.2: Research methods

1.5.1. Desk research & expert consulting

Experts from the case study will be consulted to gain insight into the characteristics of the distribution system. These experts will have a clear overview of the distribution system and can provide advice based on all the ins and outs of this study. Desk research will be performed to collect and analyze the information from the experts and to complement this information with other data. Chapter 2 will describe the characteristics of this distribution system.

1.5.2. Literature & expert consulting

Literature will be studied in Chapter 3 to gather information regarding distribution systems. The selection of methods for this research will be based on the study. Expert consulting will complement the literature study. Experts in the field of transportation and optimization problems, such as TU Delft professors, will be consulted.

1.5.3. Modelling
A mathematical model will be constructed in Chapter 4 based on the information from the first two sub-research questions. A mathematical model is a more abstract and simplified version of reality Richardson (1979). The reality will be converted into a goal, constraints, parameters, and variables. These elements combined will result in the model. A mathematical model will enable optimization and, thus, should improve the distribution system.

Some limitations will need to be taken into account when implementing a mathematical model. First, models are based on assumptions and observations. The harder it is to determine the model's specifications, the more assumptions will need to be made. To fully understand a model's limitations, one must understand the assumptions that are made. Second, sometimes, a model is fitted on a sample of data instead of a specification of equations that is determined based on several input parameters. The model can be fitted wrongly, which in turn leads to surrealistic outcomes when new data is inserted. Third, a model is dependent on data. Inaccurate results may be the result of unrealistic data. So, the model may be perfect, but as long as there is no proper set of data, the results of the model will not be valid to use. There are several significant advantages, on the other hand, to using a mathematical model. Two considerable advantages relate to evaluating a large problem while considering all relationships within the model. Furthermore, a model is less prone to biases.

Current literature often implements linear integer programming as an optimization tool. The characterization of a linear model is that all constraints and objectives contain linear formulations. Linear integer programming is flexible, and solvers have been developed for these models. This research will use a linear integer model for the optimization too.

1.5.4. Case study
A case study will be conducted at a company in the Netherlands to put the model into practice, Chapter 5. The company is active in the retail industry, with several DCs and many stores. The company has opened a new CDC, necessitating an overhaul of its distribution system. Stores need to be reallocated to the DCs. This is an excellent opportunity to use the company's data in the modelling that will be done in this case study. The outcomes can be validated more easily with the help of experts. The mathematical model can be converted into a script that can be optimized for this case. First, a default case study will be performed, after which the model will be extended with several scenarios and configurations.

1.5.5. Software
The data will be processed in the Spyder software by programming the model in the Python language and by making use of the Gurobi solver. For Python-based scientific programming, Spyder is an open-source, cross-platform integrated development environment. Gurobi is a cutting-edge solution for problems involving linear programming, mixed-integer programming, and quadratic programming.

1.5.6. Overview
An overview of the process of this research is given in Figure 1.1. Real-world developments in the field of transportation will bring specific problems with them, as stated in Section 1.3. A literature study will be performed to address these problems and to discover the current state-of-the-art. The literature study will give insight into what is already known and what is still unknown about this topic. The case study will consist of a real-world application. The case study will be solved using a mathematical model, so first, the specifications of this model will need to be determined. These specifications will include questions, relations, and assumptions on which the model should focus. The model can be formulated when the specifications are set. This formulation will contain parameters, variables, constraints, and objectives. The model can be solved using data as input from the case study. The model will be converted into a Python script, which will be solved with the help of the Gurobi optimizer. Before any conclusions can be drawn, the results will need to be evaluated and validated. If the results turn out to be invalid, it will be essential to work towards an optimal and valid result based on an iterative process. The specifications of the model can be checked first, and from this point on, the cycle can be rerun until the results seem to be a valid outcome of the model. The research will be concluded based on the validated results, and with the help of recommendations, the research gap can be solved.

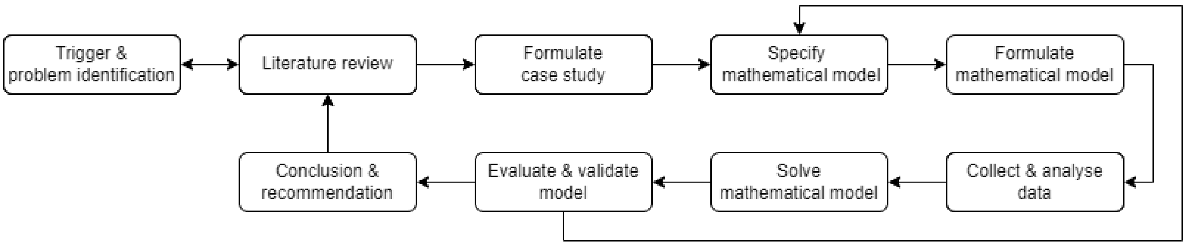


Figure 1.1: Methods and workflow overview, adapted from Pedley (n.d.)

The system that will be used in this research will be analysed in Chapter 2. Literature will be studied to gather information about models that can be used for the formulation of this distribution system, Chapter 3. The mathematical model corresponding to the system will be formulated in Chapter 4. The case study is provided in Chapter 5. The discussion, conclusion and recommendations are presented in Chapter 6.

2 System analysis

The characteristics of the distribution system that is analysed in this research are outlined in this chapter. The first sub-research question:

> What are the characteristics of this distribution system?

can be answered based on the analysis. The main characteristics are the decision level of this optimization, the system's boundaries, the locations of DCs and stores, modalities used for the distribution, demand specifications, time restrictions and performance metrics.

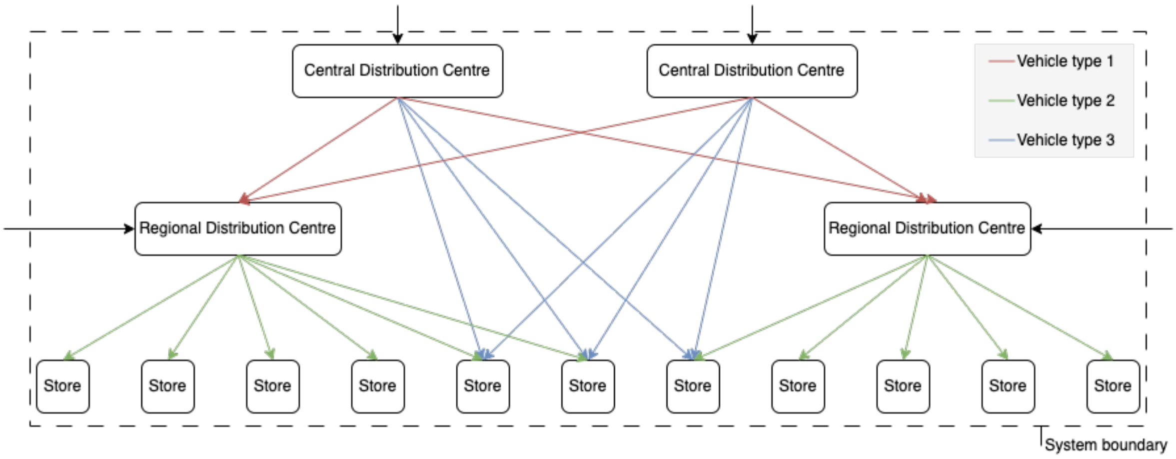


Figure 2.1: Structure of the distribution system

2.1. Decision level

Network optimization generally deals with three levels of decisions: the strategic, tactical and operational level. This research mainly focuses on the tactical decision level of a network model. The tactical decision level corresponds to medium-term decisions regarding network design. These decisions include the allocation of stores to DCs, the frequency of delivery, consolidation of deliveries and the fleet size, among others. The strategic level deals with decisions which include significant investments like locating and building new DCs. While the goal of this research is not to find an optimal location for new DCs, stores or any other long-term decision corresponding to significant investments, this decision level is not of interest. The operational level deals with short-term planning, which includes specific vehicle routes, for example. While the main goal is not to have a set of routes per vehicle, this is not the main focus point.

2.2. System boundaries

DCs and stores bound the distribution system, Figure 2.1. This means that the flow of raw materials, the production of commodities and the transport from the factories to the DCs are left out of consideration. The figure indicates the system boundary with a black dotted line, and the flows left out of consideration are indicated with black arrows. Besides these flows, the flow from the stores to the customers is left out of scope too. Only the flow within the network between the DCs and stores is considered.

2.3. Locations

The locations of all DCs and stores are known. The DCs have a fixed capacity, and the stores have a known demand. The system includes CDCs and RDCs. Stores can be supplied from the CDC directly or from the CDC via an RDC. This research focuses on the distribution of demand from the DCs to the stores. All stores receive demand from all CDCs and from one RDC. For each store, the decision must be made per CDC whether or not there is direct or indirect delivery. The delivery from RDCs will, therefore, include consolidated deliveries. This results in a balance between the costs of delivery from the CDC directly to the stores or the costs from the CDC to the RDC, consolidation costs and costs of delivery from the RDC to the stores. The combination of these two supply methods can be seen as a tree topology, Figure 2.1. Direct delivery from the CDCs to the stores is indicated with blue arrows. Delivery from the CDCs to the RDCs is indicated with red arrows. The other stores are supplied from the RDC they are allocated to, with consolidated deliveries indicated with green arrows.

2.4. Modalities

The demand for this model is transported by trucks. There are three types of trucks used for delivery. The first truck type is used for transportation from the CDCs to the RDCs. The second truck type is used for transportation from the RDC to the stores. The third truck type is used for direct transportation from the CDCs to the stores. This results in a heterogeneous fleet. The capacities and costs of these trucks may differ by type. For transport from a CDC to an RDC, there is a fixed cost for transport and a process cost per unit of demand. A variable amount per kilometre is charged for direct delivery from a CDC to a store. An end-of-route cost is charged for direct delivery from a CDC to the stores, while it is assumed that this is outsourced. The fleet size of this system is variable and may depend on the optimal solution.

2.5. Demands

The demand for stores changes over time. Stores are supplied multiple times per week. Routing schedules can be constructed per day of delivery. The demand is multi-commodity while it has an origin, a destination and a number of products, which differ per store. The trucks are loaded with demand for the stores on their route, so the routes will not change when started. This results in a static and deterministic model. Transport of demand can be consolidated so that multiple stores can be supplied by one truck.

2.6. Time restrictions

The delivery to stores may be restricted by certain times. For example, these restrictions can be set due to strategic decisions by the company or municipalities. Some municipalities restrict truck delivery to morning delivery only. On the other hand, other stores may prefer a late delivery, for example. This results in specific time windows for vehicles to supply the stores. If there are restrictions on time, the system will be time-dependent. Since converting a model to a time-space network will significantly impact the computational forces for optimising the model and the assumption is made that the time constraints will not considerably influence the optimal outcome, this is not considered for the time being. This is for ease of simplification in the model.

2.7. Distribution

The four possible distribution options of a system containing two CDCs and two RDCs are visualised in 2.2. There are four types of products in combination with a DC. The origin of type a is at one of the CDCs. The origin of type b is at another CDC. The origin of types c and d are at one of the RDCs. Scenario 1: Product types a and b are delivered to the stores from the CDC directly, and types c and d are delivered from one of the RDCs. Scenario 2: Product type b is delivered directly to the stores from the CDC, and type a is first transported to one of the RDCs to consolidate the deliveries. Then, a, c and d are delivered from one RDC to the stores. Scenario 3: Product types a and b are first transported to one of the RDCs to consolidate the deliveries. Then, all types are delivered from an RDC to the stores. Scenario 4: Product type a is delivered to the stores from the CDC directly, and type b is first transported to one of the RDCs to consolidate the deliveries. Then, b, c and d are delivered from one RDC to the stores.

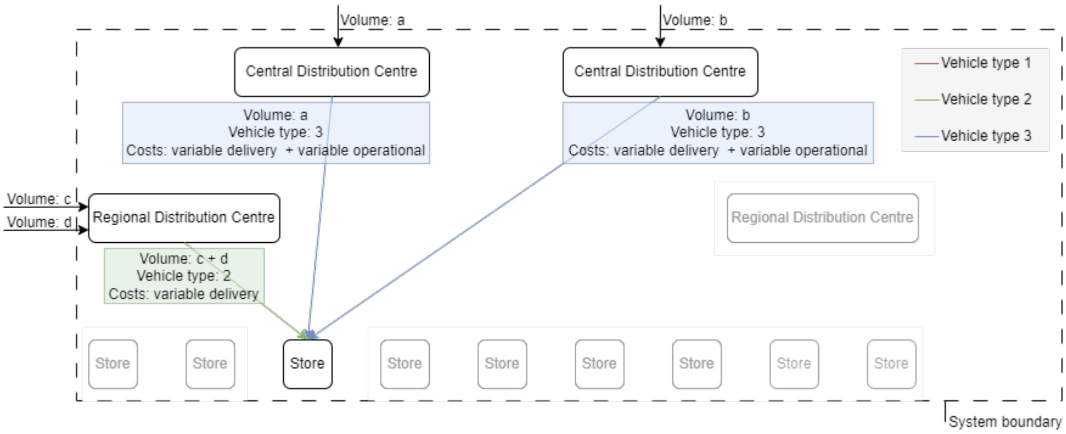
2.8. Performance metrics

Performance metrics of this distribution system were determined using Key Performance Indicators (KPIs). The most important KPI is the total cost of transport. The total cost of transport includes the routing costs and the processing costs. The routing costs consist of transportation costs between a CDC and an RDC, a CDC and a store, an RDC and a store and between two stores. The processing costs are variable and based on the size of demand transported from the CDCs to the RDCs. The second KPI is the total distance driven by all vehicles. The third KPI equals the total number of vehicles needed to transport all demand. Table 2.1 shows an overview of these KPIs.

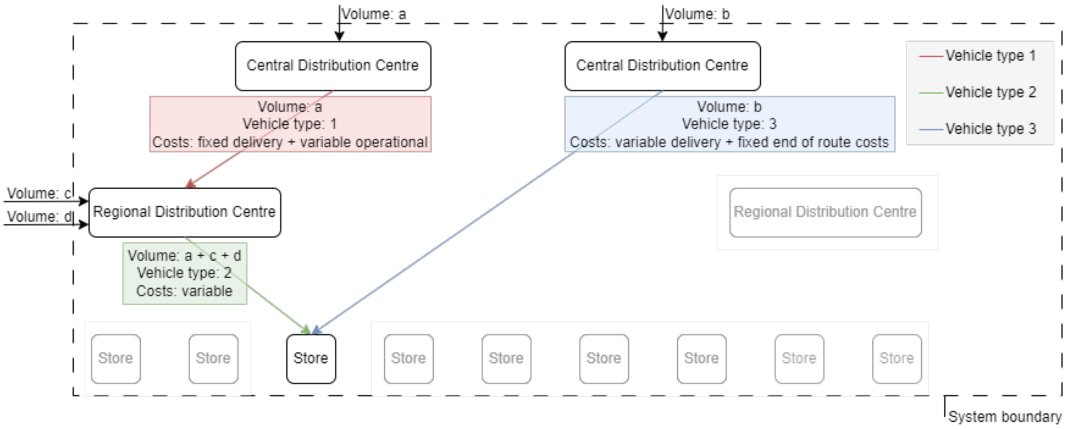
Key Performance Indicators
1. Total costs of transport [€]
2. Total distance [km]
3. Total number of vehicles [-]

Table 2.1: Overview of KPIs

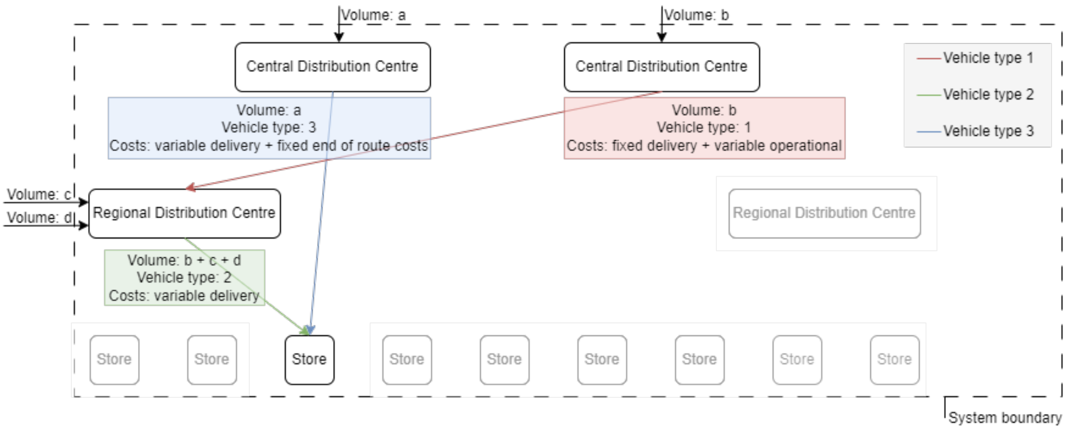
Four other metrics are used to evaluate the performance of the model. These metrics are the costs per container, load per vehicle, drop size per stop and the number of stops per vehicle. The costs per container are preferably as high as possible, and the load per vehicle, drop size per stop and stops per vehicle as low as possible.



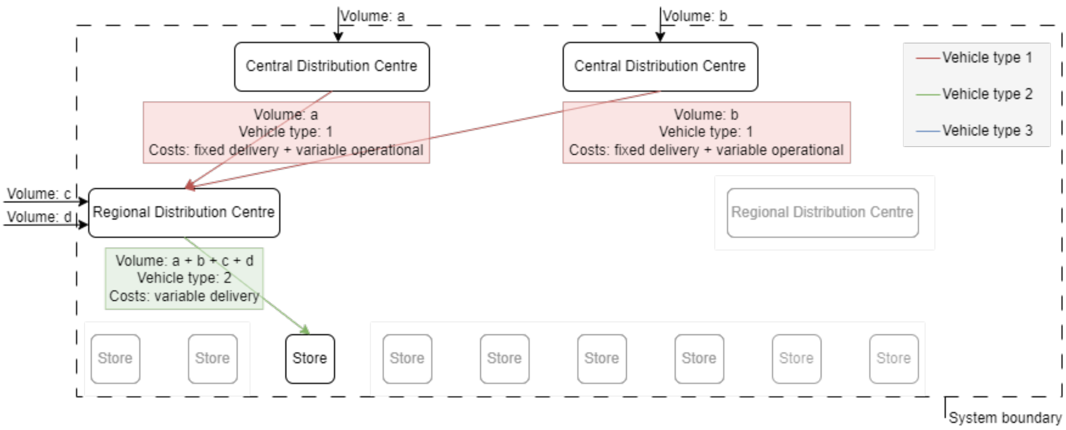
(a) Option 1



(b) Option 2



(c) Option 3



(d) Option 4

Figure 2.2: Visualisation of distribution options

3
Literature

Literature can be studied based on the system analysis of Chapter 2 and the resulting answer to the first sub-research question. The literature can provide insight into distribution systems. The second sub-research question can be answered based on the literature:

> Which methods can be used to formulate a planning model regarding this distribution system?

Network design is a general term for various distribution system problems. The planning levels of distribution systems can be classified into three categories: strategic, tactical and operational (Crainic & Laporte, 1997). Strategic (long-term) decisions shape the strategies and determine general development. Tactical (medium-term) decisions relating to the design of a network. Operational (short-term) decisions relate to detailed representations of the assets, facilities and activities. Network designs contain four decision layers: topology, location, allocation and routing decisions. The topology layer decides the structure of the network. The location layer determines where the facilities must be located in the network. The allocation layer allocates customers to open facilities. The routing layer determines the routes for the vehicles to satisfy the demand.

3.1. Network design

Network designs are widely used in distribution system problems (Crainic, 2000). Network design deals with strategic decisions that may contain significant investments and focus on the long term. These formulations are defined on graphs, with nodes connected by links. When the links are directed, they are represented by arcs. The nodes can represent origins and destinations. The links may have costs, length and capacity. The main objective of network designs is to select links in the network to satisfy all demands for distribution by minimizing the total costs. A commonly used version of the network design is the linear cost, incapacitated, multi-commodity (MCND) network design. Multi-commodity networks include two or more commodities that must be distributed from a specific origin to a destination (Salimifard & Bigharaz, 2022). Multi-modality networks use multiple modes of transportation for the distribution of demand. Combinations of these modes can be made to reduce the total costs of the transport. Another advantage of multi-modal transportation corresponds to the sustainability of the transport, which can be increased.

3.2. Service network design

A service network design focuses mainly on the tactical level but includes some strategic and operational characteristics. This aligns with the decision level of the distribution system from this research. A service network design aims to plan the resources and activities to satisfy the demand (Crainic & Hewitt, 2021). The operation of a vehicle is called a service. Service network design usually takes place in transportation based on consolidation. Consolidated transportation considers capacities and service schedules. Several orders can be combined within the same vehicle, and multiple vehicles may be used for one delivery from origin to destination. The service contains a route and physical as well as operational characteristics. The physical characteristics include the vehicle type and vehicle capacity, for example. The operational characteristics include the costs, total trip time and departure time. The goal of tactical planning is to create a distribution plan and schedule to minimise the negative effects of consolidation, meet customer demand and service-quality standards and be profitable and effective. It discusses system-wide operational planning to choose and schedule services and transfer and consolidate activities at terminals. The service network design combines two sets of choices. The first set corresponds to the frequencies or schedules of the operations of the services. The second set includes the routes. The routes contain an origin, destination and intermediate stops. A service network design contains various characteristics. As stated, the design can be capacitated or incapacitated (Andersen et al., 2009), one single or multiple commodities may be used, and the network may allow consolidation of deliveries (Andersen et al., 2009), so it can consist of direct and/or indirect deliveries (Section 3.5). Other characteristics are the flow type of the model, which can be arc or path-based (Section 3.4), fixed charge or variable charged (Andersen et al., 2009), and time-based so that frequencies can be set (Section 3.6). Many different closed-form methodologies have been used to solve transportation issues. However, finding an explicit optimal solution becomes computationally costly and nearly impossible to establish when more complicated restrictions are considered. For this reason, academics have investigated several heuristics. The framework used by Amorim et al. (2014) employs an Adaptive Large Neighbourhood Search (ALNS) metaheuristic optimization method inspired by Large Neighbourhood Search (LNS). ALNS enhances solutions iteratively by making large-scale modifications, alternately deleting and repairing the solution to avoid local optima. ALNS offers benefits like handling various limitations and exploring diverse search spaces but has drawbacks in computational complexity and parameter sensitivity (Amorim et al., 2014), (Ropke & Pisinger, 2006). Osvald and Stirn (2008) utilize a sequential constructive heuristic followed by a Tabu Search (TS) improvement heuristic. TS uses memory structures to learn from previous solutions and avoid cycling but is sensitive to parameter selection and prone to getting stuck in local optima (Osvald & Stirn, 2008), (Fu et al., 2005), (Vidal et al., 2013). Tarantilis and Kiranoudis (2002) implements a List-Based Threshold Accepting (LBTA) algorithm, a modification of Threshold Accepting (TA) that stores candidate solutions in a list. LBTA is suitable for resolving issues in large and complex search spaces but shares some downsides with TA, including sensitivity to parameters and the potential for suboptimal solutions (Tarantilis & Kiranoudis, 2002), (Maringer, 2005).

3.3. Network structure

Hubs connect many nodes by using a relatively small number of links in Facility Location - Network Design (FL-ND) (Maknoon, 2022). Consolidation can take place at hub facilities. The network design and economy of scale rules often affect the costs. All locations are represented by nodes, the infrastructure by edges and the route of products by arcs. An origin, a destination, and a volume characterize the commodities. The connection layout of the network can be grouped into six design basics: a line topology, a star topology, a ring topology, a tree topology, a mesh topology and a hybrid topology. Hubs are connected by a single line in the line topology. All nodes are linked to a single central hub through which all traffic passes. Each hub is connected to two other hubs in a ring topology. This results in a low probability of failure. In a mesh topology, hubs are partially or fully connected. In the hierarchy tree topology structure, all locations are arranged hierarchically. The network of Ambrosino and Scutella (2005) is an example of a tree analysis, Figure 3.1. This network comprises a plant, multiple central depots, multiple regional facilities and clients. The use of depots and facilities helps to reduce the last mile delivery costs as low as possible, while transport between facilities is often against a significantly lower cost. Distribution centres can also function for resilience against disruptions (Alikhani et al., 2021). The flow of commodities can be re-routed easily when a network consists of multiple distribution centres.

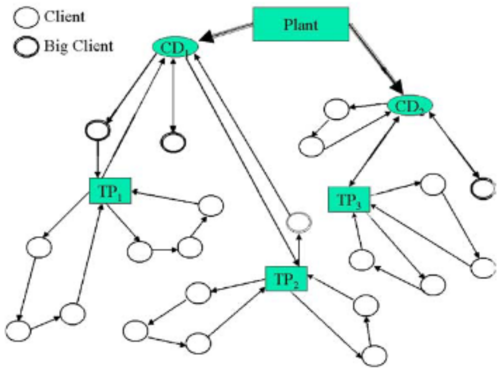


Figure 3.1: Network structure of Ambrosino and Scutella (2005)

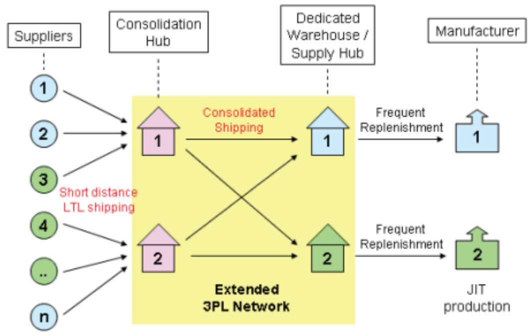
The hybrid topology is a combination of several other topologies. To create the proper network structure, four steps can be followed. The first two steps are linked to the design decisions, and the second two to the operational decisions. The design decisions consist of location decisions, so what locations should be considered as a hub and topology decisions, so which link needs to be selected. The operational decisions consist of allocation decisions, so the assignment of supply and demand nodes to hubs and routing decisions, so how demand is routed between the origin and destination. This study focuses on the operational decisions of the distribution system. The stores must be allocated to the DCs, and the commodities must be routed through the network. The tree topology is assumed to best fit as a network structure. It is believed that the design decisions have already been made.

3.4. Flow type

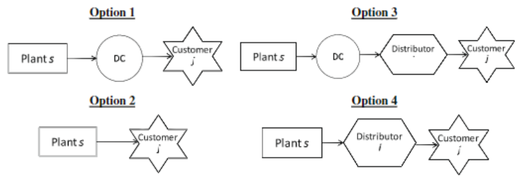
The flow in a service network design can be modelled path-based or arc-based (Ohmori, Yoshimoto, et al., 2019). Arc (or link) based modelling focuses on all individual links between the nodes in the network (Andersen et al., 2009). The design and optimization are performed on this level. Path (or route) based modelling focuses on all routes that connect the origin and destination nodes. The design and optimization are performed on this level of connecting the origin and destination with paths. This research implements an arc-based model.

3.5. Direct and indirect delivery

Some networks include direct and indirect deliveries, just like the distribution system from this research. The distinction can be made based on the size of the demand of specific locations. When a network consists of plants and warehouses, it may be more cost-effective to first transport orders from a plant to a warehouse, where they can be consolidated and delivered simultaneously. Trucks can be used with a higher load factor. A tree topology can be used to apply various layers of distribution centres (Munasinghe & Rupasinghe, 2016). With this, direct and indirect supplies can be achieved. Consolidating deliveries at a cross-dock (Sung & Song, 2003) can make demand delivery more efficient. Demand is distributed from origin to destination via distribution centres. The demand arrives at a distribution centre and is directly loaded in another vehicle for the final delivery. The delivery of demand that originates from the same place but has delivery locations in different regions may be more efficient by splitting this delivery. The network, including direct and indirect deliveries, may differ in structure. The network of Cheong et al. (2007) contains several suppliers, consolidation hubs, warehouses and manufacturing plants, Figure 3.2a. All suppliers are linked to a single consolidation hub, and all manufacturers are connected to a dedicated warehouse. Consolidated shipping takes place between the consolidation hubs and the warehouses. Another network, which consists of the flow of consumer goods, is designed by Cintron et al. (2010). This network deals with four options for the transportation of goods. A combination of direct and indirect deliveries reduces the total transportation costs, Figure 3.2b.



(a) Network structure of Cheong et al. (2007)

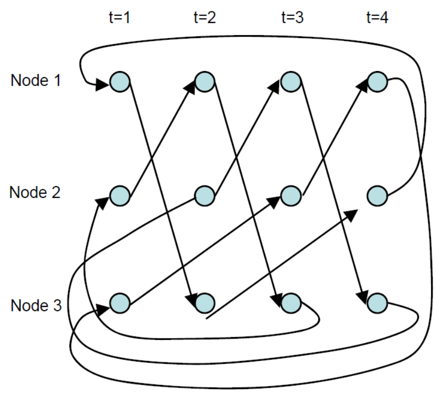


(b) Network structure of Cintron et al. (2010)

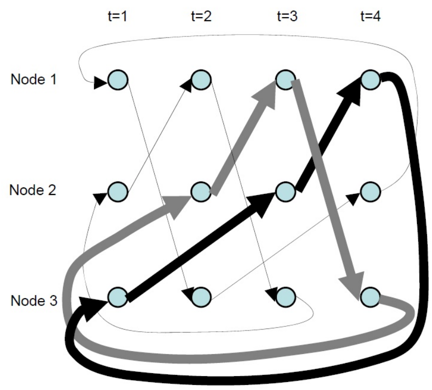
Figure 3.2: Examples of direct and indirect delivery networks

3.6. Time-space representation

A time-space network replicates the nodes in each period of time (Andersen et al., 2009). The scheduled length is partitioned into periods; each time step can include a specific action. The routes of the service or flow of products are included in this time-space network, while at each time step, the location is indicated. Changes over time are indicated by an arrow which connects the nodes. An example of a time-space representation is visualized in Figure 3.3. This example consists of three nodes and four time periods. Figure 3.3a represents all possible services, and a feasible solution is given in Figure 3.3b. Frequency constraints can be added to the service network design, which sets the number of occurrences of each arc (Andersen et al., 2009). This frequency is set in a time-space setting. Lower and upper bounds set the frequency of the services. Route length constraints can be added to the model to create repetition in the asset routes (Andersen et al., 2009). This repetition results in some cycles in the network for specific routes. A multiple of the time horizon bounds the length of these routes. An extensive network with a detailed representation of time may result in a difficult-to-solve model. The assumption has been made that time can be disregarded for this research, but it is worth understanding how this would affect the model and how it could be applied in a more in-depth study.



(a) A network with four time periods and three nodes



(b) Feasible example of a service

Figure 3.3: Example of a time-space representation by Andersen et al. (2009)

4 Mathematical model

This chapter introduces the mathematical model of the distribution system. First, the scope of this mathematical model is formulated. Second, the assumptions are listed to simplify the real-world problem. Third, the model is developed based on sets, parameters, decision variables, an objective and constraints. This mathematical model can be formulated based on the system analysis of Chapter 2, the literature of Chapter 3 and the resulting answers to the first and second sub-research questions. Fourth, an explanation is stated of how this mathematical model is implemented as a computer model. Fifth, the model is verified based on a dummy situation. Sixth, computational information is gathered like the Gurobi input parameters, computational times and computational limits. The model can be used to answer the third sub-research question:

How do you evaluate the performance of the system?

4.1. Scope

The goal of this research is to design a large distribution system for the allocation of stores and the routing of vehicles. The scope of this research is based on the system analysis of Chapter 2. The movement between DCs and stores defines the system's boundaries. The locations, which include CDCs, RDCs, and stores, are all known in advance. Stores are assigned to all CDCs, but they are only assigned to a single RDC. Three different vehicle types comprise the fleet: vehicle type 0 transports demand from a CDC to an RDC, vehicle type 1 transports demand from an RDC to a store and vehicle type 2 transports demand from a CDC to a store. This model converts the real-world situation to a mathematical formulation. The model must make several decisions:

- On which arcs will (parts of) a commodity flow to reach the destination?
- How many services are required to meet the demand on the network?
- From which CDCs are the stores supplied directly?

4.2. Assumptions

It is important to map the assumptions and state the input and output needed for this model. Several assumptions are made for the model:

- The volume of a commodity may be larger than the capacity of a truck, so split delivery is possible.
- A split delivery of a commodity must always originate from the same DC. So the complete commodity is delivered from the CDC to the stores, or from the CDC via an RDC to the stores, or from the RDC to the stores.
- Consolidation of commodities is possible for delivery. This allows it to operate with a higher truckload.

- Transportation is possible between a CDC and an RDC. There is no transportation possible between CDCs and between RDCs.
- Vehicle type 0 may start and end at a CDC and only drive to an RDC. Vehicle type 1 may start and end at an RDC and may only drive to stores. Vehicle type 2 may begin at a CDC and may only drive to stores.
- The demand of all stores is met.
- DCs have no capacity limit as it is assumed that the demand of all stores does not exceed this.
- A maximum number of deliveries is set per store based on the demand size and the number of DCs it is supplied from.

4.3. Formulation

This model is based on the methods from the papers of Crainic and Hewitt (2021) and Ambrosino and Scutella (2005). Constraints 4.2, 4.9, 4.17 are inspired by the formulations of Crainic and Hewitt (2021). Constraints 4.3, 4.5, 4.10, 4.11 are inspired by the formulations of Ambrosino and Scutella (2005). The objective function 4.1 and constraints 4.4, 4.6, 4.7, 4.8, 4.12, 4.13, 4.14, 4.15, 4.16, 4.18 are a potential contribution of this paper to the literature. Table 4.1 describes the sets, parameters and decision variables.

4.3.1. Sets

The sets of this model can be divided into five categories. The first category contains all sets of nodes. The second category includes all sets of arcs. The third category contains a set of commodities. The fourth category includes sets corresponding to the vehicles. The fifth and last category contains the set of route types.

Nodes

The model consists of four sets of nodes. The first set of nodes includes all stores (N^s). All stores have demand which needs to be transported from the CDCs (N^{cdc}) and an RDC (N^{rdc}). Finally, all sets of nodes are combined into a complete set of nodes (N).

Arcs

The model consists of ten sets of arcs. The CDCs are linked to all RDCs. The number of arcs from the CDCs to the RDCs equals: $|A^{dc^+}|= |A^{dc^-}|= |N^{cdc}|\cdot|N^{rdc}|$. All stores are linked to one RDC. The number of arcs from the RDCs to the stores equals: $|A^{rdcs^+}|= |A^{rdcs^-}|= |N^s|$. All stores are linked to both CDCs. The number of arcs from the CDCs to the stores equals: $|A^{cdcs^+}|= |A^{cdcs^-}|= |N^{cdc}|\cdot|N^s|$. The stores are linked only if they are connected to the same RDC. Therefore, the total number of store arcs can not be determined based on a generic formula. The number of arcs between stores which are supplied by the same RDC equals: $|A^s|= |N^s|^2-|N^s|$. There is a complete set of arcs, which consists of all other sets of arcs (A). Finally, there are two sets of arcs which contain the outward ($N^+(i)$) as well as the inward ($N^-(i)$) arcs of a specific node i. These sets contain $|N|$ sets of arcs of which the sets of arcs differ in size. The size depends on the number of arcs linked to node i.

Commodities

In a system containing two CDCs, all stores have three commodities linked to them: the first one is demand from the first CDC, the second one is demand from the second CDC, and the third one is demand from the RDC. The commodities are defined as $100000 +$ store ID, $200000 +$ store ID and $300000 +$ store ID respectively. Therefore, the number of commodities equals $|P|= 3 \cdot |N^s|$.

Vehicles

The set of vehicles is determined based on the volumes of the commodities per location and the vehicle capacity of vehicle type 1 (the vehicle used for transportation from RDCs to stores). The number of vehicles equals $|V|= \sum_{i\in N^s} \frac{\sum_{p\in(P:i=d^p)} vol^p}{cap^1}$. This number assumes the extreme situation where everything is delivered via the RDC, and the number of vehicles is rounded up per store. The set of vehicle types consists of three types, so $|C|= 3$. Vehicle type 0 transports demand from a CDC to an RDC, vehicle type 1 transports demand from an RDC to a store and vehicle type 2 transports demand from a CDC to a store.

Routes

The set of route types consists of two types, so $|Q|= 2$. Route type 0 is for indirect delivery from a CDC, so the demand is first transported to an RDC. Route type 1 is for direct delivery from a CDC to a store.

4.3.2. Parameters

The parameters of this model can be divided into three categories. The first category contains all parameters regarding the commodities. The second category includes all parameters regarding the vehicles. The third and last category contains parameters regarding the routes.

Commodities

Commodities consist of a volume (vol^p), origin (o^p) and destination(d^p). Direct delivery from one of the CDCs to a store can be prohibited for several reasons. If direct delivery is prohibited, this is indicated with a 1. This is determined per commodity (rdd^p).

Vehicles

The capacity of a vehicle is set per vehicle per type (cap^{vc}). In other words, the maximum capacities are assigned per type of vehicle for each vehicle number. The transportation cost per kilometre (tc^c), transportation cost per hour (hc^c), fixed loading time (flt^c), variable loading time (vlt^c), fixed unloading time (fut^c) and variable unloading time (vut^c) are set per vehicle type. Therefore, there are three values per parameter. The times are, again, converted to a number of hours. The delivery costs from one of the CDCs to one of the RDCs (fc_{ij}) are fixed per combination of DCs. The processing costs (pc) per unit of demand for commodities transported from a CDC to a store via an RDC is a fixed value and the same for all locations. Lastly, end-of-route costs (erc) are accounted for using vehicle type 2.

Routes

Distances are determined for all arcs (dis_{ij}). These distances are based on actual distances and not as the crow flies distances. Travel times are also determined for all arcs ($times_{ij}$). These times are based on actual travel times for the given distances. The times are converted to a number of hours, so $01:23:12$ becomes 1.39 hours.

4.3.3. Objective

Objective: Minimize the total sum of costs. The total costs consist of a fixed cost for delivery between a CDC and an RDC, a variable processing cost based on the size of the demand from the CDC at the RDC, a variable operating cost based on the distance driven for delivery from the DCs to the stores, a variable operating cost based on the travel time for delivery from the DCs to the stores, a fixed cost for the loading of a vehicle at DCs, a fixed cost for the unloading of a vehicle at a store, a variable costs for the loading and unloading of a vehicle per unit of demand and a fixed cost for the end of a route of vehicle type 2.

$$
\begin{aligned}
min \sum_{(i,j)\in A^{dc+}} \sum_{v\in V} fc_{ij} \cdot y_{ij}^{v0} + \sum_{(i,j)\in A^{dc+}} \sum_{p\in P} \sum_{v\in V} pc \cdot x_{ij}^{pv0} + \\
\sum_{(i,j)\in (A^{rdcs+}\cup A^{rdcs-}\cup A^{s})} \sum_{v\in V} (tc^1 \cdot dis_{ij} + hc^1 \cdot time_{ij}) \cdot y_{ij}^{v1} + \sum_{(i,j)\in A^{rdcs+}} \sum_{v\in V} hc^1 \cdot flt^1 \cdot y_{ij}^{v1} + \\
\sum_{(i,j)\in (A^{rdcs+}\cup A^{s})} \sum_{v\in V} hc^1 \cdot fut^1 \cdot y_{ij}^{v1} + \sum_{(i,j)\in A^{rdcs+}} \sum_{p\in P} \sum_{v\in V} hc^1 \cdot (vlt^1 + vut^1) \cdot x_{ij}^{pv1} + \\
\sum_{(i,j)\in (A^{cdcs+}\cup A^{s})} \sum_{v\in V} (tc^2 \cdot dis_{ij} + hc^2 \cdot time_{ij}) \cdot y_{ij}^{v2} + \sum_{(i,j)\in A^{cdcs+}} \sum_{v\in V} hc^2 \cdot flt^2 \cdot y_{ij}^{v2} + \\
\sum_{(i,j)\in (A^{cdcs+}\cup A^{s})} \sum_{v\in V} hc^2 \cdot fut^2 \cdot y_{ij}^{v2} + \sum_{(i,j)\in A^{cdcs+}} \sum_{p\in P} \sum_{v\in V} hc^2 \cdot (vlt^2 + vut^2) \cdot x_{ij}^{pv2} + \\
\sum_{(i,j)\in A^{cdcs+}} \sum_{v\in V} erc \cdot y_{ij}^{v2}
\end{aligned}
\tag{4.1}
$$

Sets		Description
Nodes	N^s	Set of store nodes
	N^{rdc}	Set of Regional Distribution Centre nodes
	N^{cdc}	Set of Central Distribution Centre nodes
	N	Set of all nodes ($N^s \cup N^{rdc} \cup N^{cdc}$)
Arcs	A^{dc^+}	Set of arcs from CDCs to RDCs
	A^{dc^-}	Set of arcs from RDCs to CDCs
	A^{cdcs^+}	Set of arcs from CDCs to stores
	A^{cdcs^-}	Set of arcs from stores to CDCs
	A^{rdcs^+}	Set of arcs from RDCs to stores
	A^{rdcs^-}	Set of arcs from stores to RDCs
	A^s	Set of arcs between stores
	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s$)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i
Commodities	P	Set of commodities
Vehicles	V	Set of vehicles
	C	Set of vehicle types
Routes	Q	Set of route types $\{\text{indirect delivery}, \text{direct delivery} : 0, 1\}$
Parameters		**Description**
Commodities	vol^p	Volume of commodity p
	o^p	Origin of commodity p
	d^p	Destination of commodity p
	rdd^p	Indicates whether direct delivery of commodity p is permitted
Vehicles	cap^{vc}	Capacity of vehicle of type c
	tc^c	Transportation cost per kilometer of vehicle of type c
	hc^c	Transportation cost per hour of vehicle of type c
	flt^c	Fixed loading time of vehicle of type c
	vlt^c	Variable loading time of vehicle of type c
	fut^c	Fixed unloading time of vehicle of type c
	vut^c	Variable unloading time of vehicle of type c
	fc_{ij}	Fixed cost for delivery between a CDC i and an RDC j
	pc	Processing costs per unit of demand from CDC at RDC
	erc	End of route costs for vehicle v of type 2
Routes	dis_{ij}	Distance between node i and j
	$time_{ij}$	Travel time between node i and j
	M	Very large number
Decision variables		**Description**
	x_{ij}^{pvc}	Non-negative real number representing the demand volume of commodity p transferred on arc (i,j) by vehicle v of type c
	y_{ij}^{vc}	Binary variable, 1 if vehicle v of type c is selected for design arc (i,j), 0 otherwise
	z_i^{pq}	Binary variable, 1 if either commodity p is transported from a CDC to an RDC (z_i^{p0}) or if commodity p is transported from a CDC to a store (z_i^{p1}), 0 otherwise

Table 4.1: Sets, parameters and decision variables

4.3.4. Design constraints

Constraint: Design balance conservation.

$$\sum_{j \in N^+(i)} y_{ij}^{vc} - \sum_{j \in N^-(i)} y_{ji}^{vc} = 0 \quad \forall i \in N, v \in V, c \in C \tag{4.2}$$

Constraint: Vehicle type 1 may only start from one of the RDCs.

$$\sum_{i \in N^{rdc}} \sum_{j \in N^+(i)} y_{ij}^{v1} \leq 1 \quad \forall v \in V \tag{4.3}$$

Constraint: Vehicle type 1 cannot enter or leave a CDC.

$$\sum_{(i,j) \in (A^{dc+} \cup A^{dc-} \cup A^{cdcs+} \cup A^{cdcs-})} y_{ij}^{v1} = 0 \quad \forall v \in V \tag{4.4}$$

Constraint: Vehicle types 0 and 2 may only start from one of the CDCs.

$$\sum_{i \in N^{cdc}} \sum_{j \in N^+(i)} y_{ij}^{vc} \leq 1 \quad \forall v \in V, c \in [0, 2] \tag{4.5}$$

Constraint: Vehicle type 0 cannot drive from a DC to a store.

$$\sum_{(i,j) \in (A^{cdcs+} \cup A^{cdcs-} \cup A^{rdcs+} \cup A^{rdcs-} \cup A^{s})} y_{ij}^{v0} = 0 \quad \forall v \in V \tag{4.6}$$

Constraint: Vehicle type 2 cannot drive to an RDC and, therefore, drive not from an RDC to a store.

$$\sum_{(i,j) \in (A^{dc+} \cup A^{dc-} \cup A^{rdcs+} \cup A^{rdcs-})} y_{ij}^{v2} = 0 \quad \forall v \in V \tag{4.7}$$

Constraint: Restrict the maximum number of vehicles allowed to deliver at a store. This maximum is based on the volume of demand and the number of DCs it is supplied from. For example, when a store is supplied from both CDCs directly, and an RDC, and the volume per DC is less than the capacity of a vehicle, the maximum number of vehicles to deliver equals three. So, the number of deliveries is based on the number of DCs and the minimum number of vehicles needed to transport all demand, taking the capacity of the vehicles into account.

$$\begin{aligned} \sum_{i \in N^-(j)} \sum_{v \in V} \sum_{c \in C} y_{ij}^{vc} \leq \sum_{i \in N^{cdc}} \sum_{p \in (P: i = o^p, j = d^p)} z_i^{p1} \cdot \frac{vol^p}{cap^{0\,2}} + \\ \frac{\sum_{i \in N^{cdc}} \sum_{p \in (P: i = o^p, j = d^p)} (1 - z_i^{p1}) \cdot vol^p + \sum_{p \in (P: o^p \in N^{rdc}, j = d^p)} vol^p}{cap^{0\,1}} + 0.99 \quad \forall j \in N^s \end{aligned} \tag{4.8}$$

4.3.5. Flow constraints

Constraint: The flow conservation constraint ensures that the commodity flow on all incoming arcs of a node equals the commodity flow of all outgoing arcs. There are two exceptions: the origin node and the destination node of a commodity. At these places, the commodity flow equals the volume of the demand.

$$\sum_{j \in N^+(i)} \sum_{v \in V} \sum_{c \in C} x_{ij}^{pvc} - \sum_{j \in N^-(i)} \sum_{v \in V} \sum_{c \in C} x_{ji}^{pvc} = \begin{cases} vol^p, & i = o^p \\ -vol^p, & i = d^p \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N, p \in P \tag{4.9}$$

Constraint: Make sure that a commodity can not switch from a vehicle.

$$\sum_{j \in N^+(i)} x_{ij}^{pvc} - \sum_{j \in N^-(i)} x_{ji}^{pvc} = 0 \quad \forall i \in (N^{cdc} \cup N^s : i \neq o^p, i \neq d^p), p \in P, v \in V, c \in C \tag{4.10}$$

Constraint: Satisfy all demands.

$$\sum_{i \in N^-(d^p)} \sum_{v \in V} \sum_{c \in C} x_{id^p}^{pvc} = vol^p \quad \forall p \in P \tag{4.11}$$

Constraint: Demand can not be sent back to a DC.

$$\sum_{p \in P} x_{ij}^{pvc} = 0 \quad \forall (i,j) \in (A^{dc^-} \cup A^{cdcs^-} \cup A^{rdcs^-}), v \in V, c \in C \tag{4.12}$$

Constraint: The total volume of a commodity must be delivered via the same DCs while considering split deliveries. So, if the commodity is originated at a CDC, it can be delivered from the CDC directly or via one of the RDCs. Equation 4.13 checks whether a (part of) commodity p is transported from a CDC to an RDC. Equation 4.14 indicates whether direct delivery from a CDC is permitted. Equation 4.15 checks whether a (part of) commodity p is transported from a CDC to a store. Equation 4.16 makes sure that a commodity is either delivered directly or indirectly from the CDC.

$$\sum_{v \in V} x_{ij}^{pv0} \leq M \cdot z_i^{p0} \quad \forall i \in N^{cdc}, j \in N^{rdc}, p \in P \tag{4.13}$$

$$rdd^p \leq z_i^{p0} \quad \forall i \in N^{cdc}, p \in P \tag{4.14}$$

$$\sum_{j \in N^s} \sum_{v \in V} x_{ij}^{pv2} \leq M \cdot z_i^{p1} \quad \forall i \in N^{cdc}, p \in P \tag{4.15}$$

$$z_i^{p0} + z_i^{p1} = 1 \quad \forall i \in (N^{cdc} : i = o^p), p \in P \tag{4.16}$$

4.3.6. Design and flow integrated constraints

Constraint: A service's capacity cannot be exceeded. Therefore, if one or multiple commodities are transported by a vehicle, they must be big enough to carry them.

$$\sum_{p \in P} x_{ij}^{pvc} \leq cap^{vc} \cdot y_{ij}^{vc} \quad \forall (i,j) \in A, v \in V, c \in C \tag{4.17}$$

Constraint: A vehicle may not leave a DC without a commodity.

$$y_{ij}^{vc} \leq \sum_{p \in P} x_{ij}^{pvc} \quad \forall (i,j) \in (A^{dc^+} + A^{cdcs^+} + A^{rdcs^+}), v \in V, c \in C \tag{4.18}$$

4.3.7. Variable constraints

$$x_{ij}^{pvc} \geq 0 \quad \forall (i,j) \in A, p \in P, v \in V, c \in C \tag{4.19}$$

$$y_{ij}^{vc} \in \{0,1\} \quad \forall (i,j) \in A, v \in V, c \in C \tag{4.20}$$

$$z_i^{pq} \in \{0,1\} \quad \forall i \in N^{cdc}, p \in P, q \in Q \tag{4.21}$$

4.4. Implementation

The model is translated to code using the Spyder software. The code is written in the Python language. First, all data is converted from several Excel files to the correct format of the sets and parameters. Second, the converted data is input for the model described in Section 4.3. The model is solved using Gurobi. This process is executed several times for the different verification steps and eventually for the case study. An HP ZBook Studio G4 with an Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz 2.81 GHz and 16 GB of Random-Access Memory (RAM) is used as hardware for running the model. The computational limit linked to this RAM is described in Section 4.6.1.

4.5. Verification

A verification is performed to see if the model is a correct conversion of the real-world problem. Different verification steps focus on another part of the model to check this. First, the model's outcomes are compared with self-calculated outcomes for a small-scale network. Second, two parameters are changed, and the model's behaviour is assessed. Third, the store arcs have been added to check for demand consolidation. Finally, a large-scale network is inserted to see if the model performs as expected.

4.5.1. Step 1. Output

The first step of the verification consists of a check of the output values. The output results of the model that are manually calculated are the total costs, total distance driven by all vehicles and total number of vehicles used for delivery. These values are checked for a small network which consists of two CDCs, two RDCs and three stores, Table 4.2. To limit the number of outcomes, there are no arcs between stores. Section B.1 contains all input data of this verification step.

Sets	
N^s	{3001, 3002, 3003}
N^{cdc}	{10001, 10002}
N^{rdc}	{20001, 20003}
N	$N^s \cup N^{rdc} \cup N^{cdc}$

Table 4.2: Set of nodes step 1

All possible results are given in Table B.5. The optimal solution from the model consists of a total cost of €$1,098$, a total distance of 534 km and a total number of vehicles of 4. The same optimal solution results from manually calculating all possible solutions, Table 4.3.

Description	Optimal solution
Total costs [€]	$1,098$
Total distance [km]	534
Total number of vehicles [-]	4
CDC 10001 to store 3001	Indirect
CDC 10002 to store 3001	Indirect
CDC 10001 to store 3002	Indirect
CDC 10002 to store 3002	Indirect
CDC 10001 to store 3003	Indirect
CDC 10002 to store 3003	Indirect

Table 4.3: Optimal solution step 1

The routes of the vehicles and the routes of the commodities are tracked to discover any inaccuracies in the outcome. The vehicle routes are given in Table B.6 and the commodity routes in Table B.7. No inaccuracies were found.

4.5.2. Step 2. Parameters

Now that it is established that the output values are calculated correctly, the behaviour of the model can be studied of this same small network in step 2, Table 4.2.

Increasing fc_{ij}

Currently, the most optimal solution is to deliver indirectly from both CDCs. To check the behaviour for this choice, the fixed costs for delivery between the CDCs and RDCs (fc_{ij}) can be increased to an amount at which the model should switch to direct delivery. The fc_{ij} values are increased by a factor of 10 to make sure the model should switch from indirect delivery to direct delivery, Table B.8.

All possible results are given in Table B.9. The optimal solution from the model consists of a total cost of €$1,439$, a total distance of 515 km and a total number of vehicles of 4. The same optimal solution results from manually calculating all possible solutions, Table 4.4. The expectation is met while all demand from the CDCs is delivered directly.

Description	Optimal solution
Total costs [€]	$1,439$
Total distance [km]	515
Total number of vehicles [-]	4
CDC 10001 to store 3001	Direct
CDC 10002 to store 3001	Direct
CDC 10001 to store 3002	Direct
CDC 10002 to store 3002	Direct
CDC 10001 to store 3003	Direct
CDC 10002 to store 3003	Direct

Table 4.4: Optimal solution step 2 - increasing fc_{ij}

Increasing the demand

The behaviour for the direct and indirect delivery from the CDCs can be investigated by increasing some of the demand. The demand from CDC 10001, and from the RDCs is increased by a factor of 10. The expectation is that the demand from this CDC is delivered directly and indirectly from the other CDC.

All possible results are given in Table B.13. The optimal solution from the model consists of a total cost of €$6,567$, a total distance of $3,066$ km and a total number of vehicles of 18. The same optimal solution results from manually calculating all possible solutions, Table 4.5. The expectation is met while all demand from CDC 10001 is delivered directly and from CDC 10002 indirectly.

Description	Optimal solution
Total costs [€]	$6,567$
Total distance [km]	$3,066$
Total number of vehicles [-]	18
CDC 10001 to store 3001	Indirect
CDC 10002 to store 3001	Direct
CDC 10001 to store 3002	Indirect
CDC 10002 to store 3002	Direct
CDC 10001 to store 3003	Indirect
CDC 10002 to store 3003	Direct

Table 4.5: Optimal solution step 2 - increasing the demand

4.5.3. Step 3. Adding the store arcs

Adding the store arcs to the model includes the possibility of consolidating demand for delivery to multiple stores, Table B.16 and Table B.17. The expectation is that when the distance between stores is relatively small compared to their distance to a DC, both stores will be supplied by the same vehicle if the capacity allows this. This will result in lower costs and a lower number of vehicles.

The optimal solution from the model consists of a total cost of €$1,086$, a total distance of 532 km and a total number of vehicles of 3, Table 4.6. The expectation is met while the total costs and number of vehicles are lower than the total costs and the total number of vehicles of step 1, equal to €$1,098$ and 4, respectively.

Description	Optimal solution
Total costs [€]	$1,086$
Total distance [km]	532
Total number of vehicles [-]	3
CDC 10001 to store 3001	Indirect
CDC 10002 to store 3001	Indirect
CDC 10001 to store 3002	Indirect
CDC 10002 to store 3002	Indirect
CDC 10001 to store 3003	Indirect
CDC 10002 to store 3003	Indirect

Table 4.6: Optimal solution step 3

The routes of the vehicles and the routes of the commodities are tracked to discover any inaccuracies in the outcome. The vehicle routes are given in Table B.18 and the commodity routes in Table B.19. No inaccuracies were found.

4.5.4. Step 4. Large scale

This step consists of a check of the behaviour of the model on a large scale. The output values are checked for a network which consists of two CDCs, two RDCs and ten stores, Table 4.7. Section B.4 contains all input data of this verification step.

Sets	
N^s	{3004, 3001, 3005, 3006, 3007, 3002, 3003, 3008, 3009, 3010}
N^{cdc}	{10001, 10002}
N^{rdc}	{20001, 20003}
N	$N^s \cup N^{rdc} \cup N^{cdc}$

Table 4.7: Set of nodes step 4

A time limit is 7 hours, which equals $25,200$ seconds. Since the purpose of this step is not to discover the most optimal outcome but to check the operation of the model, a time limit is set. The solution from the model consists of a total cost of €$3,136$, a total distance of $1,090$ km and a total number of vehicles of 12, Table 4.9. This solution is linked to an optimization gap of 17.9%.

The routes of the vehicles and the routes of the commodities are tracked to discover any inaccuracies in the outcome. The vehicle routes are given in Table B.24 and the commodity routes in Table B.25. No inaccuracies were found.

4.5.5. Conclusion

The five verification steps were successful, and no inaccuracies were found in the model results, Table 4.8. With that, it is assumed that the model works properly and can be implemented in the case study.

Description	Successful verification?
Step $1.$ Output	Yes
Step $2.$ Parameters - increasing fc_{ij}	Yes
Step $2.$ Parameters - increasing the demand	Yes
Step $3.$ Adding the store arcs	Yes
Step $4.$ Large scale	Yes

Table 4.8: Overview of verification steps

Description	Solution
Total costs [€]	$3,136$
Total distance [km]	$1,090$
Total number of vehicles [-]	12
CDC 10001 to store 3001	Indirect
CDC 10002 to store 3001	Indirect
CDC 10001 to store 3002	Indirect
CDC 10002 to store 3002	Indirect
CDC 10001 to store 3003	Indirect
CDC 10002 to store 3003	Indirect
CDC 10001 to store 3008	Direct
CDC 10002 to store 3008	Direct
CDC 10001 to store 3009	Indirect
CDC 10002 to store 3009	Indirect
CDC 10001 to store 3010	Indirect
CDC 10002 to store 3010	Indirect
CDC 10001 to store 3005	Indirect
CDC 10002 to store 3005	Indirect
CDC 10001 to store 3006	Indirect
CDC 10002 to store 3006	Indirect
CDC 10001 to store 3007	Indirect
CDC 10002 to store 3007	Indirect
CDC 10001 to store 3004	Indirect
CDC 10002 to store 3004	Indirect

Table 4.9: Solution step 4

4.6. Computation

This model is focused on large networks, and with that, it also becomes very complex. Therefore, an understanding of computational complexity is given first. Then, the Gurobi input parameters are tuned based on the output files of these networks. Improvements in time are achieved due to this tuning of input parameters. Finally, a callback function is formulated to limit the optimization time.

4.6.1. Computational Limits

The model quickly becomes very complex to solve. This is due to the number of variables required. The complexity influences the computational forces needed to solve the model. The goal is to find the optimal solution for the allocation of stores and the routing of vehicles. The Gurobi solver aims to find a solution in which the lower and upper bound of the solution is equal to 0%. Although the optimality gap can be adjusted, the problem is still very complex to solve. The more complex the model, the longer it takes to solve it, and the more RAM is needed to store the values. Several elements influence computational forces. The optimization in Gurobi is based on certain variables. Five variable types can be implemented (Gurobi, 2022a): continuous, general integer, binary, semi-continuous, and semi-integer. Continuous variables between the lower and upper bound can take any value. General integer variables can also take integral values but are more constrained than continuous variables. Binary variables take either the value 0 or 1. Semi-continuous and semi-integer variables can take 0 as a value or a value between the lower and upper bound. Binary variables take the least memory and continuous variables the most. There is one general integer, and there are two binary variables used in the model. The number of variables needed to solve the problem can be determined based on the indices of the different variables and can be calculated with Equation 4.22.

$$\text{number of variables} = |A|\cdot|P|\cdot|V|\cdot|C|+|A|\cdot|V|\cdot|C|+|N^{cdc}|\cdot|P|\cdot|Q| \tag{4.22}$$

To give an insight into the complexity of this model, a network consisting of 2 CDCs, 4 RDCs and 100 stores is chosen. This network results in $832 \cdot 10^6$ variables needed to solve the problem. A problem containing this number of variables can be interpreted as a very complex model.

$$\text{number of variables} = 829,013,130 + 2,791,290 + 1,188 = 831,805,608 \tag{4.23}$$

A computational limit for the hardware that contains 16 GB RAM is found for a model containing an approximate number of variables of $27 \cdot 10^6$ in total.

4.6.2. Start solution

In the Gurobi optimiser, it is possible to add a start vector (Gurobi, 2022b). The MIP solver will then try to build an initial solution. Adding this vector can save time searching for the initial solution and possibly reduce the optimisation area. It is possible to add parts or all of a start vector. If the entire vector is added, a choice has thus been made in advance in the z_i^{p0} variable, and a choice has thus been made whether a CDC supplies a store directly or indirectly. This vector is added and can be enabled at will.

4.6.3. Input parameters

When optimizing using the Gurobi tool, there are default input parameters set that affect this process. These parameters can be adjusted to match the model better. This model can be characterized as Mixed Integer Programming (MIP). Before changing them, it is essential to have a reference situation. Therefore, optimization times are determined for various network sizes. All networks consist of 2 CDCs, 1 RDC and a certain number of stores, Table 4.10. These run times correspond to a gap of 0%. The networks contain no store arcs due to the high solving time. The numbers of variables are calculated using Equation 4.22. All stores have the same demand from the three DCs. They are supplied directly from the first CDC and indirectly, so via the RDC, from the second CDC.

Num. of stores [-]	**Num. of variables [-]**	**Num. of constraints [-]**	**Computational time [s]**
1	$1,068$	743	0.04
5	$44,220$	$11,455$	0.58
10	$282,840$	$58,460$	3.59
20	$1,991,280$	$367,120$	26.36

Table 4.10: Computational times before tuning the Gurobi input parameters

The parameters are tuned in two steps. First, the Gurobi tuning tool is used to find the first improvements that can be made regarding the input parameters. Second, the input parameters are adjusted based on information supplied by Gurobi.

Two parameters are set for the Gurobi tuning tool. The first one is the time limit, and the second one is the number of seeds. The time limit is set to -1 so the tuning tool can determine a proper tuning time (Gurobi, 2022c). The number of seeds is set to 3, so everything is checked with three sets of values (Gurobi, 2022d). This reduces the chance of giving parameter values that only work well for a single input data set. The Gurobi tuning tool resulted in one improved parameter set, Appendix C.0.2. The optimal input parameter is stated in Table 4.11.

Input parameter	**Value [-]**
Heuristics	0.0001

Table 4.11: Input parameters Gurobi tuning tool

The heuristics input parameter determines the time spent on heuristics (Gurobi, 2022e). More and better feasible solutions are often received using a significant heuristic time value. These parameter adjustments result in a decrease in solving time, Table 4.12. These run times correspond to a gap of 0%.

Num. of stores [-]	Num. of variables [-]	Num. of constraints [-]	Computational time [s]
1	$1,068$	743	0.03
5	$44,220$	$11,455$	0.36
10	$282,840$	$58,460$	3.71
20	$1,991,280$	$367,120$	24.27

Table 4.12: Computational times after tuning with the Gurobi tool

Several other input parameters are selected based on the information supplied by Gurobi. Combinations of these parameters are used to determine what combination of parameters has a positive influence on the computational time. A selection of these parameters is made based on their improvements compared to the computational times of the tuning tool, Table 4.13.

Input parameter	Value [-]
Heuristics	0.0001
Cuts	2
MIPFocus	1
PreSparsify	2

Table 4.13: Input parameters manual tuning

The global cut control focuses on the creation and application of global cutting planes, which are effective tools for enhancing the problem's linear relaxation and enlarging the range of possible solutions (Gurobi, 2022f). The aggressive cut generation (2) seems best for this model. Depending on the objectives, the high-level solution strategy can be adjusted using the MIPFocus parameter (Gurobi, 2022g). The solver finds a balance between locating new solutions and proving that the existing solution is the best. Setting this to 1 enables finding workable solutions rapidly. The presolve sparsify reduction can potentially reduce the number of non-zero values in the presolved model (Gurobi, 2022h). All model types must execute this step when the setting is set to 2. These parameter adjustments result in a decrease in solving time, Table 4.14. These run times correspond to a gap of 0%.

Num. of stores [-]	Num. of variables [-]	Num. of constraints [-]	Computational time [s]
1	$1,068$	743	0.05
5	$44,220$	$11,455$	0.71
10	$282,840$	$58,460$	2.18
20	$1,991,280$	$367,120$	11.88

Table 4.14: Computational times after manual tuning

Some other input parameters were tested but did not result in a lower computational time. The method is the algorithm for the initial root relaxation of a MIP model (Gurobi, 2022i). The barrier method (2) seemed best. The model can be presolved to make it smaller and easier to solve (Gurobi, 2022j). The conservative solve setting (1) seemed best, while the aggressive setting (2) may lead to a tighter model. The value represents the maximum number of cutting plane passes made during the production of the root cut (Gurobi, 2022k). Setting this option to 3 seemed the best for large distribution systems. The cover cut generation can be controlled, and the best setting seemed to be a moderate cut generation (1) for this model (Gurobi, 2022l).

4.6.4. Callback

In some cases, the Gurobi tool puts an extreme amount of time into proving the optimal answer but with little development in the outcome. Here, the gap decreases in minimal quantity, as does the incumbent solution. To avoid taking up an extreme amount of time for this in significant optimisations, a callback function is implemented that avoids this (Gurobi, 2022m). The callback function keeps track of time and gaps. If the gap development is greater than 1%, the optimiser saves the iteration time. If the development in the gap is smaller than 1%, the tool does not do so. The optimiser gets terminated if the last iteration time is longer than 20 min ago, i.e. if the change in gap development has been smaller than 1% for longer than 20 min. After this, these outcomes are included as results of this optimisation.

4.6.5. Conclusion

This section reflects the complexity of the model. The number of variables rapidly increases dramatically, which can also quickly reach the hardware limit. A start solution can be added to save time and reduce the optimisation area. Also, the input parameters of the model have been changed to better suit its implementation. For this purpose, the heuristic, cuts, MIP focus and pre-sparsify have been set. Finally, a callback functionality is used that reduces optimisation time. All these methods together improve the computation.

5 Case study

A case study at a company in the Netherlands is provided in this chapter. This case fits the generic distribution problem, as described in the system analysis, Chapter 2, and specifies data and configuration according to the company's distribution network. The fourth research question can be answered based on the case study of this chapter:

What is the performance of the system, given the data from the case study?

First, the case study is explained in detail. Here, it focuses mainly on the characteristics of the distribution system. Based on these characteristics, insight is given into the data belonging to the case study. The data is presented per a group of sets, parameters and decision variables. Next, a computational plan is formulated. This plan examines several cases with different computational improvements. This computational plan shows that the adjustments to the Gurobi optimization tool have a positive influence on the run time. Then, the experimental plan is introduced. It consists of a base case, validation of the base case, and five experimental cases. Finally, the experimental plan cases are compared and a relationship between routing decisions, distances and volumes is sought. The base case represents the company's current situation. It uses the existing data as input for the model, but also the company's route choices. Next, the outcome of this case is validated with the company's calculations concerning the KPIs regarding the same input. There is a difference in the main KPI, the total costs of transport of 7.3%. This difference is expected to be the result of the callback function that is used and the clustering of the data. The callback function stops the optimisation when a particular criterion is met. A difference between the incumbent and best-bound solution is the result. The clustering disables the opportunity of consolidating the demand. This leads to less efficient transport, while the truckload is expected to be lower. Based on this, it is assumed that the model performs correctly. Five experimental cases follow the validation. The first case is an optimisation based on the current data. Here, only the route choices are redefined by the model. The first case has an improvement on the total costs of transport of 5.3% compared to the base case. The two other KPIs, the total distance and total number of vehicles, perform slightly less than the base case. The other four metrics perform better. This indicates that the routing decisions of this case are better than those of the company. This also suggests that optimisation is a better approximation for route choices than the current models. The second case introduces the possibility of consolidation of demand at the CDCs. It is expected that it will become more attractive to consolidate demand at the CDCs and then deliver directly rather than consolidate at RDCs. The metrics of this case are all positive and so an improvement compared to the base case. In addition, it is also an improvement on the first case because all the results are an improvement. In the third case, transport costs between CDCs and RDCs are reduced five times. This reduction reflects the current developments regarding the increase in the costs of last-mile delivery and the possibility of making shuttle journeys cheaper and more efficient because this is owned and regulated by the company. This case provides insight into the behaviour of the model concerning route choices. The model behaves as expected. When transportation costs from the CDCs to the RDCs reduce, the number of direct deliveries decreases and the number of indirect deliveries increases. Then, the fourth experiment considered is changing the volume. Since the volumes of stores vary continuously, it is essential to discover how this affects the route choices made. Again, this case provides insight into the model's behaviour concerning route choices. When the volume decreases, the number of direct deliveries decreases and the number of indirect deliveries increases. When the volume increases, the number of direct deliveries increases, and the number of indirect deliveries decreases. Finally, an attempt is made to solve the model without clustering for a more realistic outcome. For this, the model is converted from the Gurobi to the PuLP tool and the company's server is used to run the code. Unfortunately, the server also proved to be not powerful enough. As a result, it was impossible to compare the performance of clustering with the situation where the data was not clustered. The robustness of the model was also examined to analyse the behaviour of the outcomes. This showed that the model is sensitive to changes in volume regarding the performance metrics but not really sensitive regarding the routing decisions. The analysis also showed that the model is not really sensitive to changes in fixed costs (fc_{ij}) regarding the performance metrics but that it is sensitive to changes in routing decisions. The computational values were also tracked to understand the performance of the optimisation. The average run time per case equals 29 hours. The results are compared from three perspectives. First, they are compared based on the performance metrics. The main KPI is the most important metric to compare the cases on, but the other metrics need to be considered, too. Consolidation of demand at the CDCs is proven to be a successful improvement, while an improvement in costs of 5.4% can be achieved. The total costs influence the third and fourth cases, so these cases can not be compared with the others. What can be concluded from these cases is that the metrics of both perform as expected. When lowering the total costs of transportation between the CDCs and RDCs, the third case, the percentage of indirect delivery will increase. When increasing the volume, in the fourth case, the direct delivery increases too. And when decreasing the volume, the direct delivery decreases too. Second, the results are compared based on their routing decisions. In some cases, indirect delivery is more attractive, and in others, direct delivery. There is a difference in the postal codes with the most significant influence on the routing decisions. Besides this fact, two postal codes pop up in all cases as postal codes with a considerable influence on the total costs, considering a difference in routing decisions. These postal codes are 48 and 94. Therefore, it is advised to change the routing decisions of these postal codes. Third, the relationship between routing decisions, distances and volumes is discovered. As a result, there are no convincing relationships between the routing decisions and distances on its own and between the routing decisions and volumes. However, a relationship between all three can be determined using a logistic regression. Here, with an accuracy between 63.1% for the second case and 78.0% for the third case, advice can be given for the routing decision based on the distance of a store to the CDCs, the distance of a store to an RDC plus the distance of this RDC to the CDCs, and the volume of demand of this store.

5.1. General description

A case study is carried out for the distribution system of a company in the Netherlands. The company is active in the retail industry, with several DCs and many stores. New locations are opened every year, resulting in a continuous need for improvement of the transport and logistics network. To continue to meet growing demand, processes need to be made more efficient, and automation plays a major role in this. Therefore, a fully automated distribution centre will be opened. A large proportion of all products will be stored in this CDC. The CDC will deliver slow-movers, which consist of products with a particular shelf life. The RDCs will deliver fast movers, while the delivery time needs to be as low as possible. A store will receive demand from both CDCs and from one of the RDCs. Now, the question is whether it is more cost-effective to first transport demand from the CDC to the RDC before delivery to the stores. The locations are known in advance and include two CDCs, four RDCs and a large number of stores. The goal is to allocate the stores to one or multiple DCs. While the stores will always be supplied from an RDC, the main goal is to determine whether or not they are supplied from the CDC directly or whether this demand is consolidated at an RDC and transported from there. The order sizes of the stores are different per day of delivery. While the company is interested in a fixed allocation of the stores to the DCs, which may not differ per day, the demand for a representative day is considered for the optimization. This is for ease of simplification in the model. Currently, there are $16 \cdot 10^3$ containers routed through the retail network. The transportation cost for this container volume amounts to €$135 \cdot 10^3$. The overall transportation costs can be categorized into three sections: routing costs from CDCs to RDCs, routing costs from CDCs to stores, and routing costs from RDCs to stores.

The shuttle costs, which refer to the routing costs from CDCs to RDCs, amount to €$14 \cdot 10^3$. Additionally, the transportation costs from CDCs to stores sum up to €$46 \cdot 10^3$, while the transport costs from RDCs to stores reach €$76 \cdot 10^3$. Consequently, the average transportation cost per unit of demand is calculated as €3.09. Based on these figures, it can be concluded that 30% of stores are directly supplied by the first CDC, while 85% receive their supplies directly from the second CDC.

5.2. Data

This section provides an overview of the data used for the case study. It should be noted that this data is used as a starting point, but the data can be altered for different scenarios.

5.2.1. About the data

The data provided by the company is based on a representative day. The aim is to find out which stores are supplied directly or indirectly by the CDCs. This allocation will be the same for every day of the week, so it is essential to assume a representative day. The data is based on all stores and includes node, arc, commodity, vehicle and route sets, and commodity, vehicle and route parameters, Table 4.1.

5.2.2. Complexity of the model

The model used in the case study quickly becomes very complex, as stated in Chapter 4.6.1. This is due to the size of the problem and, therefore, the amount of variables used in the model. The number of variables needed to solve the problem can be determined based on the indices of the different variables and can be calculated with Equation 4.22.

$$\text{number of variables} \;=\; 1,659,549,691,800 \,+\, 773,682,840 \,+\, 8,580 \;=\; 1,660,323,383,220 \tag{5.1}$$

A problem containing $2 \cdot 10^{12}$ variables can be interpreted as a very complex model, and simplification is needed to solve this. Simplification of this problem is achieved via clustering of data, Section 5.2.3.

5.2.3. Clusters

The distribution network is clustered two times due to the complexity. First, all data is divided into four groups, one group per RDC. The stores (N^s) are assigned to one of the groups based on the allocation. While all stores are supplied from both CDCs (N^{cdc}), these are left out of this clustering. Second, all locations within the RDC cluster are grouped based on their postal code. Postal codes in the Netherlands have the following format: 1234AB. First, The Netherlands is divided based on the first two digits of the postal codes (12). Second, the areas containing the same first two digits of the postal codes are divided even more by the second two digits (34). A final division is achieved based on the two letters assigned to the postal codes (AB). The first two digits are used to group all locations of this distribution network. A small set of locations is not clustered based on their postal code due to their size of demand. A total of 14 locations are left out of the clusters. Clustering these locations would have too big of an influence on optimising locations in the same postal code area. So, all locations are assigned to an RDC group and a postal group, resulting in a total number of 137 clusters.

5.2.4. Sets

The sets of this case study are divided into five categories: the nodes, the arcs, the commodities, the vehicles and the routes.

Nodes

The total set of nodes (N) can be split into five categories, among which there are 2 CDCs (10001, 10002), 4 RDCs (20001, 20002, 20003, 20004), and a large number of stores. A distinction is made between regular stores and large stores, which have a significantly larger demand.

Arcs

The first optimization step consists of finding the optimal solution to supply all postal code areas from the dedicated RDC they are assigned to and the CDCs. The A^{rdcs} set consists of arcs between the postal code areas and the RDC from which they are supplied. The set of arcs between the postal code areas (A^s) is empty. In the second step of finding the optimal solution, the A^{rdcs} set consists of arcs between the stores of a postal code area and the RDC from which they are supplied. The set of arcs between the stores in this postal code area (A^s) contains all possible arcs.

Commodities
All stores have three commodities linked to them. These commodities contain information on the orders from the stores to both CDCs and one of the RDCs.

Vehicles
Three types of trucks are considered in this case study. The first type drives between CDCs and RDCs, the second between RDCs and stores and the third between CDCs and stores.

Routes
There are two sets of routes. The first route indicates indirect delivery from a CDC, so the delivery is done via an RDC. The second route indicates direct delivery from a CDC.

5.2.5. Parameters

The parameters of this case study are divided into three categories: the commodities, the vehicles and the routes.

Commodities
All commodities contain a volume, origin and destination. The volume of a store may be 0, while not all stores place an order on each day of the week. There is a chance that direct delivery from a CDC may be restricted due to the location of the store. This may be caused by a store located on an island, an environmental zone in a city, a General Municipal Regulation restriction or simply because a store cannot be reached by a large truck.

Vehicles
All of the company's vehicles have an equal capacity. The transportation cost per kilometre, transportation cost per hour, fixed loading time, variable loading time, fixed unloading time and variable unloading time are set per vehicle type, and they differ from vehicles that are linked to a CDC to those that are linked to an RDC. The processing costs at all DCs are assumed to be equal. A partner executes direct delivery from the CDCs. An end-of-route cost is accounted for because of this.

Routes
The company supplied data containing absolute distances and times between all locations in the network. These distances and times are linked to routes the vehicles can drive and not to the as the crow flies distance between two locations. New distances and times are needed for the postal code areas. These are calculated based on the average value of all locations involved. Various methods have been devised to do this differently. Still, they are mainly suitable for situations where the coordinates are known of all locations and a realistic estimate of distance and subsequent travel time needs to be made (Daganzo, 1984). Since, in this case, the actual distances and times are known, it is assumed that the average of these values sufficiently matches the actual values. It is taken into account that small differences in these distances do not have a significant impact on the outcome of this optimisation step. Here, direct or indirect delivery from the CDC is considered for an entire postal code area, and here, a few kilometres do not have a significant impact on the outcome.

5.2.6. Solving the model

Solving the problem is done in several steps. While the data is clustered two times, the system is solved in five steps. First, the stores are clustered based on the allocation to one of the RDCs. Second, all locations in each RDC cluster are grouped based on the first two digits of the postal codes. Exceptions for these postal code groups are the larger stores. Third, the model is optimized for the postal code clusters per RDC. The result of this optimization step is the allocation of the postal code clusters to the CDCs or the RDCs. This decision is taken into account in step four. Fourth, the model is optimized per postal code of an RDC. The result of this optimization step is the costs, distance and number of vehicles needed for delivery. Benefits can be achieved by consolidating demand from various stores per vehicle. Fifth, the total costs, distance and number of vehicles for the delivery to all clusters can be calculated based on all results of step four.

5.3. Computational plan

A computational plan was drawn up to test the various methods on the company's inputs from Section 4.6. This analysis consists of a base configuration, i.e. without tuning, and four other configurations. Three different sets of input data are compared for each configuration. The second optimization step considered in this plan is creating a set of routes for all stores within one postal.

5.3.1. Base configuration

No adjustments are made to the model for the base configuration. So, no start solution, input parameters and callback are added. A maximum run time limit is set to three hours ($10,800$ s) to limit the computation. The results of the different configurations can be compared by considering the maximum time limit.

		Postal A	**Postal B**	**Postal C**
	RDC	20003	20003	20003
	Number of stores [-]	3	6	9
Obj. bound	Start [€]	-	-	-
	Heuristic [€]	360	$1,308$	$1,974$
	Upper-bound (root relaxation) [€]	330	706	$1,134$
	Incumbent [€]	360	818	$1,339$
	Best-bound [€]	360	818	$1,165$
	Gap [%]	0	0	13.0
Work	Explored nodes [-]	63	$25,698$	$9,673$
	Simplex iterations [-]	$1,440$	$9,750,065$	$15,273,241$
	Run time [s]	0.26	$1,540.60$	$10,800.14$

Table 5.1: Results of base configuration

5.3.2. First configuration, start solution

A start solution is added in the first configuration. This start solution can save much time in the first optimisation phase for large distribution systems. The Gurobi tool can continue to find a better solution based on this initial solution. The maximum time limit is also set for this configuration.

		Postal A	**Postal B**	**Postal C**
	RDC	20003	20003	20003
	Number of stores [-]	3	6	9
Obj. bound	Start [€]	563	$1,476$	$2,366$
	Heuristic [€]	396	$1,308$	$1,974$
	Upper-bound (root relaxation) [€]	330	706	$1,134$
	Incumbent [€]	360	818	$1,339$
	Best-bound [€]	360	778	$1,166$
	Gap [%]	0	4.8	12.9
Work	Explored nodes [-]	40	$119,525$	$13,518$
	Simplex iterations [-]	$1,562$	$73,604,413$	$18,190,849$
	Run time [s]	0.29	$10,800.12$	$10,800.18$

Table 5.2: Results of first configuration

5.3.3. Second configuration, input parameters

Input parameters are added in the second configuration. These parameters can be adjusted to better match the model with the settings of the Gurobi tool. The maximum time limit is also set for this configuration.

		Postal A	Postal B	Postal C
	RDC	20003	20003	20003
	Number of stores [-]	3	6	9
Obj. bound	Start [€]	-	-	-
	Heuristic [€]	-	-	-
	Upper-bound (root relaxation) [€]	330	706	1,134
	Incumbent [€]	360	818	1,338
	Best-bound [€]	360	818	1,236
	Gap [%]	0	0	7.7
Work	Explored nodes [-]	19	3,442	20,694
	Simplex iterations [-]	1,491	463,422	17,632,610
	Run time [s]	0.35	38.00	10,800.11

Table 5.3: Results of second configuration

5.3.4. Third configuration, callback function

A callback function is added in the third configuration. This function terminates the model if the change in gap development has been smaller than 1% for longer than 20 min. The maximum time limit is also set for this configuration.

		Postal A	Postal B	Postal C
	RDC	20003	20003	20003
	Number of stores [-]	3	6	9
Obj. bound	Start [€]	-	-	-
	Heuristic [€]	360	1,308	1,974
	Upper-bound (root relaxation) [€]	330	706	1,134
	Incumbent [€]	360	818	1,339
	Best-bound [€]	360	818	1,150
	Gap [%]	0	0	14.1
Work	Explored nodes [-]	63	25,698	1,163
	Simplex iterations [-]	1,440	9,750,065	2,655,780
	Run time [s]	0.27	1,551.95	2,405.70

Table 5.4: Results of third configuration

5.3.5. Fourth configuration, combination of all configurations

The adjustments of all previous configurations are combined in this configuration. So, a start solution, input parameters and callback are added. The maximum time limit is also set for this configuration.

		Postal A	Postal B	Postal C
	RDC	20003	20003	20003
	Number of stores [-]	3	6	9
Obj. bound	Start [€]	563	1,476	2,366
	Heuristic [€]	396	1,308	1,974
	Upper-bound (root relaxation) [€]	330	706	1,134
	Incumbent [€]	360	818	1,341
	Best-bound [€]	360	818	1,154
	Gap [%]	0	0	14.0
Work	Explored nodes [-]	19	3,286	1,336
	Simplex iterations [-]	1,491	486,649	2,421,573
	Run time [s]	0.44	62.99	2,246.28

Table 5.5: Results of fourth configuration

5.3.6. Comparative results

The start solution added in the first configuration has no significant influence on the results. Besides that, the model has difficulties finding a start solution, which results in a situation where it can take hours or days to find the first solution. The second configuration includes adjustments to the input parameters of the Gurobi tool. It can be noted that the optimization time reduces drastically, especially for the more extensive networks. The gap difference decreases for the extensive network too. A callback function is added in the third configuration. This callback function is helpful for more extensive networks where the gap stabilises more or less after a while. The time limit is reached in the first two configurations and the base configuration, but the run time does not reach its limit in the third configuration. Combining all methods results in the most optimal situation for all types of networks. The start solution is needed to avoid difficulties in finding the first solution, the input parameters are added for a higher solving efficiency, and the callback ensures that the model does not stay around the same gap for a long time. In the end, the gap and run-time are important factors to reduce, and that is achieved in the combination of all methods.

5.4. Experimental plan

The company's distribution network is analyzed in a base case, a validation and five new cases. Optimal results are determined, and insight into the network can be gained from these cases and their scores on the three KPIs from Table 2.1:

1. Total costs of transport [€]
2. Total distance [km]
3. Total number of vehicles [-]

The five new cases contain different scenarios and configurations. A case may consist of a new scenario, a new configuration, or a combination of both. Scenarios are characterized by all the (input) data from external influences (store demands, etc) and configurations by design alternatives (network design, procedures, etc).

5.4.1. Base case

The base case concerns the company's current situation. This situation will serve as a reference for the follow-up scenarios. This will allow us to see if changes in the model are an improvement over the base case and thus result in a more optimal situation that the company can implement. In this process, the KPIs will be set against each other, Section 5.5. The z_i^{pq} decision variable, which indicates whether or not a commodity p is transported from a CDC to an RDC or if commodity p is transported from a CDC to a store, is set by the current decisions of the company.

Mathematical model adjustment

There is one adjustment in the decision variables. The decision variable indicating the routing decisions (z_i^{pq}) is known already and therefore converted to a parameter, Table 5.6.

from

Decision variables		Description
	z_i^{pq}	Binary variable, 1 if either commodity p is transported from a CDC to an RDC (z_i^{p0}) or if commodity p is transported from a CDC to a store (z_i^{p1}), 0 otherwise

to

Decision variables		Description
	z_i^{pq}	Routing decisions by the company, 1 if either commodity p is transported from a CDC to an RDC (z_i^{p0}) or if commodity p is transported from a CDC to a store (z_i^{p1}), 0 otherwise

Table 5.6: Parameters adjustment

Results

The results of the base case are stated in Table 5.7. The total costs of transport for the base case equals €$147 \cdot 10^3$.

	Description	Value
KPIs	Total costs of transport [€ $\cdot 10^3$]	147
	Total distance [km $\cdot 10^3$]	55
	Total number of vehicles [- $\cdot 10^3$]	0.5
	Costs per container [€]	3.35
	Load per vehicle [containers]	15
	Drop size per stop [containers]	8
	Stops per vehicle [-]	0.6

Table 5.7: Results base case

An overview of the results divided per RDC is given in Table 5.8. What can be concluded is that more than half of the demand from the CDCs is sent directly.

	Total costs [€ $\cdot 10^3$]	CDC costs [€ $\cdot 10^3$]	Transfer costs [€ $\cdot 10^3$]	RDC costs [€ $\cdot 10^3$]	Direct [%]	Indirect [%]
RDC 20001	42	14	6	22	57.4	42.6
RDC 20002	38	10	8	19	42.5	57.5
RDC 20003	38	14	3	21	69.8	30.2
RDC 20004	29	9	5	15	54.5	45.5
Total	147	48	22	77	57.4	42.6

Table 5.8: Results per RDC of base case

The routing decisions are divided per CDC and visualized in Figures D.1 and D.2.

5.4.2. Validation

To validate the model, the company's routing decisions are adopted into the model and optimised. The optimal values are compared with the results of the company's models concerning this equal input value. From these models, it is not possible to extract the total distance and total number of vehicles. However, total costs and the proportion of direct and indirect deliveries can be compared.

Results

The total costs are €$135 \cdot 10^3$, Table 5.9. There is a difference in the total costs of €$11 \cdot 10^3$ compared to the base case. As explained earlier, run times are reduced using the callback function. If the best-bound value is adopted as the optimal value, it amounts to €$133 \cdot 10^3$. Here, there is only a difference of €$3 \cdot 10^3$ in the final cost. Based on this and the other analysis results, it is assumed that the results are valid. The main difference can be found in the transfer costs. Higher transfer costs can be a result of optimizing the model in clusters. While the models of the company assume a high load factor per vehicle, this can be lower while the transport from the CDCs to the RDCs can not be combined from different postal codes. More vehicles are assumed to be needed, and therefore, higher transfer costs are linked to this. Other differences can occur quickly because of the difference in the accuracy of calculations. Given the large total demand, small costs can have a large impact, and the discrepancies are assumed to be realistic. The outcomes have also been checked by experts from the company and assumed to be valid. The total distance can not be determined in the company's model and is therefore left out of consideration of this validation.

Description		Value	Difference [%]
KPIs	Total costs of transport [€ $\cdot 10^3$]	135	7.3
	Total distance [km $\cdot 10^3$]	-	-
	Total number of vehicles [- $\cdot 10^3$]	0.5	5.6
	Costs per container [€]	3.09	7.3
	Load per vehicle [containers]	16	5.1
	Drop size per stop [containers]	8	9.1
	Stops per vehicle [-]	0.8	17.6

Table 5.9: Results validation

The main increase in costs is linked to the transfer costs, Table 5.10. This increase is expected due to the given reasoning. The percentages corresponding to the number of direct and indirect deliveries from both CDCs are equal in the base case and the validation.

	Total costs [€ $\cdot 10^3$]	**CDC costs [€ $\cdot 10^3$]**	**Transfer costs [€ $\cdot 10^3$]**	**RDC costs [€ $\cdot 10^3$]**	**Direct [%]**	**Indirect [%]**
RDC 20001	39	14	4	21	57.4	42.6
RDC 20002	35	9	6	20	42.5	57.5
RDC 20003	38	15	2	21	69.8	30.2
RDC 20004	24	7	3	14	54.5	45.5
Total	135	46	14	76	57.4	42.6

Table 5.10: Results per RDC of validation

5.4.3. First case, optimization by new model

The first case can be characterized as a change in configuration and no change in scenario, while the routing decision from the CDCs may differ compared to the base case. There is no other change in the model, so the model's effectiveness compared to the current situation, the base case, can be investigated.

Results

The results of this case are stated in Table 5.7. There is a positive difference in the main KPI value from this configuration compared to the base case. The total costs are €$8 \cdot 10^3$ lower. The KPI values of the total distance and total number of vehicles are slightly higher.

Description		Value	Improvement [%]
KPIs	Total costs of transport [€ $\cdot 10^3$]	139	5.3
	Total distance [km $\cdot 10^3$]	55	-1.2
	Total number of vehicles [- $\cdot 10^3$]	0.5	-1.0
	Costs per container [€]	3.17	5.3
	Load per vehicle [containers]	15	2.6
	Drop size per stop [containers]	10	13.6
	Stops per vehicle [-]	0.6	5.1

Table 5.11: Results first case (a positive percentage equals an improvement)

There is a difference in routing decisions too, Table 5.12. There is a shift from direct to indirect delivery of 18%.

	Total costs [€ $\cdot 10^3$]	**CDC costs [€ $\cdot 10^3$]**	**Transfer costs [€ $\cdot 10^3$]**	**RDC costs [€ $\cdot 10^3$]**	**Direct [%]**	**Indirect [%]**
RDC 20001	40	8	9	22	39.5	60.5
RDC 20002	37	9	8	19	45.8	54.2
RDC 20003	35	8	6	22	44.7	55.3
RDC 20004	27	3	8	16	23.1	76.9
Total	139	29	31	80	39.4	60.6

Table 5.12: Results per RDC of first case

The ten clusters with the biggest cost differences are added to Table 5.13. The route difference percentages are split into the transportation from CDC 10001 and from CDC 10002. The percentages indicate the number of stores with a different route option than the base case compared to the total number of stores in the postal. The cost difference shows the increase or decrease due to the difference in routing options. A positive cost difference means lower costs for the current case. The distance from the postal to the CDCs and from the postal to the RDC is stated too.

Postal [-]	**RDC**	10001 **[%]**	10002 **[%]**	**Cost diff. [€]**	**CDC dist. [km]**	**RDC dist. [km]**
56	20001	0	77.8	732	32	10
48	20004	33.3	100	613	26	2
94	20002	0	87.5	271	64	7
91	20002	0	50.0	232	73	30
62	20001	0	88.9	225	66	44
63	20001	0	33.3	219	66	45
95	20002	0	25	211	75	19
88	20002	0	50	197	60	36
82	20003	33.3	100	187	25	32
27	20003	100	100	181	18	12

Table 5.13: Route option differences first case

The routing decisions are divided per CDC and visualized in Figures E.1 and E.2.

5.4.4. Second case, consolidation of demand at CDCs

The second case can be characterized as a change in configuration and no change in scenario, while the network design differs from the base case. Both CDCs are located next to each other. Currently, the commodities there are not merged due to internal restrictions. It is expected that combining commodities at the CDCs will benefit direct delivery to stores. Stores can then, for example, be supplied with a vehicle that contains products from both DCs. This may eliminate the benefit of consolidating demand at an RDC and potentially change the allocation choice. Demand from both CDCs can be consolidated at CDC 10002. So if the decision is made that the vehicle will deliver both demands, the demand will be first picked up at CDC 10001 and then pick up the demand from CDC 10002.

Mathematical model adjustments

The sets of arcs are extended by including the arcs between the CDCs (A^{cdc^+} and A^{cdc^-}), Table 5.14. The route sets include the distances and times of these new arcs too. The objective function and several constraints are adjusted. Two new constraints are added.

from

Sets		Description
Arcs	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s$)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i
Parameters		**Description**
Routes	dis_{ij}	Distance between node i and j
	$time_{ij}$	Travel time between node i and j

to

Sets		Description
Arcs	A^{cdc^+}	Set of arcs between CDCs
	A^{cdc^-}	Set of arcs between CDCs
	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s \cup A^{cdc^+} \cup A^{cdc^-}$)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i (for the new A)
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i (for the new A)
Parameters		**Description**
Routes	dis_{ij}	Distance between node i and j (including the distances for the new $A^{cdc^+} and A^{cdc^-}$)
	$time_{ij}$	Travel time between node i and j (including the times for the new $A^{cdc^+} and A^{cdc^-}$)

Table 5.14: Sets and parameters adjustment

Adjusted objective function, Equation 4.1.

$$\begin{aligned} min \sum_{(i,j)\in A^{dc^+}} \sum_{v\in V} fc_{ij} \cdot y_{ij}^{v0} + \sum_{(i,j)\in A^{dc^+}} \sum_{p\in P} \sum_{v\in V} pc \cdot x_{ij}^{pv0} + \\ \sum_{(i,j)\in (A^{rdcs^+} \cup A^{rdcs^-} \cup A^s)} \sum_{v\in V} (tc^1 \cdot dis_{ij} + hc^1 \cdot time_{ij}) \cdot y_{ij}^{v1} + \sum_{(i,j)\in A^{rdcs^+}} \sum_{v\in V} hc^1 \cdot flt^1 \cdot y_{ij}^{v1} + \\ \sum_{(i,j)\in (A^{rdcs^+} \cup A^s)} \sum_{v\in V} hc^1 \cdot fut^1 \cdot y_{ij}^{v1} + \sum_{(i,j)\in A^{rdcs^+}} \sum_{p\in P} \sum_{v\in V} hc^1 \cdot (vlt^1 + vut^1) \cdot x_{ij}^{pv1} + \\ \sum_{(i,j)\in (A^{cdcs^+} \cup A^s \cup A^{cdc^+})} \sum_{v\in V} (tc^2 \cdot dis_{ij} + hc^2 \cdot time_{ij}) \cdot y_{ij}^{v2} + \sum_{(i,j)\in (A^{cdcs^+} \cup A^{cdc^+})} \sum_{v\in V} hc^2 \cdot flt^2 \cdot y_{ij}^{v2} + \\ \sum_{(i,j)\in (A^{cdcs^+} \cup A^s)} \sum_{v\in V} hc^2 \cdot fut^2 \cdot y_{ij}^{v2} + \sum_{(i,j)\in (A^{cdcs^+} \cup A^{cdc^+})} \sum_{p\in P} \sum_{v\in V} hc^2 \cdot (vlt^2 + vut^2) \cdot x_{ij}^{pv2} + \\ \sum_{(i,j)\in A^{cdcs^+}} \sum_{v\in V} erc \cdot y_{ij}^{v2} \end{aligned} \tag{5.2}$$

Adjusted constraint Vehicle type 1 cannot enter or leave a CDC, Equation 4.4.

$$\sum_{(i,j)\in (A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{cdc^+} \cup A^{cdc^-})} y_{ij}^{v1} = 0 \quad \forall v \in V \tag{5.3}$$

Adjusted constraint: Vehicle type 0 may only start from one of the CDCs, Equation 4.5.

$$\sum_{i\in N^{cdc}} \sum_{j\in N^+(i)} y_{ij}^{vo} \leq 1 \quad \forall v \in V \tag{5.4}$$

New constraint: Vehicle type 2 may only drive on arcs in set A^{cdc^+} or to the stores.

$$\sum_{(i,j)\in A^{cdc^+}} y_{ij}^{v2} + \sum_{(i,j)\in A^{cdcs}} y_{ij}^{v2} \leq 1 \quad \forall v \in V \tag{5.5}$$

New constraint: Vehicle type 2 may only drive from one of the CDCs to the stores.

$$\sum_{i \in N^{cdc}} \sum_{j \in N^s} y_{ij}^{v2} \leq 1 \quad \forall v \in V \tag{5.6}$$

Adjusted constraint: Vehicle type 0 cannot drive from a DC to a store and cannot drive between CDCs, Equation 4.6.

$$\sum_{(i,j) \in (A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s \cup A^{cdc^+} \cup A^{cdc^-})} y_{ij}^{v0} = 0 \quad \forall v \in V \tag{5.7}$$

Adjusted constraint: Vehicle type 2 cannot drive to an RDC and therefore drive not from an RDC to a store and cannot drive back from a CDC to another CDC, Equation 4.7.

$$\sum_{(i,j) \in (A^{dc^+} \cup A^{dc^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^{cdc^-})} y_{ij}^{v2} = 0 \quad \forall v \in V \tag{5.8}$$

Adjusted constraint: Demand can not be sent back to a DC, Equation 4.12.

$$\sum_{p \in P} x_{ij}^{pvc} = 0 \quad \forall (i,j) \in (A^{dc^-} \cup A^{cdcs^-} \cup A^{rdcs^-} \cup A^{cdc^-}), v \in V, c \in C \tag{5.9}$$

Adjusted constraint: Checks whether a (part of) commodity p is transported from a CDC to a store, Equation 4.15.

$$\sum_{j \in N^s} \sum_{v \in V} x_{ij}^{pv2} + \sum_{k \in (N^{cdc}: k \neq i, (i,k) \in A^{cdc^+})} \sum_{j \in N^s} \sum_{v \in V} x_{ij}^{pv2} \leq M \cdot z_i^{p1} \quad \forall i \in N^{cdc}, p \in P \tag{5.10}$$

Results

The results of this case are stated in Table 5.15. There is a positive difference in all KPI values from this configuration compared to the base case. The total costs are €$8 \cdot 10^3$ lower. The KPI values of the total distance and total number of vehicles are improved by 4.9% and 3.5%, respectively.

Description		Value	Improvement [%]
KPIs	Total costs of transport [€ $\cdot 10^3$]	139	5.4
	Total distance [km $\cdot 10^3$]	52	4.9
	Total number of vehicles [- $\cdot 10^3$]	0.5	3.5
	Costs per container [€]	3.17	5.4
	Load per vehicle [containers]	15	2.6
	Drop size per stop [containers]	9	4.5
	Stops per vehicle [-]	0.6	0

Table 5.15: Results second case (a positive percentage equals an improvement)

There is a small difference in routing decisions too, Table 5.16. There is a shift from direct to indirect delivery of 2.4%. In this shift, 72.0% of the demand is transported from CDC 10001 to CDC 10002.

	Total costs [€ $\cdot 10^3$]	CDC costs [€ $\cdot 10^3$]	Transfer costs [€ $\cdot 10^3$]	RDC costs [€ $\cdot 10^3$]	Direct [%]	Indirect [%]
RDC 20001	41	13	7	21	60.7	39.3
RDC 20002	36	12	6	18	65.9	34.1
RDC 20003	35	10	5	21	52.3	47.7
RDC 20004	27	5	7	15	37.7	62.3
Total	139	39	25	75	55.0	45.0

Table 5.16: Results per RDC of second case

The ten clusters with the biggest cost differences are added to Table 5.17. One of the postal codes, postal 39, has no difference in routing decisions. The improvement in costs is due to the consolidation of demand at CDC 10002 from CDC 10001.

Postal [-]	RDC	10001 [%]	10002 [%]	Cost diff. [€]	CDC dist. [km]	RDC dist. [km]
48	20004	33.3	100	613	26	2
94	20002	0	87.5	271	64	7
39	20003	0	0	250	9	15
89	20002	100	0	240	63	27
91	20002	0	50.0	232	73	30
62	20001	0	88.9	225	66	44
83	20002	100	0	220	41	25
63	20001	0	33.3	219	66	45
95	20002	0	25.0	211	75	19
68	20001	33.3	0	206	24	23

Table 5.17: Route option differences second case

The routing decisions are divided per CDC and visualized in Figures F.1 and F.2. The second case performs slightly better on the main KPI compared to the first case (0.1%). The performance on the other two KPIs, the total distance and the total number of vehicles, is higher (6.1% and 4.5% respectively). There is an overlap in the difference in routing decision compared to the top 10 of the first case too, postal codes 48, 94 (RDC 20002), 91 (RDC 20002), 62, 63 and 95.

5.4.5. Third case, altered shuttle costs

The third case can be characterized as a change in configuration and no change in scenario while fixed costs for delivery between a CDC i and an RDC j differs compared to the base case. There are two expectations regarding transportation costs. First, normal trucks are currently used for transportation between CDCs and RDCs. There will be a change over time where the normal trucks will get an extra trailer for more efficient transportation. Of these, the cost will be relatively low (per unit of demand) compared to the current cost of commuting. Second, delivery restrictions are emerging in more and more places. Think of this as combining demand from different stores to make a maximum number of deliveries on a street for all stores combined. In addition, there may also be restrictions on the type of vehicles that can deliver demand in a downtown area. There is a trend here whereby the cost of last-mile delivery continues to rise. Therefore, it is interesting to investigate how the model responds to changes in fixed costs between CDC and RDC (fc_{ij}). To discover this relationship, these costs are reduced by 5% five times. This allows allocation differences to be construed and to see how the allocation might look in the future.

Mathematical model adjustments
There is one change compared to the base case, Table 5.18. The fixed delivery costs between the CDCs and RDCs parameters (fc_{ij}) are adjusted. These costs are five times reduced by 5%.

from

Parameters		Description
Vehicles	fc_{ij}	Fixed cost for delivery between a CDC i and an RDC j, 100% of the costs

to

Parameters		Description
Vehicles	fc_{ij}	Fixed cost for delivery between a CDC i and an RDC j, 95%, 90%, 85%, 80%, 75% of the costs

Table 5.18: Parameters adjustment

Results
The results of this case are stated in Table 5.19. Costs are not representative for this case while the costs are lower for direct delivery per configuration.

Description		$fc_{ij} = 0.95$	$fc_{ij} = 0.90$	$fc_{ij} = 0.85$	$fc_{ij} = 0.80$	$fc_{ij} = 0.75$
KPIs	Total costs of transport [€ $\cdot 10^3$]	136	134	133	131	129
	Total distance [km $\cdot 10^3$]	55	55	55	55	57
	Total number of vehicles [- $\cdot 10^3$]	0.5	0.5	0.5	0.5	0.5
	Costs per container [€]	3.12	3.06	3.03	2.98	2.94
	Load per vehicle [containers]	16	16	16	16	16
	Drop size per stop [containers]	10	10	10	10	10
	Stops per vehicle [-]	0.6	0.6	0.6	0.6	0.6

Table 5.19: Results third case (a positive percentage equals an improvement)

Differences in routing decisions for all RDCs are stated per fc_{ij} percentage in Table 5.20. The ratio of direct and indirect delivery changes with an expected trend. When the costs of transportation from the CDCs to the RDCs reduce, the number of direct deliveries decreases and the number of indirect deliveries increases.

		Direct [%]	**Indirect [%]**
$fc_{ij} = 0.95$	RDC 20001	27.9	72.1
	RDC 20002	45.8	54.2
	RDC 20003	43.5	56.5
	RDC 20004	23.1	76.9
	Total	35.6	64.4
$fc_{ij} = 0.90$	RDC 20001	26.2	73.8
	RDC 20002	37.7	62.3
	RDC 20003	40.6	59.4
	RDC 20004	17.5	82.5
	Total	31.4	68.6
$fc_{ij} = 0.85$	RDC 20001	26.2	73.8
	RDC 20002	37.7	62.3
	RDC 20003	38.2	61.8
	RDC 20004	17.5	82.5
	Total	30.7	69.3
$fc_{ij} = 0.80$	RDC 20001	26.2	73.8
	RDC 20002	37.7	62.3
	RDC 20003	35.5	64.5
	RDC 20004	13.4	86.6
	Total	29.1	70.9
$fc_{ij} = 0.75$	RDC 20001	21.7	78.3
	RDC 20002	28.9	71.1
	RDC 20003	26.3	73.7
	RDC 20004	13.4	86.6
	Total	23.1	76.9

Table 5.20: Results per RDC of third case

The routing decisions are divided into direct and indirect delivery and are shown in Figure 5.1 and Figure 5.2 respectively.

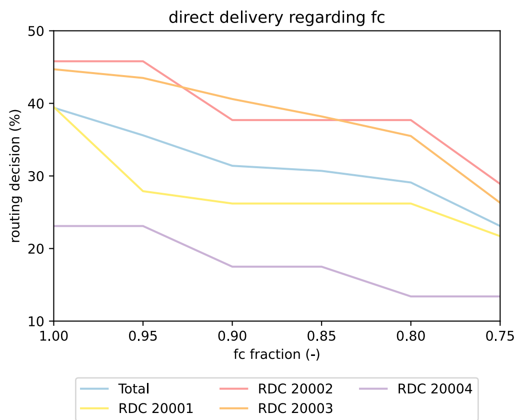


Figure 5.1: Direct delivery regarding fc_{ij}

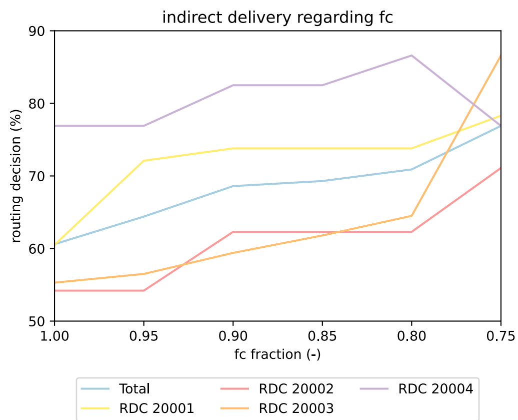


Figure 5.2: Indirect delivery regarding fc_{ij}

The ten clusters with the biggest differences in costs are added per fc_{ij} percentage to Table 5.21. When a postal exists in multiple RDC clusters, the cluster is indicated with the RDC ID. There is an overlap in postal codes 56, 48, 57, 94 (RDC 20002), 62.

$fc_{ij} = 0.95$ **[-]**	$fc_{ij} = 0.90$ **[-]**	$fc_{ij} = 0.85$ **[-]**	$fc_{ij} = 0.80$ **[-]**	$fc_{ij} = 0.75$ **[-]**
56	56	56	56	56
48	48	48	48	48
57	57	57	57	57
94 (20002)	94 (20002)	94 (20002)	43	43
52	52	54	94 (20002)	94 (20002)
91 (20002)	54	52	54	54
62	34	34	52	34
50	50	50	34	62
63	62	62	26	26
95	91 (20002)	91 (20002)	62	91 (20002)

Table 5.21: Postal codes of maximal route option differences third case

The performance of this case can not directly be compared with the other cases while there is a difference in transportation costs. There is an overlap in the difference in routing decisions compared to the top 10 of the first and second scenarios, postal codes 48, 94 (RDC 20002) and 62.

5.4.6. Fourth case, altered volumes

The fourth case can be characterized as a change in scenario and no change in configuration while the volume of demand differs compared to the base case. There is an expected change in volume because of two reasons. First, the volume is based on a representative day. While by default not all stores have the same demand during the week, this indicates differences already. Second, the company expects growth during the coming years. Because of this growth, it is important to see what changes in routing decisions will be the result. The volumes are adjusted five times to discover the influence. This allows allocation differences to be construed and to see how the allocation might look in the future.

Mathematical model adjustments

There is one change compared to the base case, Table 5.22. The volumes (fc_{ij}) are adjusted five times by a certain percentage. A new set of vehicles and their corresponding capacities are a result of this adjustment.

from

Sets		Description
Vehicles	V	Set of vehicles
Parameters		**Description**
Commodities	vol^p	Volume of commodity p, 100% of the volume
Vehicles	cap^{vc}	Capacity of vehicle of type c

to

Sets		Description
Vehicles	V	Set of vehicles, adjusted set based on new total volume
Parameters		**Description**
Commodities	vol^p	Volume of commodity p, 125%, 115%, 105%, 95%, 85% of the volume
Vehicles	cap^{vc}	Capacity of vehicle of type c, adjusted based on new set of vehicles (V)

Table 5.22: Sets and parameters adjustment

Results

The results of this case are stated in Table 5.23. Costs are not representative of this case, while the costs are strongly dependent on the volume of demand (vol^p).

Description		$vol^p = 0.85$	$vol^p = 0.95$	$vol^p = 1.05$	$vol^p = 1.15$	$vol^p = 1.25$
KPIs	Total costs of transport [€ $\cdot 10^3$]	128	136	148	157	165
	Total distance [km $\cdot 10^3$]	51	54	58	61	63
	Total number of vehicles [- $\cdot 10^3$]	0.5	0.5	0.5	0.5	0.6
	Costs per container [€]	3.36	3.36	3.15	3.05	2.97
	Load per vehicle [containers]	14	15	16	16	16
	Drop size per stop [containers]	9	10	10	10	11
	Stops per vehicle [-]	0.6	0.6	0.6	0.6	0.6
	Number of containers [-]	14	16	18	19	21

Table 5.23: Results fourth case (a positive percentage equals an improvement)

Differences in routing decisions for all RDCs are stated per vol^p percentage in Table 5.24. The ratio of direct and indirect delivery changes with a trend. When the volume decreases, the number of direct deliveries decreases and the number of indirect deliveries increases. When the volume increases, the number of direct deliveries increases and the number of indirect deliveries decreases.

		Direct [%]	**Indirect [%]**
$vol^p = 0.85$	RDC 20001	44.8	55.2
	RDC 20002	44.5	55.5
	RDC 20003	34.1	65.9
	RDC 20004	35.8	64.2
	Total	39.8	60.2
$vol^p = 0.95$	RDC 20001	38.1	61.9
	RDC 20002	41.2	58.8
	RDC 20003	45.6	54.4
	RDC 20004	18.7	81.3
	Total	37.4	62.6
$vol^p = 1.05$	RDC 20001	55.0	45.0
	RDC 20002	49.7	50.3
	RDC 20003	33.9	66.1
	RDC 20004	32.8	67.2
	Total	43.3	56.7
$vol^p = 1.15$	RDC 20001	56.0	44.0
	RDC 20002	50.0	50.0
	RDC 20003	43.8	56.2
	RDC 20004	29.5	70.5
	Total	46.0	54.0
$vol^p = 1.25$	RDC 20001	47.4	52.6
	RDC 20002	53.9	46.1
	RDC 20003	43.8	56.2
	RDC 20004	27.2	72.8
	Total	43.9	56.1

Table 5.24: Results per RDC of fourth case

The routing decisions are divided into direct and indirect delivery and are shown in Figure 5.3 and Figure 5.4 respectively.

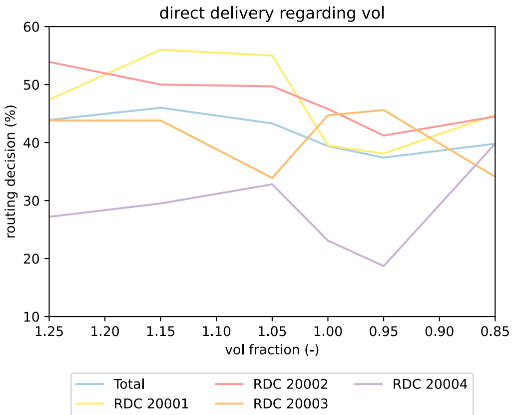


Figure 5.3: Direct delivery regarding vol^p

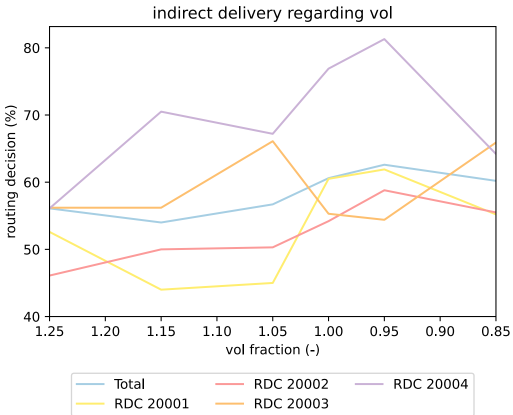


Figure 5.4: Indirect delivery regarding vol^p

The ten clusters with the biggest differences in costs are added per vol^p percentage to Table 5.25. When a postal exists in multiple RDC clusters, the cluster is indicated with the RDC ID. There is an overlap in postal codes 48 and 94 (RDC 20002). For the smaller volumes, there is an overlap in the postal codes 48 and 94 (RDC 20002). For the larger volumes, there is an overlap in the postal codes 48, 91 (RDC 20002), 94 (RDC 20002) and 94 (RDC 20004).

$vol^p = 0.85$ [-]	$vol^p = 0.95$ [-]	$vol^p = 1.05$ [-]	$vol^p = 1.15$ [-]	$vol^p = 1.25$ [-]
48	56	48	48	48
57	48	52	91 (20002)	91 (20002)
56	57	94 (20002)	94 (20002)	94 (20004)
38 (20003)	75	91 (20002)	94 (20004)	56
11	43	95	31 (20003)	38 (20001)
94 (20002)	94 (20002)	88	29 (20004)	90 (20004)
89 (20002)	52	28 (20004)	38 (20001)	94 (20002)
43	25 (20003)	14	90 (20004)	31 (20003)
50 (20004)	50	27	40 (20003)	36 (20001)
46	91 (20002)	20 (20003)	36 (20001)	20 (20004)

Table 5.25: Postal codes of maximal route option differences fourth case

The performance of this case can not directly be compared with the other cases while there is a difference in volumes. There is an overlap in the difference in routing decisions compared to the top 10 of the first and second scenarios, postal codes 48 and 94 (20002).

5.4.7. Fifth case, optimization on the company's server

The fifth case can be characterized as a combination of change of configuration and no change in scenario while only the network is adjusted. The data is clustered because of the large amount of data. The clustering ensures that consolidation of demand cannot be properly accounted for in the optimization and the model will therefore never be able to determine the most optimal outcome. To still be able to perform a more optimal optimization once, an optimization is performed on the company's servers. This will allow a one-time look at a more optimal outcome and compare it to the outcomes of the current clustered model. This comparison provides insight into the effectiveness of the clustered model. The optimization that will be performed concerns a model per RDC. For each RDC, store arcs will be added to examine more possibilities. The optimization on the company's servers needs to be done with the help of another solver named PuLP (PuLP, 2022). Therefore, the Gurobi model needs to be rewritten to a PuLP model.

Mathematical model adjustments

There is one change compared to the base case, Table 5.26. The set of store arcs (A^s) is extended. Arcs are added for every store based on the 5 nearest stores. A new set of all arcs (A), outward arcs ($N^+(i) = \{j \in N : (i,j) \in A\}$) and inward arcs ($N^-(i) = \{j \in N : (j,i) \in A\}$) are a result of this adjustment.

from

Sets		Description
Arcs	A^s	Set of arcs between stores
	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s$)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i

to

Sets		Description
Arcs	A^s	Set of arcs between stores, each store is connected to the five nearest stores
	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s$), adjusted based on new set of store arcs (A^s)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i, adjusted based on new set of store arcs (A^s)
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i, adjusted based on new set of store arcs (A^s)

Table 5.26: Sets and parameters adjustment

Computation

Besides converting the model from Gurobi to PuLP, the input parameters, start solution and callback also need to be changed. It has been found as not possible to apply the input parameters to the PuLP optimisation. However, a time limit of two hours was set. This limits the load on the company's server and will not hold up other work too much. The start solution was converted to PuLP to save time in optimisation. Finally, it has also not been found possible to implement the callback in the new model.

Results
The PuLP optimisation was started several times on the company's server. First, the model with arcs from stores to the 5 nearest stores was started. This gave a memory error. This was also tried for 4, 3 and finally 2 arcs between stores. In the end, this still proved too powerful a model to run. The server capacity is loaded and, as shown in figure 5.5, a memory problem arises. As a result, it is not possible to compare the outcomes of this case with previous cases to determine the effectiveness of clustering. This again shows the complexity in size of the problem.

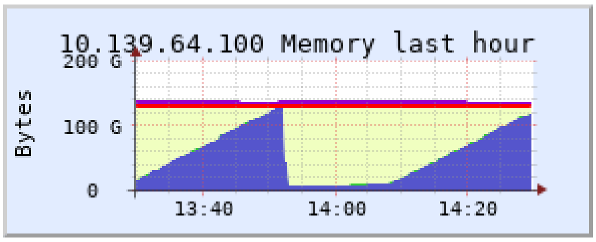


Figure 5.5: Server capacity

5.4.8. Sensitivity analysis
The robustness of the model can be investigated with the help of a sensitivity analysis. A sensitivity analysis can provide insight into the behaviour of the model. Some input values may change easily over time and other input values may be uncertain. This analysis gives insight into how much the values affect the outcomes of the model. Volumes are expected to change frequently and easily. In addition, the assumption has been made that transport costs are uncertain. Therefore, this analysis is divided into two parts. The first analysis focuses on the sensitivity of the volume. The second analysis focuses on the sensitivity of the costs of transportation from the CDCs to the RDCs.

The sensitivity of the model to costs can be analysed in a number of ways. First, the percentage difference in costs can be plotted against the percentage difference in outcomes, in addition, route differences can be assessed against the current situation. Finally, the postal codes with the largest changes on a cost basis can be assessed for similarities.

First analysis, sensitivity of volume
The case study focuses on a representative day regarding the volumes. The representative day can be used for a long-term, so strategic, allocation decision. The demand of stores varies while the orders of a store depend on the sales. The volumes also depend on seasons such as Christmas. Besides these fluctuations, there is an expected growth of volume because of the company's goals to continue to grow as a company and thus increase the number of products sold. Five percentage adjustments of volume are considered in this analysis, 85%, 95%, 105%, 115% and 125% of the volume respectively. Data for this analysis is extracted from the first and from the fourth case of the case study, Section 5.4.3 and Section 5.4.6 respectively.

The three KPIs are sensitive regarding the difference in the volumes, Table 5.4.6.2. The total costs, distance and number of vehicles increase when the volume increases. The three KPIs decrease when the volume decreases. They do not increase or decrease as much as the volume does, but they follow the same trend. The load per vehicle and drop size per stop have somewhat the same behaviour. The costs per container are not really sensitive to volume changes. Overall, the costs per container are lower when the total volume is higher, but they are within a 7% window compared to the base case. The stops per vehicle are more or less constant for all volumes and therefore not really sensitive to changes. The sensitivity of the route choices is quite low, Table 5.20. For the first 5% in reduction, the difference is 2.1%, 2.7% for the second reduction, 3.9% for the third reduction, 2.0% for the fourth reduction and finally 2.4% for the fifth reduction. There is no trend in the differences, Figure 5.7, but it can be concluded that these costs have a relatively low impact on the outcome.

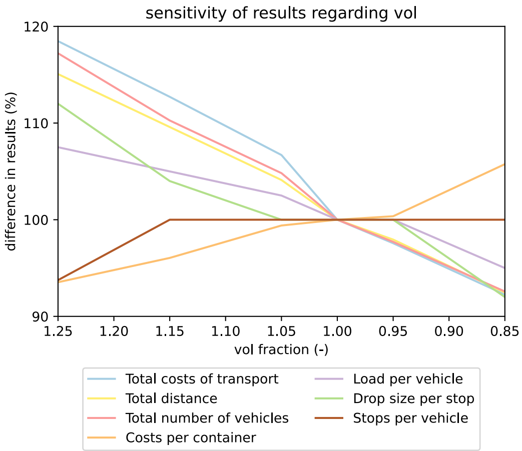


Figure 5.6: Sensitivity of the results compared to the base case regarding vol^p

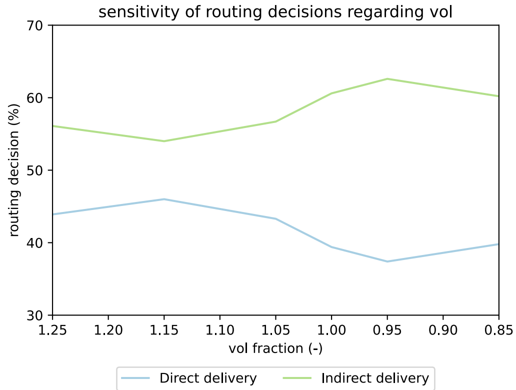


Figure 5.7: Sensitivity of routing decision regarding vol^p

Unfortunately, there is a small overlap in the postal codes that involves a large cost difference for the change in volume, Table 5.25. So these route choices are affected by the changes in volume and will result in quite some differences.

Overall it can be concluded that the model is sensitive to changes in the volume. It is, therefore, important to consider route choices given the expected changes in these volumes and to observe the lack of robustness of the model to these changes.

Second analysis, sensitivity of costs

The costs of transportation are assumed to be uncertain while they depend on third parties and fluctuations in prices of fuel prices. Besides these differences, there is an expectation that the costs of transportation from the CDCs to the RDCs will decrease over time while this can be optimized. Furthermore, increasing restrictions on last-mile deliveries results in more expensive solutions. The effect of all these uncertainties are combined in the costs of transportation from the CDCs to the RDCs. Five percentage adjustments of costs are considered in this analysis, 95%, 90%, 85%, 80% and 75% of the fc_{ij} costs respectively. Data for this analysis is extracted from the first and from the third case of the case study, Section 5.4.3 and Section 5.4.5 respectively.

The main KPI, the total costs of transportation, is sensitive regarding the difference in the fixed cost for delivery between a CDC i and an RDC j, Table 5.4.5.2. This cost and the costs per container, which follow the same trend, decrease almost linearly between the 1.00 and 0.75 fractions of fc_{ij}. The total distance, total number of vehicles and load per vehicle are not sensitive regarding fc_{ij} while they do not differ that much. The number of stops per vehicle decrease with lower costs and the corresponding drop size per stop increase. These are highly influenced between the 0.95 and 0.90 fractions and stabilise after.

The sensitivity of the route choices is quite high, Table 5.20. For the first 5% in reduction, the difference is 3.8%, 4.2% for the second reduction, 0.7% for the third reduction, 1.6% for the fourth reduction and finally 6% for the fifth reduction. There is no trend in the differences, Figure 5.9, but it can be concluded that these costs have a significant impact on the outcome.

Despite the fact that there are differences in route choices compared to the base case, there is a great similarity in the postal codes that change, which involve a large cost difference, Table 5.21. So, these route choices are affected by the reduction in costs, but the biggest changes remain somewhat constant.

Overall, it can be concluded that the model is sensitive to changes in the fixed cost for delivery between a CDC i and an RDC j. It is, therefore, important to consider route choices given the expected changes in these costs and to observe the lack of robustness of the model to these changes.

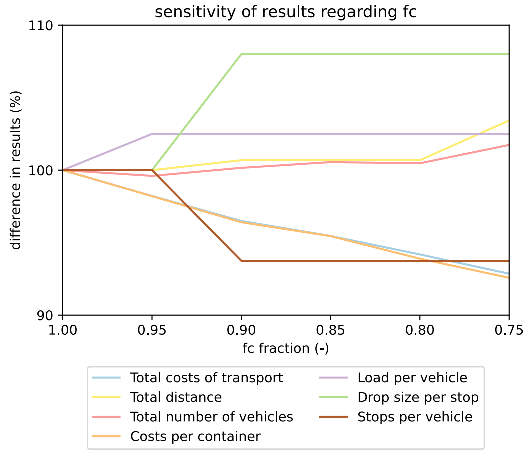


Figure 5.8: Sensitivity of the results compared to the base case regarding fc_{ij}

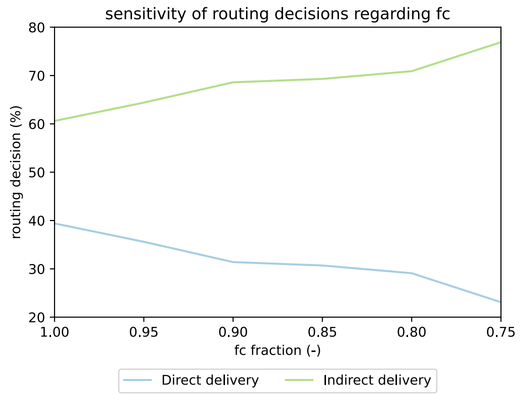


Figure 5.9: Sensitivity of routing decision regarding fc_{ij}

5.4.9. Computation

Several adjustments have been made to the model related to computational limits. These modifications include the start solution, input parameters and a callback function. Adding a callback function affects the gap remaining after optimisation. As a result, there may be a difference in the incumbent solution and the best bound. The gap can be calculated using the incumbent and best-bound using the following equation:

$$\text{gap } (\%) = \frac{|\text{best-bound} - \text{incumbent}|}{\text{incumbent}} \tag{5.11}$$

To gain insight into what influence this has had, these numbers are given in Table 5.27. While the best bound is the best objective value derived from relaxing the problem at a certain node in the search tree, the incumbent is the currently best-known solution. The best bound serves as an indicator of the effectiveness of the problem's current relaxation. The best objective value among the nodes that have been investigated in the relaxation and have been bound is the best bound. It acts as a lower bound on the objective value of the best solution. The incumbent solution is equal to the optimal values of the cases. A gap range can be discovered between 6.76% and 10.3%. This means that the final solution is always sub-optimal, while the gap is not equal to 0% for all of the experimental cases.

Case	**Run time [h]**	**Incumbent [€ $\cdot 10^3$]**	**Best-bound [€ $\cdot 10^3$]**	**Gap [%]**
Base	35	147	133	9.6
First	33	139	127	8.7
Second	39	139	125	10.3
Third, $fc_{ij} = 0.95$	32	136	126	8.0
Third, $fc_{ij} = 0.90$	33	134	125	7.2
Third, $fc_{ij} = 0.85$	32	133	123	7.3
Third, $fc_{ij} = 0.80$	32	131	121	7.2
Third, $fc_{ij} = 0.75$	27	129	120	6.8
Fourth, $vol^p = 1.25$	28	165	153	7.3
Fourth, $vol^p = 1.15$	31	157	143	8.7
Fourth, $vol^p = 1.05$	31	148	135	9.3
Fourth, $vol^p = 0.95$	30	136	125	7.9
Fourth, $vol^p = 0.85$	32	128	115	10.3

Table 5.27: Computation values of all cases

5.5. Comparative results

The objective of this case study is to ascertain the most efficient outcomes for the distribution system. Achieving optimality within the system can be assessed from various perspectives. Firstly, performance metrics can serve as a basis for comparing different scenarios. Ultimately, the primary aim is to minimize the overall transportation costs. However, routing decisions may also consider alternative metrics, such as vehicle load, for instance. Secondly, we can compare routing decisions to explore their influence on performance metrics. We focus on identifying the routing decisions that exert the greatest impact on costs. Thirdly, examining the relationship between routing decisions, distances and volumes between postal codes, CDCs, and RDCs can yield valuable insights, providing a practical guideline for routing decisions. These three comparative analyses offer a comprehensive overview of the system's performance, enabling the extraction of optimal results.

An important note to consider is the fact that the data is aggregated because of the clustering of all locations. This will, therefore, result in an error and uncertainty in the outcome. The statements made about the results, therefore, relate to the current outcomes, but it is important to take into account deviations that are not included. Besides the aggregation of the data, sub-optimal results also arise due to the gaps that are not equal to 0% for any experimental cases. These are elements that should be taken into account when interpreting the results and conclusions.

5.5.1. Comparing the performance metrics

The performance metrics of all cases are combined in Table 5.28. The metrics can not directly be compared among all cases. This is while for the third case, the fixed costs between CDC and RDC (fc_{ij}) are reduced five times which automatically results in lower costs. For the fourth case, this is due to the volume adjustments which will also automatically result in different costs. What can be concluded from this overview is that consolidation of demand, the second case, will have a positive influence on the main KPI, the total costs of transport. This adjustment in the distribution system will also positively influence the total distance driven, the total number of vehicles, the costs per container and the load per vehicle. In general, the model performs better compared to the routing decisions of the company, while the total costs of transport for the first case are lower than those of the base case. In the third case there is a decreasing trend in costs of transport and an increasing trend in the total distance and total number of vehicles. In the fourth case, the costs of transport per container decrease when the volume increases and increase when the volume decreases. The total costs of transport, total distance driven, total number of vehicles, load per vehicle and drop size per stop show the opposite behaviour. They increase when the volume increases too.

	Total costs of transport [€ $\cdot 10^3$]	Total distance [km $\cdot 10^3$]	Total number of vehicles [- $\cdot 10^3$]	Costs per container [€]	Load per vehicle [containers]	Drop size per stop [containers]	Stops per vehicle [-]
Base case	147	55	476	3.35	15	8	0.6
First case	139	55	481	3.17	15	4	0.6
Second case	139	52	459	3.17	15	9	0.6
Third case, $fc_{ij} = 0.95$	136	55	479	3.12	16	10	0.6
Third case, $fc_{ij} = 0.90$	134	56	482	3.06	16	10	0.6
Third case, $fc_{ij} = 0.85$	133	56	484	3.03	16	10	0.6
Third case, $fc_{ij} = 0.80$	131	56	483	2.98	16	10	0.6
Third case, $fc_{ij} = 0.75$	129	57	489	2.94	16	10	0.6
Fourth case, $vol^p = 1.25$	165	63	564	2.97	16	11	0.6
Fourth case, $vol^p = 1.15$	157	61	530	3.05	16	10	0.6
Fourth case, $vol^p = 1.05$	148	58	504	3.15	16	10	0.6
Fourth case, $vol^p = 0.95$	136	54	470	3.18	15	10	0.6
Fourth case, $vol^p = 0.85$	128	51	445	3.36	14	9	0.6

Table 5.28: Comparing the performance metrics

5.5.2. Comparing the routing decisions

The routing decisions and corresponding costs of all cases are combined in Table 5.29. The interesting difference between the first and the base case is that in the base case, 57.4% of all postal codes is delivered directly from the CDC and in the first case only 39.4%. When consolidating the demand at the CDCs, in the second case, the routing decision shifts back to direct delivery. That is an expected transition while the vehicles from CDCs to the stores can have a higher load. A trend can be discovered in the third case, the lower the fixed transportation costs from CDC to RDC, the higher the indirect delivery. Demand can be transported to the RDC at a lower cost, where it can be consolidated and a vehicle can operate with a higher load. The fourth case shows the same behaviour as in the second case. When the volume increases, the direct delivery percentages increase too. This is the result of the possibility of increasing the truckload for delivery from the CDCs to the stores. It can be seen, then, that the most optimal outcome of the model has to do with the truckload. The higher the truckload, the cheaper the transport. Consolidation of demand is thus an important element of distribution. When consolidating at the RDCs, there will be a trade-off between the transport costs from the CDC and the shuttle costs plus the processing costs at the RDC. So overall, there is a main difference between the base case and the first case, but the model behaves as expected in the other cases.

	Total costs [€ $\cdot 10^3$]	**CDC costs [€ $\cdot 10^3$]**	**Transfer costs [€ $\cdot 10^3$]**	**RDC costs [€ $\cdot 10^3$]**	**Direct [%]**	**Indirect [%]**
Base case	147	48	22	77	57.4	42.6
First case	139	29	31	80	39.4	60.6
Second case	139	39	25	75	55.0	45.0
Third case, $fc_{ij} = 0.95$	136	27	32	80	35.6	64.4
Third case, $fc_{ij} = 0.90$	134	23	33	81	31.4	68.6
Third case, $fc_{ij} = 0.85$	133	22	33	81	30.7	69.3
Third case, $fc_{ij} = 0.80$	131	21	34	75	29.1	70.9
Third case, $fc_{ij} = 0.75$	129	17	36	84	23.1	76.9
Fourth case, $vol^p = 1.25$	165	38	32	95	43.9	56.1
Fourth case, $vol^p = 1.15$	157	38	30	89	46.0	54.0
Fourth case, $vol^p = 1.05$	148	36	29	84	43.3	56.7
Fourth case, $vol^p = 0.95$	136	27	31	78	37.4	62.6
Fourth case, $vol^p = 0.85$	128	27	29	72	39.8	60.2

Table 5.29: Comparing the routing decisions and costs

The postal codes of maximal route option differences of all cases are combined in Table 5.30. The postal codes on the left have the highest cost differences compared to the base case and are in decreasing order to the right. When a postal exists in multiple RDC clusters, the cluster is indicated with the RDC ID. If the postal codes reappear in several or even all cases, this shows an improvement that can be made. In that case, the model suggests each time that changing the routing decision has a positive impact on the costs concerning transport. Between the first and second cases, there is an overlap in six postal codes, 48, 94 (RDC 20002), 91 (RDC 20002), 62, 63 and 95. Within the third case, there is an overlap in postal codes 56, 48, 57, 94 (RDC 20002), 62. Between the first, second and third cases, there is an overlap in postal codes 48, 94 (RDC 20002) and 62. Within the fourth case, there is an overlap in postal codes 48 and 94 (RDC 20002). For the smaller volumes, there is an overlap in the postal codes 48 and 94 (RDC 20002). For the larger volumes, there is an overlap in the postal codes 48, 91 (RDC 20002), 94 (RDC 20002) and 94 (RDC 20004). Finally, between all cases, there is an overlap in postal codes 48 and 94 (RDC 20002). Therefore, it is advised to change the routing decisions of these postal codes while they have a positive influence on all configurations and scenarios.

	postal codes									
First case	56	48	94 (20002)	91 (20002)	62	63	95	88	82 (20003)	27
Second case	48	94 (20002)	39 (20003)	89 (20002)	91 (20002)	62	83	63	95	68
Third case, $fc_{ij} = 0.95$	56	48	57	94 (20002)	52	91 (20002)	62	50	63	95
Third case, $fc_{ij} = 0.90$	56	48	57	94 (20002)	52	54	34	50	62	91 (20002)
Third case, $fc_{ij} = 0.85$	56	48	57	94 (20002)	54	52	34	50	62	91 (20002)
Third case, $fc_{ij} = 0.80$	56	48	57	43	94 (20002)	54	52	34	26	62
Third case, $fc_{ij} = 0.75$	56	48	57	43	94 (20002)	54	34	62	26	91 (20002)
Fourth case, $vol^p = 1.25$	48	91 (20002)	94 (20004)	56	38 (20001)	90 (20004)	94 (20002)	31 (20003)	36 (20001)	20 (20004)
Fourth case, $vol^p = 1.15$	48	91 (20002)	94 (20002)	94 (20004)	31 (20003)	29 (20004)	38 (20001)	90 (20004)	40 (20003)	36 (20001)
Fourth case, $vol^p = 1.05$	48	52	94 (20002)	91 (20002)	95	88	28 (20004)	14	27	20 (20003)
Fourth case, $vol^p = 0.95$	56	48	57	75	43	94 (20002)	52	25 (20003)	50	91 (20002)
Fourth case, $vol^p = 0.85$	48	57	56	38 (20003)	11	94 (20002)	89 (20002)	43	50 (20004)	46

Table 5.30: Comparing the postal codes of maximal route option differences

5.5.3. Comparing the relationship between routing decisions, distances and volumes

The total costs depend mainly on distances and volumes. Therefore, three relationships are examined: the routing decisions based on distances, the routing decisions based on the number of containers and the relationship between the two elements using logistic regression.

The relationship between routing decisions and distances

The first relationship explored is between routing decisions and distance from the store to the CDCs. The second relationship is between the routing decision and the distance from the store to the RDC plus the distance from the RDC to the CDCs. These relationships are first examined for the base case, Figures 5.10 and 5.11. For the routing choices based on distance from the CDC, it can be seen that up to 75 km, there is a stronger preference for direct delivery. After 75 km, the choice is almost at the same level. For the distance to the RDC, there is a preference for direct delivery up to 140 km, and after that, the choice is again around the same level.

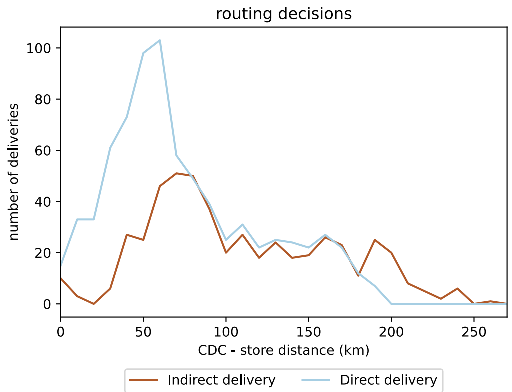


Figure 5.10: Routing decisions per set of CDC - store distances of the base case

Figure 5.11: Routing decisions per set of CDC - RDC + RDC - store distances of the base case

As a next step, the relationships were also examined for the first case, Figures 5.12 and 5.13. Here, it can be seen for both relationships that indirect delivery generally takes place more than direct delivery. Furthermore, no areas can be identified with a clear strength preference for a routing decision.

Figure 5.12: Routing decisions per set of CDC - store distances of the first case

Figure 5.13: Routing decisions per set of CDC - RDC + RDC - store distances of the first case

Unfortunately, for both situations, no convincing route choice can be recommended based on the distances. This assumes that no relationship can be found based on either of the distances alone.

The relationship between routing decisions and volumes

The second relationship explored is the relationship between routing decisions and volumes of demand. This relationship is first examined for the base case, Figure 5.14. A strong preference can be seen for direct delivery for volumes between 5 and 10 containers for a store. Here, there is a peak in preference at volumes of 6 containers. Here, the choice is almost 6 times more likely for direct delivery. For the first case, this relationship is, unfortunately, less intense, Figure 5.15. Here, between 1 and 15 containers, there is a slight preference for indirect delivery, but the number of choices is still enormously close here. Between 6 and 13 containers, there is an increase in this preference, but it remains small.

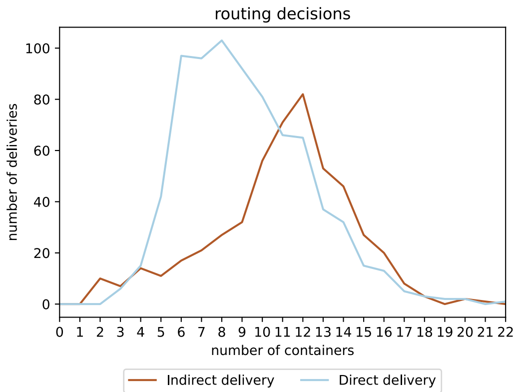


Figure 5.14: Routing decisions per set of volumes of the base case

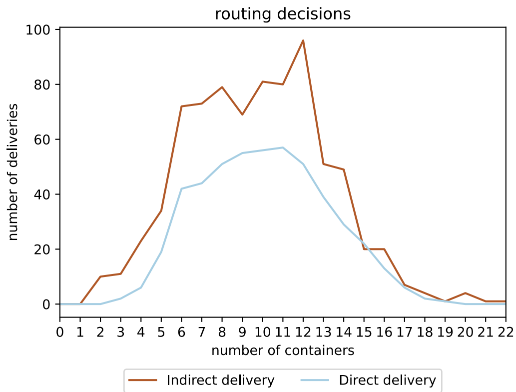


Figure 5.15: Routing decisions per set of volumes of the first case

Despite a preference for direct delivery for the base case within a specific range of containers, no advice can be given outside this range. For the first case, this is even more general as no convincing advice can be provided for almost no number of containers. It is thus assumed here that this general relationship between the number of containers and route choice cannot be found either.

The relationship between routing decisions, distances and volumes using a logistic regression model

The third relationship explored is the relationship between routing decisions, distances and volumes. A correlation between these could allow an assumption to be made for route choice. Given the high run time, it could be advantageous to estimate the routing decision with a rule of thumb. This rule-of-thumb was determined using a logistic regression (Yu et al., 2011). A logistic regression was chosen as a method, given the possibility of applying binary outcomes. For this case study, the binary outcome 1 equals direct delivery from the CDC, and 0 equals indirect delivery. The objective function is most influenced by the distance to stores and by the volume of demand. Therefore, these elements are included to fit the data using the logistic regression. A regression function is formulated:

$$x = a \cdot dis_{cdc,store} + b \cdot \sum_{p \in (P : o^p = cdc, d^p = store)} vol^p + c \cdot (dis_{cdc,rdc} + dis_{rdc,store}) + d \tag{5.12}$$

The coefficients a, b, c and d are fitted on the data. A probability of the routing decision can be determined from the result of Equation 5.12 where the coefficients are fit to the data and the distances and volume of the new store can be entered:

$$\mathcal{P}(x) = \frac{1}{1 + e^{-x}} \tag{5.13}$$

If the probability is 0.5 or higher, the advice is to deliver directly from the CDC. If the probability is lower than 0.5 it is advised to deliver indirectly. The data is split into a train set and a test set. This division is done with a ratio of $80 : 20$.The coefficients are determined for the base case, and the accuracy of this regression equals 63.5%. While this regression function is influenced by three variables, a 3D graph should be used to give instant insight into all routing decisions. For ease of use, the average value is chosen so that a constant value for the volume can be assumed. For the base case, the average volume equals 4 containers. A 2D graph can be plotted based on this average volume so that easy insight can be gained into the advice for the routing decision, Figure 5.16.

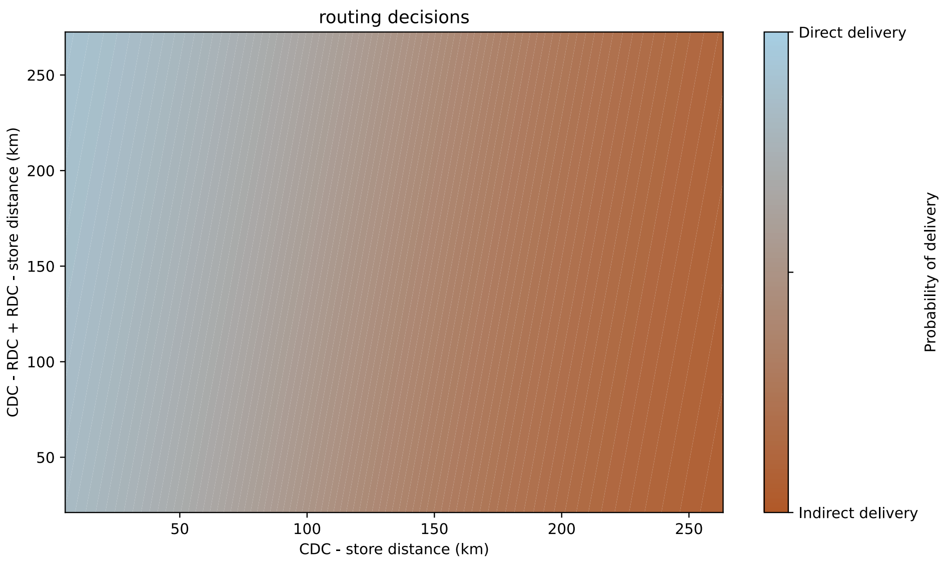


Figure 5.16: Regression of routing decisions of the base case

For the base case, the average volume equals 4 containers too, and the accuracy of this regression equals 68.9%. This regression is visualized in Figure 5.17.

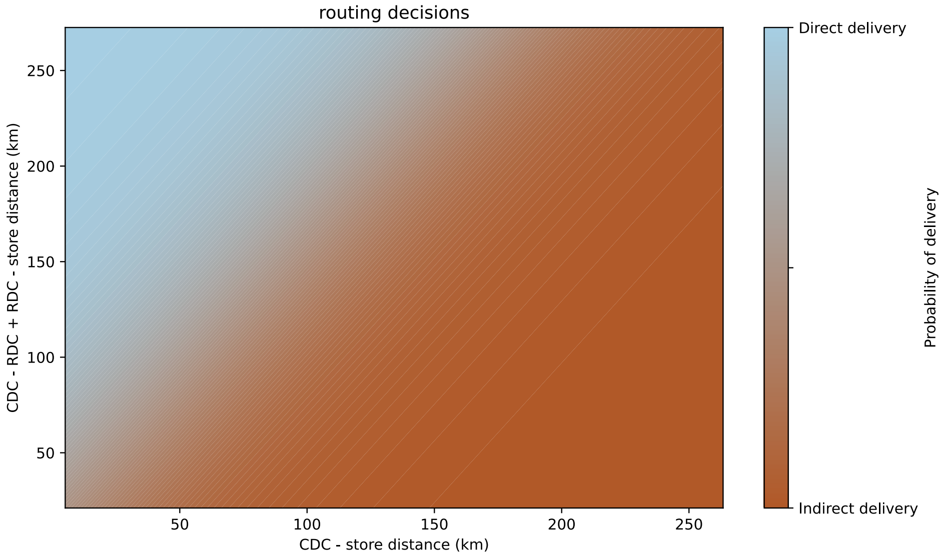


Figure 5.17: Regression of routing decisions of the first case

An overview of the coefficients and accuracies of all cases is shown in Table I.1. Corresponding figures of the regressions for the average volume are added to Appendix I. To make this rule-of-thumb more general, matrices can be created where advice can be obtained for each distance and volume combination. This makes it possible to give advice for the routing decision with reasonable certainty without immediately performing another two-day run.

6 Conclusion, discussion and recommendation

This chapter will first conclude the study using the main and sub-research questions formulated earlier. Next, a discussion will compare this research with the literature, and it will also indicate the limitations of the study. The chapter will conclude with a recommendation.

6.1. Conclusion

The goal of this research was to design a large distribution system for the allocation of stores and the routing of vehicles. Four sub-research questions have been formulated to help answer the main research question that is linked to the goal of this study. The first sub-research question can be answered based on the system analysis:

> What are the characteristics of this distribution system?

The distribution system analysed in this research focuses mainly on the tactical decision level. The decisions corresponding to this level include fleet size, frequency of delivery, allocation of stores to DCs and consolidation of deliveries. The flow between DCs and stores bounds the system. The locations are all known in advance and consist of CDCs, RDCs and stores. Stores are always allocated to one RDC and to all CDCs. The fleet consists of three types of trucks. The first type drives between CDCs and RDCs, the second between RDCs and stores and the third between CDCs and stores. The demand of stores may change over time, and the stores are supplied multiple times a week. Deliveries may be restricted to a certain time window in which the delivery must be done. A set of KPIs is formulated to track the performance of the network. The KPIs are stated as follows: total cost of transport, total distance and total number of vehicles. A case study is performed at a company in the Netherlands. The company has opened a new CDC, necessitating an overhaul of its distribution system. Stores need to be reallocated to the DCs. Three types of trucks must be taken into account for this case study. While the company wants the stores to be allocated to a single set of DCs, the network can be optimized for a representative day, and time does not need to be considered. Now that the characteristics of the system are known, the second sub-research question can be answered based on the literature:

> Which methods can be used to formulate a planning model regarding this distribution system?

Various properties of network designs have been investigated in the literature. The distribution system from this research focuses mainly on the tactical decision level. Therefore, the system can be formulated as a service network design. The stores can be supplied directly and indirectly, and consolidation can occur at several DCs. An arc-based representation aligns best with the consolidated deliveries, while the commodities and services can be distinguished per arc. For now, time can be left out of consideration, but the arc-based formulation allows us to include this in more in-depth research. The models of Crainic and Hewitt (2021) and Ambrosino and Scutella (2005) contain several characteristics of the distribution system from this research and are therefore used as an input for constructing the model. The other parts of the mathematical model, which mainly focus on the routing of the vehicles, are considered a contribution to the literature because they are focused on the structure of the distribution system of this research. The third research question can be answered based on the characteristics of the system and the methods to formulate the model:

How do you evaluate the performance of the system?

The performance can be tracked using the mathematical model. First, the mathematical model was constructed using some assumptions and the literature reviewed earlier. This model was then verified. This is an essential step for the continuation of the study. It was found that this model quickly runs into computational complexities, and therefore, the limits were examined. Improvements have been made based on a start solution, Gurobi input parameters, a callback function and a start solution. The main KPI of this research indicates the performance of the system, so the total cost of transportation. This KPI is the objective function of the model too. The computational characteristics of the model are essential to track. The complexity and run times influence the usability of the model. The fourth research question can be answered based on the answer to the previous research question.

What is the performance of the distribution system, given the data from the case study?

A computational plan was carried out to investigate the impact of various solution enhancements on the model. This showed that for all types of networks, combining all strategies yields the best outcome. The callback makes sure that the model does not remain in the same gap for an extended period of time, the input parameters are included for a higher solving efficiency, and the start solution is required to avoid issues in finding the first solution. When these methods are used, it is possible to reduce the gap and run time, which are crucial variables. The combination of these methods and the clustering of data enables us to solve a model of this size. This combination, which is specified for the distribution system of this research, is considered a contribution to the literature. What must be noted here is that the callback function and the clustering result in a sub-optimal and somewhat surrealistic output, which needs to be interpreted correctly. One needs to ensure that the gap between the given solution and the best solution and data aggregation results in an error and uncertainty in the results. Five cases are examined in the experimental plan. The base case consists of the company's input and route choices, the outcome of which has been recalculated by our model. This was then verified with the company's current calculations. Here, a difference emerged, explained by clustering data in this study's model. Based on this explanation and an expert check, it was assumed that the model works correctly. The first case involves optimising the model with respect to the current situation. From this, an improvement was seen regarding the main KPI, total cost, of 5.3%. The two other KPIs, the total distance and total number of vehicles, perform slightly less than the base case. The second case introduces the possibility of consolidation of demand at the CDCs. This case results in improvements on all metrics compared to the base case, with an improvement on the total costs of 5.4%. It is also an improvement over the first case with a small reduction in cost but a significant improvement for the other two KPIs. In the first case, demand from the CDCs cannot yet be consolidated. Consolidation is possible at the RDCs and this can explain why indirect deliveries have a higher share. There is a more optimal transport from the RDCs due to the higher load rates. Therefore, in the second situation, you see a strong transition because demand can be consolidated at the CDCs. Thus this transport also improves with a higher load factor as follows. Transport costs between CDCs and RDCs are reduced five times in the third case. This reduction is due to the increasing costs of last-mile delivery and improvements in shuttle journeys. This case provides insight into the behaviour of the model with respect to route choices. The model behaves as expected. When the costs of transportation from the CDCs to the RDCs reduce, the number of direct deliveries decreases and the number of indirect deliveries increases. Changes in volumes represent real-world behaviour and are included in the fourth case. Again, this case provides insight into the behaviour of the model with respect to route choices. When the volume decreases, the number of direct deliveries decreases and the number of indirect deliveries increases. When the volume increases, the number of direct deliveries increases, and the number of indirect deliveries decreases. The truckload is, therefore, essential to take into consideration. Transport costs decrease with increasing load rate. Thus, a key component of distribution is the consolidation of demand. The shuttle costs plus the processing costs at the RDC must be less than direct transport from the CDCs while combining at the RDCs. As a fifth case, the model is tried to solve without clustering on the the company's server by converting the model from Gurobi to PuLP. Unfortunately, the server also proved to be not powerful enough. As a result, it was impossible to compare the performance of clustering with the situation where the data was not clustered. Through a sensitivity analysis, the model was found to be sensitive in the performance metrics to changes in volume but not as sensitive to changes in route choices. For changes in transport costs between CDCs and RDCs, an inverse relationship was found. The computational values were also tracked to understand the optimisation performance, and the average run time per case equals 29 hours. The study evaluated results from three angles. First, it compared performance metrics, with the main metric, the total costs, being the most important. Consolidating demand at CDCs proved successful, resulting in a 5.4% cost improvement. However, the third and fourth cases couldn't be directly compared to others while they have a direct influence on the total costs. It was observed that lowering transportation costs between CDCs and RDCs increased indirect deliveries, while higher volumes led to more direct deliveries. Second, the study examined results based on routing decisions, noting differences in attractiveness between direct and indirect deliveries. Specific postal codes, especially 48 and 94 (RDC 20002), consistently influenced total costs and are recommended routing changes. Third, the study explored the relationship between routing decisions, distances, and volumes. While no direct links were found between routing decisions and distances or volumes individually, logistic regression offered accurate advice (ranging from 63.1% to 78.0%) for routing decisions based on store distance to CDCs, store distance to RDCs plus RDC-to-CDC distance, and store demand volume. Now that we also know the performance of the distribution system, we can conclude the study and answer the main research question:

> How can a large distribution system be designed for the allocation of stores and routing of vehicles?

In conclusion, this research aimed to design an effective distribution system for store allocation and vehicle routing, addressing the main research question. Four sub-research questions guided the study, delving into system characteristics, planning model formulation, system performance, and the generalizability of findings. The analysis revealed that consolidating demand at CDCs can lead to significant cost improvements because of the higher truckloads. Changing transport costs between CDCs and RDCs and adjusting order volumes can influence delivery strategies. Moreover, the study identified specific postal codes as crucial in routing decisions. Despite the absence of direct relationships between routing decisions, distances, and volumes, logistic regression models provide guidance. Notably, the findings suggest that this distribution model can be adapted to various scenarios with different network structures, making a general contribution to the literature. Overall, this research contributes valuable insights into the design of large-scale distribution systems, offering a foundation for more efficient and cost-effective store allocation and vehicle routing strategies.

6.2. Discussion

First, the generalisation of the case study's findings is formulated. Second, the results of this study will be compared with the literature to determine its contribution to the literature. Then, the limitations of this study will also be recalled. Finally, environmental considerations in the route decisions are discussed.

6.2.1. Generalisation of findings

The model can easily be implemented in other situations. However, this requires the structure of this distribution system to be of the same form. Thus, the network consists of CDCs, RDCs and stores. Here, three different types of vehicles can be used, each on its own part of the network. However, the characteristics of these vehicles can be adjusted. In addition, it is not necessarily necessary to receive orders from both CDCs and RDCs as the volumes can also be set to 0. The number of CDCs and RDCs is variable and can be reduced or increased. The same applies to stores. Thus, it can be seen that this model is generalised and can be easily reapplied. However, the model can be adapted to specific preferences, and a time element can be added, for example.

6.2.2. Comparison with literature

One of the results of this study indicates that distance from the DCs and volumes influence the routing decisions. Especially with a high truckload and a small distance to the CDC, it indicates direct delivery. Hiohi et al. (2015) discovered the same relationship between direct and indirect delivery. In case the delivery nodes are at a relatively close distance to the DCs, and if the demand size is large enough to ensure a high load of the vehicle, direct delivery is often advised. If one of these arguments does not hold, indirect delivery is more appropriate often. The same relationship between the volume and consolidation is stated by Crainic and Hewitt (2021), Bakir et al. (2021) and Crainic and Kim (2007). They state that no consolidation will take place when the size of demand is large enough compared to the capacity of the vehicle. Geurs (2022) experienced an equal problem regarding large-scale optimization. The customers are aggregated to the postal code 2 level too. An expectation in difference in costs is stated in this study and this aligns with the finding of this research, while there is a difference in the results of the base case and the validation. Abbasi et al. (2019) faced the same problem regarding the computational complexity of a large-scale problem. They mention that with an increase in nodes, the size of the scale to solve the problem quickly grows. They implement a Variable Neighborhood Search (VNS) algorithm to avoid this problem.

6.2.3. Limitations

This study contains several limitations like the exclusion of time, the exclusion of capacity limits of DCs, restrictions on the computation, the use of a representative day and clustering of data. First, time is assumed to have little influence on the optimal outcome. But to make the model align better with the real world, it is essential to include the time dimension. Restrictions on delivery time can be included, and vehicles can be selected for multiple routes per day. Second, the model tries to find an optimal solution for the entire distribution system. All DCs have a minimum and a maximum capacity. The minimum capacity is linked to the cost of using this DC. A DC must trade a specific size of demand to be valuable and net something. The maximum capacity is set more or less by the physical limitations. These capacity limits may influence the outcome. Third, restrictions are imposed for the optimization, like a callback function. The optimiser gets terminated if the change in gap development has been smaller than 1% for longer than 20 min. This results in a difference between the best-bound and the incumbent solution. So, a more optimal solution can be found without the callback function. Fourth, the data is based on a representative day. While the volume has a significant influence on the routing decisions, this can result in different outcomes when changes occur. The model, therefore, may be optimized for several volumes to have a more realistic set of decisions. Fifth, the data and hence optimisation are clustered to reduce complexity and thus stay within limits. With this, there is a sub-optimal outcome shown in Figure 6.1.

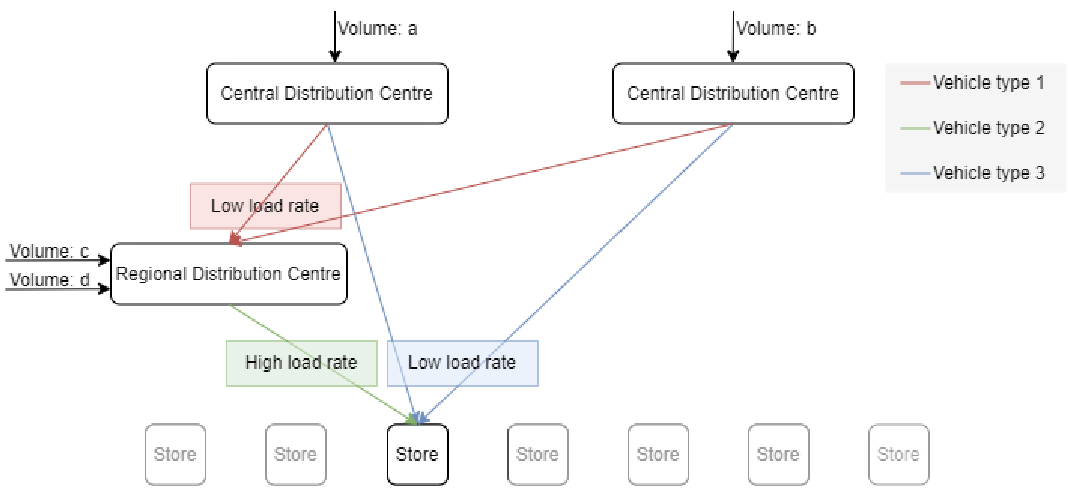


Figure 6.1: Sub-optimality in transport due to clustering

Two of the three transport flows are sub-optimal because of clustering. Route choices are determined by postal code, and then, based on that choice, transport is set up for all stores within that postal code. This, therefore, allows demand to be consolidated only for stores within that postal code. From both CDCs, only demand coming from there can be transported and thus cannot be consolidated with other demand from other DCs. However, vehicles can drive past multiple stores. At the RDC, demand from that RDC and from both CDCs can be consolidated if it comes through this DC first. So here, the load factor is in all likelihood higher, and this transport is more optimal. So, this flow is reasonably optimal compared to the other two. However, the flows can still be a lot more optimal, and thus the load factor higher if the transport is set up without clustering. Then, more demand can be consolidated. This is also one of the reasons why the share of indirect delivery is currently expected to be higher.

6.2.4. Environmental considerations in route decisions

In the current model, the total distance driven and average truckload are determined and included in the results, but no constraints are based on them. Given the recent developments regarding environmental awareness and the corresponding new regulations being drawn up for transport, it is essential to consider this. Total distance and truckload are somewhat linked. The higher the truck load, the fewer vehicles are needed, and this is likely to have a positive impact on the total distance driven. Now, of course, the cheaper option may not imply a smaller distance to be driven, but in general, it can be concluded that fewer vehicles result in fewer kilometres. In addition, empty truck movement is also present in the system. This is not directly included at the moment but is also definitely something to consider. Vehicle type 2, which drives from CDCs to the stores, are rented, and thus the empty truck movement of this type is not considered. For vehicle types 0 and 1, this does matter. When the demand is delivered to the stores, they drive back partly empty. However, stores have packaging that needs to be returned to the DCs, so the vehicles generally never drive back without any load. This could be an interesting follow-up study to include this in the optimisation. So, the choice may ultimately be made to positively influence the total distance driven, total number of vehicles and truckload in the objective function with which the cost may therefore be higher. This adjustment contributes to a better environment.

6.3. Recommendation

Recommendations may apply to current or upcoming research. The practice recommendations are discussed first. These guide the actions needed to put the study's findings into practice. Following that, suggestions for additional research will be made.

6.3.1. Practice recommendations

There are several recommendations for the company to use this model. Given the accuracy of this model to plan transport at vehicle level and to include actual distances, times and costs, route choices can be made even better. This avoids the more general assumptions for the number of stores visited by a vehicle in the company's model, for example. An important thing to note is that due to clustering, the results are more or less surrealistic. It is expected that the number of indirect deliveries is over-valued, while the load rate is higher from RDCs to the stores in the model. This is all due to the clustering and is, therefore, essential to avoid. More server capacity is needed to run the model, so this is something to focus on. While the model takes quite some time to get to the optimal solution, the logistic regression function can be used. The company may solve the model for several scenarios and calculate the coefficients of this function. This function can then be used for half a year, for example. Based on the distances to the stores and the volumes, the routing choice can be determined quickly when using this function. Furthermore, based on the results, it can be noted that consolidation at the CDCs is beneficial for the costs of transportation. Therefore, it is strongly advised to start planning to make this possible. In addition, based on the expectation of the increase in last-mile delivery costs and the decrease in transferring costs, it can be seen that this will involve a shift from direct to indirect delivery. This shift should, therefore, be factored into planning for the coming years.

6.3.2. Further research

The advice for further research is linked to the limitations of this current study. First, it is advised to include the time dimension in the model. This constraint can be written in the following form (Andersen et al., 2009) for example:

$$T = \{t\} = \{1, ..., T_{max}\} \tag{6.1}$$

$$\sum_{(i,j)\in A:T_i\leq t\leq T_j} y_{ij}^{vc} - \delta^{vc} = 0 \quad \forall v \in V, c \in C \tag{6.2}$$

$$\delta^{vc} \in 0,1 \quad \forall v \in V, c \in C \tag{6.3}$$

Here, the planning period is divided into time periods, Equation 6.1. The vehicles can only be used in one activity per time period, Equation 6.2. The δ^{vc} indicates whether a vehicle is used or not, Equation 6.3. Vehicles can be used multiple times and stores can be supplied within specific periods of time by including this. Second, the capacity of the DCs can be added quite easily. The total outgoing demand from the DCs must be between two boundaries, a minimum and a maximum. This constraint can be written in the following form for example:

$$\sum_{j\in N^+(i)} \sum_{p\in P} \sum_{v\in V} \sum_{c\in C} x_{ij}^{pvc} > min^i \quad \forall i \in (N^{cdc} \cup N^{rdc}) \tag{6.4}$$

$$\sum_{j\in N^+(i)} \sum_{p\in P} \sum_{v\in V} \sum_{c\in C} x_{ij}^{pvc} < max^i \quad \forall i \in (N^{cdc} \cup N^{rdc}) \tag{6.5}$$

In these formulas, the min^i and max^i can be set per DC. This ensures that all DCs remain valuable to keep in use. Third, avoiding the callback function is possible when more time is available for the optimization. This function is only added to limit the run time, but if the run time matters less, it can be switched off. In addition, a larger server to run the code on will also reduce the run time so again the time can be limited with an optimal outcome. Fourth, clustering has the most significant impact on the outcome. Therefore, it would be of great value to avoid this. This can be avoided by running the code on a large server. The fifth case of this case study already tries to avoid the clustering, but it still seemed too complex to solve. Therefore, another larger server needs to be used actually to run this. Here, choices can be made to connect all stores or, for example, only create arcs between stores for the five closest ones. Also, the choice can be made to optimise per RDC, thus reducing the size of the problem somewhat. The recommendation is to optimise per RDC with a restriction on the store arcs because it is expected that this alone will contribute significantly to the outcome and is already a reasonable approximation of the actual optimal situation.

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A

Paper

The design of a large distribution system for the allocation of stores and the routing of vehicles

A Dutch case study

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Abstract

Efficient distribution is crucial in many industries today, especially in retail, where customer demand grows rapidly. The rise of online shopping and automation has led to a shift towards using Central Distribution Centers (CDCs). Retailers are part of this trend, which requires them to rethink how they handle their distribution. This study focuses on two main challenges: first, changing the distribution system to adapt to new CDCs, and second, dealing with the computational complexities when optimizing the distribution on a large scale. The aim is to design a large distribution system for the allocation of stores and the routing of vehicles. In this research, we introduce new elements to the mathematical model, which are mostly focused on the routing of vehicles. We also improve the computation by implementing a start solution, adjusting the parameters of the Gurobi optimization tool and including a callback function which bounds the run time. This model will determine the routing decisions for demand from several DCs to the stores and find the optimal vehicle routes. Through a case study at a company in the Netherlands, this paper provides insights into the changing world of distribution systems. It offers practical solutions for businesses trying to make their distribution processes better. The analysis revealed that consolidating demand at CDCs can lead to significant cost improvements while altering transport costs between CDCs and RDCs and adjusting order volumes can influence delivery strategies. Despite the absence of direct relationships between routing decisions, distances, and volumes, a logistic regression model provides guidance.

Keywords: Distribution System Design, Location-Routing Problem, Transportation, Retailer-Depot Allocation

1 Introduction

Distribution plays a significant role in many industries due to growing demand (Munasinghe & Rupasinghe, 2016). The same is true for the retail industry. Retailers must ensure that their supply chains are effective and can fulfil consumer needs. To do this, a focus is needed on inventory management and transportation (Lagorio & Pinto, 2021). Several factors influence the need for innovation in inventory management. First, as companies struggle to recruit employees, they are forced to automate their inventory systems (PWC, 2023). Second, customers are increasingly getting used to a flexible market where last-minute changes can be made to their orders and orders can be delivered at short notice (Tarry, 2022). This plays a significant role in the growth of e-commerce in the retail industry (Morgan Stanley, n.d.). To increase the efficiency of a company's inventory system and transportation, there is a trend of more and more companies returning to automated Central Distribution Centres (CDCs) where they can easily handle a large proportion of orders at one location. Bol.com and Albert Heijn are examples of such companies where this trend has also impacted and which has also brought to life an automated CDC that (partially) replaces several non-automated Distribution Centres (DCs) (Stad, 2022). The building of new CDCs forces companies to adjust their transport schedules. Stores need to be reassigned to the Regional Distribution Centre (RDC) they are currently assigned to or to the new CDC. The demand can be delivered from the CDC directly or from the CDC via an RDC. Consolidation of demand can take place at an RDC, which can make it attractive to deliver from that DC. Here, financial and environmental aspects need to be taken into account by the company. The financial aspect is important to remain competitive in the market. Higher costs in transport and logistics will eventually be passed on to the customer. Today, the environment is becoming an increasingly important aspect to consider. Society and the government are both encouraging improvements in this area. As a direct result, companies must have this focus too.

A challenge that arises from the described developments is the need for a change of structure of the distribution system. This challenge can be split into two parts: an industrial challenge and a computational challenge. The industrial challenge arises due to the opening of the new DC. The distribution system must be adjusted to reallocate the stores to the DCs. The computational challenge arises due to the size of the distribution system. The larger the distribution system, the higher the number of options for allocation and routing, which all need to be evaluated. The complexity influences the computational forces needed to solve the model. Therefore, the purpose of this study is to provide an answer to the following question: *How can a large distribution system be designed for the allocation of stores and routing of vehicles?*

Although there have been numerous papers written on this subject and its variations, there is still room for research. The mathematical model for the distribution system of this research will be built on existing models. New parts will be added to the combination of existing parts as a scientific contribution. The new parts will involve various modalities that can be deployed on limited parts of the system. Adjustments regarding the input parameters of the optimization tool, a callback function and math-based heuristics will be implemented to avoid reaching the computational limits. This study will provide a solution to the more complex allocation of stores and help businesses further design their distribution system.

A case study will be conducted in the Netherlands. The company has several DCs and a large number of locations. The company has opened a new CDC, necessitating an overhaul of its distribution system. Stores need to be reallocated to the DCs.

2 System analysis

To correctly represent the distribution system in a mathematical formulation, it is essential to analyse the system. The main characteristics are the decision level of this optimization, the system's boundaries, the locations of DCs and stores, modalities used for the distribution, demand specifications, time restrictions and performance metrics.

This study mainly focuses on the tactical decision level of a network model. The tactical decision level corresponds to medium-term decisions regarding the design of a network. These decisions include the allocation of stores to DCs, the frequency of delivery, consolidation of deliveries and the fleet size, among others. DCs and stores bound the distribution system, Figure 1. This means that the flow of raw materials, the production of commodities and the transport from the factories to the DCs are left out of consideration. The system boundary is indicated in the figure with a black dotted line, and the flows left out of consideration are indicated with black arrows. Besides these flows, the flow from the stores to the customers is left out of scope too. Only the flow within the network between the DCs and stores is considered.

The locations of all DCs and stores are known. The DCs have a fixed capacity, and the stores have a known demand. The system includes CDCs and RDCs. Stores can be supplied from the CDC directly or from the CDC via an RDC. The focus of this research is on the distribution of demand from the DCs to the stores. All stores receive demand from all CDCs and from one RDC. For each store, the decision must be made per CDC whether or not there is direct or indirect delivery. The delivery from RDCs will, therefore, include consolidated deliveries. This results in a balance between the costs of delivery from the CDC directly to the stores or the costs from the CDC to the RDC, consolidation costs and costs of delivery from the RDC to the stores. Usually, the demand for stores changes over time. Stores are supplied multiple times a week. Routing schedules can be constructed per day of delivery. But while this study focuses on the tactical level, a representative day will be chosen to allocate the stores. The demand is multi-commodity while it has an origin, a destination and a type and number of products which differ per store. A store receives demand from all CDCs and from one of the RDCs. There are three possible scenarios for delivery. In the first scenario, the demand from all CDCs and the RDC is delivered directly. No consolidation of demand takes place in this scenario. In the second scenario, demand is delivered directly from a part of the CDCs. The other CDCs send demand to the RDC, where it will be consolidated for delivery to the store. In the third scenario, all demand from the CDCs is sent to the RDC for complete delivery consolidation. The demand for this model is transported by trucks. There are three types of trucks used for delivery. The first truck type is used for transportation from the CDCs to the RDCs. The second truck type is used for transportation from the RDC to the stores. The third truck type is used for direct transportation from the CDCs to the stores.

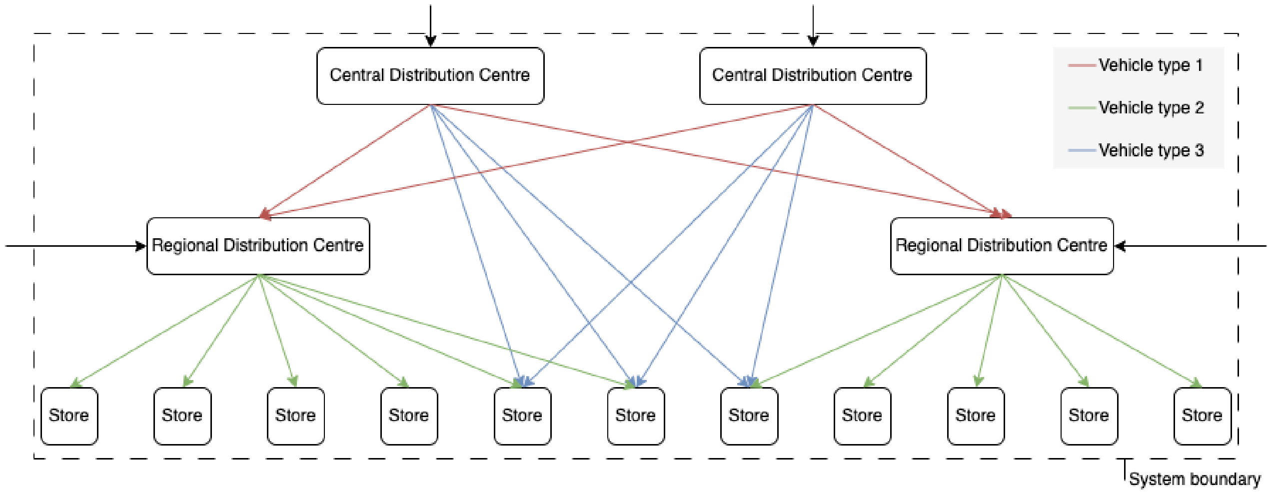

Figure 1: Structure of the distribution system

This results in a heterogeneous fleet. The capacities and costs of these trucks may differ by type. The delivery to stores may be restricted by certain times. Since converting a model to a time-space network will have a significant impact on the computational forces for optimising the model and the assumption is made that the time constraints will not have particularly large influences on the optimal outcome, this is left out of consideration.

Performance metrics of this distribution system are determined using Key Performance Indicators (KPIs). The most important KPI is the total cost of transport. The total cost of transport includes the routing costs and the processing costs. The routing costs consist of costs of transportation between a CDC and an RDC, a CDC and a store, an RDC and a store and between two stores. The processing costs are variable and based on the size of demand transported from the CDCs to the RDCs. The second KPI is the total distance driven by all vehicles. The third KPI equals the total number of vehicles needed to transport all demand. Four other metrics are used to evaluate the performance of the model. These metrics are the costs per container, load per vehicle, drop size per stop and the number of stops per vehicle. The costs per container are preferably as high as possible, and the load per vehicle, drop size per stop and stops per vehicle as low as possible.

3 Literature study

Literature can be studied now that the system characterizations are specified. Methods from previous studies can be analysed to determine which methods suit best for this study.

Network design is a general term for various distribution system problems. Network designs contain four decision layers: topology, location, allocation and routing decisions. The topology layer decides the structure of the network. The location layer determines where the facilities must be located in the network. The allocation layer allocates customers to open facilities. The routing layer decides the routes for the vehicles to satisfy the demand. Network designs are widely used in distribution system problems (Crainic, 2000). Network design deals with strategic decisions which may contain significant investments and which focus on the long term. These formulations are defined on graphs, which have nodes that are connected by links. When the links are directed, they are represented by arcs. The nodes can represent origins and destinations. The links may have costs, length and capacity. The main objective of network designs is to select links in the network to satisfy all demands for distribution by minimizing the total costs. A commonly used version of the network design is the linear cost, incapacitated, multi-commodity (MCND) network design. Multi-commodity networks include two or more commodities that must be distributed from a specific origin to a destination (Salimifard & Bigharaz, 2022). Multi-modality networks use multiple modes of transportation for the distribution of demand. Combinations of these modes can be made to reduce the total costs of the transport. Another advantage of multi-modal transportation corresponds to the sustainability of the transport, which can be increased.

A service network design focuses mainly on the tactical level but includes some strategic and operational characteristics. This aligns with the decision level of the distribution system from this research. A service network design aims to plan the resources and activities to satisfy the demand (Crainic & Hewitt, 2021). The operation of a vehicle is called a service. Service network design usually takes place in the context of transportation based on consolidation. Consolidated transportation considers capacities and service schedules. Several orders can be combined within the same vehicle, and multiple vehicles may be used for one delivery from origin to destination. The service contains a route and physical as well as operational characteristics. The physical characteristics include the vehicle type and vehicle capacity, for example. The operational characteristics include the costs, total trip time and departure time. The goal of tactical planning is to create a distribution plan and schedule that will minimise the negative effects of consolidation, meet customer demand and service-quality standards and be profitable and effective to run. It discusses system-wide operational planning to choose and schedule services and transfer and consolidate activities at terminals. The service network design combines two sets of choices. The first set corresponds to the frequencies or schedules of the operations of the services. The second set includes the routes. The routes contain an origin, destination and intermediate stops. A service network design contains various characteristics. As stated, the design can be capacitated or incapacitated (Andersen et al., 2009), one single or multiple commodities may be used, and the network may allow consolidation of deliveries (Andersen et al., 2009), so it can consist of direct and/or indirect deliveries. Other characteristics are the flow type of the model, which can be arc or path-based, fixed charge or variable charged (Andersen et al., 2009), and time-based so that frequencies can be set.

Hubs are used to connect a large number of nodes by using a relatively small number of links in Facility Location - Network Design (FL-ND) (Maknoon, 2022). Consolidation can take place at hub facilities. The costs are affected by the network design and economy of scale rules often. All locations are represented by nodes, the infrastructure by edges and the route of products by arcs. An origin, a destination and a volume characterize the commodities. The connection layout of the network can be grouped into six design basics: a line topology, a star topology, a ring topology, a tree topology, a mesh topology and a hybrid topology. Hubs are connected by a single line in the line topology. All nodes are linked to a single, central, hub through which all traffic passes. Each hub is connected to two other hubs in a ring topology. This results in a low probability of failure. In a mesh topology, hubs are partially or fully connected. In the hierarchy, tree topology, structure all locations are arranged in a hierarchal way. The network of Ambrosino and Scutella (2005) is an example of a tree analysis. This network consists of a plant, multiple central depots, multiple regional facilities and clients. The use of depots and facilities helps to reduce the last mile delivery costs as low as possible, while transport between facilities is often against a significantly lower cost. Distribution centres can also function for resilience against disruptions (Alikhani et al., 2021). The flow of commodities can be re-routed easily when a network consists of multiple distribution centres.

The hybrid topology is a combination of several other topologies. To create the proper network structure, four steps can be followed. The first two steps are linked to the design decisions, and the second two to the operational decisions. The design decisions consist of location decisions, so what locations should be considered as a hub and topology decisions, so which link needs to be selected. The operational decisions consist of allocation decisions, so the assignment of supply and demand nodes to hubs and routing decisions, so how demand is routed between the origin and destination. This study focuses on the operational decisions of the distribution system. The stores need to be allocated to the DCs, and the commodities need to be routed through the network. The tree topology is assumed to best fit as a network structure. It is believed that the design decisions have already been made.

The flow in a service network design can be modelled path-based or arc-based (Ohmori, Yoshimoto, et al., 2019). Arc (or link) based modelling focuses on all individual links between the nodes in the network (Andersen et al., 2009). The design and optimization are performed on this level. Path (or route) based modelling focuses on all routes that connect the origin and destination nodes. The design and optimization are performed on this level of connecting the origin and destination with paths. This research implements an arc-based model.

Some networks include direct and indirect deliveries, just like the distribution system from this research. The distinction can be made based on the size of the demand of particular locations. When a network consists of plants and warehouses, it may be more cost-effective to first transport orders from a plant to a warehouse, where they can be consolidated and delivered simultaneously. Trucks can be used with a higher load factor. A tree topology can be used to apply various layers of distribution centres (Munasinghe & Rupasinghe, 2016). With this, direct and indirect supplies can be achieved. Demand delivery can be made more efficient by consolidating deliveries at a cross-dock (Sung & Song, 2003). Demand is distributed from origin to destination via distribution centres. The demand arrives at a distribution centre and is directly loaded in another vehicle for the final delivery. The delivery of demand that originates from the same place but has delivery locations in different regions may be more efficient by splitting this delivery. The network, including direct and indirect deliveries, may differ in structure. The network of Cheong et al. (2007) contains several suppliers, consolidation hubs, warehouses and manufacturing plants. All suppliers are linked to a single consolidation hub, and all manufacturers are linked to a dedicated warehouse. Consolidated shipping takes place between the consolidation hubs and the warehouses. Another network, which consists of the flow of consumer goods, is designed by Cintron et al. (2010). This network deals with four options for the transportation of goods. A combination of direct and indirect deliveries reduces the total transportation costs.

The distribution system from this study focuses mainly on the tactical decision level. Therefore, the system can be formulated as a service network design. The stores can be supplied directly and indirectly, and consolidation can occur at several DCs. An arc-based representation aligns best with the consolidated deliveries, while the commodities and services can be distinguished per arc.

4 Mathematical model

The goal of this study is to design a large distribution system for the allocation of stores and the routing of vehicles. The scope of this study is based on the system analysis. The movement between DCs and stores defines the system's boundaries. The locations, which include CDCs, RDCs, and stores, are all known in advance. Stores are assigned to all CDCs, but they are only assigned to a single RDC. Three different vehicle types comprise the fleet: vehicle type 0 transports demand from a CDC to an RDC, vehicle type 1 transports demand from an RDC to a store and vehicle type 2 transports demand from a CDC to a store. This model converts the real-world situation to a mathematical formulation using the methods from the literature study.

Several assumptions are made for the model. First, the volume of a commodity may be larger than the capacity of a truck, so split delivery is possible. Second, a split delivery of a commodity must always originate from the same DC. So the complete commodity is delivered from the CDC to the stores, or from the CDC via an RDC to the stores, or from the RDC to the stores. Third, consolidation of commodities is possible for delivery. This allows it to operate with a higher truckload. Fourth, transportation is possible between a CDC and an RDC. There is no transportation possible between CDCs and between RDCs. Fifth, vehicle type 0 may start and end at a CDC and only drive to an RDC. Vehicle type 1 may start and end at an RDC and may only drive to stores. Vehicle type 2 may start at a CDC and may only drive to stores. Sixth, the demand of all stores is met. Seventh, DCs have no capacity limit as it is assumed that the demand of all stores does not exceed this. Eight, a maximum number of deliveries is set per store based on the demand size and the number of DCs it is supplied from.

This model is based on the methods from the papers of Crainic and Hewitt (2021) and Ambrosino and Scutella (2005). Constraints (2, 9, 17) are inspired by the formulations of Crainic and Hewitt (2021). Constraints (3, 5, 10, 11) are inspired by the formulations of Ambrosino and Scutella (2005). The objective function (1) and constraints (4, 6, 7, 8, 12, 13, 14, 15, 16, 18) are a potential contribution of this paper to the literature.

4.1 Sets, parameters and variables

The sets of this model can be divided into five categories. The first category contains all sets of nodes. The second category includes all sets of arcs. The third category contains a set of commodities. The fourth category comprises sets corresponding to the vehicles. The fifth and last category includes the set of route types. The parameters of this model can be divided into three categories. The first category contains all parameters regarding the commodities. The second category includes all parameters regarding the vehicles. The third and last category contains parameters regarding the routes. Table 1 describes the sets, parameters and decision variables.

4.2 Formulation

$$
\begin{aligned}
min \sum_{(i,j)\in A^{dc+}} \sum_{v\in V} fc_{ij} \cdot y_{ij}^{v0} + \sum_{(i,j)\in A^{dc+}} \sum_{p\in P} \sum_{v\in V} pc \cdot x_{ij}^{pv0} + \sum_{(i,j)\in (A^{rdcs+} \cup A^{rdcs-} \cup A^{s})} \sum_{v\in V} (tc^1 \cdot dis_{ij} + hc^1 \cdot time_{ij}) \cdot y_{ij}^{v1} + \\
\sum_{(i,j)\in A^{rdcs+}} \sum_{v\in V} hc^1 \cdot flt^1 \cdot y_{ij}^{v1} + \sum_{(i,j)\in (A^{rdcs+} \cup A^{s})} \sum_{v\in V} hc^1 \cdot fut^1 \cdot y_{ij}^{v1} + \sum_{(i,j)\in A^{rdcs+}} \sum_{p\in P} \sum_{v\in V} hc^1 \cdot (vlt^1 + vut^1) \cdot x_{ij}^{pv1} + \\
\sum_{(i,j)\in (A^{cdcs+} \cup A^{s})} \sum_{v\in V} (tc^2 \cdot dis_{ij} + hc^2 \cdot time_{ij}) \cdot y_{ij}^{v2} + \sum_{(i,j)\in A^{cdcs+}} \sum_{v\in V} hc^2 \cdot flt^2 \cdot y_{ij}^{v2} + \sum_{(i,j)\in (A^{cdcs+} \cup A^{s})} \sum_{v\in V} hc^2 \cdot fut^2 \cdot y_{ij}^{v2} + \\
\sum_{(i,j)\in A^{cdcs+}} \sum_{p\in P} \sum_{v\in V} hc^2 \cdot (vlt^2 + vut^2) \cdot x_{ij}^{pv2} + \sum_{(i,j)\in A^{cdcs+}} \sum_{v\in V} erc \cdot y_{ij}^{v2}
\end{aligned}
\tag{1}
$$

Subject to the following constraints:

$$
\sum_{j\in N^+(i)} y_{ij}^{vc} - \sum_{j\in N^-(i)} y_{ji}^{vc} = 0 \quad \forall i \in N, v \in V, c \in C \tag{2}
$$

Sets		Description
Nodes	N^s	Set of store nodes
	N^{rdc}	Set of Regional Distribution Centre nodes
	N^{cdc}	Set of Central Distribution Centre nodes
	N	Set of all nodes ($N^s \cup N^{rdc} \cup N^{cdc}$)
Arcs	A^{dc^+}	Set of arcs from CDCs to RDCs
	A^{dc^-}	Set of arcs from RDCs to CDCs
	A^{cdcs^+}	Set of arcs from CDCs to stores
	A^{cdcs^-}	Set of arcs from stores to CDCs
	A^{rdcs^+}	Set of arcs from RDCs to stores
	A^{rdcs^-}	Set of arcs from stores to RDCs
	A^s	Set of arcs between stores
	A	Set of all arcs ($A^{dc^+} \cup A^{dc^-} \cup A^{cdcs^+} \cup A^{cdcs^-} \cup A^{rdcs^+} \cup A^{rdcs^-} \cup A^s$)
	$N^+(i) = \{j \in N : (i,j) \in A\}$	Outward arcs of node i
	$N^-(i) = \{j \in N : (j,i) \in A\}$	Inward arcs of node i
Commodities	P	Set of commodities
Vehicles	V	Set of vehicles
	C	Set of vehicle types
Routes	Q	Set of route types $\{\text{indirect delivery}, \text{direct delivery} : 0, 1\}$
Parameters		**Description**
Commodities	vol^p	Volume of commodity p
	o^p	Origin of commodity p
	d^p	Destination of commodity p
	rdd^p	Indicates whether direct delivery of commodity p is permitted
Vehicles	cap^{vc}	Capacity of vehicle of type c
	tc^c	Transportation cost per kilometer of vehicle of type c
	hc^c	Transportation cost per hour of vehicle of type c
	flt^c	Fixed loading time of vehicle of type c
	vlt^c	Variable loading time of vehicle of type c
	fut^c	Fixed unloading time of vehicle of type c
	vut^c	Variable unloading time of vehicle of type c
	fc_{ij}	Fixed cost for delivery between a CDC i and an RDC j
	pc	Processing costs per unit of demand from CDC at RDC
	erc	End of route costs for vehicle v of type 2
Routes	dis_{ij}	Distance between node i and j
	$time_{ij}$	Travel time between node i and j
	M	Very large number
Decision variables		**Description**
	x_{ij}^{pvc}	Non-negative real number representing the demand volume of commodity p transferred on arc (i,j) by vehicle v of type c
	y_{ij}^{vc}	Binary variable, 1 if vehicle v of type c is selected for design arc (i,j), 0 otherwise
	z_i^{pq}	Binary variable, 1 if either commodity p is transported from a CDC to an RDC (z_i^{p0}) or if commodity p is transported from a CDC to a store (z_i^{p1}), 0 otherwise

Table 1: Sets, parameters and decision variables

$$\sum_{i \in N^{rdc}} \sum_{j \in N^{+}(i)} y_{ij}^{v1} \leq 1 \quad \forall v \in V \tag{3}$$

$$\sum_{(i,j) \in (A^{dc+} \cup A^{dc-} \cup A^{cdcs+} \cup A^{cdcs-})} y_{ij}^{v1} = 0 \quad \forall v \in V \tag{4}$$

$$\sum_{i \in N^{cdc}} \sum_{j \in N^{+}(i)} y_{ij}^{vc} \leq 1 \quad \forall v \in V, c \in [0,2] \tag{5}$$

$$\sum_{(i,j) \in (A^{cdcs+} \cup A^{cdcs-} \cup A^{rdcs+} \cup A^{rdcs-} \cup A^{s})} y_{ij}^{v0} = 0 \quad \forall v \in V \tag{6}$$

$$\sum_{(i,j) \in (A^{dc+} \cup A^{dc-} \cup A^{rdcs+} \cup A^{rdcs-})} y_{ij}^{v2} = 0 \quad \forall v \in V \tag{7}$$

$$\sum_{i \in N^{-}(j)} \sum_{v \in V} \sum_{c \in C} y_{ij}^{vc} \leq \sum_{i \in N^{cdc}} \sum_{p \in (P: i = o^p, j = d^p)} z_i^{p1} \cdot \frac{vol^p}{cap^{0\,2}} + \frac{\sum_{i \in N^{cdc}} \sum_{p \in (P: i = o^p, j = d^p)} (1 - z_i^{p1}) \cdot vol^p + \sum_{p \in (P: o^p \in N^{rdc}, j = d^p)} vol^p}{cap^{0\,1}} + 0.99 \quad \forall j \in N^s \tag{8}$$

$$\sum_{j \in N^{+}(i)} \sum_{v \in V} \sum_{c \in C} x_{ij}^{pvc} - \sum_{j \in N^{-}(i)} \sum_{v \in V} \sum_{c \in C} x_{ji}^{pvc} = \begin{cases} vol^p, & i = o^p \\ -vol^p, & i = d^p \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N, p \in P \tag{9}$$

$$\sum_{j \in N^{+}(i)} x_{ij}^{pvc} - \sum_{j \in N^{-}(i)} x_{ji}^{pvc} = 0 \quad \forall i \in (N^{cdc} \cup N^s : i \neq o^p, i \neq d^p), p \in P, v \in V, c \in C \tag{10}$$

$$\sum_{i \in N^{-}(d^p)} \sum_{v \in V} \sum_{c \in C} x_{id^p}^{pvc} = vol^p \quad \forall p \in P \tag{11}$$

$$\sum_{p \in P} x_{ij}^{pvc} = 0 \quad \forall (i,j) \in (A^{dc^-} \cup A^{cdcs^-} \cup A^{rdcs^-}), v \in V, c \in C \tag{12}$$

$$\sum_{v \in V} x_{ij}^{pv0} \leq M \cdot z_i^{p0} \quad \forall i \in N^{cdc}, j \in N^{rdc}, p \in P \tag{13}$$

$$rdd^p \leq z_i^{p0} \quad \forall i \in N^{cdc}, p \in P \tag{14}$$

$$\sum_{j \in N^s} \sum_{v \in V} x_{ij}^{pv2} \leq M \cdot z_i^{p1} \quad \forall i \in N^{cdc}, p \in P \tag{15}$$

$$z_i^{p0} + z_i^{p1} = 1 \quad \forall i \in (N^{cdc} : i = o^p), p \in P \tag{16}$$

$$\sum_{p \in P} x_{ij}^{pvc} \leq cap^{vc} \cdot y_{ij}^{vc} \quad \forall (i,j) \in A, v \in V, c \in C \tag{17}$$

$$y_{ij}^{vc} \leq \sum_{p \in P} x_{ij}^{pvc} \quad \forall (i,j) \in (A^{dc^+} + A^{cdcs^+} + A^{rdcs^+}), v \in V, c \in C \tag{18}$$

$$x_{ij}^{pvc} \geq 0 \quad \forall (i,j) \in A, p \in P, v \in V, c \in C \tag{19}$$

$$y_{ij}^{vc} \in \{0,1\} \quad \forall (i,j) \in A, v \in V, c \in C \tag{20}$$

$$z_i^{pq} \in \{0,1\} \quad \forall i \in N^{cdc}, p \in P, q \in Q \tag{21}$$

The objective function minimizes the total sum of the costs (1). The total costs consist of a fixed cost for delivery between a CDC and an RDC, a variable processing cost based on the size of the demand from the CDC at the RDC, a variable operating cost based on the distance driven for delivery from the DCs to the stores, a variable operating cost based on the travel time for delivery from the DCs to the stores, a fixed cost for the loading of a vehicle at DCs, a fixed cost for the unloading of a vehicle at a store, a variable costs for the loading and unloading of a vehicle per unit of demand and a fixed cost for the end of a route of vehicle type 2. Constraint (2) is the design balance conservation. Constraints (3 - 7) indicate routing restrictions. Vehicle type 1 may only start from one of the RDCs and cannot enter or leave a CDC, vehicle types 0 and 2 may only start from one of the CDCs. Vehicle type 0 cannot drive from a DC to a store, and vehicle type 2 cannot drive to an RDC and, therefore, drive not from an RDC to a store. The maximum number of vehicles allowed to deliver at a store is set by constraint (8). This maximum is based on the volume of demand and the number of DCs it is supplied from. For example, when a store is supplied from both CDCs directly, and an RDC, and the volume per DC is less than the capacity of a vehicle, the maximum number of vehicles to deliver equals three. So, the number of deliveries is based on the number of DCs and the minimum number of vehicles needed to transport all demand, taking the capacity of the vehicles into account. Constraint (9) is the flow conservation. It ensures that the commodity flow on all incoming arcs of a node equals the commodity flow of all outgoing arcs. There are two exceptions: the origin node and the destination node of a commodity. At these places, the commodity flow equals the volume of the demand. Constraint (10) restricts commodities from switching from vehicles. All demand is satisfied by constraint (11). Constraint (12) ensures that demand can not be sent back to a DC. Constraints (13 - 16) force that the total volume of a commodity must be delivered via the same DCs while considering split deliveries. So, if the commodity is originated at a CDC it can be delivered from the CDC directly or via one of the RDCs. Constraint (17) makes sure that a service's capacity cannot be exceeded, therefore, if one or multiple commodities are transported by a vehicle, the vehicle must be big enough to carry these. A vehicle may not leave a DC without a commodity due to constraint (18). Constraints (19 - 21) are the variable constraints.

4.3 Implementation

The model is translated to code using the Spyder software. The code is written in the Python language. The model is solved using Gurobi. An HP ZBook Studio G4 with an Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz 2.81 GHz and 16 GB of Random-Access Memory (RAM) is used as hardware for running the model.

4.4 Verification

A verification is performed to see if the model is a correct conversion of the real-world problem. Different verification steps focus on other parts of the model to check this. First, the model's outcomes are compared with outcomes calculated by hand for a small-scale network. Second, two parameters are changed, and the model's behaviour is assessed. These parameters consist of the costs of transportation between the CDCs and RDCs and the demand. Third, the store arcs have been added to check for consolidation of demand. Finally, a large-scale network is inserted to see if the model performs as expected. The five verification steps are all successful, and no inaccuracies are found in the model results. With that, it is assumed that the model works properly and can be implemented in the case study.

4.5 Computation

The model quickly becomes very complex to solve. This is due to the number of variables required. The complexity influences the computational forces needed to solve the model. The goal is to find the optimal solution for the allocation of stores and the routing of vehicles. The Gurobi solver aims to find a solution in which the lower and upper bound of the solution is equal to 0%. Although the optimality gap can be adjusted, the problem is still very complex. The more complex the model, the longer it takes to solve it, and the more RAM is needed to store the values. The number of variables needed to solve the problem can be determined based on the indices of the different variables and can be calculated with Equation 22.

$$\text{number of variables} = |A| \cdot |P| \cdot |V| \cdot |C| + |A| \cdot |V| \cdot |C| + |N^{cdc}| \cdot |P| \cdot |Q| \tag{22}$$

To give an insight into the complexity of this model, a network consisting of 2 CDCs, 4 RDCs and 100 stores is chosen. This network results in $832 \cdot 10^6$ variables needed to solve the problem. A computational limit for the hardware that contains 16 GB RAM is found for a model containing an approximate number of variables of $27 \cdot 10^6$ in total. To reduce the complexity of this model and the corresponding run times, a start solution can be added, the Gurobi input parameters can be adjusted, and a callback function can be implemented. A start solution vector can save time searching for the initial solution and possibly reduce the optimisation area. When optimizing using the Gurobi tool, there are default input parameters set that affect this process. These parameters can be adjusted to match the model better. This model can be characterized as Mixed Integer Programming (MIP). The heuristic, cuts, MIP focus and pre-sparsify have been set to 0.0001, 2, 1 and 2, respectively. Finally, a callback functionality is used that reduces optimisation time. The optimiser gets terminated if the last iteration time is longer than 20 min ago, i.e. if the change in gap development has been smaller than 1% for longer than 20 min. All these methods together improve the computation.

5 Case study

The case study is carried out for the distribution system of a company in the Netherlands. New locations are opened yearly, resulting in a continuous need to improve the transport and logistics network. To continue to meet growing demand, processes need to be made more efficient, and automation plays a significant role in this. Therefore, a fully automated distribution centre will be opened. A large proportion of all products will be stored in this CDC. A store will receive demand from both CDCs and from one of the RDCs. Now, the question is whether it is more cost-effective to first transport demand from the CDC to the RDC before delivery to the stores. The locations are known in advance and include two CDCs, four RDCs and a large number stores. The goal is to allocate stores to one or multiple DCs. While the stores will always be supplied from an RDC, the main goal is to determine whether or not they are supplied from the CDC directly or whether this demand is consolidated at an RDC and transported from there. The order sizes of stores are different per day of delivery. Still, while the company is interested in a fixed allocation of the stores to the DCs, the demand for a representative day is considered for optimization. The distribution network is clustered two times due to the complexity. First, all data is divided into four groups, one group per RDC (N^{rdc}). The stores (N^s) are assigned to one of the groups based on the allocation. The first two digits of the postal codes of all locations are used to group this distribution network. A small set of locations is not clustered based on their postal code due to their size of demand. A total of 14 locations are left out of the clusters. Clustering these locations would have too big of an influence on the optimization of locations in the same postal code area.

5.1 Computational plan

A computational plan is drawn up to test the various methods on the company's inputs. This analysis consists of a base configuration, i.e. without tuning, and four other configurations. Three different sets of input data are compared for each configuration. The start solution added in the first configuration has no significant influence on the results besides that the model has difficulties with finding a start solution, which results in a situation where it can take hours or days to find the first solution. The second configuration includes adjustments to the input parameters of the Gurobi tool. It can be noted that the optimization time reduces drastically, especially for the more extensive networks. The gap difference decreases for the large network too. A callback function is added in the third configuration. This callback function is helpful for more extensive networks where the gap stabilises more or less after a while. The time limit is reached in the first two configurations and the base configuration, but the run time does not reach its limit in the third configuration. Combining all methods results in the most optimal situation for all types of networks. The start solution is needed to avoid difficulties in finding the first solution, the input parameters are added for a higher solving efficiency, and the callback ensures that the model does not stay around the same gap for a long time. In the end, the gap and run-time are important factors to reduce, and that is achieved in the combination of all methods.

5.2 Experimental plan

The distribution network is analyzed in a base case, a validation and five new cases. Optimal results are determined, and insight into the network can be gained from these cases and their scores on the performance metrics. The base case represents the company's current situation. It uses the existing data as input for the model, but also the company's route choices. Next, the outcome of this case is validated with the company's calculations concerning the KPIs regarding the same input. There is a difference in the main KPI, the total costs of transport of 7.3%. This difference is expected to be the result of the callback function that is used and the clustering of the data. The callback function stops the optimisation when a specific criterion is met. A difference between the incumbent and best-bound solution is the result. The clustering disables the opportunity of consolidating the demand. This leads to less efficient transport, while the truckload is expected to be lower. Based on this, it is assumed that the model performs correctly. Five experimental cases follow the validation. The first case is an optimisation based on the current data. Here, only the route choices are redefined by the model. The first case has an improvement on the total costs of transport of 5.3% compared to the base case. The two other KPIs, the total distance and total number of vehicles, perform slightly less than the base case. The other four metrics perform better. This indicates that the routing decisions of this case are better than the current ones. This also suggests that optimisation is a better approximation for route choices than the current models. The second case introduces the possibility of consolidation of demand at the CDCs. It is expected that it will become more attractive to consolidate demand at the CDCs and then deliver directly rather than consolidate at RDCs. The metrics of this case are all positive and so an improvement compared to the base case. In addition, it is also an improvement on the first case because all the results are an improvement. In the third case, transport costs between CDCs and RDCs are reduced five times. This reduction reflects the current developments regarding the increase in the costs of last-mile delivery and the possibility of making shuttle journeys cheaper and more efficient because this is owned and regulated by the company. This case provides insight into the behaviour of the model concerning route choices. The model behaves as expected. When the costs of transportation from the CDCs to the RDCs reduce, the number of direct deliveries decreases and the number of indirect deliveries increases. Then, the fourth experiment considered is changing the volume. Since the volumes of stores vary continuously, it is essential to discover how this affects the route choices made. Again, this case provides insight into the model's behaviour concerning route choices. When the volume decreases, the number of direct deliveries decreases and the number of indirect deliveries increases. When the volume increases, the number of direct deliveries increases, and the number of indirect deliveries decreases. Finally, an attempt is made to solve the model without clustering for a more realistic outcome. For this, the model is converted from the Gurobi to the PuLP tool and the company's server is used to run the code. Unfortunately, the server also proved to be not powerful enough. As a result, it was impossible to compare the performance of clustering with the situation where the data was not clustered. The robustness of the model was also examined to analyse the behaviour of the outcomes. This showed that the model is sensitive to changes in volume regarding the performance metrics but not really sensitive regarding the routing decisions. The analysis also showed that the model is not really sensitive to changes in fixed costs (fc_{ij}) regarding the performance metrics but that it is sensitive regarding changes in routing decisions. The computational values were also tracked to understand the performance of the optimisation. The average run time per case equals 29 hours, Table 2.

Case	**Run time [h]**	**Incumbent [€ $\cdot 10^3$]**	**Best-bound [€ $\cdot 10^3$]**	**Gap [%]**
Base	35	147	133	9.6
First	33	139	127	8.7
Second	39	139	125	10.3
Third, $fc_{ij} = 0.95$	32	136	126	8.0
Third, $fc_{ij} = 0.90$	33	134	125	7.2
Third, $fc_{ij} = 0.85$	32	133	123	7.3
Third, $fc_{ij} = 0.80$	32	131	121	7.2
Third, $fc_{ij} = 0.75$	27	129	120	6.8
Fourth, $vol^p = 1.25$	28	165	153	7.3
Fourth, $vol^p = 1.15$	31	157	143	8.7
Fourth, $vol^p = 1.05$	31	148	135	9.3
Fourth, $vol^p = 0.95$	30	136	125	7.9
Fourth, $vol^p = 0.85$	32	128	115	10.3

Table 2: Computation values of all cases

5.3 Results

The results are compared from three perspectives. It must be considered that the results are impacted by the clustering of data, and sub-optimal outcomes are present because certain gaps in the data are not fully eliminated in the experiments, affecting result interpretation. This error and uncertainty are based on the level of aggregation of data and the gap size of the solution.

First, they are compared based on the performance metrics. The main KPI is the most important metric to compare the cases on, but the other metrics need to be taken into account too, Table 3. Consolidation of demand at the CDCs is proven to be a successful improvement, while an improvement in costs of 5.4% can be achieved. The total costs influence the third and fourth cases, so these cases can not be compared with the others. What can be concluded from these cases is that the metrics of both perform as expected. When lowering the total costs of transportation between the CDCs and RDCs, the third case, the percentage of indirect delivery will increase. When increasing the volume, in the fourth case, the direct delivery increases too. And when decreasing the volume, the direct delivery decreases too.

	Total costs of transport [€ $\cdot 10^3$]	Total distance [km $\cdot 10^3$]	Total number of vehicles [-]	Costs per container [€]	Load per vehicle [containers]	Drop size per stop [containers]	Stops per vehicle [-]
Base case	147	55	476	3.35	15	8	0.6
First case	139	55	481	3.17	15	4	0.6
Second case	139	52	459	3.17	15	9	0.6
Third case, $fc_{ij} = 0.95$	136	55	479	3.12	16	10	0.6
Third case, $fc_{ij} = 0.90$	134	56	482	3.06	16	10	0.6
Third case, $fc_{ij} = 0.85$	133	56	484	3.03	16	10	0.6
Third case, $fc_{ij} = 0.80$	131	56	483	2.98	16	10	0.6
Third case, $fc_{ij} = 0.75$	129	57	489	2.94	16	10	0.6
Fourth case, $vol^p = 1.25$	165	63	564	2.97	16	11	0.6
Fourth case, $vol^p = 1.15$	157	61	530	3.05	16	10	0.6
Fourth case, $vol^p = 1.05$	148	58	504	3.15	16	10	0.6
Fourth case, $vol^p = 0.95$	136	54	470	3.18	15	10	0.6
Fourth case, $vol^p = 0.85$	128	51	445	3.36	14	9	0.6

Table 3: Comparing the performance metrics

Second, the results are compared based on their routing decisions, Table 4. In some cases, indirect delivery is more attractive, and in others direct delivery. There is a difference in the postal codes with the highest influence on the routing decisions. Besides this fact, two postal codes pop up in all cases as postal codes with a significant influence on the total costs considering a difference in routing decisions. These postal codes are 48 and 94. Therefore, it is advised to change the routing decisions of these postal codes.

	Total costs [€ $\cdot 10^3$]	CDC costs [€ $\cdot 10^3$]	Transfer costs [€ $\cdot 10^3$]	RDC costs [€ $\cdot 10^3$]	Direct [%]	Indirect [%]
Base case	147	48	22	77	57.4	42.6
First case	139	29	31	80	39.4	60.6
Second case	139	39	25	75	55.0	45.0
Third case, $fc_{ij} = 0.95$	136	27	32	80	35.6	64.4
Third case, $fc_{ij} = 0.90$	134	23	33	81	31.4	68.6
Third case, $fc_{ij} = 0.85$	133	22	33	81	30.7	69.3
Third case, $fc_{ij} = 0.80$	131	21	34	75	29.1	70.9
Third case, $fc_{ij} = 0.75$	129	17	36	84	23.1	76.9
Fourth case, $vol^p = 1.25$	165	38	32	95	43.9	56.1
Fourth case, $vol^p = 1.15$	157	38	30	89	46.0	54.0
Fourth case, $vol^p = 1.05$	148	36	29	84	43.3	56.7
Fourth case, $vol^p = 0.95$	136	27	31	78	37.4	62.6
Fourth case, $vol^p = 0.85$	128	27	29	72	39.8	60.2

Table 4: Comparing the routing decisions and costs

Third, the relationship between routing decisions, distances and volumes is discovered. As a result, there are no convincing relationships between the routing decisions and distances, and between the routing decisions and volumes. However, a relationship between all three can be determined using a logistic regression, Figure 2.

$$x = a \cdot dis_{cdc,store} + b \cdot \sum_{p \in (P: o^p = cdc, d^p = store)} vol^p + c \cdot (dis_{cdc,rdc} + dis_{rdc,store}) + d \tag{23}$$

The coefficients a, b, c and d are fitted on the data. A probability of the routing decision can be determined from the result of Equation 23 where the coefficients are fit to the data and the distances and volume of the new shop can be entered:

$$\mathcal{P}(x) = \frac{1}{1 + e^{-x}} \tag{24}$$

Here, with an accuracy between 63.1% for the second case and 78.0% for the third case, advice can be given for the routing decision based on the distance of a store to the CDCs, the distance of a store to an RDC plus the distance of this RDC to the CDCs, and the volume of demand of this store.

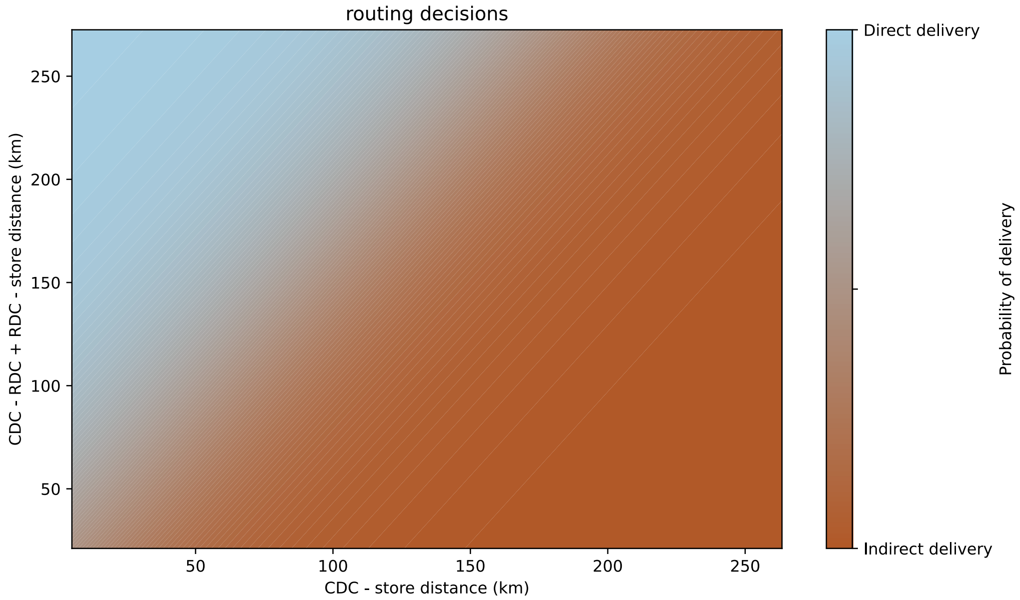

Figure 2: Regression of routing decisions of the first case

6 Conclusions

This study aimed to design an effective distribution system for store allocation and vehicle routing, addressing the main research question. Four topics guide the study: delving into system characteristics, planning model formulation, system performance, and the generalizability of findings. The analysis revealed that consolidating demand at CDCs can lead to significant cost improvements while altering transport costs between CDCs and RDCs and adjusting order volumes can influence delivery strategies. Moreover, the study identified specific postal codes as crucial in routing decisions. Despite the absence of direct relationships between routing decisions, distances, and volumes, logistic regression models provide guidance. Notably, the findings suggest that this distribution model can be adapted to various scenarios with different network structures, making a general contribution to the literature. Overall, this research contributes valuable insights into the design of large-scale distribution systems, offering a foundation for more efficient and cost-effective store allocation and vehicle routing strategies.

7 Discussion

The model is adaptable to similar distribution systems involving CDCs, RDCs, and stores. It allows for variations in vehicle types, order sources (CDCs and RDCs), and the number of CDCs, RDCs, and stores. While the model is versatile and can be applied broadly, it can also be customized for specific preferences, including the addition of time-related factors.

One key finding of this study shows that routing decisions are influenced by distance from DCs and volumes of demand. High truckloads and short distances to CDCs favour direct delivery, aligning with the findings of Hiohi et al. (2015). In cases where these conditions are not met, indirect delivery is more suitable. This relationship between volume of demand and consolidation is consistent with studies by Crainic and Hewitt (2021), Bakir et al. (2021) and Crainic and Kim (2007). The results of this study align with the experience of Geurs (2022), revealing cost differences between the base case and the validation. Handling computational complexity in large-scale problems, Abbasi et al. (2019) successfully implemented a Variable Neighborhood Search (VNS) algorithm to improve efficiency as the number of nodes increased.

This study contains several limitations like the exclusion of time, the exclusion of capacity limits of DCs, restrictions on the computation and clustering of data. First, time should be included to align the model better with the real world. Second, a minimum and maximum capacity should be set for the DCs to make sure that a DC is financially interesting to keep open and to make sure that the physical capacities are considered. Third, a more optimal solution with a smaller gap can be found without the callback function. Fourth, while the volume has a significant influence on the routing decisions, it may be optimized for several volumes to have a more realistic set of decisions. Fifth, data clustering should be avoided so that no sub-optimal outcome is generated.

In the current model, the total distance driven and average truckload are calculated but not directly constrained. These factors need to be considered, given the growing environmental concerns and emerging transport regulations. Higher truckloads generally lead to fewer vehicles and a reduced total distance, which leads to more sustainable transportation. Moreover, it is essential to address empty truck movements within the system, as some vehicles may return partially empty after deliveries. Including these aspects in the optimization may result in higher costs but offers a greener, more eco-friendly solution.

8 Recommendation

There are several practical recommendations for the company. Given the accuracy of this model to plan transport at vehicle level and to include actual distances, times and costs, route choices can be made even better. An important thing to note is that due to clustering, the results are more or less surrealistic. It is expected that the number of indirect deliveries is over-valued while the load rate is higher from RDC to the stores in the model. More server capacity is needed to run the model. While the model takes some time to get to the optimal solution, the logistic regression function can be used. The company may solve the model for several scenarios and calculate the coefficients. Based on the distances to the stores and the volumes, the routing choice can be determined quickly when using this function. Furthermore, based on the results, it can be noted that consolidation at the CDCs is beneficial for the costs of transportation.

The advice for further research is linked to the limitations of this current study. First, it is advised to include the time dimension in the model. Second, the capacity of the DCs can be added quite easily. The total outgoing demand from the DCs must be between two boundaries, a minimum and a maximum. Third, avoiding the callback function is possible when more time is available for the optimization. This function is only added to limit the run time, but if the run time matters less, it can be switched off. In addition, a larger server to run the code on will also reduce the run time, so again, the time can be limited with an optimal outcome. Fourth, clustering has the biggest impact on the outcome. Therefore, it would be of great value to avoid this. This can be avoided by running the code on a large server.

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