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Estimating primaries by sparse inversion and application to near-offset data reconstruction

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ABSTRACT

Accurate removal of surface-related multiples remains a challenge in many cases. To overcome typical inaccuracies in current multiple-removal techniques, we have developed a new primary-estimation method: estimation of primaries by sparse inversion (EPSI). EPSI is based on the same primary-multiple model as surface-related multiple elimination (SRME) and also requires no subsurface model. Unlike SRME, EPSI estimates the primaries as unknowns in a multidimensional inversion process rather than in a subtraction process. Furthermore, it does not depend on interpolated missing near-offset data because it can reconstruct missing data simultaneously. Sparseness plays a key role in the new primary-estimation procedure. The method was tested on 2D synthetic data.

INTRODUCTION

Although correct elimination of surface-related multiples is possible theoretically (Berkhout, 1982; Berkhout and Verschuur, 1997; Weglein et al., 1997), in practice it has many hurdles. Often, the surface-related multiple elimination (SRME) method is implemented as a prediction and subtraction process (Verschuur and Berkhout, 1997) in which the subtraction process is assumed to compensate for the prediction errors. However, in real situations, many factors limit the success of SRME, such as limited sampling, 3D effects (Dragoset and Jeričević, 1998), and distortion of primaries during subtraction (Guitton and Verschuur, 2004). Therefore, we propose avoiding the prediction and subtraction process and considering the primaries as unknowns in an inversion process.

A similar approach is described by van Borselen et al. (1996), but it estimates primaries under the assumption of known source properties and with a minimum-energy constraint. Biersteker (2001) estimates shallow primary reflections from multiples, again under a minimum-energy constraint. Hargreaves (2006) and van Groenestijn and Verschuur (2006) further elaborate on Biersteker's work by estimating missing offsets in the seismic data. However, in all of these cases, the minimum-energy constraint limits the quality of the estimated data.

Herrmann et al. (2007) propose estimating the primaries using the sparse curvelet domain but still rely on predicted multiples to guide the separation process. Alternatively, Amundsen (2001) redefines SRME as a multidimensional division of the upgoing and downgoing wavefields. The disadvantage is that a multidimensional deterministic division is not trivial for field data because of edge effects and stabilization issues. Wang (2004) introduces a primary-estimation method based on explicit matrix inversions, so that missing near-offset data do not pose a problem. However, it remains a prediction and subtraction process.

We introduce estimation of primaries by sparse inversion (EPSI), which, like all of the above-mentioned methods, is based on the primary-multiple relationship. We propose a solution through an iterative inversion process and introduce a sparseness constraint to the obtained primary impulse response.

We discuss the primary-multiple model behind SRME and EPSI and recapitulate iterative SRME. Then we explain the algorithm of EPSI. Small modifications are made to this algorithm to reconstruct missing near-offset data.

THE PRIMARY-MULTIPLE MODEL AND SRME

In the detail-hiding operator notation for 2D data (Berkhout, 1982), a bold quantity represents a prestack data volume for one frequency, columns represent monochromatic shot records, and rows represent monochromatic common-receiver gathers. Using this notation, we can express the upgoing data at the surface $\mathbf{P}$ as

$$\mathbf{P} = \mathbf{X}_0\mathbf{S} + \mathbf{X}_0\mathbf{R}\mathbf{P}, \qquad (1)$$

where the primary impulse responses $\mathbf{X}_0$ multiplied by the source properties $\mathbf{S}$ equals the primaries, $\mathbf{P}_0 = \mathbf{X}_0\mathbf{S}$. Note that what is called "primaries" in this paper refers to all events that do not reflect at the

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surface, including internal multiples. The matrix multiplication of $\mathbf{X}_0$ by the reflection operator at the surface $\mathbf{R}$ and the total data yields the surface multiples $\mathbf{M} = \mathbf{X}_0\mathbf{RP}$.

From equation 1, one can derive that surface multiples can be predicted by a convolution of the primaries with the data:

$$\mathbf{M} = \mathbf{P}_0\mathbf{AP}, \quad (2)$$

where surface operator $\mathbf{A} = \mathbf{S}^{-1}\mathbf{R}$. Iterative SRME estimates the primaries with the aid of (Berkhout and Verschuur, 1997)

$$\hat{\mathbf{P}}_{0,i+1} = \mathbf{P} - \hat{A}_{i+1}\hat{\mathbf{P}}_{0,i}\mathbf{P}, \quad (3)$$

where i represents the iteration number, $\mathbf{A}$ is replaced by an angle-independent approximation $A(\omega)\mathbf{I}$, and $\hat{\mathbf{P}}_{0,1} = \mathbf{P}$. To avoid the use of an obliquity factor (Weglein et al., 1997), no source deghosting is done. The assumed dipole characteristic of the source thus includes the obliquity factor (see Berkhout, 1982). Because equation 3 contains more unknowns ($\hat{\mathbf{P}}_{0,i+1}$ and $\hat{A}_{i+1}$) than knowns ($\mathbf{P}$), an extra constraint is needed. Typically, the primaries are assumed to have minimum energy (the L2-norm). This constraint is used when $\hat{A}_{i+1}$ is estimated as a filter that matches the predicted multiples $\hat{\mathbf{M}}_i = \hat{\mathbf{P}}_{0,i}\mathbf{P}$ to the data in the time domain, resulting in the new primary estimation $\hat{\mathbf{P}}_{0,i+1}$ (Verschuur and Berkhout, 1997).

The minimum-energy norm often leads to the satisfactory subtraction result, but it does not work properly in all cases (Nekut and Verschuur, 1998). Guitton and Verschuur (2004) and van Groenestijn and Verschuur (2008) show that other minimization norms, such as the L1-norm or a sparseness norm, can lead to different and sometimes better subtraction results.

ESTIMATION OF PRIMARIES BY SPARSE INVERSION

In this section, we propose to estimate primaries through full-waveform inversion, avoiding an explicit multiple-subtraction step. The algorithm is based on the same primary-multiple model discussed above. If we take $\mathbf{S}(\omega) = S(\omega)\mathbf{I}$ (meaning a constant-source wavelet for all shots) and $\mathbf{R} = -\mathbf{I}$, equation 1 becomes

$$\mathbf{P} = \mathbf{X}_0 S - \mathbf{X}_0\mathbf{P}. \quad (4)$$

Because this equation has more unknowns ($\mathbf{X}_0$ and S) than knowns ($\mathbf{P}$), an extra constraint is needed to solve it. We use the constraint that $\mathbf{X}_0$ is sparse in the time domain. This is valid because $\mathbf{X}_0$ consists of a series of spikes in the time domain, each representing a reflection event from a subsurface boundary. The objective function J is introduced as

$$J_i = \sum_{\omega}\sum_{j,k} |\mathbf{P} - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P}|^2_{j,k}, \quad (5)$$

where i denotes the iteration, $\Sigma_{j,k}$ indicates a summation over all of the elements of the matrix, and Σ_ω indicates a summation over all of the frequencies. Unlike in SRME, the objective function will go to zero during the iterations. Thus, primaries and their resultant multiples are estimated simultaneously. We set the initial values of $\hat{\mathbf{X}}_0$ and $\hat{S}$ at zero.

First, $\hat{\mathbf{X}}_0$ is updated. The updated $\Delta\mathbf{X}_0$ is a steepest-descent step:

$$\Delta\mathbf{X}_0 = (\mathbf{P} - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P})(\hat{S}_i\mathbf{I} - \mathbf{P})^H, \quad (6)$$

where $(\hat{S}_i\mathbf{I} - \mathbf{P})^H$ is the complex adjoint of $(\hat{S}_i\mathbf{I} - \mathbf{P})$. The term $(\mathbf{P} - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P})$ can be seen as the unexplained data or the residual. Because $\hat{S}_i\mathbf{I}$ is a diagonal matrix, a matrix multiplication $(\mathbf{P} - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P})(\hat{S}_i\mathbf{I})^H$ will bring information from a column of the residual to the same column of $\Delta\mathbf{X}_0$. This can be interpreted as information of the primaries in the residual that is transferred to the primary impulse responses. The matrix multiplication $(\mathbf{P} - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P})(-\mathbf{P})^H$ will bring information from all of the residual columns to a column of $\Delta\mathbf{X}_0$. This can be interpreted as multiples in the residual providing information to the primary impulse responses. This mechanism is similar to the one described in Berkhout and Verschuur (2006), in which multiples are transformed into primaries.

We use a synthetic data set based on a 2D subsurface model (Figure 1) to illustrate this method. Figure 2a shows one shot gather, and Figure 2b shows the first update step $\Delta\mathbf{X}_0$ for the same shot position. Because $\hat{\mathbf{X}}_0$ and $\hat{S}$ are zero in the first iteration step, the first step equals a multidimensional correlation of the data with itself, $-\mathbf{PP}^H$.

In van Groenestijn and Verschuur (2008), a sparseness term is added to the objective function of equation 5 to force $\hat{\mathbf{X}}_0$ to be sparse. However, this causes the solution to converge very slowly (more than 1000 iterations for a 1D plane-wave example). In this paper, we propose to enforce sparseness on the update of $\hat{\mathbf{X}}_0$, which is achieved in a separate step.

A window is placed over the update of $\hat{\mathbf{X}}_0$ in the time domain, and the biggest event(s) per trace is selected. Figure 2c shows the result of this process. Increasing the size of the window in each iteration improves convergence. As much as possible, in each iteration, the window should exclude strong events in the update that are not associated with primaries (such as water-bottom multiples).

Next, the sparse update $\Delta\hat{\mathbf{X}}_0$ is added to the primary impulse response:

$$\hat{\mathbf{X}}_{0,i+1} = \hat{\mathbf{X}}_{0,i} + \alpha\Delta\bar{\mathbf{X}}_0, \quad (7)$$

where α is a positive frequency-independent factor that scales the update step. If the value of the objective function in equation 5 is increased after replacing $\hat{\mathbf{X}}_{0,i}$ with $\hat{\mathbf{X}}_{0,i+1}$, then $\hat{\mathbf{X}}_{0,i+1}$ should be recalculated with a smaller α. We have chosen to halve α. In the beginning, α should be set deliberately too high.

Next, $\hat{S}_{i+1}$ is estimated as a filter obtained by least-squares matching the primary impulse responses $\hat{\mathbf{X}}_{0,i+1}$ to $(\mathbf{P} + \hat{\mathbf{X}}_{0,i+1}\mathbf{P})$ in the time domain. Wavelet $\hat{S}_{i+1}$ is estimated for all shots. Figure 3 shows how the primary estimation $\hat{\mathbf{X}}_{0,i}\hat{S}_i$ builds up during the iterations.

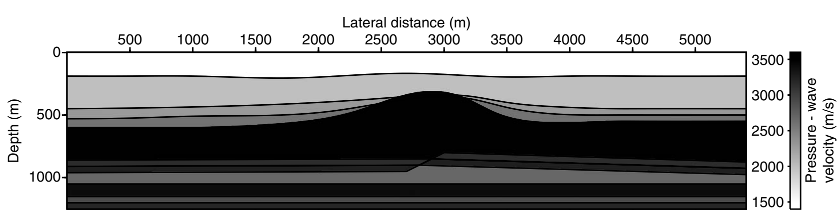

Figure 1. Synthetic salt model with water as the top layer.

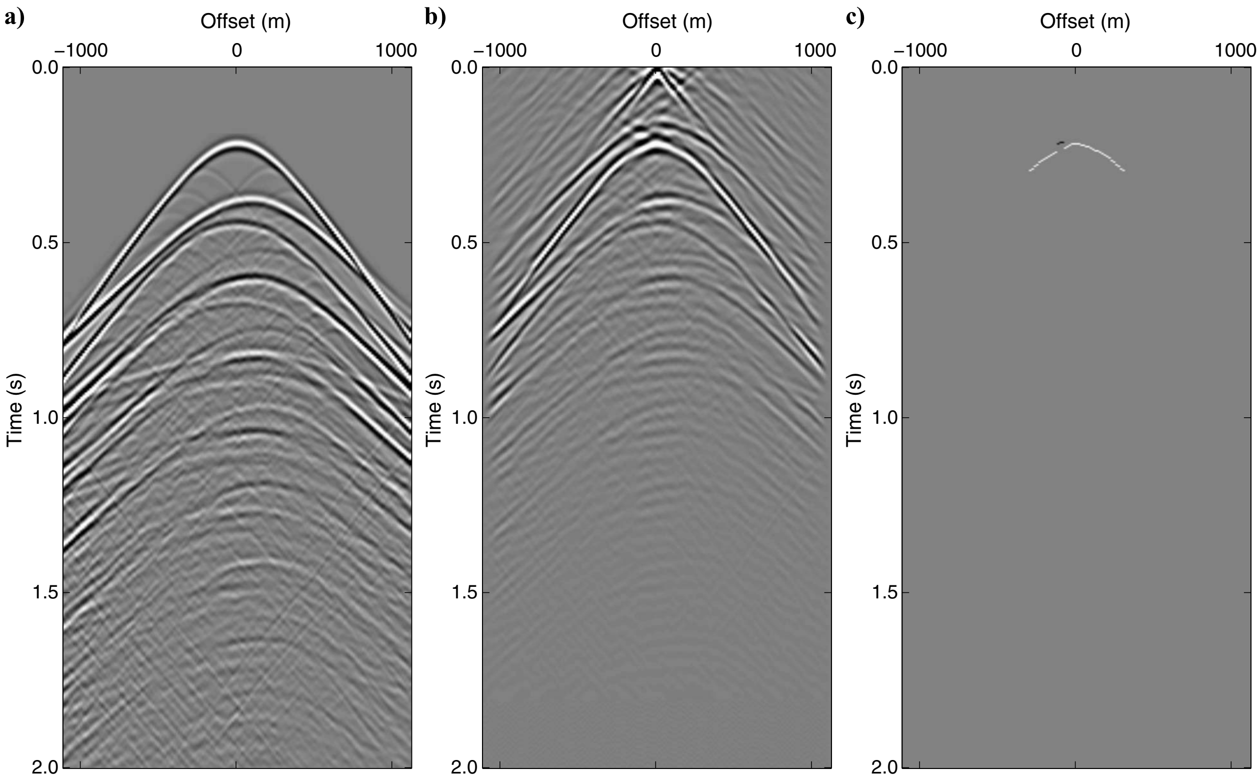

Figure 2. (a) Shot gather for a source at $x = 2685$ m with all multiples. (b) The corresponding update of the primary impulse response before imposing sparseness $\Delta \mathbf{X}_0$. (c) Corresponding update of the primary impulse response after imposing sparseness $\Delta \bar{\mathbf{X}}_0$.

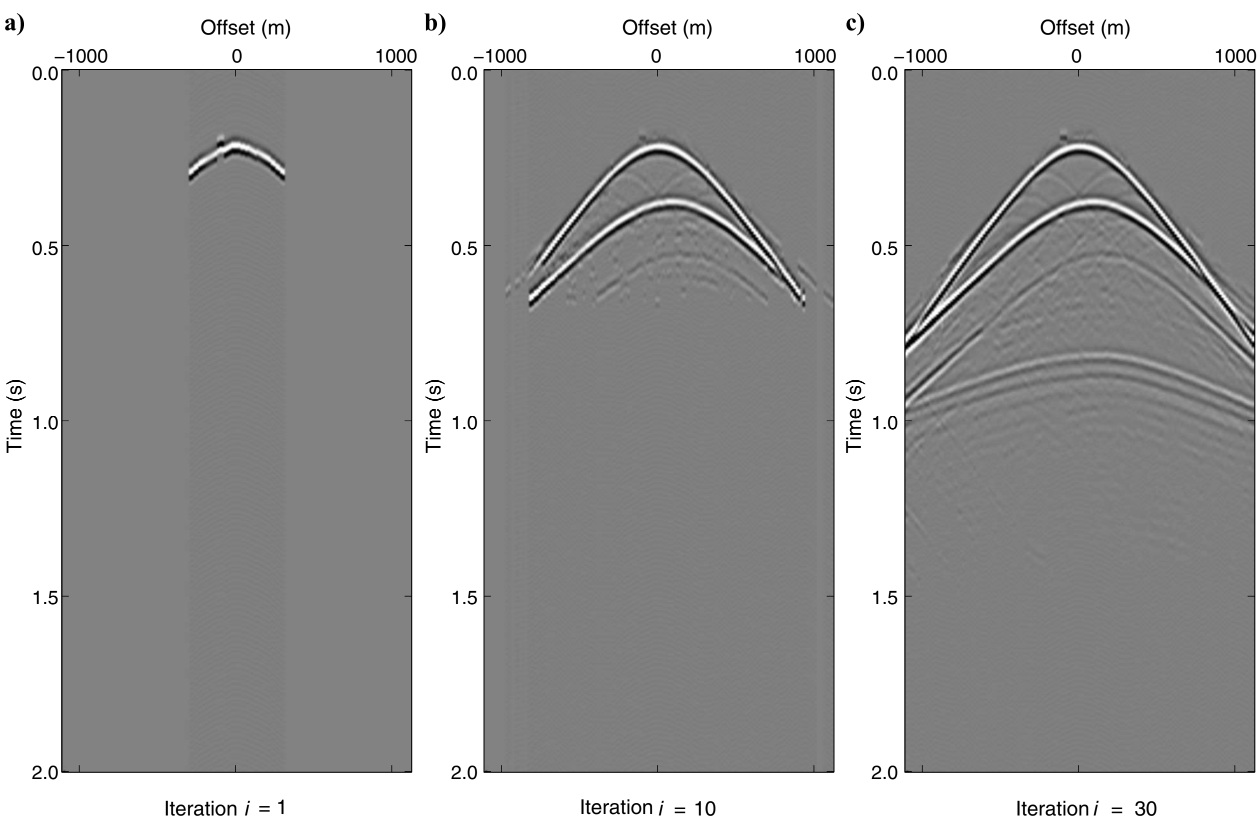

Figure 3. The primary estimation $\hat{\mathbf{X}}_{0,i}\hat{S}_i$ during different iterations at shot location $x = 2685$ m. This figure has the same gain and clipping values as Figure 2a.

Finally, the estimates of $\hat{\mathbf{X}}_0$ and $\hat{S}$ can be used in two ways to obtain a primary estimation: directly by convolving the estimated (spiky) impulse responses with the estimated wavelets, $\hat{\mathbf{P}}_0 = \hat{\mathbf{X}}_0\hat{S}$, and conservatively by creating the multiples and subtracting them from the total data, $\hat{\mathbf{P}}_0 = \mathbf{P} - \hat{\mathbf{M}} = \mathbf{P} + \hat{\mathbf{X}}_0\mathbf{P}$. The estimated multiples are subtracted from the data without any matching filter.

Figure 4a shows the result of the direct approach; Figure 4b shows its conservative counterpart. Figure 4c shows the true primaries. Note that for this ideal, noise-free situation, both primary results are virtually identical. The weak events below 1.0 s are internal multiples, which this process is not expected to remove. Residual surface multiples are hardly visible.

Imposing the sparseness on the update is necessary to give the primary impulse response $\mathbf{X}_0$ its spiky behavior in the time domain. It also ensures that strong primaries — and their corresponding multiples — are resolved before weaker reflections are recovered. In that respect, EPSI resembles a matching-pursuit approach. However, EPSI is robust in the sense that during iterations, it can repair errors made in earlier iterations. As an example, iteration artifacts in Figure 3 are no longer visible in Figure 4a.

RECONSTRUCTING MISSING NEAR-OFFSET DATA

In this section, we include the reconstruction of missing near-offset data in EPSI. Equation 1 shows how data are built from sources, total data (secondary sources), and primary impulse responses. Although the missing near-offset data are not measured, the consequences of firing the secondary sources in the near-offset data are measured in the multiples. Therefore, the near-offset data can be reconstructed theoretically from the multiples. We will discuss how the algorithm for data with missing near offsets differs from the one for data without missing near-offsets.

The data obtained in iteration i consist of two subsets: $\mathbf{P}_i = \mathbf{P}'' + \hat{\mathbf{P}}_i'$. Subset $\mathbf{P}''$ is measured data outside the near-offset gap, with matrix elements within the gap being zero. Subset $\hat{\mathbf{P}}_i'$ is the estimated near-offset data for iteration i, with matrix elements outside the gap being zero. In the objective function, $\hat{\mathbf{P}}'$, $\hat{\mathbf{X}}_0$, and $\hat{S}$ are the unknowns:

$$J_i = \sum_{\omega}\sum_{j,k} |\mathbf{P}_i - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P}_i|_{j,k}^2. \quad (8)$$

Initially, the values for $\hat{\mathbf{P}}'$, $\hat{\mathbf{X}}_0$, and $\hat{S}$ are set to zero. The update of $\hat{\mathbf{X}}_0$ is the same as in equation 6, except that we use the total data $\mathbf{P}_i$ (with reconstructed near-offset data) instead of $\mathbf{P}$:

$$\Delta\mathbf{X}_0 = (\mathbf{P}_i - \hat{\mathbf{X}}_{0,i}\hat{S}_i + \hat{\mathbf{X}}_{0,i}\mathbf{P}_i)(\hat{S}_i\mathbf{I} - \mathbf{P}_i)^H. \quad (9)$$

Note that the first update of $\hat{\mathbf{X}}_0$ does not rely on information from inside the missing near-offset gap because $\hat{\mathbf{P}}'_{i=0} = 0$. To obtain $\hat{S}_{i+1}$, $\hat{\mathbf{X}}_{0,i+1}$ is least-squares matched to $\mathbf{P}_i + \hat{\mathbf{X}}_{0,i+1}\mathbf{P}_i$ in the time domain. Finally, the update of $\mathbf{P}_i'$ is given by a steepest-descent step:

$$\Delta\mathbf{P}' = -(\mathbf{I} + \hat{\mathbf{X}}_{0,i+1})^H(\mathbf{P}_i - \hat{\mathbf{X}}_{0,i+1}\hat{S}_{i+1} + \hat{\mathbf{X}}_{0,i+1}\mathbf{P}_i). \quad (10)$$

Noncausal events ($t < 1.5$ s) are removed from $\Delta\mathbf{P}'$ in the time do-

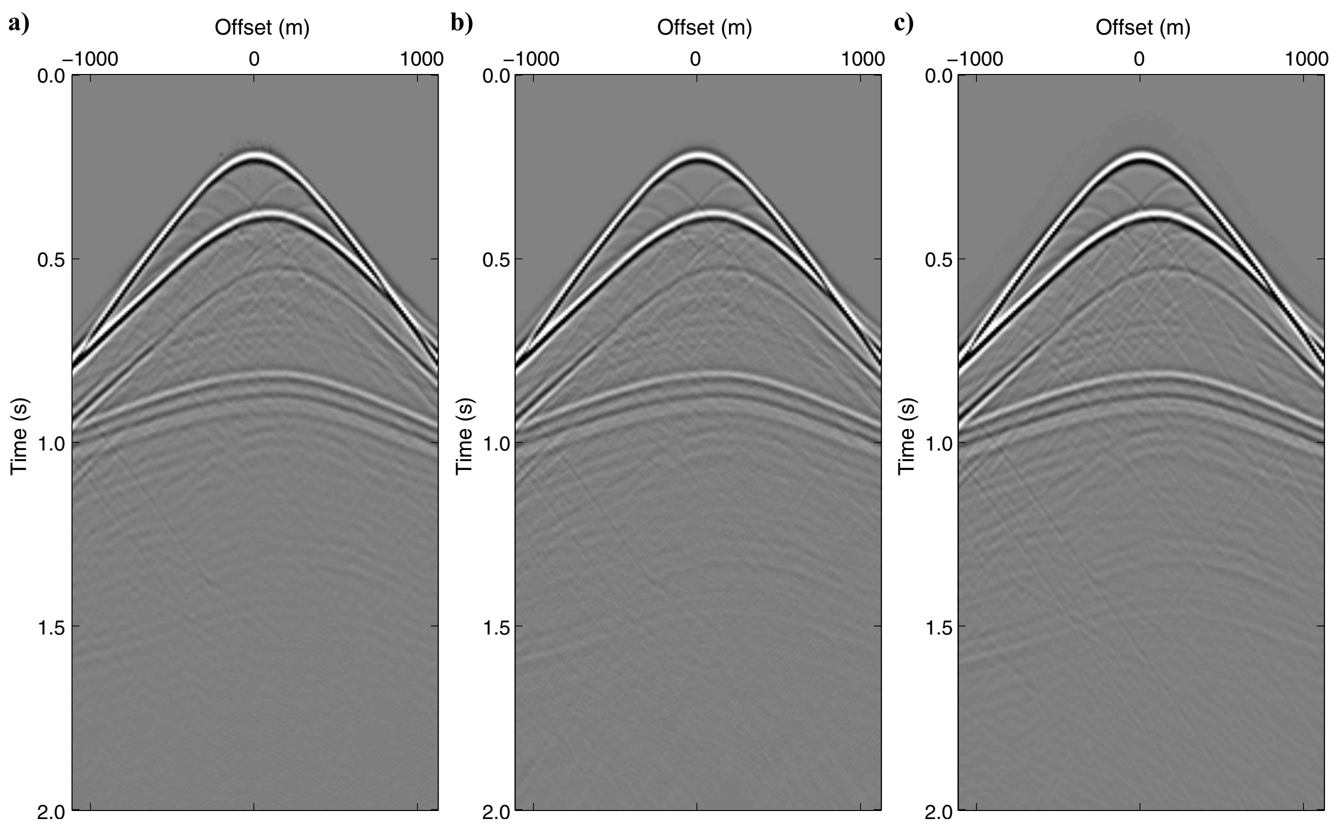

Figure 4. (a) Direct and (b) conservative estimation of the primaries of a source at $x = 2685$ m. (c) The true primaries. This figure has the same gain and clipping values as Figure 2a.

main. The update of $\hat{\mathbf{P}}'$ is scaled to ensure that the next objective function value of equation 8 is lower than the previous one.

To demonstrate this, we again use the data related to the model in Figure 1 and remove near-offsets. Figure 5a shows one input shot record for the same source location as in Figures 2, 3, and 4. Figure 5b and c shows the direct and conservative primary-estimation results. Figure 5c has missing near-offsets because it was obtained by subtracting the multiples from the data having missing near-offsets. Note the excellent resemblance with the results in Figure 4a and b and the quality of the reconstructed near offsets in Figure 5b. Figure 6a and b shows the zero-offset section of the true data and the reconstructed total data. Except for some minor artifacts, the reconstruction is very good. The inversion method also yields the primary estimate at the near-offsets (Figure 6c), which appears very satisfactory as well. Especially note the accurately reconstructed diffraction events.

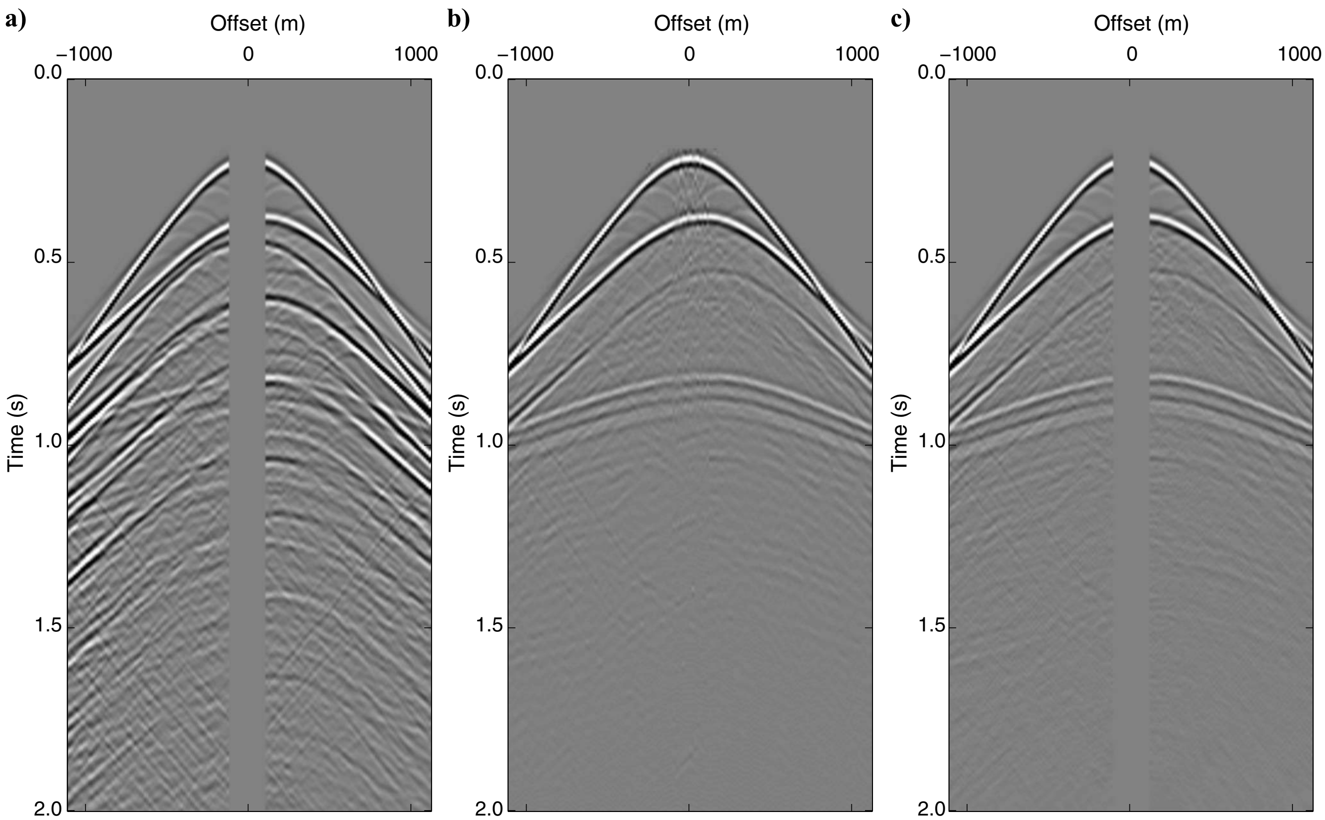

Figure 5. (a) Input data with missing near-offsets for a source at $x = 2685$ m. (b) Direct and (c) conservative primary-estimation result. This figure has the same gain and clipping values as Figure 2a.

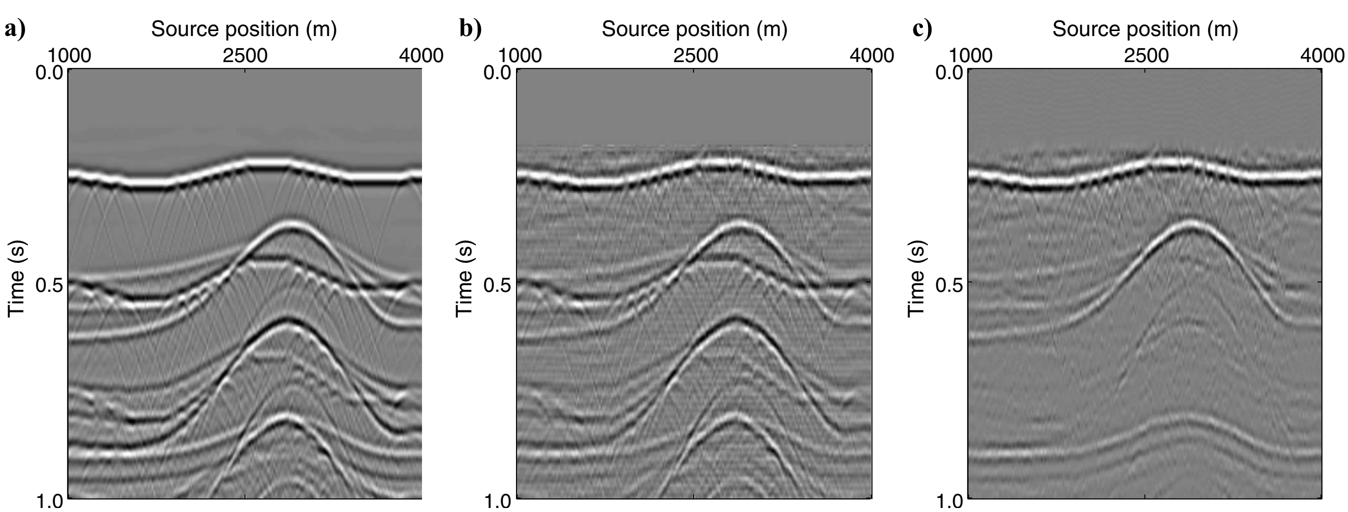

Figure 6. (a) Zero-offset section of the true data, (b) the reconstructed data, and (c) the estimated primaries $\hat{\mathbf{X}}_0\hat{S}$. This figure has the same gain and clipping values as Figure 2a.

DISCUSSION

Although the number of iterations in EPSI (typically 60) is higher than in SRME (typically three), there are reasons to assume that the calculation time does not have to be 20 times greater. First, because $\mathbf{X}_0$ is spiky in the time domain, its convolution with the total data can be carried out in the time domain, eliminating the need to perform a Fourier transform. Second, the correlation $\mathbf{PP}^H$ need be calculated only once.

We believe there is large scope for applying this new combined primary and near-offset estimation method to shallow-water situations because that is where SRME has difficulties (e.g., Verschuur, 2006). For the current implementation, we assume a constant-source wavelet for all shots. However, EPSI can be extended to include estimation of source-to-source wavelet variations and source directivity.

Although all tests were performed on 2D data, we believe that EPSI can be of value for the full 3D situation because EPSI only needs to store the spikes of impulse responses. Furthermore, the missing data between the streamer lines might be treated similarly to the missing near-offsets.

CONCLUSIONS

We have presented a new primary-estimation method, EPSI. The primaries are considered to be unknowns in a full-waveform inversion process. There are two interesting differences between SRME and EPSI: EPSI has no adaptive subtraction step that matches multiples to data, and EPSI does not need interpolated near-offsets to estimate primaries. Results for a synthetic 2D data set show the feasibility of EPSI.

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