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Extracting Heterogeneous Compliance of a Single Fracture from Seismic Scattering Coupled with Perturbation Theory

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SUMMARY

We have presented a new methodology to model the scattered wavefield due to a heterogeneous distribution of compliance along a single fracture and then to invert this compliance distribution or its power spectral density (PSD) from the scattered seismic response. We illustrate the validity through numerical examples using Gaussian PSD. First, we show that the perturbation theory offers very close result to that obtained from exact relation (wdSDD approach), by considering only the first 3 orders of perturbation. By solving the inverse scattering problem we can estimate the heterogeneous compliance distribution quite accurately. Next, in order to estimate the PSD of the heterogeneous compliance distribution, the normalized power of the stress field is used. We use the result from the wdSDD approach to represent the observed data. We show the possibility of successful extraction of the correct PSD by curve-fitting. The techniques presented here can lead to a conceptually new and accurate approach for estimating heterogeneous compliance distribution and the corresponding spatial variation in roughness and aperture along the fracture plane. This may, in turn, be related to the crucial fluid flow properties in the fractured subsurface.

Introduction

The elastic compliance is a major determinant of the seismic response of a fracture. The long-wavelength assumption and assigning a single value for the fracture compliance are usual. These lead to a total response of multiple fractures (e.g., effective medium theory). However, the compliance along a single fracture surface may be sensed by the seismic wave as heterogeneous and spatially varying. Pyrak-Nolte and Nolte (1992) showed that the assumption of the spatially heterogeneous compliance distribution along the fracture surface explains better the experimental data. Brown and Scholz (1985) showed that the fracture compliance depends on the parameter representing the roughness of the fracture surface and the normal stress applied to the fracture. Hudson et al. (1996) derived the normal and tangential compliance from the scattered wavefield; the scattered wavefield is shown to be dependent on the distribution of the contact area and the fluid-filled aperture of the fracture. Spatial variations and heterogeneity in roughness, normal stress and aperture along the fracture surface are the key factors that control the fracture compliance distribution. Furthermore, as suggested by Brown and Fang (2012), fracture surface roughness can and does control the fluid flow. Therefore, estimating the spatial variations of compliance distribution can be crucial in estimating the fluid flow properties. Figure 1(a) shows an example of a simulated heterogeneous compliance distribution along x-axis for a single fracture. Such a variation is presumably a result of the spatial variation of the microstructures.

In this research, we have concentrated on developing a methodology for reliable characterization of the spatially varying fracture compliance using the scattered seismic wavefield. Extending further the work of Nakagawa et al. (2004) and Leiderman et al. (2007), we employ the perturbation theory to obtain the dominant information and the power spectral density (PSD) of the heterogeneous fracture compliance. We illustrate this using numerical examples of the modeled scattering response and the estimated compliance distribution and its PSD through fitting the power spectrum of the scattered wavefield.

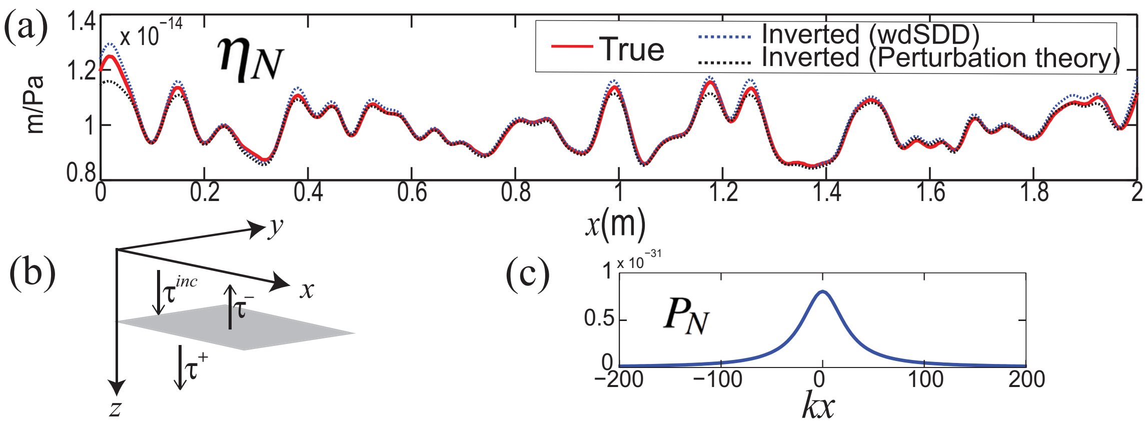

__Figure 1__ (a) Simulated heterogeneous normal compliance distribution (red line). Black and blue dotted lines are estimated from the inverse scattering using wdSDD and the first-order perturbation theory, respectively. (b) Orientation of the considered fracture plane. (c) Corresponding Gaussian PSD.

Method

(a) Wavenumber-domain seismic displacement-discontinuity method (wdSDD)

The linear-slip model of a fracture is expressed as $[\mathbf{u}](x,y) = \mathbf{Z}(x,y)\boldsymbol{\tau}(x,y)$, where $[\mathbf{u}]$ and $\boldsymbol{\tau}$ are the displacement-discontinuity vector and the stress traction vector on the fracture plane, respectively. Here we consider the fracture to be in the $z-$plane in Cartesian coordinates (Figure1b). The diagonal compliance matrix $\mathbf{Z}$ is defined as $\mathbf{Z}(x,y) = diag(\eta_T(x,y), \eta_T(x,y), \eta_N(x,y))$, where η_T and η_N are tangential and normal compliances, respectively. We assume these compliances as functions of space (x,y) along the fracture plane. Nakagawa et al. (2004) derived the plane-wave response of the heterogeneous compliance distribution in the $f-k$ domain. We assume a downgoing incident wave in the upper medium and no upgoing wave in the lower medium (Figure1b). Consequently, we can construct the transmission/reflection problem. Solving the problem for the stress field yields the following relation at the fracture surface:

$$\hat{\mathbf{H}}(\hat{\boldsymbol{\tau}}^{+}-\hat{\boldsymbol{\tau}}^{inc}) = i\omega\hat{\mathbf{Z}} * \hat{\boldsymbol{\tau}}^{+}, \tag{1}$$

where the superscript ˆ indicates that the parameters are in the $f-k$ domain. $\hat{\boldsymbol{\tau}}^{inc}$ and $\hat{\boldsymbol{\tau}}^{+}$ are the incident stress vector and the downgoing stress vector in the lower medium, respectively. $\hat{\mathbf{H}}$ is made of the composition matrix: $\hat{\mathbf{H}} = \mathbf{L}_1^{+}(\mathbf{L}_2^{+})^{-1} - \mathbf{L}_1^{-}(\mathbf{L}_2^{-})^{-1}$, where we assume the same medium parameter for the upper and the lower medium. $\mathbf{L}_1^{\pm}$ and $\mathbf{L}_2^{\pm}$ are derived by eigenvalue decomposition of the first-order differential operator of the wave equation (Wapenaar and Berkhout, 1989). Equation 1 includes convolution "$*$" of the Fourier-transformed compliance matrix $\hat{\mathbf{Z}}$. With the known incident stress field ($\hat{\boldsymbol{\tau}}^{inc}$) and the compliance matrix, we can estimate the downgoing stress field ($\hat{\boldsymbol{\tau}}^{+}$) by using the wavenumber-discretized form of equation 1 (Nakagawa et al., 2004). Having estimated the stress field, we can then interchange the particle velocity vector ($\hat{\mathbf{v}}$), the stress vector ($\hat{\boldsymbol{\tau}}$) and the upgoing/downgoing potential vector ($\hat{\mathbf{D}}^{\pm}$) by using the composition matrices. Note that the variables are evaluated on the fracture plane. Therefore, we can calculate the reflection response (upgoing wavefied) at a desired position by applying a phase-shift operator in order to propagate the potential wavefield: $\hat{\mathbf{D}}^{-}_{obs} = \hat{\mathbf{W}}^{-}\hat{\mathbf{D}}^{-}$, where $\hat{\mathbf{D}}^{-}_{obs}$ is the observed reflection response and $\hat{\mathbf{W}}^{-}$ is the phase-shift operator including the propagation velocity of the medium and the distance of propagation (Δz) from the fracture plane (Wapenaar and Berkhout, 1989).

(b) Application of perturbation theory

We have derived the perturbed solution of equation 1 by assuming the form $\mathbf{Z}(x,y) = \mathbf{Z}_0 + \varepsilon\mathbf{Z}_1(x,y)$, where $\mathbf{Z}_0$ is a constant background compliance matrix, $\mathbf{Z}_1(x,y)$ is the perturbation from the background value. $\mathbf{Z}_1(x,y)$ is scaled such that it has the same order as $\mathbf{Z}_0$. ε is the magnitude of perturbation. By expanding the wavefield into a power-series of ε, we obtain the transmission/reflection response for various orders of ε. When we solve them in terms of the upgoing stress field in the upper medium (reflected wave), the zero-th order gives the response for constant compliance, while the n-th order ($n > 0$) yields:

$$\hat{\boldsymbol{\tau}}^{-(n)} = \left(\hat{\mathbf{H}} - i\omega\mathbf{Z}_0\right)^{-1}\hat{\mathbf{S}}^{(n)}, \tag{2}$$

where $\hat{\mathbf{Z}}_1$ is the Fourier-transformed perturbed compliance matrix. The source term $\hat{\mathbf{S}}^{(n)}$ is constructed from the $\hat{\mathbf{Z}}_1$ and the stress field of the one-lower order as $\hat{\mathbf{S}}^{(n)} = i\omega\hat{\mathbf{Z}}_1 * \hat{\boldsymbol{\tau}}^{(n-1)}$. One can calculate the total response as, e.g. $\hat{\boldsymbol{\tau}}^{-} = \hat{\boldsymbol{\tau}}^{-(0)} + \varepsilon\hat{\boldsymbol{\tau}}^{-(1)} + \varepsilon^2\hat{\boldsymbol{\tau}}^{-(2)}\cdots$. In order to construct finally the observed wavefield, the calculated wavefields are phase-shifted, as discussed above.

(c) Inverse scattering solution to estimate the compliance distribution

Following Leiderman et al. (2007), we estimate the compliance distribution from seismic scattering response. Given the incident stress field $\hat{\boldsymbol{\tau}}^{inc}$, elastic medium parameters and the background compliance $\mathbf{Z}_0$, we calculate the perturbed compliance distribution $\varepsilon\mathbf{Z}_1$ from the upgoing stress field at the fracture $\hat{\boldsymbol{\tau}}^{-}$. Note that obtaining the stress field at the fracture requires a knowledge of the fracture position. The forward problem (equation 1 or 2) can be reformulated as follows: $\hat{\mathbf{A}}(k_x) = i\omega\varepsilon\hat{\mathbf{Z}}_1(k_x) * \hat{\mathbf{B}}(k_x)$, where all parameters, for the time being, are assumed to be 1-dimensional (k_x domain). Through inverse Fourier transformation, we calculate each component of $\varepsilon\mathbf{Z}_1(x)$ as:

$$\varepsilon\eta_T^1(x) = \frac{\mathbf{A}(x)\cdot\mathbf{e}_x}{i\omega\mathbf{B}(x)\cdot\mathbf{e}_x}, \varepsilon\eta_T^1(x) = \frac{\mathbf{A}(x)\cdot\mathbf{e}_y}{i\omega\mathbf{B}(x)\cdot\mathbf{e}_y}, \varepsilon\eta_N^1(x) = \frac{\mathbf{A}(x)\cdot\mathbf{e}_z}{i\omega\mathbf{B}(x)\cdot\mathbf{e}_z}, \tag{3}$$

where $\varepsilon\eta_T^1(x)$ and $\varepsilon\eta_N^1(x)$ are, respectively, perturbed tangential and normal compliance distributions, and $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors in x, y and z directions, respectively. Here we utilize the fact that $\mathbf{Z}_1(x)$ is a diagonal matrix. The functions $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are defined according to the chosen forward approach, i.e., wdSDD method (equation 1) or the first-order perturbation theory (equation 2 with $n = 1$).

(d) Fitting the power spectrum

When the compliance distribution is a random in space, it can be characterized by the power spectral density (PSD). In the special case of plane wave incidence with wavenumber $\mathbf{k}^{inc}$, i.e., $\hat{\boldsymbol{\tau}}^{inc} = \hat{\boldsymbol{\tau}}^{c}\delta(\mathbf{k}-\mathbf{k}^{inc})$, we obtain a simple equation for the first-order solution using equation 2. Furthermore, calculating the PSD of the upgoing stress field of the first-order solution using the explicit form of $\hat{\mathbf{H}}$ given by Nakagawa et al. (2004) yields the following linear function of PSD for the perturbed normal compliance distribution:

$$\lim_{X\to\infty} E < \frac{|\varepsilon\hat{\boldsymbol{\tau}}^{-(1)}|^2}{X} >= \lim_{X\to\infty} E < \frac{\varepsilon^2(\hat{\boldsymbol{\tau}}^{-(1)})^{\dagger}\hat{\boldsymbol{\tau}}^{-(1)}}{X} >= f(\mathbf{k};\omega,\mathbf{k}^{inc},V_S,V_P,\rho,\hat{\boldsymbol{\tau}}^{c},\nu)P_N(\mathbf{k}), \quad (4)$$

where $E < \cdot >$ is an ensemble averaging, $\dagger$ denotes Hermitian conjugation, X is total length of the compliance distribution, $P_N(\mathbf{k})$ is the PSD of the perturbed normal compliance distribution ($\varepsilon\eta_N^1$) and the function $f(\mathbf{k})$ is defined by the medium parameters (V_P, V_S, ρ), incident stress, frequency, incident wavenumber and the compliance ratio ν (assuming the simple relation between the compliances as $\eta_T = \nu\eta_N$). The PSD of the total upgoing stress field at the fracture up to the first-order ($\lim_{X\to\infty} E < |\hat{\boldsymbol{\tau}}^{-(0)} + \varepsilon\hat{\boldsymbol{\tau}}^{-(1)}|^2/X >$) shows that the non-specular component is a function of $P_N(\mathbf{k})$ due to the presence of $\hat{\boldsymbol{\tau}}^{-(0)}$. Therefore, one can estimate the PSD of the perturbed compliance distribution from the non-specular components of the PSD of total upgoing stress field using the first-order perturbation theory.

Numerical examples

We illustrate the 1-D application of this new method using numerical examples. The heterogeneous compliance distribution is modeled using a given background compliance ($\mathbf{Z}_0$ or η_T^0 and η_N^0) and the PSD for the perturbed compliance distribution ($\varepsilon\mathbf{Z}_1$ or $\varepsilon\eta_T^1$ and $\varepsilon\eta_N^1$), assuming a stationary random process. We use the Gaussian PSD of the form $P_N(k_x) = \varepsilon^2\sqrt{\pi}l_c e^{-k_x^2 l_c^2/4}$ (Figure1c) with the corresponding autocorrelation function $\gamma_N(x) = \varepsilon^2 e^{-x^2/l_c}$, where ε and l_c are the standard deviation (deviation from the background value) and the correlation length, respectively. We then create the model (red line in Figure1a) using the method in the Fourier domain (Pardo-Iguzquiza and Chica-Olmo, 1993) with $l_c = 4cm$ and $\varepsilon = 10\%$ from the mean ($\eta_N^0 = 1\times10^{-14}m/Pa$). Next, we calculate the reflection response using wdSDD approach (equation 1) and compare the result with that using the perturbation theory (equation 2). The wavenumber domain is discretized by 601 wavenumbers in the range $[-942, 942]$. We assume $V_P = 6.3km/s, V_S = 3.4km/s, f = 600kHz(lab frequency), \nu = 1$ and calculate the scattered response 1m above the fracture plane. Figure 2(a) shows the modeled reflection response (P to P reflection) at different wavenumbers for normal incidence ($k_x^{inc} = 0$). Note the non-zero amplitude for the non-specular component, indicating scattering due to perturbation of the compliance distribution. Figure 2(b) shows the amplitude in $f-x$ domain obtained by inverse Fourier transformation of the result in Figure 2(a) followed by substitution of $x = 0$. From Figures 2(a) and 2(b), it is clear that after third-order, the perturbation solution leads very close to the wdSDD solution. We estimate the compliance distribution solving inverse scattering problem (equation 3), with the assumption that we can successfully estimate the stress field at the fracture from the backscattered data. For this purpose, we need the observed vector wavefield, medium parameters and the position of the fracture plane known. For our numerical test, we consider data from wdSDD approach to be representing the observed data. The results of the estimated compliance distribution from the exact relation (wdSDD equation) and that from the first-order perturbation theory are compared in Figure 1(a). The result using the exact relation (black dotted line in Figure 1a) obviously better estimates the compliance distribution than the first-order perturbation theory (blue dotted line in Figure 1a). Because of the absence of higher order of scattered wave, the first-order perturbation theory estimates lower magnitude of the variation . We use the equation 4 to estimate the PSD of the perturbed compliance distribution. We fit the curve of the normalized power of the stress field ($d(k_x) = |\hat{\boldsymbol{\tau}}^-|^2/X$, Figure 2c) by minimizing the misfit function as $E(\varepsilon, l_c) = \sum_{k_x}|d(k_x) - d^{est}(k_x)|^2$, where d^{est} is calculated from equation 4 with varying ε and l_c, and the summation is taken along the non-specular wavenumber component. We used a grid-search algorithm to estimate the best fit curve.

Figure 2(c) and 2(d) show the fitted Gaussian PSD and the value of the energy function. The appropriate values of ε and l_c are obtained from this fitting ($l_c = 0.039 \pm 0.001$ and $\varepsilon = 0.095 \pm 0.005$).

Conclusions

We have presented a new methodology to model the scattered wavefield due to a heterogeneous distribution of compliance along a single fracture and then to invert this compliance distribution or its PSD from the scattered seismic response. We illustrate the validity through numerical examples using Gaussian PSD. First, we show that the perturbation theory offers very close result to that obtained from exact relation (wdSDD approach), by considering only the first 3 orders of perturbation. By solving the inverse scattering problem we can estimate the heterogeneous compliance distribution quite accurately. Next, in order to estimate the PSD of the heterogeneous compliance distribution, the normalized power of the stress field is used. We use the result from the wdSDD approach to represent the observed data. We show the possibility of successful extraction of the correct PSD by curve-fitting. The techniques presented here can lead to a conceptually new and accurate approach for estimating heterogeneous compliance distribution and the corresponding spatial variation in roughness and aperture along the fracture plane. This may, in turn, be related to the crucial fluid flow properties in the fractured subsurface.

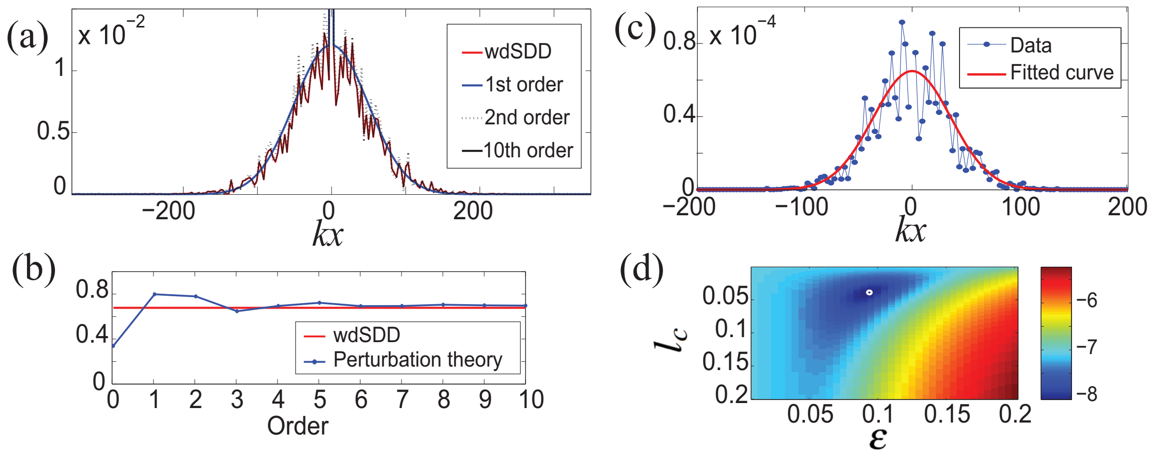

Figure 2 *(a) PP seismic scattering amplitude from the wdSDD approach and the perturbation theory. (b) Amplitude in the space domain for different order of the perturbation theory. (c) Representative observed data (normalized upgoing stress field) and the fitted curve. (d) Energy function from the grid-search algorithm.*

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