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Impact Assessment of Extreme Storm Events Using a Bayesian Network

Case: Dutch Coast

MASTER OF SCIENCE THESIS

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Abstract

The most densely populated and economical most valuable areas in The Netherlands lie below mean sea level. These areas are protected against the sea by a coastal dune system. The vital importance of this dune system is reflected in the extensive collection of Dutch legal regulations that ensure the safety level of the dunes.

Current safety assessment method for dunes, prescribed by these regulations, is based on conducting large numbers (thousands) of simulations to estimate dune erosion at individual locations along the Dutch coast. With the use of more complex dune-erosion models (i.e. Xbeach; [Roelvink et al. \(2009\)](#)) in the safety assessment, the method gets computationally more intensive. This means that conducting large numbers of simulations for a dune safety assessment are not feasible. Therefore new probabilistic approaches (e.g., Bayesian approach) are needed in order to apply state-of-the-art insights and models for dune erosion in a safety assessment.

Aim

The aim of this study was to gain an insight in the applicability of the Bayesian network approach for dune safety assessment against extreme storm events on the Dutch coast.

Method

First a database was generated that serves as input to the Bayesian network. The content of this dataset is obtained in a way similar to current assessment method. This means, data-sources (wave conditions, bottom profiles) and the dune erosion model (DUROS+; [Vellinga \(1986\)](#), [van Gent et al. \(2008\)](#)) to simulate the dune erosion process were equal.

Qualitatively the Bayesian network is represented by nodes (variables) and arrows (relations). Variables and relations were selected such, they were physically related to the dune erosion process and could be obtained out of the database. Quantitatively the Bayesian network is described by a conditional probability table defined by a expectation-maximization training algorithm. For this training process, cases (a case is a record in the database and represents a storm event) out of the dataset were selected.

Sensitivity analysis of the required number of training data, as well the determining variable of this Bayesian network were made.

Results

Results show the Bayesian network is capable of reproducing the dune erosion process (given this set of data) for 89%. The loss in skill is a consequence of the discretization of the variables in the Bayesian network. The number of trainig-cases needed to make reliable predictions that are more accurate then predictions made with the use of the prior probability, is approximately 5,000 cases.

Results of the sensitivity analysis indicates water level information is a determining term

regarding the prediction skill. However, when other hydraulic conditions (wave height, peak period) were known the water level was redundant. Indicating water level information is captured by the variables wave height and/or peak period.

Dune erosion volumes predictions for locations at the Holland coast, shows a high amount of uncertainty and results in unreliable predictions (negative log-likelihood ratio) when using a Bayesian network trained on cases representing the Wadden coast. Probably coastal features between both coastal regions are too diverse.

Furthermore, a Bayesian network is a useful tool to improve insight into data. Expected relations between variables can be investigated and visualized in the Bayesian network quickly.

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1. Introduction

1.1. Background

The favourable location of The Netherlands in a delta area of large rivers brought The Netherlands their prosperity. Without solid water-defence two-third of the country would be highly vulnerable to flooding ([Figure 1.1](#)). Moreover, in this most vulnerable area nine million people live and 65% of the national income is earned. Therefore a proper sea-defense is of vital importance. Especially the condition of the primary defence against water from the sea, main rivers and big lakes is crucial. They form the first line of defense and protect The Netherlands against flooding.



Figure 1.1.: Areas in the Netherlands threatened by sea and river (light-green). Higher grounds are indicated with a dark-green colour. The primary water defense is indicated in red. ([HR2006, 2007](#))

1.1.1. Historical Background

In the beginning of February 1953 the Netherlands suffered from a severe storm event and in several parts of the Netherlands (mainly Holland and Zeeland) the sea defence could not withstand the high water and collapsed. The damage was enormous, 1800 people lost their lives and around 15,000 ha land inundated. The failure of the dikes was caused by the unprecedented high level of the North Sea at that particular time and place: the water overtopped the dike, the back side (inner slope) was gradually washed away and finally the dike collapsed.

Since it was apparent that the coastal defense was insufficient, the government appointed a committee (the so-called Delta committee) to recommend on an appropriate measures to pre-

vent a disaster like that. The problem was studied from an economic but also from a physical point of view. The economic analysis compared the costs of building dikes of a certain height with the risks (risk = costs of inundation \times probability) of a possible inundation given that specific height.

Nowadays, almost 60 years later, both the number of observations of sea levels and the statistical methodology have improved considerably. This has led, among others, to a new approach in which the strength of coastal defense is probabilistically determined. The probability that there is a flood in a given year equals p , where the number p has to be determined by the politics ranging between 10^{-4} and 10^{-3} (depending on the importance of the area to protect)¹. Despite of the fact that only dikes collapsed during the 1953 storm event, in this research attention will be paid to dunes solely.

Dunes

Because the major part of the Dutch coastal defence consist of dunes (approximately 250 km), an appropriate safety assessment method for dunes is necessary. In order to evaluate whether safety level of the dune system is conform the standard a five-yearly safety assessment of the dune system is executed (see [Section 2.1](#)). If the actual safety level does not meet the agreed standard, reinforcement measures should be taken.

In order to assess the dune safety against flooding a dune-erosion model is necessary to simulate the dune-erosion process. In the 1970s and 1980s a lot of research has been done with respect to dune-erosion due to heavy storms ([van de Graaff \(1977, 1986\)](#) and [Vellinga \(1982, 1986\)](#)). Despite of the complex dune-erosion processes, the final result can be described with a relative simple model DUROS ([Vellinga, 1986](#)) and updated to DUROS+ ([van Gent et al., 2008](#)). Moreover, this research also showed the amount of dune-erosion is determined by a relative large number of parameters. Due to this large number of process-determining variables a probabilistic approach to assess the dune safety against flooding was followed ([TAW, 1984](#)).

The storm characteristics associated by the prescribed safety level are never observed in front of the Dutch coast and therefore needs to be obtained out of the available data. This data are collected about water levels at (mainly) five stations along the Dutch coast since the end of the 19th century. This collection contains many observations of water levels at different locations over a time period of more than 100 years. A far extrapolation of this data has to be made to include the storm characteristics of the design storm (i.e., 1/10,000 year). With this extrapolation a certain amount of uncertainty is introduced. Current research on the enlargement of the data collection and on better extrapolation measures try to minimize this uncertainty ([Baart et al., 2011](#)).

1.1.2. Interpretations of probability

There have been two main contending views about how to understand probability ([Korb and Nicholson, 2004](#)). One asserts that probabilities are fundamentally dispositional properties of non-deterministic physical systems, such as dice. This view is particularly associated with *frequentism*, identifying probabilities with long-run frequencies of events. The main complaint

¹for dike-ring 14, the Holland coast, a probability of exceedance of 10^{-4} once in ten-thousand years is held

is that short-run frequencies do not match probabilities (e.g., if we toss a coin only once, we would hardly conclude that its probabilities is either one or zero). Moreover, the distinction between short-run and long-run is vague, leaving the commitments of this frequentist interpretation unclear.

The traditional alternative to this concept is to think of probabilities as reporting our subjective degrees of belief. This view was expressed by Thomas Bayes ([Bayes and Price, 1763](#)) and Pierre Simon Laplace two hundred years ago. The origin of *Bayesianism* lies in an interpretation of *Bayes' theorem*. The Bayesian approach is a more general account of probability in that one has subjective belief in a huge variety in propositions.

1.2. Motivation

Current safety assessment method is based on conducting large numbers (thousands) of simulations to estimate dune erosion at individual locations along the Dutch coast. More complex dune-erosion models (i.e. XBEACH; [Roelvink et al. \(2009\)](#)) are computationally more intensive. This means that conducting large numbers of simulations for a dune safety assessment are not feasible. Therefore new probabilistic approaches (e.g., Bayesian approach) are needed in order to apply state of the art insights and models for dune erosion in a safety assessment. This research focusses on the use of a Bayesian probabilistic approach in order to assess the dune safety against storm events.

Bayesian approach

The Bayesian approach, formally described in previous section, is one example in the family of probability interpretations. Bayesian statistics are not new. The field of Bayesian statistics is rapidly evolving and new approaches for model construction and sampling have been utilized recently in a wide variety of disciplines to combine information ([Wikle and Berliner, 2007](#)). In the field of coastal engineering recent work has been done by [Plant and Holland \(2011b\)](#); and [Plant and Holland \(2011a\)](#). In their paper a relatively well-understood problem (wave-height evolution in the surf zone) demonstrated some very powerful capabilities that are offered by a Bayesian modelling approach. These include:

1. reduction of model and data dimension;
2. propagation of model and input uncertainty;
3. inverse estimation of model boundary-conditions;
4. assimilation of diverse observational inputs, and
5. identification of gaps in understanding.

The focus of this research is not to wave evolution in the surf zone but these advantages have a more general character and might be useful here as well. In conclusion, the application of the Bayesian approach for the Dutch coastal safety assessment is worth to be tested for feasibility.

1.3. Research Objective

The overall objective of this research can be formulated as follows:

Obtain insight into the applicability, with respect to current probabilistic approaches, of the Bayesian approach for the dune safety assessment against flooding due to extreme storm events.

1.3.1. Scope and assumptions

Main objective is to investigate whether the Bayesian approach is profitable with respect to the current approach for the assessment of dune safety. Therefore, the dune erosion model used in dune safety assessment nowadays, DUROS+ (van Gent et al., 2008), will be used for the Bayesian approach as well. In Section 2.1.2 a detailed description of this dune erosion model will be given, but loosely speaking, DUROS+ is a single dimension dune erosion model that approximates the post-storm profile shape assuming conservation of volume in cross-shore direction.

This research makes use of the output obtained by this empirical dune erosion model. The lack of sufficient data limits the possibilities to verify the results based on actual measurements. However, it should be clear that this thesis aims at a feasibility study of an alternative probabilistic methodology and not at a method that accounts for all relevant (dune erosion) phenomena. So, results of the relatively simple, empirical dune erosion model will serve as input for the Bayesian network. In Bayesian words the outcome of the dune erosion model is handled as *truth* to the Bayesian network.

1.3.2. Sub-questions

The aim of the study is to obtain an insight in the applicability of a Bayesian network approach for the dune safety assessment. In order to reach this objective, four intermediate research questions have been formulated:

SQ1 Is it possible to appropriately reproduce the current safety assessment for dunes in a Bayesian network?

SQ2 What is an appropriate way to characterize the dune erosion process in a Bayesian network?

- Which variables play an important role in the dune erosion process and how are they correlated?
- What is the best way to end up with a robust and fast Bayesian network, regarding the discretization of the variables available in the Bayesian network?

SQ3 What is the relation between the amount of data used for training of the Bayesian network and the prediction skill?

SQ4 Is it possible to forecast the dune erosion process outside the spatial domain described by the Bayesian network?

1.4. Methodology

The application of the Bayesian approach is rather complex due to the limited knowledge on the subject and the few applications in the field of coastal engineering. Therefore, one needs to look in other fields (e.g., computer science, artificial intelligence) to learn the Bayesian fundamentals. Literature about the theory and successful applications of Bayesian networks is largely available. Therefore the first step of this research was to start with a thorough literature study. Cross-shore bottom profiles along the Dutch coast were selected, together with a set of hydraulic boundary conditions. With the use of a dune erosion model the dune erosion process was simulated for appropriate (with respect to used dune erosion model) JARKUS transects for the Dutch coast. The input and output of this model was stored and will be used as input to the Bayesian network. The Bayesian network is used for predictions of the dune safety levels along the Dutch coast. The output of a Bayesian network is an updated probability distribution given evidence.

1.5. Organisation of the report

This document is chronologically structured based on the research process. Any background information or secondary actions taken are described in the appendices. The context of the research, including current and past studies concerning dune erosion and legal regulations, is briefly described in [Chapter 2](#). In this chapter also an introduction in Bayesian network will be given together with the basic aspects of probabilistics.

[Chapter 3](#) gives mainly an overview of earlier work with respect to the data used in the dune safety assessment. In [Chapter 4](#) the construction of the actual Bayesian network is described in detail. This chapter also includes a validation of the Bayesian network.

Results of the predictions in the Bayesian network are described in [Chapter 5](#). Out of the dataset described in [Chapter 3](#) several samples were obtained and used for training and/or testing of the Bayesian network. A discussion corresponding to the results is made in [??](#). To conclude, conclusions and recommendations are presented in [Chapter 6](#).

2. Literature Study

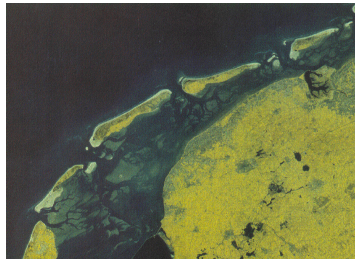
2.1. Dutch Dune Safety Assessment

In the Netherlands, the Law on Water Defences prescribes a five yearly assessment of the primary water defences by the administrators of those water defences. The Dutch government prescribes the safety assessment regulation for this purpose in Dutch: ‘Voorschrift Toetsen Veiligheid’ or VTV ([WL | Delft Hydraulics, 2007b](#)). Because the major part of the Dutch coastal defence consist of dunes, an appropriate assessment method for dunes is necessary. In the Guideline Dune Erosion ([TAW, 1984](#)), the safety assessment of dunes as water defences is described.

During daily circumstances one might think the dunes are no part of the active coastal processes. Hence, the water level and the waves do not reach the dunes. Only some aeolian transport is observed. However, during heavy storms on the North Sea high waves are generated and a water level set-up can be expected. Especially when the wind is out of Northern direction this set-up can reach several meters with respect to the mean sea level. This storm surge together with the astronomical tide can lead to high water levels in front of the coast. When those water levels are accompanied by high waves the dunes will be affected. Sand from the dunes will be transported to deeper parts and the dune-front will move landwards. In the 1970s and 1980s a lot of research has been done with respect to the dune-erosion due to heavy storms ([van de Graaff \(1977, 1986\)](#) and [Vellinga \(1982, 1986\)](#)). Despite of the complex dune-erosion processes, the final result can be described with a relative simple model DUROS ([Vellinga, 1986](#)) and updated to DUROS+ by ([van Gent et al., 2008](#)). Given some characteristics of the storm (maximum water level during the flood h_{max} and the significant wave height in deep water H_{0s}) and the grain diameter of the sand, the height and the shape of the upper part of the cross-shore dune profile after the flood can be determined. This upper part of the cross-shore profile is often indicated as dune erosion profile. Given the location of the cross-shore profile before the storm, the amount (volume) of dune-erosion can be calculated. An important assumption made is the closed sand balance, the amount of sand in the profile is constant (no losses or gains!) This closed sand balance procedure, which is realistic in case of a straight coast, determines the position of the dune retreat profile in horizontal direction. Moreover, the research also showed the amount of dune-erosion is not only determined by the water level but also by a relative large number of parameters. Due to this large number of process-determining variables a probabilistic approach to assess the dune safety was recommended. This recommendation is followed and formally written in the ([TAW, 1984](#)). In case of a probabilistic approach the probability of failure in case of a ‘just’ safe water defence is explicitly determined beforehand. The Delta-committee determined the probability of exceedance belonging to the design water levels. In addition, the Delta-committee pointed out emphatically that at water levels belonging to the design-level still ‘complete’ safety against failure of the water defence must be present. Therefore, the probability of failure is ten times smaller than the probability of the design water level (belonging to the extreme storm

e.g. 1/10,000 years event).

2.1.1. Study Area



(a) Wadden Sea and barrier islands



(b) 'Delta area'



(c) Holland coast

Figure 2.1.: The three typical features of the Dutch coasts

The Dutch coast can roughly be divided in three areas (from North to South): Wadden Sea and barrier islands (2.1a), Holland coast (2.1c) and the 'Delta' area (2.1b). When one looks to Figure 2.1 several complex features can be recognized (e.g. curved coastlines). Due to this assumption the dune erosion model has less applicability at both the *Wadden barrier islands* and the *Delta region*. In Section 3.2.1 a selection of transects (i.e., cross-shore bottom profiles) that meet the assumptions is made.

2.1.2. Dune erosion model DUROS+

This section describes the basics of the dune erosion model DUROS+: what goes in, how does it work and what comes out. In the official safety assessment method prescribed by the Dutch government, several model additions and assumptions were made to deal with uncertainties in model input (e.g. storm duration) and output (erosion volume). Here a description of the model itself will be given and minor attention will on the assessment method will be paid.

After the 1953 storm surge disaster important research was done to better understand the response of dunes to storm surges. This work resulted in the Guideline for Dune Erosion (TAW, 1984) that provides a relatively simple equilibrium approach (commonly referred to as the DUROS model (Vellinga, 1986)). Later research indicated a slightly greater peak period that was previously assumed. Additional laboratory tests and studies were performed (van Gent et al. (2008); van Thiel de Vries et al. (2008)) and those updated insights were integrated in the previous model leading to the currently applied DUROS+ model. The DUROS+ model assumes an equilibrium profile being developed during a storm, with an average duration (see 2.2b).

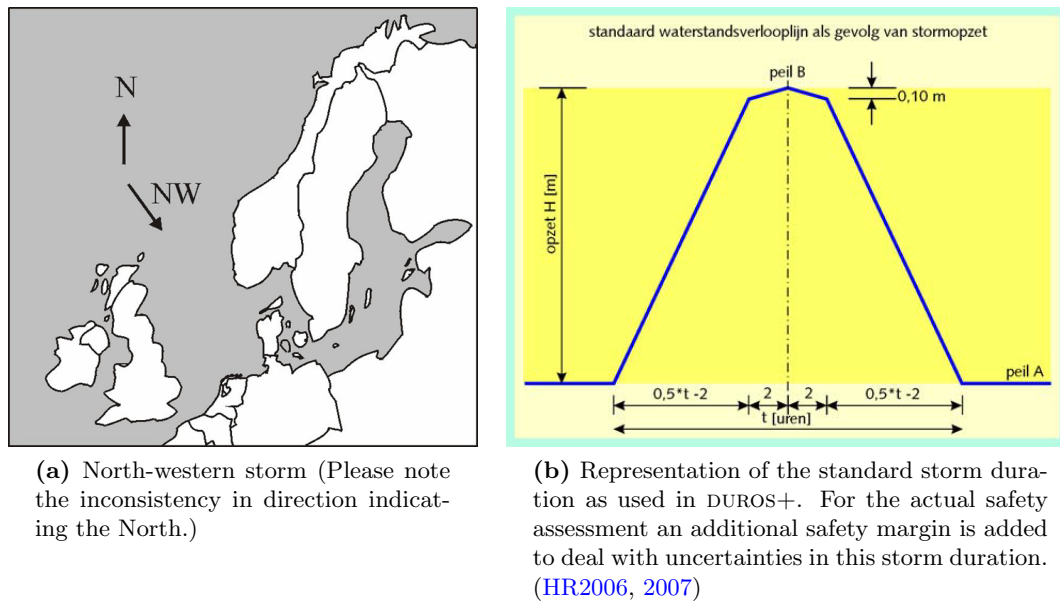


Figure 2.2.: Typical characteristics of a storm in front of the Dutch coast. A storm out of North-western direction will result in high waves (long fetch) and high water levels (wind induced set-up).

DUROS+ does not simulate the profile development in time, it approximates the post-storm profile shape. The cross-shore position of the post-storm profile is found by assuming conservation of volume in cross-shore direction. This 1D approach, implies an underlying assumption of alongshore uniformity. The post-storm profile consist out of three elements (Figure 2.3)

It is important to realise that DUROS+ is developed for dune *erosion*. Inundation or overwash processes (Sallenger, 2000), are not included in the model. Therefore, cases in which the water level is higher than maximum dune height of the first dune row will give no results and are not included in the dataset described in Chapter 3.

Required DUROS+ input

The post-storm cross-shore profile described by Equation A.1 depends on: (1) significant wave height (2) wave peak period (3) grain size (see Equation A.4). Together with the required pre-storm profile information they form the necessary input to DUROS+ model. In Section 3.2 an extensive description of each variable and the way of obtaining the data (how is a value

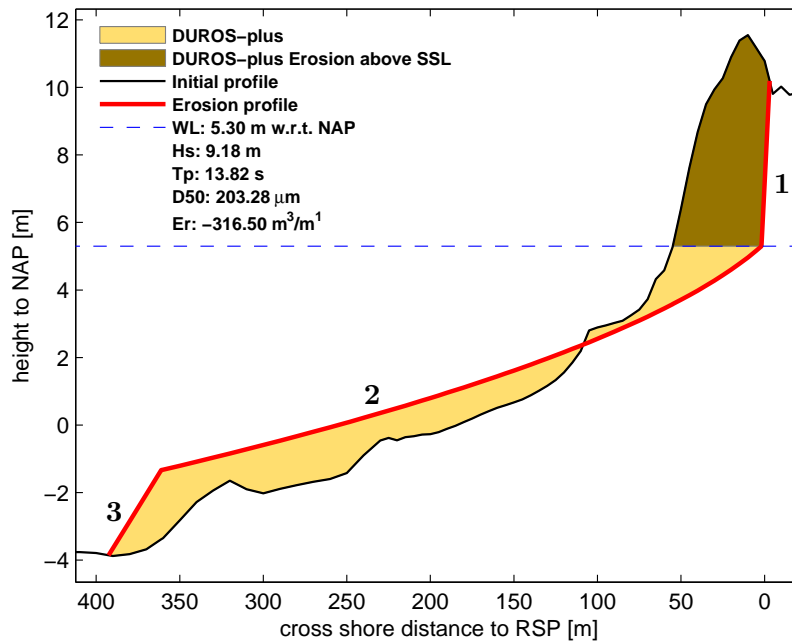


Figure 2.3.: Example of a DUROS+ output for an arbitrary case. The red line indicate the post-storm profile and the numbers indicate the three elements the post-storm profile is built of.

selected or sampled and where does it come from).

The set of all findings entered into the nodes of a single Bayesian network is referred to as a *case*. A case provides information about a particular event or scenario. For example, a case could represent a storm event characterized by a water level, wave height and peak period. Case files consist of many cases acting as a database, in which each case is record. In [Section 4.3](#) such a file of cases will be used to learn the conditional probability tables of the Bayesian network.

2.2. Theoretical Introduction

2.2.1. Probability Theory

Before a start will be made with the introduction to the Bayesian network approach the basic probability calculus will be given. Some rules and axioms are presented together with a few words regarding notation.

For a clarification of the basic notations; random variables are denoted by capital letters A, B, C , etc. The values taken by these variables are denoted by lower case letters a_i, b_j, c_k , etc. The probability for A to assume the value a_i is denoted by $P(A = a_i)$ or in short $P(a_i)$. Where P is a number in the interval $[0, 1]$. Lower-case $p()$ refer to a probability density function. So, $p(a)$ is the probability density function of an observed quantity (e.g. heads in case of flipping a coin). Formal notation of the probability of a joint event A and C is $P(A \cap C)$, however here, in Bayesian statistics more often used, $P(A, B)$ will be used. In the end it means the same¹.

¹fundamental rule for variables

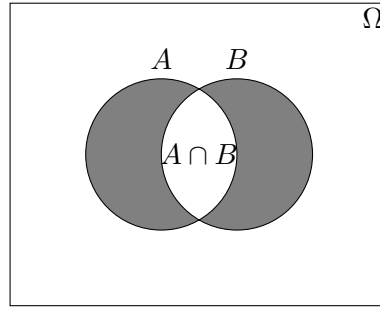


Figure 2.4.: Venn diagram of two events A and B in sample space Ω . The intersection of A and B is indicated with $A \cap B$.

2.2.2. Conditional probability

Knowing that an event has occurred sometimes forces us to reassess the probability of another event, the new probability is called *conditional probability*. In general, computing the probability of an event A , given that an event C occurs, means finding which fraction of the probability C is also in the event A . The conditional probability of A given C is given by:

$$P(A|C) = \frac{P(A, C)}{P(C)} \quad (2.1)$$

Provided $P(C) > 0$.

From this definition a useful rule can be derived by multiplying left and right by $P(C)$, this is called the product or multiplication rule:

$$P(A, C) = P(A|C)P(C) = P(C|A)P(A) \quad (2.2)$$

Computing the $P(A, C)$ can hence be decomposed into two parts, computing $P(C)$ and $P(A|C)$ separately, which is often easier than computing $P(A, C)$ directly.

Using both, definition of conditional probability ([Equation 2.1](#)) and the multiplication rule ([Equation 2.2](#)) one can derive *Bayes' rule*²:

$$P(C|A) = \frac{P(A|C)P(C)}{P(A)} \quad (2.3)$$

Under its usual Bayesian interpretation, it asserts that the probability of a hypotheses C conditioned upon some evidence A is equal to the *likelihood* $P(A|C)$ times its probability prior to any evidence $P(A)$, *normalized* by dividing $P(C)$ (so that the conditional probabilities of all hypotheses sum to 1). [Equation 2.3](#) is the traditional form of Bayes' theorem, after the English mathematician and Presbyterian minister who derived this in the 18th century. Bayes' theorem is fundamental in Bayesian statistics, and is widely applied in fields including engineering, science, medicine and law. An application of Bayes' theorem can be found in a Bayesian network ([Section 2.3](#)) and will be used in this thesis.

²In fact it was the French mathematician and astronomer Pierre-Simon Laplace who adopted Bayes' ideas in this formula. However, a half century of usage forces us to give Bayes' name to what was really Laplace's achievement.

Independence

Because independency plays an important role in Bayesian networks, attention will be paid to the independence in statistics. If the conditional probability equals what the probability was before, the events evolved are called *independent*. In formula form: An event A is called independent of C if

$$P(A|C) = P(A) \quad (2.4)$$

By application of the multiplication rule (Equation 2.2), if A is independent of C , then $P(A, C) = P(A|C)P(C) = P(A)P(C)$. Finally, by definition of conditional probability, if A is independent of C , then

$$P(C|A) = \frac{P(A, C)}{P(A)} = \frac{P(A)P(C)}{P(A)} = P(C)$$

that is, C is independent of A . This works in reverse, too, so we have:

$$A \text{ independent of } C \iff C \text{ independent of } A$$

This statement says that in fact, independence is a *mutual property*. Therefore one may say ‘ A is independent of C ’ or ‘ A and C are independent’.

2.2.3. Probabilistic Graphical Models

A Bayesian network is one example in the family of *probabilistic graphical models*. Probabilistic graphical models are a combination of probability theory and graph theory. The definition of a probabilistic model can be found in (Pearl, 1988): “A set U of discrete random variables together with a joint probability distribution $P()$ defined over these variables.” A probabilistic graphical model provides a natural tool for dealing with two problems occurring in engineering: *uncertainty* and *complexity*. Fundamental in the idea of a graphical model is the notion of modularity; a complex system is built by combining simpler parts. The probability theory provides the glue whereby the parts are combined, ensuring that the system as a whole is consistent, and providing ways to interface models to data. Graphical models provide an intuitively appealing interface by which humans can model highly interacting sets of variables.

2.2.4. Difference between Bayesian network and other probabilistic graphical models

To put the Bayesian network in the context of other probabilistic graphical models the main differences between the Bayesian network and two other often used models will briefly be discussed below.

- *Markov model* or *hidden Markov model*: also a graphical probabilistic model comparable to a Bayesian network. However, a Markov net is capable to represent cyclic dependencies. In a regular Markov model, the state is directly visible to the observer, and therefore the state transition probabilities are the only parameters. In a *hidden* Markov model, the state is not directly visible, but output, dependent on the state, is visible. The main difference between a Markov model and a Bayesian network is that a Markov model concept does not have *history* (i.e., prior information).

- *Neural Network*: An (artificial) neural network is a system based on the operation of biological neural networks, in other words, it is an emulation of biological neural system. Often people refer to Neural networks as a ‘black box’ due to the difficulty in understanding the decision making process of the Neural network.

There are more graphical models (e.g. restricted Boltzmann machine, junction (or clique) tree) but they are less applied in probabilistic theory and will therefore not be discussed here.

2.3. Bayesian Network approach

A Bayesian network is a method of reasoning using probabilities³, where the *nodes* represent variables and *arrows* represent direct influence between the nodes. Nodes can represent any kind of variable: a measured parameter, a latent (hidden) variable or a hypothesis. The graph itself is a *directed acyclic graph (DAG)* which means that arrows have a direction (they point from one node to another) and there are no cycles or loops in the network (to prevent new information would be affected by its own result).

If there is an arrow from node A to another node B , A is called a *parent* of B , and B is a *child* of A . The set of parent nodes of a node x_i is denoted by $parents_{x_i}$. In a Bayesian network the joint probability distribution of the node variables can be written as the product of the local distributions of each node and its parents as

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i | parents_{x_i}) \quad (2.5)$$

In this sense Bayesian networks have built-in independence assumptions. In summary (Wikle and Berliner, 2007): one has prior belief (information) in a certain event, collects data, and then updates that belief given that new data (information). In the Bayesian world, the definition of belief is simply the conditional probability given the evidence. So, the task of the Bayesian network is to compute the posterior distribution function. This section gives a general introduction in Bayesian networks, in Chapter 4 the Bayesian network approach is applied.

The best way to understand a Bayesian network is to illustrate it with an example. Here, the burglar-earthquake alarm example (adopted from Pearl (1988)) will be used. Consider the network in Figure 2.5. This network represents a distribution on 5 variables $P(B, E, A, C, R)$. The intuition behind the model is that either burglaries or earthquakes (Pearl lived in Los Angeles) can trigger an alarm. If the alarm rings your neighbour may call you at work to let you know. On your rush way home you hear a radio report of an earthquake, the degree of confidence (i.e. belief) that there was a burglary will diminish. This problem is captured in Figure 2.5. Nodes and edges form the qualitative part, the quantitative part of the network consist out of a set of conditional probability distributions. The table in Figure 2.5 shows the conditional probability for the node ‘alarm’. Together both parts form a unique distribution in factored form (see also Equation 2.5):

$$P(B, E, A, C, R) = P(B)P(E)P(A|E, B)P(R|E)P(C|A) \quad (2.6)$$

³Several names are used to describe the method of reasoning using probabilities: belief networks, Bayesian networks, knowledge maps, probabilistic causal networks, and so on. In this work Pearl’s name, Bayesian networks (Pearl, 1988), is adopted, on the grounds that the name is completely neutral about the status of the network (do they really represent beliefs, causality or what?).

E	B	$P(A E, B)$	
e	b	$P(A e, b)$	$P(\neg A e, b)$
e	$\neg b$	$P(A e, \neg b)$	$P(\neg A e, \neg b)$
$\neg e$	b	$P(A \neg e, b)$	$P(\neg A \neg e, b)$
$\neg e$	$\neg b$	$P(A \neg e, \neg b)$	$P(\neg A \neg e, \neg b)$

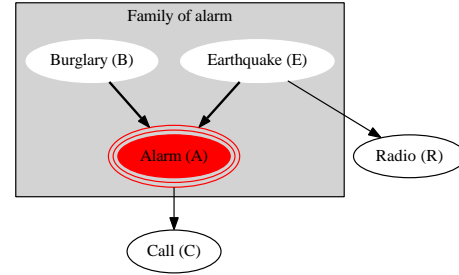


Figure 2.5.: Example of a Bayesian network considering an alarm that can be triggered by either an earthquake or a burglar. Left: the conditional probability table for the node ‘alarm’, \neg means ‘not’, so $\neg b$ means no burglary. Right: Bayesian network of the situation with in the box the family alarm.

Via the conditional probability definition (Equation 2.5) one can see that a node is independent of its *ancestors* given its *parents*. In this example this means: the node ‘call’ (child) is independent of ‘burglary’ and ‘earthquake’ (ancestors) given the node ‘alarm’ (parent). So, given ‘alarm’, ‘call’ is conditionally independent of ‘burglary’ and ‘earthquake’. In Bayesian terms this is called, the nodes ‘burglary’ and ‘call’ are *d-separated*. There is no *d-connecting* path between both nodes. Two nodes are *d-connected* if either there is a causal path between them, or there is evidence that renders the two nodes correlated to each other (Charniak, 1991). Together they form a ‘family’. In Figure 2.5 the family of the node ‘alarm’ is highlighted in the grey box. So, Equation 2.5 specifies the required parameters for this Bayesian network, the probability of every node given all possible combinations of its parents. Representing joint probability distributions via conditional independence is very useful, it reduce the dimensionality of the network from full joint probability table, $2^5 - 1 = 31$ to $1 + 1 + 4 + 2 + 2 = 10$ parameters (Equation 2.6). This savings might not seem great, but if the net is doubled in size the number of parameters become $2^{10} - 1 = 1023$. One can imagine that in case of larger nets and non-boolean nodes (not just two possible classes, non-dichotomous) the number of parameters will grow exponentially.

2.3.1. Bayesian inference

Bayesian inference is the application of Bayes’ theorem to calculate how the degree of belief in a proposition changes due to evidence. For illustration, let X denote unobservable quantities of interest and Y the data. The full probability model can always be factored in components $p(x, y) = p(y|x)p(x) = p(x|y)p(y)$. Applying Bayes’ rule (Equation 2.3) gives

$$p(x, y) = \frac{p(y|x)p(x)}{p(y)},$$

provided $0 < p(y) < \infty$. It is good to examine each component of Bayes’ rule separately (Wikle and Berliner, 2007).

- Data distribution, $p(y|x)$

Distribution of the data, given the unobservables. When viewed as a function of X for fixed y , it is known as a *likelihood function*, $L(x|y)$. A key is one thinks of the data *conditioned* on x . For example, if Y represents imperfect observations of temperature,

and X is the true temperature, then $p(x|y)$ quantifies the distribution of measurement errors in observing temperature, reflecting instrument error as well as possible biases.

- Prior distribution, $p(x)$
This distribution quantifies the *a priori*⁴ understanding of the unobservable quantity of interest. For example, if X corresponds to a temperature, then one might base this prior distribution on historical information (climatology). A choice of a prior distribution can be subjective and forms an integral part of the Bayesian inference.
- Marginal distribution, $p(y) = \int p(y|x)p(x)dx$
For the observations Y , $p(y)$ can be thought of as the ‘normalizing constant’ in Bayes’ rule. Unfortunately, it is only for specific choices of the data and prior distribution that one can solve this integral analytically (Cooper, 1990).
- Posterior distribution, $p(x|y)$
This distribution of the unobservables given the data is for this thesis the primary interest for inference. The posterior is the *update* of our prior knowledge about x given the actual observations y .

2.3.2. Practical application of the Bayesian network

Previous section introduces the general aspects of the Bayesian network. In this section a more pragmatic part will be discussed, how can the Bayesian network be of use in order to assess dune safety level? First an overview of practical advantages that may be valuable will be given:

1. Modular representation of knowledge. A complex system is built by combining simpler parts. What is known and how are the variables mutual correlated.
2. Local, distributed algorithms for inference and learning. Lower time complexity (less time for inference)
3. Intuitive (possible causal) interpretation.
4. Factored representation may have exponentially fewer parameters than full joint representation (remember the burglar alarm example) Lower sample complexity (less data for learning).

Once a Bayesian network is constructed it can be used for several applications:

1. Posterior probabilities: probability of an event given any evidence
2. Most likely explanation: scenario that explains evidence
3. Rational decision making: maximize expected utility
4. Effect of intervention: causal analysis

Main objective of this thesis is to update prior probabilities with new information. Therefore primary use will be made of Bayesian feature to compute posterior probabilities given evidence. However, also the other features will be taken into account.

⁴a priori (lat.): beforehand

3. Simulation Data

The requirements to construct a useful Bayesian network are data that measures the inputs (morphology and hydrodynamics) and output (coastal response, here erosion volume) during actual or simulated storm conditions. Since this thesis focusses on extreme conditions (1/10,000 year storm conditions), observations are not available and numerical simulations will be made to fill the database. This chapter describes the used dataset. In order to compare both approaches, current safety assessment method (described in [WL | Delft Hydraulics \(2007a,b\)](#)) with the Bayesian network approach, data will be obtained from the same sources. This means: hydrodynamics are obtained out of [HR2006 \(2007\)](#), coastal profiles from JARKUS and the dune erosion model DUROS+ will be used to simulate dune erosion. More information about these data-sources can be found in [den Heijer et al. \(2011\)](#) and [WL | Delft Hydraulics \(2007b\)](#). Here an overview of the content of the dataset will be given. This chapter gives an overview of the content of the dataset, to enable a clear distinction between the source data and the Bayesian network, as discussed in the next chapter.

3.1. Content Dataset

The assumptions and simplifications made by using DUROS+ largely determine the content of the Bayesian input dataset. Variables that play a role in DUROS+ (e.g. water level) can be found in the dataset, while, on the other hand, variables not considered in DUROS+ (e.g. wave direction) will be ignored. The amount of dune erosion, computed by the DUROS+ model, depends on a number of parameters and factors ([Section 2.1.2](#)), namely:

- The shape of the initial cross-shore profile
- The diameter of the dune material
- The maximum storm surge level
- The significant wave height
- The (peak) wave period

This set of variables can be categorized in profile information (first two) and hydraulic boundary conditions (last three). Together they form the load and strength part during a storm event.

Detailed *profile information*, including foreshore, beach and first dune rows is essential to enable the safety assessment. In the Netherlands a well established monitoring program that measures the coastal cross-shore profiles on a yearly basis is active ([Rijkswaterstaat - Ministry of Infrastructure and Environment, 2010](#)). Called, JARKUS transects (JAaRlijkse KUSTmetingen; yearly coastal measurements). In order to assess the safety level against flooding at a particular point in time (static) transects of one year will be used. Here, the

most recent measurements (2010) serve as input to the DUROS+ simulations.

The *hydraulic boundary conditions*, water level, wave height and wave period, are derived from the time series of (directional) offshore wave buoys. In the Dutch case a number of offshore wave buoys is available with decades of high resolution time series. For a location of these buoys see [Figure 3.1](#). Because the time series are relative short with respect to the design conditions (100 years vs. a once in ten-thousand year storm event) extrapolation is necessary. A distribution is fitted to the observed data such that extreme values (that are never observed) are associated with a probability of occurrence. The stochastic description of this hydraulic boundary conditions can be found in (in Dutch:) ‘Hydraulische Randvoorwaarden 2006’ or [HR2006 \(2007\)](#)).

Finally, a *response variable* is added to describe the coastal response after an arbitrary storm event. Several measures (or *indicators*) can be used to describe the coastal response, for instance: retreat distance, change in dune height, total eroded volume, etc. Since DUROS+ have been used to simulate dune erosion process the response indicator must be obtained out of DUROS+ output. More about the DUROS+ output can be found in [Section 2.1.2](#). The choice for an appropriate response indicator(s) is elaborated in [Section 4.2.2](#). In summary, the dataset can be classified in three categories: ([Table 3.1](#)).

<i>Type</i>	<i>Data</i>	<i>Source</i>
measured	profile information	JARKUS
extrapolated	hydraulic boundary conditions	HR2006
modelled	coastal response to a storm	DUROS+

Table 3.1.: Types of data that serve as input for the Bayesian network

3.2. Input Variables duros+

Previous section describes which variables are included in the dataset. In this section attention will be paid to the figuration of the data (i.e. what sort of water levels go in the dataset). A start will be made with an description of the profile information followed by an extensive description of the hydraulic boundary conditions.

3.2.1. Profile information

Due to the balancing procedure in calculating the volume of erosion, the shape of the initial profile, the profile as present just before the storm event starts, is a determining factor for the amount of dune erosion ([WL | Delft Hydraulics, 2007b](#)). Since a profile measurement taken just before the storm surge will hardly ever be available, the uncertainty in the actual profile and thus the sediment content in the erosion-accretion zone is a relevant parameter also. Here the assumption is made that storm impact is relatively large with respect to seasonal fluctuations. The JARKUS data contains in total 2178 cross-shore transects which are measured yearly since 1965. For this research the most recent (2010) transect measurements have been used. The transects have an alongshore spacing ranging from 150 m to 250 m. The orientation of the transects is approximately shore normal and therefore seaward directed.

Selection of transects

Due to the underlying assumptions of the DUROS+ model a selection is made of transects that meet these assumptions. Table 3.2 gives an overview of transects that will be *excluded*. After

<i>Exclude transect when</i>	<i>Location</i>	<i>DUROS+ assumption</i>
Landward directed	Mainly around the heads of the Wadden islands	Alongshore uniformity
Outside the range of the offshore measuring stations that serve as source for the hydraulic boundary conditions	Southwards of Hoek van Holland (Delta region)	Wave measurement data of sufficient quality
Contains a hard structure (harbours are left out)	i.e. Hondsbossche and Pettermeer Zeewering	DUROS+ is valid for sand only
Contains only the underwater profile up to the beach, or only contain part of the beach and/or dune	Various	A post-storm erosion-sedimentation balance can not be computed

Table 3.2.: Overview of the selection of transects. Because DUROS+ is not able to model all parts of the Dutch coast certain areas are excluded.

this filtering 960 transects remain and they will form the cross-shore bottom profile input of the DUROS+ simulation. Figure 3.1 gives an overview of the location of those transects. The remaining transects are cropped at the landward side if more than one dune row is present in the data. This means that only dune erosion (or failure) of the most seaward dune row is calculated. Two main reasons have led to this choice (den Heijer et al., 2011). Firstly, this gives the best comparable results between two arbitrary transects. Secondly, after failure of the first dune row, any further erosion or overwash processes should be considered 2D.

3.2.2. Grain Diameter

The diameter of the sediment of the dune is an important parameter since it affects the shape of the erosion profile (see both Equation A.1 and Equation A.4). Grain size distributions haven been derived from sediment samples at 146 locations along the Dutch coast (TAW, 1984). For every location a normal distribution of grain sizes (D_{50}) is given (possible to have different grain sizes for the same transect). Mean grain sizes vary between 159 and 277 μm and the standard deviations vary between 8 and 37 μm . Although it is not known where in the cross-shore these samples have been taken, they are assumed to be representative for the entire cross-shore profile (den Heijer et al., 2011). Depending on the location of the transect a representative grain diameter will be obtained and serves as input for a DUROS+ simulation. Figure 3.2 shows the distribution of grain sizes in alongshore direction together with an example of a normal distribution at a certain location.

3.2.3. Water level

It is clear that dangerous water levels occur in combination with severe storm surges. The maximum surge to be reached during an arbitrary storm event is the resultant of two stochas-

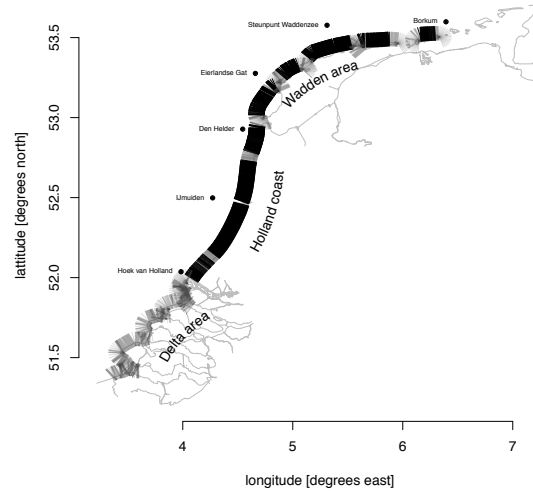


Figure 3.1.: Overview transects that were selected as suitable for DUROS+ (black). Not selected transects are shown in gray. (den Heijer et al., 2011)

tically independent phenomena, namely the astronomical tide and the wind set-up. The astronomical tide is a periodic predictable component due to the relative movements of sun, moon and earth. The set-up is the result by weather circumstances (mainly wind direction and wind speed) and therefore unpredictable. So, the set-up can be computed by: observed high tide level minus the astronomical tide level at that moment. Together, they form the *storm surge level* (ssl). Hereinafter the terms storm surge level and water level are used interchangeably, the meaning is for both the same and equal to aforementioned description of the storm surge level.

With the available water level data a distribution is fitted, use is made of the so-called conditional Weibull distribution function (see Roskam et al. (2000) and WL | Delft Hydraulics (2007a) for more details). The distribution describes the frequency of exceedance of the highest level (h) during a storm surge (per year). The water level with the highest probability will match with the mean water level and lie around the N.A.P.¹ +2 m. (see red histograms in Figure 3.3) Here the mean value represents the normal daily condition. However, the focus does not lie on those conditions but on more extreme conditions. Those extreme conditions have a very low probability of occurrence and lie in the outer tail of the *probability density function* (pdf). Thus, when randomly select a water level out of this pdf the probability of such a value is quite small. Therefore a lot of samples² are needed to guarantee also the extreme conditions are present. However, there is a statistical method to focus the low-probability values, this method is called *Importance Sampling* (IS). This technique make sure that the samples taken are near the area of interest only. The area of interest in this thesis are the extreme water levels. Since only a certain area in the probability space is observed, it is possible that one or more parts of the failure space are not represented in the dataset.

¹N.A.P. (Normaal Amsterdams Peil) is the Dutch reference level, at present approximately corresponding to mean sea level.

²rule of thumb $400/p_f$ (CUR, 1997)

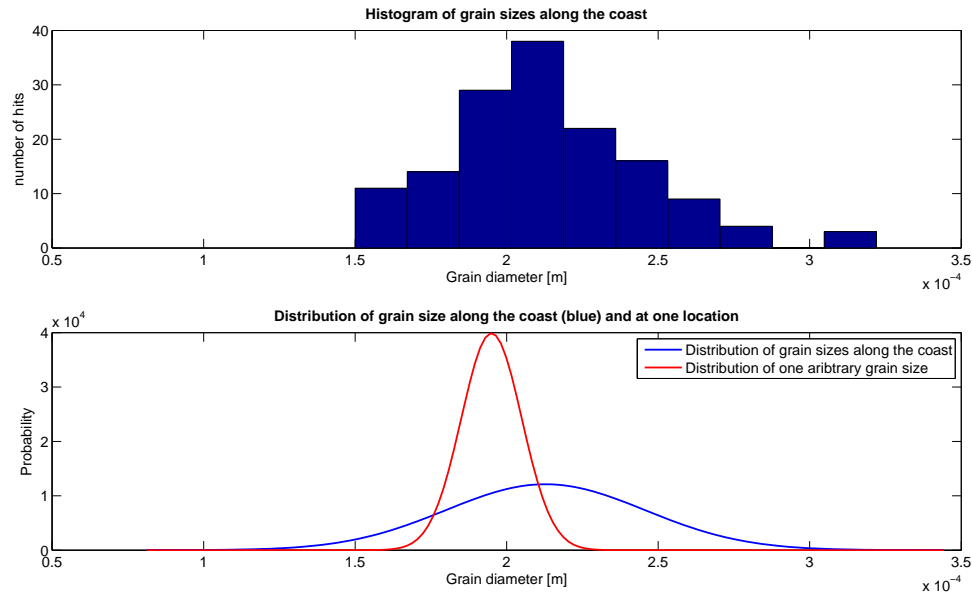


Figure 3.2.: Upper figure: histogram of grain sizes along the Dutch coast. Lower figure: Normal fitted probability density distribution grain sizes with $\mu = 2.10 \cdot 10^{-4}$ and $\sigma = 3.08 \cdot 10^{-5}$. The mean and standard deviation of the grain size at a single location is also given: $\mu = 1.90 \cdot 10^{-4}$ and $\sigma = 1.0 \cdot 10^{-5}$.

However, one would not expect problems since the low water levels, probably, will not influence the failure rate.

With the use of IS less samples are needed and that reduces the computational time considerably. Result of the use of the IS technique is the shift of the probability density function to higher water levels (see blue bins [Figure 3.3](#)). With the use of IS the probability of an extreme water level is high with respect to the non-IS probability. So, there are less samples needed to make sure also extreme values are present. [den Heijer et al. \(2011\)](#) assumed that 100 samples leads to satisfying results for the purpose of their study. An enormous reduction in comparison with the 40,000,000 before. In line with this findings, 100 samples per selected transect will be made.

3.2.4. Wave height

The main driving force during storms is wind. This make the wave height is correlated to the water level. The highest water level and wave height are expected when a storm is moving from North-Western direction. High wind speeds active over a long fetch could cause major set-up and high waves in the basin-shaped North Sea (see [Figure 2.2](#)).

According to ([TAW, 1984](#)) the stochastic properties of the wave height are described by a normal distribution. The mean value is related to the water level by a relation obtained from [Stijnen et al. \(2005\)](#). So, given a water level, with this relation the mean wave height can be computed. This mean wave height is normally distributed (with a fixed standard deviation) and a wave height can be sampled.

In [Figure 3.4](#) the probability density distribution of the wave height is given. Due to the shift

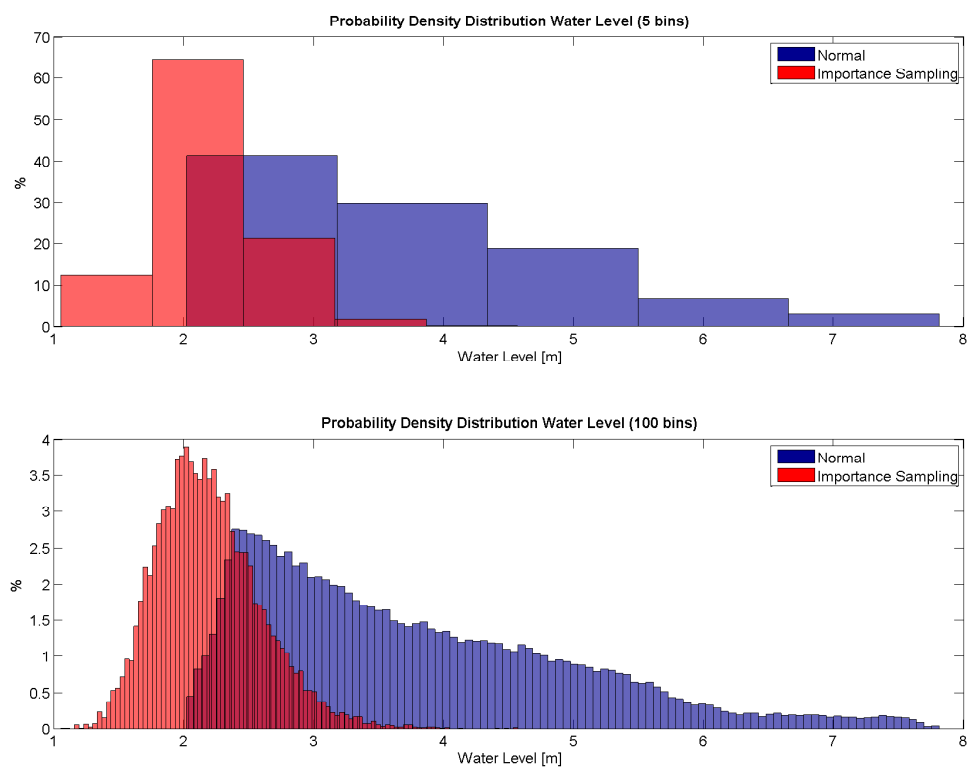


Figure 3.3.: Water level distribution with Importance Sampling

of the water level due to importance sampling to more high water levels, also the distribution of the wave height will be skewed to higher wave heights.

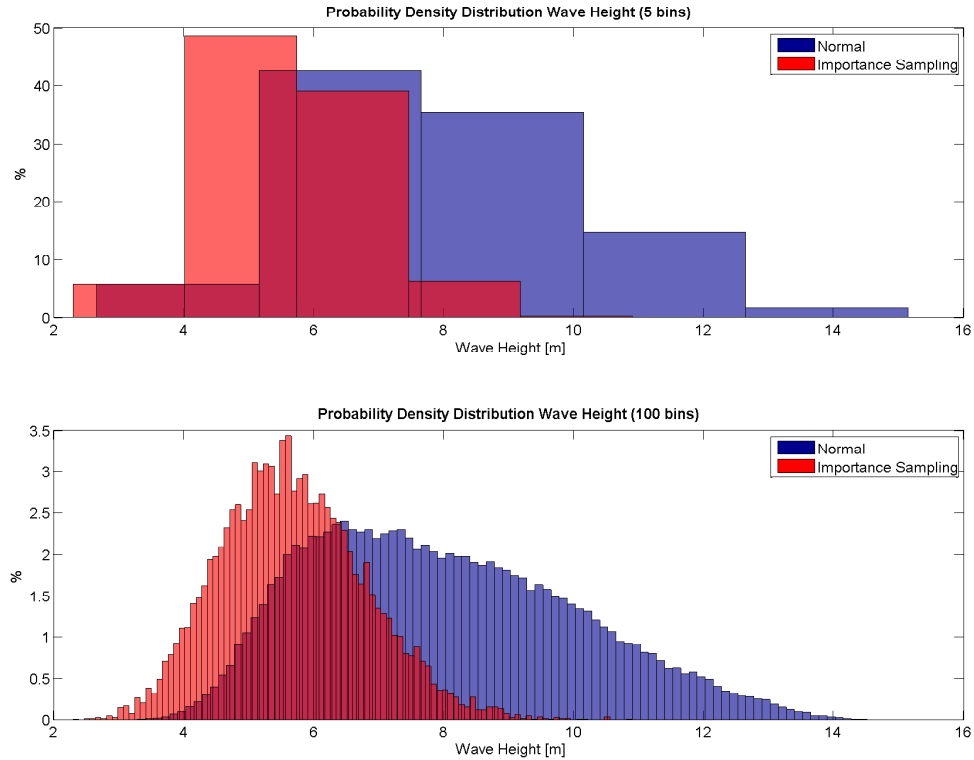


Figure 3.4.: Wave height distribution with Importance Sampling

3.2.5. Wave period

The wave period is correlated to the wave height, and thus indirectly to the water level. Physically, this relation holds that the wave steepness cannot be larger than a certain degree (around $1/20$ for wind waves). Comparable to the approach discussed in the previous section, also the wave period is normally distributed with a fixed standard deviation. So, again, given a wave height, a wave period can be obtained. In line with previous two variables also the probability density distribution of the peak period is presented in [Figure 3.5](#). Still a shift of wave periods to higher values is recognizable, however less strong than with the wave height. This illustrates there is a statistical relation between water level and wave height, but this relation is less strong than the relation between water level and wave height.

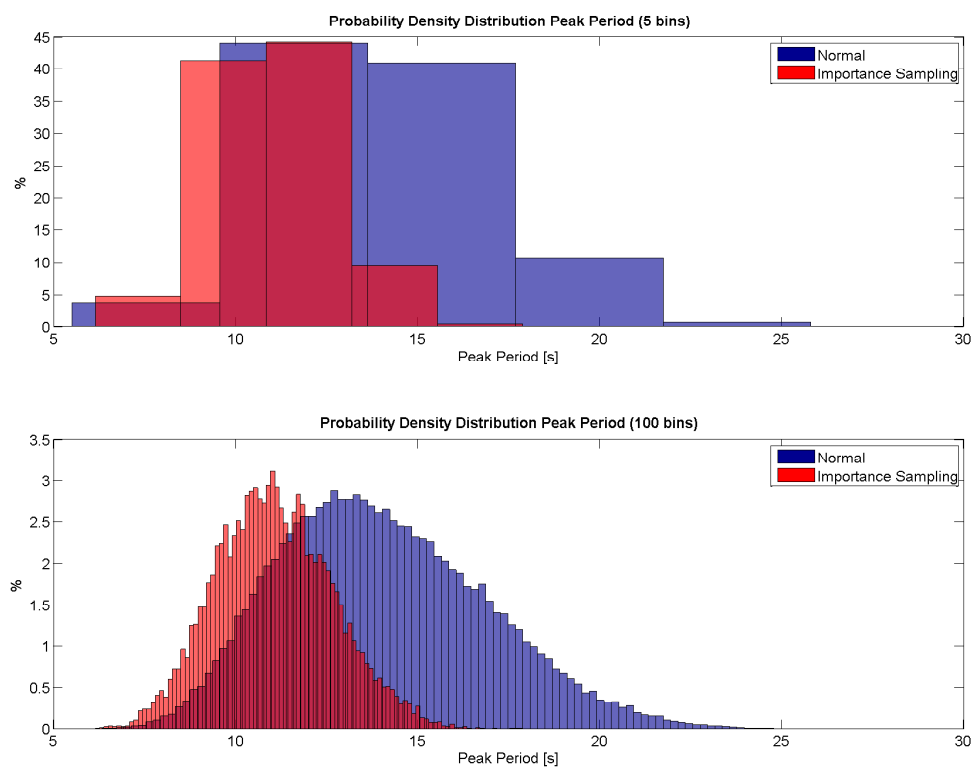


Figure 3.5.: Peak Period distribution with Importance Sampling

4. Bayesian model Set-up

4.1. Introduction

This chapter describes the set-up of the Bayesian network. This set-up can roughly be divided in three actions, respectively: construction, training and validity check. The first step (Section 4.2) is characterized by the actual construction of the network. Nodes and arrows are selected and form the actual lay-out of the Bayesian network. One has to decide which variables to include in the Bayesian network (nodes), select correlations between the variables (arrows) and discretize all the variables in a set of bins (histogram). The next step is to find the conditional distributions of the unobservable quantities given the observed data. In other words, given the variables and correlations of interest, the underlying conditional probability tables get filled with the use of data. The dataset constructed in Chapter 3 will be used for this iterative process. The feeding of information to the Bayesian network in order to construct the conditional probability tables is called *training* and will be described in Section 4.3. Finally, an evaluation of the fit of the model and its ability to adequately characterize the process of interest is made. In other words, is the Bayesian network capable to cover a representative sample of the behaviour domain to be measured and is the Bayesian network able to scientifically answer the questions it is intended to answer? In statistics this is called *validity* and will be described in Section 4.4.

For the construction, training and running of the Bayesian network the software package NETICA (www.norsys.com) was used. Netica is a widely used and relatively user friendly software program. Moreover, there is a Java version available that makes it possible to run inside a technical computing language (i.e. Matlab, Python). After all, the approach is in line with the work of Plant and Holland (2011a) and Plant and Stockdon (2012).

4.2. Construction of the Bayesian network

First step in the construction of the Bayesian network is to decide which variables play a role in the dune erosion process. The selected variables of interest are represented by nodes in the Bayesian network (see Figure 4.1). The decision which variables to include in the Bayesian network is based on two factors: first, the Bayesian network model is consistent with the knowledge of underlying physical processes. Second, because DUROS++ was used to simulate the erosion process, the selected variable are part of the DUROS++ simulation as well. (See Section 2.1.2 for an overview of variables available in DUROS++.) A variable must satisfy both statements to be selected. For instance, wind velocity has impact on dune erosion (REF), however, it is not part of the DUROS++ simulation so not selected as variable in the Bayesian network. Variables which are selected are illustrated in Figure 4.1. A correlation between two variables in a Bayesian network is illustrated with a directed arrow (see Figure 4.1). In a Bayesian network correlation means direct influence and can be called *statistical dependency* indicating the dependency does not need to be causal or physical.

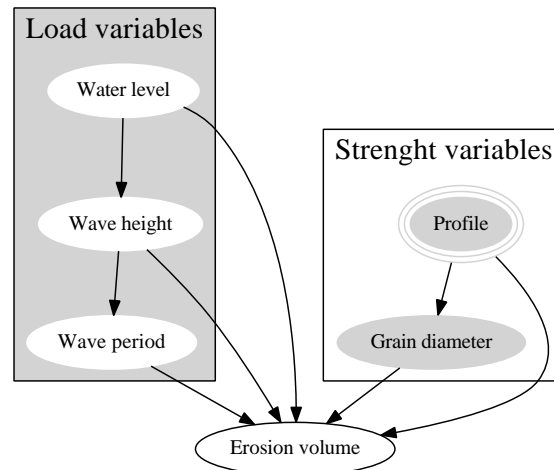


Figure 4.1.: Schematisation of simplified Bayesian network. Each node represents a physical variable and the arrows show direct influence.

(more about the correlations in [Section 2.3](#)) An arrow is drawn when a physical relation (i.e. dependency) is according to our knowledge of the underlying scientific processes. This means an arrow is drawn between water level and wave height (a physical relation exist) but no arrow between water level and grain size for instance. Finally, an overview of nodes and arrows is presented in [Figure 4.1](#). Here the nodes are grouped in a load and strength side. Together they result in coastal response, erosion volume in this case. The node ‘profile’ is highlighted to indicate profile itself is not a single variable. Next section will give more information about this issue.

4.2.1. Profile Characterization

Cross-shore bottom profiles in front of the Dutch coast (JARKUS transects) are described by vertical (altitude) and horizontal (cross-shore distance) vectors. In order to capture a bottom profile precisely, all x, z coordinate needs to be described by a single node in the Bayesian network. This takes probably a lot of effort and the resulting network will consist out of an enormous number of nodes that slow down the time for inference. Therefore, the initial bottom profiles will be characterized into several *profile indicators*. Each indicator represents a characteristic of the profile. Due to this spatial simplification some information will get lost. To characterize the profile four profile indicators are defined. Then numbers hereafter correspond to the numbers of [Figure 4.2](#).

PI 1 *Profile volume, V_{prof}*

Profile volume enclosed by horizontal reference levels NAP +3m (dune foot) and NAP −4m, together with the vertical boundaries landward and seaward of the intersection of the horizontal reference levels with the initial bottom profile. (Green box in [Figure 4.2](#).) This area indicates the potential accretion volume and is independent of dune height.

PI 2 *Slope of V_{prof} , s_{vol}*

Slope of the volume as indicated in PI 1 (purple line). This slope is comparable to the foreshore slope and is added to give more information of the shape of the profile volume.

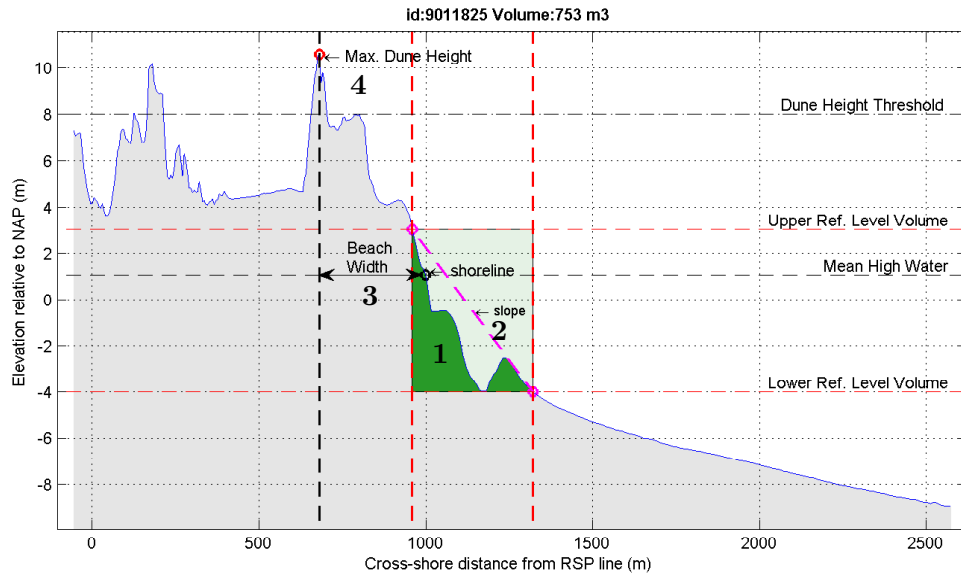


Figure 4.2.: Profile indicators for a random selected pre-storm bottom profile. The numbers correspond to the numbers in the list of profile indicators described in this section.

PI 3 Beach width, W_b

Beach width is the cross-shore distance between maximum dune height first dune row and the shoreline. So, in fact here the beach width represents the regular beach width extended with half the dune first dune row dune width. Beach width in this sense is a somewhat confusing term since it is not the common definition.

PI 4 Maximum crest level of first dune row, z_c

Maximum crest level of the first dune row. The dune height is an important indicator of the strength of a dune, since it gives information about the dry volume of sand.

4.2.2. Response Indicator

The response indicator represent the morphological response due to a storm event. During calm conditions no activity or volume changes at the upper dry beach is expected (maybe some aeolian losses). This means that for most cases the response will be negligible. However, during a severe storm event a change in dune volume could be expected. In [Section 3.1](#) a few examples of profile indicators are already given. Here the choice is made to use the erosion volume above storm surge level (hereinafter referred to as erosion volume or erosion volumeSSL in figures). The dune-erosion above storm surge level is the volume of erosion of the initial dune above the storm surge level threshold. This indicator is typical for storm regimes attacking the Dutch coast because there are always accompanied by a severe surge.

4.2.3. Discretization

Because Netica works with categories instead of numbers, it is required to represent the continuous physical processes with a small number of discrete categories. This process (hereafter referred to as *discretization*) results in a number of bins, representing the estimated probability density distribution (i.e. *histogram*) for each variable. With the discretization process

some information get lost. For example a bin with an interval of $[1, 2]$ cannot distinguish a value of 1.1 and 1.9, both numbers fall in the same bin. Following the definition in [Stevens \(1946\)](#), discretizing results in a change in measure scale from *ratio* to *nominal*. There is not one single method to discretize a continue signal in discrete categories. Hereafter, the choice for the number of bins and the range per bin will be described.

The number of bins affects the dimensionality and complexity of the Bayesian network as follows. If there are M variables and each variable has N bins, the number of correlations that are required grows (exponentially) to a size of N^M , which can lead to enormous memory requirements. The choice for the number of bins is a balance between a couple of interest and will be explained below. If there are too *many* bins relative to the size of the training data set, very few observations (or hits) will fall in some of the probable states and they, therefore, will not be well constrained. Too *few* bins, however, may not resolve the variability or threshold values of the variables, blurring important sensitivities.

Theoretically, the number of bins can vary between two (so-called, dichotomous data¹) and N , the number of values in the dataset of that variable. The last mentioned scenario describes the case for which the number of bins equals the number of unique values and one can distinguish all possible scenarios. However, mostly the number of bins lies somewhere between those outer edges.

Practically, the guidelines defined by [Plant and Holland \(2011a\)](#) will be followed initially.

1. The intervals should be as wide as possible to minimize computation effort.
2. The intervals should be narrow enough to resolve the expected uncertainty and provide forecast utility. For instance, if the expected rms prediction error were σ_H , then the interval should be no larger than σ_H .
3. The intervals need to be wide enough to collect at least a few hits from the available data. If an interval contains only a small number of samples, the histogram is not well constrained and will, at worst, return a uniform distribution as a result.

There is some discussion about the required number of hits (among others, [Yates \(1934\)](#)) necessary to well constrain the interval. Another possibility to guarantee the number of hits in an interval is sufficient, is to weight the data. For example, multiply all the data with, say, a factor 5. Now one hit in an interval counts for five and the node is less sensitive to the prior distribution. It is clear that the design of the discretization requires balancing of a number of competing interests. This research adheres to these design guidelines using an adaptive method, described below.

Number of bins

In determining the number of bins it is the aim to minimize the number of bins in order to obtain a statistically robust network that is computationally efficient [Plant and Stockdon \(2012\)](#). Following aforementioned guidelines a start is made with 5 bins per variable. For each variable the *mean*, *standard deviation* and *range* is computed ([Table 4.1](#)). Data used for the discretizing process is the same as the data described in [Chapter 3](#). The probability

¹dichotomous data are data that can be divided in two categories (e.g. dead -alive, failure - non-failure), where each value must belong to one part or the other (jointly exhaustive) and nothing can belong simultaneously to both parts (mutually exclusive)

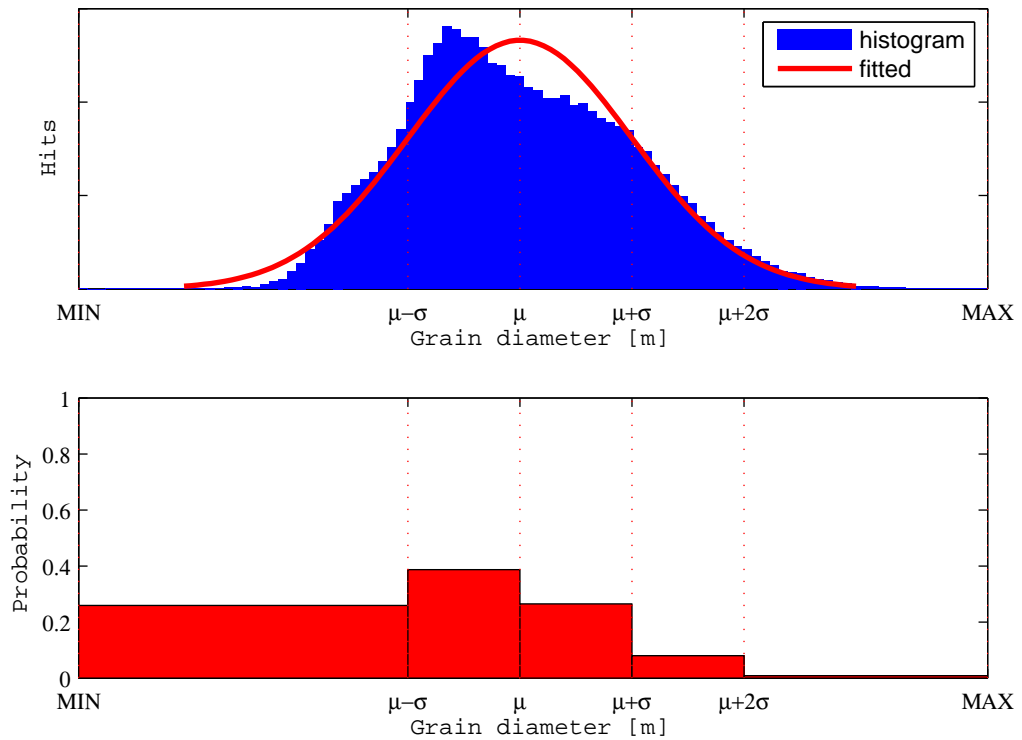


Figure 4.3.: Top figure: Histogram of measured grain sizes along the Dutch coast with a fitted distribution. This distribution serves as starting point to determine the bin ranges. Bottom figure: prior probability distribution for the five bins. The procedure for the other variable is the same.

Variable	Mean (μ)	Std. (σ)	Range	
			min	max
grain diameter [m]	$2.1e^{-4}$	$3.1e^{-5}$	$8.8e^{-5}$	$35e^{-5}$
water level [m]	3.8	1.2	2.0	7.8
wave height [m]	8.0	2.0	2.6	15.2
peak period [s]	14.1	2.9	5.4	25.9
erosion volume above SSL [m^3]	122	183	0	1425
<i>Profile indicators</i>				
volume [m^3]	1410	1044	170	9235
slope [–]	0.0197	0.0077	0.0037	0.0794
dune-crest height [m]	15.5	4.1	8.0	34.0
beach width [m]	218	179	30	1571

Table 4.1.: Mean, standard deviation and range of the distributions of the input variables. For the distributions see [Appendix B](#). The mean value determines the center of bins, the standard deviation determines the minimum range per bin. Together the bins capture all the data for the specific variable (min to max).

distributions for each single Bayesian network input variable is illustrated in [Appendix B](#). The following assumptions and choices are made in order to define resolution of bins per variable:

- Because for some variables (e.g. water level) the area of interest is located in the tail of the probability distribution no data will be excluded, thus all data points will fall in a bin.
- It is assumed distribution's center of gravity lies around the mean value (true for a more-or-less normal distribution). Therefore the bin resolution is high around the mean.
- The minimum bin width equals the standard deviation ([Plant and Holland, 2011b](#)). Therefore the area with the highest density of data (around the mean value) will get the smallest possible bin range.

These decisions result in a discretized variable consisting out of 5 bins [Figure 4.3](#). For each variable two bins are located next to the mean value, $\mu + \sigma$ and $\mu - \sigma$. The third bin with an interval of σ is either located above or below the mean value depending on the area of interest (in case of water level above, in case of dune height below). The remaining two bins reach the minimum or maximum value of the distribution and will have the largest interval for most of the cases. Assuming a normal distribution the number of hits in those areas is relatively low. With a larger interval one guarantees enough hits in those bins. [Table 4.2](#) gives an overview of the interval for each variable.

The only node that will not be discretized is the node containing the areanames (or in Dutch: *Kustvakken*). The entire dataset consist out of seven areas, for each variable a bin is made (see [Figure 4.5](#)). In this way it is possible to study areas separately. A higher probability in for this variable can be explained as more transects in the dataset. For instance, *Kustvak Noord-Holland* consist out of more transects than *Kustvak Texel*.

Variable		bin 1	bin 2	bin 3	bin 4	bin 5
Grain diameter ($m * 10^{-5}$)	min	8.8	18.8	22.1	25.4	28.7
	max	18.8	22.1	25.4	28.7	34.0
Water level (m)	min	2.0	4.05	5.30	6.55	
	max	4.05	5.30	6.55	7.80	
Wave height (m)	min	2.7	6.0	8.0	10.0	12.0
	max	6.0	8.0	10.0	12.0	15.1
Peak period (s)	min	5.5	11	14	17	20
	max	11	14	17	20	25.8
Erosion volume above SSL (m^3)	min	0	200	400	600	800
	max	200	400	600	800	1425
<i>Profile indicators</i>						
Profile volume (m^3)	min	172	360	1410	2460	3510
	max	360	1410	2460	3510	9234
Slope (—)	min	0	0.004	0.012	0.02	0.028
	max	0.004	0.012	0.02	0.028	0.08
Dune-crest height (m)	min	7.0	11.0	15.0	19.0	23.0
	max	11.0	15.0	19.0	23.0	34.0
Beach width (m)	min	30	210	390	570	750
	max	210	390	570	750	1580

Table 4.2.: Overview of the bin range for each variable. To keep numbers orderly sometimes rounding off has applied.

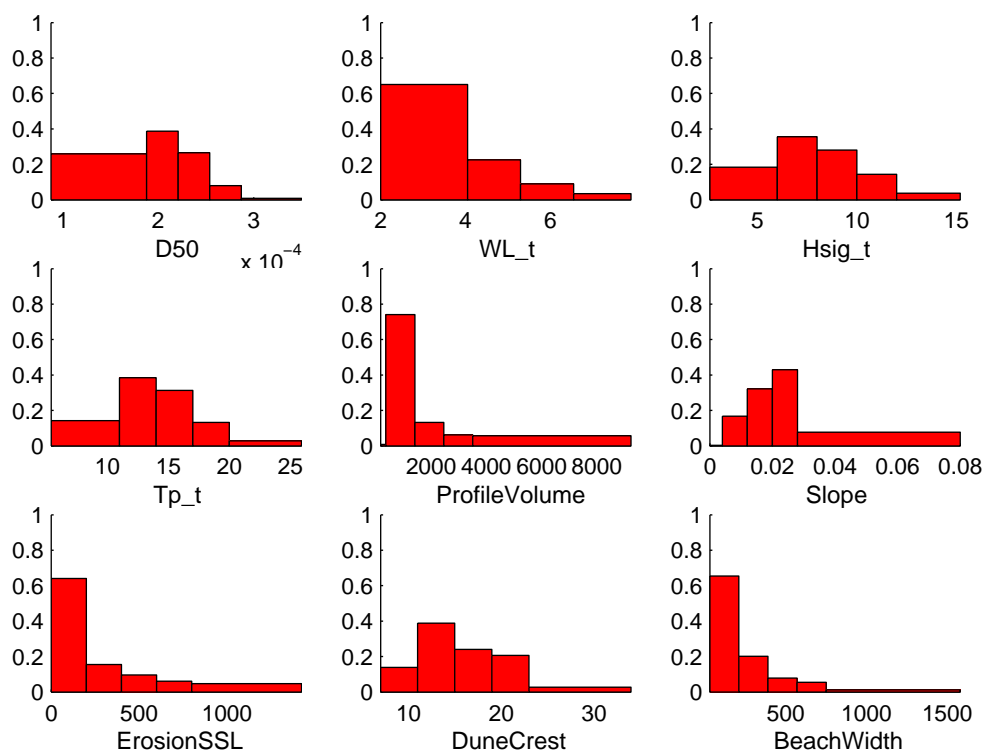


Figure 4.4.: Prior probability distributions of Bayesian network's variables. The height of each bar indicates the probability in each discrete bin and the width of each bar indicates the bin widths.

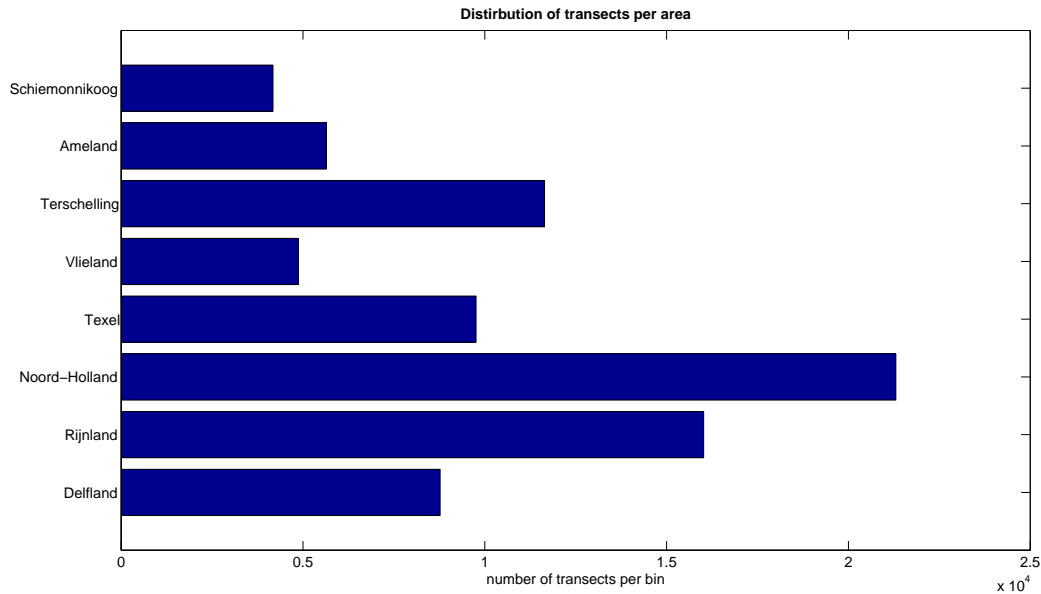


Figure 4.5.: Distribution of areanames. Each bin represents an coastal area ('Kustvak'). The height of each bin indicates the probability that a random selected transects will fall in that bin. A higher probability means that an area (e.g. 'Noord-Holland' consist out of more transects than another one (e.g. Schiermonnikoog)).

Now the Bayesian network is constructed it can be trained with data. *Training* is the process of automatically determining a representative Bayesian network net given data and will be described in next section [Section 4.3](#). Because the construction of the Bayesian network is based on a couple of somewhat *subjective* choices, it depends on the constructor which variables he/she includes and what relations are drawn, a sensitivity analysis will be conducted (see [Section 5.4](#)). In conclusion, [Figure 4.6](#) shows the nodes and correlations of the Bayesian network.

4.3. Parameter learning

In order to fully specify the Bayesian network and thus fully represent the joint probability distribution, it is necessary to specify for each node X the probability distribution for X conditional upon the parents of X . This quantification of the relationships between nodes is done by specifying a conditional probability distribution for each node in a *conditional probability table* (CPT). One of the most popular algorithms for doing parameter estimation is the Expectation-Maximization (EM) algorithm ([Dempster et al., 1977](#)). The EM algorithm is a general algorithm for finding maximum likelihood estimates for a set of parameters Θ when one is faced with an incomplete data set. The algorithm basically alternates between a so-called expectation step and a maximization step. The theory behind it is rather complex, but loosely speaking: in the expectation step the algorithm computes expected values of the unobserved variables conditional on observed data. In the maximization step the computed expected values are used to find the maximum likelihood assuming the calculated expected value are correct.

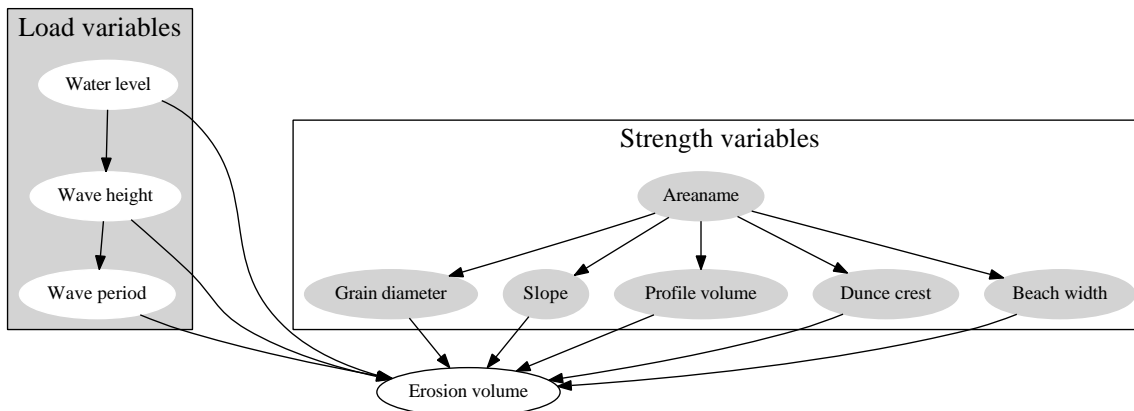


Figure 4.6.: Schematisation of the Bayesian network with all variables and correlations. Each node represents a physical variable and the arrows represent direct influence. This net is constructed in the Bayesian software package Netica.

Before the first iteration of the learning algorithm starts, the conditional probability distributions are uniformly distributed. The Bayesian network has no information yet. With each iteration the conditional probability distributions gets updated and the shape might change. A sufficient number of iterations (cases) need to be conducted to constrain the bins in a good way. Initially the Bayesian network will be trained on the entire dataset 82,196 cases (=iterations) this number seems sufficient ([Plant and Stockdon \(2012\)](#)) however in [Section 5.4](#) a varying number of cases will be used as training and the effects will be analysed.

4.4. Validity

Intermezzo Terminology

There are different types of validity (e.g., content validity, construct validity, empirical validity, predictive validity, factorial validity, see also Cronbach and Meehl (1955)), established by different types of research and requiring different interpretation. In science and statistics, validity has no single agreed definition but generally refers to the extent to which a concept, conclusion or measurement is well-founded and corresponds accurately to the real world. Because this research is active in the fields of numerical models and statistics^a, confusion about the terminology seems obvious. Therefore one needs to be consistent what kind of validity is meant. Oreskes et al. (1994) describes the problem as follows:

Verification and validation of numerical models of natural systems is impossible. This is because natural systems are never closed and because model results are always non-unique. Models can be confirmed by the demonstration of agreement between observation and prediction, but confirmation is inherently partial. ... Models can only be evaluated in relative terms, and their predictive value is always open to question. The primary value of models is heuristic.

So, following Oreskes et al. (1994) using the term validity in this research is not legitimate. Nevertheless, since this study does not focus on a numerical simulation solely and includes statistics as well, the term validity will be used. The precise meaning of validity is described in text below.

^aand involves people with a different background

The term validity, in this case, defines the degree to which the Bayesian network can represent the Dutch coastal indicators. Here the test is designed to measure dune erosion as result of a storm event. When the test appears reasonable on its face to model the real system, then the model has *face validity*. A test can be said to have face validity if it 'looks like' it is going to measure what it is supposed to measure. For instance one would expect (on average) a finer grain size at the *Wadden barrier islands* with respect to the *North-Holland* coast. In addition, one expects a high erosion volume accompanied by a high water level. If the Bayesian network meets this expectations one can say the Bayesian network has face validity. The role played by the correlations between variables in this Bayesian network is illustrated by constraining some of the inputs and observing how the probabilities are updated for the unconstrained variables. Here five interesting cases will be made in order to investigate the model is adequate and has face validity.:

1. Constrain the location to one of the *Wadden barrier islands* (Figure 4.7).
2. Constrain the location to a location with expected differences with respect to the previous one, say *North-Holland* (Figure 4.8).
3. Constrain the water level to a height representing a severe storm (Figure 4.9).
4. Constrain the output of the Bayesian network, erosion volume, to a high volume (Figure 4.10).

5. Constrain the wave height and peak period to an unexpected combination (Figure 4.11).

Several conclusions can be drawn regarding these figures:

1. Looking at Figure 4.7 and Figure 4.8 one can note that:
 - The updated probabilities of the hydraulic boundary conditions (WLt, Hsigt and Tpt) are equal to the prior distributions. This indicates there is no direct correlation between the profile indicators and the hydraulic boundary conditions. This is conform the Bayesian model set-up of Figure 4.6.
 - The grain sizes at *Schiermonnikoog* all fall in the smallest grain size bin.
 - The average grain size at *North-Holland* is larger but also broader distributed with respect to the grain size distribution at *Schiermonnikoog*.
 - There is direct correlation between profile volume and slope. (remember the definition of both indicators Section 4.2.1). A large profile volume corresponds to a mild slope and vice versa.
 - Beach widths are smaller at the *North-Holland* coast then at *Schiermonnikoog*.
 - On the other hand, on average the height of the dune crest is lower at *Schiermonnikoog* than in *North-Holland*.
 - The erosion volume distribution corresponding to the *Schiermonnikoog* constraint is more stretched, slightly increase of the outer bins. Opposite to a narrowing of the erosion volume distribution for *North-Holland*, a slightly decrease of the outer bins.
2. An observed water level of NAP +6m (Figure 4.9) indicates that:
 - A change in water level distribution does not affect the updated distributions of the profile indicators. Indicating water level is not correlated to location. In line with aforementioned notation.
 - A high water level shifts the wave height and peak period distribution to more extreme conditions as well. This indicates a high water level coincides with a higher wave height as well peak period.
 - A high water level reduces the probability of an erosion volume below the 200 m^3 (bin 1) drastically.
3. Given an erosion volume of 1000 m^3 (Figure 4.10) one can see:
 - An update in erosion volume affects all the variables in the net. Indicating a relation between two nodes in a Bayesian network works in two directions, both predictive and diagnostic.
 - A high erosion volume coincides with (relatively): high profile volumes (high availability of sand), smaller grain sizes, higher water levels. Moreover, the probability to observe this erosion volume increases, with respect to the prior distributions, at the *Wadden barrier islands*.
4. The unexpected combination wave height - peak period combination of Figure 4.11 has the following results:

- A uniform distribution of erosion volumes. This indicates this variable is *uniformed*, resulting in a equal probability for every bin (uniform). This means, this combination of wave height and peak period is according to the Bayesian network not likely.
- One is almost certain (99%) of a water level in bin 1. So this variable is *informed*, in contrast to the variable erosion volume. Reason for this is the way the Bayesian network is constructed. Water level is *conditionally independent* given a specific wave height. ($P(Wl|H_s, T_p) = P(Wl|H_s)$)

In summary, one may conclude the Bayesian network is able to capture several expected (strong) correlations. Moreover, this constraints show a high level of face validity. What you expect is what you get. However, it is important to mention that the face validity is a starting point, but should never be assumed to be provably valid for any given purpose. Next chapter will hindcast erosion volumes for a set of testcases in order to quantify the Bayesian prediction skill.

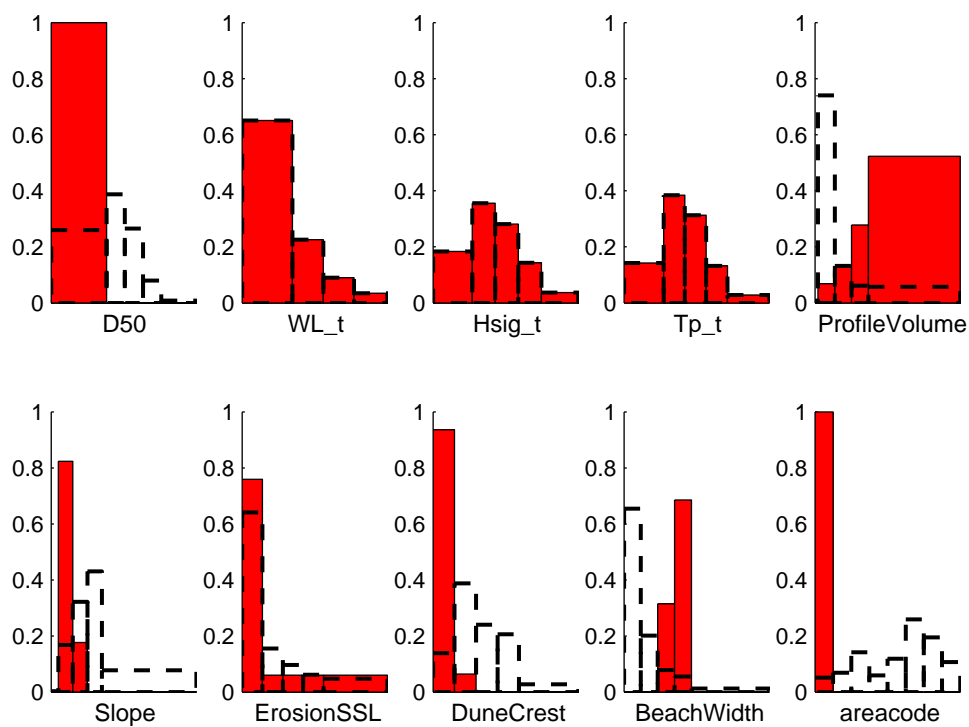


Figure 4.7.: Constrain the location to the Schiermonnikoog and update the unconstrained variables. Updated probabilities in red. Dashed lines depict the prior distributions. Hence, grain size is not constrained!

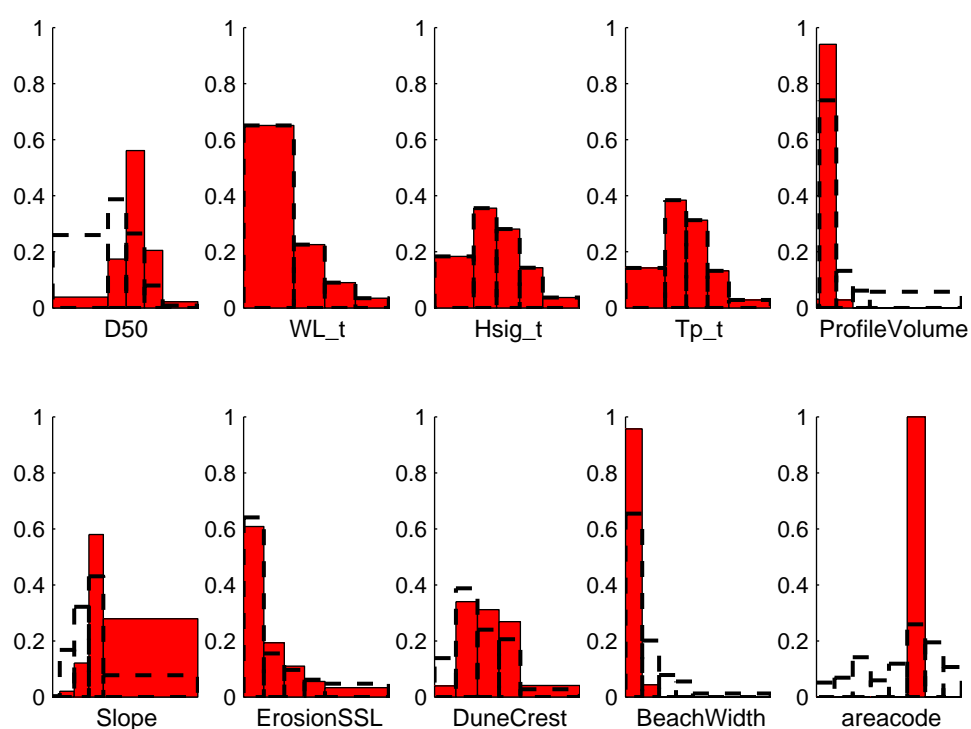


Figure 4.8.: Constrain the location to the North-Holland and update the unconstrained variables. Note the difference with previous figure.

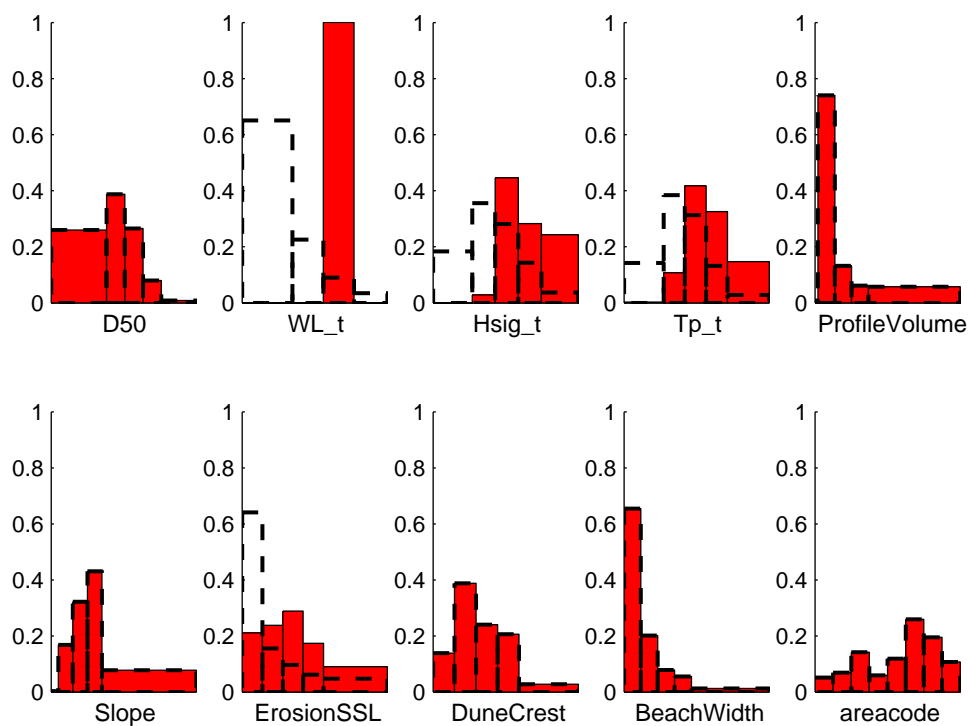


Figure 4.9.: Constrain the water level to a height (N.A.P. +6m) representing a severe storm and update the unconstrained variables.

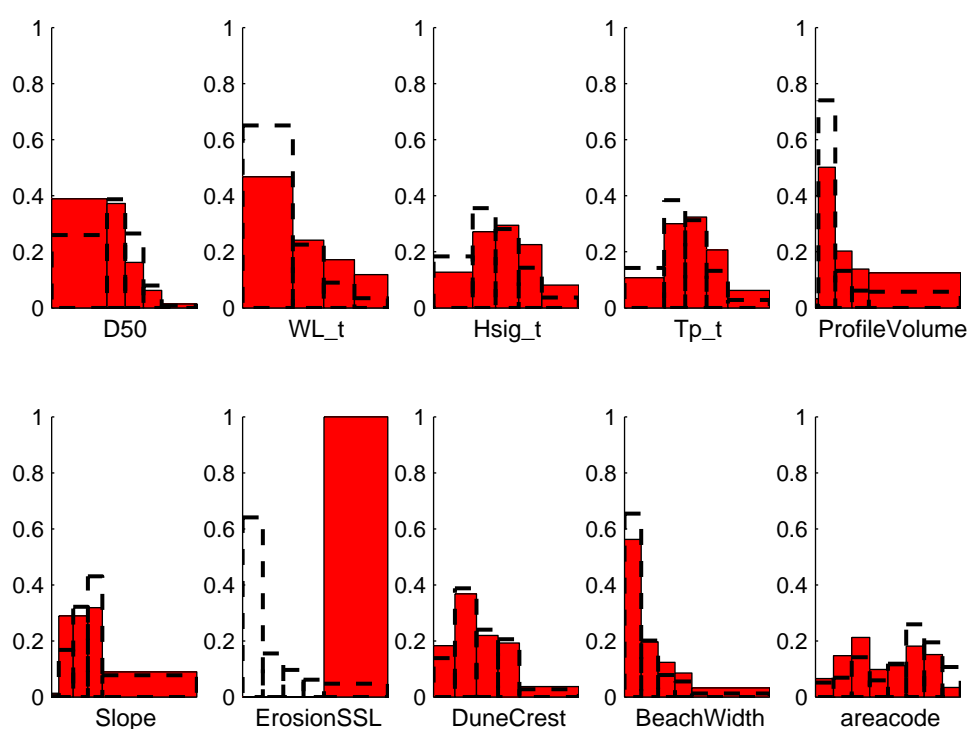


Figure 4.10.: Constrain the erosion volume to 1000 m^3 , falling in the bin with the highest erosion volumes, and update the unconstrained variables.

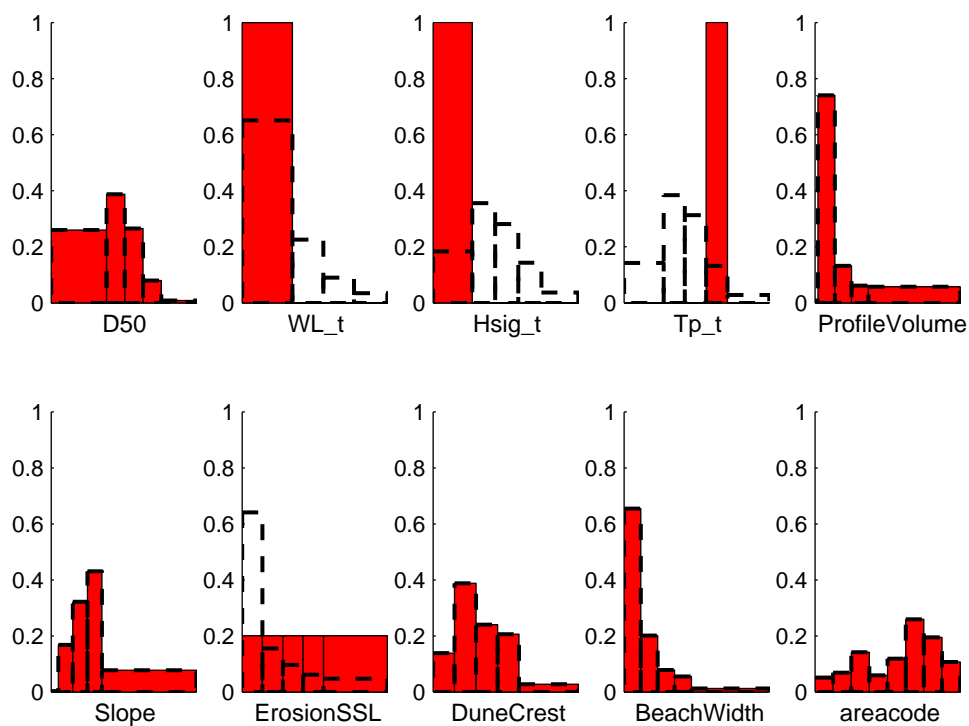


Figure 4.11.: Constrain the wave height (+NAP 3m) and peak period (18s) to a (physically speaking) unexpected combination. Hence, the water level is unconstrained!

5. Results

5.1. Introduction

Once the Bayesian network was constructed and trained, it can be used for prediction by updating the prior probabilities of the input variables with data. Erosion volumes at several locations along the Dutch coast were hindcasted with testcases obtained out of the dataset described in [Chapter 3](#). A *testcase* is a set of observations representing a scenario and serves as input for the Bayesian network in order to make a prediction for the erosion volume. In each case of this dataset an observed erosion volume is available, so a comparison between an *observed* and *predicted* erosion volume is possible. For this comparison two methods are defined, *skill* and *log-likelihood ratio*, and are described in [Section 5.2](#). In this chapter the lay-out of the Bayesian network (variables and relations) is fixed and therefore also the conditional probability tables, however data used to train and/or test the model will vary. In [Section 5.3.1](#) the Bayesian network is trained on the entire dataset (i.e., all data present in the dataset [Figure 5.1](#)). So, cases used for training will be used for testing as well. Reason for validating the model with input data is to assess the capability of the Bayesian network to reproduce the input data. The next step is to use half the dataset for training while the other half is used for predictions. In [Section 5.3.2](#) three different half data samples will be used for training and testing. Here, the set of testcases is not the same as training cases. The testcases can be seen as ‘new’ information to the Bayesian network. However, cases for training and testing are obtained out of the same *population* (so they are not independent). For instance, the dataset consists out of just 960 cross-shore bottom profiles and there are over 80,000 cases, so many cases have the same bottom profile.

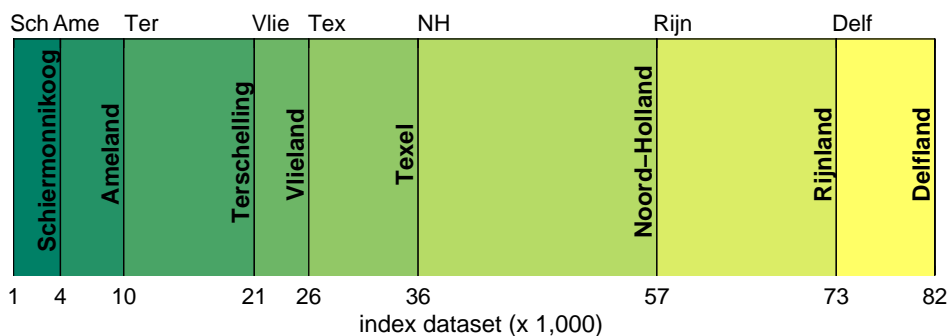


Figure 5.1.: Visualization of the dataset consisting out of more than 82,000 unique cases. As one can see the dataset is ordered per location, starting with case 1 at Schiermonnikoog and ending with case 82,196 at Delfland. The abbreviation of coastal areas is similar to the evaluation plots below ([Figure 5.4](#) and further)

In [Section 5.4](#) a sensitivity analysis of the subjective choices (nodes, relations, discretization thresholds) made in the construction of the Bayesian network, will be executed. Reference case for these analyses is the check case. The *check case* is the best possible prediction the Bayesian network can make. Here, observed erosion volumes are input to the Bayesian network as well.

The hindcast predictions, presented as confidence regions, are compared to the observations at the specific location ([Figure 5.4](#)). Before a start can be made with the evaluation of the Bayesian network first two measures to quantify the Bayesian prediction will be introduced.

5.2. Quantify prediction evaluation

To determine the degree to which the Bayesian network can recover the simulated erosion volume (morphological change) and whether its uncertainty estimates are consistent with the actual prediction errors two evaluation methods will be used. First, the prediction skill will be quantified using a simple linear regression model relating predictions to the actual observations¹. Second, the prediction skill is quantified by evaluating the likelihood of each observation, as predicted, and comparing this likelihood to the prior likelihood. Both methods will be explained in detail below. The methods will be applied to hindcast predictions using the training dataset to update some of the variables (e.g. water levels, wave height, profile volume and dune crest).

5.2.1. Simple regression model

Goal of this regression model is to quantify the prediction skill relating predictions to the actual observations. Therefore we wish to find the equation of the straight line

$$\hat{y} = \alpha + \beta \bar{x} \quad (5.1)$$

Here, \hat{y} is the regression estimate based on \bar{x} , the Bayesian-mean predicted value, and includes corrections for bias α and β . The Bayesian-mean value is

$$\bar{x}_i = \sum_{j=1}^J p[\Delta D_j | inputs_i] \Delta D_j, \quad (5.2)$$

where the summation is over the $j = 1, 2, \dots, J$ discrete bins from the prediction obtained for each case i . Here $inputs_i$ represents a ‘case’ (a particular combination of the possible input variables). The regression skill for this prediction is

$$s = 1 - \frac{\sum_{i=1}^I \sigma_{x,i}^{-2} [y_i - \hat{y}_i]^2}{\sum_{i=1}^I \sigma_{x,i}^{-2} [y_i]^2}, \quad (5.3)$$

where the summation is over all considered cases, $i = 1, 2, \dots, I$. The measure skill describes a weighted percentage of observed variance that is explained by the Bayesian-mean prediction.

¹Here, observation is not meant literally (there are no real-world observations of erosion volumes in the dataset) but a observation is a simulated erosion volume out of DUROS+ that will be ‘observed’ by the Bayesian network

The weighting factors are the prediction variance around the Bayesian-mean value and do not depend on the observations. The variances are computed as

$$\sigma_{x,i}^2 = \sum_{j=1}^J p[\Delta D_j | inputs_i] [\Delta D_j - \bar{x}_i]^2. \quad (5.4)$$

The Bayesian-mean value represents a robust predicted value (Plant and Stockdon, 2012) and the variance of the prediction provides a measure of prediction uncertainty used as weighting term in the regression and skill estimates. However, the Bayesian mean value is strictly appropriate if the result is more-or-less normally distributed. When a result is bi-modal for instance (see Figure 5.2) the Bayesian-mean value will fall between the two most likely outcomes and is a horrible prediction. Therefore, another method to quantify the prediction skill will be used as well.

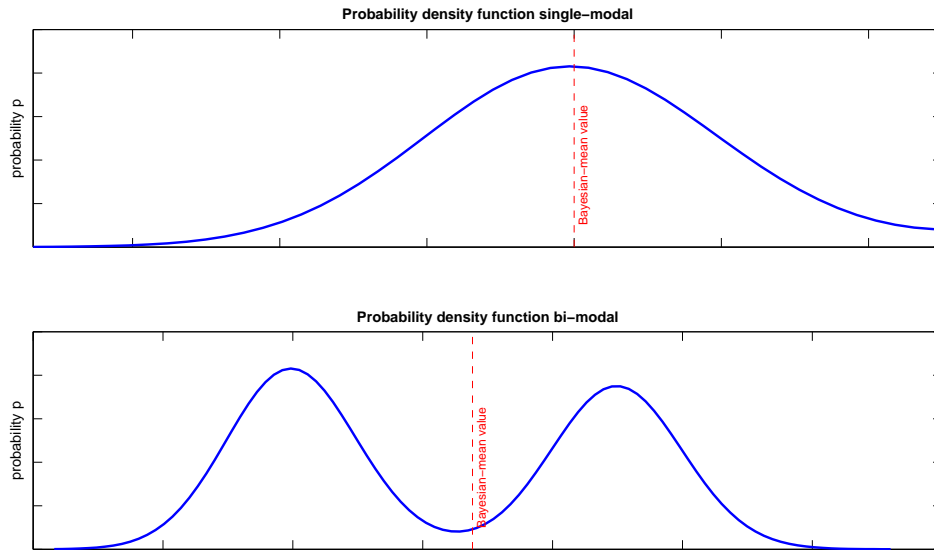


Figure 5.2.: Example of two possible probability density distributions describing the result. In case of a modal distribution Bayesian-mean value is a good method to quantify the prediction skill but it gets bad when the distribution is more-or-less bi-modal.

5.2.2. Log-likelihood ratio

It is possible to describe the skill of the updated probability distributions by evaluating the likelihood of each observation, and comparing this likelihood to the prior likelihood. Likelihood is a measure of how likely an event is. For instance for a forecast F with given observables O , $p(F|O)$, the likelihood function is the probability of the observations if the forecast is known: $p(O|F)$. The likelihood term can include both model and observation errors. That is, if the model and measurements were error free, than an observation would be likely only if it equalled the forecast value. In reality, there are numerous errors causing spread in the likelihood function.

Here, with likelihood the maximum likelihood function is meant. This function describes the likelihood of a set of parameter values given some observed outcomes is equal to the probability of those observed outcomes given those parameter values. Since, one is interested in the *most* likely parameters the likelihood is labelled as *maximum* likelihood. In this case the log-likelihood ratio will be used (Equation 5.5) to describe the maximum likelihood. A logarithmic function will be used to make the process of differentiating (to find the *maximum* likelihood) easier. Moreover, because the logarithm is an increasing function, the likelihood function and the log-likelihood function attain their extreme values for the same values.

$$L = \log \left\{ p[\Delta D_j | inputs_i]_{\Delta D_j = \Delta D_i} \right\} - \log \left\{ p[\Delta D_j]_{\Delta D_j = \Delta D_i} \right\} \quad (5.5)$$

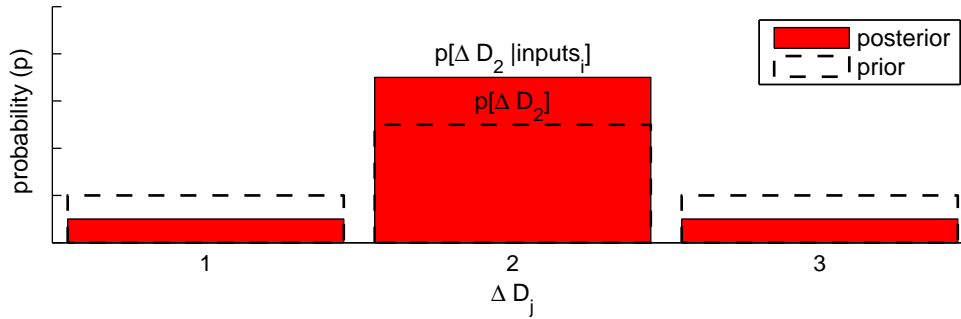


Figure 5.3.: Given $\Delta D_2 = \Delta D_i$ this figure shows the prior probability of a value in bin ΔD_2 together with the posterior probability (after update with evidence $inputs_i$) of the same bin. The posterior probability is higher than the prior probability, the log-likelihood formulated in Equation 5.5 is positive, so one is more certain about the outcome.

The first term on the right is the updated probability evaluated at the discrete bin that matches the observed erosion volume for the i^{th} case. If the updated probability is higher than the prior probability for the observed value, then the prediction is improved compared to the prior and the log-likelihood ratio is greater than zero. This indicates that the updated probability distribution is both different compared to the prior distribution and more accurate. On the other hand, if the updated probability of the observed value is lower than the prior probability, it indicates that the updated probability is either more uncertain than the prior or it is more confident but actually wrong.

Thus, the likelihood ratio scores the ability of the Bayesian network to make skilful estimates of both mean value and uncertainty. A summation of the log-likelihood ratio over all observed cases provides a measure of how much better (or worse) the Bayesian network prediction performed over the entire data set.

5.3. Testing

This section describes the hindcast of erosion volumes for a set of testcases. Testcases are sampled out of the 82,196 cases of the dataset described in Chapter 3. This dataset is chronologically ordered with respect to alongshore coastal location, Figure 5.1 gives an overview of

Number of testcases	Skill	LR	time (order of magnitude)
9	0.71	2	few seconds
83	0.88	19	30 seconds
822	0.93	218	3 minutes
8220	0.91	2102	15 minutes

Table 5.1.: Prediction skill and LR as function of the number of testcases. For all the runs a number of 83 testcases is held.

the structure of this dataset. It is clear to see coastal areas with a longer coastline consist out of more cases.

For every single prediction in the Bayesian network all the input variables² are updated with observations corresponding to the case. For every prediction the observed erosion volume is known and can be used for a comparison with the predicted erosion volume. The hindcast predictions, presented as confidence regions (5.4a), are compared to the observations for each case (5.4b).

One single prediction is not representative for evaluation of a Bayesian network, therefore a set of testcases will be used to evaluate the Bayesian network. In next sections different sets of testcases will be used, but the number of testcases in a set is constant. This is because the log-likelihood ratio (hereafter referred to as LR) is a summation over the number of testcases (see also Equation 5.5). So, the number of testcases determines the LR value.

In Appendix D different numbers of testcases were used in order to predict the erosion volumes. The Bayesian network was trained on the entire dataset and testcases were sampled with constant interval (e.g. case number:1;1001;2001 etc.) out of the same dataset. Results are presented in Table 5.1. It is clear to see the LR increases more or less linear with respect to the amount of testcases. This indicates the LR for a single prediction is more or less the same. Considering the skill one does not see this linear relation. In Figure D.1 it is clear to see 9 testcases are insufficient, there is just one observation outside the first bin. The case with 822 Figure D.3 shows the best skill, however more testcases Figure D.4 will not increase the skill, indicating the maximum skill situated around the 0.9. Moreover, more testcases make it hard to distinguish individual observations. In addition, more testcases are more time consuming as well. Therefore, the number of testcases to test the Bayesian network is fixed to 83 testcases for all evaluations.

5.3.1. Bayesian network trained on entire dataset

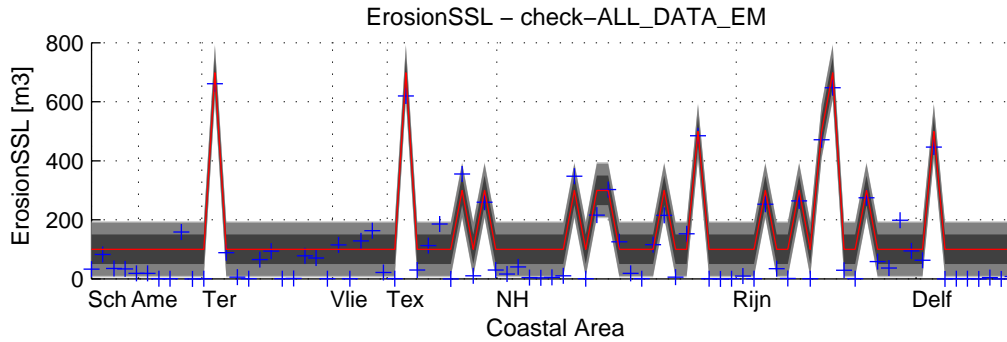
Check case

To investigate the prediction scores for different sets of testcases, first a reference case representing the best possible prediction was obtained. The highest possible prediction skill and likelihood ratio is determined by performing a prediction where the observed erosion volume (the quantity we are predicting) was supplied into the Bayesian network. This hindcast test called the ‘check’ (Figure 5.4).

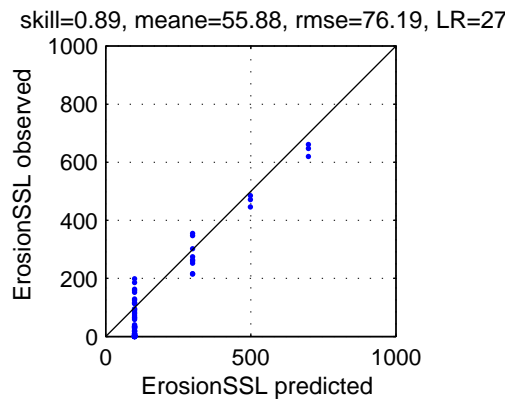
The number of cases in the dataset presented in Figure 5.1 is 82,196. Starting at case one, and using a stepsize of 1000 cases, one get 83 testcases equally sampled over the dataset length. Thus, every coastal area is represented in the evaluation of the Bayesian network (see

²water level, wave height, peak period, areaname, dune height, beach width, profile volume and slope

horizontal axis 5.4a).



(a) Predicted erosion volumes. The red line indicates Bayesian-mean predicted value and the blue dots the observations. The shading correspond to 50% (darkest), 90%, and 95% (lightest) confidence regions.



(b) Regression of Bayesian-mean predicted value against the observed erosion volumes

Figure 5.4.: Predictions of erosion volumes (a) and correlations to observations (b) obtained with the Bayesian network trained on the entire dataset. This figure represents the check case, erosion volume observations are supplied to the Bayesian network as well

5.4a indicates each of the 83 predictions fall in the bin corresponding to the observed erosion volume. For example, the prediction of the small erosion volumes observed at *Schiermonnikoog* and *Ameland* fall for 95% certain in bin one [0-200 m^3]. However, the probability density distribution inside a bin is not known, therefore given an observation the Bayesian-mean predicted value is always equal to the average of a bin. This can also be seen in 5.4b, here the predicted erosion volumes (horizontal axis) are divided in four bins with a predicted erosion volume equal to the middle of that particular bin. So, each observed erosion volume in the range 0-200 m^3 is predicted to have a value of 100 m^3 (middle of bin one).

Since the observed erosion volumes are input to the Bayesian network, introduced uncertainty is only due to the discretization of the Bayesian network. The choice for these bin ranges results in a prediction skill of 0.89. In other words, with the use of this Bayesian network 11% of information gets lost.

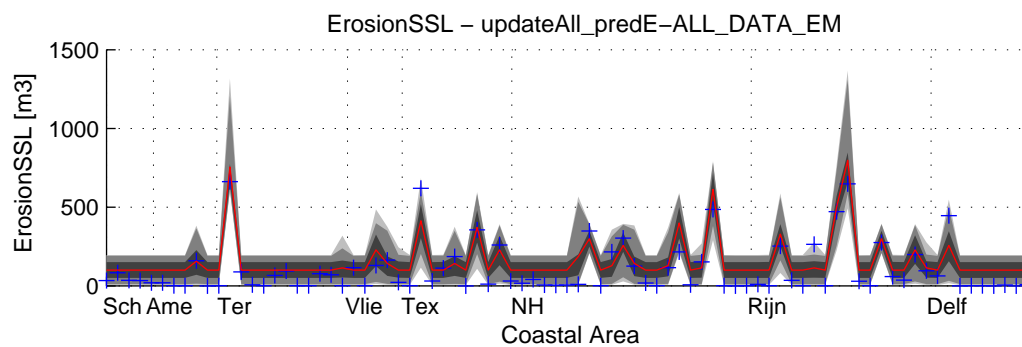
Prediction case

Now the scores of a check case are available, a prediction of erosion volumes for the same Bayesian network can be made (Figure 5.5). In fact, the only difference between this test and the check case is observed erosion volumes are no input to the Bayesian network. The set of testcases is the same.

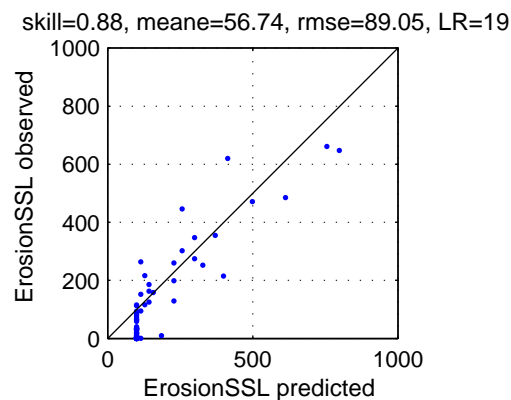
A comparison between Figure 5.4 and Figure 5.5 indicates:

- The rather broad probability distributions of 5.5a, indicated by higher confidence peaks, results in a decrease of the LR (from 27 to 19).
- Bayesian-mean predicted values are no longer fixed to the average of a bin, no vertically lined group of observations in 5.5b.
- Both prediction skills are more-or-less the same (0.88 vs. 0.87), indicating the Bayesian network is capable in predicting the erosion volumes belonging to these testcases³.

³Important to note: here the testcases were used for training as well!



(a) Predicted erosion volumes. The red line indicates Bayesian-mean predicted value and the blue dots the observations. The shading correspond to 50% (darkest), 90%, and 95% (lightest) confidence regions.



(b) Regression of Bayesian-mean predicted value against the observed erosion volumes

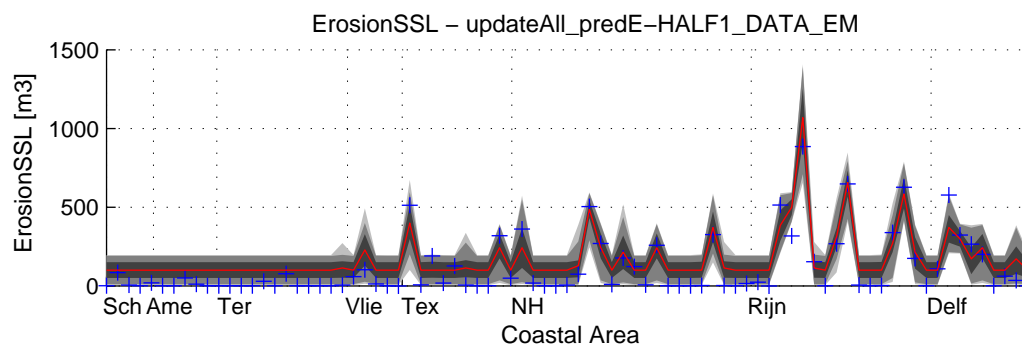
Figure 5.5.: Predictions of erosion volumes (a) and correlations to observations (b) obtained with the Bayesian network trained on the entire dataset. Here all the input variables are updated with the conditions corresponding to the 83 observations in order to predict the erosion volumes.

5.3.2. Bayesian network trained on half dataset

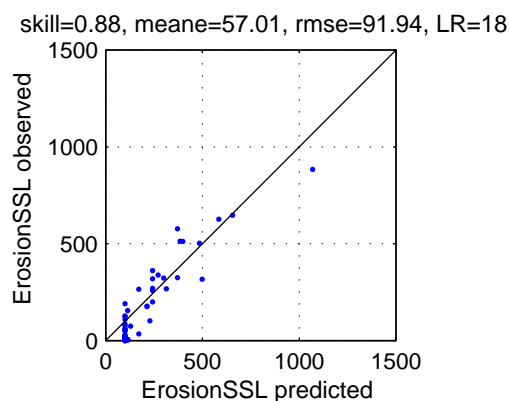
In order to prevent training and testing on the same cases, the dataset of 82,196 cases is divided into two sub-sets. One sub-set will be used for training of the Bayesian network, while the other will be used for testing of the Bayesian network. Here, the sampling of the entire dataset into two equally sized sub-sets will be done in three ways (see [Figure 5.1](#) for visualization of the dataset):

1. *Odd and evens* ([Figure 5.6](#))
Use the odd numbered cases (1,3,...,82195) for training and the even numbered cases (2,4,...,82196) for testing. Thus, both samples cover the complete coast we are studying.
2. *Split halfway* ([Figure 5.7](#))
Cut the dataset in two parts, use case 1-41,098 for training and case 41,099-82,196 for testing. Thus, the Bayesian network gets trained on the *Wadden barrier islands* plus a part of the *North-Holland* coast and tested on the remaining *Holland* coast.
3. *Randomize and split halfway* ([Figure 5.8](#))
First shuffle all the cases so they are not ordered per coastal area and then cut the dataset in two. Such, one does not know what the location of both the training and testcases is. This method will be used in [Section 5.3.3](#) to sample sets of training cases of different size.

For all three Bayesian networks 83 testcases will be used to update the probabilities. Since the length of the sub-set (41,099 cases), from which the testcases are sampled, is half the length of the entire dataset (82,196 cases), the stepsize is also halved (from 1,000 to 500). So, in case of odd and evens, testcases out of each coastal area will be tested. Meanwhile, the location of testcases used in [Figure 5.7](#) are sampled out of *Holland coast* cases only. Finally, for the shuffled half dataset it is not possible to locate a testcase (the dataset is unordered). Therefore coastal areas will be not provided in [Figure 5.8](#).

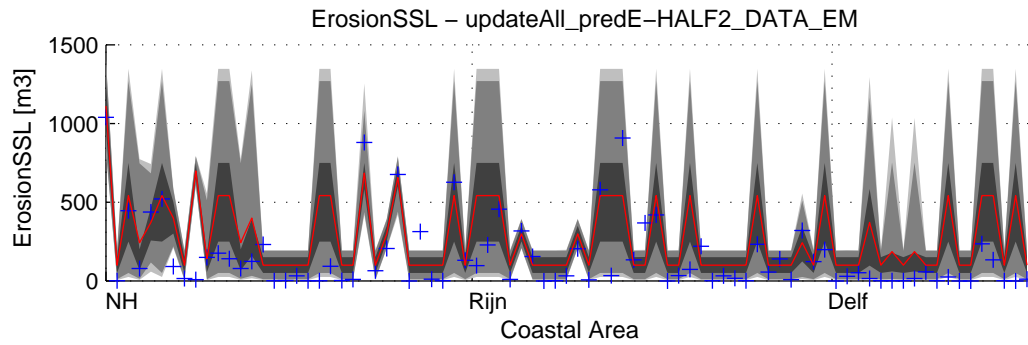


(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.

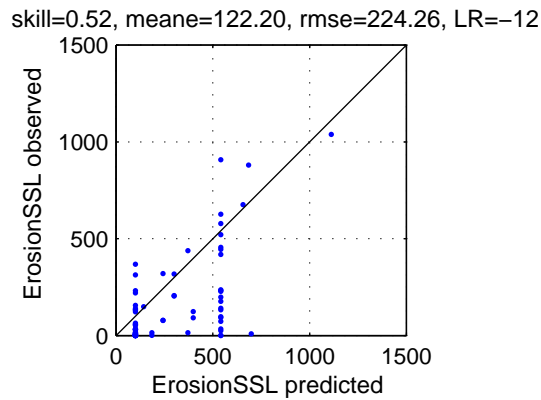


(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure 5.6.: This prediction evaluation represents the Bayesian network trained on the half dataset and tested on 83 observations obtained out of the other half of the non-training data set. Here all the input variables are updated with the conditions corresponding to the 83 observations in order to predict the erosion volumes.

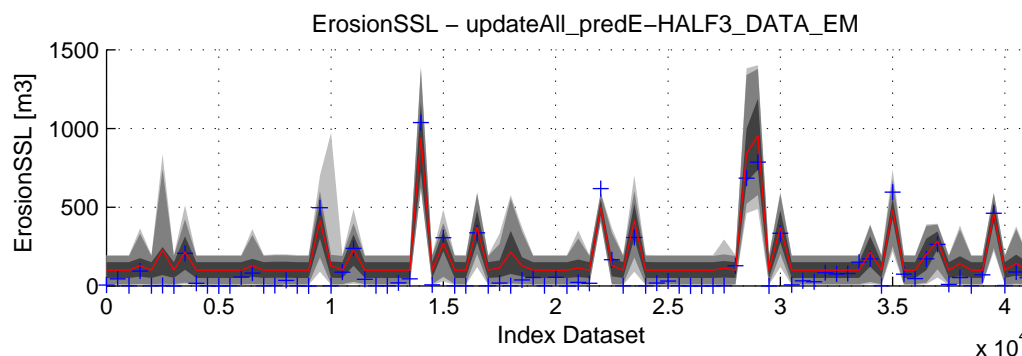


(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.

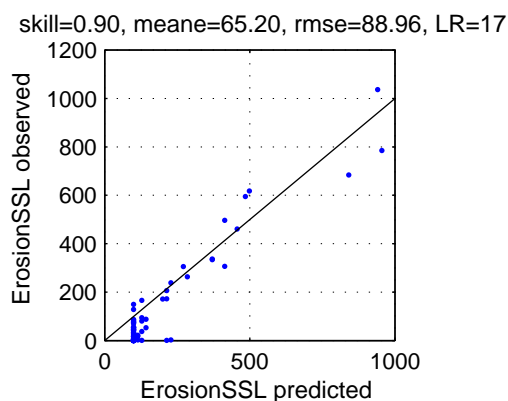


(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure 5.7.: This prediction evaluation represents the Bayesian network trained on the half dataset and tested on 83 observations obtained out of the other half of the non-training data set. Here all the input variables are updated with the conditions corresponding to the 83 observations in order to predict the erosion volumes.



(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.



(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure 5.8.: This prediction evaluation represents the Bayesian network trained on the half dataset and tested on 83 observations obtained out of the other half of the non-training data set. Here all the input variables are updated with the conditions corresponding to the 83 observations in order to predict the erosion volumes.

Trained on:	Tested on:		Skill	LR
All data	All data	check	0.89	27
		pred	0.88	19
Half1a	Half1b	check	0.89	29
		pred	0.88	18
Half2a	Half2b	check	0.91	33
		pred	0.52	-12
Half3a	Half3b	check	0.93	27
		pred	0.90	17

Table 5.2.: Prediction scores for different sampled training and testcases. Because the Bayesian network differs per set of testcases a ‘check’ case per Bayesian network is provided as well.

Prediction scores corresponding to the various sets of testcases are presented in [Table 5.2](#). When one looks at [Figure 5.6](#) - [Figure 5.8](#) the following observations can be mentioned:

- It is clear to see that a Bayesian network trained on *Wadden barrier islands* cases and tested on *Holland* cases, result in bad prediction compared to the other half datasets. Specifically, the low LR score is a result of the predictions having rather broad probability distributions (visualized with the high gray peaks in the confidence plot of [Figure 5.7](#)). This indicates the Bayesian network is not capable of predicting erosion volumes at the *Holland* coast, with *Wadden barrier islands* information only.
- Prediction scores for the other two Bayesian networkstrained on 41,099 cases are more-or-less equal (see also [Table 5.2](#)). This indicates the

5.3.3. Bayesian network trained on small datasets

This section investigates the correlation between the amount of data (i.e., number of cases) used for training and the prediction scores. Prediction evaluation with the entire and half dataset were conducted already. Before the dataset was divided in smaller parts, first the entire dataset was randomized comparable to half dataset 3. After this the dataset is not spatially ordered any more. Now the randomized dataset ([Figure 5.1](#)) is divided by 4 (20,549 cases), 8 (10,275 cases), 16 (5138 cases) and 32 (2567 cases). These (small) datasets will be used for training and the remaining cases will be used for testing. Thus, it is not possible to train and test on the same case. Results visualized in the well-known plots are presented in [Appendix E](#), prediction scores are presented in [Table 5.3](#). Please remember that all Bayesian networksare tested on 83 testcases obtained out of remaining cases not used for training.

When one looks at [Table 5.3](#) one can see the prediction skill is more-or-less equal to the prediction skill of the corresponding ‘check’ case. Only the training dataset with a length of 1/32 of the original dataset length shows a reduction in skill of 15%. The low log-likelihood ratio is a result of predictions having a rather broad probability distribution that are not strongly different from the prior distributions. The negative LR of the last set of predictions indicates the updated probabilities are worse (i.e., $LR_i < 0$) than using prior probabilities.

Trained on:	Number of training cases:	Tested on:		Skill	LR
All data	82,196	All data	check pred	0.89 0.88	27 19
Half3a	41,098	Half3b	check pred	0.93 0.90	27 17
1/4	20,549	3/4	check pred	0.91 0.93	27 20
1/8	10,275	7/8	check pred	0.91 0.85	31 11
1/16	5,137	15/16	check pred	0.89 0.89	31 10
1/32	2,569	31/32	check pred	0.91 0.76	38 -11

Table 5.3.: Prediction scores for different sets of testcases. For every set of testcases 83 predictions are made. Because the Bayesian network differs per set of testcases a ‘check’ case per Bayesian network is provided as well.

5.4. Sensitivity Analysis

The Bayesian network design relied on several somewhat subjective choices that may affect network performance. The choices include (1) selecting variables to include in the network, (2) deciding which of the possible correlations between the variables should be resolved, and (3) choosing the bin resolution for each variable. In this section a sensitivity analysis for the first two of these choices will be addressed.

5.4.1. Variable selection

In order to identify which variables (or combination of variables) contributed to improved skill, a series of additional hindcast prediction test and evaluations were performed in which different combinations of input variables were provided or withheld.

Single input variable

First, the role played by each input variable in making skilful predictions the Bayesian network will be tested on every variable individually. For this testing the Bayesian network trained on the entire dataset, described in [Section 5.3.1](#), was used. Again the erosion volume will be predicted, but this time with only evidence from one variable. For instance, given a value for the water level, predict the most probable erosion volume given that water level. Not only single nodes are used for prediction, also two combinations of nodes are evaluated. The three hydraulic boundary conditions will be tested as group, as well as the six profile indicators. Those two groups of nodes can be viewed as real-life available scenarios, for instance one knows the storm characteristics or the location along the coast. In [Appendix F](#) plots presenting updated probabilities after testing on one variable only are given. A summary of prediction scores for every evaluation is given in [Table 5.3](#).

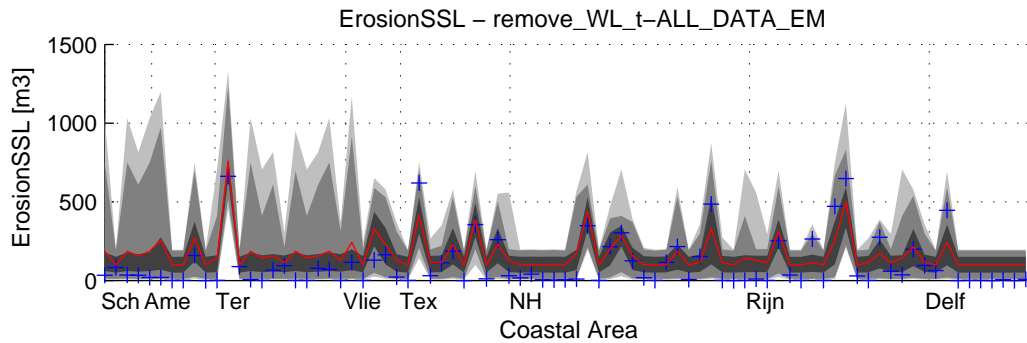


Figure 5.9.: Predictions of erosion volumes without water level information.

All input variables minus one

Next step is to remove just one variable out of the set of input variables. For instance, predict erosion volumes by updating all input variables except for water level. Results in the common plots are presented in [Appendix G](#). Here an interesting case (remove the water level) will be highlighted ([Figure 5.9](#)). These results show a couple of features of the prediction that are valuable. First, it is obvious to see predictions made at the *Wadden barrier islands* show a large amount of uncertainty (i.e., rather broad updated probability distributions). Second, despite of the fact the predictions are more uncertain (lower LR) the predictions are more accurate (higher skill).

This last result (i.e., water level is not important) illustrates an important but purely statistical conclusion, as it is quite likely that water level is important to driving dune erosion processes. However, for this analysis, based on typical Dutch storm characteristics, water level was strongly correlated with wave height and peak period, such that water level is redundant and largely unnecessary if wave height and peak period are available, and vice versa. A potential advantage of this redundancy is that if there are errors in either estimates of water level, wave height, peak period (this is always true), then the variables complement each other and the impact of these errors can be reduced through the inherent data assimilation capability of the Bayesian network approach ([Plant and Holland, 2011a,b](#)).

Variable	Only		Without	
	Skill	LR	Skill	LR
Check	0.89	27	0.89	27
Water level	0.38	5	0.93	15
Wave height	0.33	4	0.88	18
Peak period	0.22	3	0.93	18
Areaname	0.01	1	0.88	19
Dune crest height	0.01	0	0.90	17
Beach width	0.00	1	0.85	18
Profile volume	0.00	1	0.90	13
Slope	0.00	1	0.93	14
Grain size	0.01	0	0.92	17
<i>HBC</i>	0.41	6		
<i>PI's</i>	0.09	5		

Table 5.4.: Prediction scores in order to examine the relative importance of every variable included in the Bayesian network

6. Conclusions and Recommendations

6.1. Introduction

This study investigated the application of the Bayesian approach for the dune safety assessment against extreme storm events. With the dune erosion model DUROS+ a dataset was generated that serves as input (training) and as reference (testing) to the Bayesian network. Aim of the Bayesian network was to predict dune erosion volumes above storm surge level given a set of observations.

Predicted dune-erosion volumes (output Bayesian network) were compared to observed dune-erosion volumes (output DUROS+). The quantification of this comparison was done by two prediction scores, regression skill score and the log-likelihood ratio.

First the observed erosion volumes are input to the Bayesian network, in order to illustrate the loss of information with the use of a Bayesian network solely. After that several analysis were made to show the capabilities of the Bayesian network in order to predict dune erosion volumes.

During this research one single dataset was used in order to train and test the Bayesian network. The cases in this dataset are unique but not independent. For every case (i.e. simulation) data was sampled or selected out of the same sources (HR2006, and JARKUS (2010)) and the same dune-erosion model (DUROS+) was used to compute dune erosion volume. Therefore, this dataset can be viewed as one population. With the interpretation of the conclusions as described below, this should be taken into account.

6.2. Conclusions

The research presented in this thesis improved our understanding of the Bayesian network approach. Since the Bayesian approach is relatively new in the field of coastal engineering the whole process will be evaluated. The sub-questions (described in [Section 1.3.2](#)) will be discussed below.

SQ1 Is it possible to appropriately reproduce the current safety assessment for dunes in a Bayesian network?

The Bayesian network is capable of reproducing dune erosion volumes which are generated by the dune erosion model DUROS+. This is illustrated by the validity check in [Section 4.4](#) and by the ‘check’ case described in [Section 5.3.1](#).

In the validity check no predictions were conducted, only the effects of constraining some variables were studied. For example, the constraint of the water level to a range of 4-6 m illustrates a shift of the probability distributions to higher values for: wave height, peak period and erosion volume respectively. With the use of the four profile indicators several expected features can be illustrated, smaller grains and lower dunes at the Wadden islands with respect to the the Holland coast for instance. This illustrates the Bayesian network is capable of adequately representing expected correlations

regarding the Dutch coast.

Secondly, for the reproduction in the ‘check’ case also values of the variable of interest (erosion volume) were input to the Bayesian network, such the effect of the *application* of the Bayesian network can be illustrated. The loss in prediction skill (regression skill less than one) is the result of the discretization of the variables in the Bayesian network. The Bayesian network cannot distinguish values in one particular bin. When the range of a bin is $[200 - 300] \text{ m}^3$ for instance, the Bayesian network cannot distinguish an erosion volume of 201 m^3 and 299 m^3 , they both fall in the same bin. Now, when the erosion volume serves as input, the Bayesian network knows the bin for sure. By definition the predicted erosion volume is the mean value of this bin. Assume the observed erosion volume is 201 m^3 , the Bayesian network is certain that the predicted value is situated in the bin ranging from $[200-300] \text{ m}^3$, and therefore the predicted erosion volume is 250 m^3 . Compare the observed (201 m^3) and predicted erosion volume (250 m^3), this is the reason for loss in the prediction skill. The reduction in prediction skill of 11% is solely due to the thresholds used to discretize the variables. This, again, indicates the importance of the discretization of the input variables of the Bayesian network.

SQ2 What is an appropriate way to characterize the dune erosion process in a Bayesian network?

The choice to select and connect nodes corresponding to our physical knowledge of the dune erosion process worked out quite well. The updated probabilities in [Section 4.4](#) indicates the Bayesian network meets our expectations. For instance, a high water level results in a skewed probability distribution to higher erosion volumes, and vice versa.

- *Which variables play an important role in the dune erosion process and how are they correlated?*

In [Section 5.4.1](#) the effect of adding and removing a variable has been tested. Due to the number of correlations between the nodes in the Bayesian network (no node is singly connected), it is hard to point out what variable is decisive in the prediction of erosion volumes. For instance, due to the direct correlation between water level and wave height, water level information is also stored in the wave height conditional probability table. So, even if water level evidence is not available to update the probabilities it indirectly still affects the prediction.

- *What is the best way, to end up with a robust and fast Bayesian network, regarding the discretization of the variables available in the Bayesian network?*

Several times the comment was made the discretization of variables included in the Bayesian network is an important step in the model set-up. The discretization itself is responsible for a reduction in the prediction skill of 11%. Moreover, the discretization plays an important role in the dimensionality of the Bayesian network (size of conditional probability table grows exponentially with an increase of number of bins) respectively, and in the robustness of the Bayesian network (too many bins may result in an insufficient number of hits in a particular bin). However, no analyses have been made with a different bin resolution. This could be an interesting topic for future work possibly (see also [Section 6.3](#)).

SQ3 What is the relation between the amount of data used for training of the Bayesian network and the prediction skill?

A Bayesian network trained on 2,567 cases and tested on 83 observations results in a negative log-likelihood ratio (LR). This indicates that using the prior distributions would result in a better prediction score ($LR = 0$) with respect to updating the input variables. Therefore one can draw the conclusion that the input of 2,567 (1/32 of database size) is not sufficient in order to adequately predict erosion volumes for new cases.

SQ4 Is it possible to forecast the dune erosion process outside the spatial domain described by the Bayesian network?

(Predictions were made for dune erosion volumes for locations at the *Holland* coast using a Bayesian network trained on data belonging to the *Wadden barrier islands*.) Yes, it is possible but it results in bad prediction scores. It is possible because originally the variables in the Bayesian network were discretized on data of both, the *Holland* and *Wadden* coast, therefore no cases fall outside the range of a particular variable. The low prediction scores indicates both coastal areas are too diverse and cannot be used for prediction. A negative LR indicates that a prediction made after updating the input variables is less accurate than using the prior probabilities. Apparently, relations learned during training phase (with *Wadden* coast cases) are not representative for the prediction of erosion volumes at the *Holland* coast.

6.3. Recommendations

R1 Discretization of the Bayesian network

In the process of constructing the Bayesian network, discretization became an important step in the Bayesian analysis. However, the results presented in [Chapter 5](#) are all based on a Bayesian network with one set of discretization thresholds (described in [Section 4.2.3](#)). First action in future work on the Bayesian network approach must be a change in bin resolution (range and number of bins per variable). However, before this it is worthwhile to first optimize the Bayesian network in a more efficient way described in next recommendation.

R2 Build the Bayesian network in a more efficient way (reduce time for inference)

To reduce the time to evaluate multiply connected networks (more than one connections from one node to another), one could turn the network into an equivalent singly connected one. There are a few ways to perform this task. The most common ways are variations on a technique called *clustering*. In clustering, one combines nodes until the resulting graph (Bayesian network) is singly connected. By combining two nodes in one new node one can create a single connected network (also called a *polytree*). A singly connected network here has no more than one path between two nodes. An example to reduce the dimension of a Bayesian network could be to use the dimensionless fall velocity parameter $\frac{H}{T_w}$ ([Vellinga, 1986](#)) instead of using the variables: wave height, peak period and grain diameter. This makes the network more efficient due to the use of special singly connected algorithms. Those are often faster and not exact (not NP-hard). However, a problem arises if the node one creates have a large numbers of values. Then, the search for an appropriate algorithm that approximates the conditional probabilities best continues.

Since a large number of people are now using Bayesian networks, there is a great deal of research and efficient exact solution methods as well as a variety of approximation schemes.

R3 Dune safety assessment

In order to say something about the application of the Bayesian network approach in the safety assessments of dunes, the Bayesian network model presented in this research needs to be extended. Here the Bayesian network was used to predict erosion volumes. For an actual safety assessment a definition of failure needs to be added to the Bayesian network. Preferably the failure definition is determined before construction of the Bayesian network, such that determining variables are included and adjusted. This research used an erosion volume above storm surge level, which is dynamic and cannot be compared with a profile indicator for instance.

R4 Distinguish more coastal features

The four profile indicators used in this research are sufficient in order to evaluate complete coastal areas (in Dutch: ‘Kustvakken’) but is not sufficient to assess the dune safety level when zooming in on a couple or even a single transect. The profile indicators profile volume and slope are highly correlated and it could be that just using one of both is sufficient in order to predict erosion volume. More research on the way of characterizing a bottom profile (or profiles) should be executed to draw a final conclusion.

R5 Use of different types of data

Basis of a Bayesian network is data used to learn relations between the variables of interest. The database used in this research can be viewed as one population (all the cases are generated in the same way). It might be interesting to add different kind of data to the database like: real-world observations, results of lab experiments or the output of an other dune-erosion model. In this way the Bayesian network prediction is less sensitive to a possible bias of one single dune erosion model. In addition, it is possible to weight data in the Bayesian network. For instance the priority of a real-world observation case can be increased by a weighting factor.

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A. Shape of the erosion profile

Here the shape of the erosion profile computed with DUROS+ is described. Formulae are derived from [WL | Delft Hydraulics \(2007b\)](#) and [van Gent et al. \(2008\)](#). The shape of the three elements of the post-storm profile ([Section 2.1.2](#)) will be described below. The numbers correspond to the numbers indicated in [Figure 2.3](#).

1. Dry dune front

The dune-foot is defined as the place where the steep dune devolves in the much more gentle dry beach, at storm surge level. The slope of the eroded dune-front is 1:1.

2. Parabolic equilibrium post-storm beach profile

From the dune-foot ($x = 0; y = 0$) the shape of the erosion profile is parabolic (perpendicular to the coast) in seaward direction.

$$\left(\frac{7.6}{H_{0s}}\right) y = 0.4714 \left[\left(\frac{7.6}{H_{0s}}\right)^{1.28} \left(\frac{12}{T_p}\right)^{0.45} \left(\frac{w}{0.0268}\right)^{0.56} x + 18 \right]^{0.5} - 2.0 \quad (\text{A.1})$$

till the point for which holds:

$$x_{max} = 250 \left(\frac{H_{0s}}{7.6}\right)^{1.28} \left(\frac{0.0268}{w}\right)^{0.56} \quad (\text{A.2})$$

so

$$y_{max} = \left[0.4714 \left(250 \left(\frac{12}{T_p}\right)^{0.45} + 18 \right)^{0.5} - 2.0 \right] \left(\frac{H_{0s}}{7.6}\right) \quad (\text{A.3})$$

In the formulas [Equation A.1](#), [Equation A.2](#) and [Equation A.3](#) the definition of the symbols is:

- H_{0s} : significant wave height on deep water [m].
- T_p : wave period at the peak of energy density spectrum [s].
- w : the fall velocity of a sand particle in seawater of 5° Celsius [m/s].
- x : the distance from the new dune foot [m].
- y : the depth beneath storm surge level [m].

The formulas [Equation A.1](#) and [Equation A.2](#) are applicable whenever the peak period is between the interval of $12s < T_p < 20s$. Else, for $T_p < 12$ the T_p retains 12s. And for $T_p > 20s$, the T_p retains 20s.

The fall velocity (w) in seawater with a temperature of 5° Celsius can be calculated using the following formula:

$$^{10}\log\left(\frac{1}{w}\right) = 0.476 (\log D_{50})^2 + 2.180^{10} \log D_{50} + 3.226 \quad (\text{A.4})$$

In which:

w : fall velocity of (dune)sand in seawater [m/s].

D_{50} : measure for the grain diameter of the dune sand [m].

3. Transition slope connecting the post-storm beach profile with the initial profile

Seawards of the point x_{max}, y_{max} the profile pass into a straight line with a slope of 1:12.5 till the point where it crosses the original profile.

B. Probability Density Distributions - HBC

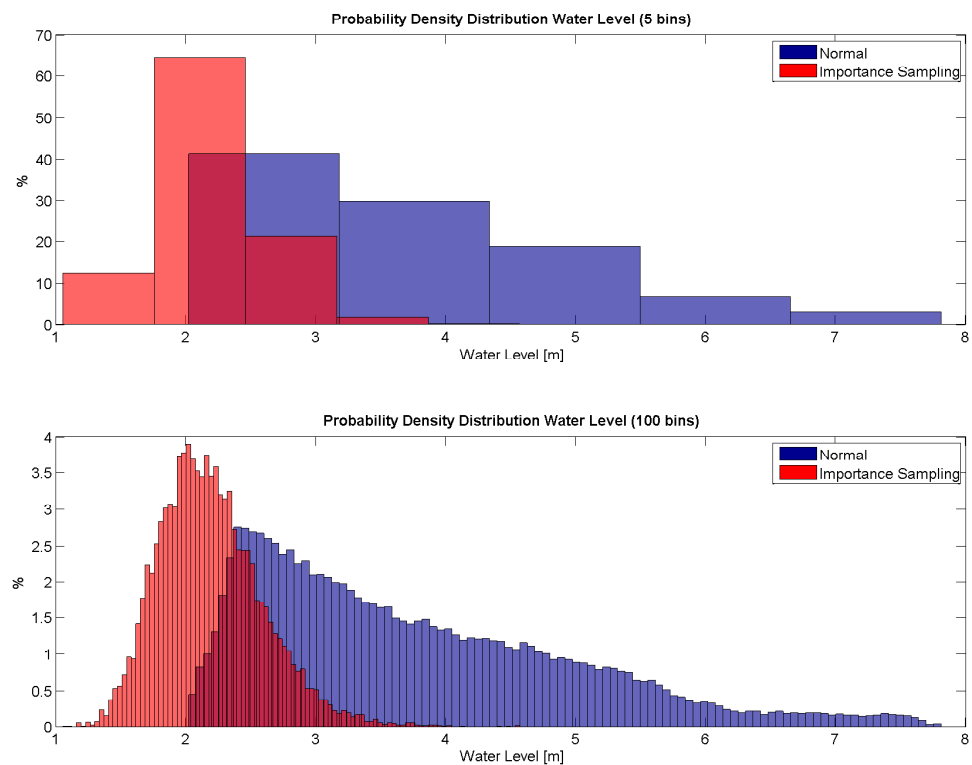


Figure B.1.: Water level distribution with Importance Sampling

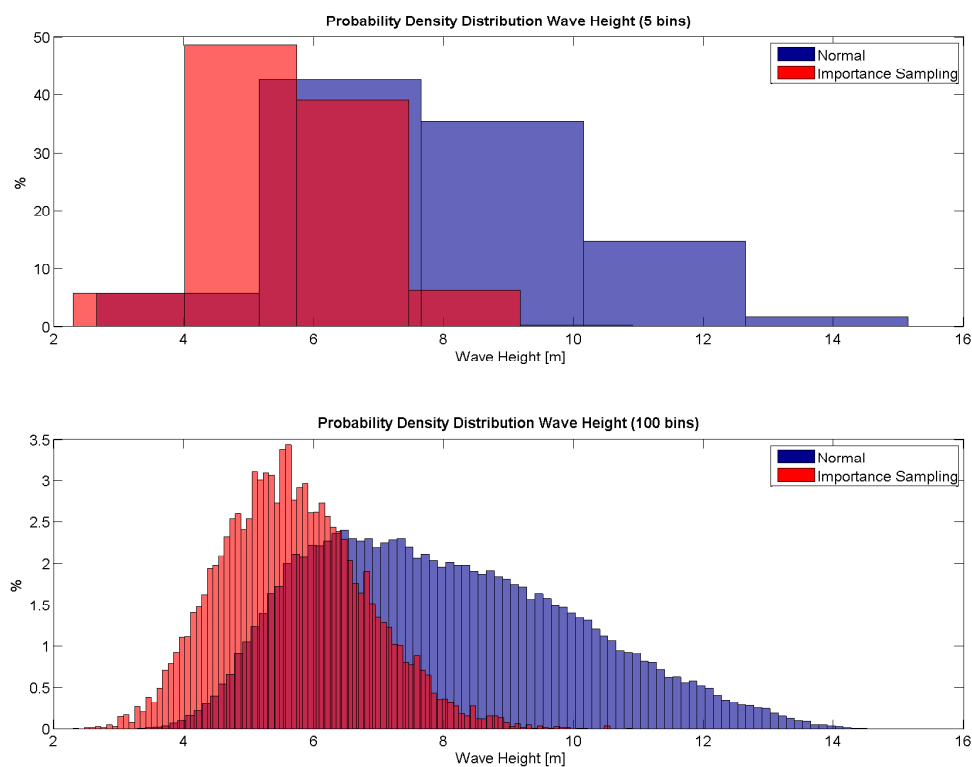


Figure B.2.: Wave height distribution with Importance Sampling

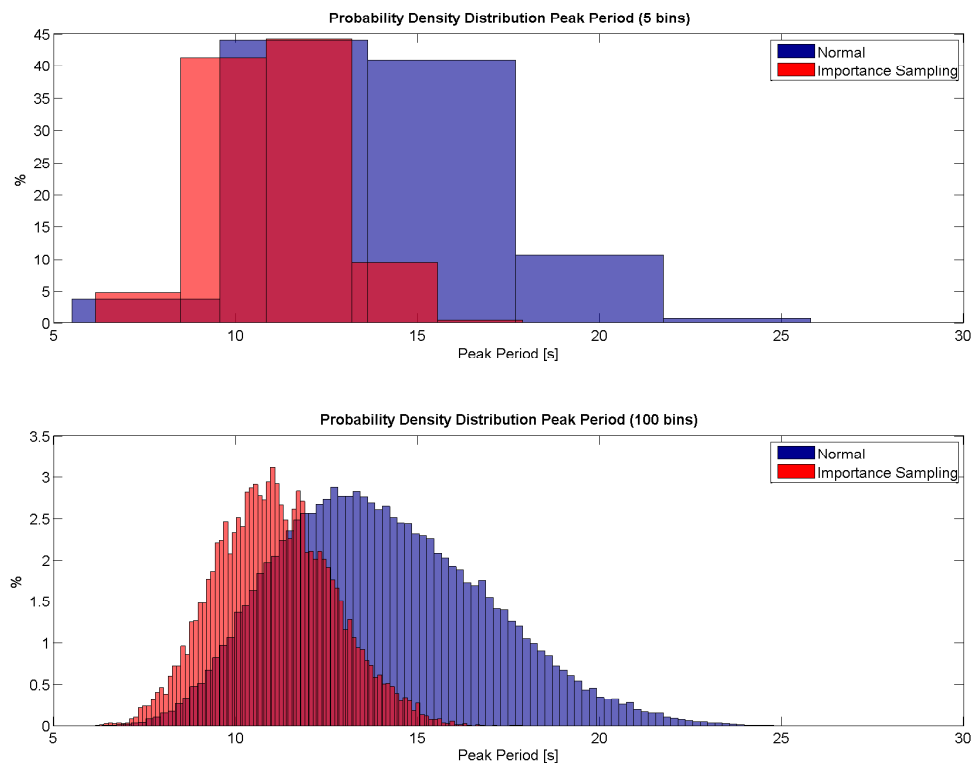


Figure B.3.: Peak Period distribution with Importance Sampling

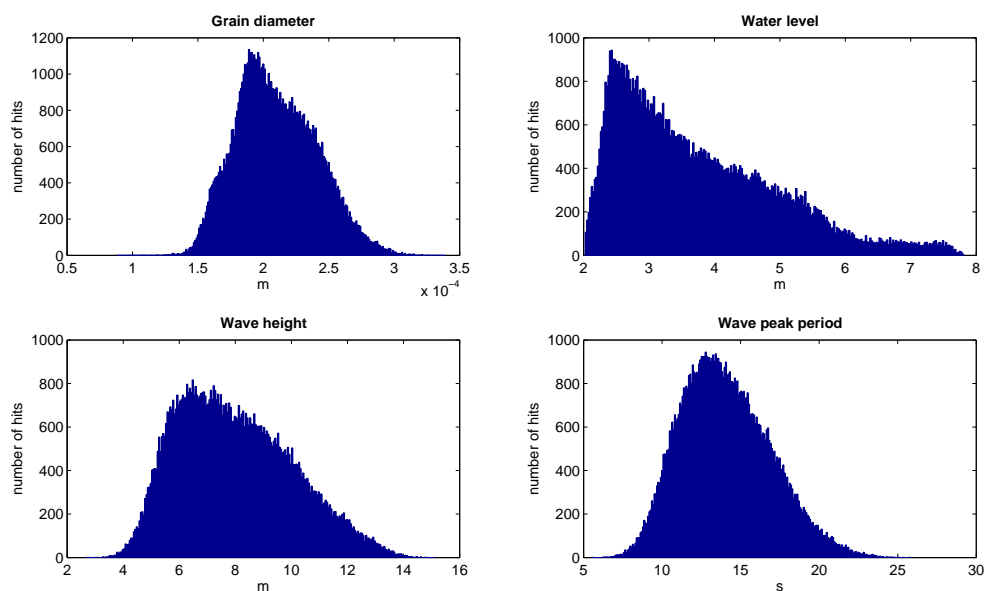


Figure B.4.: Input distributions of the Bayesian network

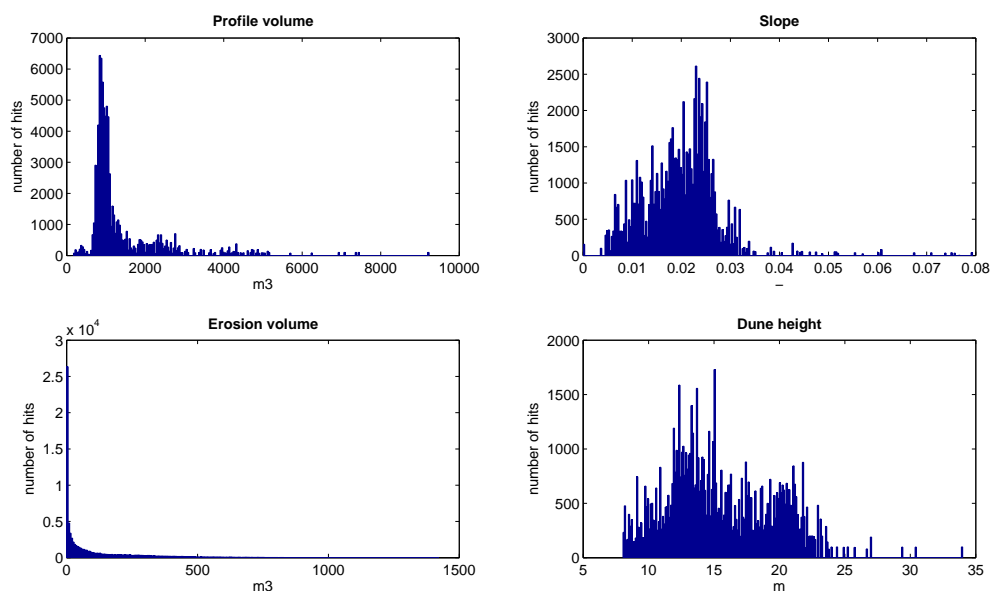


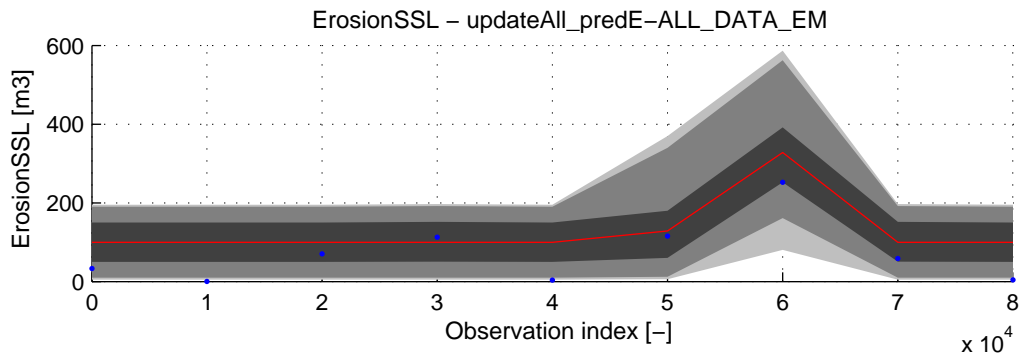
Figure B.5.: Input distributions of the Bayesian network

C. Bayesian Input Dataset

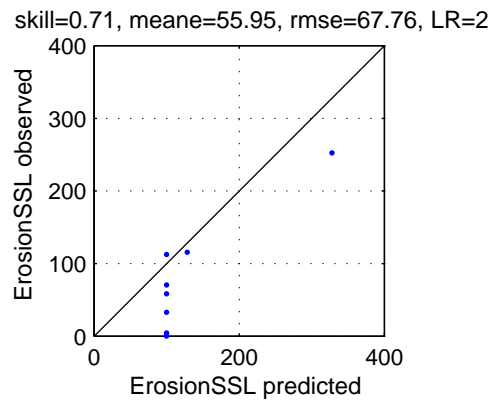
case	transid	D50	WL _t	H _{sigt}	T _{pt}	...
1	2.000501e+006	1.721635e-004	4.214463e+000	8.875190e+000	1.649850e+001	↑
2	2.000501e+006	1.755222e-004	3.190794e+000	7.747158e+000	1.385386e+001	↑
...	↑
82,195	9.011825e+006	2.655463e-004	3.300653e+000	6.792742e+000	1.272397e+001	↑
82,196	9.011825e+006	2.624021e-004	2.640258e+000	5.098103e+000	1.057869e+001	↑
...	ProfileVolume	Slope	ErosionSSL	DuneCrest	BeachWidth	areacode
↑	1.645744e+003	1.473050e-002	3.291941e+001	1.020000e+001	5.080000e+002	2.000000e+000
↑	1.645744e+003	1.473050e-002	8.910825e-001	1.020000e+001	5.080000e+002	2.000000e+000
↑
↑	7.529970e+002	1.936960e-002	1.815077e+001	1.060000e+001	3.180000e+002	9.000000e+000
↑	7.529970e+002	1.936960e-002	5.439811e+000	1.060000e+001	3.180000e+002	9.000000e+000

Table C.1.: Example of first and last two cases of the complete dataset described in [Chapter 3](#). The table is cut into two pieces in order to fit on one page. Each colour indicates one complete case.

D. Hindcast Results - Number of data

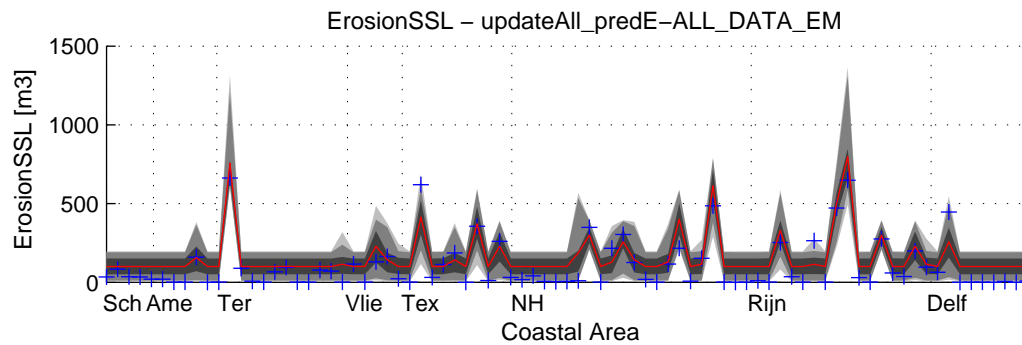


(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.

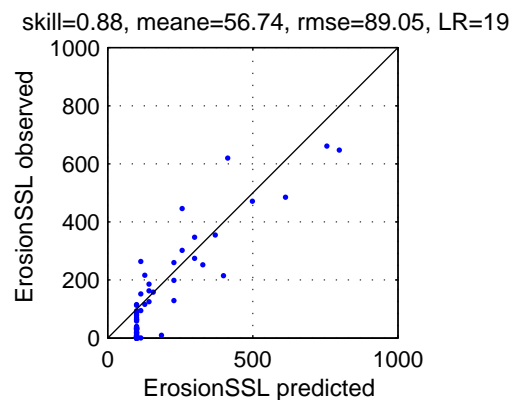


(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure D.1.: This prediction evaluation represents the Bayesian network trained on the entire dataset and tested on 9 observations obtained out of the training data set. Here all the input variables are updated with the conditions corresponding to the 9 observations in order to predict the erosion volumes.

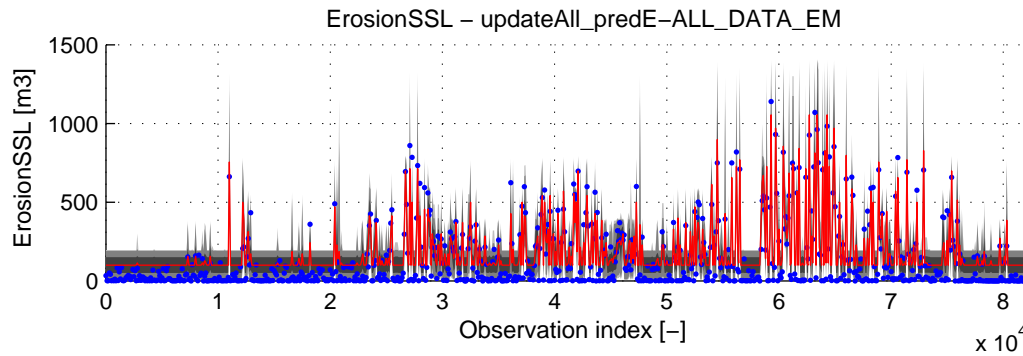


(a) Updated probabilities for erosion volume compared to the observed value (blue dots). Inputs out of training data set are used for testing. The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.

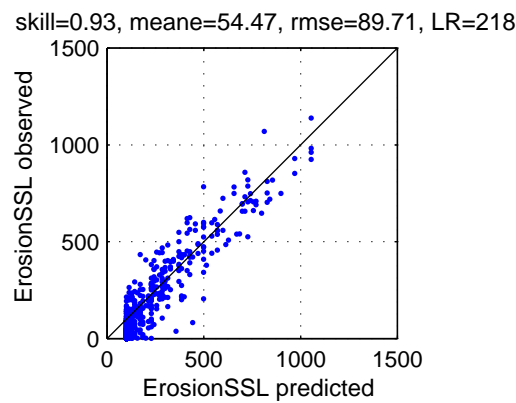


(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure D.2.: This prediction evaluation represents the Bayesian network trained on the entire dataset and tested on 83 observations obtained of the training data set. Here the observed erosion volumes are given as prediction.

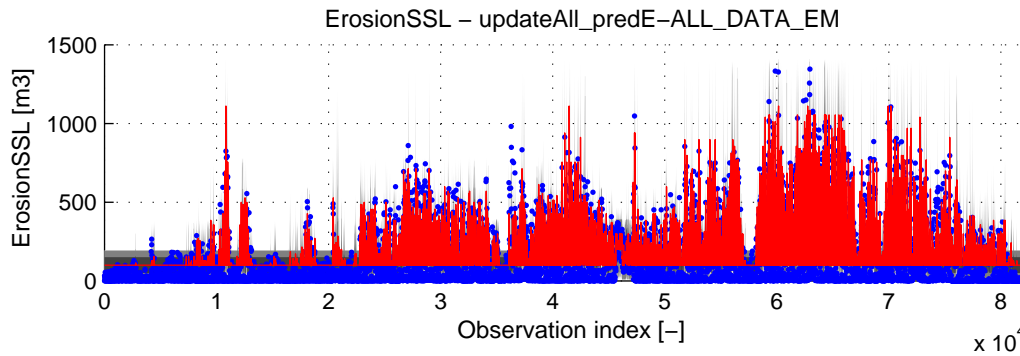


(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.

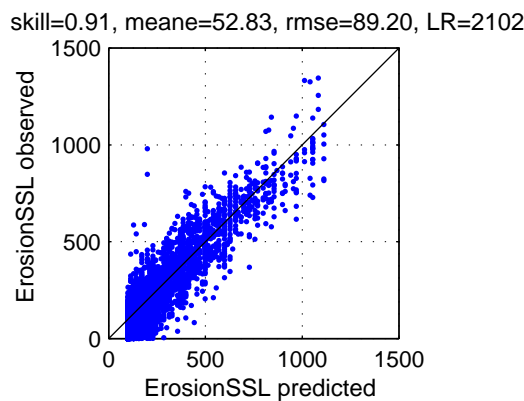


(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure D.3.: This prediction evaluation represents the Bayesian network trained on the half dataset and tested on 822 observations obtained out of the non-training data set. Here all the input variables are updated with the conditions corresponding to the 822 observations in order to predict the erosion volumes.



(a) Updated probabilities for erosion volume compared to the observed value (blue dots). The shading correspond to 50% darkest, 90%, and 95% (lightest) confidence regions.



(b) Regression of the Bayesian-mean prediction against the observed erosion volumes

Figure D.4.: This prediction evaluation represents the Bayesian network trained on the half dataset and tested on 8220 observations obtained out of the non-training data set. Here all the input variables are updated with the conditions corresponding to the 8220 observations in order to predict the erosion volumes.

E. Hindcast Results - Small set of Testcases

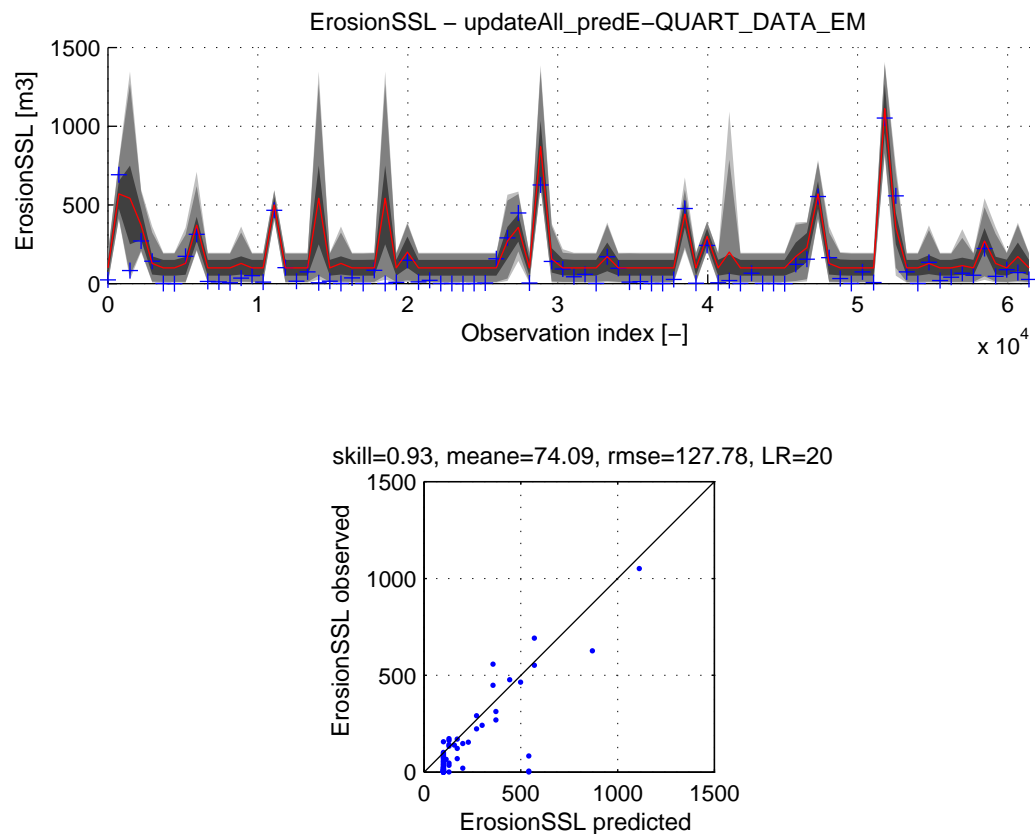


Figure E.1.: Bayesian network trained on $1/4$ of the dataset.

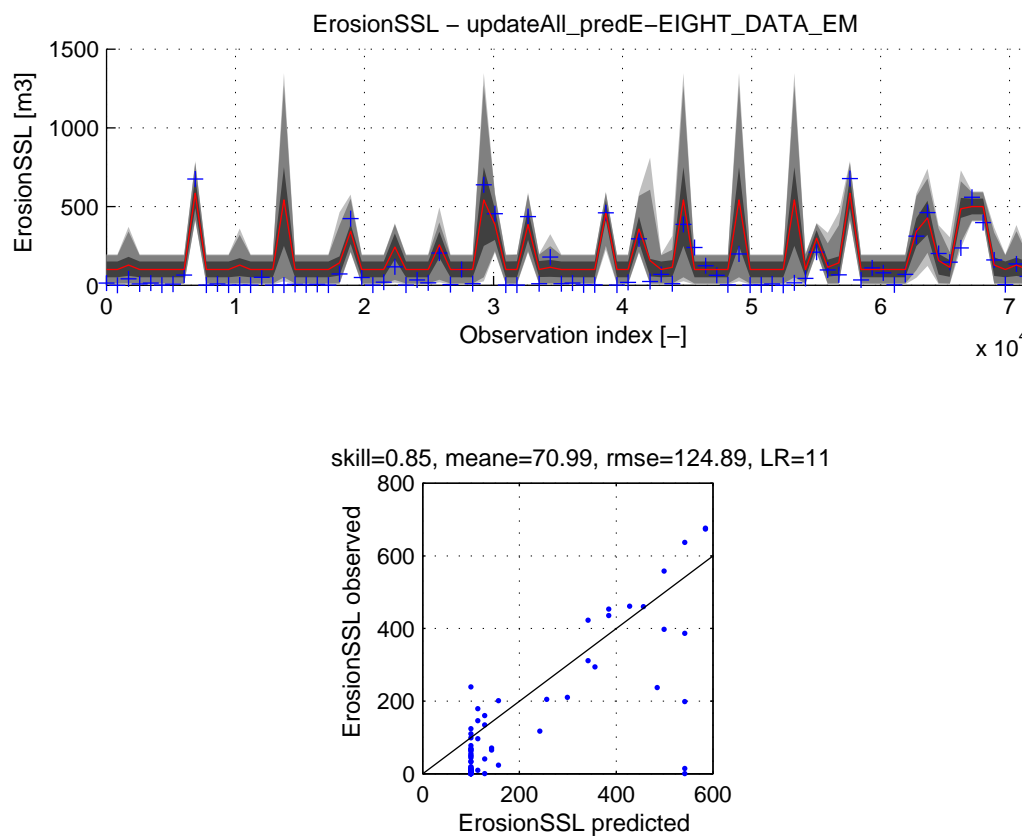


Figure E.2.: Bayesian network trained on $1/8$ of the dataset.

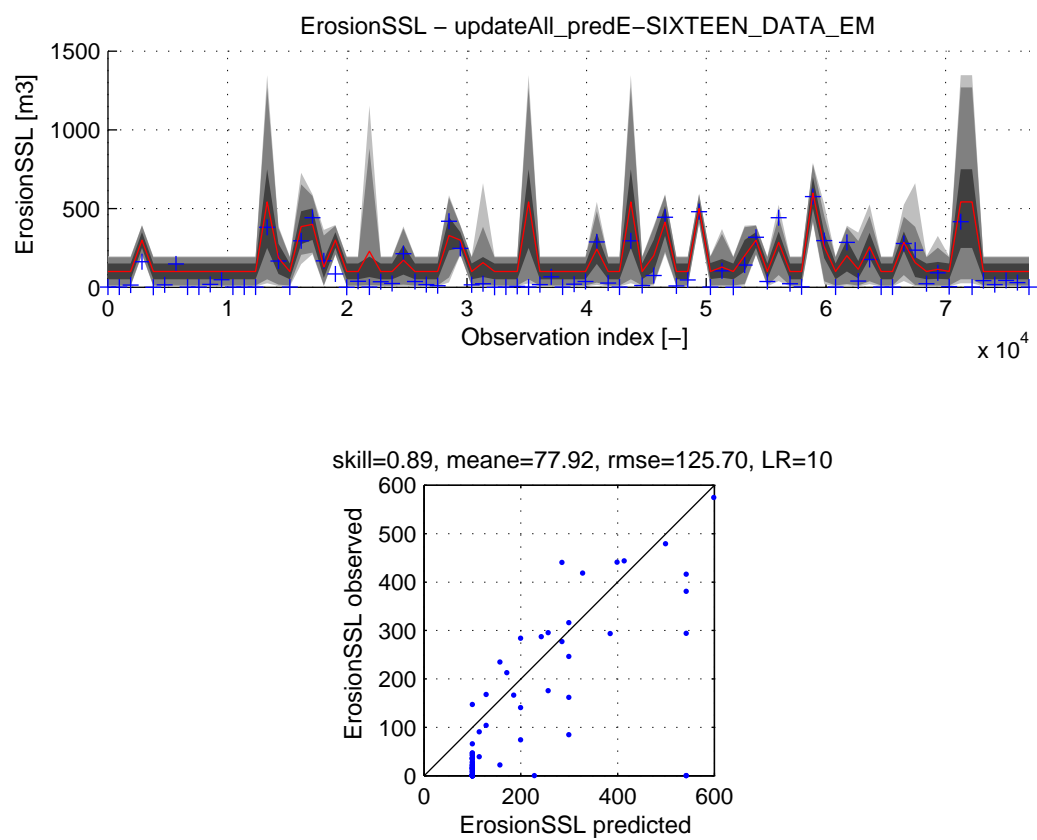


Figure E.3.: Bayesian network trained on $1/16$ of the dataset.

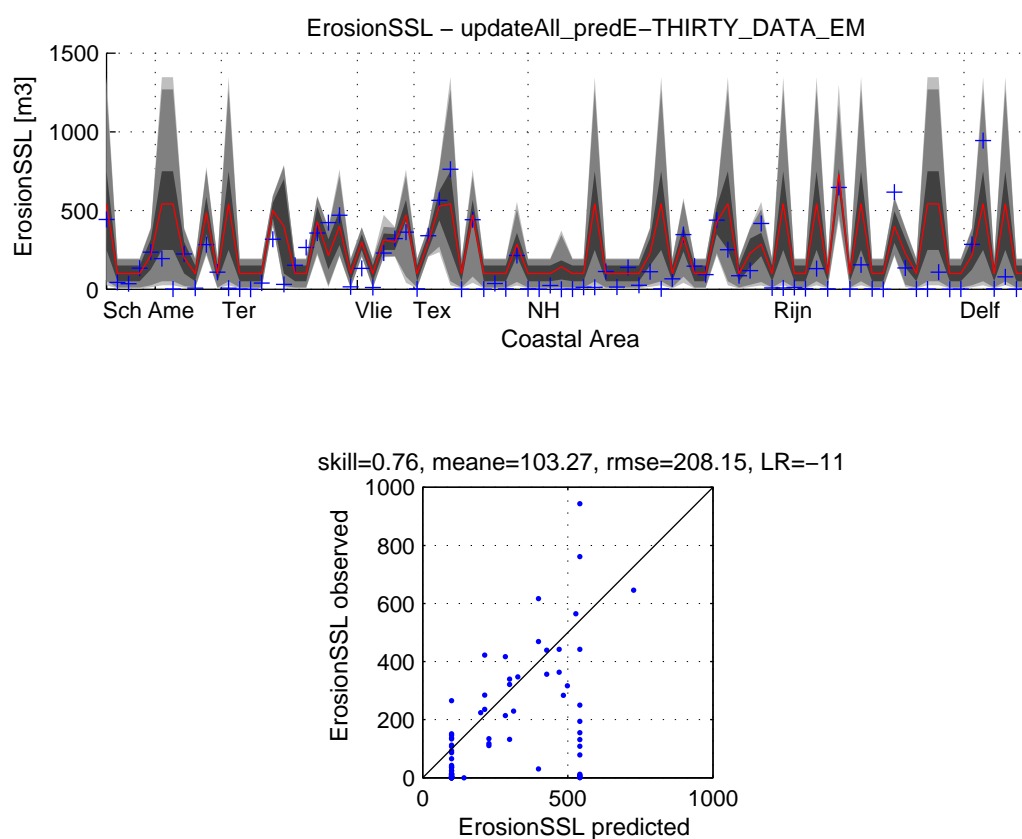


Figure E.4.: Bayesian network trained on $1/32$ of the dataset.

F. Hindcast Results - Relative importance single variable only

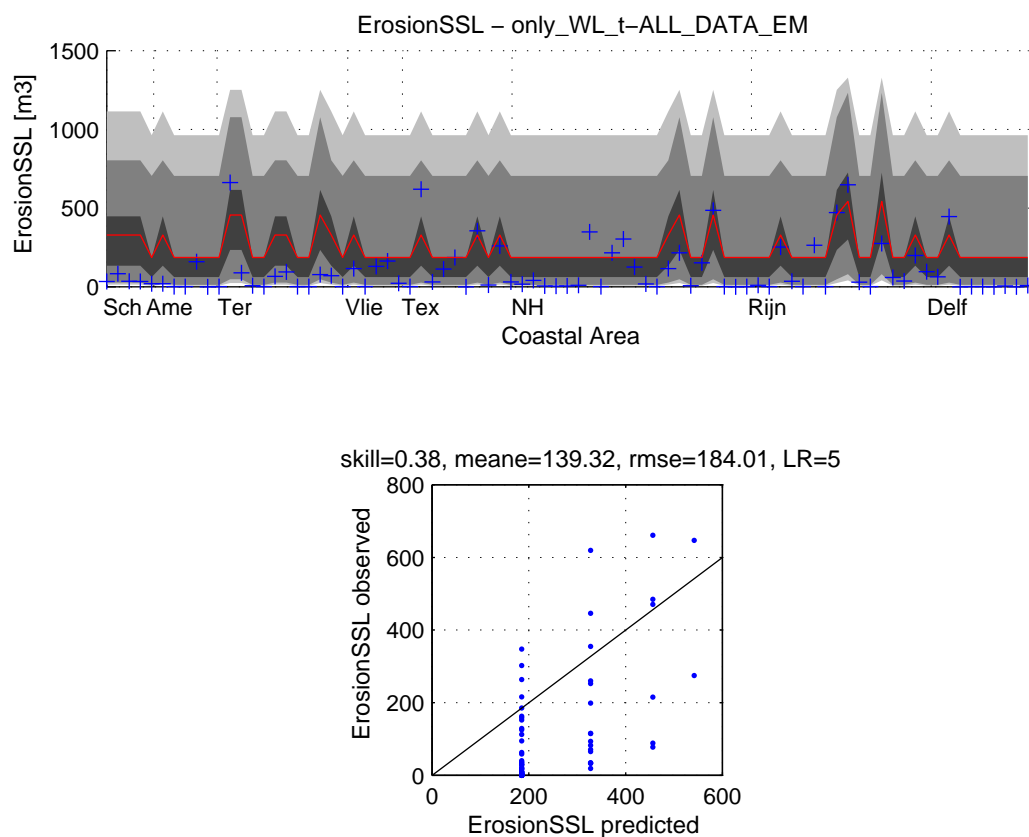


Figure F.1.: Bayesian network trained on entire dataset and tested on only water level information

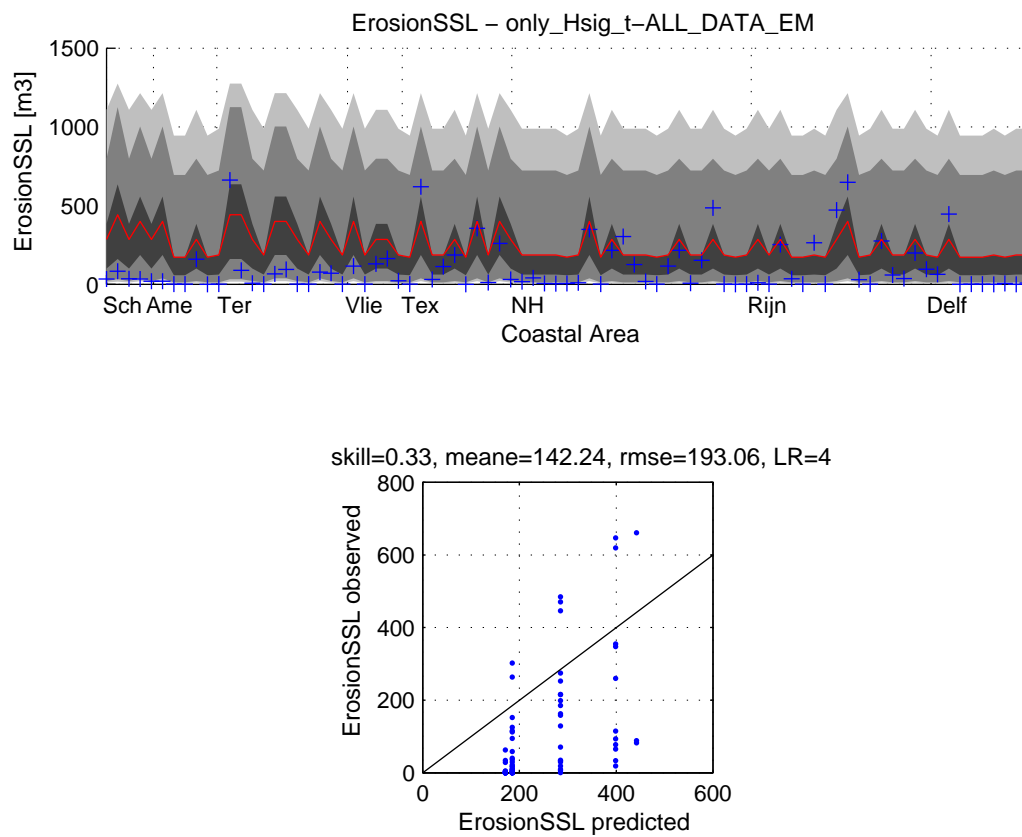


Figure F.2.: Bayesian network trained on entire dataset and tested on only wave height information

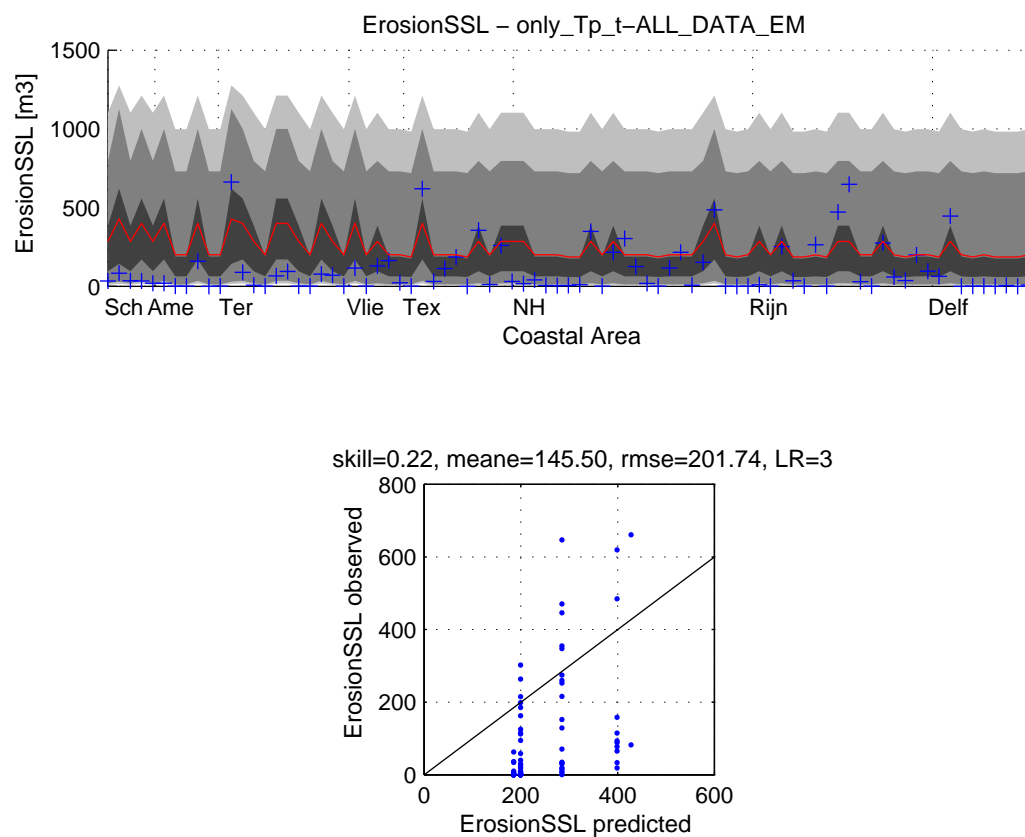


Figure F.3.: Bayesian network trained on entire dataset and tested on only peak period information

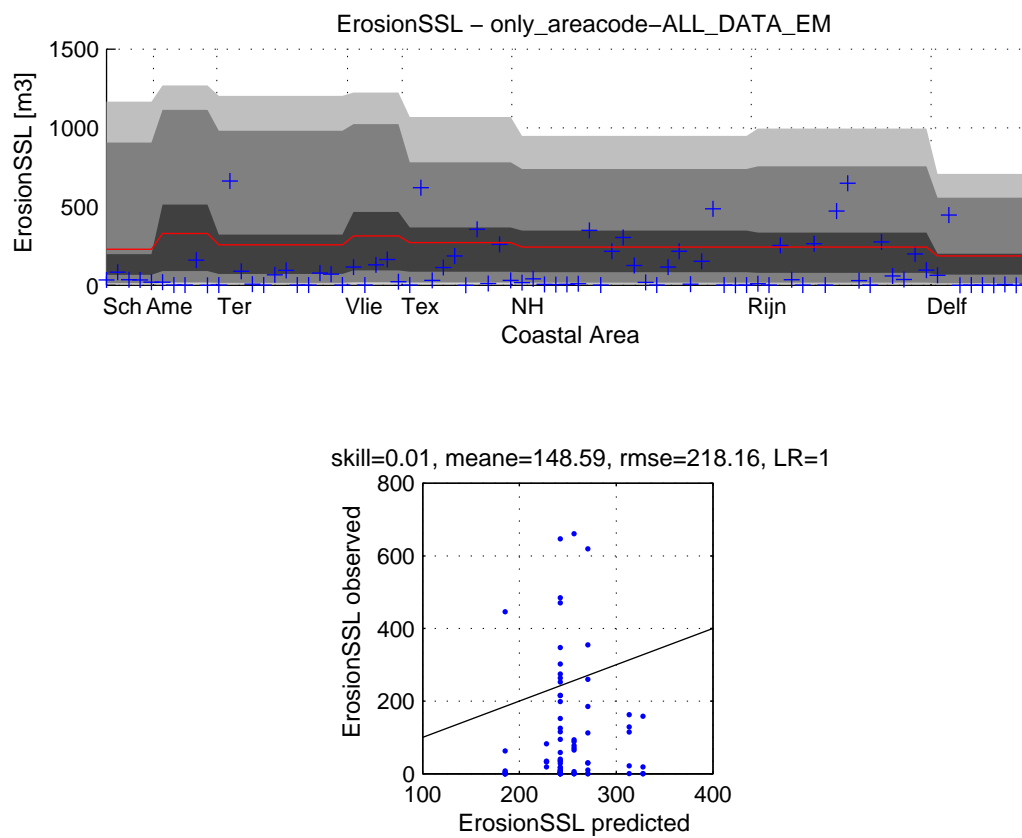


Figure F.4.: Bayesian network trained on entire dataset and tested on only areacodes

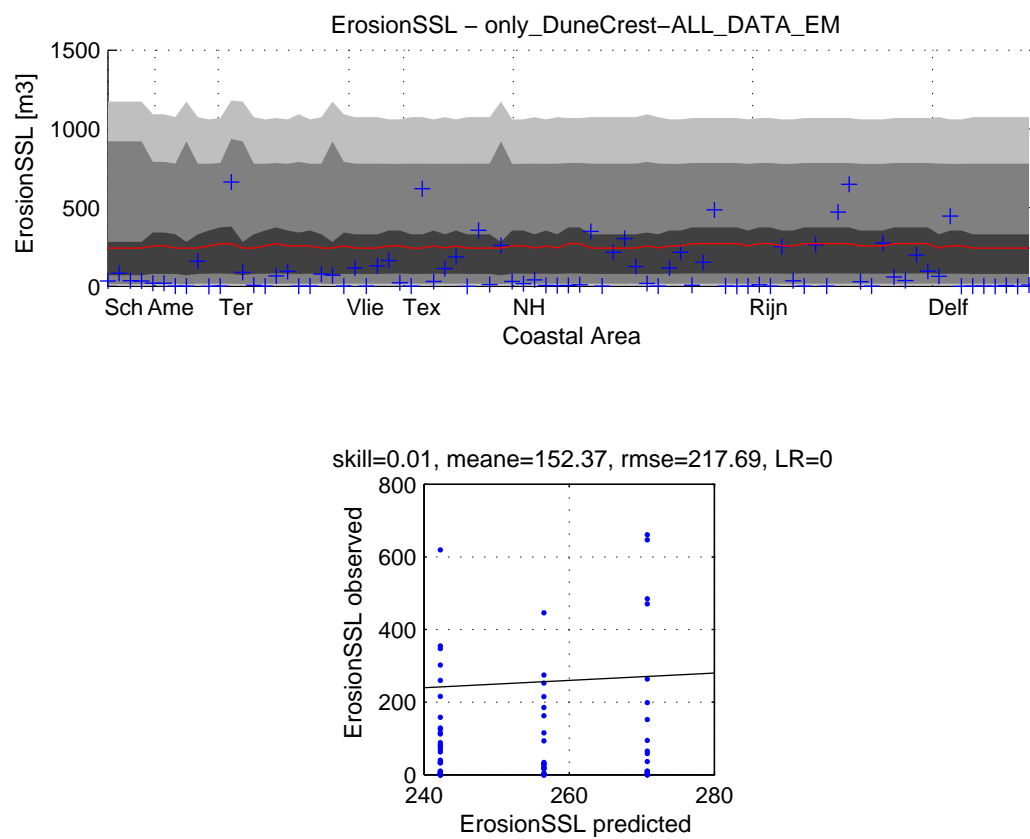


Figure F.5.: Bayesian network trained on entire dataset and tested on only dune crest information

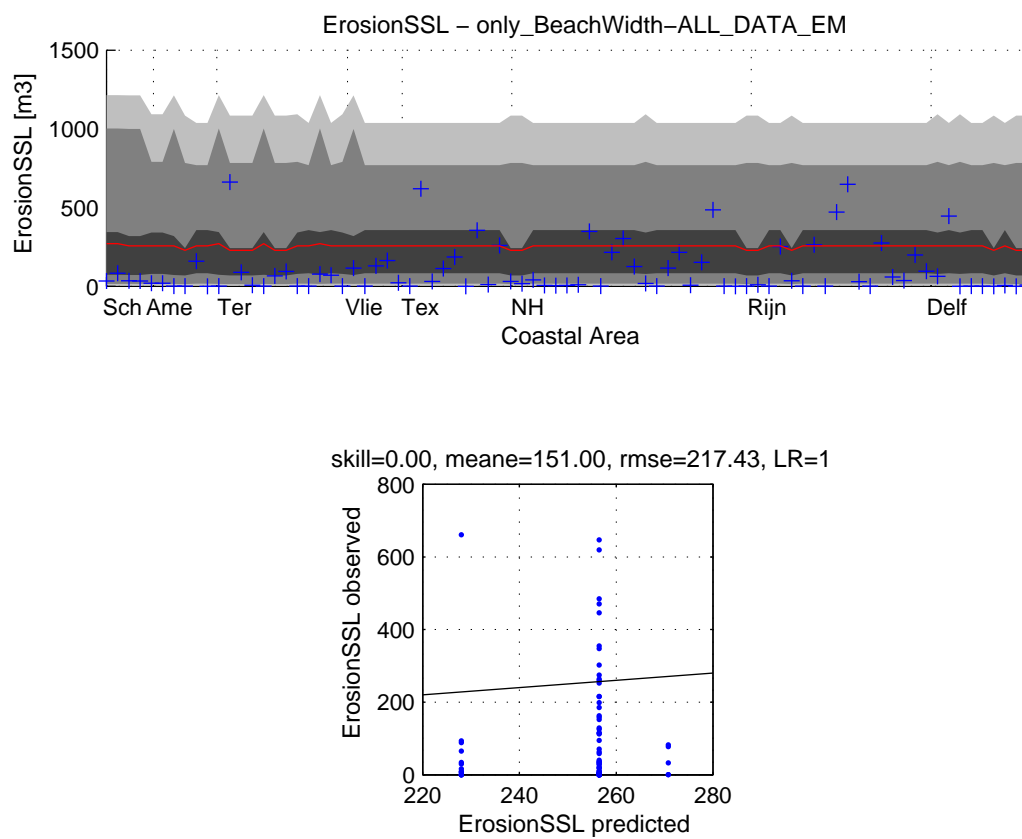


Figure F.6.: Bayesian network trained on entire dataset and tested on only beach width information

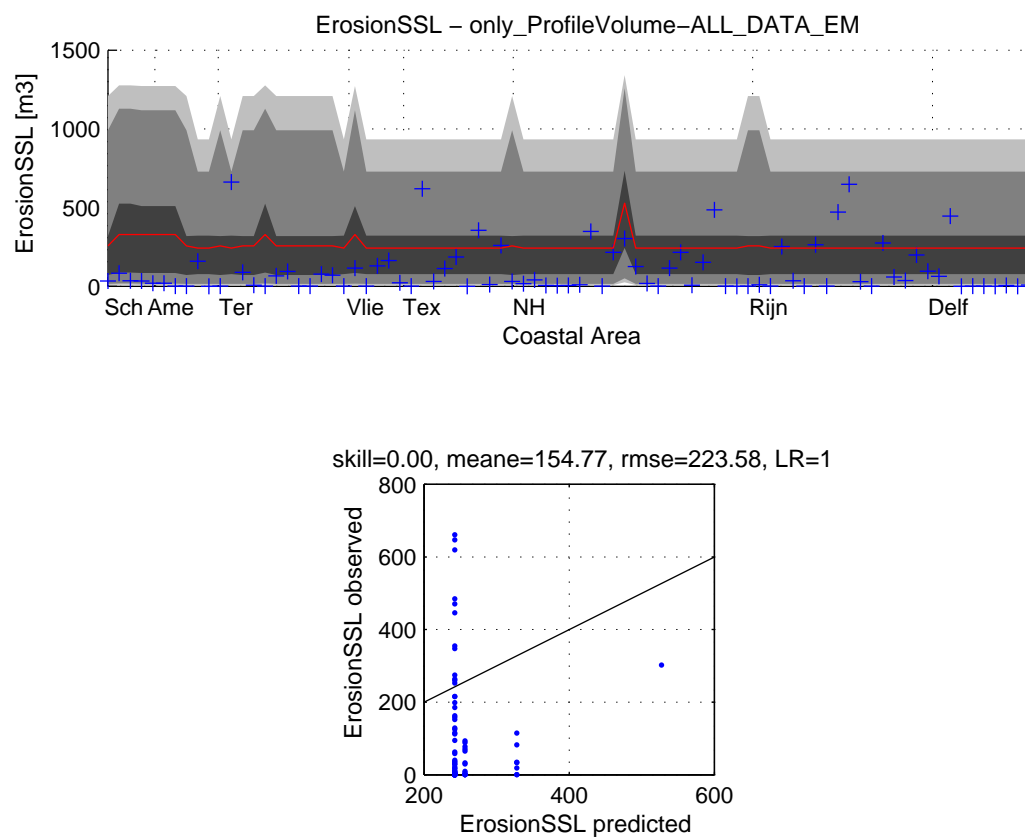


Figure F.7.: Bayesian network trained on entire dataset and tested on only profile volume information

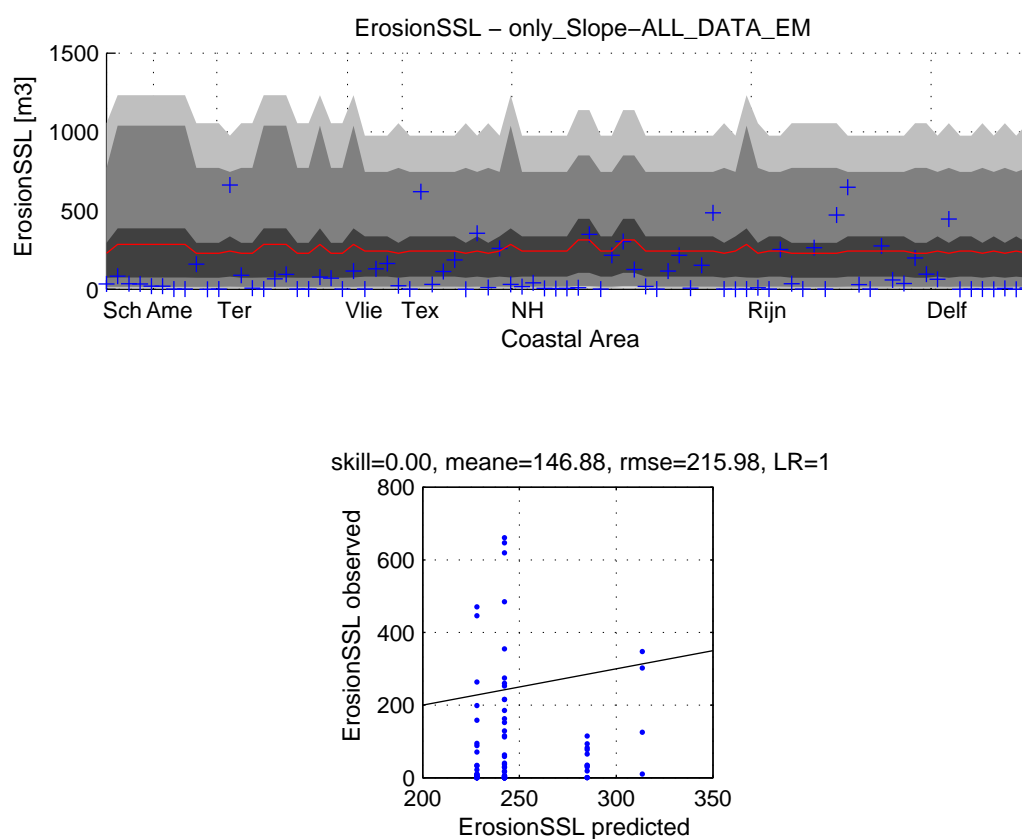


Figure F.8.: Bayesian network trained on entire dataset and tested on only slope information

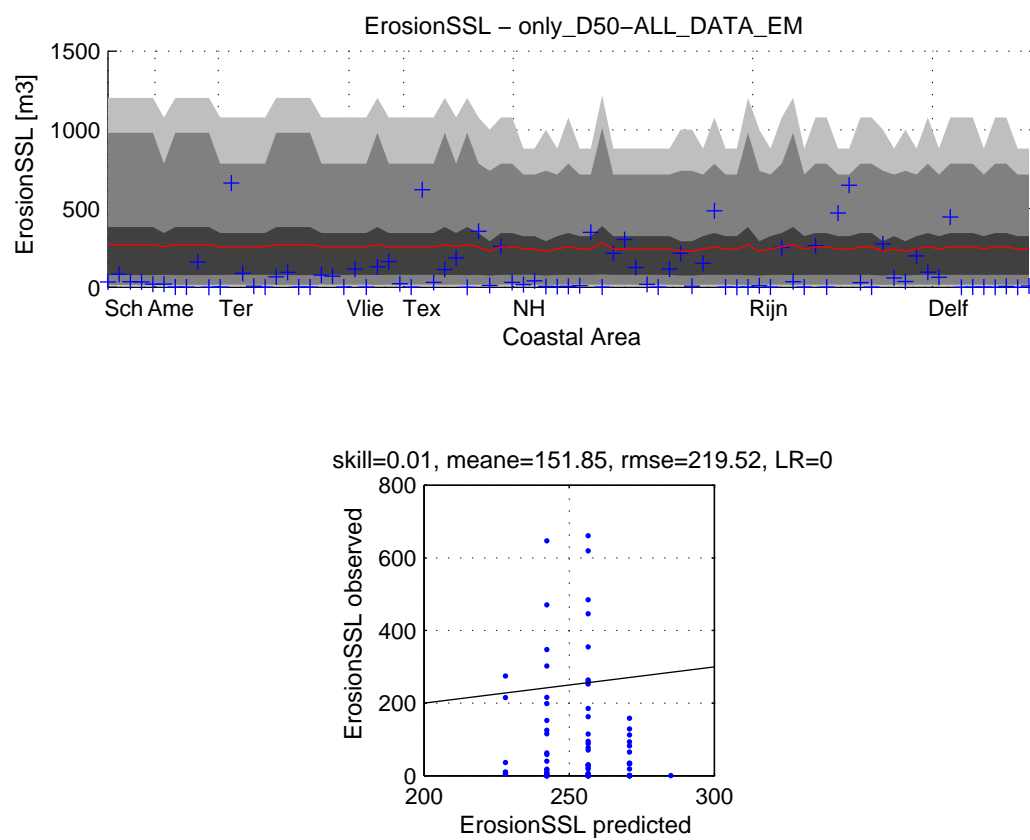


Figure F.9.: Bayesian network trained on entire dataset and tested on only grain size information

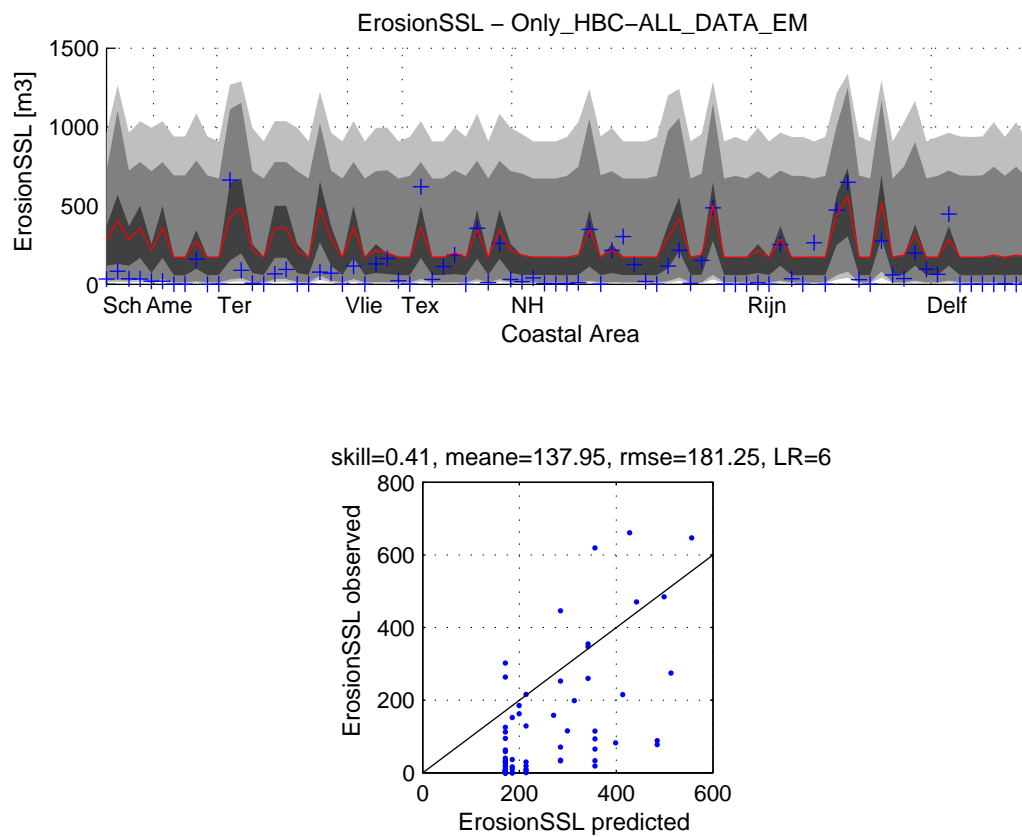


Figure F.10.: Bayesian network trained on entire dataset and tested on only hydraulic boundary conditions

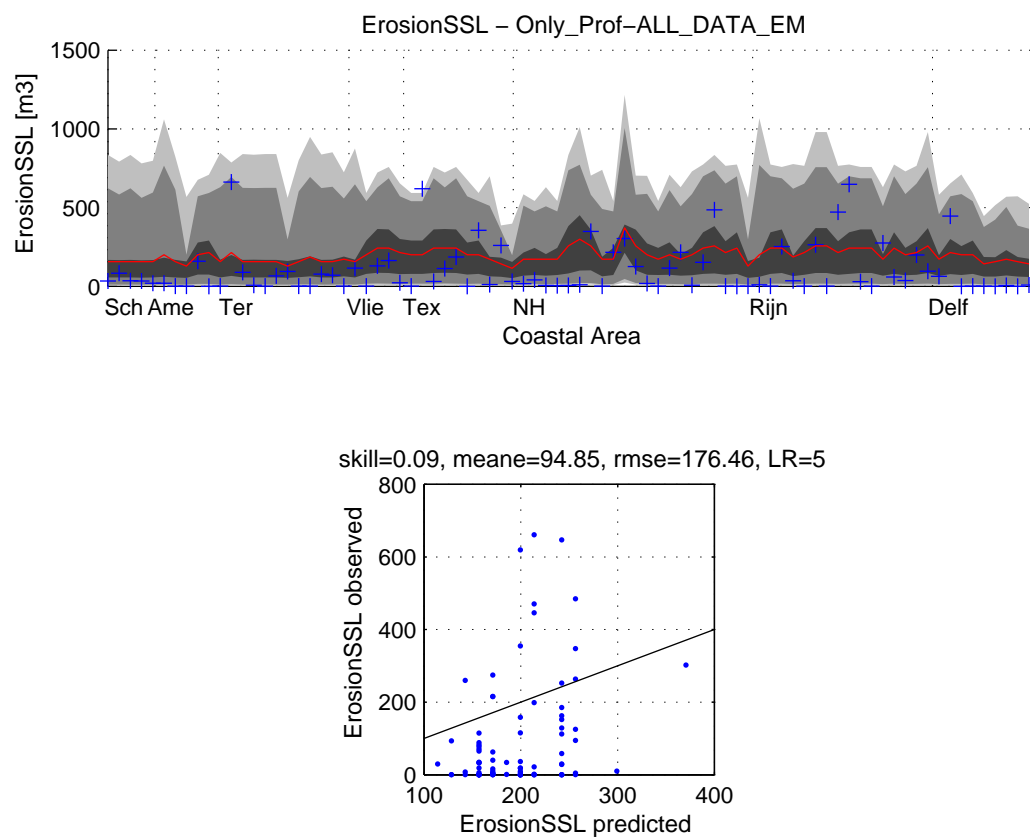


Figure F.11.: Bayesian network trained on entire dataset and tested on only profile indicator information

G. Hindcast Results - Relative importance remove single variable

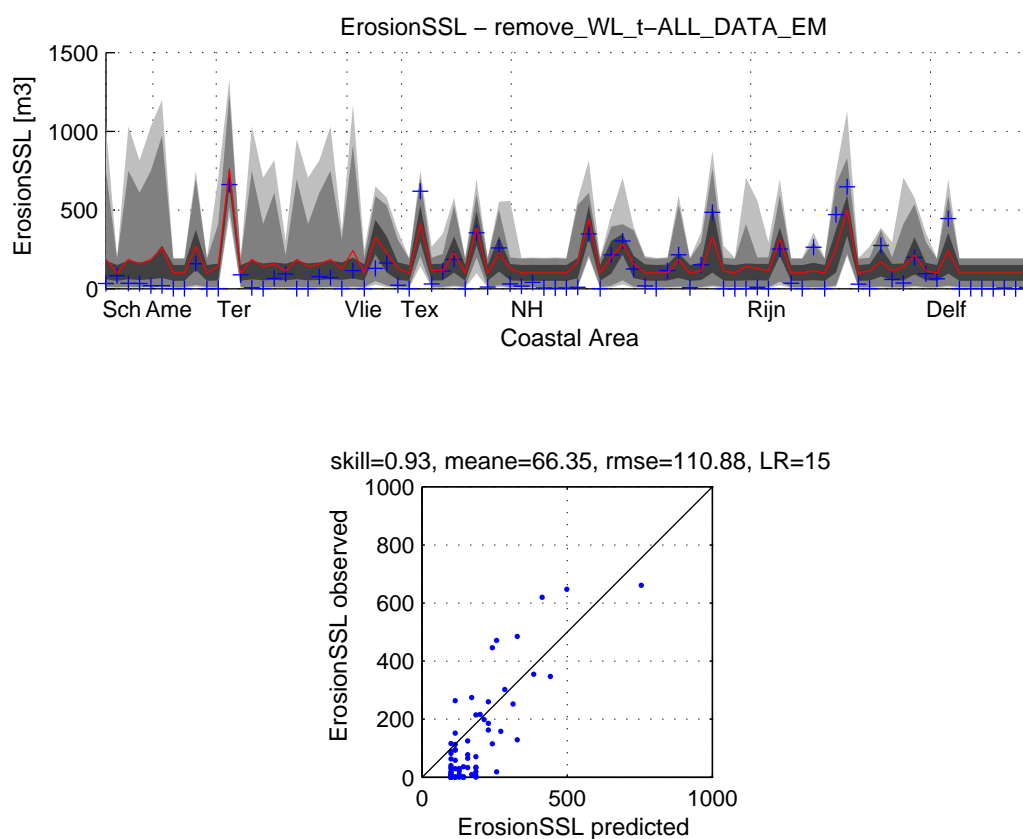


Figure G.1.: Bayesian network trained on entire dataset and tested without water level.

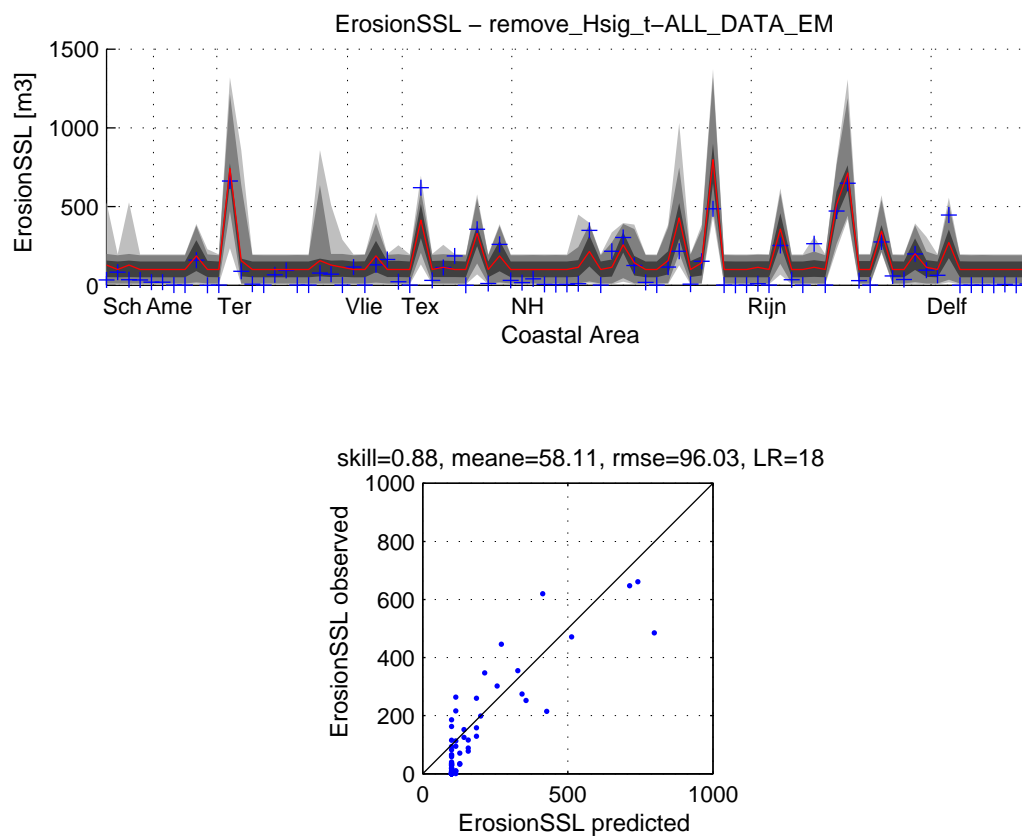


Figure G.2.: Bayesian network trained on entire dataset and tested without wave height.

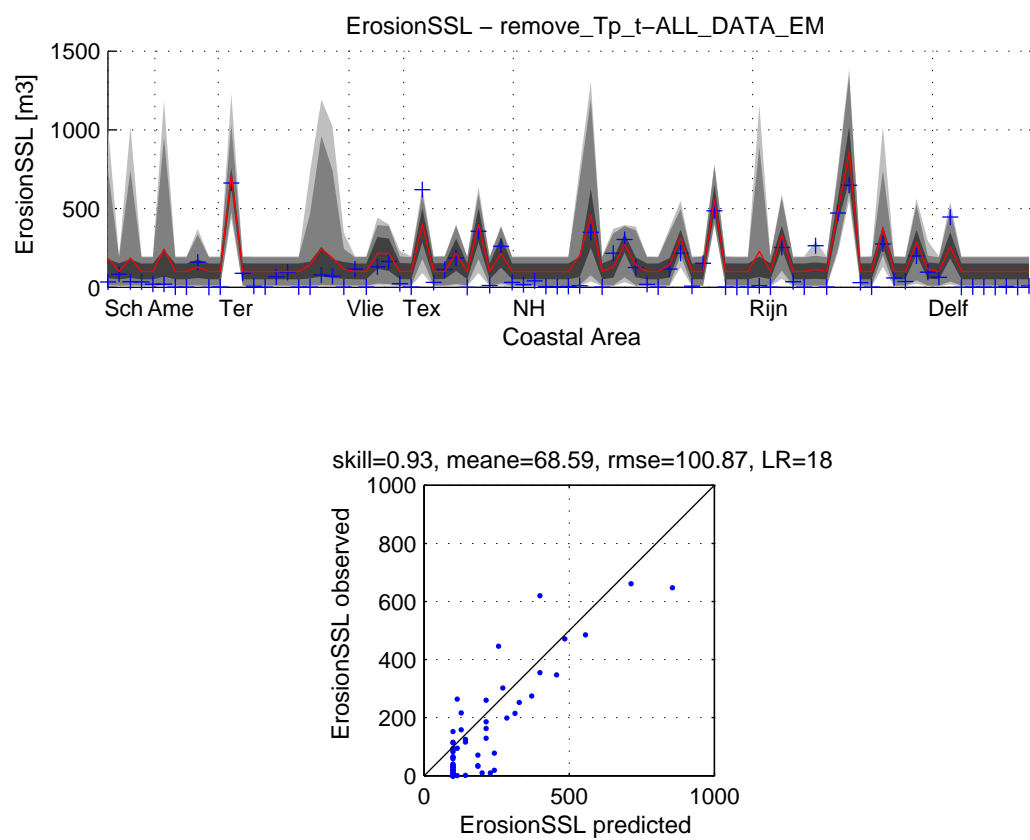


Figure G.3.: Bayesian network trained on entire dataset and tested without peak period.

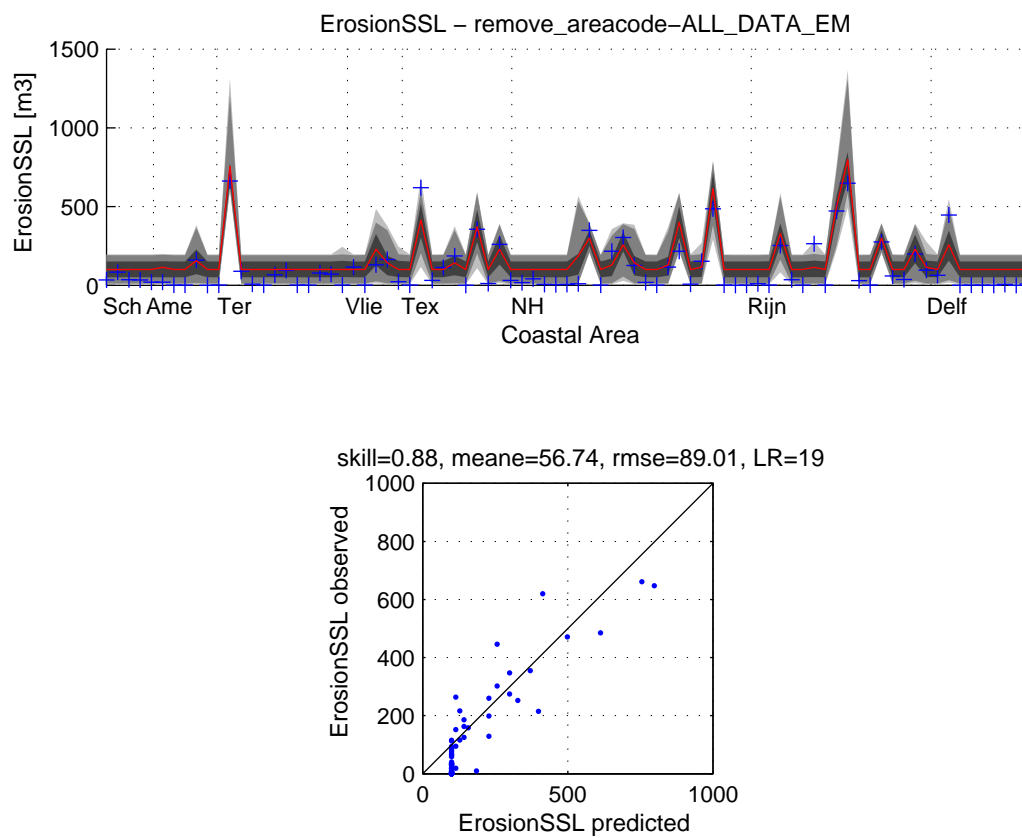


Figure G.4.: Bayesian network trained on entire dataset and tested without areacode.

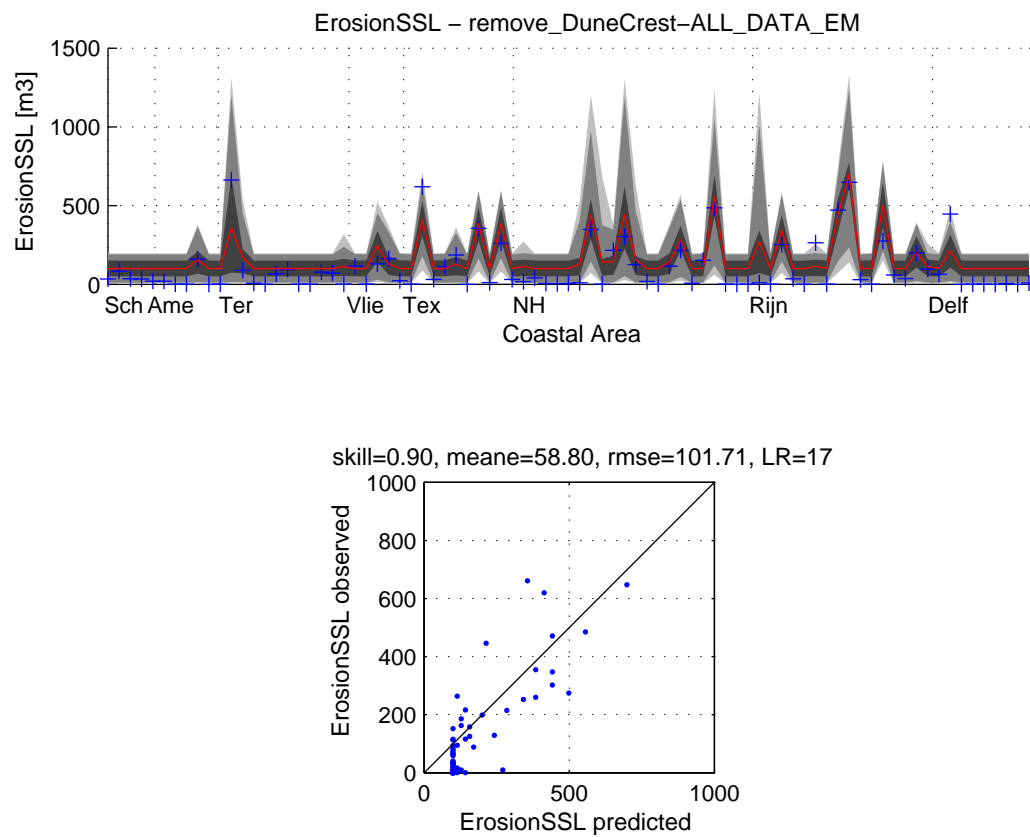


Figure G.5.: Bayesian network trained on entire dataset and tested without dune crest information.

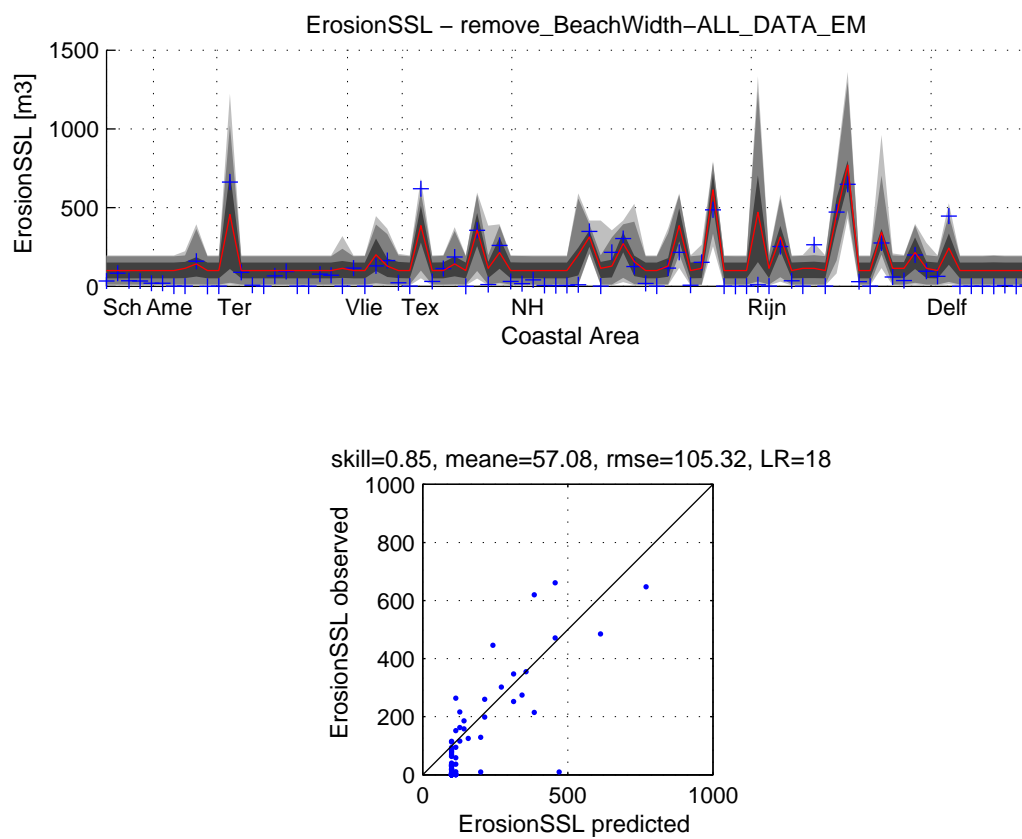


Figure G.6.: Bayesian network trained on entire dataset and tested without beach width information.

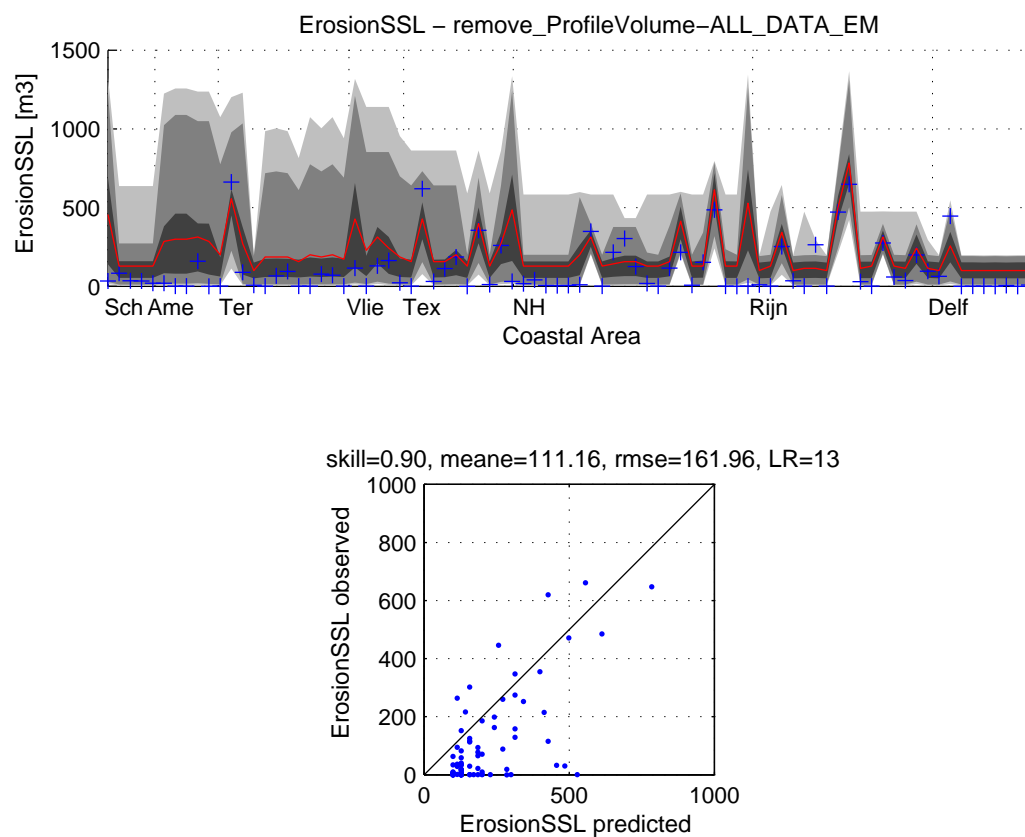


Figure G.7.: Bayesian network trained on entire dataset and tested without profile volume information.

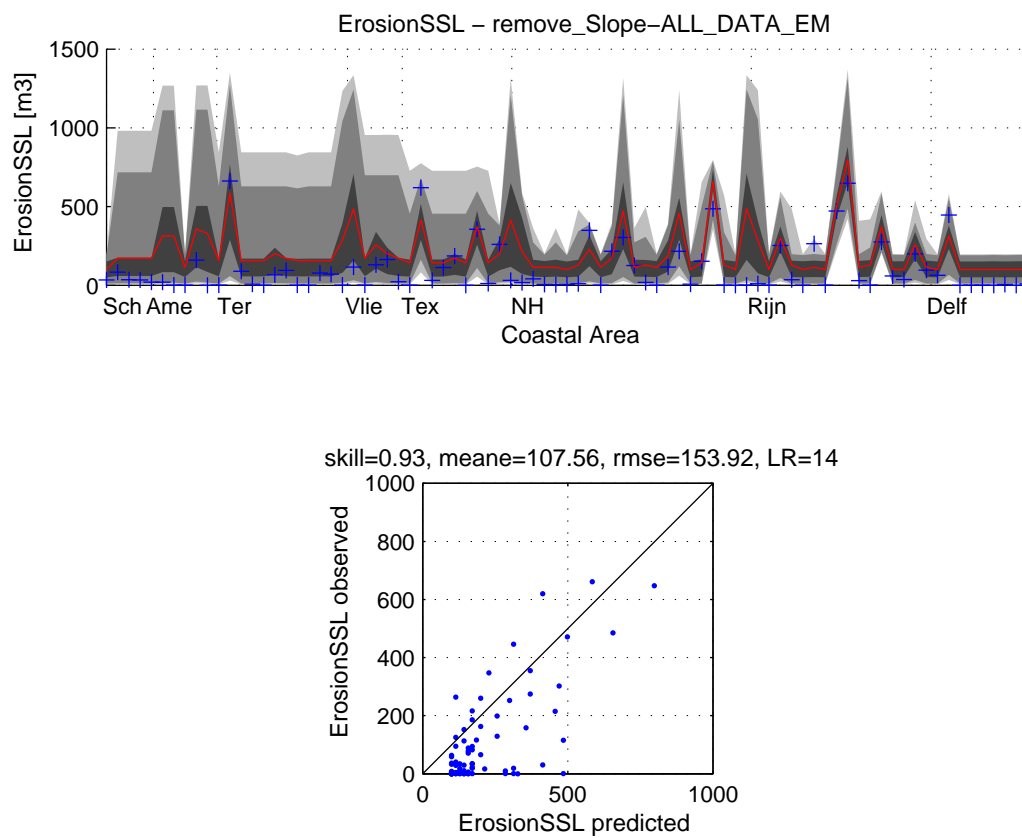


Figure G.8.: Bayesian network trained on entire dataset and tested without slope information.

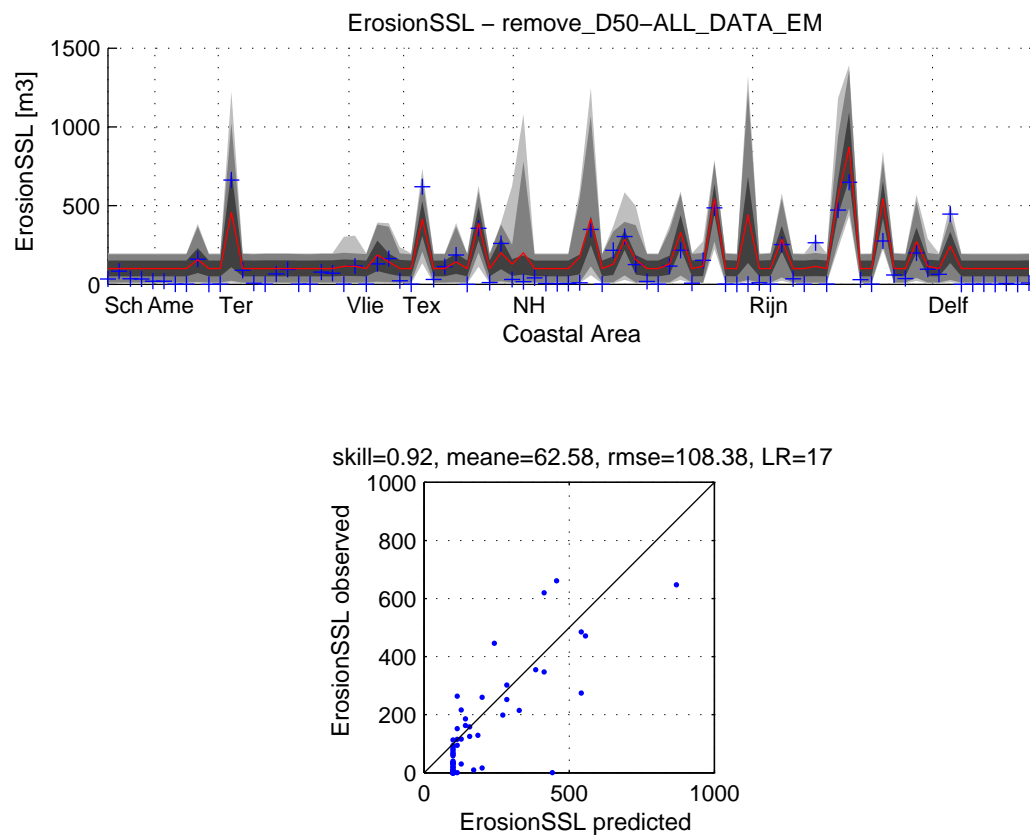


Figure G.9.: Bayesian network trained on entire dataset and tested without grain size information.