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Frequency band analysis and comparison of localisation techniques for drones using microphone array measurements

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Abstract: This study covers three aspects of acoustic localisation of drones using a microphone array. First, it assesses a grid-free approach, using differential evolution, to estimate the three-dimensional position of a drone. It is found that this is indeed possible for the drone in the near-field. For larger distances, it still provides the angular position of the drone. Second, the study emphasizes the essence of localisation over small frequency bands with the bands jointly spanning a large frequency range to reveal the presence of multiple sound sources and maximise the drone localisation range. Third, it addresses the localisation ranges for six different drones.

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1. Introduction

With the increasing use of (consumer) drones, there is an increasing need to monitor their presence and location in the airspace. Reasons stem from traffic regulator purposes, privacy, safety, and security.

So far, research primarily focused on three techniques. First, radar-based techniques, second radio frequency-based techniques, and third optical sensing techniques. However, all of these have limitations regarding their applicability. The radar-based technique is an active technique that additionally requires relatively heavy and expensive sensors. It might have trouble detecting small objects, like drones, due to a low radar cross section.[1] Radio frequency–based techniques only work if drones are controlled manually, as they detect the communication link between the drone and operator.[2] Optical sensors have limited field-of-view and fail to work in weather conditions like rain and fog.[1]

Another way to localise drones is using the noise emitted by the drones, i.e., through passive acoustics. Acoustic sensors, like microphone arrays, are lightweight, cheap, and are able to sense in all directions.[3] Therefore, they can fill the gaps of previously mentioned techniques. This is beneficial in a real-world application for drone localisation, when, ideally, a combination of different sensors is used. Nonetheless, acoustic localisation of drones has limitations as well, such as comparably lower localisation ranges.[4] This reduces even further when the signal-to-noise ratio decreases.[1]

Two strategies can be used to overcome these limitations. First, a multitude of arrays can be positioned in and around areas of interest. By doing so, observations from different arrays can be combined and triangulated to increase the localisation range. Second, the measured microphone signals can be subjected to advanced signal processing. The purpose of this is to filter out undesired noise and focus on relevant information of the signal only.

The presented research concentrates on the second strategy, building upon previous studies that have shown the potential of this strategy. Most of the previous studies apply beamforming on a predefined grid of potential source locations,[5–11] as this technique is robust and intuitive.[12] However, these studies have not yet identified the full potential of array-based acoustic localisation. For example, Blanchard *et al.*[9] used time-domain beamforming instead of frequency-domain beamforming, therefore not optimally using the frequency content typical for drones. Furthermore, while some studies employ frequency-domain beamforming, they do so by choosing a broad range of frequencies, which again does not explore the use of different frequencies.[6,11] In addition, two studies do not test the localisation limits of the method or only perform indoor measurements.[9,10] Last, localisation of multiple sound sources is discussed in some studies;[5,8] however, this required an additional algorithm and assumptions on the number of sources. Nevertheless, the results show high

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potential for acoustic source localisation. Based on the previous studies, frequency-domain beamforming is selected for the presented study.

The aims of this paper are threefold. First, the current contribution considers a grid-free approach, which can easily be extended to a three-dimensional (3D) search, estimating the distance between the drone and the microphone array. A drone is usually flying in the far-field relative to the microphone array. Therefore, the sound waves arriving at the array are planar waves. Due to this, the distance cannot be estimated. Three-dimensional beamforming can be used to find the distance for situations with the drone in the near-field. This requires an exhaustive search in the 3D space. For this reason, a grid-free approach using differential evolution (DE), a variant of the well-known genetic algorithm, will be tested. The DE-based approach searches in a predefined solution space for a sound source location by minimising a cost function. This cost function can be designed as desired and all parameters that are affecting the output of the function can be estimated. It is thus possible to include the distance as unknown parameter. DE has already proven its potential for general applications of sound source localisation[13,14] and for drone localisation.[15] In this study, DE is used to jointly estimate the azimuth angle, elevation angle, and distance to a drone for a variety of drones.

The second goal is to investigate whether the use of relatively narrow frequency bands, which together span a large range of frequencies, can increase the localisation range while identifying and filtering out undesired noise sources. The rationale behind this investigation is that the analysis within a frequency band of limited width is anticipated to enhance the localisation range compared to a more broadband analysis, potentially dominated by contributions from other noise sources. Focus will be on identifying those frequency intervals that contain the most pronounced drone signal for this purpose. To aid this research, an extensive outside measurement campaign was executed and measurements of the noise from a variety of drones were taken. An important difference of this campaign compared to the previously mentioned ones is that data were collected, which include both a drone and aircraft flyover. This study is new in the sense that it shows the benefit of acoustic localisation in smaller frequency bands, but spanning a large frequency range, for discrimination of the different sound sources and as such, maximising the range for which the drone can be localised.

The third novelty of this study is that it obtains a realistic indication of the range limit for acoustic localisation for a variety of drones.

The remainder of this contribution is ordered as follows. Section 2 provides the details of the outdoor measurements. The principles of beamforming and DE are discussed in Sec. 3. Results and discussion are provided in Sec. 4. The paper is concluded in Sec. 5.

2. Experimental setup

The data used in this research was obtained during two different measurement campaigns. The first campaign took place at the Dutch military base "Luitenant-Generaal Bestkazerne." The second campaign took place at the Unmanned Valley, which is a field lab in The Netherlands for drone-related activities.

During both campaigns, a microphone array with 64 microphones (PUI Audio 665-POM-2735P-R, Dayton, OH) was used, with the microphones distributed in an Underbrink spiral configuration.[16,17] The dimensions of the array are 4×4 m and the microphone spacing varies from 0.11 to 3.4 m. Absorbing foam and windshields were used to minimise reflections and noise from wind, respectively. The sample frequency of the microphones was 50 000 Hz. The average background noise level during the first and second campaign was 52 and 51 dB, respectively.

Data from six drones are used. Five drones are quadcopters and one drone is an electric vertical take-off and landing quadplane. The quadcopters were flown during the first campaign at an altitude of 30 m and the quadplane was flown during the second campaign at an altitude of 60 m. Details of the drones and the wind speed are found in Table 1. The blade passing frequency (BPF) is determined using the power spectral density at the time instant for which the sound level was the highest.

For each drone, one flight is analysed, except for drone 1, from which two flights are analysed. Thus, in total, seven flights are analysed. Per flight, a 45 s segment is chosen in which the drone performs a flyover over the array. Last, the second flight of drone 1, as well as the flight of drone 6, contains significant background noise, as an aircraft flyover is present in the recordings.

Table 1. Details on drone numbering, type, takeoff weight, blade passing frequency, velocity, and wind speed.

Numbering	Drone type	Weight (g)	BPF (Hz)	Velocity (m/s)	Wind speed (m/s)
Drone 1 (two flights)	DJI Mavic 3	895	153	10	1–2
Drone 2	Autel EVO II	1191	177	10	1–2
Drone 3	DJI Mini 2	242	354	5	1–2
Drone 4	DJI Phantom 3	1216	171	10	5–7
Drone 5	DJI Phantom 4	1380	159	5	5–7
Drone 6	Avy Aera (Quadplane)	18 250	225	21	6

3. Methodology

This section describes the two localisation techniques considered, i.e., beamforming using a predefined grid of potential source locations and the grid-free DE approach.

3.1 Localisation based on grid search

Conventional frequency-domain beamforming is an often used acoustic imaging technique,[18] as it is robust and intuitive.[12] Although Merino-Martínez *et al.*[12] present a variety of high-resolution beamforming types, this study uses the conventional frequency-domain variant due to computational efficiency. Beamforming typically employs an exhaustive grid search. In this research, the grid is expressed in azimuth and elevation angles. The azimuth angle ϕ ranges from 0° to 360°, with a step of 1°. The elevation angle θ ranges from 0° to 90° with respect to the ground, with a step of 0.5°. The origin of the grid is the centre of the array. The array is placed on the ground with the plane of the array parallel to the ground.

We consider the Fourier transform of the pressure signals measured by the N microphones, contained in the complex pressure vector denoted as $\boldsymbol{p}$. This pressure vector is used to compute the cross-spectral matrix (CSM), $\boldsymbol{C}$,

$$\boldsymbol{C} = \mathbb{E}\left[\boldsymbol{p}\boldsymbol{p}^H\right], \tag{1}$$

where $(\cdot)^H$ is the Hermitian, or complex conjugate transpose, and $\mathbb{E}[\cdot]$ is the expectation operator. The CSM is often computed through averaging over several frames. Nevertheless, due to the non-stationarity of the drones, as well as the requirement to have sufficient frequency resolution, in this work, the CSM is computed using a single frame. In addition to the CSM, a steering vector is computed. The elements of this steering vector, $\boldsymbol{g}_j(f)$, for grid point, j, and frequency, f, are computed as

$$g_{n,j}(f) = e^{-2\pi i f r_{n,j}/c}. \tag{2}$$

In this equation, c is the speed of sound and $r_{n,j}$ is the distance between microphone n and grid point j. The distance $r_{n,j}$ is expressed in terms of azimuth ϕ and elevation θ angles as follows:

$$r_{n,j} = r_j\sqrt{1 + \left(\frac{r_n}{r_j}\right)^2 - \frac{2}{r_j}(x_n\cos\phi\cos\theta + y_n\sin\phi\cos\theta + z_n\sin\theta)}, \tag{3}$$

where x_n, y_n and z_n are the x-, y- and z-coordinate of microphone n, r_n is the distance from the origin to microphone n, and r_j is the distance from the origin to grid point j. It should be noted that the exact value of r_j is unknown. However, in this study, the exact value is not relevant. This is because for most of the measurements and frequencies considered, the drones are flying in the far-field of the array. The wave fronts intercepted by the array can be considered planar, i.e., the phase variation over the microphones is independent of the distance between source and array. The distance does influence the received sound level, but for localisation purposes, this is not important. To confirm this, a variety of values for r_j have been tested, ranging from 0 to 200 ms and the difference in results was found to be negligible.

After computing the steering vector and CSM, the beamform output is determined for each grid point and frequency, f, as

$$B_j(f) = \frac{\boldsymbol{g}_j^H \boldsymbol{C} \boldsymbol{g}_j}{||\boldsymbol{g}_j||^4}. \tag{4}$$

This is according to the formulation of Merino-Martínez *et al.*[12] where the denominator ensures independence of the number of microphones. A total of 12 frequency bands with a width of 200 Hz are chosen and defined as: [0–200, 200–400,…, 2000–2200, 2200–2400]. It is expected that for higher frequencies, the localisation performance is limited due to atmospheric attenuation. In each band, the two frequencies with the summed highest level over all microphones after the Fourier transform are selected for beamforming. This amount of frequencies was chosen after testing different values, due to a combination of good performance and low run time. Both beamform outputs are then averaged incoherently. Finally, the grid point with the maximum beamform output is taken as the location of the drone for the frequency band under consideration. This process is repeated for each individual signal block of 2500 samples with overlap of 50%, leading to a time step of 25 ms.

3.2 Localisation using DE

In addition to beamforming on a grid, this study also explores differential evolution for drone localisation. DE is a variant of the genetic algorithm.[19] A population of candidate solutions is initialised randomly within a solution space. Each candidate solution is a vector containing all unknown parameters. These candidates evolve over generations toward a final, optimised solution. How they evolve is based upon a cost function. The objective of DE is to minimise this cost function. Throughout the generations, the candidate solutions thus probe the solution space and based upon the computed cost converge toward a solution with minimum cost. The cost function is designed according to the Bartlett cost function.[13] It is formulated as the negative value of Eq. (4), as the DE implementation searches for a minimum

$$E_{Bartlett}(f) = -B_j(f). \tag{5}$$

There is no limit on the number of parameters that can be estimated as long as they affect the value of the cost function. Consequently, two potential problem definitions are tested in this study. The first one includes azimuth angle ϕ and elevation angle θ. The second one also includes the distance r_j. Thereby, the effects of including the distance in the optimisation on the localisation performance can be examined.

The settings for DE in this research are as follows: The solution space for the azimuth angle is defined between 0° and 360°, for the elevation angle between 0° and 90°, and for the range between 0 and 200 m. The size of the population is 12 and the number of generations is 100.

4. Results and discussion

In this section, the performance of grid-based beamforming and grid-free DE are compared. Furthermore, by examining the two flights with high levels of background noise in further depth, the significance of the localisation for individual frequency bands, with the bands spanning a large range of frequencies, is demonstrated.

4.1 Beamforming and DE performance comparison

The localisation performance is assessed by determining the maximum achieved localisation range. This is the maximum distance at which the azimuth and elevation angle of the drone can be estimated correctly, based on their agreement with the drone global positioning system (GPS). The localisation range is determined for all seven flights, for all 12 frequency bands, and for azimuth and elevation direction separately. This is done for three techniques: grid-based beamforming, DE with angle estimates only, and DE with angles and range estimates. For each drone, the maximum possible localisation range is established by comparing the accomplished ranges for all techniques and frequency bands. Table 2 provides a summary of the results, including the technique and frequency band that provide the maximum range for both directions.

Table 2 clearly shows that beamforming and DE perform similar. Not one technique outperforms the other. In fact, beamforming achieves six times the best performance whereas DE achieves eight times the best performance. On top of this, the run times of the three approaches differed minimally. It took approximately 9 min to execute each approach on a basic desktop computer with 64 GB of random access memory (RAM) and an Intel(R) (Intel Corp., Santa Clara, CA) Xeon(R) central processing unit (CPU) E5-1620 v3 processor. However, this could have been reduced for the DE techniques by optimising the number of generations, while it could have been reduced for beamforming by optimising the matrix operations and coarsening the grid. The latter, however, will result in a decrease in performance accuracy.

Furthermore, none of the DE approaches performs better than the other, which is a beneficial result. It indicates that performance does not deteriorate even if additional parameters are solved within the DE problem statement. As additional parameters give extra information, it is useful to include more parameters. The range should therefore always be included when estimating the position of the drone using DE. The actual performance of the range estimates will be discussed later.

4.2 Frequency spectrum analysis

The frequency bands yielding the longest localisation range vary a lot according to Table 2. There are two reasons for this spread. The first one is the frequency content of the drone signal. As shown in Table 1, the blade passing frequency is different for each drone. Furthermore, while the velocity of the drones was assumed to be constant in this study; this is

Table 2. Maximum achieved localisation range for the seven analysed flights in both azimuth and elevation direction, including the frequency band and technique. B, beamforming; DE_A, differential evolution angle only; DE_{AR}, differential evolution angle and range; DE, both DE approaches; Max. maximum.

Flight	Max distance azimuth (m)	Technique	Frequency band (Hz)	Max distance elevation (m)	Technique	Frequency Band [Hz]
Drone 1 Flight 1	93	DE_{AR}	200–400	92	BF	200–400
Drone 1 Flight 2	110	DE_A	1000–1200	109	DE_A	1400–1600
Drone 2	126	DE_A	1400–1600	126	DE_A	1400–1600
Drone 3	103	DE	1000–1200	103	DE	1000–1200
Drone 4	133	BF	600–800	102	BF	400–600
Drone 5	108	BF	1000–1200	107	DE_A	800–1000
Drone 6	360	BF	1800–2000	360	BF	1800–2000

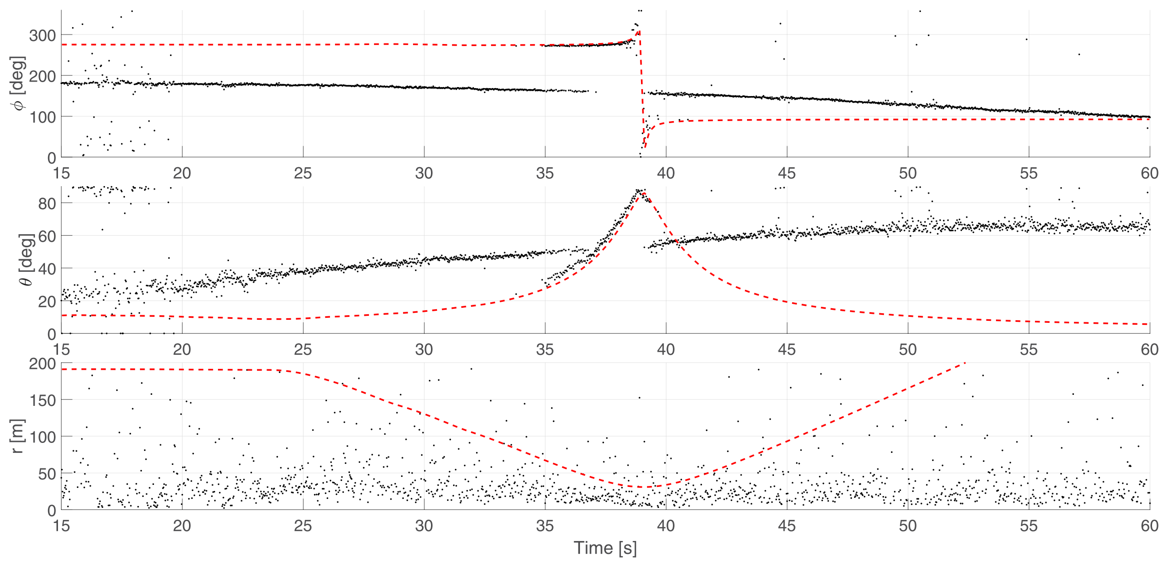

Fig. 1. Estimates of the azimuth and elevation angle and range using differential evolution for drone 1, flight 2 in the frequency band 200 - 400 Hz (dots). The dashed line indicates the drone location relative to the array according to the drone GPS.

typically not the case, and the BPF changes accordingly. Therefore, it is essential to look at different frequency bands, as it is unknown in which band the BPF or a higher harmonic is present.

The second reason is the frequency content of the background noise. To demonstrate this, Figs. 1–4 are included. These figures show the location estimates over time using DE for the drone flyovers of drone 1 (second flight) and drone 6.

Figure 1 shows the estimates in the frequency band from 200 to 400 Hz for the second flight of drone 1. It is clearly visible that the aircraft flyover dominates the estimates. The drone can only be localised when the range to the array is low, otherwise the noise of the aircraft overpowers the noise of the drone. A similar observation is visible in Fig. 3, which shows the estimates for the same frequency band for drone 6. This is especially true in the phase where the drone flies toward the array, from 120 to 140 s. It is clear that in this frequency band, the estimates of the drone position are accompanied by estimates for the aircraft position.

On the other hand, the aircraft localisations are completely eliminated when examining higher frequencies. Figure 2 shows the estimates for drone 1, flight 2, in the frequency band of 1000–1200 Hz and Fig. 4 shows the estimates for drone 6 in the frequency band of 1800–2000 Hz. In both plots, no sign of the aircraft is visible. At the same time, both figures show better estimates especially from 40 to 45 s and 145 to 155 s for drone 1 and drone 6, respectively.

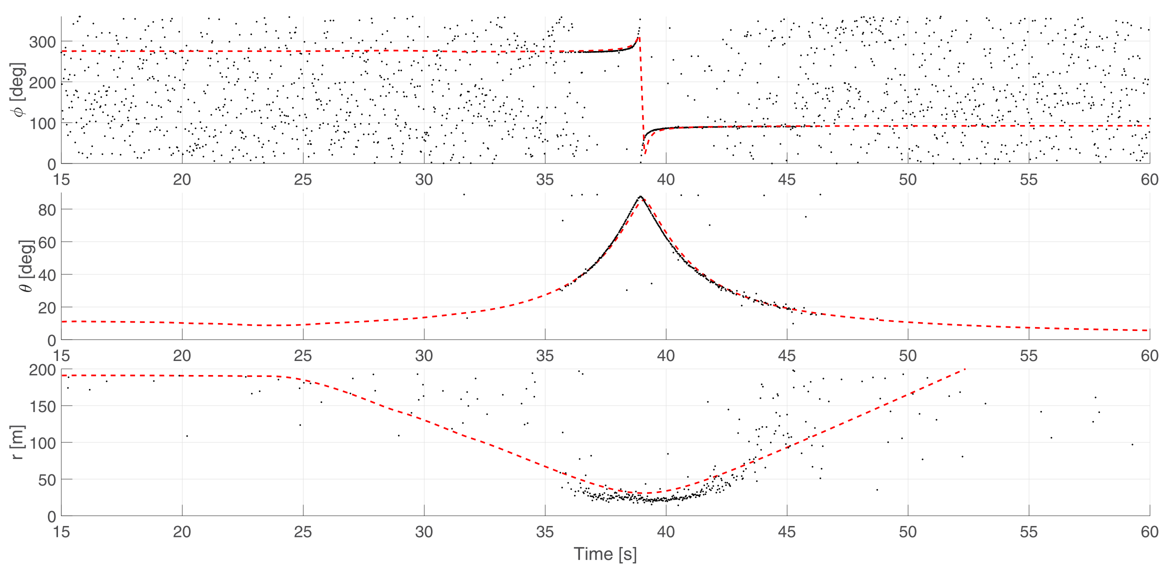

Fig. 2. Estimates of the azimuth and elevation angle and range using differential evolution for drone 1, flight 2 in the frequency band 1000 - 1200 Hz (dots). The dashed line indicates the drone location relative to the array according to the drone GPS.

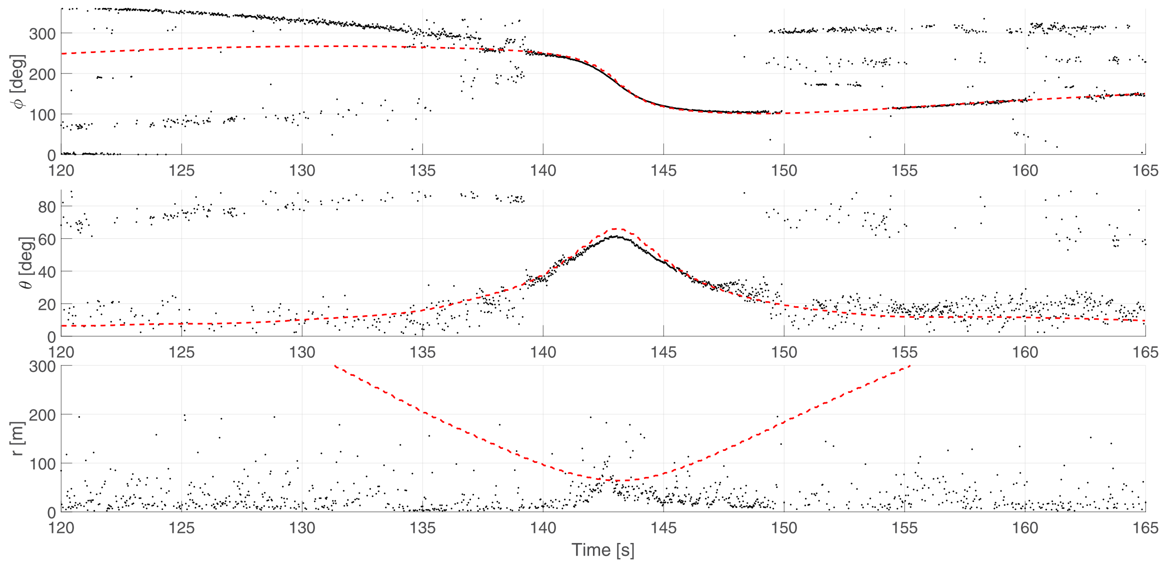

Fig. 3. Estimates of the azimuth and elevation angle and range using differential evolution for drone 6 in the frequency band 200 - 400 Hz (dots). The dashed line indicates the drone location relative to the array according to the drone GPS.

The results of the different frequency bands show the essence of frequency band analysis. As shown in this research, the benefit of using bands of 200 Hz is that different bands show different sound sources. It is, thus, not only possible to localise a drone, but also other surrounding noise sources. The frequency range in which a sound source is visible, as well as noise metrics such as the sound level, can provide information on the type of sound source.

Finally, the four figures give insight in the performance of the range estimates of DE. Only Fig. 2 shows a correct estimate for a short period when the drone is at its closest to the array, about 40 m away. According to the Fresnel distance,[20] the drone is flying in far-field from 40.4 m. Similar results were observed for the other flights. It can therefore be concluded that DE can estimate the range correctly if the drone is in the near-field compared to the array.

5. Conclusions

The first aim of this paper was to compare the localisation performance of grid-based beamforming and grid-free DE for the purpose of drone localisation. It can be concluded that both approaches perform similarly. Furthermore, if the range is estimated alongside the azimuth and elevation angle, the performance of DE remains the same. If the drone is flying in

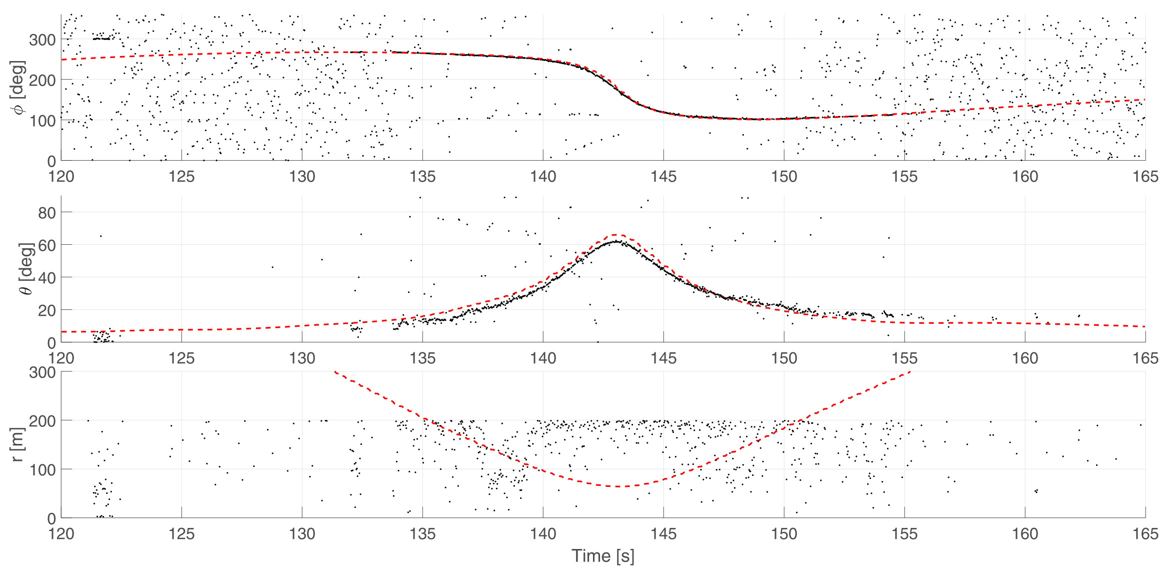

Fig. 4. Estimates of the azimuth and elevation angle and range using differential evolution for drone 6 in the frequency band 1800 - 2000 Hz (dots). The dashed line indicates the drone location relative to the array according to the drone GPS.

the near-field of the array, the range can be estimated correctly. Nevertheless, it can be concluded that it is always beneficial to estimate the range when using DE, as the performance for the angle estimates does not deteriorate. Another aim is to show the importance of frequency band analysis. It can be concluded that it is essential to cut the frequency-domain into smaller bands while ensuring that these bands jointly span a large range of frequencies to discriminate between drones and background noise and to account for the variation in sound signatures among different drones. Finally, localisation ranges are found to range from several tens of meters to hundreds of meters. For many applications, a distributed network of arrays will be needed to ensure timely or continuous localisation of drones.

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Author Declarations

Conflict of Interest

The authors have no conflicts to disclose.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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