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## An Experimental Perspective

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# Exploring the Real-World Challenges and Efficacy of Internal Coupling in Metastructures: An Experimental Perspective

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***Abstract*—Metastructures with internally coupled resonators promise enhanced vibration control and energy harvesting capabilities by theoretically enabling multiple bandgaps. This paper investigates the feasibility of these theoretical benefits under practical constraints, particularly the challenge of merging multiple bandgaps in continuous systems. Employing a closed-form analytical approach alongside FEM simulations and experimental validation, the study reveals that while internal coupling can modify bandgap behavior, achieving precise stiffness alignment and bandgap merging remains challenging. The findings indicate that practical applications may not fully realize the predicted advantages and also present more challenges in merging multiple bandgaps created in such metastructures, even for metastructures with advanced manufacturing precision and design optimization. The paper contributes to the understanding of the dynamic behavior of internally coupled metastructures and outlines directions for future research to bridge the gap between theory and application.**

***Index Terms*—Locally Resonant Metastructure, Experimental Exploration, Bandgap Engineering, Internal Coupling, Vibration Suppression**

## I. Introduction

Recent advancements in the field of metastructures have opened new frontiers in materials science, particularly in the realms of vibration suppression and energy harvesting. They have garnered substantial research attention due to their potential in low-frequency applications at scales smaller than the wavelength. However, a major limitation is the relatively narrow widths of their band gaps, which restricts their utility in environments experiencing broadband spectrum vibrations. To address this challenge, researchers have investigated various strategies to broaden the band gap of metastructures. These strategies include the creation of novel structural configurations aimed at producing multiple band gaps [1], combining Bragg Scattering and local resonance band gaps [2], and incorporating nonlinearities for broadband capabilities [3].

Among the various innovations, the concept of internal coupling within resonators presents a promising avenue for enhancing the performance of these structures. Theoretically, internal coupling facilitates the formation of multiple bandgaps, potentially broadening the bandgap width and offering substantial benefits in terms of energy dissipation and conversion efficiency [4, 5].

This phenomenon has been highlighted in seminal works, such as those by Hu et al. [6], who have demonstrated the potential of internal coupling on the dynamic properties of metastructures. Li et al. [7] have demonstrated the coherent internally coupled distant magnonic resonators via superconducting circuits, for integrated magnonic networks that can operate coherently at quantum-compatible scales. Oyelade and Oladimeji [8] also contributed by introducing a novel metastructure with a multiresonator mass-in-mass lattice system, where the internal coupling was through a linear spring, leading to the formation of two additional bandgaps over conventional designs.

A wider bandgap allows resonators to operate over a broader frequency range, enhancing their effectiveness. Integrating sensors like piezoelectric devices can transform this mechanical energy into electrical energy, increasing the efficiency of energy harvesting. Thus, a wider bandgap not only improves vibration control but also enhances the metastructure's energy harvesting capabilities.

Despite theoretical advancements and computational validations, a significant gap remains in the experimental investigation of metastructures with internal coupling. The theoretical benefits of such designs, including enhanced vibration suppression and energy harvesting, rely on precise internal coupling mechanisms which, if not accurately implemented, may not yield the expected performance improvements in practical settings. Moreover, the real-world applicability of merging multiple bandgaps to extend the bandgap width remains underexplored, leaving unanswered questions about the feasibility and effectiveness of these advanced metastructural designs under operational conditions.

Addressing these challenges requires a focused investigation into the practical implementation of internally coupled resonators within metastructures. This study aims to bridge the gap between theoretical predictions and experimental realities, offering insights into the challenges of realizing the proposed benefits of internal coupling in metastructural designs. By examining the limitations and potential discrepancies in the performance of these structures, this research contributes to a deeper understanding of the factors that influence the efficacy of metastructures in achieving desired vibration control and energy harvesting outcomes.

This work asserts that the theoretical benefits of using internally coupled resonators in metastructures, such as enhanced vibration suppression and energy harvesting, are currently limited by practical constraints in bandgap creation mechanism.

The primary contributions of this paper are summarized as follows:

- Utilizes a closed-form formulation for analyzing bandgaps in metastructures with internally coupled resonators, moving away from traditional methods like Bloch theory and dispersion curve analysis;
- Illustrates that the bandgap observed in experiments aligns with those in standard metastructures, underscoring its importance for structural dynamics and wave manipulation;
- Demonstrates the practical challenges associated with achieving the theoretical benefits of internally coupled resonators in metastructures, as discussed by researchers in earlier studies;
- Provides empirical evidence on the difficulties of merging multiple bandgaps to increase the overall bandgap width in continuous metastructures with internally coupled resonators;
- Analyzes the effectiveness of internally coupled resonators in real-world applications, questioning the practicality of their implementation for vibration suppression and energy harvesting;
- Offers insights into the limitations and considerations necessary for the successful application of internally coupled resonators in distributed or continuous systems.

The remaining sections of this paper are as follows:

The structure of the remainder of this paper is organized as follows: In Section 2, the Methodology is presented, outlining the experimental design, analytical techniques, and the steps taken to investigate internal coupling effects in metastructures. Section 3, Results and Discussion, presents the data from the experiments and FEM simulations, analyzes the dynamic behavior of metastructures with varying coupling, and assesses the findings against the backdrop of existing theories and their practical implications. The final section, Section 4, Conclusion, encapsulates the main discoveries, situates them within the broader research landscape, and proposes for future inquiry.

## II. METHODOLOGY

This section outlines the experimental setup designed to investigate the creation of band gaps and their impact on the dynamic behavior of metastructures. The methodology is bifurcated into two primary investigative thrusts: firstly, to validate the theoretical predictions concerning the standard metastructure through tangible observations; and secondly, to delineate the practical challenges and limitations inherent in the implementation of internally coupled resonators within real-world applications.

Before diving into the experimental setup, it's crucial to understand the theoretical underpinnings that guide the investigation of mechanical metastructures and their dynamic behaviors. Mechanical metastructures with local resonances manipulate wave propagation and dynamic responses by combining structural modes and integrated resonators. The modeling methodology employs modal analysis within a distributed parameter model to accurately describe the intricate interplay of mass, damping, and stiffness. The expression, which represents the transfer function of the m-th mode in response to an external force, is detailed as [9]:

$$\frac{Z_m(s)}{Q_{b_m}(s)} = \frac{1}{s^2\left(1+\frac{\mu(2\zeta_r\omega_r s+\omega_r^2)}{s^2+2\zeta_r\omega_r s+\omega_r^2}\right)+2\zeta_m\omega_m s+\omega_m^2} \quad (1)$$

Here, $s$ is the Laplace transform's complex frequency variable, $Z_m(s)$ denotes the transverse vibration displacement of the main structure, $Q_{b_m}(s)$ represents the Laplace Transform of the external force applied to the m-th mode of the main structure, and the parameters $\mu$, $\zeta_r$, $\zeta_m$, $\omega_r$, and $\omega_m$ represent the mass ratio, damping ratios, and natural frequencies of both the resonators and the modal structure. This equation highlights how resonator properties impact the metastructure's resonance behavior, allowing for the design of systems with desired dynamics, such as band gaps for wave control and vibration suppression.

Ignoring the damping ratios of both the structure and the resonator, the transfer function in (1) features two poles at the origin, reflecting the fundamental response dynamics of the system, and reveals a bandgap within the frequency spectrum due to its unique pole-zero configuration. Specifically, it features zeros at $s = \pm i\omega_r$ and poles at $s = \pm i\omega_r\sqrt{1+\mu}$, with a second-order pole at $s = 0$. This arrangement ensures that no poles between $\omega_r$ and $\omega_r\sqrt{1+\mu}$, defining a bandgap in $\omega_r < \omega < \omega_r\sqrt{1+\mu}$. The transfer function formulation represents a shift from traditional metastructure analysis methods like Bloch theory towards a more practical approach. By developing a closed-form solution that leads to a transfer function model, makes it easier to tailor these structures for specific applications by directly relating input forces to system behavior, offering a useful tool for engineers. Although the approach enhances practical analysis, it's an evolutionary step in metastructure research, focusing on application rather than theoretical novelty.

### A. Experimental Setup

Experimental investigations were carried out using a carefully designed cantilever beam arrangement, aiming to provide empirical validation for the theoretical findings discussed earlier and the numerical ones discussed later.

*1) Standard Metastructure Validation:* For the initial phase of the experiment, a standard metastructure prototype without internal coupling was constructed to serve as a baseline. The beam's dimensions and material properties are specified in Table I. The experiment was conducted using a cantilever beam setup (see Fig. 1). The cantilever beam was fabricated from aluminum and had the following dimensions: 3mm thickness, 4cm width, and 0.91m length. To adjust the resonator's natural frequency, a nut and screw with a combined mass of 19 grams were attached to the tip of the beam. This modification successfully achieved a natural frequency of 64 Hz. For generating base motion, we utilized a 100N TIRA 51110 Shaker. Acceleration at the beam tip was measured using a Dytran Accelerometer 3055D21, which has a sensitivity of 100 mV/g. To measure transmissibility, another accelerometer of the same model was positioned at the base of the cantilever beam. The input signals to the shaker were amplified using the Power Amplifier BAA 120. The Vibration Controller VR9500 was employed for base control and monitoring of vibrational inputs and responses throughout the experiment.

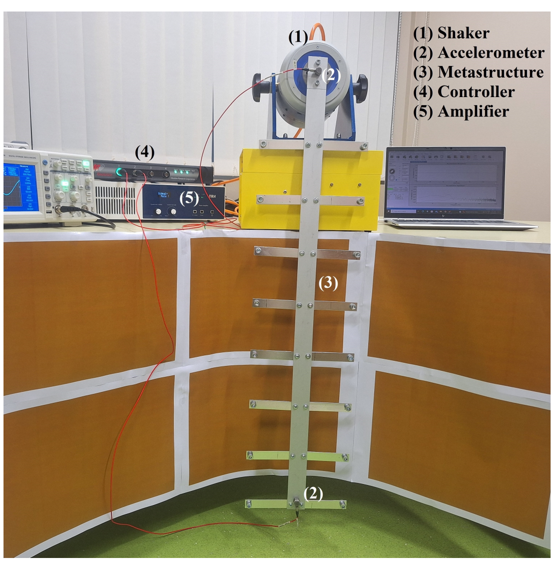


Fig. 1. Experimental setup of a metastructure prototype equipped with 8 resonators, each fine-tuned to a 64 Hz natural frequency using adjustable mass at the tip. Measurement accuracy is ensured with two Dytran Accelerometers, model 3055D2, linked by low-resistance, high-fidelity wires.

*2) Investigating Internally Coupled Resonators:* Furthermore, an additional experiment involving the metstructure with internally coupled resonators is detailed in Fig. 2. Each resonator was meticulously crafted and integrated into the metastructure, with particular attention paid to the precision of internal coupling to examine its impact on the system's dynamic response. In this setup, each internally coupled resonator is composed of pure aluminum, featuring a thickness of 2 mm, a width of 20 mm, and a length of 11.3 mm.

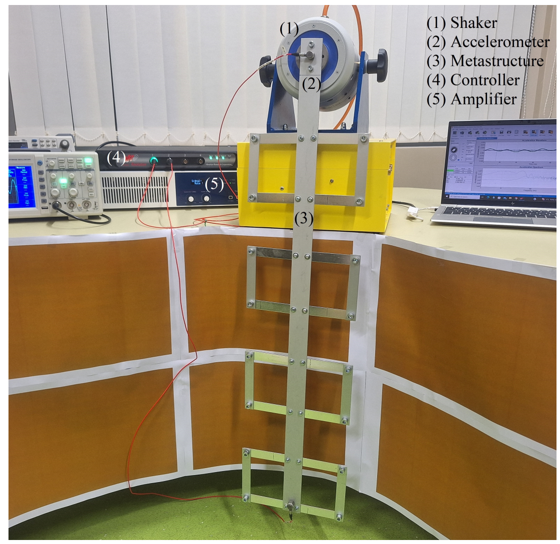


Fig. 2. Experimental Design for a Prototype Metastructure Comprising Four Unit Cells of Internally Linked Resonators, Constructed from Pure Aluminum. Each Coupled Beam has a Thickness of 2 mm, a Width of 20 mm, and a Length of 113 mm.

TABLE I.
EXPERIMENTAL PARAMETERS

| Symbol | Parameter | Value |
|---|---|---|
| $L$ | Length of the beam | 91 cm |
| $b$ | Width of the beam | 4 cm |
| $h$ | Thickness of the beam | 3 mm |
| $E$ | Young's modulus of the beam | 70 GPa |
| $\rho$ | Density of the beam | 2700 kg/m$^3$ |
| $\omega_r$ | Resonator's natural frequency | 64 Hz |
| $\omega_{r_\kappa}$ | Coupled resonator's natural frequency | 85 Hz |
| $N_r$ | Number of Resonators | 8 |

*3) Standard Metastructure Transmittance Measurements:* The experimental results displayed in Fig. 3 demonstrate the transmittance spectrum, which corresponds to the resonant frequencies of the standard metastructure with $\mu = 1.2$. The regions of low transmittance, which signify the bandgaps, commence at a frequency of $\omega_r = 64$ Hz, in line with theoretical predictions. Additionally, the observed width of the bandgap is consistent with the anticipated value of $(1+\mu) = 2.2$. This data shows the existence of a bandgap between frequencies $\omega$ and $\omega_r\sqrt{1+\mu}$, corresponding to the calculated bandgap limit of $\sqrt{1+\mu} = 1.484$. This observation confirms the presence of the primary bandgap, illustrating the dynamic behavior of the system across the spectrum.

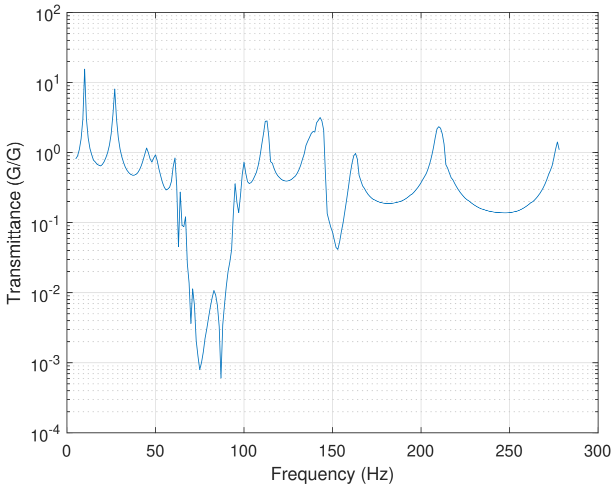


Fig. 3. Experimental transmittance data versus excitation frequency for the metastructure, with $\mu = 1.2$. The plot highlights the bandgap region between 64 to 95 Hz, which corresponds to the theoretical bandgap boundary $\omega_r\sqrt{1+\mu}$.

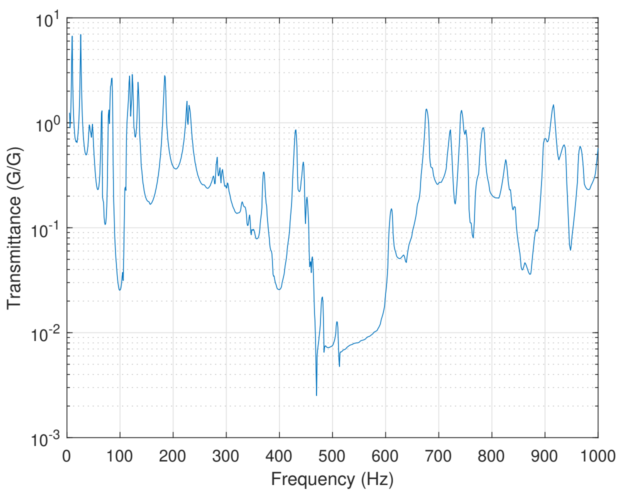


Fig. 4. Experimental transmittance results for the metastructure with internally coupled resonators. The first bandgap is observed between 90-110 Hz. This shift can be attributed to the enhanced stiffness of the unit cell, which is composed of a pair of resonators. Measurement devices are located at the base and the tip of the metastructure to capture the full spectrum of its response.

*4) Internally Coupled Metastructure Transmittance Measurements:* While the experimental outcomes for the standard metastructure corroborated the theoretical forecasts, the scenario markedly diverged with the introduction of internally coupled resonators. Fig. 4 encapsulates the experimental transmittance data, evidencing a distinct behavioral pattern for the metastructure endowed with internal coupling mechanisms. The manifestation of the initial bandgap at 85 Hz, slightly higher than what is observed in the standard metastructure (Fig. 3), aligns with the natural frequency of the coupled resonators. This is indicative of increased stiffness within the unit cell, a direct result of the resonators' collective configuration.

This measurement uncovers a scenario marked by chaos and irregularities in the transmittance spectrum, diverging from the uniform patterns expected based on theoretical projections by researchers in earlier studies, as highlighted in the introduction. Such manifestations underscore the sensitivity of the metastructure's dynamic behavior to the precise integration and configuration of internally coupled resonators, highlighting the challenges inherent in translating theoretical advantages into practical applications. It raises questions about the practical realization of internally coupled resonator benefits, such as significant bandgap widening or enhanced energy dissipation. The results imply that while the concept of internal coupling holds promise in theory, the transition to tangible applications faces challenges that may limit the effectiveness of such designs in real-world vibration control scenarios. Further investigation and refinement of the metastructure design and manufacturing processes are necessary to harness the full potential of internal coupling in metastructures for practical vibration suppression and energy harvesting applications. This claim is further supported by FEM analysis in the following section.

## III. Finite Element Study

Following the experimental investigation, the focus moves to Finite Element Method (FEM) simulations, aimed to offer an analytical view complementary to the experimental insights, especially concerning metastructures with internally coupled resonators. This shift towards numerical modeling serves as a crucial phase in corroborating experimental findings, with the primary aim of substantiating the observed behaviors in experiments, thereby deepening our comprehension of the metastructure's dynamic characteristics.

### A. *Observations from FEM Analysis:*

Fig. 5 depicts the transmissibility across different internal coupling stiffness, $\kappa$, as a function of normalized frequency. These results highlight the appearance of a pronounced second bandgap at a specific internal coupling stiffness, $\kappa$, matched to the resonator's stiffness ($\kappa/\omega_r = 0.003$), pinpointing this condition as essential for optimal bandgap definition (see bottom left corner subplot). Such precise matching between the internal coupling and resonator stiffness is key to achieving the desired dynamic behavior in the metastructure.

However, deviations from this optimal $\kappa$ value lead to significantly disordered responses, underlining the metastructure's acute sensitivity to variations in internal coupling stiffness. Such behavior showcases the challenges associated with achieving and maintaining this precision in stiffness alignment in practical applications. The observed irregularities and chaotic dynamics for non-optimal $\kappa$ values highlight potential difficulties in predictability and replicability of the metastructure's performance in real-world settings.

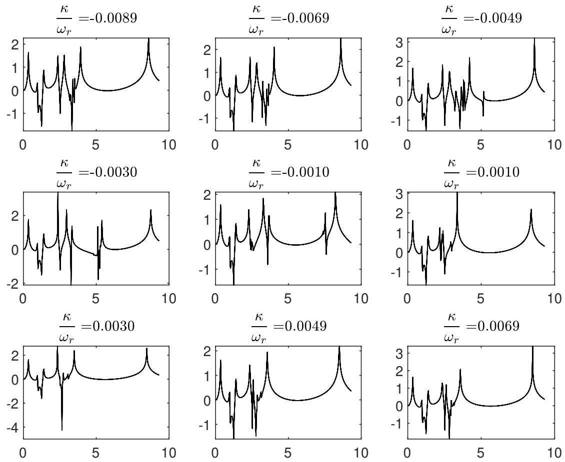


Fig. 5. Transmissibility of a cantilever beam for varying stiffness ratios $\kappa$, showing system sensitivity and its effects. Subplots detail responses at different $\kappa/\omega_r$ ratios, highlighting a critical condition at $\kappa = k_r$ in the bottom left corner subplot for optimal internal coupling. The y-axis is absolute displacement of beam tip to base displacement, $\ln |w_a(L)/w_b|$, and the x-axis is normalized frequency $\omega/\omega_r$.

Figure 6 confirms the importance of precisely tuning the internal coupling stiffness $\kappa$ in metastructures to achieve effective vibration isolation. The contour plot shows significant transmittance variations and bandgap formations, represented by cooler colors, which are crucial for blocking wave propagation. This visualization emphasizes the need for meticulous parameter optimization, as even small deviations from the ideal stiffness ratio can substantially alter the system's behavior.

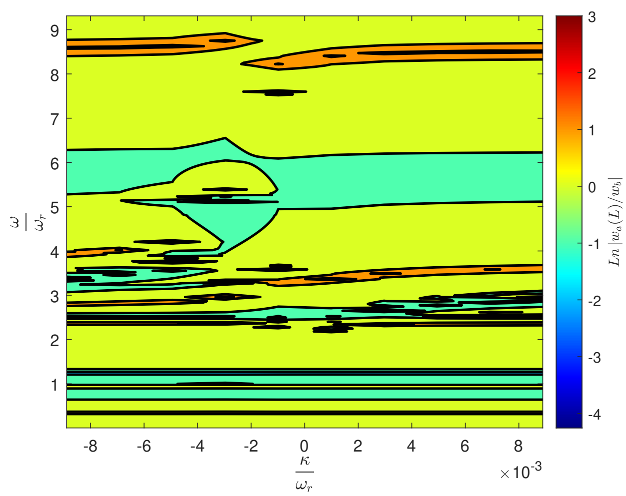


Fig. 6. Contour plot of transmittance for varying $\kappa$, illustrating the frequency-dependent formation and bifurcation of the bandgap. The plot captures the perturbations and potential destabilization inherent to varying internal coupling stiffness, underscoring the need for precise $\kappa$ calibration.

Achieving the exact bandgap properties requires careful adjustment of the metastructure's internal stiffness. While analytical models predict clear transitions and bandgap formations, the observed data might show more gradual changes and less distinct boundaries between bandgap regions. This disparity highlights the challenges in translating theoretical models into experimental or real-world scenarios. The irregularities and variations presented in the FEM results underscore the imperative for experimental studies to authenticate and fine-tune the theoretical models, thereby confirming their relevance and effectiveness in real-world applications.

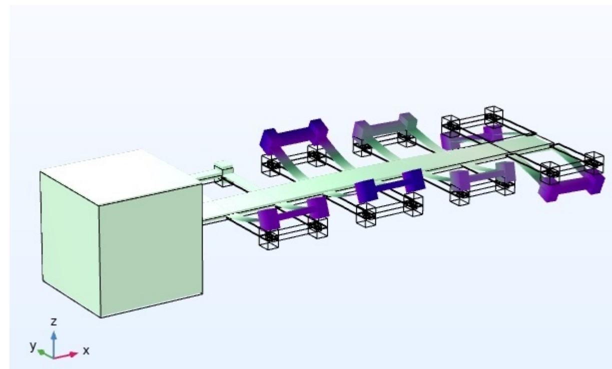

Fig. 7. Metastructure with internally coupled resonators configured as unit cells.

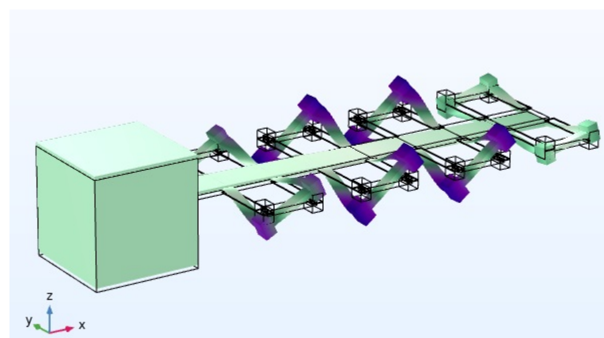

Fig. 8. Visual representation of the mechanism leading to the second bandgap in the internally coupled metastructure.

In Figs. 7 and 8, an internal coupling mechanism is demonstrated at work within the metastructure, where each pair of resonators acts as a unit cell. Fig. 7 demonstrates the initial out-of-plane oscillations that give rise to the primary bandgap, analogous to the behavior observed in conventional metastructures. As the excitation frequency increases, the system temporarily reverts to normal vibrational modes before encountering a specific frequency where the resonators within each cell commence vibration in opposing directions. This antiphase motion, depicted in Fig. 8, signifies the onset of the secondary bandgap due to the stiffness matching between the resonator and the internal coupling. However, the crucial insight is that despite the exact stiffness alignment (resonators and internal couple stiffness), merging the primary and secondary bandgaps to expand the bandgap width is not feasible. The inherent nature of the secondary bandgap's formation in such metastructures prevents the amalgamation of multiple bandgaps, thus questioning the practical application of internally coupled resonators in continuous metastructures.

The experimental outcomes for standard metastructures aligned well with theoretical predictions, confirming the models' reliability. However, experiments with internally coupled metastructures revealed discrepancies, emphasizing the need for precise matching of internal coupling and resonator stiffness ($\kappa$ to $k_r$). Practical implementation faced challenges due to manufacturing limitations. Additionally, attempts to merge multiple bandgaps into a broader one were hindered by the inherent characteristics of the second bandgap, questioning the feasibility of using internally coupled resonators in practical applications.

## IV. Conclusion

This paper has addressed the practical implications of utilizing internally coupled resonators within continuous metastructures for enhanced vibration suppression and energy harvesting. Through the experimental validation, and Finite Element Method simulations, we have illuminated both the potential advantages and the notable challenges posed by the implementation of internal coupling mechanisms. While the pioneering research by Hu et al. [6], and related studies [10], have highlighted the theoretical benefits of internal coupling in creating secondary bandgaps and boosting energy harvesting efficiency, our findings underscore the difficulties faced when translating these concepts into practical applications. The challenges identified, such as the precision required in assembly and the limitations in merging multiple bandgaps, were substantiated through experimental observations and reinforced by FEM analysis, revealing a nuanced understanding of the real-world applicability of internally coupled resonator metastructures.

The contributions of this paper are:

- Demonstrated the practical challenges in implementing internally coupled resonators within continuous metastructures.
- Provided evidence that achieving the theoretical benefits of such systems is non-trivial and highly sensitive to precise manufacturing and assembly conditions.
- Shown that while internal coupling can indeed create additional bandgaps, merging these to broaden the overall bandgap width remains problematic due to inherent structural behavior.
- Confirmed that despite identical stiffness of resonators and internal coupling, the anticipated increase in bandgap width may not be practical for real-world applications, as evidenced by experimental and FEM analysis.
- The exploration of internal coupling in metastructures presents a unique case study of how advanced material concepts transition from theory to practical realization. While our findings have highlighted several limitations, they also pave the way for future research opportunities.

Future research could focus on developing new manufacturing techniques or material configurations that mitigate the current limitations. Innovations in precision engineering and design optimization may hold the key to successfully harnessing the full potential of internally coupled resonators. Further studies could also explore alternative mechanisms for bandgap manipulation that may offer more practical and flexible solutions for real-world applications.

In closing, we must acknowledge that while our findings are promising, they are not without their caveats. The results should not be overinterpreted as the complexities of real-world applications may yield different outcomes. Additionally, while our data is robust, we caution against speculation and inflation of these results. We must recognize the limitations of our current study and refrain from drawing conclusions not fully supported by the data. Instead, we should consider these findings as stepping stones towards more comprehensive and applied research in the field of metastructures.

## Acknowledgment

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