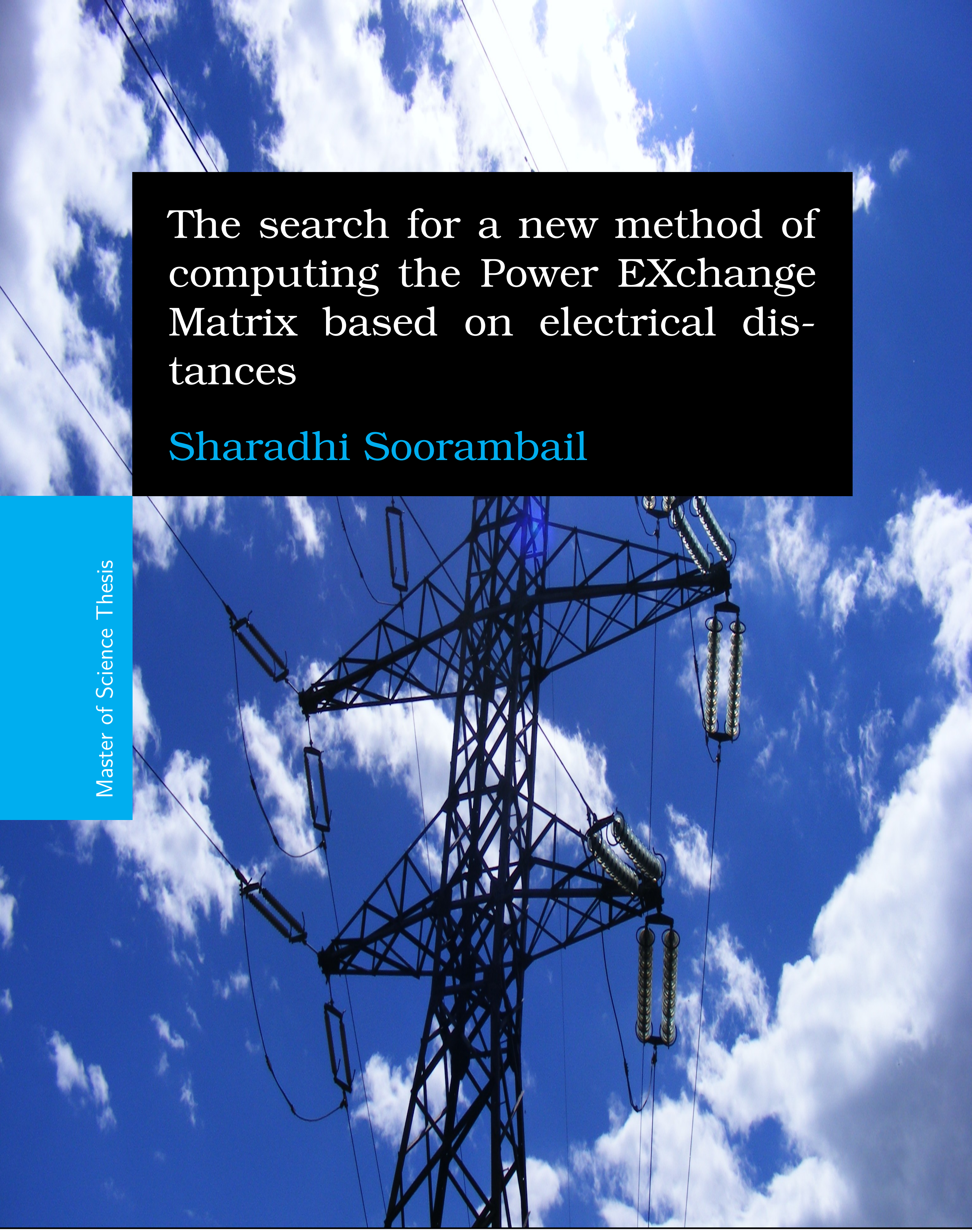

The search for a new method of computing the Power EXchange Matrix based on electrical distances

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by
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Abstract

Despite the uncertainties on the generation end and the consumption end, the transmission network thrives to be reliable. This includes its effort to predict the power flows in its lines and maintain the flows under a safety margin. Due to the said uncertainties and approximations, predicted power flows in the lines can exceed the acceptable limits. Under such a scenario, the responsible TSO has to take actions to ensure that this predicted violation won't occur in the reality. It needs to be noted that the flow in the line that mandated these actions could be caused partly by power exchanges outside the borders of the TSO. The responsible TSO bears the costs of these actions initially. The incurred expenses can be split in a fair manner by partitioning the power flow in the line into power flows caused by the power exchanges of all zones.

Full Line Decomposition(FLD) is an application developed by the FB4INV team at TenneT that calculates the power flow partitions. To compute the power flow partitions, it requires all power exchanges occurring in the system. The existing method of computing power exchanges(called the Bialek method) models the power network as a directed graph based on power flows in the branches. It traces the flow of power from the generator to the load(or vice-versa) along the paths available. If a power flow path is not available due to the directed modelling, an error of zero power exchange is introduced.

There exists multiple ways of computing the power exchanges. This paper attempts to find an unquestionable method. A method of computing power exchanges based on electrical distance reflects the behaviour of the power system in distributing flows. Through the literature review, the superposition method was found that makes use of circuit theory to find generator-to-load power contribution. In the course of the thesis, the superposition method was found to have a flaw in the equations. A new approach was developed in this thesis where power exchange is computed by defining it as an optimisation problem. The objective function of the optimisation problem reflects the excepted behaviour of the power system. The optimisation process was implemented using a genetic algorithm. The existing and developed methods were tested on an IEEE 30 bus system.

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Definitions

Electricity market related terminologies

1. **Individual Grid Model(IGM):**

   A data set describing power system characteristics (generation, load, grid topology and electrical characteristics) prepared by the responsible TSOs, to be merged with other individual grid model components in order to create the common grid model[1].

2. **Common Grid Model(CGM):**

   An Union-wide data set agreed between various TSOs describing the main characteristic of the power system (generation, load, grid topology and electrical characteristics)[1].

3. **Operational Security Analysis:**

   The entire scope of the computer based, manual and combined activities performed in order to assess Operational Security of the Transmission System, including but not limited to: processing of telemetered real-time data through State Estimation, real-time load flows calculation, load flows calculation during operational planning, Contingency Analysis in real-time and during operational planning, Dynamic Stability Assessment, real-time and offline short circuit calculations, System Frequency monitoring, Reactive Power and voltage assessment[2].

4. **Capacity allocation:**

   The attribution of cross zonal power transfer capacity[3].

5. **Bidding zone:**

   Bidding zone is the largest geographical area within which market participants are able to exchange energy without capacity allocation[3].

6. **Net Position:**

   The netted sum of electricity exports and imports for each market time unit for a bidding zone [1].

7. **Generation Shift keys:**

   It is the MW contribution of each generator in a bidding zone per MW change of the net position of that bidding zone.

8. **Day-ahead market gate closure time:**

   The point in time until which orders are accepted in the day-ahead market[1].

9. **Day-ahead market time-frame:**

   The time-frame of the electricity market until the day-ahead market gate closure time, where, for each market time unit, products are traded the day prior to delivery.

10. **Intraday cross-zonal gate opening time:**

    The point in time when cross-zonal capacity between bidding zones is released for a given market time unit and a given bidding zone border[1].

11. **Intraday cross-zonal gate closure time:**

    The point in time after which cross-zonal capacity allocation is no longer permitted for a given market time unit[1].

12. **Intraday market time-frame:**

    The time-frame of the electricity market after intraday cross-zonal gate opening time and before intraday cross-zonal gate closure time, where for each market time unit, products are traded prior to the delivery of the traded products[1].

13. **Critical network element:**

    A network element either within a bidding zone or between bidding zones that is taken into account in the capacity calculation process, as a possible limiting element for the amount of power that can be exchanged between bidding zones[3].

14. **Redispatching:**

    A measure activated by one or several system operators by altering the generation and/or load pattern in order to change physical flows in the transmission system and relieve a physical congestion[3].

15. **Power transfer distribution factor:**

    The MW change of flow in a network element per MW change of nodal power injection or per MW change of net position.

16. **Countertrading:**

    A cross zonal exchange initiated by system operators between two bidding zones to relieve physical congestion[3].

Graph theory related terminologies

Matrix definitions are provided for a network of n nodes with n_b branches. An element corresponding to row index i and column index j of a matrix M is denoted by M_{ij}.

1. **Graph, G:**

   A graph is a set of n nodes (or vertices or points) and n_b branches (or edges or lines) where each branch connects 2 nodes. A graph can be directed/undirected and weighted/unweighted.

2. **Directed Graph :**

   A graph in which the branches have a direction. One node of the branch corresponds to the sending node while the other node corresponds to the receiving node. The branch is directed from the sending node to the receiving node.

3. **Weighted Graph :**

   A weighted graph is a graph in which each branch is given a numerical weight. A weighted graph is therefore a special type of labeled graph in which the labels are numbers (which are usually taken to be positive)[4].

4. **Incidence matrix, C:**

   The incidence matrix of a graph G is a $n \times n_b$ matrix defined either as

   Undirected Incidence matrix

$$C_{ij} = \begin{cases} 1 & \text{if node } i \text{ is directly connected to branch } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

   or

   Directed Incidence matrix

$$C_{ij} = \begin{cases} 1 & \text{if node } i \text{ is the sending node of branch } j \\ -1 & \text{if node } i \text{ is the receiving node of branch } j \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

5. **Adjacency matrix, D:**

   The adjacency matrix of a graph G is a $n \times n$ matrix defined either as

   Unweighted adjacency matrix

$$D_{ij} = \begin{cases} 1 & \text{if node } i \text{ and node } j \text{ are connected by a branch} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

   or

   Weighted adjacency matrix

$$D_{ij} = \begin{cases} w_{ij} & \text{if node } i \text{ and node } j \text{ are connected by a branch} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

   where w_{ij} is the weight of the branch connecting nodes i and j.

6. **Electrical power system graph:**

   The electrical power system graph is a weighted graph that can be directed or undirected. If the weight of the branch is the power flow in the branch, the graph is directed with the corresponding power flow direction. If the weight of the branch is the reactance of the branch, the graph is undirected.

7. **Nodal admittance matrix, Y_{bus}:**

   The admittance matrix of an electrical power system graph is a $n \times n$ matrix [5].

   It is defined as:

$$Y_{bus_{ij}} = \begin{cases} \sum_{j=1}^{n} y_{ij} & \text{for all } i = j \\ -y_{ij} & \text{for all } i \neq j \end{cases} \tag{5}$$

   where y_{ij} is the admittance of the branch connecting nodes i and j. If nodes i and j are not connected by a branch, the corresponding y_{ij}=0.

8. **Power EXchange matrix, PEX:**

   The PEX matrix is a $n \times n$ matrix that holds a possible distribution of power from generators to loads. Each element PEX_{ij} equals the calculated MW power that is produced at node i and consumed at node j.

   - $PEX_{ij} \geq 0$
   - Total production at node i is $\sum_{j=1}^{n} PEX_{ij}$
   - Total consumption at node j is $\sum_{i=1}^{n} PEX_{ij}$

Chapter 1

Introduction

1-1 Motivation

Towards the end of the 20^{th} century, the goal of increasing the total social-economic welfare brought about deregulation of the electricity grid in many European countries. Competition was introduced between energy suppliers to provide a reduced energy price to the consumers. More recently, the goals of low-carbon emission increased the penetration of renewable energy sources in energy generation. Further, market coupling has been introduced which is limited by the physical power flows and transmission capacity of the critical lines. The connected countries coordinate their operations using a systematic step-by-step approach to keep the grid secure. From the perspective of this thesis, the relevant processes and the corresponding time frames are shown in Figure 1-1.

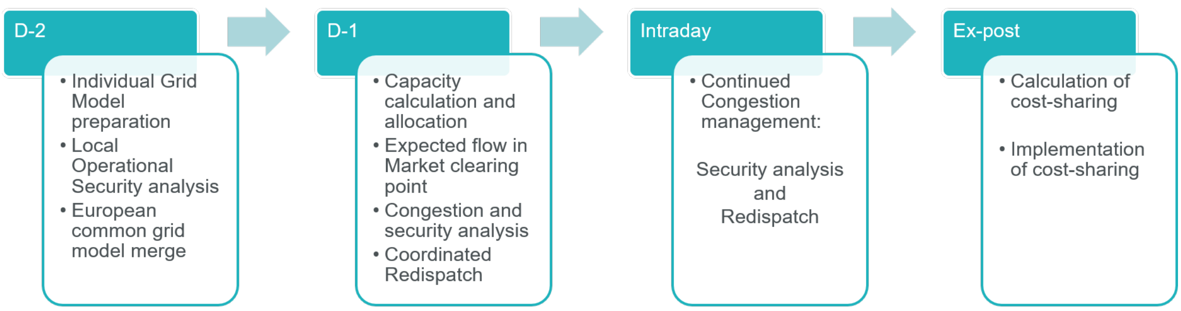


Figure 1-1: Time-frames for actions

In the 2 days ahead time-frame, every TSO performs security analysis on its own network. If security threats exist, they are removed by the TSO through suitable actions. After the removal of security threats, every TSO provides its Indiviudal Grid Model(IGM) to form the merged Common Grid Model(CGM) for Europe. In the day-ahead time frame, the capacities of the CGM that can be made available to the market are calculated. The available capacities incorporated in the CGM are provided to the electricity market. Electricity is then traded in the day-ahead wholesale market within the constraints of the available zone-to-zone capacities.

After this stage the values for dispatch of generators are known. To ensure operational security, the resulting expected power flows are recalculated for the CGM with the known generator's dispatches and the load's consumptions. Due to approximations and uncertainties, the forecasted power flows over critical transmission lines may exceed the available capacity. Under such a condition, the power grid is said to be congested. In case of insufficient control measures in this situation, the grid could become severely disturbed and may even face a blackout. The control measures are called Remedial Actions(RAs). RAs are performed to lower the power flows in the congested lines. Out of the possible RAs [Art. 22,[6]], choice and order of RAs are based on regulations defined in [Art. 21,[6]]. For economic efficiency, the TSOs first take the non-costly RAs. Only if these non-costly RAs cannot resolve the issue(or cannot resolve the issue without creating a new one), costly RAs- such as countertrading and redispatching -are taken[1]. The costs of the RAs are initially borne by the TSO that executes the RAs. The root cause for the congestion may, however, partly lay outside the zone of that TSO. In order to recover the incurred costs, a "polluter pays" principle is desired[7]. In a "polluter pays" settlement system, the RAs costs are split among the TSOs of the generators and the loads causing the congestion. The details of expected features of such a cost-sharing methodology are mentioned in [Art. 74, [1]]. The task of developing such methodologies was assigned by ENTSO-E to its member TSOs.

The *Flow Based for Investment (FB4INV)* team at TenneT TSO B.V developed one such methodology called *Full Line Decomposition(FLD)*. In brief, this method decomposes the MW power flow in a critical line into individual MW flows caused by each generator-to-load exchange. This method will the discussed in Chapter 2. FLD requires the calculation of a generator's MW share in the power consumed by all the loads. This MW share is the nodal Power EXchange or "nodal PEX".

1-2 Literature review

This section briefly describes published research into finding a generator's share in the power consumed by the load. Most of this research has the goal of finding a better way of determining transmission prices. Transmission pricing should be such that it retrieves the costs of operation and maintenance while also serving as a good indicator for planning[8]. This section also explains why the existing transmission pricing mechanisms are unsuitable for the present-day electricity network and why power exchanges become a necessary parameter in transmission price determination.

[1]In accordance with the definition in Art. 2 (26) of EU Regulation 543/2013, the costs for redispatching include the costs of multilateral remedial actions (MRAs), interruptible loads, feed-in management of renewables and activation of reserve power

In the privatised and restructured power grid, generator's compete for use of the network to supply the demands. The dispatch of these generators is based on the bids in the electricity market irrespective of their location. While some generator-to-load power transfers utilise the transmission network efficiently, others may "wheel the power through the breadth of the network"[9]. "Wheeling is the use of some party's transmission system(s) for the benefit of the other parties"[10]. With the advent of unbundling of electric services, the wheeling transactions have a higher percentage than before. So it is only fair that the transmission pricing takes into consideration individual generator's utilisation of the transmission network. For fair pricing, transparency is necessary. The need for transparency gave rise to questions such as "how an individual generator causes power flows in lines?", "how the individual generator contributes to line losses?" and "how the individual generator contributes to load's consumption?". This thesis aims at answering the last question.

Pro-rata (also called postage stamp), contract path and MW-mile are the orthodox methods of transmission pricing in power system economics. The next few paragraphs give a brief description the methods.

In the pro-rata method, generators or loads pay a flat rate per MW per hour for the use of the transmission network of a zone[11]. The charge is dependent on the MW rating of the generator and the peak demand of the load. The charge is independent of the location of the generator and the loads within the zone. Therefore, generator located close to main load centres pay the same price as the remotely located generators[12]. This characteristic makes the method unfair to centrally located generators. If the generator and load are located in different zones, each of the transmission networks along the path from the generator to the load needs to be paid a price. This is often referred to as "pancaking" of the rates as the prices of the zone of origin, the zone of delivery and the intervening zones together can produce a high price. This method reflects the average usage of the entire transmission network and doesn't indicate the use of transmission facilities by a specific generator and/or load[13].

The contract path method charges the generators and loads based on the power flow over the transmission facilities[11]. The contract path corresponds to the shortest of the possible electrically continuous paths between a generator and a load. It is assumed that power flows along this path. A price is paid for the use of elements along this path. This method is suitable in a highly radial network. However, in a meshed network there are multiple flow paths between the generator to the load. The flow of power along these paths is governed by laws of physics. Thus the results produced by this method cannot be considered accurate[12]. Unlike the postage stamp method, the contract path method considers the distance between the generator and the load. However, since the shortest electrical path is a rough estimate of the real use of the network, this method does not produce accurate results for retrieving the transmission costs.

Distance based MW-Mile allocates a cost to generators and/or loads by evaluating the product of the MW exchanged during the transaction and the geographical distance between the generator and the load. Similar to the contract path method, the considered distance may not bare a resemblance to the actual distance between the generators and the loads thereby producing questionable results.

There exist well-established methodologies for understanding the impact of generators and loads on lines through sensitivity analysis. This entails computation of factors which describe the effect on lines when a production and/or consumption changes. However, this provides information on *change* and not the *actual* value of a generator's contribution[14]. "Sensitivity analysis answer the question of how would line flows change following a change in nodal generation and/or demand ; it does not answer the question where the generation goes"[14].

All the existent transmission pricing methodologies were unsuitable to the needs of the deregulated electricity network. When the Electrical Supply Industry in England and Wales was privatised, the Electricity Pool Rules[15] stated that it is not possible to trace electricity in a meshed network[2]. One of the first methods that was developed to prove this statement incorrect was that of Bialek's [16]. He mentions that the "orthodox economic theory" was unsuitable and suggested a new paradigm that was sensitive to electrical distances. His work was the beginning of power flow tracing methodologies.

Bialek proposed an approach where every incoming flow at the node is considered to get distributed proportionally in the outgoing lines. This was termed *Proportional Sharing Principle*. The principle can neither be proved nor disproved though it has been rationalised in [17, 18]. It is an intuitive approach that satisfies Kirchhoff's Current Law. Bialek combined the proportional sharing principle with a methodology from [19] to develop a new generalised method for tracing the flow of electricity[14, 20]. It uses the power flow over the branches obtained from a load flow solution to trace power from the generators to the loads(called downstream tracing) or loads to generators(called upstream tracing). The tracing is done based on a set of linear equations explained in Chapter 2. Since power flows over the branches are used to calculate the generators contribution to the load, if the power flow solution doesn't provide a path of positive power flow from generator to the load, there is no provision of calculating this generator-to-load power exchange. This disadvantage is referred to as the "directional effect". In literature, the dependency on proportional sharing principle and the requirement to invert a sparse matrix are addressed as undesirable characteristics of the method. However, the directional effect is a more significant problem than the acceptance of the trivial proportional sharing principle or the inversion of a large sparse matrix. This will further be discussed in Chapter 2.

Another method developed shortly later was the Kirschen method[21]. This method forms a state graph based on the power flow solution. Buses in the power system are the vertices of the graph. Transmission lines and transformers are the edges of the graph. Direction of power flow gives the edge direction. The generators and loads are organised in domains and commons[21, 22, 23]. "The domain of a generator is defined as the set of buses which are reached by power produced by this generator. A common is defined as a set of contiguous buses supplied by the same generators"[21]. For each common, the proportional sharing principle is applied to obtain generator's power contribution to load. However, [24] points out that considering the share of every generator in each common to be same is a disadvantage. Further, a change in power flow solution requires reconfiguration of the common[24]. Similar to the Bialek method, this methodology also makes use of power flow solution to form a weighted directed graph. Thus the directional effect of Bialek method also exists in Kirschen method.

Subsequently many attempts to modify the Bialek method and the Kirschen method were made. However, these modifications attempt to solve the non-existent "Matrix inversion problem". The power contributions are calculated by constructing contribution factor matrices from the network incidence matrix, the adjacency matrix and the power flow solution in [25].

[2]Untraceable nature of electricity is clarified in Section 2-2 of Chapter 2

In [26] power flow solution(or state estimation) is used to compute an extended incidence matrix and a matrix of generations. These matrices are used to compute a distribution factor which is then used for power exchange calculations [26]. The method of nodal generation distribution factor[27] calculates an extraction factor from the power flow solution and utilises a search algorithm to calculate generator to load contributions.

The Equivalent Bilateral Exchanges method[9] computes power exchanges based on generator dispatch and load consumption alone. It considers every generator's production to feed all the loads proportionally. Similarly, every load receives a fraction of every generator's production. This method will be explained further in Chapter 2. These computations are independent of the distance between a generator and a load. The author of [9] believes that a lack of location dependence is more desirable as it provides no option for manipulation or volatility in price calculation. However this thesis considers locations of generator's and load's to be a important parameter in power transfer computations. A generator is expected to have a higher MW contribution to a load at a shorter distance.

In [28] a method of combining power flow tracing and classical postage stamp method is developed that uses power flow tracing compliant optimisation while minimising overall deviation from the postage stamp allocation.

A trend that emerged next was to trace power flow using circuit theory[29, 30, 31]. It aims at considering one generator at a time and analysing the effect the generator has on the power flows in lines, power loss in lines and load's power consumption. With the use of Kirchhoff's law, Ohm's law and Superposition theorem, the flow of power is traced. Because the superposition theorem is only applicable to linear power sources, the generators are modelled as a current source. This method will be analysed in-depth in this thesis in Chapter 3 and Chapter 4.

1-3 Identification of the research gap and research questions

To recover expenses of remedial actions according to the polluter-pays principle, it is required to find the contributions to the power flows that are caused by the generators and the loads. This requires the share of generators in power consumed by the loads(Power Exchange). Neither the orthodox transmission pricing methodologies (e.g. Postage Stamp, Contract path and MW-Mile) nor the conventional power system analysis tools(e.g. Sensitivity analysis) provide the option of calculating power exchanges. Power flow tracing methodologies based on proportional sharing principle(Bialek[14] and Kirschen[21]) are capable of calculating power exchanges. However, these[14, 21] tracing methodologies are applied on a directed weighted graph of the electrical network. The electrical network's lines are modelled as graph's edges. The power flows in the lines are modelled as weights of the graph edges and the direction of power flows in the lines form the the edge's directions. The tracing methods will not find any power exchange between a generator and a load if the load is not reachable from the generator in the directed graph, even when the load is close to the generator. Thus, the idea of modelling the electrical network as a *directed graph* can produce some questionable results. This will be explained further in Chapter 2.

The application of a cost-sharing methodology affects many organisations making it important to use an unambiguous method. An unambiguous method should reflect the physical reality of the system. The main question is: “How to develop a methodology of finding the share of a generator in the power consumed by load such that it *reflects the physical reality* of power flows in the electrical network?”. The physical reality is that the power flows in any electrical network are based on the electrical configuration and laws of physics (Ohm’s and Kirchoff’s law); not on the wishes of the market participants[12]. The laws of physics distribute generator’s power among lines based on impedances of the paths.

To replicate the behaviour of the transmission network, it is possible to model the network as a *weighted but undirected* graph. A graph with the nodes corresponding to buses, edges corresponding to transmission lines and edge weights corresponding to the impedance of transmission lines. From the undirected graph of the network, to obtain the generator-to-load contribution, their proximity is required. The proximity is termed “electrical distance”.

This thesis aims to find a methodology of determining generator’s MW contribution to a load based on their electrical distance, and wants to answer the following questions.

1. How to quantify electrical distance between any two nodes for a generic network?
2. Are any of the previously developed methods of computing power exchanges based on electrical distance? If such a method is identified but it has a drawback, can it be rectified?
3. How to define a metric for comparison of various PEX computation methodologies?

1-4 Scope and Approach

The methods discussed in this thesis are intended to be used in the High Voltage transmission network of Europe. The initial developments and testing will be carried out on a sample network of a smaller size such as the IEEE 30 bus test case.

During the MSc internship, the answer to the question “How to define the electrical distance?” was found. It was decided that the electrical distance between two points is the Thevenin impedance between them. Through the literature review, the method based on circuit theory was found to be relevant[29, 30, 31]. This method is henceforth referred to as the Superposition method. The Superposition method indirectly imposes the electrical distance between two points using the voltage-current relations of circuit theory equations. However, through the course of the thesis the Superposition method was found to depict an unexplainable characteristic. Therefore, a new methodology called the Optimal PEX method was developed that made use of the prior computed electrical distance directly. To facilitate a comparison of various methodologies, a performance metric called PEX_{loss} was defined.

1-5 Contributions

- A new approach to compute power exchanges through optimisation while considering the electrical distances was developed.
- A conference paper will be published.

1-6 Outline

Chapter 1 starts with explaining how the need for this project arises. It further discusses the relevant work was done in the past and addresses how this is insufficient for the purpose of the thesis.

Chapter 2 provides the necessary background to understand the application of the thesis. It highlights why the calculation of power exchange is not a typical quantity of the power system studies. It further discusses two well-established methods of calculating power exchanges, the equivalent bilateral exchange method and the Bialek method.

Chapter 3 introduces a new parameter called PEX_{loss} that will serve two purposes. To compare various PEX matrices and to be the basis of an optimisation method. This chapter further discusses two methods of computing PEX. First, the superposition method that was found in the literature review is discussed. As it has a drawback, a second method called optimal PEX was developed.

Chapter 4 starts with demonstrating the drawback of the superposition method using a 4 bus sample network. It further compares the 4 methods using the results of the IEEE 30 bus test system. The methods are applied to the transmission network of The Netherlands.

Chapter 5 provides the conclusions and suggests some possibilities for future work in terms of reducing the computation time and finding the global minimum for the optimal PEX method.

Chapter 2

Previous developments

This chapter will show that a causation-based cost-sharing method must be capable of relating the MW power flow in a line to the MW exchanges between generators and loads[1]. Full Line Decomposition (FLD) [32] is one such methodology which is explained in this chapter. The FLD application utilises power exchanges. The challenge of calculating power exchanges, the present methodology and its disadvantages are explained next.

2-1 Cause-effect relation in power systems

One of the duties of a TSO is to maintain the power flows in its critical lines within the secure limits. Some portion of the power flows in the critical lines will be caused by generators and loads located outside the borders of the TSO. In the cause-effect based cost-sharing, *causes are the MW exchanges between generators and loads inside and outside the border of the TSO* while the *effect is the MW power flow in the critical line driven by these generator-to-load exchanges.*

2-1-1 Full Line Decomposition

A causation-based cost-sharing method called Full Line Decomposition [32] has been developed by the FB4INV team of TenneT Netherlands.

Element-wise description of the matrices involved in FLD are explained first.

Power Transfer Distribution Factor ($PTDF$) : is a known sensitivity analysis tool that depicts a cause-effect relation. "PTDF indicates the incremental change in real power that occurs on transmission lines due to real power transfers between two regions. These regions can be defined by areas, zones, super areas, single buses, injection groups or the system slack.

[1]Note: Generator-to-load exchanges refer to the MW power exchanges between generators and loads. Net generations and net consumptions are summated to obtain zone-to-zone exchanges power exchange at a zonal level. The terms may be used interchangeably in some scenarios.

These values provide a linearized approximation of how the flow on the transmission lines change in response to a transaction between the source and the sink[33]."

At the single bus level, the source is a generator and the sink a load.

$PTDF_{i,l}$ holds the effect on the power flow in line l if 1MW is injected at the generator node i and consumed at the slack node.

$PTDF_{j,l}$ holds the effect on the power flow in line l if 1MW is injected at the load node j and consumed at the slack node.

Power Exchange Distribution Factor ($PEDF$) : Subtracting the PTDF of the generator node and the PTDF of the load node for a specific line gives the Power Exchange Distribution Factor($PEDF$) for the line due to this generator-to-load MW exchange.

$$PEDF_{l_{i,j}} = PTDF_{i,l} - PTDF_{j,l} \tag{2-1}$$

$PEDF_{l_{i,j}}$ holds the effect on the power flow in line l if 1MW is injected at the generator node i and consumed at the load node j. This parameter is independent of the slack node.

To highlight, $PEDF_{l_{i,j}}$ provides the power flow in line l caused by **unit MW exchange** between the generator node i and the load node j.

$PEDF_{l_{i,j}}$ still needs to be multiplied by **actual MW exchange** between the generator node i and the load node j to get the power flow in line l due actual power exchanged between the generator node i and the load node j.

Power EXchange (PEX) :The actual power exchanges are elements of the Power EXchange(PEX) matrix.

PEX_{ij} holds the contribution of the generator node i to the power consumed by the load node j.

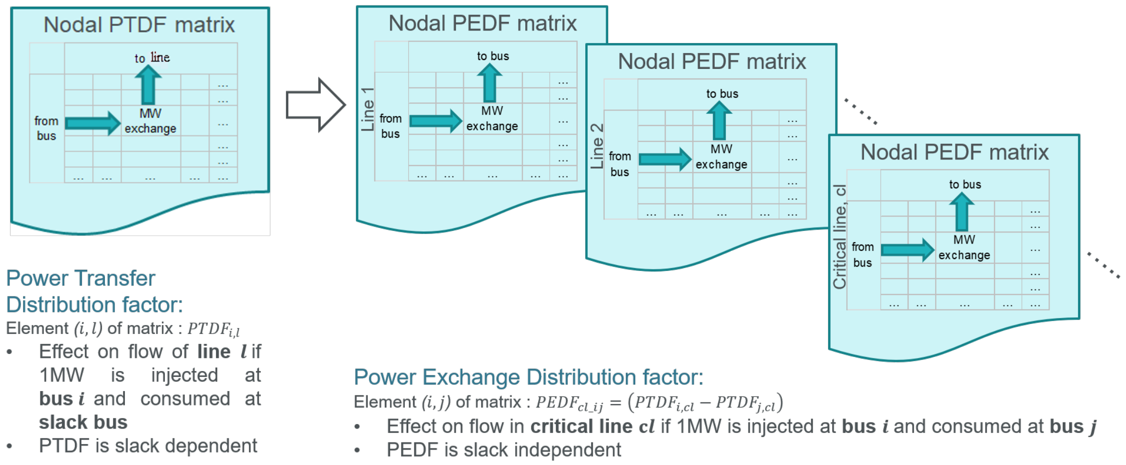
- $PEX_{ij} \geq 0$
- Total production at node i is $\sum_{j=1}^{n} PEX_{ij}$
- Total consumption at node j is $\sum_{i=1}^{n} PEX_{ij}$

Power Flow Partition (PFP):

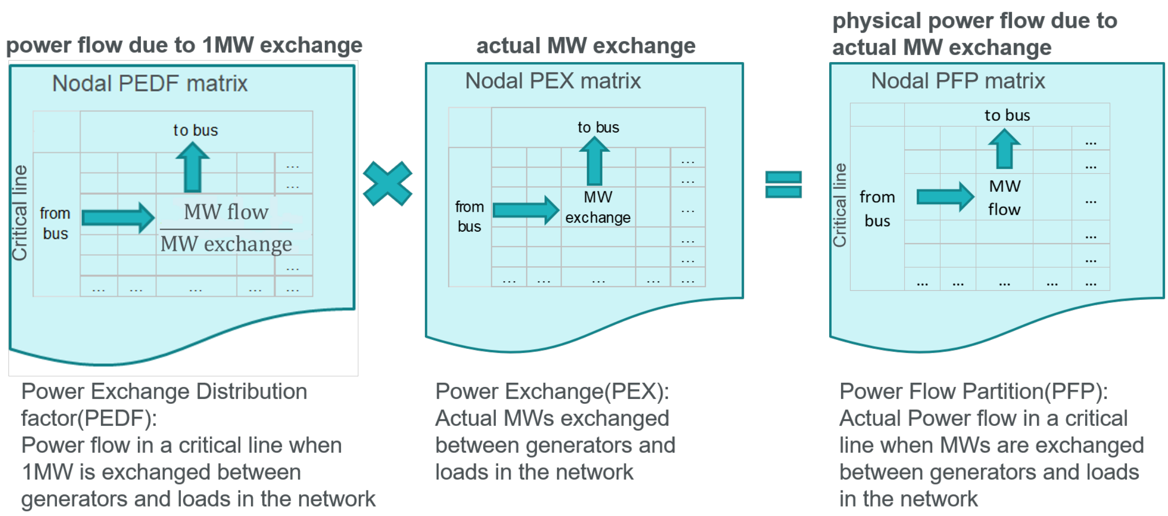
$PFP_{l_{i,j}}$ holds the power flow in line l due to power transferred from the generator node i to the load node j.

Power flow in line caused by generator-to-load power exchange is the product of power flow in line caused by generator-to-load 1MW exchange and the actual generator-to-load MW exchange.

$$PFP_{l_{i,j}} = PEDF_{l_{i,j}} \times PEX_{ij} \tag{2-2}$$



(a) Conversion of PTDF to PEDF



(b) PFP from PEDF and PEX

Figure 2-1: Simplified FLD process

Figure 2-1 provides a visual aid of the simplified FLD process. Because the calculation of $PEDF$ is straightforward, the challenge in implementing FLD comes down to calculating Power Exchanges.

2-2 Challenge of calculating power exchanges

This section shows 3 cases of power flows in electrical networks. The first case is a 2-bus radial network. For the first case, power exchange is obvious from the power flow solution. The second case is a 4-bus radial network with 2 generators and 2 loads. The third case is a generic electrical network with multiple generators and multiple loads. For the second and third case, power exchange is not obvious from the power flow solution. The following paragraphs show why.

case (i)

In Figure 2-2, calculation of the contribution of generator G1 to power consumed by the load D1 is straightforward because it is known that all the production of generator G1 flows through line 1 and feeds load D1.

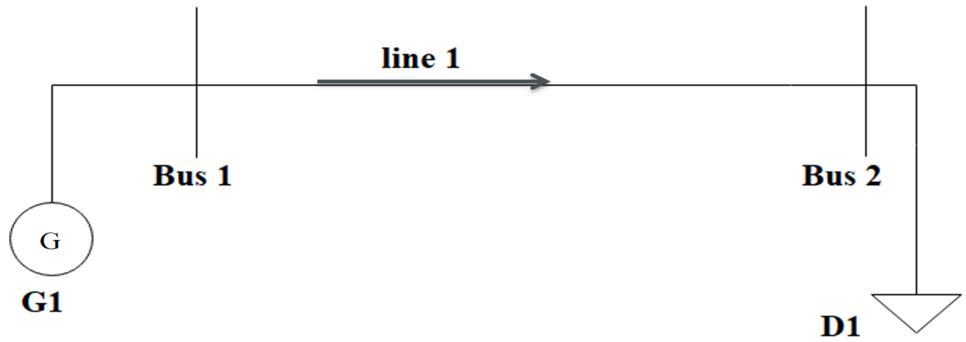


Figure 2-2: A simple 2 bus network with 1 generator and 1 load

case(ii)

In Figure 2-3, the production of generator G1 at bus 1 flows through line 1. At bus 2, the flow in line 1 mixes with the power injected by generator G2. The power flow in line 2 which thereby feeds the demands is a mix of both the generator's productions. There is no obvious relation between the productions and consumptions.

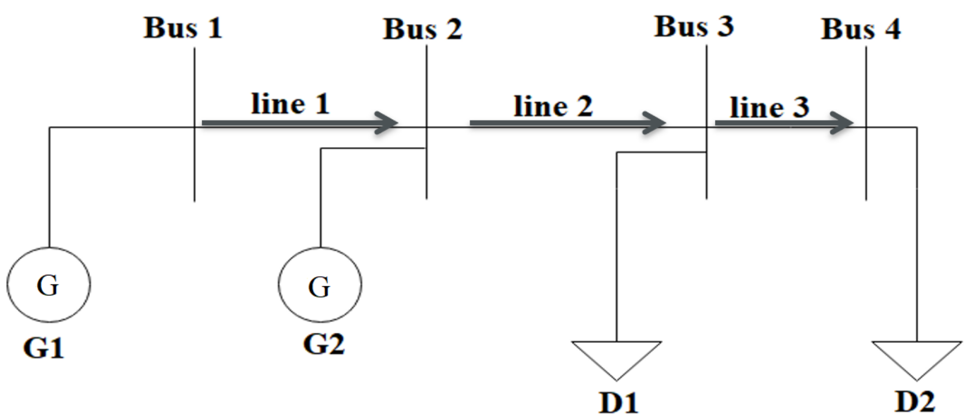


Figure 2-3: A simple 4 bus network with 2 generators and 2 loads

case(iii)

In Figure 2-4, every generator's production goes through the electrical network prior to feeding the load. In the electrical network, the production of all the generators are mixed.

An exclusive relation between individual generations and individual consumptions is not observable from the power flow solution when 2 or more generators and 2 or more loads are present in the electrical network.

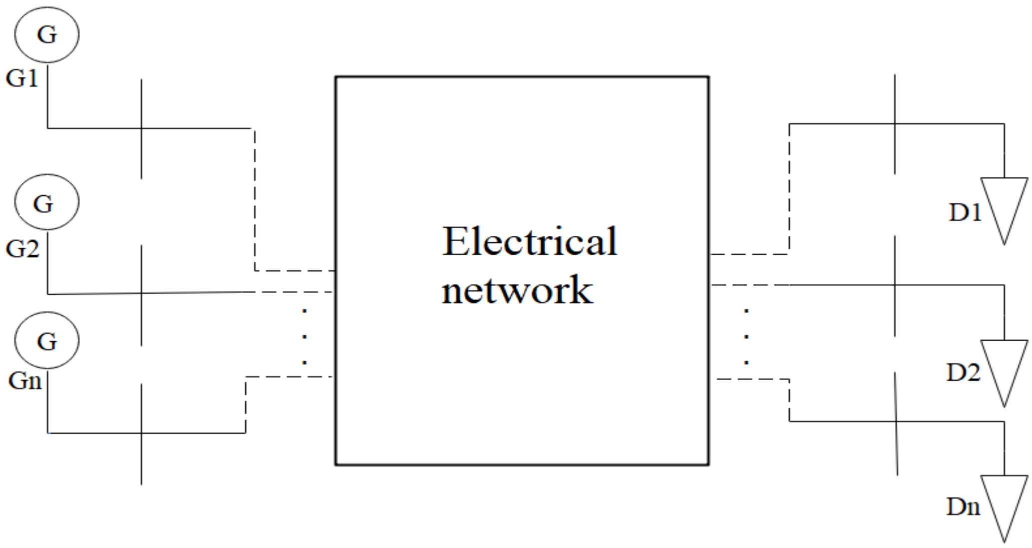


Figure 2-4: A network with multiple generators and loads

2-3 Equivalent Bilateral Exchanges

The simplest way to solve the PEX calculation challenge is to assume every generator contributes to every load in a proportionate manner[9]. Each demand is assigned a fraction of each generation and each generator is assigned a fraction of each demand in a uniform manner[9].

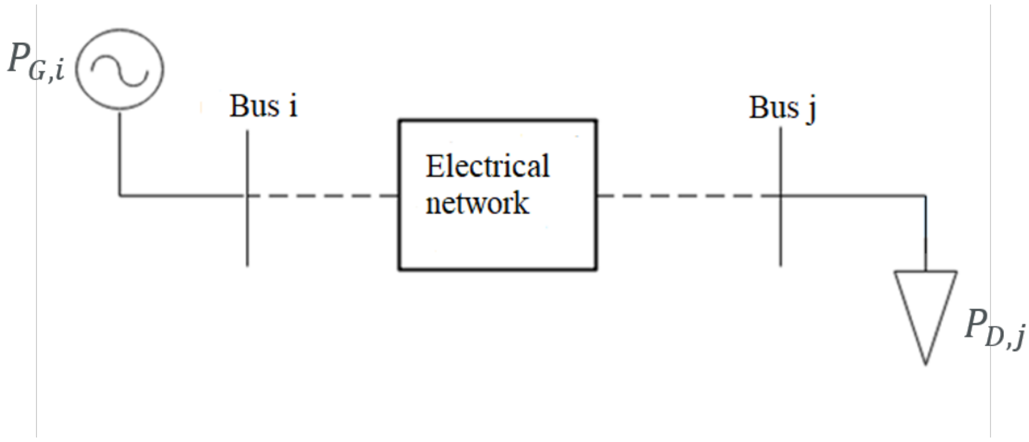


Figure 2-5: An electrical network

In any electrical network with n_G generators and n_D loads, if a generator at node i generates power $P_{G,i}$ and a load at node j consumes power $P_{D,j}$,

$$PEX_{ij} = \frac{P_{G,i} \times P_{D,j}}{\text{Total Exchange}} \tag{2-3}$$

where Total Exchange$=\sum_{i=1}^{n_G} P_{G,i} = \sum_{j=1}^{n_D} P_{D,j}$

2-3-1 Application to example network: Results, advantages and disadvantages

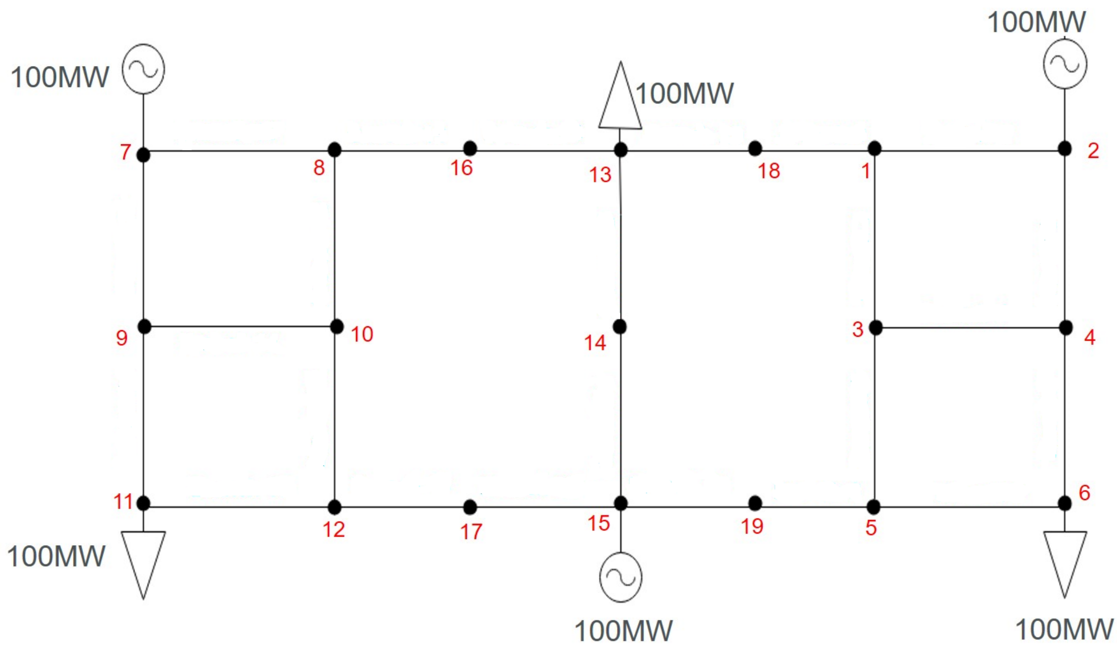


Figure 2-6: Symmetric 19 bus network

Consider the symmetric 19-bus system in Figure 2-6. The choice of this system is based on its symmetry. This system would be used for illustration of various methods in this chapter and the chapter to follow as well. In Figure 2-6, the numbers in red indicate the bus number. Numbers over the branches indicate the active power flow in the lines obtained from the power flow solution. There are 3 generators of 100MW each at buses 2, 7 and 15; and 3 loads of 100 MW each at buses 6, 11 and 13. All the 24 branches of the system have zero resistance, zero susceptance and 1Ω line reactance.

Result:

Using Eq. (2-3) for Figure 2-6, resulting PEX is shown in Table 2-4.

PEX (in MW)		Loads			Production of generator
		Bus 6	Bus 11	Bus 13	
Generators	Bus 2	33.3333	33.3333	33.3333	100
	Bus 7	33.3333	33.3333	33.3333	100
	Bus 15	33.3333	33.3333	33.3333	100
Consumption of load		100	100	100	300

Table 2-1: Generator to load contribution for symmetric 19-bus network using equivalent bilateral exchanges method

The rows indicate the buses containing the generators while the columns indicate the buses with the loads. Matrix elements indicate the contribution of generator to power consumed by the load.

Advantage:

- Simplicity in calculation makes it understandable to everyone
- Unique solution
- Independent of Generation Shift Keys
- Independent of slack bus location
- Satisfies PEX definition
  - Generator to load contribution always positive
  - Adds up to generation and demand such that
    $\sum_{i=1}^{nG} PEX_{ij} = P_{D_j}$
    $\sum_{j=1}^{nD} PEX_{ij} = P_{G_i}$

Disadvantage:

Because value of network reactances are not considered, electrical distance between generator and load is disregarded. If the generator and load are located far away but the generation and consumption are high, resultant exchange is high.

In the example network, the contribution of generator at bus 2 is the same to loads at bus 6, bus 11 and bus 13. Based on the electrical distance, it is expected that the contribution of generator 2 is highest to bus 6, lesser to bus 13 and the least to bus 11.

2-4 The Bialek method

The method currently used for calculating power exchanges in FLD is called the Bialek method. The rest of this section describes the underlying principle of Bialek method. Section 2-4-1 describes the set of equations used for PEX computations in any general network. Section 2-4-2 discusses the results of the method emphasising on its advantages and disadvantages. Detailed description of the methodology can be found in [32, 14, 20, 34].

To deal with the power exchange challenge, Bialek applies a Perfect Mixer(or Proportional Sharing) principle at each node[2]. Because it is impossible to tell which incoming electron goes to which out-flowing line, it is pragmatic to assume that incoming flows get distributed proportionally in the out-flowing lines[20].This is an intuitive approach that can be neither proved or disproved. It is rationalised in [17, 18] using game theory and information theory.

Example: To demonstrate the principle, an example is provided.

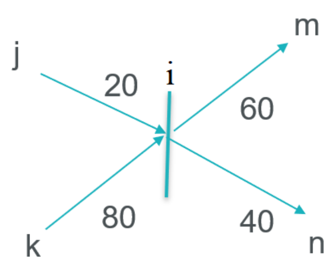


Figure 2-7: Example demonstrating perfect mixer node i

In Figure 2-7, node i has inflowing lines j and k carrying 20MW and 80MW; and out-flowing lines m and n carrying 60MW and 40MW.

The outgoing flow in line m and line n can be split as flows due to incoming lines j and k. This is called downstream tracing. The corresponding calculation is shown in Table 2-2.

[2] Not to be confused with the proportionate exchanges in the method of Equivalent Bilateral Exhanges

	Flow in line m = 60MW	Flow in line n = 40MW
Due to line j	$\frac{20}{60+40} \times 60 = 12\text{MW}$	$\frac{20}{60+40} \times 40 = 8\text{MW}$
Due to line k	$\frac{80}{60+40} \times 60 = 48\text{MW}$	$\frac{80}{60+40} \times 40 = 32\text{MW}$

Table 2-2: Downstream tracing for power at node i

The incoming flow in line j and line k can be split as flows due to out-flowing lines m and n. This is called upstream tracing. The corresponding calculation is shown in Table 2-3.

	Flow in line j = 20MW	Flow in line k = 80MW
Due to line m	$\frac{60}{20+80} \times 20 = 12\text{MW}$	$\frac{60}{20+80} \times 80 = 48\text{MW}$
Due to line n	$\frac{40}{20+80} \times 20 = 8\text{MW}$	$\frac{40}{20+80} \times 80 = 32\text{MW}$

Table 2-3: Upstream tracing for power at node i

2-4-1 The matrix approach

Instead of applying proportional sharing principle to every node one after another, Bialek proposed the systematic matrix approach for a general network using a set of linear equations. These equations involve the incidence matrix, the adjacency matrix, the nodal powers and the branch flows from the power flow solution[32, 34]. The matrix approach is proven to produce a unique solution[32].

For a network with n nodes and n_b branches, $\mathbf{P}$,$\mathbf{P_G}$ and $\mathbf{P_D}$ are nodal power matrix, nodal generation matrix and nodal demand matrix respectively. $\mathbf{P}$, $\mathbf{P_G}$ and $\mathbf{P_D}$ are of order $n \times 1$. $\mathbf{F}$ is a $n_b \times 1$ vector of branch flows obtained from the power flow solution.

The directed incidence matrix $\mathbf{C}$ of the network is split into matrix $\mathbf{C_u}$ consisting of -1's and matrix $\mathbf{C_d}$ consisting of 1's.

$\mathbf{F_d}$ is a matrix of downstream branch flows formed using branch flows $\mathbf{F}$,

$$\mathbf{F_d} = -\mathbf{C_d^T} \mathbf{F} \mathbf{C_u} \tag{2-4}$$

Row i and column j of $\mathbf{F_d}$ indicates the downstream flow in branch ij towards node j. Row i and column j of $\mathbf{F_d^T}$ indicates the upstream flow in branch ij towards node i.

$$\begin{aligned}\text{Nodal power } \mathbf{P} &= \text{generator's power injection} + \text{sum of flow on incoming lines} \\ &= \mathbf{P_G} + \mathbf{F_d^T} 1 \\ &= \text{load's power consumption} + \text{sum of flow on outflowing lines} \\ &= \mathbf{P_D} + \mathbf{F_d} 1 \end{aligned} \tag{2-5}$$

From (2-5), nodal demand matrix $\mathbf{P_D}$ and nodal generation matrix $\mathbf{P_G}$ is calculated as

$$\begin{aligned}\mathbf{P_D} &= \mathbf{P} - \mathbf{F_d}1 \\ &= \mathbf{P} + \mathbf{C_d^T}\text{diag}(\mathbf{F})\mathbf{C_u}1 \\ &= [\mathbf{I} + \mathbf{C_d^T}\text{diag}(\mathbf{F})\mathbf{C_u}\text{diag}(\mathbf{P^{-1}})]\mathbf{P} \\ &= \mathbf{A_d}\mathbf{P}\end{aligned} \tag{2-6}$$

$$\begin{aligned}\mathbf{P_G} &= \mathbf{P} - \mathbf{F_d^T}1 \\ &= \mathbf{P} + \mathbf{C_u^T}\text{diag}(\mathbf{F})\mathbf{C_d}1 \\ &= [\mathbf{I} + \mathbf{C_u^T}\text{diag}(\mathbf{F})\mathbf{C_d}\text{diag}(\mathbf{P^{-1}})]\mathbf{P} \\ &= \mathbf{A_u}\mathbf{P}\end{aligned} \tag{2-7}$$

$$\begin{aligned}\text{where } \mathbf{A_d} &= \mathbf{I} + \mathbf{C_d^T}\text{diag}(\mathbf{F})\mathbf{C_u}\text{diag}(\mathbf{P^{-1}}) \\ \mathbf{A_u} &= \mathbf{I} + \mathbf{C_u^T}\text{diag}(\mathbf{F})\mathbf{C_d}\text{diag}(\mathbf{P^{-1}})\end{aligned}$$

It was found that $\mathbf{A_d}$ and $\mathbf{A_u}$ can also be calculated directly from the power flow solution as

$$[\mathbf{A_u}]_{\mathbf{ij}} = \begin{cases} 1 & \text{for } i = j \\ -\frac{|P_{ji}|}{P_j} & \text{for } j\epsilon\alpha_i^{(u)} \\ 0 & \text{otherwise} \end{cases} \tag{2-8}$$

$$[\mathbf{A_d}]_{\mathbf{ij}} = \begin{cases} 1 & \text{for } i = j \\ -\frac{|P_{ji}|}{P_j} & \text{for } j\epsilon\alpha_i^{(d)} \\ 0 & \text{otherwise} \end{cases} \tag{2-9}$$

where $\alpha_i^{(u)}$ is the set of nodes supplying node i directly
$\alpha_i^{(d)}$ is the set of nodes supplied directly from node i
P_{ji} is the power flow in line connecting nodes i and j
P_j is flow nodal power at node j

Nodal power $\mathbf{P}$ is related to nodal generation $\mathbf{P_G}$ and nodal demand $\mathbf{P_D}$ as

$$\begin{aligned}\mathbf{P} &= \mathbf{A_u^{-1}}\mathbf{P_G} \\ &= \mathbf{A_d^{-1}}\mathbf{P_D}\end{aligned} \tag{2-10}$$

For better understanding, the calculation of one element of the $\mathbf{P}$ is described. Consider a load at node j. Power consumed at node j is given by

$$\begin{aligned}\mathbf{P_j} &= \sum_{k=1}^{n} [\mathbf{A_u^{-1}}]_{\mathbf{jk}}\mathbf{P_{Gk}} \\ &= \sum_{k=1}^{n} [\mathbf{A_d^{-1}}]_{\mathbf{jk}}\mathbf{P_{Dk}}\end{aligned} \tag{2-11}$$

where

$$[\mathbf{A_u^{-1}}]_{\mathbf{jk}} = \text{contribution of node } k \text{to nodal power of node} j$$
$$[\mathbf{A_d^{-1}}]_{\mathbf{jk}} = \text{contribution of node } j \text{ to nodal power of node } k$$

$$\text{Nodal demand at node } j,\ \mathbf{P_{Dj}} = \frac{\mathbf{P_{Dj}}}{\mathbf{P_j}}\mathbf{P_j} = \frac{\mathbf{P_{Dj}}}{\mathbf{P_j}}\sum_{k=1}^{n}[\mathbf{A_u^{-1}}]_{\mathbf{jk}}\mathbf{P_{Gk}}$$
$$\text{Nodal generation at node } k,\ \mathbf{P_{Gk}} = \frac{\mathbf{P_{Gk}}}{\mathbf{P_j}}\mathbf{P_j} = \frac{\mathbf{P_{Gk}}}{\mathbf{P_j}}\sum_{k=1}^{n}[\mathbf{A_d^{-1}}]_{\mathbf{kj}}\mathbf{P_{Dj}} \tag{2-12}$$

For a particular generator at node k supplying a load at node j, power exchange between them is given by

$$\mathbf{P_{Gk,Dj}} = \frac{\mathbf{P_{Dj}}}{\mathbf{P_j}}[\mathbf{A_u^{-1}}]_{\mathbf{jk}}\mathbf{P_{Gk}}$$
$$= \frac{\mathbf{P_{Gk}}}{\mathbf{P_k}}[\mathbf{A_d^{-1}}]_{\mathbf{kj}}\mathbf{P_{Dj}} \tag{2-13}$$

2-4-2 Application to example network: Results, advantages and disadvantages

Result

Consider the symmetric 19-bus system in Figure 2-8 again.

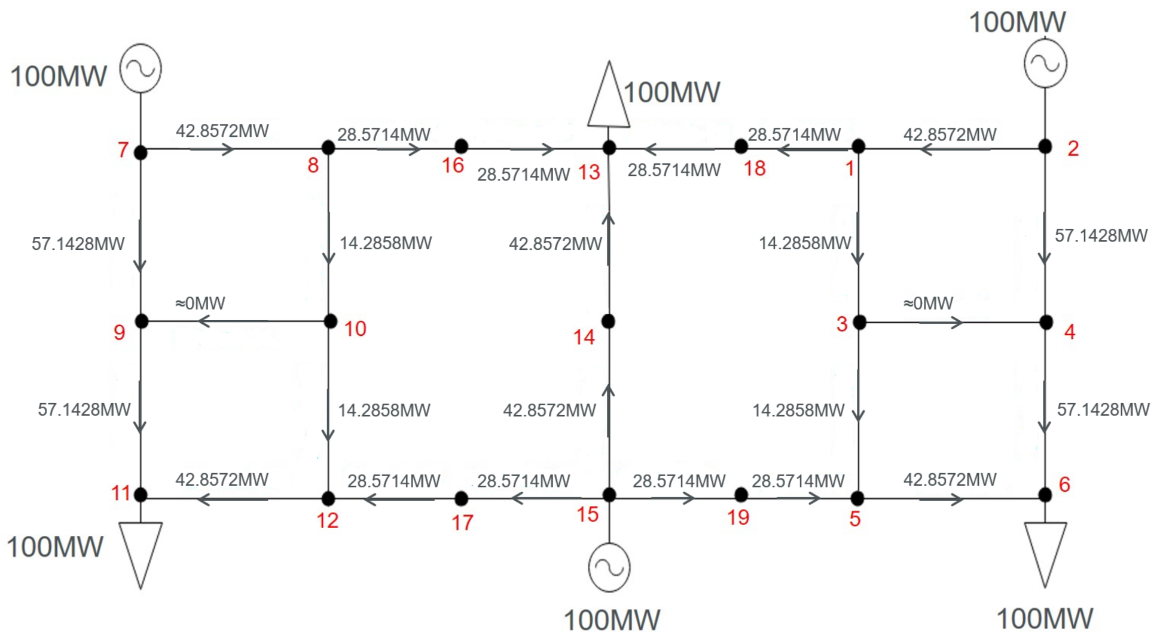


Figure 2-8: Symmetric 19 bus network

After obtaining the load flow solution, the matrix-approach method is applied. The resultant PEX matrix(in MW) is

PEX (in MW)		Loads			Production of generator
		Bus 6	Bus 11	Bus 13	
Generators	Bus 2	71.4286	0	28.5714	100
	Bus 7	0	71.4286	28.5714	100
	Bus 15	28.5714	28.5714	42.8572	100
Consumption of load		100	100	100	300

Table 2-4: Generator to load contribution for symmetric 19 bus network using equivalent bilateral exchanges method

Advantages

- Unique solution
- Independent of Generation Shift Keys
- Independent of slack bus location
- Satisfies PEX definition
  - Generator to load contribution always positive
  - Adds up to generation and demand such that
    $\sum_{i=1}^{nG} PEX_{ij} = P_{D_j}$
    $\sum_{j=1}^{nD} PEX_{ij} = P_{G_i}$
- **Proximity effect:** Referring to Figure 2-8 and the resultant PEX matrix, it can be seen that the load at bus 6 gets most of its power from the generator at bus 2. Thus, the expectation that the load gets more power contribution from the generators at a shorter distance is met. In [32] the method was also tested for realistic European transmission network models and the desired proximity effect is maintained.

Disadvantages

- **Directional effect:** Referring to Figure 2-8, it's seen that there is no power flow path from generator at bus 2 to load at bus 11 with a positive power flow in every branch. The same can also be observed for the power flow from generator at bus 7 to load at bus 6. The tracing methods will not find any power exchange between these generators and loads as the loads are not reachable from the generators in the directed graph.

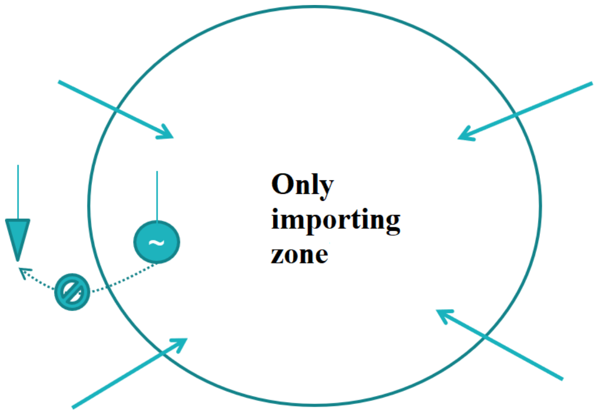


Figure 2-9: A zone with only import(Negative net position)

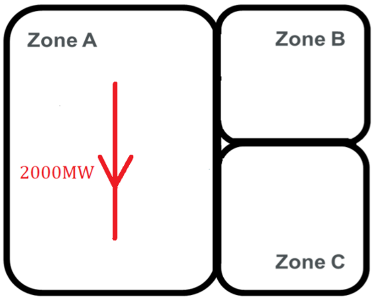
If there exists a zone with only imports on a large scale (Figure 2-9), the method doesn't find the contribution of a generator within this zone to a load outside the zone because there is no outward flow path available or tracing. This effect is called the "directional effect".

A more desirable feature would be to have the provision of calculating every possible contribution even if the result provides a zero or a negligible value.

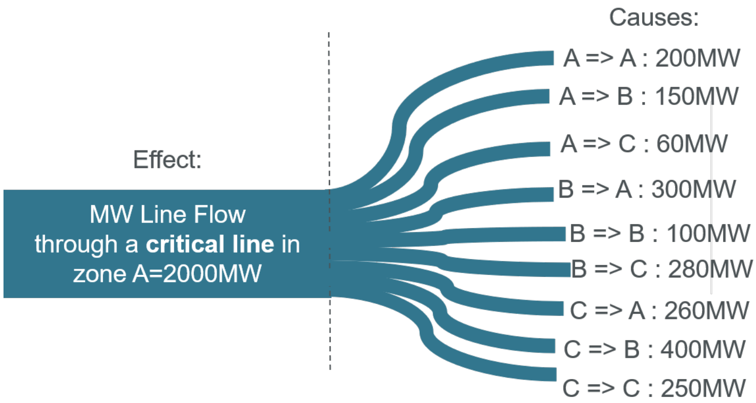
2-5 Demonstration of FLD

This section demonstrates FLD with an example. The purpose of this section is to provide a broad picture of the capability of FLD. Please note that numerical values used in this section are an example to facilitate a better understanding.

Consider a system with zones A, B and C (Figure 2-10a). Consider a flow of 2000MW in a critical line in zone A that exceeds the secure limit. Using FLD, it is possible to decompose(or partition) the 2000MW flow in the critical line into components of MW flows caused by each zone-to-zone exchange. The components indicate the sending zone, the receiving zone and the power exchange between them through the critical line. Inversely, sum of each of these components provide the 2000MW in the critical line. The decomposition of the effect into causes is shown in Figure 2-10b.



(a) Example with a critical line in zone A



(b) The cause-effect relation: Partition of power flow of the critical line in zone A into zone-to-zone power exchange. Note: The numerical values are example numbers provided for a better understanding

Figure 2-10: Partition of power flow in the critical line

Each decomposed(or partitioned) power flow can be described as an internal flow, a loop flow, an export flow, an import flow, a transit flow or a cycle flow. Cycle flows can be caused by HVDC links, PSTs or FACTS devices. FLD identifies the cycle flows caused by PSTs. In the future versions FLD will also be capable of identifying flows due to FACTS devices and HVDC links. ENTSO-E is yet to formally define the cycle flows. ENTSO-E's definitions[35] for internal flow,loop flow,export flow, import flow or transit flow are based on the locations of the source, the sink and the considered line (Table 2-5).

ENTSO-E flow types:

Internal flow: the source, the sink and the line are located in the same zone.

Loop flow: the source and the sink are located in the same zone, but the line is in another zone.

Export flow: the source and the line are located in the same zone, but the sink is in another zone.

Import flow: the sink and the line are located in the same zone, but the source is in another zone.

Transit flow: the source, the sink and the line are all located in different zones.

Source zone	Sink zone	Line in zone	Flow Type
A	A	A	Internal
B	B	A	Loop
C	C	A	Loop
A	B or C	A	Export
B or C	A	A	Import
B	C	A	Transit
C	B	A	Transit

Table 2-5: ENTSO-E flow types for the example network

Applying the flow type definitions(Table 2-5) to the partitioned power flow in the critical line (Figure 2-10b), the Power Flow Partition (PFP) matrix is obtained. For the considered example, the resulting PFP matrix is shown in Table 2-6. The row of the PFP matrix indicates the sending zone, column indicates the receiving zone and the matrix element indicates the power exchanged between the two zones through the critical line.

Power flow in a critical line in zone A is 2000 MW	zone A	zone B	zone C
zone A	200: Internal	150: Export	60: Export
zone B	300: Import	100: Loop	280: Transit
zone C	260: Import	400: Transit	250: Loop

Table 2-6: PFP of the example network

As the name suggests, the Power Flow Partitioning matrix partitions the power flow in a line into power flows through the line caused by zone-to-zone exchanges.

2-6 Summary

This chapter described why the less-known parameter Power EXchange is required in the FLD application for partitioning the power flows in lines based on their causes(generators-to-loads MW exchange).

Because the flow of electrons cannot be "dyed" or measured, methodologies that extrapolate the power exchange values based on the understanding of the behaviour of a power system are required.

The first method, Equivalent Bilateral Exchange, computes the power exchange between the generator and the load without taking into account the topology of the electrical network between the corresponding buses.

The second method, Bialek method, takes into account the topology of the network. However, it computes the power exchange using a directed model of the electrical network. This produces some counter-intuitive results.

Therefore, the third chapters attempts to find a new method of computing power exchanges based on the behaviour of the power system.

Chapter 3

Proposed methods

Section 3-1 describes how the power exchanges between nodes are related to their electrical distances. This chapter further discusses 2 methods that consider electrical distances in power exchange computations. The first method is discussed in Section 3-2 called *the superposition method*. Due to certain drawbacks of the superposition method, a new method was required. Since there exists multiple ways of calculating the PEX matrix but no defined way of comparing them, a new performance metric called PEX_{loss} is defined in Section 3-3. This PEX_{loss} metric also serves as the basis of the second method called *the optimal PEX method*. This method is explained in Section 3-4.

3-1 Power EXchange and electrical distance

This thesis aims at finding a method that preserves the proximity effect but eradicates the directional effect. Reminder: Directional effect is dependency on availability of a power flow path to trace the power from the generator to the load. Proximity effect is when a generator provides a higher power contribution to the load located electrically closer to it.

The idea is to utilise the electrical distance between the generator and the load to calculate the power exchanged between them.

Electrical distance between two buses is defined as the Thevenin impedance between them denoted by Z_{th}.

Electrical distance between node i and node j is computed as:

$$\begin{aligned}\text{Electrical distance}_{ij} &= Z_{th,ij} \\ &= Z_{bus,ii} + Z_{bus,jj} - 2Z_{bus,ij} \\ &= \left.\frac{V_i}{I_i}\right|_{\text{only current source } i \text{ considered}} \\ &\quad + \left.\frac{V_j}{I_j}\right|_{\text{only current source } j \text{ considered}} \\ &\quad - 2\left.\frac{V_i}{I_j}\right|_{\text{only current source } j \text{ considered}}\end{aligned} \tag{3-1}$$

Eq. (3-1) indicates that the electrical distance between 2 nodes is calculated considering one the current injection at a time and using the bus impedance matrix Z_{bus} to compute voltage produced at both nodes.

To attain proximity effect, the power exchange between nodes should be inversely proportional to the electrical distance between them.

$$PEX_{ij} \propto \frac{1}{\text{Electrical distance}_{ij}} \tag{3-2}$$

Therefore it is possible to have two methods of calculating PEX matrix based on the electrical distance. The superposition method considers electrical distance indirectly. Each generator's current source is considered at a time. The voltages induced at the buses are used in power exchanges computations. The optimal PEX method utilises electrical distance directly to compute power exchanges.

3-2 The superposition method

This section describes the superposition method. It models the generators as current sources, loads as impedances and utilises circuit theory to compute power exchanges. This method was also found to have a drawback which will be elaborated on in the next chapter. This section focuses on describing the steps of the method.

3-2-1 Superposition theorem

"In any linear circuit having more than one source, the response across any element is the sum of the responses obtained from each source considered separately with all other sources suppressed."

Superposition theorem is based on the concept of linearity between the response and the excitation of an electrical circuit.

$$\begin{aligned} P &= VI \\ &= I^2R \\ &= \frac{V^2}{R} \end{aligned} \tag{3-3}$$

As the relation of power flow with voltage and current are non-linear, power sources(generators) need to be the modelled as current sources based on the injected powers and the corresponding bus voltages; and power sinks(loads) need to be modelled as impedances based on the consumed powers and the corresponding bus voltages.

3-2-2 Procedure

Major steps in the method are shown in the flowchart of Figure 3-1.

The equations corresponding to each step are described in this section.

Power flow solution is computed first to obtain the power injected by the generators, the power consumed by the loads, voltage magnitudes at all the buses and voltage angles at all the buses.

Consider a network with n buses, n_G generators, a generator at bus i and a load at bus j.

The apparent power injected by generator i is

$$S_{i,G} = P_{i,G} + jQ_{i,G} \tag{3-4}$$

Current injected by generator i is calculated as

$$I_{i,G} = \left(\frac{S_{i,G}}{V_{i,G}}\right)^* \tag{3-5}$$

where $V_{i,G}$ is the voltage at bus i.

The load j is replaced by the corresponding admittance given by

$$Y_{j,L} = \frac{1}{\frac{|V_{j,L}|^2}{P_{j,L} - jQ_{j,L}}} \tag{3-6}$$

where $V_{j,L}$ is the voltage at bus j, $P_{j,L}$ is the active power consumed by the load j and $Q_{j,L}$ is the reactive power consumed by the load j.

Nodal admittance matrix(Y_{bus}) is formed using Eq. (5).

The load admittances are added to corresponding diagonal elements of Y_{bus} to get Y_{matrix}.

$$Y_{matrix} = Y_{bus} + Y_{loads} \tag{3-7}$$

$$Z_{matrix} = \frac{1}{Y_{matrix}} \tag{3-8}$$

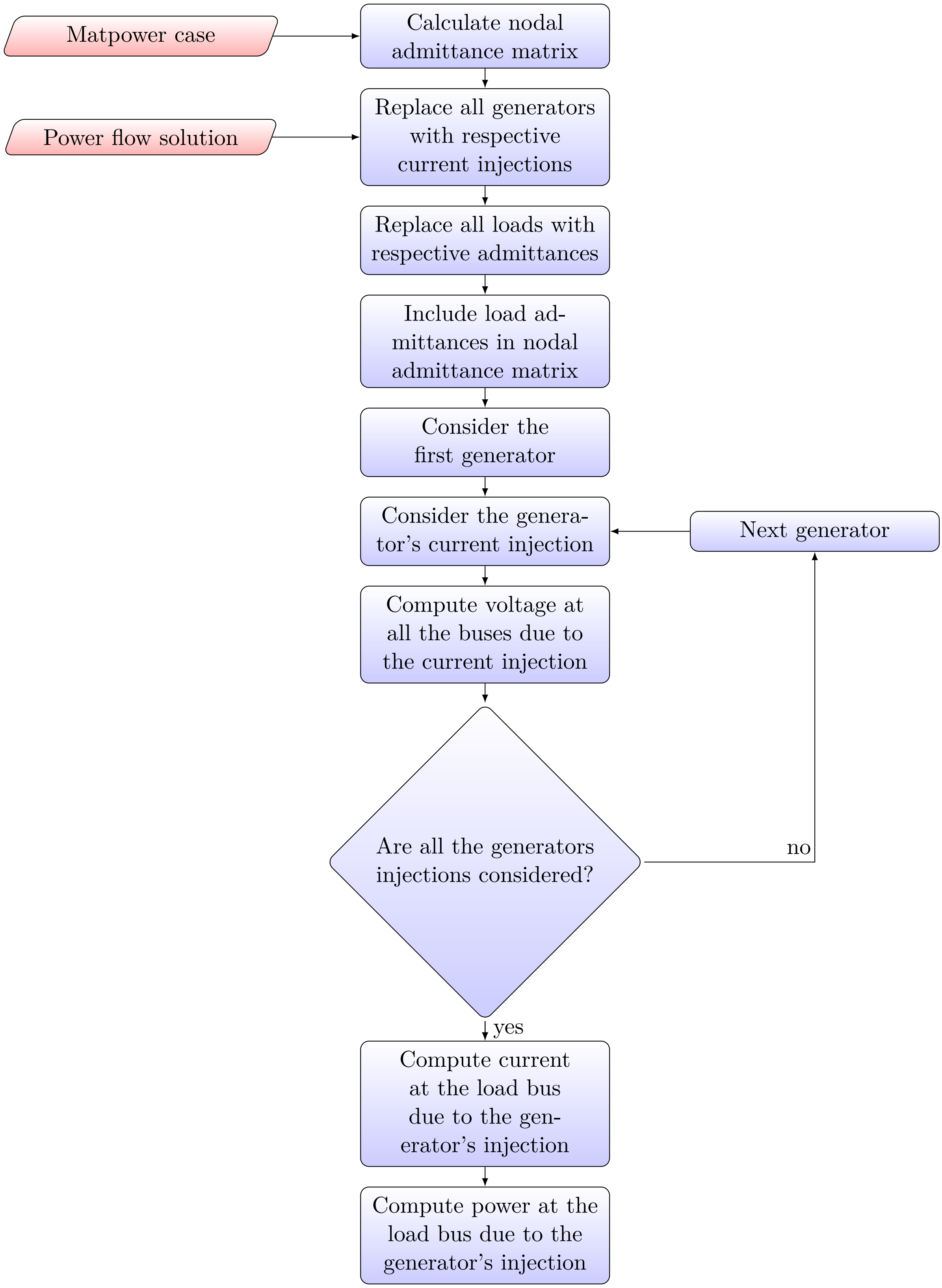


Figure 3-1: Flowchart for superposition method

Considering only one generator injection at a time, voltage at all the buses due to the generator injection are calculated as:

Element-wise:

$$v_j^{i,G} = Z_{matrixij} \times I_{i,G} \tag{3-9}$$

Matrix-wise:

$$\begin{matrix} Bus\ 1 \\ : \\ : \\ : \\ Bus\ n \end{matrix} \begin{bmatrix} v_1^{i,G} \\ : \\ : \\ : \\ v_n^{i,G} \end{bmatrix} =$$

$$\begin{matrix} \\ Bus\ 1 \\ : \\ : \\ : \\ Bus\ n \end{matrix} \begin{matrix} Bus\ 1 \quad .. \quad .. \quad .. \quad Bus\ n \\ \begin{bmatrix} & & : & & \\ & & : & & \\ .. & .. & Z_{matrix} & .. & .. \\ & & : & & \\ & & : & & \end{bmatrix} \end{matrix} \times \begin{matrix} 1 \\ : \\ : \\ : \\ Bus\ n \end{matrix} \begin{bmatrix} : \\ 0 \\ I_{i,G} \\ 0 \\ : \end{bmatrix}$$

Generator contribution to loads

Current at load j due to generator i is

$$i_{L,j}^{G,i} = v_{L,j}^{G,i} \times Y_{j,i} \tag{3-10}$$

The sum of voltage at bus j due to all generators add up to the actual bus voltage at node j(same as in the power flow solution).

$$V_j = \sum_{i=1}^{n_G} v_{L,j}^{G,i} \tag{3-11}$$

Contribution of the generator i to the power consumed by the load j is given by

$$P_{L,j}^{G,i} = real(V_j \times i_{L,j}^{G,i*}) \tag{3-12}$$

Generator contribution to branch losses

For a branch connected between nodes a and b with the branch admittance y_{ab},

Current through the branch from bus a to bus b due to the generator at bus i is

$$i_{ab}^{i} = (v_a^i - v_b^i)(y_{ab}) \tag{3-13}$$

Power flow in the branch from bus a to bus b due to the generator at bus i is

$$P_{ab}^{i} = real(V_a \times (i_{ab}^{i})^{*}) \tag{3-14}$$

Power flow in the branch from bus b to bus a due to the generator at bus i is

$$P_{ba}^{i} = real(V_b \times (i_{ab}^{i})^{*}) \tag{3-15}$$

Power loss in the branch due to the generator at bus i is

$$P_{loss,ab}^{i} = P_{ab}^{i} - P_{ba}^{i} \tag{3-16}$$

3-2-3 Need for scaling

Generator's contribution to power consumed by load is

$$\begin{aligned} P_{j,L}^{i,G} &= real(V_j i_j^{i*}) \\ &= real(V_j (V_j^i y_j)^{*}) \\ &= real(V_j (z_{ji} I_i y_j)^{*}) \end{aligned} \tag{3-17}$$

Power contribution from generator i to load $j \propto$

- Voltage at the load bus j, V_j
- Current injection at the generator bus i, I_i
- Load to generator element of Z_{bus} matrix, z_{ji}
- Load admittance, y_j

For a significantly high load consumption(P_j), the corresponding load admittance(y_j) is also large. Usually $y_j >> z_{ji}$. The calculation of generator to load power exchange gets dominated by load admittance with little role of the network impedance. This challenge is overcome by scaling all the network line reactances by the same factor. Scaling increases the weights of network reactances in power exchange calculations.

A physical equivalent of this step would be to elongate every line by an equal scaling factor. Ideally, a large or infinite scaling factor would be desirable.

The PEX matrix obtained from this method was found to disobey the PEX definitions. This will be elaborated in the chapter to follow. Due to the drawback, a new method was required.

3-3 PEX_{loss}: A performance indicator for the PEX matrix

Different ways of calculating power exchanges have been proposed by researchers. As the topic is gaining increased attention, many more PEX calculation methods are expected to be developed in the future.

As long as the resultant PEX matrix satisfies the following definitions of PEX, it is correct:

For generator i and load j, the power exchange between them is PEX_{ij}

- $PEX_{ij} \geq 0$
- $\sum_{i=1}^{n} PEX_{ij} = P_{G,i}$, Total production at node i
- $\sum_{j=1}^{n} PEX_{ij} = P_{D,j}$, Total consumption at node j

The only existing tool that facilitates the comparison of resulting $PEXs$ from different methods is their intuitiveness. Therefore, a quantifiable measure of a PEX's correctness is desired. A performance measure called PEX_{loss} is defined here.

Using equations Eq. (3-4) to Eq. (3-9), a matrix that provides voltages at all the loads due to each generator is obtained. This is called the voltage distribution matrix and is denoted by Υ.

The voltage at load node j caused by generator node i is given by Υ_{ij}. The electrical distance between the generator node i and load node j be given by $X_{th,ij}$. The power exchange between these 2 nodes is given by PEX_{ij}.

$$Loss_{ij} = \Big(\frac{PEX_{ij}}{\Upsilon_{ij}}\Big)^2 \times X_{th,ij} \tag{3-18}$$

For a network of n nodes,

$$PEX_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} Loss_{ij} \tag{3-19}$$

A lower value of PEX_{loss} indicates a better preforming PEX matrix[1] .

A PEX matrix with least PEX_{loss} is defined as the *optimal PEX* matrix. This method of calculating PEX is explained in Section 3-4.

3-4 The Optimal PEX method

In section 3-3, a performance metric called PEX_{loss} was defined that facilitates comparison of PEX matrices. Based on the original definitions of a PEX matrix, linear constraints for the elements of PEX matrices can also be defined. This section proposes a new method of calculating the generator-to-load power exchange by directly utilising the performance metric(which is in turn based on electrical distance) and a set of constraints. The objective of this method is to find a PEX matrix that has a minimal PEX_{loss}. The resulting PEX matrix will provide high power exchanges between nodes with low electrical distance and low power exchanges between nodes with high electrical distance.

3-4-1 PEX matrix calculation as a linear optimisation problem

Consider a network with n_G generator nodes and n_L load nodes. The PEX matrix has the order of $n_G \times n_L$.

[1]Note: $Loss_{ij}$ and PEX_{loss} serve as a metric for comparison of PEXs. They don't indicate the physical losses in the system.

Variables:

Every element of the PEX matrix is a variable whose optimal value needs to be determined. Therefore, number of variables = $n_G \times n_L$.

Objective function:

$$\text{Fitness function} = min(PEX_{loss}) \tag{3-20}$$

where

$$PEX_{loss} = \sum_{i=1}^{n_G} \sum_{j=1}^{n_L} Loss_{ij}$$

and

$$Loss_{ij} = \Big(\frac{PEX_{ij}}{\Upsilon_{ij}}\Big)^2 \times X_{th,ij}$$

PEX_{ij} is power exchanged between generator i and load j. $X_{th,ij}$ is the electrical distance between generator i and load j. Υ_{ij} is the voltage at load node j caused by generator node i.

Linear equality constraints:

For all generators,

$$\sum_{j=1}^{n_L} PEX_{ij} = P_{Gi} \tag{3-21}$$

and

for all loads,

$$\sum_{i=1}^{n_G} PEX_{ij} = P_{Lj} \tag{3-22}$$

Eq. (3-21) indicates that the MW contribution of generator i to all the loads add up to the actual MW production of the generator i(in alignment with load flow solution). Similarly, Eq. (3-22) indicates that the MW contribution of all the generators to the power consumed by load j add up to the actual MW consumption of load j(in alignment with load flow solution).

There exists $n_G + n_L$ equations that represent the nodal generation and the nodal consumption.

Bound constraints:

The lower bound is defined by the fact that the contribution of a generator to a load cannot be negative. The upper bound is defined by the fact that the contribution of a generator to a load has to be less than or equal to its nodal production and nodal consumption.

$$0 \leq PEX_{ij} \leq min(P_{Gi}, P_{Dj}) \tag{3-23}$$

3-4-2 Unsuitability of deterministic optimisation approach for PEX problem

The optimisation problem can be solved using a deterministic solver or a heuristic solver.

Deterministic solvers find the best solution to a problem by generating every possible solution over the solution space and evaluating each solution. This "exhaustive search" is bound to find the best solution eventually. For determining the best PEX matrix, an exhaustive search will have to generate each possible PEX matrix over the solution space of $n_G \times n_L$ dimensions and evaluate PEX_{loss} for each possibility. Due to the enormous number of PEX matrices possible, generating and evaluating each possibility won't be feasible. When the deterministic solvers attempt to solve such a wide solution spaced problem, they confine themselves to an area and/or a direction thereby making the likeliness of an accurate solution low. They either provide a local minimum as the best solution or just fail to converge.

To deal with this, it is desired to have an algorithm that chooses to evaluate a few points in every part of the solution domain instead of attempting to cover every possibility. It should traverse through the solution space in varying step sizes in multiple directions simultaneously. The varying step size and changing directions can be attained by incorporating a factor of arbitrariness. The randomness in the size and direction of steps will force the solver to make jumps to an unexplored area of the solution space. This increases the chances of landing in the vicinity of the global minimum. Genetic algorithm is one such solver.

3-4-3 Introduction to Genetic Algorithm

Genetic algorithm is a metaheuristic solver that mimics biological evolution to search through the solution space for an optimal solution. It begins with randomly generating a set of possible solutions. Each solution is called an individual. The set of individuals is called a population. The individuals carry the values of all variables which are called the chromosomes. The best performing individuals out of the population are selected as parents. The parents are bred to form a new generation of offspring. In the next generation, the offspring form the updated population. This entire process is repeated till a predefined stopping criteria is met. After multiple generations, the solver "evolves" towards the optimal solution.

3-4-4 Terminology relating genetic algorithm and PEX problem

Genetic algorithm	PEX problem	Optimisation
Chromosome	An element of the PEX matrix	A variable
Individual	A possible PEX matrix	A possible solution
Population	A set of possible PEX matrices	A set of possible solutions
Generation	Iteration	Iteration
Fitness: The optimal solution has the highest fitness.	PEX_{loss}: The optimal solution has the least PEX_{loss}	Objective function
Expectation	The value of PEX_{loss} transformed to a rank	-
Parent	Chosen PEX matrix	-
Offspring	A modified PEX matrix	-

3-4-5 Steps and operators

The algorithm works with maintaining a constant population size and updating the individuals in the population over multiple generations. Starting from a randomly generated diverse population in the first generation, it converges to a global minimum in the last generation. The process of updating the population is done using the genetic operators- selection, mutation and cross-over. This section describes each of the steps in the genetic algorithm process.

Initial population

The first step is to generate a population(i.e. a group of possible solutions) to the problem. It is critical to maintain diversity in the generated population. This is done by randomly generating multiple individuals that can lie anywhere in the solution space. A set of correct PEXs were generated in MATLAB using the linear programming function and a random number generator.

Fitness evaluation

The value of PEX_{loss} for each PEX matrix in the population is evaluated. This provides the fitness of the PEX matrices. Fitness values are transformed into expectations to provide a better scale of comparison for relative performances. These expectations are used further in the selection stage.

Selection

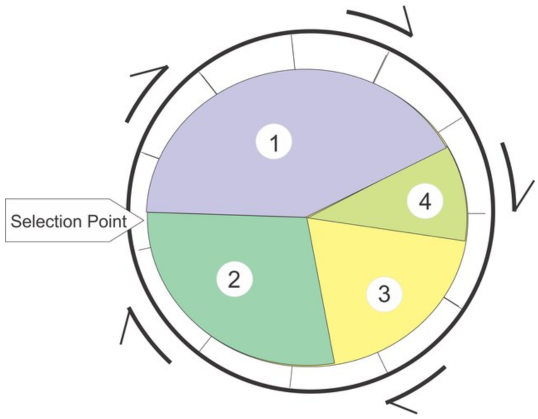


Figure 3-2: Selection based on roulette wheel

This stage chooses some individuals from the population to breed the offspring. Selection can be - extinctive or preservative. In extinctive selection, the strategy of "survival of the fittest" is used. This step improves the average quality of the population by letting the individuals with low fitness to die while it chooses the individuals with a high fitness to breed the offspring. Thus focusing on the region of the search space that contains the promising individuals. However, this also kills some individuals that could have provided a better offspring. Therefore, a preservative selection scheme is used. In preservative selection, even the low-fit individuals are allowed a small possibility to be selected as a parent. This way diversity is maintained. Based on the expectation, every individual is assigned a surface area on the roulette wheel. The roulette wheel is spun as many times as the number of parent required. In each spin, the individual the selection point lands on is chosen as a parent.

As the high-fit individual covers a larger surface area of the roulette wheel than the low-fit individual, the probability of the high-fit individual chosen as a parent is higher.

Children

The population of individuals are updated in every iteration by replacing the current population with the offspring population. Offspring are generated using 3 processes: elitism, cross-over and mutation.

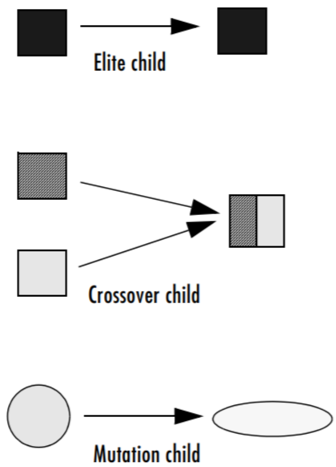


Figure 3-3: Process of generating offspring

- *Elitism:* Some of the most fit individuals of each generation are directly forwarded to the next generation without any genetic modification. These are the "elite individuals". This step is done to keep breeding the best fit individual in the following generations. However, retaining an elite individual can also cause loss of diversity as the super-fit individual keeps dominating. As a result, the algorithm can get stuck in a local minimum and converge prematurely.

- *Cross-over:* In the crossover process, offspring are generated by selecting 2 parents and combining their genetic information. The child may have a better or a worse fitness based on whether it inherited the desirable or the undesirable characteristics of the parents.

  $PEX_{child} = \frac{(w_1 * PEX_{mom}) + (w_2 * PEX_{dad})}{w_1 + w_2}$

  where w_1 and w_2 are weights assigned to the parent PEX matrices such that

  $0 < w_1 \leq 1$

  $0 < w_2 \leq 1$

- *Mutation:* It is the process of random modification of a chromosome(variable) in the individual. It is similar to an offspring being born with some abnormality. While it would most likely harm the functioning of the child, in rare situations, it also provides an improved capability to the offspring.

A random change of one variable will produce an incorrect PEX. While generating a new PEX matrix, care must be taken that it satisfies all the PEX definitions. Therefore, a delta-operator is applied to a randomly chosen pair of 2 generators and 2 loads. The rest of elements of PEX matrix remain unchanged.

$$PEX_{original} = \begin{array}{c} \\ \vdots \\ G_p \\ \vdots \\ G_q \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccc} ... & L_j & ... & L_k & ... \end{array} \\ \begin{bmatrix} & \vdots & & \vdots & \\ ... & PEX_{pj} & ... & PEX_{pk} & ... \\ & \vdots & & \vdots & \\ ... & PEX_{qj} & ... & PEX_{qk} & ... \\ & \vdots & & \vdots & \end{bmatrix} \end{array}$$

$$\text{Delta-operator} = \begin{bmatrix} \Delta & -\Delta \\ -\Delta & \Delta \end{bmatrix}$$

$$PEX_{mutated} = PEX_{original} + \text{Delta-operator}$$

$$= \begin{array}{c} \\ \vdots \\ G_p \\ \vdots \\ G_q \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccc} ... & L_j & ... & L_k & ... \end{array} \\ \begin{bmatrix} & \vdots & & \vdots & \\ ... & PEX_{pj} + \Delta & ... & PEX_{pk} - \Delta & ... \\ & \vdots & & \vdots & \\ ... & PEX_{qj} - \Delta & ... & PEX_{qk} + \Delta & ... \\ & \vdots & & \vdots & \end{bmatrix} \end{array}$$

Usage of this delta operator to mutate the PEX matrix preserves the data of row sum being equal to nodal generation and column sum being equal to nodal consumption. However, care still needs to be taken that the third definition of non-negative PEX elements is also complied with.

The appropriate value of Δ in the delta operator is based on the values of the elements in the sub-PEX matrix.

PEX element values	Value of Δ
All PEX elements are positive: PEX_{pj},PEX_{pk}, PEX_{qj} and PEX_{qk} are positive	Δ_1=-min(PEX_{pj},PEX_{qk}) Δ_2= min(PEX_{pk},PEX_{qj}) Δ is a random number chosen between Δ_1 and Δ_2
PEX_{pj} and PEX_{qk} are positive	Δ_1=-min(PEX_{pj},PEX_{qk}) Δ is a random number chosen between Δ_1 and 0.
PEX_{pk} and PEX_{qj} are positive	Δ_2=min(PEX_{pk},PEX_{qj}) Δ is a random number chosen between 0 and Δ_2.
Other conditions	NA. Can't apply mutation without violating PEX definition.

Stopping criterion

The iterations are stopped when the specified number of iterations are reached or when no variation in PEX_{loss} of the best individual occurs over multiple iterations. This thesis used the specific number of iterations.

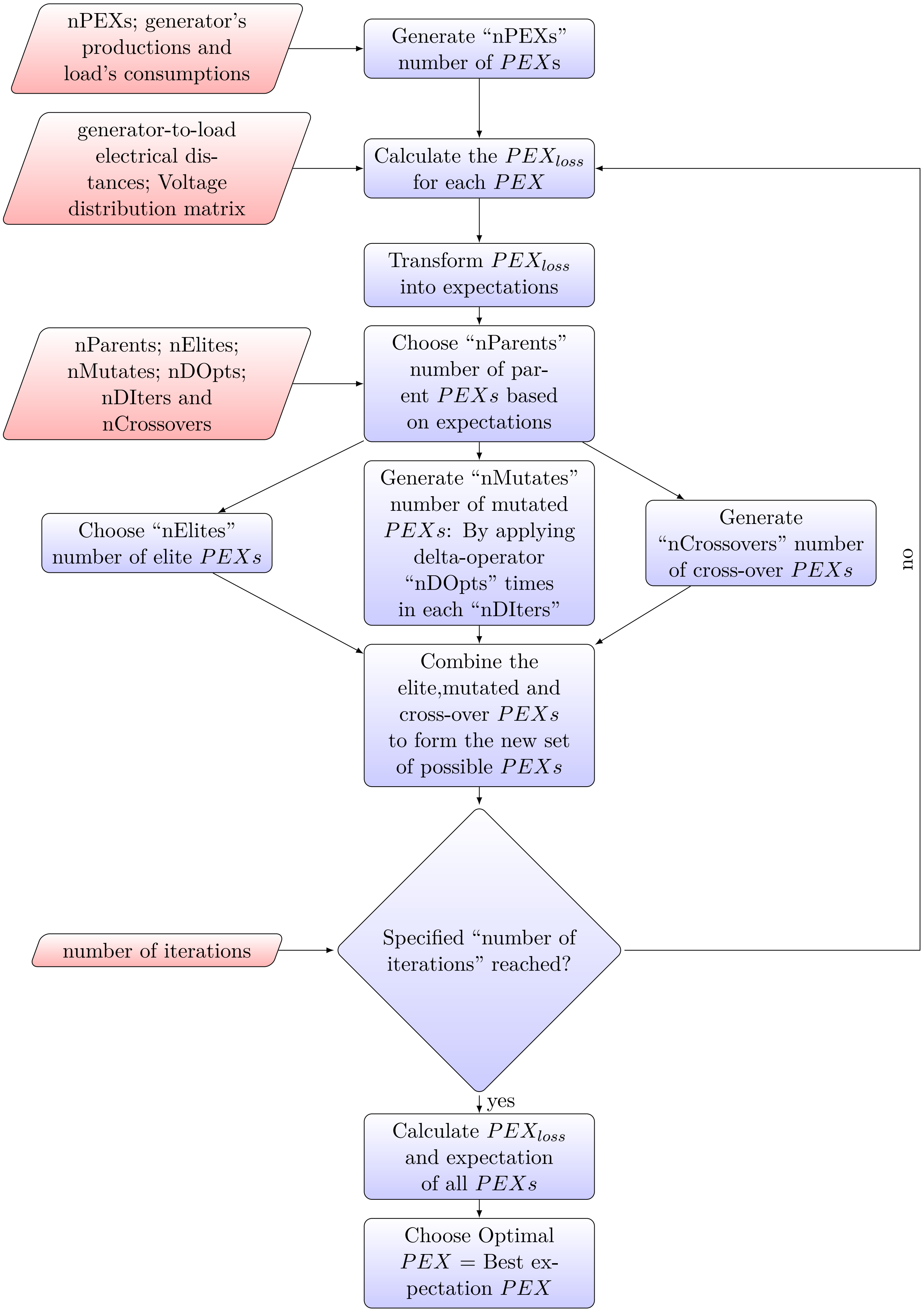


Figure 3-4: Flowchart for PEX computation using Genetic Algorithm

Chapter 4

Results and analysis

This section shows the results of the analyses of the methods proposed in Chapter 3. Both methods were implemented in MATLAB 2017a. The system used a i5-7200U CPU @2.50GHz 2.71GHz. Section 4-1 describes the application of the superposition method to the 4-bus test system. Section 4-2 shows the results of the existing and proposed methods for the IEEE 30 bus test system. Section 4-3 shows the results of the superposition method for a snapshot of the Netherlands transmission network.

4-1 4 bus test system

Figure 4-1 shows a simple 4 bus test system that is used to explain the shortcoming of the superposition method. The system has 2 generators and 2 loads of 100MW and 200MW. Further data corresponding to the buses and branches is found in the power flow solution (Table 4-1,Table 4-2).

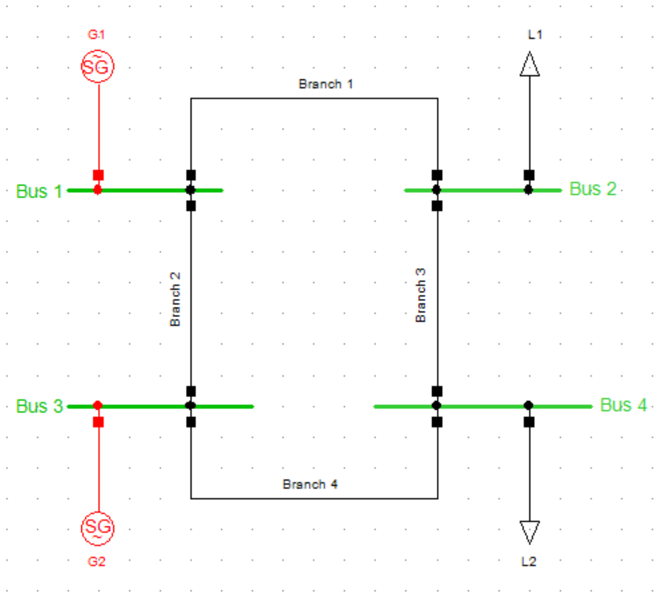


Figure 4-1: 4 bus test network

Bus number	Bus type	Power demand	Power generated	Voltage magnitude	Voltage angle
1	Slack	0	200	1	0
2	PQ	100	0	0.9913	-7.1735
3	PV	0	100	1	-2.3786
4	PQ	200	0	0.9913	-9.5732

Table 4-1: Power flow solution- Bus data

Branch number	From Bus	To Bus	Resistance	Reactance	Susceptance	Power flow
2	1	3	0	0.0826	0	50.2175
4	3	4	0	0.0826	0	150.2175
1	1	2	0	0.0826	0	149.7825
3	2	4	0	0.0826	0	49.7825

Table 4-2: Power flow solution- Branch data

The PEX matrix for the 4-bus system is shown in Table 4-3. It is obtained by using the superposition method's procedure as described in Section 3-2.

Power EXchange (in MW)		Load L1: Bus 2	Load L2: Bus 4	Production of generator
Generator	G1: Bus 1	66.492	132.693	199.185
	G2: Bus 3	33.508	67.307	100.815
Consumption of load		100.00	200.00	Total exchange: 300

Table 4-3: Power EXchange matrix for 4-bus system using superposition method

The first step is to check whether the obtained PEX matrix is "correct" as per the PEX definitions:

All the elements of the PEX matrix are non-negative	✓
Contributions of G1 and G2 to L1 add up to 100MW Contributions of G1 and G2 to L2 add up to 200MW	✓
Contributions of G1 to L1 and L2 add up to 199.185MW Contributions of G2 to L1 and L2 add up to 100.815MW	×

As per the obtained PEX, generator G1 produces 0.815MW less than it is supposed to and generator G2 produces 0.815MW more than it is should.

Generator's contribution to branch power loss (in MW)		Branch 1	Branch 2	Branch 3	Branch 4	Total branch loss
Generator	G1	0.4951	-0.3198	3.145e-06	0.6397	0.815
	G2	-0.4951	0.3198	-3.145e-06	-0.6397	-0.815
		0	0	0	0	0

Table 4-4: Contribution of the generators to the active power loss in the branches

Table 4-4 shows the contribution of the generators to the MW losses in individual branches using the formulas in Section 3-3.

PEX (in MW)		loads L1	L2	Total branch loss loss	Production of generator
generators	G1	66.492	132.693	0.815	200
	G2	33.508	67.307	-0.815	100
consumption of load		100.00	200.00	0	Total Exchange: 300

Table 4-5: Generators contributions to loads and total branch losses

Appending the contribution of generator to loads and line losses(in Table 4-5) provides the correct productions for the generators.

Generator G1 feeds 199.185MW to the loads and 0.815MW to the branch losses. Generator G2 feeds 100.815MW to the loads and -0.815MW to the branch losses.

Given the context that the lines are purely inductive, the positive and negative MW contributions of generators to the non-existent active power loss is a strange phenomenon. The reason for this becomes evident upon looking at the equations used for calculating generator's contribution to branch power loss.

Non-zero power loss over zero resistance line

This subsection describes how the positive and negative contributions to the zero branch loss occur due to a characteristic of the equations used.

Case(i) shows that the active power loss in an inductive branch is zero when all generators are present.

Case(ii) shows that the active power loss in an inductive branch becomes non-zero when only one generator is active.

Case(i): <u>All the generators are present and active</u>

Consider a branch connected across nodes a and b with reactance X_{ab}:

i_{ab} is the current flowing in branch ab when all generators are active.

V_a and V_b are the voltages at buses a and b respectively (in accordance with bus voltage obtained from power flow solution).

θ_a and θ_b are the voltages angles at buses a and b respectively.

$$
\begin{aligned}
P_{lossab} &= real((V_a\angle\theta_a - V_b\angle\theta_b) \times i_{ab}^*) \\
&= real\Big((V_a\angle\theta_a - V_b\angle\theta_b) \times \Big(\frac{V_a\angle\theta_a - V_b\angle\theta_b}{X_{ab}\angle 90^\circ}\Big)^*\Big) \\
&= real\Big(\frac{(V_a - V_b)^2}{X_{ab}\angle -90^\circ}(\angle\theta_a - \angle\theta_b - \angle\theta_a + \angle\theta_b)\Big) \qquad (4\text{-}1) \\
&= \frac{(V_a - V_b)^2}{X_{ab}}\cos(90^\circ) \\
&= 0 \qquad (4\text{-}2)
\end{aligned}
$$

The actual power loss over a line is the real part of the product of the voltage drop across the line and the conjugate of the current through the line. Because voltage drop causes current flow, the angular difference between the voltage and conjugated current cancel out. Hence the phase angle for power flow in the line is determined by the line impedance. For a purely-inductive branch, this is 90°. Therefore the active power loss in the line is zero.

Case(ii): <u>When only one generator G is active</u>

Consider a branch connected across nodes a and b with reactance X_{ab}:

i_{ab}^G is the current flowing in the line when only one generator G is active.

V_a^G and V_b^G are the voltages at buses a and b respectively caused by generator G alone.

θ_a^G and θ_b^G are the voltage angles at buses a and b respectively caused by generator G alone.

$$
\begin{aligned}
P_{lossab} &= real((V_a\angle\theta_a - V_b\angle\theta_b) \times i_{ab}^{G*}) \\
&= real\Big((V_a\angle\theta_a - V_b\angle\theta_b) \times \Big(\frac{V_a^G\angle\theta_a^G - V_b^G\angle\theta_b^G}{X_{ab}\angle 90^\circ}\Big)^*\Big) \\
&= real\Big(\frac{(V_a - V_b)(V_a^G - V_b^G)}{X_{ab}\angle -90^\circ}(\angle\theta_a - \angle\theta_b - \angle\theta_a^G + \angle\theta_b^G)\Big) \\
&= \frac{(V_a - V_b)(V_a^G - V_b^G)}{X_{ab}}\cos(90^\circ + \angle\theta_a - \angle\theta_b - \angle\theta_a^G + \angle\theta_b^G) \\
&\neq 0 \qquad (4\text{-}3)
\end{aligned}
$$

When only one generator is active, voltage drop due to one generator alone is considered to calculate the current through the line. The angle between voltage due to all generators and conjugated current due to one generator only don't cancel out. This angular difference of $\angle\theta_a - \angle\theta_b - \angle\theta_a^G + \angle\theta_b^G$ is positive or negative which adds to the otherwise 90° voltage-current phase displacement. When the V-I phase displacement is less than 90°, the inductive branch behaves like a series RL branch. The resistive part of the branch causes I^2R losses to which the generator provides a positive MW contribution. When the V-I phase displacement is above 90°, the inductive branch behaves as a RL branch where the resistance is negative. It's impossible to have a negative resistance. However, being the only plausible explanation, the negative resistance in the line produces power that is fed to the generator. Or, the generator contributes negative power to the line. This is an irrational behaviour of the method is caused by using equations that mix values from two different cases.

Workaround

This section describes how the contribution of generators to line losses can be distributed among the load proportionally.

Consider the PEX matrix where the contribution of the generators to the loads and the branch losses are appended.

$PEX=$

$$\begin{array}{c} \\ G_p \\ G_q \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccc} L_j & L_k & ... & losses \end{array} \\ \begin{bmatrix} PEX_{pj} & PEX_{pk} & ... & loss_p \\ PEX_{qj} & PEX_{qk} & ... & loss_q \\ ... & ... & ... & ... \end{bmatrix} \end{array}$$

If PEX_{mn} is an element of the PEX matrix, P_n is the power consumed by load n, and TC is the total power consumption by all the loads,

$$P_n = \sum_{m=1}^{nG} PEX_{mn}$$

$$TC = \sum_{n=1}^{nL} \sum_{m=1}^{nG} PEX_{mn} = \sum_{n=1}^{nL} P_n$$

$$\sum_{i=1}^{nG} loss_i = 0$$

Forcing the losses to be distributed among the PEX elements proportionally gives

$PEX_{new}=$

$$\begin{array}{c} \\ G_p \\ G_q \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} L_j & L_k \end{array} \\ \begin{bmatrix} PEX_{pj} + \frac{P_j \times loss_p}{TC} & PEX_{pk} + \frac{P_k \times loss_p}{TC} & ... \\ PEX_{qj} + \frac{P_j \times loss_q}{TC} & PEX_{qk} + \frac{P_k \times loss_q}{TC} & ... \\ ... & ... & ... \end{bmatrix} \end{array}$$

PEX_{new} (in MW)		Loads L1: bus 2	L2: bus 4	Production of generator
Generators	G1: bus 1	66.764	133.236	200
	G2: bus 3	33.236	66.764	100
Consumption of load		100.00	200.00	Total exchange: 300

After applying the workaround, the generated PEX_{new} for the 4 bus test system satisfies all PEX definitions:

All the elements of the PEX matrix are non-negative	✓
Contributions of G1 and G2 to L1 add up to 100MW Contributions of G1 and G2 to L2 add up to 200MW	✓
Contributions of G1 to L1 and L2 add up to 200MW Contributions of G2 to L1 and L2 add up to 100MW	✓

This workaround is not to be used as a solution as it doesn't actually fix the problem in the superposition method. This step is only done to check what results the superposition method would provide if it were to produce a correct PEX. Further, this fix could produce negative power exchanges if the fictitious losses are high. This will be shown in section 4-3.

4-2 IEEE 30 bus system

This subsection compares the generator to the load contributions of the Equivalent Bilateral Exchanges method, the superposition method, the optimal PEX method and the Bialek method.

The IEEE 30 Bus system is used as a test case. It represents a portion of the American Electric Power System (in the Midwestern US) as of December, 1961. It contains 41 branches, 6 generators and 18 loads. Further data about the system can be found in the load flow solution and Thévenin impedance matrix in Appendix A-1.

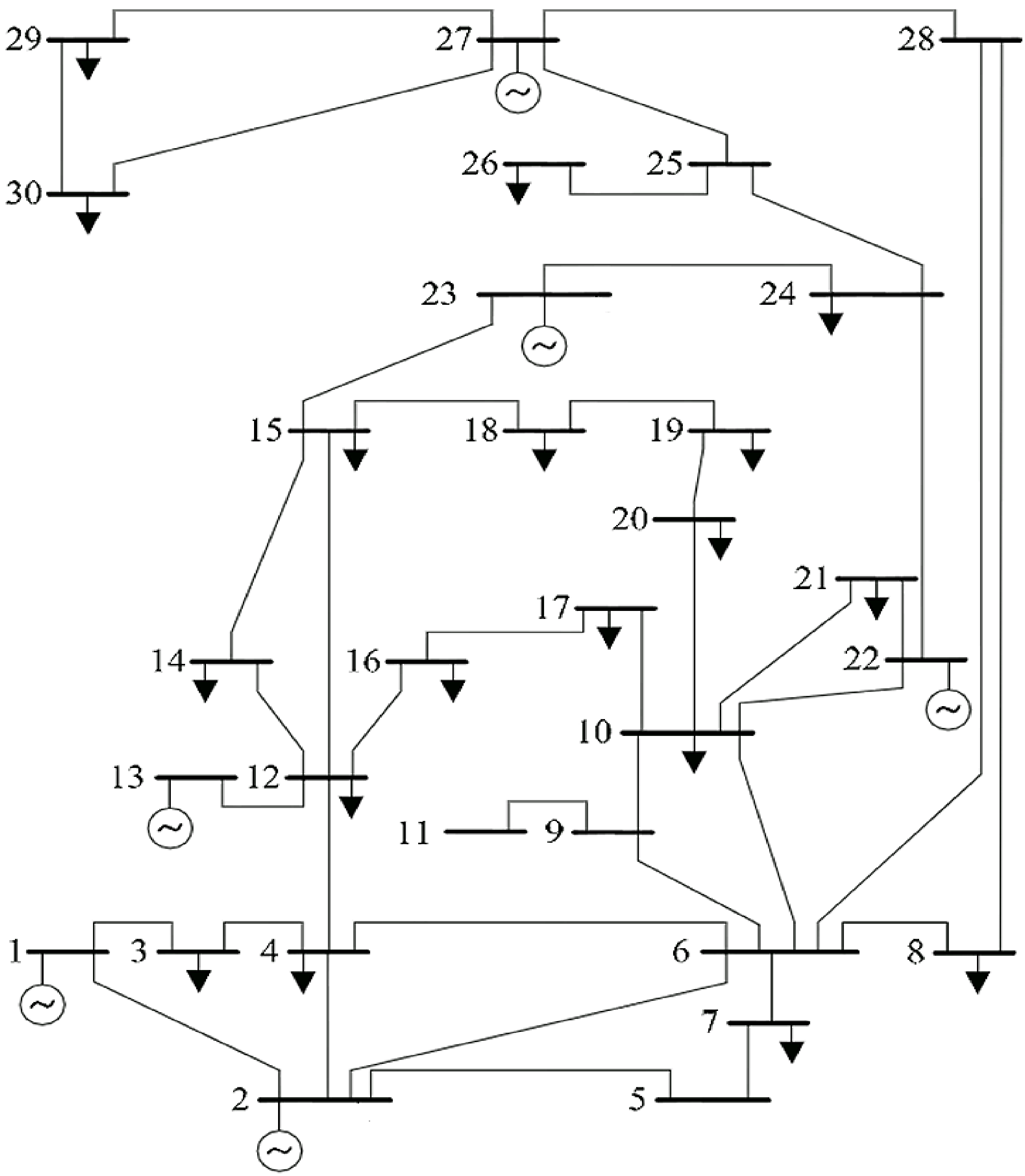


Figure 4-2: IEEE 30 bus test system

The optimal PEX method was implemented using the genetic algorithm.

The input parameters were:

$$\begin{aligned} \text{nPEXs} &= 500 \\ \text{nParents} &= 50 \\ \text{nElites} &= 15 \\ \text{nMutates} &= 15 \\ \text{nDOpts} &= 5 \\ \text{nDIters} &= 5 \\ \text{nCrossovers} &= 5 \\ \text{number of iterations} &= 2500 \end{aligned}$$

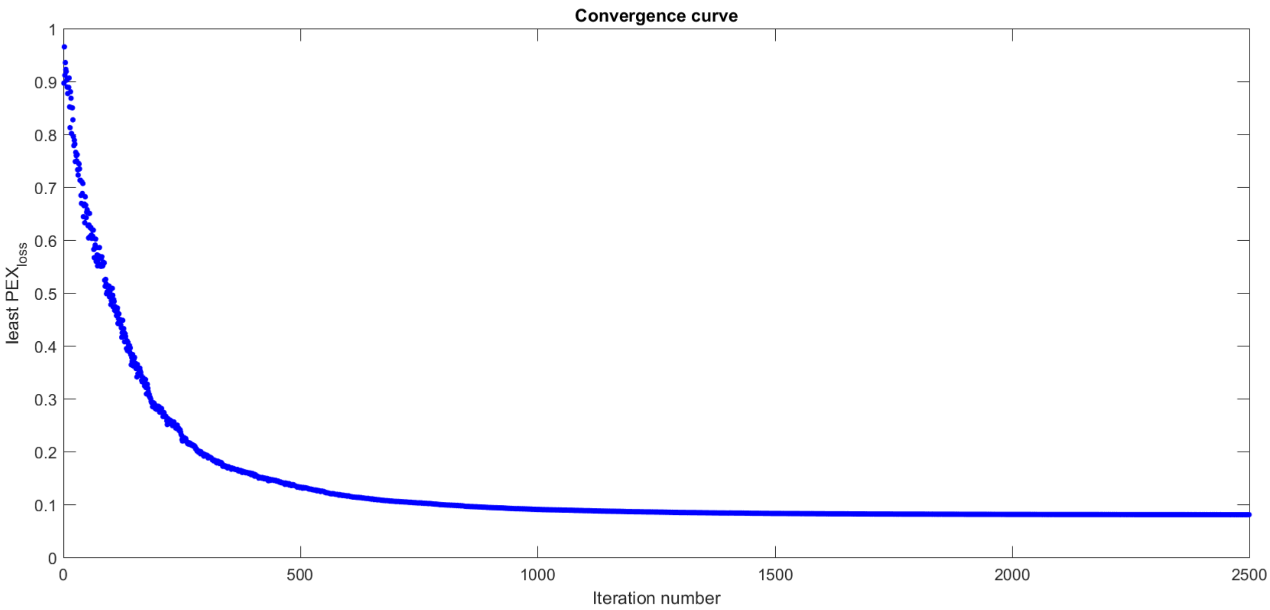


Figure 4-3: Convergence curve for IEEE 30 bus test case

This implementation consumed roughly 1.5 hrs. Figure 4-3 shows the value of PEX_{loss} after every iteration for the given input parameters. The solution is not a global minimum as the convergence curve shows a non-steady PEX_{loss} value if the y-axis is zoomed-in to the 6^{th} decimal point.

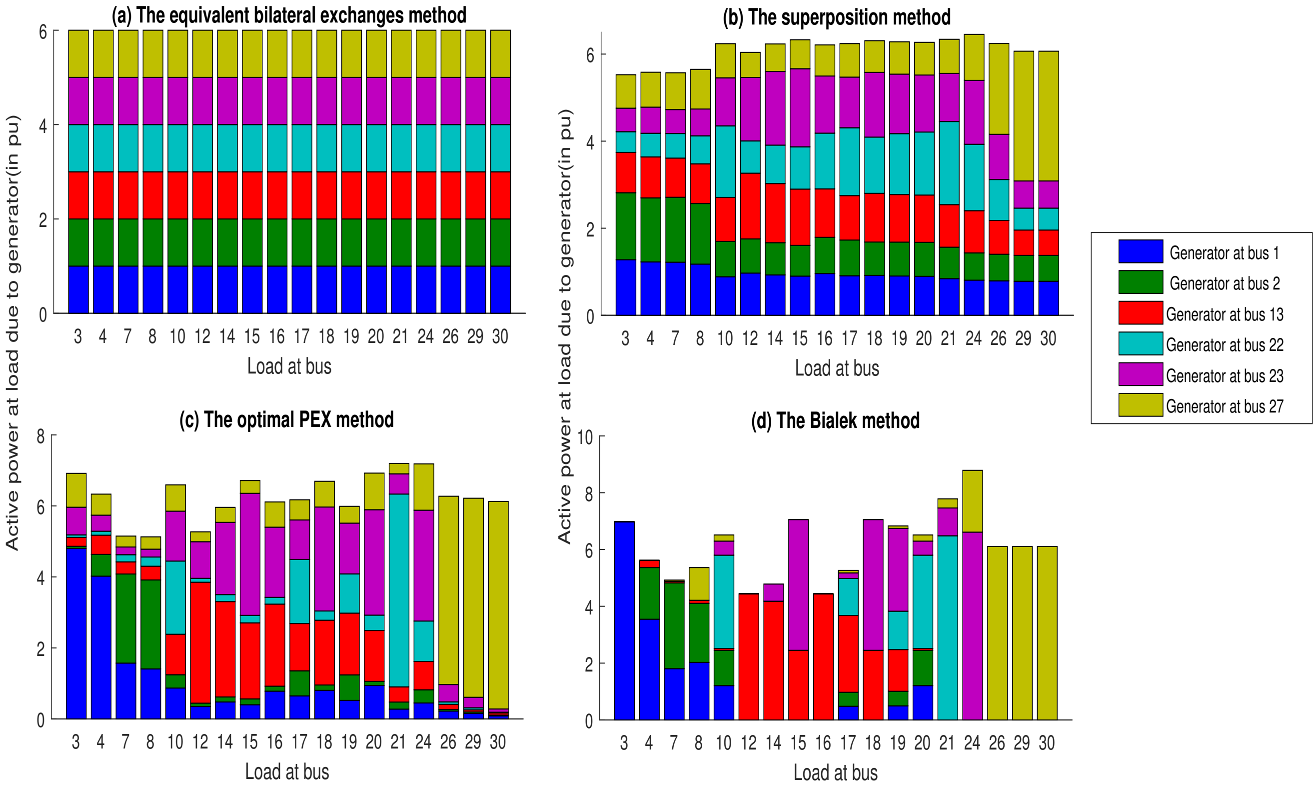


Figure 4-4: Comparison of PEX elements in IEEE 30 bus system

When the generations and the consumptions are high, the power exchanges are high. To see the distribution of the power exchanges independent of the productions and demands, the power exchanges are normalised. Distributing every generator's production proportionally among the all loads gives the base PEX. Normalisation is done by dividing the obtained power exchanges by the base PEX.

Figure 4-4 shows the normalised contribution of all the generators to the power consumed at the loads. Along the x-axis is the bus at which the load is located. The contributions are coloured based on the generator. The corresponding colours are mentioned in the legend.

Figure 4-4(a) shows the result of the method of Equivalent Bilateral Exchanges(EBE). The method of EBE also computes the contribution of generator to loads proportionally. This method does not take into consideration the topology of the network or the electrical distance between the two points in the network. Therefore, normalised power exchanges show equal shares of every generator to every load.

Figure 4-4(b) shows the result of the superposition method. The normalised generator-to-load contribution considers the electrical distance. The consideration of electrical distance is evident from, for example, load 30. Load 30 gets the highest contribution from the generator at bus 27 due to the electrical proximity of bus 27 and bus 30. Next, load 21 gets the highest power contribution from generator at bus 22 as the buses have the least electrical distance with respect to each other. However, due to the extreme electrical proximity of buses 21 and 22, even higher share of generator 22 in the consumption of load 21 is expected. It is seen that the results of superposition method still remains close to the method of equivalent bilateral exchanges. Furthermore, the implementation of superposition method utilised a workaround that remains ambiguous.

Figure 4-4(c) shows the result of the optimal PEX method. The normalised generator-to-load contribution is based on electrical distance in the optimal PEX method. The loads get the highest shares from the generators located electrically closest to them. The loads also receive a contribution from the other generators based on their proximity. This indicates the success of the method in considering electrical proximity. The optimisation procedure is implemented with a genetic algorithm. A possibility remains that a better solution(a global minimum) can be found with the use of advanced optimisation tools.

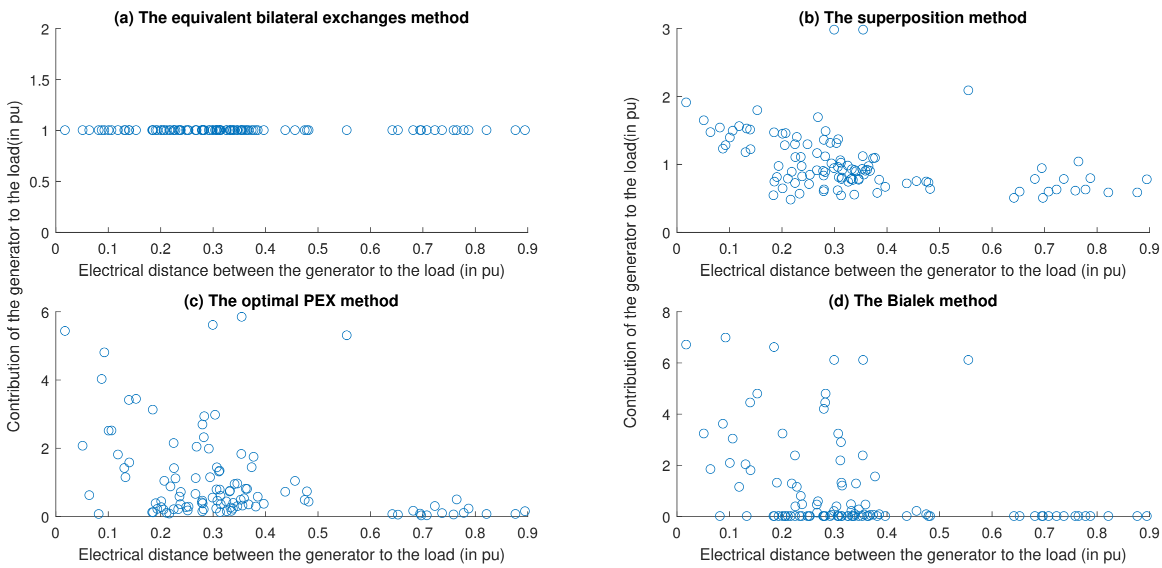


Figure 4-5: PEX between generators and loads vs electrical distance between them

Because the Bialek method considers electrical distances for finding power exchanges, a PEX-vs-electrical distance characteristic similar to Bialek's is expected from the optimal PEX method. However, it should eliminate the directional effect of Bialek where some exchanges are forced to zero and others are much higher.

Figure 4-5 shows the relation between the generator-to-load power exchanges and the electrical distance between them for the 4 methods.

Figure 4-5(a) shows the result for the method of equivalent bilateral exchanges. As mentioned before, this method is independent of the electrical distance. Hence, no variation in PEX elements with the electrical distance is seen.

Figure 4-5(b) shows the result for the superposition method. A relation between power exchange and electrical distance is not evident though there are slightly higher exchanges at electrical distances between 0-0.2 pu and reduced exchanges between 0.6-0.9 pu.

Figure 4-5(c) shows the result for the optimal PEX method. A few exchanges exist at electrical distances below 0.1 pu. Most power exchanges occur in the electrical distance of 0.1

-0.5 pu. In 0.1-0.5 pu electrical distance range, most of these power exchanges are between 0-3 pu while 3 power exchanges are higher. These higher power exchanges correspond to the contribution of generator 27 to load 26, load 29 and load 30. In the range of 0.6-0.9 pu electrical distances, relatively low power exchanges exist.

Method	PEX_{loss} in pu
Equivalent Bilateral exchanges	0.2106
Bialek	0.1195
Superposition	0.1414
Optimal PEX	0.0805

Table 4-6: Performance of various methods

Table 4-6 shows the value of the performance metric for the different methods of calculating PEX. Since the PEX_{loss} metric gives high importance to electrical distance, the Equivalent Bilateral Exchange method has the highest PEX_{loss} as this method doesn't consider electrical distance. Superposition method has comparatively lower PEX_{loss}. The next lower loss occurs in Bialek method. The optimal PEX method succeeds in its attempt to find a PEX matrix of least PEX_{loss}. As compared to the Bialek method, in the optimal PEX method the high exchanges get reduced and the zero exchanges raised. It is to be noted that the result can be further improved for the optimal PEX method by using an advanced optimisation solver.

4-3 The Netherlands transmission network

A snapshot of the transmission network of The Netherlands on September 10^{th} 2018 was used as the test case. It contained 91 generators and 1479 loads. If the optimal PEX algorithm is applied on this test network, it needs to find the optimal values of 134,589 variables. The genetic algorithm in this thesis was not efficient enough to generate a diverse initial population and to traverse through the large solution space to find the global minimum. However, the equivalent bilateral exchange method, the superposition method and the Bialek could be tested for this network.

The power exchanges using the method of equivalent bilateral exchanges, the superposition method and the Bialek method were computed for the snapshot of The Netherlands. Figure 4-6 shows a plot of the elements of the normalised power exchange matrix along the y-axis and the corresponding electrical distances along the x-axis. From such a PEX-vs-*electrical distance* curve, it is expected to see the relation that low electrical distances have high power exchanges and as the electrical distance increases, power exchanges reduce.

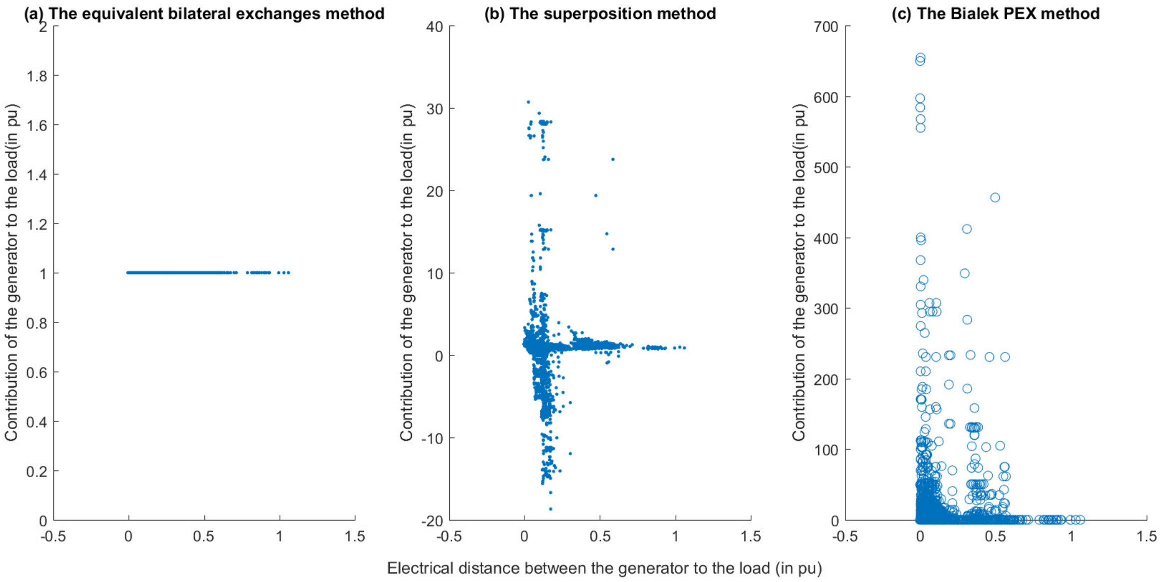


Figure 4-6: PEX vs electrical distance for The Netherlands

Figure 4-6(a) shows that the normalised power exchanges are independent of the electrical distance when PEX is calculated using the Equivalent Bilateral Exchanges method.

Figure 4-6(b) shows the result for the superposition method. Some generators provide a negative contribution to the loads. This is because the negative contributions of the generators to the total line losses are forcibly pushed into the PEX matrix in the workaround fix. It is seen that the superposition method doesn't meet the requirements of the desired PEX matrix. This proves the workaround fix and hence the superposition method to be unusable.

Figure 4-6(c) shows the result for the superposition method. As already known, Bialek power exchanges obey the basic expectations of a PEX matrix. The power exchanges have no negative values, the contributions of the generators to all the loads and contribution of all the generators to the loads matched the nodal productions and consumptions. Further, with rising electrical distance, a decrease in normalised power exchanges are observed. However, there also exist some zero power exchanges due to the directional effect. A few nodes also exchange significantly high powers at low electrical distances.

Chapter 5

Conclusion and recommendations

5-1 Conclusions

This thesis compared 2 well-established methods and 2 new methods. The first method, Equivalent Bilateral Exchanges, computes power exchange by giving no significance to the network topology whatsoever. The second method, the Bialek method, considers the electrical distance. But the result produced can have zero-exchanges due to the modelling of the electrical network as a directed graph. The third method, the Superposition method also considers the electrical distance. However, the equations used produce fictitious losses. Since the losses aren't physically present, a meaningful method to compensate for these losses was not developed. All three of these methods are computationally efficient. The final method, the optimal PEX method, produced results that are compliant with the expectations of this thesis. Therefore, the optimal PEX method serves as a proof of concept. However, it is computationally inefficient at this point as the genetic algorithm used doesn't find the global minimum despite many iterations. It requires further study and development to be made fit for use in the transmission network of Europe.

The research questions stated in Chapter 1 were answered as follows:

1. How to quantify electrical distance between any two nodes for a generic network?

   Electrical distance is the Thévenin impedance between two points. It was found to have a roughly linear correlation with the geographical distance. This work was performed during the MSc internship.

2. Are any of the previously developed methods of computing power exchanges based on electrical distance? If such a method is identified but it has a drawback, can it be rectified?

   The superposition method was found to utilise electrical distances indirectly. It was expected that the issue with the method would be the value of the load impedance becoming more significant than the network impedances in the power exchange computation. However, a flaw in the equations of the method was found that could not be rectified; rendering the method(in its original form or with the workaround) unusable. The initial steps of the superposition method were used to compute the voltage distribution at loads due to each generator. This matrix was one of the inputs to the optimal PEX method.

3. How to define a metric for comparison of various methodologies?

   A performance metric called PEX_{loss} was defined that facilitated the comparison of various PEX calculation methodologies. It utilises three matrices: the PEX matrix, Thévenin impedance matrix and voltage distribution matrix.

5-2 Recommendations

The optimal PEX method was implemented using the heuristic genetic algorithm. As the algorithm didn't succeed in finding the global minimum after a significant computation time, it is recommended to utilise other advanced or hybrid heuristic algorithms.

Figure 5-1 provides some other optimisation algorithms available. A study on the suitable heuristic algorithm for the PEX calculation problem can be conducted. The intention being to find the global minimum and reduce the computation time.

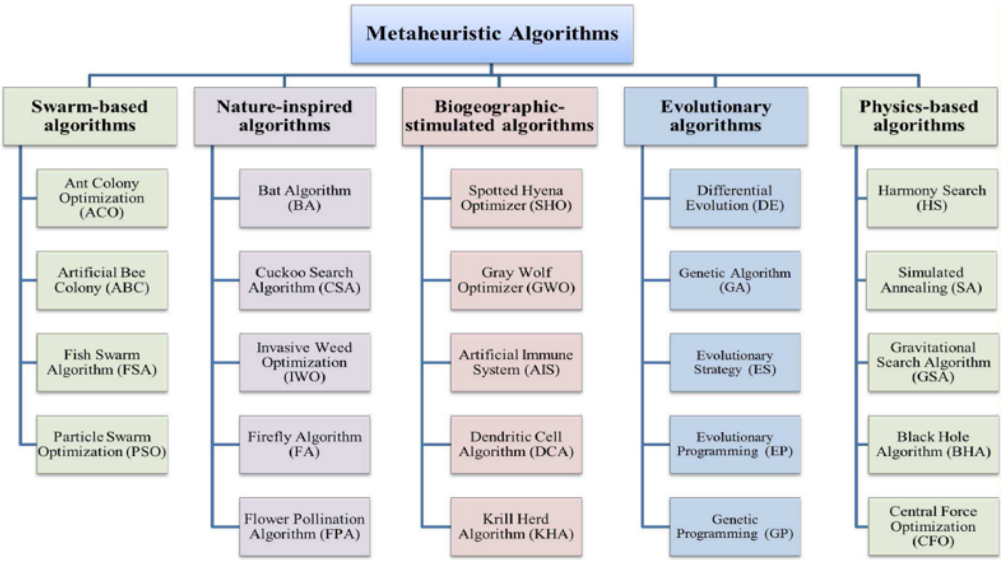


Figure 5-1: Options for future work

Appendix A

IEEE 30 bus test case

Bus number	Bus type	Power demand	Power generated	Voltage magnitude	Voltage angle
1	Slack	0	23.5386	1	0
2	PV	0	39.2676	1.00000	-0.31550
3	PQ	2.4	0	0.99930	-1.56467
4	PQ	7.6	0	0.99925	-1.83920
5	PQ	0	0	0.99926	-1.84416
6	PQ	0	0	0.99920	-2.32722
7	PQ	22.8	0	0.99915	-2.76208
8	PQ	30	0	0.99915	-2.89531
9	PQ	0	0	0.99943	-2.90567
10	PQ	5.8	0	0.99960	-3.20854
11	PQ	0	0	0.99943	-2.90567
12	PQ	11.2	0	0.99890	-1.64977
13	PQ	0	36.9977	1.00000	1.32256
14	PQ	6.2	0	0.99896	-2.46378
15	PQ	8.2	0	0.99909	-2.37796
16	PQ	3.5	0	0.99905	-2.71996
17	PQ	9	0	0.99942	-3.35446
18	PQ	3.2	0	0.99904	-3.55671
19	PQ	9.5	0	0.99917	-4.01436
20	PQ	2.2	0	0.99926	-3.87912
21	PQ	17.5	0	0.99990	-3.09141
22	PV	0	21.5887	1	-2.85740
23	PV	0	15.9990	1.00000	-1.40021
24	PQ	8.7	0	0.99985	-2.55680
25	PQ	0	0	0.99977	-1.77377
26	PQ	3.5	0	0.99968	-2.53625
27	PV	0	26.9084	1.00000	-0.85431
28	PQ	0	0	0.99923	-2.29227
29	PQ	2.4	0	0.99938	-2.30910
30	PQ	10.6	0	0.99920	-3.24926

Table A-1: Power flow solution- Bus data

From bus	To bus	Resistance	Reactance	Susceptance	Power flow
1	2	0	0.060	0	9.17735
1	3	0	0.190	0	14.3612
2	4	0	0.170	0	15.6297
3	4	0	0.040	0	11.9612
2	5	0	0.200	0	13.3286
2	6	0	0.180	0	19.4865
4	6	0	0.040	0	21.2602
5	7	0	0.120	0	13.3286
6	7	0	0.080	0	9.47134
6	8	0	0.040	0	24.7462
6	9	0	0.210	0	4.80087
6	10	0	0.560	0	2.74335
9	11	0	0.210	0	0
9	10	0	0.110	0	4.80087
12	4	0	0.260	0	1.26928
13	12	0	0.140	0	36.9977
12	14	0	0.260	0	5.45248
12	15	0	0.130	0	9.75647
12	16	0	0.200	0	9.31951
15	14	0	0.200	0	0.74751
16	17	0	0.190	0	5.81951
15	18	0	0.220	0	9.33323
18	19	0	0.130	0	6.13323
20	19	0	0.070	0	3.36677
10	20	0	0.210	0	5.56677
10	17	0	0.080	0	3.18049
21	10	0	0.070	0	2.91907
22	10	0	0.150	0	4.08402
22	21	0	0.020	0	20.4190
23	15	0	0.200	0	8.52427
24	22	0	0.180	0	2.91434
23	24	0	0.270	0	7.47475
25	24	0	0.330	0	4.13960
25	26	0	0.380	0	3.50000
27	25	0	0.210	0	7.63960
27	28	0	0.400	0	6.26877
27	29	0	0.420	0	6.04103
27	30	0	0.600	0	6.95896
29	30	0	0.450	0	3.64103
28	8	0	0.200	0	5.25383
28	6	0	0.060	0	1.01494

Table A-2: Power flow solution- Branch data

Electrical distance (in pu)		Loads								
		Bus 3	Bus 4	Bus 7	Bus 8	Bus 10	Bus 12	Bus 14	Bus 15	Bus 16
Generators	Bus 1	0.0932	0.0881	0.1407	0.1312	0.2193	0.2381	0.3533	0.2789	0.3071
	Bus 2	0.0824	0.0645	0.1068	0.1014	0.1910	0.2119	0.3268	0.2522	0.2800
	Bus 13	0.3337	0.2996	0.3682	0.3390	0.2949	0.1400	0.2803	0.2254	0.2829
	Bus 22	0.2171	0.1844	0.2341	0.2018	0.0517	0.1862	0.2828	0.1942	0.2060
	Bus 23	0.3133	0.2800	0.3384	0.3067	0.2261	0.2076	0.2691	0.1536	0.2925
	Bus 27	0.3468	0.3147	0.3567	0.3152	0.3329	0.3821	0.4827	0.3973	0.4382
		Bus 17	Bus 18	Bus 19	Bus 20	Bus 21	Bus 24	Bus 26	Bus 29	Bus 30
	Bus 1	0.2670	0.3629	0.3596	0.3416	0.2538	0.3088	0.7879	0.6822	0.7373
	Bus 2	0.2390	0.3356	0.3320	0.3138	0.2255	0.2807	0.7592	0.6531	0.7082
	Bus 13	0.3130	0.3544	0.3778	0.3742	0.3258	0.3654	0.8954	0.8221	0.8772
	Bus 22	0.1191	0.2492	0.2289	0.2017	0.0181	0.1335	0.6954	0.6424	0.6975
	Bus 23	0.2674	0.2837	0.3077	0.3044	0.2365	0.1855	0.7652	0.7235	0.7786
	Bus 27	0.3857	0.4796	0.4753	0.4568	0.3469	0.3118	0.5555	0.3000	0.3551

Table A-3: Electrical distance between generators and loads

Bibliography

[1] "Commission regulation (eu) 2015/1222 of 24 july 2015 establishing a guideline on capacity allocation and congestion management." http://data.europa.eu/eli/reg/2015/1222/oj, July 24, 2015.

[2] ENTSO-E, "Supporting document for the network code on operational security." https://www.entsoe.eu/fileadmin/user_upload/_library/resources/OS_NC/130924-AS-NC_OS_Supporting_Document_2nd_Edition_final.pdf, September 24, 2013.

[3] "Commission regulation (eu) no 543/2013 of 14 june 2013 on submission and publication of data in electricity markets and amending annex i to regulation (ec) no 714/2009 of the european parliament and of the council text with eea relevance." http://data.europa.eu/eli/reg/2013/543/oj, June 14, 2013.

[4] E. W. Weisstein, "Weighted graph." http://mathworld.wolfram.com/WeightedGraph.html.

[5] J. J. Grainger, *Power system analysis*. McGraw-Hill, 1999.

[6] "Commission regulation (eu) 2017/1485 of 2 august 2017 establishing a guideline on electricity transmission system operation." http://data.europa.eu/eli/reg/2017/1485/oj, August 2, 2017.

[7] M. Pavesi, *Partitioning the Power Flow in a Multi-zonal Transmission Network*. Eindhoven University of Technology, MSc Thesis, 2017.

[8] D. A. Lima, A. Padilha-Feltrin, and J. Contreras, "An overview on network cost allocation methods," *Electric Power Systems Research*, vol. 79, no. 5, pp. 750–758, 2009.

[9] F. D. Galiana, A. J. Conejo, and H. A. Gil, "Transmission network cost allocation based on equivalent bilateral exchanges," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1425–1431, 2003.

[10] H. M. Merrill and B. W. Erickson, "Wheeling rates based on marginal-cost theory," *IEEE Transactions on Power Systems*, vol. 4, no. 4, pp. 1445–1451, 1989.

[11] T. Krause, "Evaluation of transmission pricing methods for liberalized markets: A literature survey," tech. rep., ETH Zurich, 2003.

[12] D. S. Kirschen and G. Strbac, *Fundamentals of power system economics.* John Wiley & Sons, 2018.

[13] J. M. Lima, "Allocation of transmission fixed charges: an overview," *IEEE Transactions on Power Systems*, vol. 11, no. 3, pp. 1409–1418, 1996.

[14] J. Bialek, "Tracing the flow of electricity," *IEEE Proceedings - Generation, Transmission and Distribution*, vol. 143, pp. 313–320, July 1996.

[15] T. E. P. of England and Wales, *An introduction to Pool Rules.* prepared March 1991, updated December 1991 and April 1993, p.3.

[16] J. Bialek and D. B. Tam, "Tracing the generators' output," in *International Conference on Opportunities and Advances in International Electric Power Generation (Conf. Publ. No. 419)*, pp. 133–136, March 1996.

[17] J. W. Bialek and P. A. Kattuman, "Proportional sharing assumption in tracing methodology," *IEE Proceedings-Generation, Transmission and Distribution*, vol. 151, no. 4, pp. 526–532, 2004.

[18] Z. Jing and F. Wen, "Discussion on the proving of proportional sharing principle in electricity tracing method," in *2005 IEEE/PES Transmission & Distribution Conference & Exposition: Asia and Pacific*, pp. 1–5, IEEE, 2005.

[19] C. Macqueen and M. Irving, "An algorithm for the allocation of distribution system demand and energy losses," in *Proceedings of Power Industry Computer Applications Conference*, pp. 234–239, IEEE, 1995.

[20] J. Bialek, "Topological generation and load distribution factors for supplement charge allocation in transmission open access," *IEEE Transactions on Power Systems*, vol. 12, no. 3, pp. 1185–1193, 1997.

[21] D. Kirschen, R. Allan, and G. Strbac, "Contributions of individual generators to loads and flows," *IEEE Transactions on power systems*, vol. 12, no. 1, pp. 52–60, 1997.

[22] G. Strbac, D. Kirschen, and S. Ahmed, "Allocating transmission system usage on the basis of traceable contributions of generators and loads to flows," *IEEE Transactions on Power Systems*, vol. 13, no. 2, pp. 527–534, 1998.

[23] D. Kirschen and G. Strbac, "Tracing active and reactive power between generators and loads using real and imaginary currents," *IEEE Transactions on Power Systems*, vol. 14, no. 4, pp. 1312–1319, 1999.

[24] M. Mustafa, H. Shareef, and M. Ahmad, "An improved usage allocation method for deregulated transmission systems," in *2005 International Power Engineering Conference*, pp. 1–406, IEEE, 2005.

[25] F. F. Wu, Y. Ni, and P. Wei, "Power transfer allocation for open access using graph theory-fundamentals and applications in systems without loopflow," *IEEE transactions on power systems*, vol. 15, no. 3, pp. 923–929, 2000.

[26] K. Xie, J. Zhou, and W. Li, "Analytical model and algorithm for tracing active power flow based on extended incidence matrix," *Electric Power Systems Research*, vol. 79, no. 2, pp. 399–405, 2009.

[27] F. Gubina, D. GrgiËĞc, and I. Banic, "A method for determining the generators' share in a consumer load," *IEEE transactions on power systems*, vol. 15, no. 4, pp. 1376–1381, 2000.

[28] A. Abhyankar, S. Soman, and S. Khaparde, "Optimization approach to real power tracing: an application to transmission fixed cost allocation," *IEEE transactions on power systems*, vol. 21, no. 3, pp. 1350–1361, 2006.

[29] M. Sulaiman, M. Mustafa, O. Aliman, and I. Daut, "Determination of power flow and loss allocation using superposition theory method," *International Energy Journal*, vol. 9, no. 1, 2008.

[30] M. Sulaiman, O. Aliman, M. Mustafa, and I. Daut, "Tracing generators' output in transmission open access," in *2007 5th Student Conference on Research and Development*, pp. 1–6, IEEE, 2007.

[31] J.-H. Teng, "Power flow and loss allocation for deregulated transmission systems," *International Journal of Electrical Power & Energy Systems*, vol. 27, no. 4, pp. 327–333, 2005.

[32] M. Pavesi, J. v. Casteren, and S. A. d. Graaff, "The full line decomposition method- a further development for causation-based cost sharing," Cigre Science and Engineering, CSE 009, 2017.

[33] P. W. Corporation, "Powerworld simulator user guide." https://www.powerworld.com/WebHelp/Default.htm#MainDocumentation_HTML/Power_Transfer_Distribution_Factors.htm%3FTocPath%3DSensitivities%7CPower%2520Transfer%2520Distribution%2520Factors%7C_____1.

[34] J. W. Bialek and S. Ziemianek, "Tracing based transmission pricing of cross-border trades: fundamentals and circular flows," in *2003 IEEE Bologna Power Tech Conference Proceedings,*, vol. 3, pp. 8–pp, IEEE, 2003.

[35] ENTSO-E, "Joint task force cross border redispatch flow definitions." https://docstore.entsoe.eu/Documents/MC%20documents/150929_Joint%20Task%20Force%20Cross%20Border%20Redispatch%20Flow%20Definitions.pdf.