

DIFFRACTIVE OPTICS OF ANISOTROPIC POLARIZATION GRATINGS

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Summary

Diffractive Optics of Anisotropic Polarization Gratings

Gratings consisting of isotropic materials have been studied extensively in optics. The relative recent progress in the manufacture of micro- and nano-structures has made it possible to make gratings with periods comparable to or even smaller than the wavelength of visible light. Today diffraction gratings are being used to manipulate light in many different applications, such as in flat panel display systems, modern lighting systems, and optical recording. Diffraction gratings can be made polarization selective due to form birefringence. Wire-grid polarization gratings are a well-known example. An alternative approach to polarization selective gratings is to use birefringent materials. In this thesis, diffraction gratings consisting of at least one birefringent material are investigated. These gratings cause in general both form birefringence and material induced birefringence.

The optical properties of an anisotropic medium follow from its dielectric permittivity tensor. Compared to isotropic media whose dielectric permittivity reduces to a scalar, the permittivity tensor of an anisotropic material on some basis is in general a full 3×3 matrix. Often this matrix is real symmetric. This applies for example to all anisotropic materials that contain only linear birefringence. Linear birefringent materials have two different refractive indices for two orthogonally linearly polarized waves. In most standard textbooks, the treatment of anisotropic materials is restricted to those with symmetric real permittivity tensor. However, some materials have different refractive indices for two orthogonally circularly polarized beams. This property is called circular birefringence. For a lossless medium with circular birefringence, the dielectric permittivity tensor becomes complex hermitian instead of real symmetric. In this thesis, we therefore develop a rigorous study of light propagation in anisotropic media with in general a complex hermitian dielectric permittivity tensor of which the real symmetric tensor is only a special case. We remark that materials with a complex hermitian tensor are lossless, just as materials with a real symmetric tensor. Anisotropic media with losses are much more difficult to treat and will not be studied in this thesis.

In Chapter 2 we study Maxwell's equations for time-harmonic monochromatic plane waves in anisotropic media. We extend the classical theory of a real symmetric permittivity tensor to that of a complex hermitian tensor. In a medium with an hermitian type of permittivity tensor, the eigenstates of the electric field vector are elliptically

polarized with opposite directions of rotation. For the two components there are two in general different real refractive indices.

We consider in particular the reflection and transmission at an interface between an isotropic and an anisotropic medium whose permittivity tensor is hermitian and of special type. Due to the special form of the hermitian tensor, the eigenstates of the electric field are left- and right-handed circular polarized. At the interface, reflection and transmission coefficients are derived separately for left- and right-handed circular polarizations.

Then the classical problem of a plane wave incident on a multilayer consisting of in general anisotropic layers is studied in detail. In one or several of the layers there can occur evanescent waves. We have implemented a MatLab code to solve this problem. This code is used in particular to benchmark the rigorous finite element code Cyclop3D, which we have used extensively in our work.

In Chapter 3, gratings consisting of liquid crystal molecules oriented parallel to the grating grooves are studied for application as polarizing color separator in liquid crystal displays. It is shown that the efficiency of the separation of colors and polarizations can be substantially improved by replacing the conventional absorption based polarizers and color filters by anisotropic gratings. We study to what extent the two orthogonal linear polarizations can be separated by the grating in both normal incidence mount and in conical incidence mount. The simulations show that the diffracted transmitted intensity strongly depends on the angles of incidence but nevertheless has a high contrast ratio for all angles of interest. The transmission efficiency is approximately 2% , hence rather low. However, for a long lightguide, the total extraction efficiency will be efficiently high because the light that is reflected by the grating is recycled in the waveguide.

In Chapter 4, we characterize the diffraction efficiencies of so-called POLICRYPS (acronym for Polymer Liquid Crystal Polymer Slices) gratings which have been introduced by the LICRYL research group lead by Prof. Umeton of the University of Calabria, Italy. The POLICRYPS grating is an improved holographic polymer dispersed liquid crystal (H-PDLC) grating. The average director of the LC molecules in the grating is perpendicular to the polymer walls and parallel to the direction of periodicity. The angular dependence of the diffraction efficiency of the grating is investigated for all angles of incidence by both simulations and experiments. An experimental check of the grating efficiency at normal incidence shows a strong angular selectivity around the Bragg angle. The diffraction efficiency of the POLICRYPS is wavelength dependent. Results of a study of diffraction efficiencies for conical angles of incidence are presented for the first time. The results show a good tolerance range of conical tilting. The grating is suitable for applications in e.g. backlight for flat panel display to couple out light more efficiently. Furthermore, a uniform high diffraction efficiency can be achieved by limiting the incident angles to the cone defined by the Bragg angle.

In Chapter 5 and Chapter 6 photoanisotropic polarization gratings are studied extensively. A photoactive medium, which initially is isotropic, becomes anisotropic due to irradiation. The property of such a medium is called photoanisotropy.

Photoanisotropic polarization gratings can be manufactured with polarization holographic recording. Two interfering beams are used to make a periodic interference pattern which is then translated into a periodically modulated permittivity tensor using the photoanisotropic material. Depending on the polarization states of the two recording beams and the recording angle between them, periodic modulations of both the polarization states and the intensity can be achieved. Materials, such as, azobenzene polymers and linear photo polymers, which are sensitive to polarization can be used to record the interference pattern. When the medium is sensitive to both linear and circular polarizations, the photo-induced dielectric permittivity tensor becomes complex hermitian. A model of the writing process has been described by Kakichashvili in the 1970s. Existing literature on the topic is limited to the paraxial case, i.e. the recording angle between the recording beams is small. In this thesis, we study the subject beyond this approximation and we develop formulations of polarization holography with arbitrary polarizations and arbitrarily large recording angles. Chapter 5 is focused on the recording process of the photoanisotropic polarization gratings. In Chapter 6 the reading process is studied. In general the reading is performed at a wavelength that differs from that used for recording.

In this thesis, two special photoanisotropic polarization gratings are studied in detail, namely polarization gratings formed by two orthogonally circularly (OC) polarized recording beams and gratings formed by two orthogonally linearly (OL) polarized recording beams. The interference pattern of these two special polarized beams is studied in Chapter 5 for both small and large recording angles. In particular, the periodically varying permittivity tensors that are thus obtained are studied. Then in Chapter 6, the transmission of a perpendicular plane wave incident on these anisotropic gratings is studied. We derive an approximate transmission matrix which is expected to be accurate when the period of the permittivity tensor is sufficiently large compared to the wavelength. This approximate transmission matrix is useful because it gives insight in the diffractive properties of these holographic gratings. We compare the prediction of the transmission matrix with transmission calculation using the rigorous finite element method and we find very good agreement for surprisingly small pitches.

The OC polarization grating can be used as a circular polarization converter which transfers the incident right-handed circular polarization into left-handed and vice versa. In contrast, the OL polarization grating is suited as a linear polarization converter which transfer the incident linear polarization state to the orthogonal state of linear polarization. Rigorous simulations show that with these holographic polarization gratings very high diffraction efficiencies can be achieved, higher than 95% for appropriate thicknesses of the grating and for pitches for which there are only three diffraction orders in the transmitted field, namely the $\pm 1^{st}$ and 0^{th} orders. Furthermore, when the incident angle of the reading beam is smaller than 20° , the diffraction efficiencies do not have strong angular dependence. Simulations for polarization gratings recorded at larger angles are presented for the first time. Larger recording angles result in gratings with smaller pitch but at the cost of lower diffraction efficiencies for the same wavelength of the reading beam. In this case, reading beams with shorter wavelengths perform better. For OL polarization gratings, the circular polar-

ization sensitivity of the recording material is critical for a high diffraction efficiency. Some possible applications of photoanisotropic polarization gratings are presented at the end of Chapter 6, such as, flat panel displays, polarimetry, optical recording, quantum computing, etc.

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Delft, February 2009

Samenvatting

Diffractieve Optica van Anisotrope Polarizatiematerialen

Materialen die uit isotrope materialen bestaan, zijn uitgebreid bestudeerd in de optica. De relatief recente voortgang in de fabricage van micro- en nanostructuren heeft het mogelijk gemaakt materialen te maken met periodes vergelijkbaar met of zelfs kleiner dan de golflengte van zichtbaar licht. Tegenwoordig worden diffractiematerialen gebruikt om licht te manipuleren in vele toepassingen, zoals vlakke beeldschermen, moderne verlichtingsystemen en optical recording. Diffractiematerialen kunnen dankzij dubbele breking, veroorzaakt door de vorm van het materiaal, polarisatieselectief worden gemaakt. Wire-grid diffractiematerialen zijn hiervan een bekend voorbeeld. Een alternatieve vorm van polarisatieselectieve materialen zijn materialen gemaakt van materialen met dubbele breking. In dit proefschrift worden materialen bestaande uit ten minste één dubbelbrekend materiaal onderzocht. Deze materialen veroorzaken dubbele breking door zowel de vorm als de samenstelling van het materiaal.

De optische eigenschappen van een anisotroop medium volgen uit de diëlektrische permittiviteitstensor. Vergeleken met isotrope media, waarvan de diëlektrische permittiviteit een scalar is, wordt de permittiviteit van een anisotroop materiaal op een willekeurige basis over het algemeen gegeven door een tensor die uit een volledig volle 3×3 matrix bestaat. Vaak is deze matrix reëel symmetrisch. Dit geldt bijvoorbeeld voor alle anisotrope materialen die alleen lineaire dubbele breking vertonen. Materialen met lineaire dubbele breking hebben twee verschillende brekingsindices voor twee orthogonaal lineair gepolariseerde golven. In de meeste studieboeken wordt de behandeling van anisotrope materialen beperkt tot materialen met een reëel symmetrische permittiviteitstensor. Sommige materialen hebben echter verschillende brekingsindices voor twee circulair gepolariseerde bundels. Deze eigenschap wordt circulaire dubbele breking genoemd. Voor een verliesvrij medium met circulaire dubbele breking wordt de diëlektrische permittiviteitstensor complex hermitisch in plaats van reëel symmetrisch. In dit proefschrift bestuderen we daarom de voortplanting van licht in anisotrope media met een over het algemeen complex hermitische diëlektrische permittiviteitstensor waarvan de reëel symmetrische tensor slechts een bijzonder geval is. We merken op dat materialen met een complex hermitische tensor net als materialen met een reëel symmetrische tensor verliesvrij zijn. Anisotrope media met verliezen zijn veel moeilijker te behandelen en zullen niet worden bestudeerd in dit proefschrift.

In Hoofdstuk 2 bestuderen we de Maxwell vergelijkingen voor tijdsharmonische vlakke golven in anisotrope media. We breiden de klassieke theorie van een reëel symmetrische permittiviteitstensor uit tot die van een complex hermitische tensor. In een medium met een hermitische vorm van de permittiviteitstensor zijn de eigentoestanden van de elektrische veldvector elliptisch gepolariseerd met tegengestelde rotatierichting. Er zijn in het algemeen twee verschillende brekingsindices voor de twee componenten.

We beschouwen in het bijzonder de reflectie en transmissie aan een grensvlak tussen een isotroop en een anisotroop medium waarvan de permittiviteitstensor hermitisch is en een bijzondere vorm heeft. Ten gevolge van die bijzondere vorm van de hermitische tensor zijn de eigentoestanden van het elektrische veld links- en rechtshandige circulaire polarisaties. De reflectie- en transmissiecoëfficiënten voor links- en rechtshandige circulaire polarisaties worden afzonderlijk afgeleid.

Vervolgens wordt het klassieke probleem van een vlakke golf invallend op een multi-layer bestaande uit in het algemeen anisotrope lagen in detail bestudeerd. In één of meer lagen kunnen evanescente golven voorkomen. Om dit probleem op te lossen hebben we een Matlab-programma gemaakt. Dit programma is in het bijzonder gebruikt om de uitkomsten van het rigoreuze eindige elementen programma Cyclop3D te verifiëren, dat we intensief hebben gebruikt in ons onderzoek.

In Hoofdstuk 3 worden tralies bestudeerd bestaande uit parallel met de traliegroeven georiënteerde vloeibare kristallen bestudeerd, voor toepassingen als een kleuren scheidende polarisator in vloeibare kristallen displays. Er wordt aangetoond dat de efficiëntie van de scheiding van kleuren en polarisatie substantieel kan worden verbeterd door de conventionele op absorptie gebaseerde polarisatoren en kleurenfilters te vervangen door anisotrope tralies. We onderzoeken in welke mate de twee orthogonaal lineaire polarisaties kunnen worden gescheiden door de tralie onder zowel normale als conische inval te plaatsen. De simulaties laten zien dat de doorgelaten intensiteit sterk afhangt van de invalshoek, maar desalniettemin een hoge contrastverhouding heeft voor alle van belang zijnde hoeken. De transmissie-efficiëntie is bij benadering 2%, dus vrij laag. Voor een lange lichtgeleider zal de totale extractie-efficiëntie echter toch hoog zijn omdat het door de tralie gereflecteerde licht wordt gerecycled in de golfgeleider.

In Hoofdstuk 4 karakteriseren we de diffractie-efficiënties van de zogenaamde POLICRYPS (acroniem voor Polymer Liquid Crystal Polymer Slices) tralies welke zijn geïntroduceerd door de LICRYL research group geleid door Prof. Umeton van de University of Calabria, Italy. De POLICRYPS tralie is een verbeterde holografische tralie gemaakt met polymer dispersed vloeibare kristallen (H-PDLC). De LC-moleculen in de tralie staan loodrecht op de polymeerwanden en parallel aan de richting van de periodiciteit. De hoekafhankelijkheid van de diffractie-efficiëntie van de tralie wordt onderzocht voor alle invalshoeken door middel van zowel simulaties als experimenten. Een experimentele controle van de efficiëntie voor een loodrechte invalshoek laat een sterke hoekselectiviteit zien rond de Bragg-hoek. De diffractie-efficiëntie van de POLICRYPS is afhankelijk van de golflengte. Het is voor het eerst dat resultaten van een onderzoek naar de diffractie-efficiënties voor conische hoeken van inval worden gepresenteerd. De resultaten laten een goed tolerantiegebied voor

conical tilting zien. De tralie is geschikt voor toepassing in bijvoorbeeld backlight voor vlakke beeldschermen om het licht efficiënter uit te koppelen. Bovendien kan een uniforme hoge diffractie-efficiëntie worden bereikt door de invalshoeken te beperken tot de door de Bragg-hoek gedefinieerde kegel.

In Hoofdstuk 5 and Hoofdstuk 6 worden fotoanisotrope polarisatietralies uitgebreid onderzocht. Een fotoactief medium dat oorspronkelijk isotroop is, wordt anisotroop door de belichting. Een dergelijk medium heet fotoanisotroop. Fotoanisotrope polarisatietralies kunnen worden gefabriceerd door middel van polarisatieholografische opname. Twee interfererende bundels worden gebruikt om een periodiek interferentiepatroon te creëren dat vervolgens in het fotoanisotroop materiaal wordt omgezet in een periodiek gemoduleerde permittiviteitstensor. Afhankelijk van de polarisatietoestanden van de twee opnamebundels en de opnamehoek daartussen kunnen periodieke modulaties van zowel de polarisatietoestanden als van de intensiteit worden verkregen. Materialen zoals azobenzeenpolymeren en lineaire fotopolymeren, welke gevoelig zijn voor polarisatie, kunnen worden gebruikt om het interferentiepatroon op te nemen. Indien het medium gevoelig is voor zowel lineaire als circulaire polarisatie dan wordt de fotogeïnduceerde diëlektrische permittiviteitstensor complex hermitisch. Een model voor het schrijfproces is beschreven door Kakichashvili in de zeventiger jaren van de vorige eeuw. De bestaande literatuur over het onderwerp is beperkt tot het paraxiale geval waarbij de opnamehoek tussen de opnamebundels klein is. In dit proefschrift onderzoeken we het onderwerp zonder deze benadering en ontwikkelen we formuleringen voor polarisatieholografie met willekeurige polarisaties en willekeurig grote opnamehoeken. Hoofdstuk 5 is toegespitst op de recording van fotoanisotrope polarisatietralies. In Hoofdstuk 6 wordt het leesproces bestudeerd. Over het algemeen wordt het uitlezen verricht met behulp van een andere golflengte dan die waarmee wordt opgenomen.

In dit proefschrift worden twee speciale fotoanisotrope polarisatietralies in detail bestudeerd, namelijk polarisatietralies die worden gevormd door twee orthogonaal circulair (OC) gepolariseerde opnamebundels en tralies die worden gevormd door twee orthogonaal lineair (OL) gepolariseerde opnamebundels. Het interferentiepatroon van deze twee speciaal gepolariseerde bundels wordt voor zowel kleine als grote opnamehoeken onderzocht in Hoofdstuk 5. De zo verkregen periodieke permittiviteitstensen worden onderzocht. Vervolgens wordt in Hoofdstuk 6 de transmissie van een loodrecht invallende vlakke golf op deze anisotrope tralies bestudeerd. We leiden een benaderende transmissiematrix af die geacht wordt nauwkeurig te zijn indien de periode van de permittiviteitstensor groot genoeg is in verhouding tot de golflengte. Deze benaderde transmissiematrix is nuttig omdat deze inzicht geeft in de diffractie-eigenschappen van de holografische tralies. We vergelijken de voorspellingen van de transmissiematrix met de transmissieberekening gebruikmakend van de rigoreuze eindige elementen methode en komen tot een erg goede overeenstemming voor verrassend kleine perioden.

De OC-polarisatietralie kan worden gebruikt om invallende rechtshandige circulaire polarisatie om te zetten in linkshandige en omgekeerd. De OL-polarisatietralie daarentegen is bruikbaar om invallende lineaire polarisatie om te zetten in de orthogonale lineaire polarisatie. Resultaten van simulaties tonen aan dat met deze holografische

polarisatietralies erg hoge diffractie-efficiënties kunnen worden bereikt, hoger dan 95% voor goed gekozen dikte van de tralie en voor perioden waarvoor er slechts drie diffractieordes zijn in het doorgelaten veld, namelijk de $\pm 1^{ste}$ and 0^{de} ordes. Indien bovendien de invalshoek van de leesbundel kleiner is dan 20° , zijn de diffractie-efficiënties niet sterk hoekafhankelijk. Voor het eerst worden simulaties gepresenteerd voor polarisatietralies opgenomen onder grotere hoeken. Grotere opnamehoeken resulteren in tralies met kleinere perioden, maar dit resulteert in lagere diffractie-efficiënties bij dezelfde golflengte van de leesbundel. In dit geval geven leesbundels met kortere golflengtes een beter resultaat. Voor OC-polarisatietralies is de circulaire polarisatiegevoeligheid van het opnamemateriaal essentieel voor een hoge diffractie-efficiëntie. Enkele mogelijke toepassingen van fotoanisotrope polarisatietralies, zoals vlakke beeldschermen, polarimetrie, optical recording, quantum computing, et cetera, worden gepresenteerd aan het eind van Hoofdstuk 6.

Man Xu

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Chapter 1

Introduction

1.1 Optical anisotropy

In physics, *anisotropy* is used to describe direction-dependent properties of materials as opposed to *isotropy*, which means invariance of direction direction. The most common example of anisotropy is wood. Its properties vary in a large-scale when they are measured along the growth grain or perpendicular to it. The mechanical



Figure 1.1: Black locust is a ring-porous hardwood. The right panel shows a zoom-in look of the growth grain. The photo is taken from [1].

anisotropy of wood comes from the longitudinal arrangement or pattern of fibers as can be seen in Fig. 1.1. The spatial arrangement is the reason for the anisotropy of material. In optics, the relevant size of the spatial arrangement shrinks to the micron or nanometer scale. The macroscopic anisotropy is due to the microscopic molecular orientation and organization in space. Different materials can be grouped as mainly *isotropic* or *anisotropic*. Possible molecular structures in several optical materials are listed as follows.

- If the orientations of the molecules in space are random, or there is a mixture of several different elements, or the molecules themselves are isotropic, the media

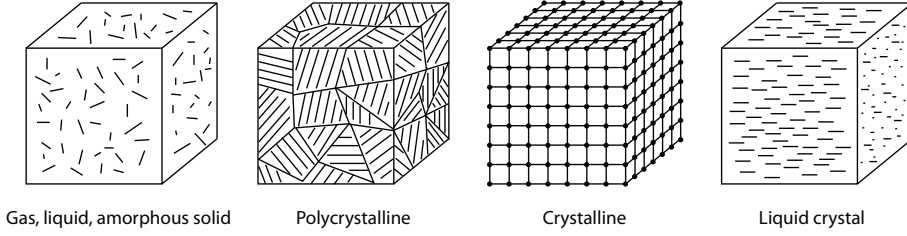


Figure 1.2: Positional and orientational order in different kinds of materials. The picture is taken from [2].

is isotropic. Gas, liquid, and amorphous solid have such kind of property and they are typical isotropic media.

- If the orientation of the molecules or arrangement in space has a certain order and preferred direction, the medium is crystalline-like, and is anisotropic. In general, crystals have such molecular organizations.
- In some materials, the crystalline structure only exists in a rather small domain. In that domain, it is anisotropic. On a larger scale, the disjointed crystalline grains have random orientations as compared to each other. Thus on the macroscopic scale, the medium is isotropic. This type of material is polycrystalline.
- Liquid crystal (LC) molecules are anisotropic, due to the rod-like or dish-like shape. The LC molecules can be lined up by using some external strength, such as, an electric field, a flow, or an alignment layer. When the molecules are aligned, the whole medium is optical anisotropic.

The optical anisotropy can be described by the refractive index n and dielectric permittivity ϵ . The relation between these two physics identities is: $\epsilon = n^2$. For isotropic materials, n and ϵ are constant, whereas for anisotropic materials, there are more than one value for n and ϵ becomes a tensor of rank 2 which we shall denote by $\underline{\underline{\epsilon}}$. For time-harmonic fields, the relative dielectric permittivity tensor connects the electric displacement vector $\mathbf{D}$ and the electric field $\mathbf{E}$,

$$\mathbf{D} = \epsilon_0 \underline{\underline{\epsilon}} \mathbf{E}, \quad (1.1)$$

where, ϵ_0 is the dielectric permittivity in a vacuum and the relative dielectric permittivity tensor $\underline{\underline{\epsilon}}$ can on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ be written as,

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}. \quad (1.2)$$

For a non-absorbing medium, the tensor is real symmetric in most cases. However, it can also be complex hermitian, i.e., $\epsilon_{ij} = \epsilon_{ji}^*$ where * denotes the complex conjugate

(in this thesis, both cases occur). In this thesis all anisotropic materials are assumed to be non-absorbing. Hence $\underline{\underline{\epsilon}}$ is either a real symmetric or an hermitian tensor. The tensor $\underline{\underline{\epsilon}}$ is a linear map $\mathbb{C}^3 \rightarrow \mathbb{C}^3$, where $\mathbb{C}^3$ is the linear space of all complex vectors $\mathbf{a} = (a_1, a_2, a_3)$ with scalar product $\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3}$ which is linear in $\mathbf{a}$ but anti-linear in $\mathbf{b}$, i.e. $\langle \lambda \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3} = \lambda \langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3}$ and $\langle \mathbf{a}, \lambda \mathbf{b} \rangle_{\mathbb{C}^3} = \lambda^* \langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3}$ for every $\lambda \in \mathbb{C}$. Because $\underline{\underline{\epsilon}}$ is hermitian, there exists an orthonormal basis $\{\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}\}$ of $\mathbb{C}^3$ with respect to which the tensor is diagonal:

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix} \quad (1.3)$$

where $\epsilon_1, \epsilon_2, \epsilon_3$ are real. The vector $\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}$ are eigenvectors of $\underline{\underline{\epsilon}}$ with eigenvalues $\epsilon_1, \epsilon_2, \epsilon_3$, respectively. When $\underline{\underline{\epsilon}}$ is real symmetric, the vectors $\{\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}\}$ have real components on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$. Then $\hat{\mathbf{u}}, \hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$ can be associated with specific directions in euclidean space. These directions are called the *principal axes* of the tensor. However in the more general case that $\underline{\underline{\epsilon}}$ is hermitian, the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ have complex components on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$. Then the unit vectors $\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{w}}$ can not be associated with specific directions in euclidean space. The numbers n_1, n_2, n_3 defined by

$$n_1 = \sqrt{\epsilon_1}, \quad n_2 = \sqrt{\epsilon_2}, \quad n_3 = \sqrt{\epsilon_3},$$

are known as the *principal refractive indices*.

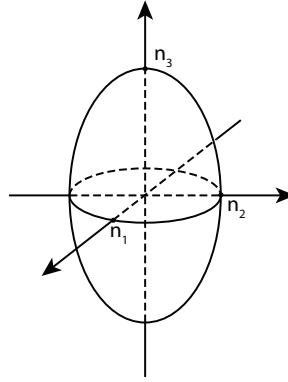


Figure 1.3: The refractive index ellipsoid on the principal axes.

When the tensor is real symmetric and two of the principal refractive indices are equal (for instance, $n_1 = n_2$), the medium is called *uniaxial*. The third direction is called the *optic axis*. When an unpolarized plane wave is incident into such a medium, then depending on the direction of the wave vector, it will 'feel' different refractive indices. A wave of which the electric field is perpendicular to both the optic axis and the wave vector, is called the *ordinary wave*. It senses the *ordinary refractive index* $n_o = n_1 = n_2$. The other wave of which the electric field is parallel to the plane

through the wave vector and the optic axis, is called the *extraordinary wave* because it experiences the *extraordinary refractive index* $n_e = n_3$. The difference between the two refractive indices $\Delta n = n_e - n_o$ is defined as the *birefringence* which results into *double refraction*.

When the three refractive indices are all different, the medium is called *biaxial*. Many crystals are optically anisotropic, and birefringence and double refraction effects can be observed. An observation is shown in Fig. 1.4 with a calcite crystal which is an example of a uniaxial birefringent material.



Figure 1.4: A calcite crystal laid upon a paper with some letters showing the double refraction. The photo is taken from [3].

In wave optics, light is represented by electromagnetic waves propagating in space. Light is often interpreted by the vibration of the electric field in time and space. In a non-absorbing isotropic medium, the electric field vector of a propagating plane wave is always perpendicular to the wave vector. There are two linearly independent directions of vibration, which can be chosen arbitrarily perpendicular to the wave vector. If the components of the electric field vector along these two vectors are totally uncorrelated, the beam is *unpolarized*. If the vibration is totally constrained in one direction, i.e. electric field oscillates parallel to a line, the light is called *linearly polarized*. As mentioned above, the electric field vectors of the ordinary and the extraordinary waves in an non-absorbing uniaxial crystal only vibrate in a single direction separately and each only sense one of the refractive index depending on the propagation direction. Those propagating light waves in anisotropic media are linearly polarized. This holds also for a biaxial medium provided the tensor $\underline{\underline{\epsilon}}$ is real symmetric. Then also the polarization of every plane wave in the medium is always linear. States of polarization are further discussed in Section 1.3.

1.1.1 Birefringence of liquid crystals

A liquid crystal (LC) is a liquid in which an ordered arrangement of molecules exists. It is an intermediate state between liquid and crystalline. It was first observed in 1881 in cholesteryl benzoate, a crystalline solid. When the material is heated to 145°C, it turns into a cloudy liquid but the anisotropy remains; when heated up further to 179°C, the liquid becomes clear and isotropic. The process is reversible by cooling. During the heating, the LC experiences different phases. For a rod-like LC, without

chiral molecules, the sequence of phase changes due to rising temperature are in general:

solid crystal $\rightarrow$ smectic liquid crystal $\rightarrow$ nematic liquid crystal $\rightarrow$ isotropic liquid.

The smectic and nematic phases of rod-like LCs are illustrated in Fig. 1.5. The vector

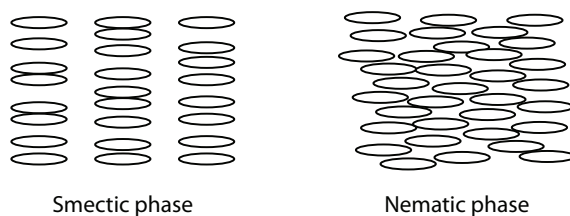


Figure 1.5: Phase of liquid crystal: on the left – smectic phase, and on the right – nematic phase.

along the orientation of the molecular axes is known as *director* of the LC and is usually denoted by $\mathbf{n}$ (not to be confused with refractive index n). The *smectic phase* is close to the solid state of the material, because the molecules maintain both translational order and orientational order. As the temperature rises, the translational order vanishes, and at the *nematic phase*, only longitudinal alignment remains. There is another LC phase, the *cholesteric phase*, which occurs when chiral molecules exist in the medium. Chiral molecules induce spontaneous twisting around a helical axis normal to the director. Depending on the chirality, the twist is either right- or left-handed. On each plane perpendicular to the helical axis, the LCs are in the nematic phase. The cholesteric phase is shown in Fig. 1.6

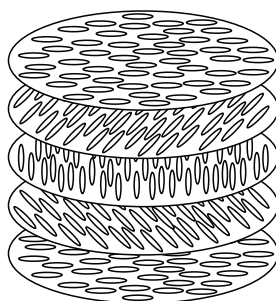


Figure 1.6: Cholesteric phase of liquid crystal.

Nematic phase LCs are most commonly used in industrial applications at the moment. The temperature window for the nematic phase varies for different compounds, for instance, hexylcyanobiphenyl (6CB) is nematic in the temperature range $15^{\circ}\text{C} < T < 29^{\circ}\text{C}$. A single LC compound offers a rather small temperature window

and often cannot fulfill the requirement of nematic behavior over a temperature range of e.g. -20 to $+80^\circ\text{C}$. A mixture of several compounds can offer a much larger temperature range of nematic phase. The melting point (start of the nematic phase) of a mixture is lower than any of the constituent compounds. The melting point depends on the mixture ratio. The clearing point (end of the nematic phase) of the LC mixture is the linear average of the clearing points of the compounds [4]. In this way a good working condition can be achieved.

LCs are mostly uniaxial. The optic axis coincide with the director of the molecules. LCs also have quite large birefringence. Typically, the ordinary refractive index of an LC is 1.5, while the extraordinary one is 1.7, which implies a birefringence of 0.2. The LC molecules can easily be reoriented, and this property enables the possibility and flexibility to have anisotropy in a specific desired direction. The application to polarization sensitive gratings will be introduced in the following section.

1.2 Polarization selective gratings

An optical grating is a periodic structure that diffracts light into a number of orders propagating in different directions. The number of orders and the direction of propagation are determined by the geometry or configuration of the grating. In optics, the period (Λ) of the grating is usually in the range of microns, sometimes even in the range of nanometers. For any incident beam, there is never a diffracted order propagates when the period is $\Lambda < \lambda/2$, where λ is the wavelength measured in the medium.

Diffraction gratings can be classified into two groups, namely thin and thick gratings. The distinction is based on the value of the so-called Klein parameter Q [5] defined by:

$$Q = \frac{2\pi\lambda d}{n_0\Lambda^2}, \quad (1.4)$$

where d is the thickness of the grating, λ is the wavelength, and n_0 is the average refractive index. In the case that $Q < 1$, the grating is considered to be thin, whereas when $Q > 10$, the grating is regarded as thick. For a thick grating, because of multiple reflections and subsequent of constructive interference, a single high diffracted order can be obtained for a certain angle of incidence. In this case, the grating is called a Bragg grating, and the specific angle of incidence is called the Bragg angle. This phenomenon does not hold in a thin grating. For such gratings several diffraction orders are observed at the same time. In either the *thin* or *thick* regime, the diffraction efficiency can be calculated simply and rather accurately by making some approximations. In the intermediate regime, rigorous diffraction theory is required.

The diffraction of a grating can be made strongly dependent on polarization by different methods. One way to achieve this effect is to use *form birefringence*, i.e. birefringence induced by the shape of the interface between the materials which are all optically isotropic. A wire-grid polarizing grating [6, 7] is a good example. The wire-grid polarizer (WGP) is a metallic grating of which the period is smaller than

the wavelength of the light used. It consists of deep rectangular slits. The WGP reflects s -polarized light of which the electric field is parallel to the slits but transmits p -polarization of which the electric field is perpendicular to the slits. WGPs are used as polarization beam splitter in optical systems.

Another approach to obtain polarization selective gratings is to use anisotropic materials. Consider a grating structure of which isotropic and anisotropic materials are stacked in a periodic alternating manner. Suppose that the materials are chosen such that the refractive index of the isotropic medium n matches the ordinary refractive index of the anisotropic medium n_o , whereas, the extraordinary refractive index n_e is significantly different from the ordinary one, i.e. the anisotropic medium has high birefringence. Due to the matching of n and n_o , the ordinary wave with respect to the anisotropic medium, will not see the grating structure. In contrast, the extraordinary wave with respect to the anisotropic medium is diffracted by the grating. Thus the two polarizations are separated.

Considering all the requirements, nematic liquid crystals are an ideal choice as uniaxial material for use in such a grating. Typically refractive indices of an LC $n_o = 1.5$ and $n_e = 1.7$. The ordinary refractive index matches that of many isotropic polymer media, which are common, frequently used materials for constructing gratings. LC molecules can be aligned in several different schemes. Some possible assemblies are shown in Fig. 1.7. Due to different mechanism, the LC molecules can point either parallel or perpendicular to the direction of the grating grooves and ridges as illustrated in Fig. 1.7 (a) and (b). By using an orientation layer, a much more sophisticated arrangement of LC patterning can be achieved. This is illustrated in Fig. 1.7 (c). We next briefly discuss the mechanism behind each assembly.

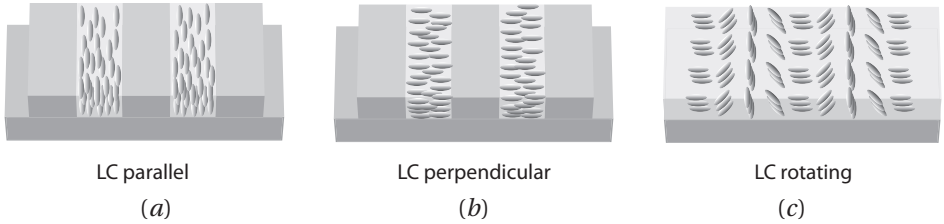


Figure 1.7: Liquid crystal arrangement in diffraction gratings.

- (a) LCs are filled in a polycarbonate grating with surface relief (grooves). Berreman's model [8] shows that for grooved surfaces, the elastic deformation energy of nematics is minimized when the directors line up parallel to the grooves. Of a nematic LC, the director indicates the orientation of the optic axis, which is in this case, thus parallel to the groove. For a light wave at perpendicular incidence, of which the electric field vector is linear polarized parallel to the grooves, senses the difference between the extraordinary refractive index of the LC and the index of the surrounding isotropic polycarbonate. Hence for this polarization, diffraction will happen. When the electric field is perpendicular to

the grooves, the refractive index is matched, i.e. there is no grating and hence there is no diffraction. This grating assembly will be studied in Chapter 3 as a polarization color-separator in the backlight for flat panel display systems.

- (b) In different circumstances, LCs can align themselves across the grating grooves and perpendicular to the walls of ridges. This is the case in polymer dispersed liquid crystals (PDLC). An initially isotropic mixture of LC compounds and polymer molecules is spread out on a glass substrate. By using holographic recording, the periodically modulated intensity pattern obtained by interference of two light beams transfers the isotropic distribution of the mixture into a grating structure with average LC directors perpendicular to the grooves. Thus in this case, the light wave at perpendicular incidence, of which the electric vector vibrates in the direction perpendicular to the grating grooves and ridges is diffracted, while the light wave with electric field parallel to the grating grooves and ridges is unaffected. An example of this kind of grating will be discussed in detail in Chapter 4.
- (c) In the Linear Photo Polymer (LPP) layer linear polarization of light can be recorded. It is used for polarization holographic recording. In polarization holography, periodic polarization modulation is obtained in addition to intensity modulation. By using this method, polarization patterns are recorded in polarization sensitive thin layer, such as, LPP. LC molecules have the ability to copy the pattern on the LPP layer which is therefore used as orientation layer. The directors of the LCs follow the spatial change of orientation of the recorded polarization in the LPP. In Fig. 1.7(c) an example is shown. Because of the periodically spatial variation of the refractive index, the whole system behaves as a diffraction grating. Since the anisotropy is induced by light, it is called photoanisotropy, and the obtained grating is called photoanisotropic polarization gratings. The subject is treated extensively in Chapter 5 and 6.

Additionally, LC molecules can be reoriented by an external electric field. Therefore, LCs can be used for devices that have to be switchable. LC materials consist of *polar* or *non-polar* molecules. Polar molecules exhibit permanent dipole moments which are due to small charge separations in the molecule. For non-polar molecules, the imposed electric field causes electric dipoles by the separation of positive and negative charges. The permanent dipoles are stronger than the induced dipoles, although they react the same when the external field is applied. Both permanent and induced electric dipoles can exist parallel or perpendicular to the longitudinal molecule axis. Depending on the direction of the dipole, the molecules react differently. If the dipole vector is parallel to the long molecule axis, the director tends to orient parallel to the electric field (see Fig. 1.8 left), while, if the dipole vector is perpendicular to the long molecule axis, the directors tend to align perpendicular to the the electric field (see Fig. 1.8 right). Note that the larger the dielectric anisotropy, the smaller the electric field needed to reorient the LC molecules. Thus an external electric field can be used to switch the grating 'on' or 'off' by reorienting the LC molecules for the polarization that is used.

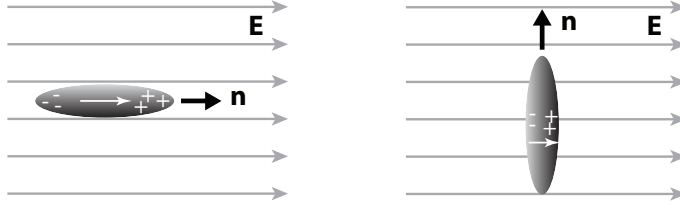


Figure 1.8: Liquid crystal molecules with longitudinal electric dipole (left) and transverse dipole (right) react with external electric field.

In the following section, we give a detailed description of the state of polarization of an time-harmonic arbitrary electric field. A local basis is defined, which varies with position, and which is used to express the local eigen-states of the electric field.

1.3 Polarization of harmonic vector waves

A time-harmonic electromagnetic field depends on time by the factor of $e^{-i\omega t}$ where $\omega > 0$ is the frequency. We consider a time-harmonic field whose spatial dependence is arbitrary. We will give the definition of the polarization state of such a field of arbitrary form (see also Ref. [9]).

Let the real and complex electric field vector be $\mathcal{E}(\mathbf{r}, t)$ and $\mathbf{E}(\mathbf{r})$, respectively, with

$$\mathcal{E}(\mathbf{r}, t) = \text{Re} \left[\mathbf{E}(\mathbf{r}) e^{-i\omega t} \right]. \quad (1.5)$$

The real electric field can also be written:

$$\mathcal{E}(\mathbf{r}, t) = \begin{pmatrix} A_x(\mathbf{r}) \cos[\phi_x(\mathbf{r}) - \omega t] \\ A_y(\mathbf{r}) \cos[\phi_y(\mathbf{r}) - \omega t] \\ A_z(\mathbf{r}) \cos[\phi_z(\mathbf{r}) - \omega t] \end{pmatrix}, \quad (1.6)$$

where A_x and ϕ_x are the amplitude and the phase of the complex number $\mathcal{E}_x$ etc. and with $-\pi \leq \phi_{x,y,z} \leq \pi$. By using $\cos(\xi - \eta) = \cos \xi \cos \eta + \sin \xi \sin \eta$, (1.6) can be written as [9]:

$$\mathcal{E}(\mathbf{r}, t) = \mathbf{p}(\mathbf{r}) \cos \omega t + \mathbf{q}(\mathbf{r}) \sin \omega t, \quad (1.7)$$

where

$$\mathbf{p}(\mathbf{r}) = \begin{pmatrix} A_x(\mathbf{r}) \cos \phi_x(\mathbf{r}) \\ A_y(\mathbf{r}) \cos \phi_y(\mathbf{r}) \\ A_z(\mathbf{r}) \cos \phi_z(\mathbf{r}) \end{pmatrix}, \quad (1.8)$$

$$\mathbf{q}(\mathbf{r}) = \begin{pmatrix} A_x(\mathbf{r}) \sin \phi_x(\mathbf{r}) \\ A_y(\mathbf{r}) \sin \phi_y(\mathbf{r}) \\ A_z(\mathbf{r}) \sin \phi_z(\mathbf{r}) \end{pmatrix}. \quad (1.9)$$

Therefore the following is obvious,

$$\mathbf{E}(\mathbf{r}) = \mathbf{p}(\mathbf{r}) + i\mathbf{q}(\mathbf{r}). \quad (1.10)$$

Now we consider the behavior of the vector $\mathcal{E}(\mathbf{r}, t)$, a function of time, at a fixed position $\mathbf{r}$ in space. We will show that $\mathcal{E}(\mathbf{r}, t)$ describes an ellipse in the plane through the vectors $\mathbf{p}(\mathbf{r})$ and $\mathbf{q}(\mathbf{r})$. In general $\mathbf{p}(\mathbf{r})$ and $\mathbf{q}(\mathbf{r})$ are not orthogonal, but as we will show, it is always possible to find two *real* orthogonal vectors $\mathbf{a}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ such that,

$$\mathbf{p}(\mathbf{r}) + i\mathbf{q}(\mathbf{r}) = [\mathbf{a}(\mathbf{r}) + i\mathbf{b}(\mathbf{r})]e^{-i\psi(\mathbf{r})}, \quad (1.11)$$

for some ψ . Eq. (1.11) implies that $\mathbf{a}$ and $\mathbf{b}$ can be expressed in terms of $\mathbf{p}$ and $\mathbf{q}$ as follows:

$$\begin{aligned} \mathbf{a}(\mathbf{r}) &= \text{Re}[\mathbf{a}(\mathbf{r}) + i\mathbf{b}(\mathbf{r})] = \text{Re}\left\{[\mathbf{p}(\mathbf{r}) + i\mathbf{q}(\mathbf{r})]e^{i\psi(\mathbf{r})}\right\} \\ &= \mathbf{p}(\mathbf{r}) \cos \psi(\mathbf{r}) - \mathbf{q}(\mathbf{r}) \sin \psi(\mathbf{r}), \end{aligned} \quad (1.12)$$

and

$$\begin{aligned} \mathbf{b}(\mathbf{r}) &= \text{Im}[\mathbf{a}(\mathbf{r}) + i\mathbf{b}(\mathbf{r})] = \text{Im}\left\{[\mathbf{p}(\mathbf{r}) + i\mathbf{q}(\mathbf{r})]e^{i\psi(\mathbf{r})}\right\} \\ &= \mathbf{p}(\mathbf{r}) \sin \psi(\mathbf{r}) + \mathbf{q}(\mathbf{r}) \cos \psi(\mathbf{r}). \end{aligned} \quad (1.13)$$

By requiring $\mathbf{a}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ to be orthogonal, i.e. $\mathbf{a}(\mathbf{r}) \cdot \mathbf{b}(\mathbf{r}) = 0$, and that $\mathbf{a}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$ are parallel to the axes of the ellipse, ψ is uniquely determined. The derivation of ψ is given in Appendix A.

By using (1.5), (1.10) and (1.11) we find

$$\begin{aligned} \mathcal{E}(\mathbf{r}, t) &= \text{Re}\left[\mathbf{E}(\mathbf{r})e^{-i\omega t}\right] = \text{Re}\left\{[\mathbf{a}(\mathbf{r}) + i\mathbf{b}(\mathbf{r})]e^{-i\omega t - i\psi(\mathbf{r})}\right\} \\ &= \mathbf{a}(\mathbf{r}) \cos[\omega t + \psi(\mathbf{r})] + \mathbf{b}(\mathbf{r}) \sin[\omega t + \psi(\mathbf{r})]. \end{aligned} \quad (1.14)$$

We define unit vectors parallel to the major and minor axes of the polarization ellipse:

$$\hat{\mathbf{e}}_1(\mathbf{r}) = \frac{\mathbf{a}(\mathbf{r})}{|\mathbf{a}(\mathbf{r})|}, \quad (1.15)$$

$$\hat{\mathbf{e}}_2(\mathbf{r}) = \frac{\mathbf{b}(\mathbf{r})}{|\mathbf{b}(\mathbf{r})|}. \quad (1.16)$$

If $|\mathbf{a}(\mathbf{r})| < |\mathbf{b}(\mathbf{r})|$, to ensure that $\hat{\mathbf{e}}_1$ is always parallel to the major axis, then we define

$$\hat{\mathbf{e}}_1(\mathbf{r}) = \frac{\mathbf{b}(\mathbf{r})}{|\mathbf{b}(\mathbf{r})|}, \quad (1.17)$$

$$\hat{\mathbf{e}}_2(\mathbf{r}) = -\frac{\mathbf{a}(\mathbf{r})}{|\mathbf{a}(\mathbf{r})|}. \quad (1.18)$$

Finally, we define $\hat{\mathbf{e}}_3(\mathbf{r}) = \hat{\mathbf{e}}_1(\mathbf{r}) \times \hat{\mathbf{e}}_2(\mathbf{r})$ so that $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ is a positively oriented orthonormal basis. $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ will be called 'the local basis along the local principal axes', or 'the local basis' for short.

The electric field vector on the local basis are defined as:

$$\left. \begin{aligned} \mathcal{E}_1(\mathbf{r}, t) &= \mathcal{E}(\mathbf{r}, t) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{a}| \cos(\omega t + \psi), \\ \mathcal{E}_2(\mathbf{r}, t) &= \mathcal{E}(\mathbf{r}, t) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = |\mathbf{b}| \sin(\omega t + \psi); \end{aligned} \right\} \quad |\mathbf{a}| \geq |\mathbf{b}|, \quad (1.19a)$$

$$\left. \begin{aligned} \mathcal{E}_1(\mathbf{r}, t) &= \mathcal{E}(\mathbf{r}, t) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{b}| \sin(\omega t + \psi), \\ \mathcal{E}_2(\mathbf{r}, t) &= \mathcal{E}(\mathbf{r}, t) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = -|\mathbf{a}| \cos(\omega t + \psi); \end{aligned} \right\} \quad |\mathbf{a}| < |\mathbf{b}|, \quad (1.19b)$$

The polarization ellipse is now described by Eq. (1.19) with major and minor axes along the directions of the unit vectors $\hat{\mathbf{e}}_1(\mathbf{r})$ and $\hat{\mathbf{e}}_2(\mathbf{r})$, respectively. Furthermore if a right-handed screwdriver rotates in the sense of the electric field, then it moves in the direction of the vector $\hat{\mathbf{e}}_3(\mathbf{r})$.

Hence the electric field vector can be written as the sum of two linear polarizations, in the direction parallel to the directions of the major and minor axes of the ellipse. Notice that, the electric field can also be written as a linear combination of two circular polarizations.

The complex electric field vector can be written as:

$$\left. \begin{aligned} E_1(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{a}(\mathbf{r})| e^{-i\psi(\mathbf{r})}, \\ E_2(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = i|\mathbf{b}(\mathbf{r})| e^{-i\psi(\mathbf{r})} = |\mathbf{b}(\mathbf{r})| e^{-i\psi(\mathbf{r}) + i\frac{2\pi}{2}}, \\ E_3(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_3(\mathbf{r}) = 0, \end{aligned} \right\} \quad |\mathbf{a}| \geq |\mathbf{b}|, \quad (1.20a)$$

$$\left. \begin{aligned} E_1(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{b}(\mathbf{r})| e^{-i\psi(\mathbf{r})}, \\ E_2(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = -i|\mathbf{a}(\mathbf{r})| e^{-i\psi(\mathbf{r})} = |\mathbf{b}(\mathbf{r})| e^{-i\psi(\mathbf{r}) - i\frac{2\pi}{2}}, \\ E_3(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_3(\mathbf{r}) = 0, \end{aligned} \right\} \quad |\mathbf{a}| < |\mathbf{b}|. \quad (1.20b)$$

With our definitions, at a certain position, the ellipticity of the polarization ellipse is given by

$$e(\mathbf{r}) = \frac{|E_2(\mathbf{r})|}{|E_1(\mathbf{r})|}. \quad (1.21)$$

On the local basis, Stokes parameters [10] can be derived. They are given by:

$$S_0 = |E_1|^2 + |E_2|^2, \quad (1.22a)$$

$$S_1 = |E_1|^2 - |E_2|^2, \quad (1.22b)$$

$$S_2 = 2|E_1||E_2|\cos\delta = 0, \quad (1.22c)$$

$$S_3 = 2|E_1||E_2|\sin\delta = 2|E_1||E_2|. \quad (1.22d)$$

where δ is the phase difference between $E_2(\mathbf{r})$ and $E_1(\mathbf{r})$, i.e. $\delta = +\pi/2$. All four quantities have the same dimension of intensity. The parameter S_0 represents the total intensity of the field. S_1 is the degree of linear polarization along the $\hat{\mathbf{e}}_1$ axis. S_2 describes the extend to which the field is linear polarized along direction at angles $\pm 45^\circ$ to the major axis of the polarization ellipse. Since $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}$ are parallel to the major and minor axes, respectively, we have $S_2 = 0$ everywhere. The last parameter S_3 represents the degree of circular polarization.

If one of the electric field components is zero, i.e. $E_1 = 0$ or $E_2 = 0$, then we have $S_1/S_0 = \pm 1$ and $S_3 = 0$. In this case, the electric field oscillates along a line, thus it is called linear polarization. When the two electric field components have the same amplitude but a $\pi/2$ phase difference, we have $S_1 = 0$ and $S_3/S_0 = 1$. The electric field in this case is defined as circularly polarized. Other cases are in between these two special extreme situations, and the electric field vector of light in general follows an ellipse as we discussed earlier. Correspondingly the polarization state is elliptic. Note that for the magnetic field there is also a polarization ellipse. But this differs from that of $\mathbf{E}$ and it does not even have the same $\hat{\mathbf{e}}_3$ -vector (i.e. the ellipses are in general not even in one plane).

The time harmonic vector field is used commonly in wave optics formulations, and they are used in this thesis to describe the electromagnetic field of the optical beams.

1.4 Content of the thesis

In Chapter 2, a plane wave that propagates through an anisotropic medium with complex hermitian tensor is studied. The treatment presented is more general than that is in standard textbooks because we consider anisotropic media with in general complex hermitian permittivity tensors instead of merely a real symmetric tensor. Then an anisotropic multilayer structures is considered. Furthermore, a systematic routine to calculate electromagnetic field distributions in a multilayer with arbitrary number layers is described. In the multilayer structure, any layer can be an anisotropic medium. This routine is used to check rigorous numerical codes that is used for non-planar grating structures consisting of anisotropic media. Finally, a general review of different rigorous numerical methods to simulate anisotropic gratings is given. We describe in particular the finite element method (FEM) which we used for simulation in our research.

In Chapter 3, one of the polarization selective gratings consisting of a LC is discussed. The orientation of the LC in this chapter is parallel to the grating grooves as illustrated in Fig. 1.7 (a). In this chapter, rigorous calculations are used and the results obtained are discussed. The application as polarizing color separator in backlight for flat-panel display systems is considered.

In Chapter 4 polarization gratings are discussed, in which the LC molecules are perpendicular to the grating grooves. The LCs assembly of this case is illustrated in Fig. 1.7 (b). The curing process to achieve such organization of LCs is presented in more details in the beginning of the chapter. In this chapter, the grating is a thick one, i.e. a volume grating, in which the Bragg diffraction is observed. The Bragg diffraction condition is discussed. In the later part of the chapter, results from simulations and comparisons with experiments are discussed.

Photoanisotropic polarization gratings of which an example is shown in Fig. 1.7(c), are investigated in Chapter 5 and 6. The recording or writing process of the grating is considered in Chapter 5. First, the interference pattern of two plane waves is described using the position dependent local basis. From this the light induced dielectric tensor is obtained using the model of Ref. [11]. Two special cases are considered: one is using two orthogonally circular polarizations as recording beams; the other is using two orthogonal linear polarizations. Materials possessing of photoanisotropy and corresponding recording processes are described at the end of Chapter 5.

After the writing of the photoanisotropic polarization grating, the diffraction that occurs in the reading process is studied in Chapter 6. An analytic formula for the approximative transmission matrix is developed for some limited paraxial case by making some approximations. Rigorous numerical simulations extend the study to the general case beyond the approximative transmission matrix method. In the paraxial

case, the results from the two methods are compared. The dependence of diffraction efficiencies on many parameters is investigated in detail.

Chapter 2

Light propagation in anisotropic media

2.1 Introduction

The propagation and distribution of light in media depends basically on the material configuration of the system. This information about the physical state of the system is described by the dielectric permittivity tensor. The variation of anisotropy of a medium can be described by changes in the components of the permittivity tensor. Those changes are critical and important because they cause modifications of the optical properties of the system and consequently of the propagation of the light. The dielectric permittivity tensor thus plays an important role.

Maxwell's equations is the most fundamental theory in wave optics. Many reference books [9, 12] explain the equations thoroughly. In most literatures that treat anisotropic media, the dielectric permittivity is often assumed to be a real symmetric tensor. For this classical case, we will not repeat the theory in this thesis since it is well known. What we will solve is that the light propagation in a non-absorption anisotropic medium with a *complex hermitian* type of dielectric tensor. This is a more general case which includes the real symmetric case as a special case. The hermitian type of tensors are known in magnetic-optical effect due to the existence of a static magnetic field, of which the study and discussion can be found in some classical books such as book by Landau and Lifshitz [13] and book by Chen [14]. In this thesis, we deal with the hermitian permittivity tensors, which depend on the electric fields of light waves.

In Section 2.2 we first discuss the time-harmonic Maxwell's equations in an anisotropic medium in complex space $\mathbb{C}^3$. Plane waves in homogeneous media are discussed in Section 2.3. Then the reflection and refraction between two layers consisting anisotropic materials is studied in Section 2.4. In that section the reflection and transmission coefficients for circular polarization are derived at the in-

interface between two anisotropic media with in general hermitian permittivity tensor. A further extension of the study to a three-layer system is followed in a later part of the section. Finally, a multilayer system is considered in Section 2.5. This problem is important because its solution is used to benchmark rigorous codes for anisotropic gratings.

In Section 2.6 we give an overview of existing rigorous numerical methods for diffraction gratings consisting of anisotropic materials. In particular we will discuss the finite element method (FEM) which is the numerical method that has been applied in our research.

2.2 Time-harmonic anisotropic Maxwell's equations

The time-harmonic Maxwell's equations in a non-magnetic anisotropic medium without sources are

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H}, \quad (2.1)$$

$$\nabla \times \mathbf{H} = -i\omega\epsilon_0\underline{\underline{\epsilon}}\mathbf{E}, \quad (2.2)$$

with ϵ_0 the dielectric permittivity and μ_0 the magnetic permeability of vacuum¹. In the equation, $\underline{\underline{\epsilon}}$ is the relative dielectric permittivity tensor. For an anisotropic loss-free material, $\underline{\underline{\epsilon}}$ is a complex hermitian tensor of rank 2. This means $\underline{\underline{\epsilon}}_{ij} = \underline{\underline{\epsilon}}_{ji}^*$ for all $i, j = 1, 2, 3$. The star * indicates the complex conjugate. This means the medium is non-absorbing for electromagnetic field. For a time-harmonic field in an anisotropic medium, the electric displacement vector $\mathbf{D}$ is connected with the electric field $\mathbf{E}$ through the dielectric permittivity as follows:

$$\mathbf{D} = \epsilon_0\underline{\underline{\epsilon}}\mathbf{E}. \quad (2.3)$$

So in general, the vector $\mathbf{D}$ is not in the same direction as $\mathbf{E}$. Because of the complex hermitian dielectric permittivity tensor of the medium, polarization can be rotated when it is propagating through. More about the property will be studied in the following parts of the thesis.

2.3 Light propagation in homogeneous anisotropic media

We seek plane wave solutions of the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{A}e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (2.4)$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{B}e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (2.5)$$

¹ $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, by definition $\mu_0 = 4\pi \times 10^{-7}$ H/m, and $c = \sqrt{1/(\epsilon_0\mu_0)} = 2.9979 \times 10^8$ m/s is the speed of light in vacuum.

where $\mathbf{k}$ is the wave vector and $\mathbf{A}$ and $\mathbf{B}$ are constant, in general complex vectors. The physical field are then

$$\mathcal{E}(\mathbf{r}, t) = \text{Re} \left[\mathbf{E}(\mathbf{r}) e^{-i\omega t} \right] = \text{Re} \left[\mathbf{A} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \right], \quad (2.6)$$

$$\mathcal{H}(\mathbf{r}, t) = \text{Re} \left[\mathbf{H}(\mathbf{r}) e^{-i\omega t} \right] = \text{Re} \left[\mathbf{B} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \right]. \quad (2.7)$$

With respect to the cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ we have

$$\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}}, \quad (2.8)$$

$$\mathbf{r} = x \hat{\mathbf{x}} + y \hat{\mathbf{y}} + z \hat{\mathbf{z}}, \quad (2.9)$$

and the dot product is given by

$$\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z. \quad (2.10)$$

The dot product is the ordinary scalar product of real vectors, i.e. vectors in $\mathbb{R}^3$. For complex vectors $\mathbf{a} \in \mathbb{C}^3$ we have

$$\mathbf{a} = a_x \hat{\mathbf{x}} + a_y \hat{\mathbf{y}} + a_z \hat{\mathbf{z}}, \quad (2.11)$$

with a_x, a_y, a_z complex. The scalar product on $\mathbb{C}^3$ is given by

$$\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3} = a_x b_x^* + a_y b_y^* + a_z b_z^* = \mathbf{a} \cdot \mathbf{b}^* \quad (2.12)$$

where the ** denotes complex conjugation. Hence if $\mathbf{a}$ and $\mathbf{b}$ are real, their complex scalar product is identical to the dot product.

We shall first seek solutions (2.4) with real wave vector $\mathbf{k}$, i.e. for which k_x, k_y and k_z are real. By substitution of Eqs. (2.4), (2.5) into Maxwell's equations, we get

$$\mathbf{H} = \frac{1}{\omega \mu_0} \mathbf{k} \times \mathbf{E}, \quad (2.13)$$

$$\mathbf{D} = -\frac{1}{\omega} \mathbf{k} \times \mathbf{H}. \quad (2.14)$$

If $\mathbf{k}$ is real, this implies that the vectors $\mathcal{D}(\mathbf{r}, t) = \text{Re} [\mathbf{D}(\mathbf{r}) e^{-i\omega t}]$, $\mathcal{H}(\mathbf{r}, t)$, $\mathbf{k}$ form a positively orientated orthogonal system.

By eliminating the magnetic field from Maxwell's equations, we obtain the vector Helmholtz equation for the electric field

$$k_0^2 \underline{\underline{\epsilon}} \mathbf{E} - \nabla \times \nabla \times \mathbf{E} = 0, \quad (2.15)$$

where $k_0 = \omega \sqrt{\epsilon_0 \mu_0}$ is the wave number in vacuum. By substituting Eq. (2.4) into Eq. (2.15) we find, that on the $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ basis we have:

$$(k_0^2 \underline{\underline{\epsilon}} - \mathbf{k} \cdot \mathbf{k}) \mathbf{A} + \mathbf{k} \cdot \mathbf{A} \mathbf{k} = 0 \quad (2.16)$$

This linear system has only a nonzero solution for $\mathbf{A}$ when the determinant vanishes.

Note that $\underline{\epsilon}$ is a linear map $\mathbb{C}^3 \rightarrow \mathbb{C}^3$. Since $\underline{\epsilon}$ is hermitian, there exists an orthonormal basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$ of $\mathbb{C}^3$ on which $\underline{\epsilon}$ is diagonal. 'Orthonormal' means in the sense of the scalar product (2.12) on $\mathbb{C}^3$, i.e.

$$\langle \hat{\xi}, \hat{\xi} \rangle_{\mathbb{C}^3} = \langle \hat{\eta}, \hat{\eta} \rangle_{\mathbb{C}^3} = \langle \hat{\zeta}, \hat{\zeta} \rangle_{\mathbb{C}^3} = 1, \quad (2.17)$$

$$\langle \hat{\xi}, \hat{\eta} \rangle_{\mathbb{C}^3} = \langle \hat{\xi}, \hat{\zeta} \rangle_{\mathbb{C}^3} = \langle \hat{\eta}, \hat{\zeta} \rangle_{\mathbb{C}^3} = 0. \quad (2.18)$$

Then for two complex vectors $\mathbf{a}, \mathbf{b}$

$$\mathbf{a} = a_{\xi} \hat{\xi} + a_{\eta} \hat{\eta} + a_{\zeta} \hat{\zeta}, \quad (2.19)$$

$$\mathbf{b} = b_{\xi} \hat{\xi} + b_{\eta} \hat{\eta} + b_{\zeta} \hat{\zeta}, \quad (2.20)$$

with

$$a_{\xi} = \langle \mathbf{a}, \hat{\xi} \rangle_{\mathbb{C}^3}, \quad b_{\xi} = \langle \mathbf{b}, \hat{\xi} \rangle_{\mathbb{C}^3}, \quad \text{etc.} \quad (2.21)$$

and

$$\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3} = a_{\xi} b_{\xi}^* + a_{\eta} b_{\eta}^* + a_{\zeta} b_{\zeta}^*. \quad (2.22)$$

If $\hat{\xi}, \hat{\eta}$ and $\hat{\zeta}$ are written on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$,

$$\hat{\xi} = \xi_x \hat{\mathbf{x}} + \xi_y \hat{\mathbf{y}} + \xi_z \hat{\mathbf{z}}, \quad (2.23a)$$

$$\hat{\eta} = \eta_x \hat{\mathbf{x}} + \eta_y \hat{\mathbf{y}} + \eta_z \hat{\mathbf{z}}, \quad (2.23b)$$

$$\hat{\zeta} = \zeta_x \hat{\mathbf{x}} + \zeta_y \hat{\mathbf{y}} + \zeta_z \hat{\mathbf{z}}; \quad (2.23c)$$

then the matrices,

$$U = \begin{pmatrix} \xi_x & \eta_x & \zeta_x \\ \xi_y & \eta_y & \zeta_y \\ \xi_z & \eta_z & \zeta_z \end{pmatrix} \quad \text{and} \quad U^{-1} = U^* = \begin{pmatrix} \xi_x^* & \xi_y^* & \xi_z^* \\ \eta_x^* & \eta_y^* & \eta_z^* \\ \zeta_x^* & \zeta_y^* & \zeta_z^* \end{pmatrix} \quad (2.24)$$

transform the coefficients of a vector or the basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$ to that on $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ and vice versa:

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = U \begin{pmatrix} a_{\xi} \\ a_{\eta} \\ a_{\zeta} \end{pmatrix}; \quad \begin{pmatrix} a_{\xi} \\ a_{\eta} \\ a_{\zeta} \end{pmatrix} = U^* \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}. \quad (2.25)$$

We write $\mathbf{k}$ and $\mathbf{A}$ on both bases:

$$\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}} = k_{\xi} \hat{\xi} + k_{\eta} \hat{\eta} + k_{\zeta} \hat{\zeta}, \quad (2.26)$$

$$\mathbf{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}} = A_{\xi} \hat{\xi} + A_{\eta} \hat{\eta} + A_{\zeta} \hat{\zeta}, \quad (2.27)$$

with

$$k_x = \mathbf{k} \cdot \hat{\mathbf{x}}, \quad k_{\xi} = \langle \mathbf{k}, \hat{\xi} \rangle_{\mathbb{C}^3}; \quad \text{etc.} \quad (2.28)$$

Note that although k_x, k_y, k_z are real, k_{ξ}, k_{η} and k_{ζ} are in general complex. We have

$$\begin{aligned} \mathbf{k} \cdot \mathbf{k} &= k_x k_x + k_y k_y + k_z k_z = k_x k_x^* + k_y k_y^* + k_z k_z^* = \langle \mathbf{k}, \mathbf{k} \rangle_{\mathbb{C}^3} \\ &= k_{\xi} k_{\xi}^* + k_{\eta} k_{\eta}^* + k_{\zeta} k_{\zeta}^*, \end{aligned} \quad (2.29)$$

$$\begin{aligned} \mathbf{A} \cdot \mathbf{k} &= A_x k_x + A_y k_y + A_z k_z = A_x k_x^* + A_y k_y^* + A_z k_z^* = \langle \mathbf{A}, \mathbf{k} \rangle_{\mathbb{C}^3} \\ &= A_{\xi} k_{\xi}^* + A_{\eta} k_{\eta}^* + A_{\zeta} k_{\zeta}^*. \end{aligned} \quad (2.30)$$

Let $\epsilon_\xi, \epsilon_\eta, \epsilon_\zeta$ be the eigenvalue of $\underline{\underline{\epsilon}}$, i.e.

$$\underline{\underline{\epsilon}}\hat{\xi} = \epsilon_\xi \hat{\xi}, \quad \underline{\underline{\epsilon}}\hat{\eta} = \epsilon_\eta \hat{\eta}, \quad \underline{\underline{\epsilon}}\hat{\zeta} = \epsilon_\zeta \hat{\zeta}. \quad (2.31)$$

The eigenvalues are real and on the basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$ we have

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_\xi & 0 & 0 \\ 0 & \epsilon_\eta & 0 \\ 0 & 0 & \epsilon_\zeta \end{pmatrix}. \quad (2.32)$$

With Eqs. (2.29), (2.30) and (2.32), it follows that on the basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$, Eq. (2.16) becomes

$$\begin{pmatrix} k_0^2 \epsilon_\xi - |k_\eta|^2 - |k_\zeta|^2 & k_\eta^* k_\xi & k_\zeta^* k_\xi \\ k_\xi^* k_\eta & k_0^2 \epsilon_\eta - |k_\xi|^2 - |k_\zeta|^2 & k_\zeta^* k_\eta \\ k_\xi^* k_\zeta & k_\eta^* k_\zeta & k_0^2 \epsilon_\zeta - |k_\xi|^2 - |k_\eta|^2 \end{pmatrix} \begin{pmatrix} A_\xi \\ A_\eta \\ A_\zeta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (2.33)$$

Setting the determinant of Eq. (2.33) equal to zero gives the well-known Fresnel equation:

$$\frac{|k_\xi|^2}{|\mathbf{k}|^2 - k_0^2 \epsilon_\xi} + \frac{|k_\eta|^2}{|\mathbf{k}|^2 - k_0^2 \epsilon_\eta} + \frac{|k_\zeta|^2}{|\mathbf{k}|^2 - k_0^2 \epsilon_\zeta} = 1. \quad (2.34)$$

If we define the unit vector

$$\hat{\mathbf{s}} = \frac{\mathbf{k}}{|\mathbf{k}|} = s_\xi \hat{\xi} + s_\eta \hat{\eta} + s_\zeta \hat{\zeta}, \quad (2.35)$$

then

$$\mathbf{k} = n k_0 \hat{\mathbf{s}} \quad (2.36)$$

with

$$n = \frac{|\mathbf{k}|}{k_0} = \frac{\sqrt{\mathbf{k} \cdot \mathbf{k}}}{k_0}. \quad (2.37)$$

Then Eq. (2.34) can be written

$$\frac{s_\xi^2}{n^2 - \epsilon_\xi} + \frac{s_\eta^2}{n^2 - \epsilon_\eta} + \frac{s_\zeta^2}{n^2 - \epsilon_\zeta} = \frac{1}{n^2}. \quad (2.38)$$

For given unit vector $\hat{\mathbf{s}}$, there are two solutions n_1, n_2 for n .

If k_ξ, k_η, k_ζ satisfy Eq. (2.34), then

$$\begin{aligned} \mathbf{A} &= A_\xi \hat{\xi} + A_\eta \hat{\eta} + A_\zeta \hat{\zeta}, \\ &= -\langle \mathbf{A}, \mathbf{k} \rangle_{\mathbb{C}^3} \left\{ \frac{k_\xi}{k_0^2 \epsilon_\xi - |\mathbf{k}|^2} \hat{\xi} + \frac{k_\eta}{k_0^2 \epsilon_\eta - |\mathbf{k}|^2} \hat{\eta} + \frac{k_\zeta}{k_0^2 \epsilon_\zeta - |\mathbf{k}|^2} \hat{\zeta} \right\}. \end{aligned} \quad (2.39)$$

The components of $\mathbf{A}$ on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ follow from Eq. (2.25)

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = U \begin{pmatrix} A_\xi \\ A_\eta \\ A_\zeta \end{pmatrix}. \quad (2.40)$$

Note that A_x, A_y and A_z are in general complex, i.e. the electric field of the plane wave is in general elliptically polarized. However, if $\underline{\underline{\epsilon}}$ is real symmetric, then the orthonormal basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$ consists of real vectors, hence the matrices U and U^T are then real. Then $\mathbf{A}$ is a real vector on the cartesian basis and hence a plane wave is then linearly polarized.

When a plane wave is incident on an interface with a second (possibly anisotropic) medium with normal parallel to the z -direction, then k_x and k_y are real but k_z could be purely imaginary in the second medium. This can also happen in one or several of the layers of a multilayer. We shall therefore now also consider a plane wave (2.4), (2.5), with wave vector $\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}}$, where the components are complex. The medium in which the plane wave propagates is still assumed to have a hermitian tensor.

We again rewrite Eq. (2.16) on the orthonormal basis $\{\hat{\xi}, \hat{\eta}, \hat{\zeta}\}$ on which $\underline{\underline{\epsilon}}$ is diagonal. We have (2.25), as before,

$$\underline{\underline{\epsilon}} = U \begin{pmatrix} \epsilon_\xi & 0 & 0 \\ 0 & \epsilon_\eta & 0 \\ 0 & 0 & \epsilon_\zeta \end{pmatrix} U^*. \quad (2.41)$$

By substituting this into Eq. (2.16) and applying U^* from the left, we find

$$\left[k_0^2 \begin{pmatrix} \epsilon_\xi & 0 & 0 \\ 0 & \epsilon_\eta & 0 \\ 0 & 0 & \epsilon_\zeta \end{pmatrix} - \mathbf{k} \cdot \mathbf{k} \right] U^* \mathbf{A} + \mathbf{k} \cdot \mathbf{A} \mathbf{v} = 0, \quad (2.42)$$

where $\mathbf{v} = U^* \mathbf{k}$. Let $\mathbf{k}^*$ be the vector which is on the cartesian basis defined by,

$$\mathbf{k}^* = k_x^* \hat{\mathbf{x}} + k_y^* \hat{\mathbf{y}} + k_z^* \hat{\mathbf{z}}. \quad (2.43)$$

Then

$$\begin{aligned} \mathbf{k} \cdot \mathbf{A} &= A_x k_x + A_y k_y + A_z k_z \\ &= A_x (k_x^*)^* + A_y (k_y^*)^* + A_z (k_z^*)^* \\ &= \langle \mathbf{A}, \mathbf{k}^* \rangle_{\mathbb{C}^3} = \langle U U^* \mathbf{A}, \mathbf{k}^* \rangle_{\mathbb{C}^3} \\ &= \langle U^* \mathbf{A}, U^* \mathbf{k}^* \rangle_{\mathbb{C}^3} \\ &= A_\xi (U^* \mathbf{k}^*)_\xi^* + A_\eta (U^* \mathbf{k}^*)_\eta^* + A_\zeta (U^* \mathbf{k}^*)_\zeta^* \\ &= \langle U^* \mathbf{A}, \mathbf{w} \rangle_{\mathbb{C}^3} \end{aligned} \quad (2.44)$$

where $\mathbf{w} = U^* \mathbf{k}^*$. Substitute this into Eq. (2.42), then

$$\left[k_0^2 \begin{pmatrix} \epsilon_\xi & 0 & 0 \\ 0 & \epsilon_\eta & 0 \\ 0 & 0 & \epsilon_\zeta \end{pmatrix} - \mathbf{k} \cdot \mathbf{k} \right] \begin{pmatrix} A_\xi \\ A_\eta \\ A_\zeta \end{pmatrix} + M \begin{pmatrix} A_\xi \\ A_\eta \\ A_\zeta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (2.45)$$

where

$$M = \begin{pmatrix} v_\xi w_\xi^* & v_\xi w_\eta^* & v_\xi w_\zeta^* \\ v_\eta w_\xi^* & v_\eta w_\eta^* & v_\eta w_\zeta^* \\ v_\zeta w_\xi^* & v_\zeta w_\eta^* & v_\zeta w_\zeta^* \end{pmatrix}. \quad (2.46)$$

Set the determinant equal to zero. Then for given real k_x, k_y we have an equation for k_z^2 which has two real solutions corresponding to. If $k_z^2 < 0$ then the wave is evanescent, otherwise it is propagating. The electric field vector corresponding to a zero determinant is given by

$$\begin{pmatrix} A_\xi \\ A_\eta \\ A_\zeta \end{pmatrix} = -\langle \mathbf{A}, \mathbf{w} \rangle_{\mathbb{C}^3} \begin{pmatrix} \frac{v_\xi}{k_0^2 \epsilon_\xi - \mathbf{k} \cdot \mathbf{k}} \\ \frac{v_\eta}{k_0^2 \epsilon_\eta - \mathbf{k} \cdot \mathbf{k}} \\ \frac{v_\zeta}{k_0^2 \epsilon_\zeta - \mathbf{k} \cdot \mathbf{k}} \end{pmatrix}. \quad (2.47)$$

The components on the Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ are then finally can be derived by Eq. (2.40).

2.4 Reflection and transmission at the interface between two layers

The reflection and transmission for the case of a real dielectric tensor is treated in many textbooks. In this and the next Section we give examples for the case of a complex dielectric tensor. Assume that we have a two-layer system with interface at $z = 0$. The upper layer is an isotropic dielectric medium with real relative permittivity ϵ_1 . The lower layer is an anisotropic medium with permittivity tensor given by:

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy}^* & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}. \quad (2.48)$$

Assume that $\epsilon_{xy} = -i\beta$ with $\beta > 0$. Then $|\epsilon_{xy}| = \beta$ and $\frac{|\epsilon_{xy}|}{\epsilon_{xy}} = i$. If furthermore $\epsilon_{zz} > \epsilon_{xx} + |\epsilon_{xy}| = \epsilon_{xx} + \beta$, then the ordered eigenvalues and corresponding normalized eigenvectors are

$$\epsilon_\xi = |\epsilon_{xx} - |\epsilon_{xy}||, \quad \hat{\xi} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}); \quad (2.49)$$

$$\epsilon_\eta = \epsilon_{xx} + |\epsilon_{xy}|, \quad \hat{\eta} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) = \hat{\xi}^*; \quad (2.50)$$

$$\epsilon_\zeta = \epsilon_{zz}, \quad \hat{\zeta} = \hat{\mathbf{z}}. \quad (2.51)$$

Suppose there is a plane wave perpendicularly incident from medium $z > 0$. In the lower layer with permittivity given by Eq. (2.48), the field of a plane wave can be written as the sum of a left and right-hand circularly polarized wave [13]. Hence the incident wave in $z > 0$ can be written as

$$\mathbf{E}^i(\mathbf{r}) = \frac{a_R^i}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{-ik_0 n_1 z} + \frac{a_L^i}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_1 z}, \quad (2.52)$$

$$\mathbf{H}^i(\mathbf{r}) = -\frac{ik_0 n_1}{\omega\mu_0} \frac{a_R^i}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{-ik_0 n_1 z} + \frac{ik_0 n_1}{\omega\mu_0} \frac{a_L^i}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_1 z}, \quad (2.53)$$

where $n_1 = \sqrt{\epsilon_1}$, a_R, a_L are the amplitudes of the decomposed right and left-hand circular polarizations and the superscript 'i' indicates the incident field. The reflected wave (as indicated by the superscript 'r') is written as

$$\mathbf{E}^r(\mathbf{r}) = \frac{a_R^r}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{ik_0 n_1 z} + \frac{a_L^r}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_1 z}, \quad (2.54)$$

$$\mathbf{H}^r(\mathbf{r}) = \frac{ik_0 n_1}{\omega \mu_0} \frac{a_R^r}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{ik_0 n_1 z} - \frac{ik_0 n_1}{\omega \mu_0} \frac{a_L^r}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_1 z}, \quad (2.55)$$

The field in $z < 0$ can be written as

$$\mathbf{E}^t(\mathbf{r}) = \frac{a_R^t}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{-ik_0 n_\xi z} + \frac{a_L^t}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_\eta z}, \quad (2.56)$$

$$\mathbf{H}^t(\mathbf{r}) = \frac{-ik_0 n_\xi}{\omega \mu_0} \frac{a_R^t}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})e^{-ik_0 n_\xi z} + \frac{ik_0 n_\eta}{\omega \mu_0} \frac{a_L^t}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_\eta z}, \quad (2.57)$$

where $n_\xi = \sqrt{\epsilon_\xi}$ and $n_\eta = \sqrt{\epsilon_\eta}$.

By matching the tangential electric and magnetic field components we find:

$$\left. \begin{aligned} a_R^i + a_R^r &= a_R^t \\ -n_1 a_R^i + n_1 a_R^r &= -n_\xi a_R^t \end{aligned} \right\} \Rightarrow \begin{aligned} a_R^r &= \frac{n_1 - n_\xi}{n_1 + n_\xi} a_R^i, \\ a_R^t &= \frac{2n_1}{n_1 + n_\xi} a_R^i, \end{aligned} \quad (2.58)$$

and

$$\left. \begin{aligned} a_L^i + a_L^r &= a_L^t \\ n_1 a_L^i - n_1 a_L^r &= n_\eta a_L^t \end{aligned} \right\} \Rightarrow \begin{aligned} a_L^r &= \frac{n_1 - n_\eta}{n_1 + n_\eta} a_L^i, \\ a_L^t &= \frac{2n_1}{n_1 + n_\eta} a_L^i. \end{aligned} \quad (2.59)$$

We introduce notation for coefficient between the reflected/transmitted field and the incident field for right and left-hand circular polarizations as follows,

$$\begin{aligned} r_R^{12} &= \frac{n_1 - n_\xi}{n_1 + n_\xi}, & t_R^{12} &= \frac{2n_1}{n_1 + n_\xi}, \\ r_L^{12} &= \frac{n_1 - n_\eta}{n_1 + n_\eta}, & t_L^{12} &= \frac{2n_1}{n_1 + n_\eta}. \end{aligned} \quad (2.60)$$

If the wave is incident from the anisotropic medium, then we find,

$$\begin{aligned} r_R^{21} &= \frac{n_\xi - n_1}{n_1 + n_\xi} = -r_R^{12}, & t_R^{21} &= \frac{2n_\xi}{n_1 + n_\xi}, \\ r_L^{21} &= \frac{n_\eta - n_1}{n_1 + n_\eta} = -r_L^{12}, & t_L^{21} &= \frac{2n_1}{n_1 + n_\eta}. \end{aligned} \quad (2.61)$$

We conclude that in a two layer system, left and right-hand circularly polarized plane wave do not mix when they propagate parallel to the z -axis.

2.4.1 Three layers

Suppose we have a special three-layer system (see Fig. 2.1) with:

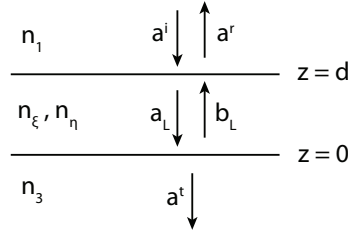


Figure 2.1: A three-layer system.

- in $z > d$ an isotropic medium with refractive index n_1 .
- for $0 < z < d$ an anisotropic medium with permittivity tensor of type (2.48).
- in $z < 0$ an isotropic medium with refractive index n_3 .

For left circularly polarized waves:

$$\left. \begin{aligned} \mathbf{E} &= \frac{a_L^i}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_1 z} + \frac{a_L^r}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_1 z}, \\ \mathbf{H} &= \frac{ik_0 n_1}{\omega\mu_0} \frac{a_L^i}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_1 z} - \frac{ik_0 n_1}{\omega\mu_0} \frac{a_L^r}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_1 z}, \end{aligned} \right\} \quad z > d, \quad (2.62)$$

$$\left. \begin{aligned} \mathbf{E} &= \frac{a_L}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_\eta z} + \frac{b_L}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_\eta z}, \\ \mathbf{H} &= \frac{ik_0 n_\eta}{\omega\mu_0} \frac{a_L}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_\eta z} - \frac{ik_0 n_\eta}{\omega\mu_0} \frac{b_L}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{ik_0 n_\eta z}, \end{aligned} \right\} \quad 0 < z < d, \quad (2.63)$$

$$\left. \begin{aligned} \mathbf{E} &= \frac{a_L^t}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_3 z} \\ \mathbf{H} &= \frac{ik_0 n_3}{\omega\mu_0} \frac{a_L^t}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})e^{-ik_0 n_3 z} \end{aligned} \right\} \quad z < 0. \quad (2.64)$$

Matching the tangential components of the electromagnetic field results in:

$$a_L = \frac{t_L^{12} e^{ik_0(n_\eta - n_1)d}}{1 + r_L^{12} r_L^{23} e^{2ik_0 n_\eta d}} a_L^i, \quad (2.65)$$

$$b_L = \frac{t_L^{12} r_L^{23} e^{ik_0(n_\eta - n_1)d}}{1 + r_L^{12} r_L^{23} e^{2ik_0 n_\eta d}} a_L^i, \quad (2.66)$$

where $r_L^{23} = \frac{n_\eta - n_3}{n_\eta + n_3}$, and

$$a_L^r = \frac{r_L^{12} 2t_L^{12} e^{-2ik_0 n_1 d} + r_L^{23} e^{2ik_0(n_\eta - n_1)d}}{1 + r_L^{12} r_L^{23} e^{2ik_0 n_\eta d}} a_L^i, \quad (2.67)$$

$$a_L^t = \frac{t_L^{12}(1 + r_L^{23})e^{ik_0(n_\eta - n_1)d}}{1 + r_L^{12} r_L^{23} e^{2ik_0 n_\eta d}} a_L^i = \frac{t_L^{12} t_L^{23} e^{ik_0(n_\eta - n_1)d}}{1 + r_L^{12} r_L^{23} e^{2ik_0 n_\eta d}} a_L^i. \quad (2.68)$$

The solution for right-hand circularly polarized waves follows from that of left-hand circularly polarized waves by the substitution: $n_\eta \rightarrow n_\xi$ everywhere (including in the reflection and transmission coefficients).

2.5 Multilayer system

To generalize the analysis, we consider an N-layer system which is sketched in Fig. 2.2. For this system, the interface planes are at $z = z_i$, $i = 1, 2, \dots, n, \dots, N - 1$ with $z_i > z_{i+1}$. For generality both the top and bottom half-infinite layer may also be anisotropic media. Notice that when a particular layer is isotropic, its permittivity tensor is diagonal and all diagonal elements are the same.

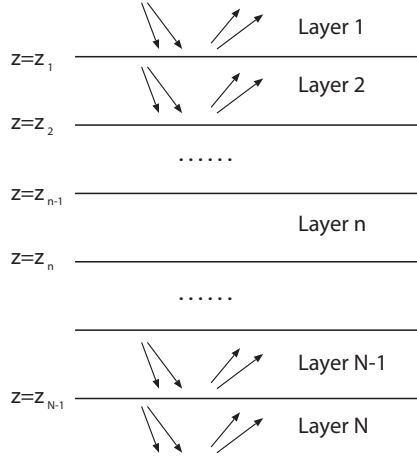


Figure 2.2: A schematic drawing of the multilayer system.

Consider an electromagnetic plane wave with field strength $\mathbf{E}, \mathbf{H}$ and wave vector $\mathbf{k}$ incident from top layer 1. The tangential components of the wave vector, k_x, k_y are the same in all layers. The requirement that the determinant of system Eq. (2.16) must vanish then yields an equation for k_z^2 , which has two real solutions. This applies to a non-absorbing medium, with the solutions for k_z being real or purely imaginary. Hence there are four solutions for k_z , namely two pairs of opposite solutions which correspond to waves propagating (or evanescent) in the positive and negative z -direction. When $k_z^2 < 0$, the wave is evanescent. Subsequently four solutions for $\mathbf{A}$ are obtained. Each $\mathbf{A}$ is normalized, i.e. $\mathbf{A} \cdot \mathbf{A}^* = 1$, and it indicates the polarization of the electric plane wave. Then for each layer, there holds,

$$\mathbf{E}(\mathbf{r}) = \sum_{j=1}^4 c_j \mathbf{A}_j e^{i\mathbf{k}_j \cdot \mathbf{r}}, \quad (2.69)$$

$$\mathbf{H}(\mathbf{r}) = \sum_{j=1}^4 c_j \frac{\mathbf{k}_j}{\omega \mu_0} \times \mathbf{A}_j e^{i\mathbf{k}_j \cdot \mathbf{r}}, \quad (2.70)$$

with c_j is a complex amplitude. The tangential electromagnetic field components at an interface are continuous. With the known incident field, matching the boundary conditions yields a linear system for the unknown amplitudes. For a N-layer system,

if there is only one plane wave source incident from the top, there are $4 \times (N - 1)$ unknown amplitudes c_j , which can be obtained from the $4 \times (N - 1)$ linear equations derived from matching the tangential components. With expansion coefficients c_j obtained, the solution is determined.

The electromagnetic field can be calculated for any position in space $\mathbf{r}$. First find in which region or layer $\mathbf{r}$ lays; then substitute the specific $\mathbf{k}_j, \mathbf{A}_j$ and corresponding c_j ($j = 1, 2, 3, 4$) into Eqs. (2.69), (2.70) to obtain the field strength at that position $\mathbf{r}$.

In the following part we show one example of the light propagation problem in a multilayer structure. Suppose that we have a three-layer system:

- the top layer (half-infinite), air with $n_1 = 1$;
- the middle layer ($8 \mu\text{m}$ thickness), anisotropic with a complex hermitian permittivity tensor, $\underline{\underline{\epsilon}} = \begin{pmatrix} 2.7 & -0.2i & 0 \\ 0.2i & 2.25 & 0 \\ 0 & 0 & 2.25 \end{pmatrix}$;
- the lower layer (half-infinite), isotropic material with $n_3 = 1.5$.

A plane wave with electric field vector $\mathbf{E} = (0, 1, 0)^T$ is perpendicularly incident from the top as shown in Fig. 2.3.

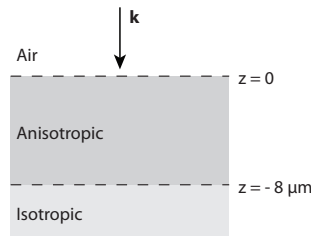


Figure 2.3: A three-layer structure with an anisotropic medium in the middle.

The tangential components of the electric field for the propagating field in the three-layer structure are plotted in Fig. 2.4. Because of the complex component in the dielectric permittivity tensor, the two orthogonally linearly polarized beams are coupled. For a linearly y -polarized incident plane wave, the x -component of the electric field is generated, so that the electric field in the medium is generally elliptically polarized. Notice that regions where the x -component has the large amplitudes are also the regions where the y -components has the small amplitudes. The polarization state rotates along the z -axis.

By using the multi-layer calculation routine that has been developed above, we find two refractive indices for propagating waves in the anisotropic layer, namely $n^{(1)} = 1.47$, $n^{(2)} = 1.67$. With respect to each refractive index, there are two propagating wave vectors, one in the positive z -direction, the other in the negative z -direction, and thus

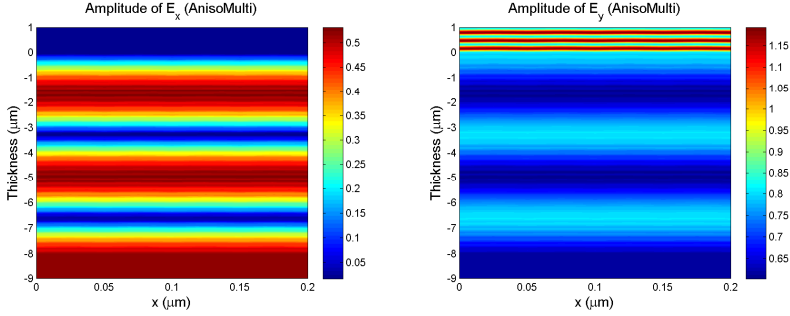


Figure 2.4: An example of light propagating in a multi-layer system consisting of anisotropic materials.

in total there are four propagating wave vectors in the middle layer. A standing wave of which the amplitude of the field varies periodically in the z -direction is formed in the middle anisotropic layer. The period of the variation is determined by the birefringence of the material, i.e. $\Delta n = n^{(2)} - n^{(1)}$ and has period $\lambda/\Delta n$ where λ is the wavelength of the plane wave in vacuum. If we look carefully into the plots, some small fringes can be observed, which is due to the interference of the up-coming and down-going waves which sense the same refractive index.

2.6 Methods to solve anisotropic grating problems

In general, Maxwell's equations have to be used to determine the interaction between any optical wave and any diffracting structure by solving the electromagnetic boundary problem. This means that the electromagnetic boundary conditions are applied at every point along the boundary. When the interaction on the boundary is properly established, the resulting field becomes a secondary source. Then propagation of the field gives the diffraction field. Analytic solutions of the electromagnetic boundary value problem only exist for a few extreme simple geometries, such as, cubes, cylinders, and spheres [15, 16]. To a closed-form analytical solution, there should exist a coordinate system on which the Maxwell's equations separate and for which boundaries of the diffracting bodies coincide with surfaces of constant coordinate values. Only a limited number of coordinate systems exist on which the Helmholtz equation separates. Analytic solutions are solutions which can be expressed in terms of tabulated special functions or a series of such functions.

Unfortunately, for most geometries analytic solutions do not exist. To solve those problems, numerical methods are necessary. A versatile numerical method can handle more complex configurations and should yield approximate solutions with sufficient accuracy when compared to analytic solutions in cases that the latter exist. Numerical solutions of Maxwell's equations in general require solving a large coupled system of linear equations. We briefly describe some of the numerical methods which

can be used for the diffraction by a possibly anisotropic grating.

In a general three-dimensional anisotropic grating problem all the electric and magnetic field components are coupled. Several authors have studied the scattering of an incident plane wave by an anisotropic grating. The Coupled Wave Method for isotropic gratings has been extended to anisotropic gratings by Rokushima et al. [17], Glytsis et al. [18] and Mori et al. [19]. The Chandezon method (or C-method) has been applied to anisotropic gratings by Harris et al. [20]. Li [21, 22] has extended his Fourier modal method (FMM) to anisotropic gratings. The FMM is a modification of the coupled wave method to improve convergence for the case of TM polarization. In Li's paper conical incident angles are allowed. The finite-difference time-domain method (FDTD) has been adapted to anisotropic media by Schneider et al. [23]. Since the 1990's, the finite element method (FEM) has been applied for solving electromagnetic scattering problem [24, 25]. FEM is the method that has been used for studying anisotropic gratings in our research. The simulation results contained in this thesis mainly are performed with our in-house code Cyclop3D based on FEM. The code has been tested for all kind of structures containing only isotropic materials (dielectric materials, metals). It has been benchmarked with other numerical methods including FDTD, FMM which are developed by different international research groups and institutes [26]. In this thesis we further test the code with hermitian complex permittivity tensor. A more detailed description of the code is given in the next subsection. Here we explain two other often used diffraction models, namely the FMM and the FDTD.

Fourier modal method

The modal method (MM) is built on the condition that the profile of the simulated diffraction optical element is periodic. This means that the geometry of the grating surface or the modulation of the index of refraction is periodic. Thus the material structure and the electromagnetic fields can be expanded in terms of series. If the field expansions consist of individual modes, and each mode represents a physical solution to Maxwell's equations in some subregion, the method is often referred to as the modal method (MM). If the fields are represented by a plane-wave expansion, then the method is called the rigorous coupled wave analysis (RCWA). If the expansion that used is Fourier expansion, then the method is called the Fourier modal method (FMM) [27]. Each plane-wave component used in the RCWA or FMM does not represent a solution to Maxwell's equations individually, but only the entire expansion is a solution. The two specific expansions of the MM and the FMM (or RCWA) result in two different formulations, however they have been proved to be equivalent by Gaylord and Moharam [28, 29].

For a 2D problem, i.e. both the geometry and also the fields are independent of one of the spatial coordinates, FMM calculates the electric field for the s-polarization, and the magnetic field for the p-polarization. While the rest of the field components, such as, magnetic field in a s-polarization case and electric field in a p-polarization case, are derived from Maxwell's equations afterwards. The configuration is divided into several layers. As an example consider a system as illustrated in Fig. 2.5. This simple structure is divided into three layers. The top layer, where $z > 0$, is a homogeneous

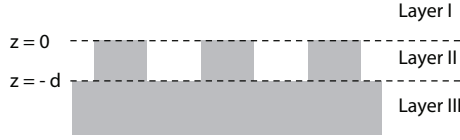


Figure 2.5: A simple grating structure which is treated as a three-layer system in FMM.

medium from which light is incident; the middle layer, $0 < z < -d$, is the grating region; the lower layer, $z < -d$, carries the transmitting field and is also a homogeneous media. The method solves an eigen-problem in every layer of the structure. By matching the boundary conditions imposed by Maxwell's equations at the interfaces between the layers, the field is reconstructed.

Finite-difference time-domain method

The FDTD method was first introduced by Yee [30]. In the following years, Taflove and his colleagues and many other researchers have made many contributions. In 1980, Taflove [31] called the method the 'Finite-difference time-domain' and introduced the acronym 'FDTD'. A further extension of the method for anisotropic media has been developed by Schneider et al. [23].

The FDTD is based on a discretization of Maxwell's equations in space and time by using central-difference approximations. The resulting finite-difference equations are solved as an initial value problem with time stepping in a leapfrog grid manner until the transient has died out and the solutions have become time harmonic. The leapfrog grid that is used in the method is a set of two separate grids, one for the electric field $\mathbf{E}$, the other for the magnetic field $\mathbf{H}$. The two grids intersect by a half cell, such that a node of one grid is in the center of the cell of the other. The material parameters of the medium are incorporated in the computation of the field components within each computational cell. This gives the flexibility to the method when complex shapes and inhomogeneous media occur.

The application of FDTD to a non-periodic structure is straightforward. When the method is used to simulate periodic gratings, periodic boundary conditions have to be applied in the direction of periodicity. Perfect matched layer (PML) [32] absorbing boundary condition is added to the FDTD mesh in the other directions.

2.6.1 Finite element method

In our work a numerical code based on Finite Element Method (FEM) [33, 34] is employed to perform the simulations. The FEM is a general numerical method for solving boundary value problems in mathematical physics. In the 2D case a computational domain in the (x, z) -plane is defined which is such that all nonplanar layers and all anisotropic layers are contained in its interior. The domain is truncated in the vertical direction by a PML [32, 35, 36]. In this computational domain, a boundary value problem for the vector Helmholtz equation for either the electric or magnetic

field is formulated. The computational domain is meshed using triangles or quadrilaterals and the electric or magnetic field is approximated by edge elements of first, second or third order.

The FEM is relatively difficult to implement, especially in 3D. However, it is very flexible because both periodic and non-periodic configurations and all kinds of materials can be modeled. In particular, anisotropic and inhomogeneous materials with arbitrary principal axes can be treated without problems. In this respect the FEM differs substantially from the Coupled Wave Method and the FMM for which the extension to anisotropic materials is highly non-trivial.

To ensure that the FEM code gives accurate results when applied to the anisotropic gratings considered in this paper, we have compared the results obtained for a flat multi-layer consisting of anisotropic media with an analytic model based on the expansion in terms of plane waves as described in Section 2.5. Such a multi-layer is a very good test case for the FEM code because, in contrast with for example the FMM, the FEM does not simplify when applied to an anisotropic multi-layer. We found that when the edge elements of lowest order are used with 50 mesh points per wavelength, the computed field is quite accurate for all conical angles of incidence.

We remark that the present model differs from the one described in [33, 34] regarding the numerical method that is used to solve the discretized linear system of equations. In [33, 34] an iterative method was used with approximate minimum fill-in reordering and a preconditioner from ILUPACK of Yousef Saad and Matthias Bollhoefer (see [37]). This preconditioning requires a lot of memory, in particular for three dimensional problems. In the present version of the code the sparse linear solver Pardiso [38, 39] is used. Due to this not only much less memory is needed, but also the computation time has been reduced drastically.

Chapter 3

Polarizing color separator for LCDs

3.1 Introduction

Liquid crystal displays (LCDs) have been adapted to a broad range of applications as convenient flat panel displays. However, the light efficiency of the current LCD systems is below 10%. The low efficiency is mainly because of the use of absorption-based polarizers and color filters. The absorption by the polarizers and color filters is approximately 50% and 70%, respectively. Therefore, it is desirable to consider alternative approaches for polarization- and color-separation. Ideally one would like to use one device which can separate both color and polarization and which is not based on absorption. By applying a diffraction grating on the lightguide in the LCDs, colors can be separated [40, 41]. If the grating is made of anisotropic material, one can achieve that light of one polarization is diffracted and light of the orthogonal polarization is not [42], as discussed in Section 1.2. The configuration of such a backlight for LCDs is shown in Fig. 3.1. This structure can be placed in a side-lit configuration with a cold-cathode fluorescence lamp (CCFL) or a light-emitting-diodes (LEDs) [43]. The grooves of the grating are filled with a birefringent material having its optic axis parallel to the grooves. The ordinary refractive index of the birefringent material is matched to the grating material, whereas its extraordinary refractive index is significantly higher. As a result, light with polarization parallel to the grooves is diffracted, whereas for the relevant angles of incidence, the light with polarization perpendicular to the grooves is totally reflected back into the lightguide.

The principle and performance of both the non-polarized and polarized version of color-separating backlights have been discussed by De Boer [42]. In a color-separating backlight the light distribution, which contains a multitude of angles and mainly three wavelengths, is diffracted into three slightly overlapping angular distributions for the main colors. When the optic axis of the anisotropic material

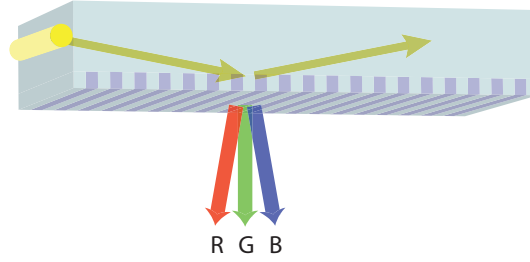


Figure 3.1: Configuration of the polarized color-separating backlight. The size is not to scale.

is parallel to the grating grooves, for polarization parallel or perpendicular to the direction of the optic axis, the refractive indices of the anisotropic medium can be depicted by equivalent extraordinary or ordinary refractive index. For this case with two uncoupled linear polarizations, the medium can be treated as 'non-birefringent' (this will be addressed in Section 3.3 by rigorous formulations). In Ref. [42], De Boer et al. have simulated the situation of which the plane of incidence is perpendicular to the grating grooves, by using a commercially available computer program [44], based on rigorous coupled wave theory for non-birefringent materials. However, it was not possible to do simulations for other (conical) directions. Note that in general a backlight contains light of several directions and all these should be taken into account to understand its efficiency.

The aim of our work is to discuss a method for simulating diffraction by birefringent gratings at conical angles, applied to the problem of polarization selective color-separating backlights. To study the performance of the diffraction gratings containing birefringent material, a numerical program is required. The numerical code based on finite element method (FEM) as described in Section 2.6.1 is employed. With our program any arbitrary conical angle of incidence can be calculated rigorously. Results of simulations are shown in this chapter for when the incident wave vectors are in the plane perpendicular to the grooves (*normal incidence*) and for wave vectors with *conical incidence* in Section 3.4.

3.2 Configuration

The geometry and configuration of the grating in our study is depicted in Fig. 3.2. A surface-relief grating with a period of 400 nm and duty cycle of 0.5 is applied onto a lightguide made of polycarbonate. The grooves of the grating are filled with a birefringent material which is liquid crystal (LC) having its optic axis along the grooves. The wavelength dependent refractive indices are shown in Table 3.1. The grating is rectangular and the depth of the grooves is 140 nm.

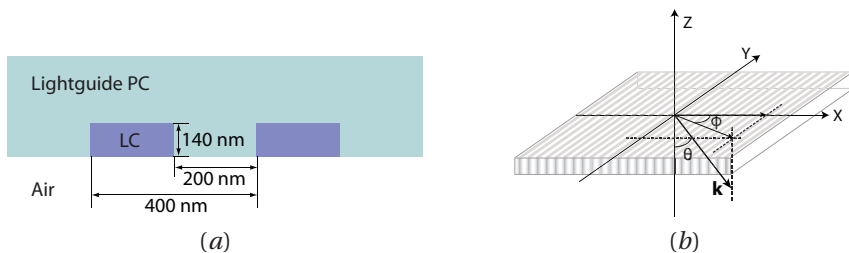


Figure 3.2: Configuration of the grating: (a) geometry of x, z cross-section; acronyms in the figure: PC – polycarbonate, LC – liquid crystal; (b) coordinate system and angles.

λ	Polycarbonate n	LC n_o	LC n_e
450	1.61	1.54	1.82
535	1.58	1.53	1.77
632	1.58	1.52	1.76

Table 3.1: The refractive indices of polycarbonate and of liquid crystal for three colors.

3.3 Anisotropic media that are translation invariant in one direction

For a configuration that consists of isotropic materials and that is translational invariant with respect to one coordinate, say the y -coordinate, it is well-known that Maxwell's equations are equivalent to a system of two coupled second order partial differential equations for only the components E_y and H_y [9]. This is also true when the sources and fields depend on the y -coordinate by a factor $e^{ik_y y}$, for some k_y . When $k_y = 0$, the two partial differential equations for E_y and H_y are uncoupled and therefore one can separate s- and p-polarization. For s-polarization, H_y vanishes whereas for p-polarization E_y is zero. When $k_y \neq 0$, the two partial differential equations are coupled and the polarizations mix.

These observations apply in particular to periodic diffraction gratings that consist of isotropic materials. In this section we want to investigate to what extent the separation in two polarizations is still possible in the case of gratings made of anisotropic materials.

We will first show that in an anisotropic medium of which the dielectric permittivity $\underline{\epsilon}$ is independent of the y -direction, all field components can be expressed in terms of E_y and H_y . The time-harmonic source-free Maxwell's equations in non-magnetic anisotropic materials are given by Eqs. (2.1) and (2.2). In the equations $\underline{\epsilon}$ is the relative dielectric permittivity of the anisotropic crystal, which in this chapter is assumed to

be a real symmetric tensor of rank 2 given by:

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}, \quad (3.1)$$

with respect to the cartesian coordinate system $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$. The configuration and the material properties are assumed to be invariant with respect to translations in the y -direction, but may vary in a cross-sectional plane $y = \text{constant}$. Hence $\underline{\underline{\epsilon}}$ may be a function of (x, z) : $\underline{\underline{\epsilon}}(x, z)$. Until stated otherwise, $\underline{\underline{\epsilon}}$ does not have to be periodic.

We consider an electromagnetic field which depends harmonically on y , i.e.

$$\frac{\partial}{\partial y} \mathbf{E} = ik_y \mathbf{E}, \quad (3.2)$$

$$\frac{\partial}{\partial y} \mathbf{H} = ik_y \mathbf{H}, \quad (3.3)$$

for some real k_y . It is easy to see that by using Maxwell's equations, the x - and z -components of the electromagnetic field can be expressed in terms of the y -components:

$$\begin{pmatrix} E_x \\ E_z \end{pmatrix} = -\frac{\omega^2 \epsilon_0 \mu_0}{D} \mathcal{N} \begin{pmatrix} \epsilon_{xy} \\ \epsilon_{zy} \end{pmatrix} E_y + \frac{i}{D} \mathcal{N} \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} + \frac{i}{D} \mathcal{Q} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} \quad (3.4)$$

$$\begin{aligned} \begin{pmatrix} -H_z \\ H_x \end{pmatrix} &= \frac{k_y}{\omega \mu_0} \begin{pmatrix} E_x \\ E_z \end{pmatrix} + \frac{i}{\omega \mu_0} \begin{pmatrix} \partial/\partial x \\ \partial/\partial z \end{pmatrix} E_y \\ &= -\frac{\omega^2 \epsilon_0 \mu_0}{D} \frac{k_y}{\omega \mu_0} \mathcal{N} \begin{pmatrix} \epsilon_{xy} \\ \epsilon_{zy} \end{pmatrix} E_y + \frac{i}{D} \frac{k_y}{\omega \mu_0} \mathcal{N} \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} \\ &\quad + \frac{i}{D} \frac{k_y}{\omega \mu_0} \mathcal{Q} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} + \frac{i}{\omega \mu_0} \begin{pmatrix} \partial/\partial x \\ \partial/\partial z \end{pmatrix} E_y, \end{aligned} \quad (3.5)$$

where

$$\begin{aligned} D &= (\omega^2 \epsilon_0 \mu_0 \epsilon_{xx} - k_y^2)(\omega^2 \epsilon_0 \mu_0 \epsilon_{zz} - k_y^2) - (\omega^2 \epsilon_0 \mu_0)^2 \epsilon_{xz} \epsilon_{zx} \\ &= (\omega^2 \epsilon_0 \mu_0)^2 (\epsilon_{xx} \epsilon_{zz} - \epsilon_{xz} \epsilon_{zx}) - \omega^2 \epsilon_0 \mu_0 k_y^2 (\epsilon_{xx} + \epsilon_{zz}) + k_y^4. \end{aligned} \quad (3.6)$$

$$\mathcal{N} = \begin{pmatrix} \omega^2 \epsilon_0 \mu_0 \epsilon_{zz} - k_y^2 & -\omega^2 \epsilon_0 \mu_0 \epsilon_{xz} \\ -\omega^2 \epsilon_0 \mu_0 \epsilon_{zx} & \omega^2 \epsilon_0 \mu_0 \epsilon_{xx} - k_y^2 \end{pmatrix}, \quad (3.7)$$

$$\mathcal{Q} = \mathcal{N} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -\omega^2 \epsilon_0 \mu_0 \epsilon_{xz} & -(\omega^2 \epsilon_0 \mu_0 \epsilon_{zz} - k_y^2) \\ \omega^2 \epsilon_0 \mu_0 \epsilon_{zx} & \omega^2 \epsilon_0 \mu_0 \epsilon_{xx} - k_y^2 \end{pmatrix}, \quad (3.8)$$

Note that D is the determinant of both the matrices $\mathcal{N}$ and $\mathcal{Q}$. It is assumed here that k_y is such that D never vanishes. The y -components of Maxwell's equations (2.1) and (2.2) imply:

$$-i\omega \mu_0 H_y = \tilde{\nabla} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_z \end{pmatrix}, \quad (3.9)$$

$$i\omega \epsilon_0 \epsilon_{yy} E_y = -i\omega \epsilon_0 (\epsilon_{yx} \epsilon_{yz}) \begin{pmatrix} E_x \\ E_z \end{pmatrix} - \tilde{\nabla} \cdot \begin{pmatrix} H_z \\ -H_x \end{pmatrix}, \quad (3.10)$$

where $\tilde{\nabla}$ is defined by,

$$\tilde{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{pmatrix}. \quad (3.11)$$

By substitution of Eqs. (3.4), (3.5) into the right-hand sides of Eqs. (3.9), (3.10) we get a coupled system for E_y and H_y ,

$$\begin{aligned} -i\omega\mu_0 H_y &= -\omega^2\epsilon_0\mu_0\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q}^T \begin{pmatrix} \epsilon_{xy} \\ \epsilon_{zy} \end{pmatrix} E_y \right] + i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q}^T \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} \right] \\ &\quad + i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{M} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} \right], \end{aligned} \quad (3.12)$$

$$\begin{aligned} i\omega\epsilon_0\epsilon_{yy}E_y &= i\frac{\omega^2\epsilon_0\mu_0}{D}\omega\epsilon_0(\epsilon_{yx}\epsilon_{yz})\mathcal{N} \begin{pmatrix} \epsilon_{xy} \\ \epsilon_{zy} \end{pmatrix} E_y + \frac{\omega\epsilon_0}{D}(\epsilon_{yx}\epsilon_{yz})\mathcal{N} \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} \\ &\quad + \frac{\omega\epsilon_0}{D}(\epsilon_{yx}\epsilon_{yz})\mathcal{Q} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} + \frac{k_y}{\omega\mu_0}\omega^2\epsilon_0\mu_0\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{N} \begin{pmatrix} \epsilon_{xy} \\ \epsilon_{zy} \end{pmatrix} E_y \right] \\ &\quad - i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{N} \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} \right] - i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega\mu_0 H_y \end{pmatrix} \right] \\ &\quad - \frac{i}{\omega\mu_0}\Delta E_y, \end{aligned} \quad (3.13)$$

where

$$\mathcal{M} = \begin{pmatrix} \omega^2\epsilon_0\mu_0\epsilon_{xx} - k_y^2 & \omega^2\epsilon_0\mu_0\epsilon_{xz} \\ \omega^2\epsilon_0\mu_0\epsilon_{zx} & \omega^2\epsilon_0\mu_0\epsilon_{zz} - k_y^2 \end{pmatrix}. \quad (3.14)$$

Note for a medium with a real symmetric permittivity tensor that $\mathcal{M} = D\mathcal{N}^{-1}$. Even when $k_y = 0$, both Eqs. (3.12) and (3.13) contain E_y and H_y .

Consider the case of a grating made of anisotropic materials which is periodic with respect to x and invariant with respect to translations in the y -direction. If the material above the grating is isotropic and the incident wave is in the (x, z) -plane perpendicular to the grooves, i.e. $k_y = 0$, then for s- or p-polarized irradiation, the total field will be a mixture of both polarizations. In this respect anisotropic and isotropic gratings totally differ.

Next we consider an anisotropic medium of which the y -axis (i.e. the axis of translational invariance) is one of the principal axes of $\underline{\underline{\epsilon}}$ for all points (x, y, z) . Then the dielectric tensor is of the form,

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & 0 & \epsilon_{xz} \\ 0 & \epsilon_{yy} & 0 \\ \epsilon_{zx} & 0 & \epsilon_{zz} \end{pmatrix}, \quad (3.15)$$

and the coupled system Eqs. (3.12) and (3.13) becomes:

$$-i\omega\mu_0 H_y = i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q}^T \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \right] + i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{M} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \right], \quad (3.16)$$

$$\begin{aligned} i\omega\epsilon_0\epsilon_{yy}E_y &= -i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D}\mathcal{N}\frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \right] - i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D}\mathcal{Q}\frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \right] \\ &\quad - \frac{i}{\omega\mu_0}\Delta E_y. \end{aligned} \quad (3.17)$$

Furthermore Eqs. (3.4) and (3.5) become

$$\begin{pmatrix} E_x \\ E_z \end{pmatrix} = \frac{i}{D}\mathcal{N}\frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} + \frac{i}{D}\mathcal{Q}\frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \quad (3.18)$$

$$\begin{aligned} \begin{pmatrix} -H_z \\ H_x \end{pmatrix} &= \frac{i}{D}\frac{k_y}{\omega\mu_0}\mathcal{N}\frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} + \frac{i}{D}\frac{k_y}{\omega\mu_0}\mathcal{Q}\frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \\ &\quad + \frac{i}{\omega\mu_0} \begin{pmatrix} \partial/\partial x \\ \partial/\partial z \end{pmatrix} E_y, \end{aligned} \quad (3.19)$$

Now assume again that $k_y = 0$, then Eq. (3.16) contains only H_y and Eq. (3.17) contains only E_y :

$$\begin{aligned} \omega^2\mu_0\epsilon_0 H_y + \frac{\partial}{\partial x} \left(\frac{\epsilon_{xx}}{\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}\epsilon_{zx}} \frac{\partial H_y}{\partial x} + \frac{\epsilon_{zx}}{\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}\epsilon_{zx}} \frac{\partial H_y}{\partial z} \right) \\ + \frac{\partial}{\partial z} \left(\frac{\epsilon_{xz}}{\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}\epsilon_{zx}} \frac{\partial H_y}{\partial x} + \frac{\epsilon_{zz}}{\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}\epsilon_{zx}} \frac{\partial H_y}{\partial z} \right) = 0, \end{aligned} \quad (3.20)$$

$$\omega^2\epsilon_0\mu_0\epsilon_{yy}E_y + \frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} = 0. \quad (3.21)$$

Then the system is uncoupled for E_y and H_y . Eqs. (3.20) and (3.21) imply that when $k_y = 0$ we can distinguish two types of polarization, namely:

S-polarization $H_y = 0, H_x \neq 0, H_z \neq 0, E_x = E_z = 0, E_y \neq 0$;

P-polarization $E_y = 0, E_x \neq 0, E_z \neq 0, H_x = H_z = 0, H_y \neq 0$.

Hence when the y-axis is the principal axis, the splitting in uncoupled s- and p-polarization for $k_y = 0$ is analogous to the isotropic case. But, E_x and E_z have other values than in the isotropic case because Eq. (3.18) depends on off-diagonal elements of the tensor.

Suppose now that the anisotropic material is uniaxial with optic axis coincides with the y-axis. The dielectric permittivity tensor is then diagonal:

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}, \quad (3.22)$$

with $\epsilon_{xx} = \epsilon_{zz} = n_o^2$ and $\epsilon_{yy} = n_e^2$. When $k_y = 0$ the s- and p-polarization are separated and one has $n = n_o$ for p-polarization and $n = n_e$ for s-polarization. However, when $k_y \neq 0$, the system is coupled as can be easily seen from Eqs. (3.16) and (3.17).

3.3.1 Birefringent gratings

The periodic grating which is applied onto the lightguide, was designed in such a way that for all angles of incidence of interest, only the -1^{st} order is transmitted and propagates. If we consider a plane wave incident from the lightguide on to the grating with wave vector $\mathbf{k}^i$ and incident angles θ^i and ϕ^i , the tangential components of the wave vector are given by,

$$k_x^i = k_0 n^i \cos \phi^i \sin \theta^i, \quad (3.23)$$

$$k_y^i = k_0 n^i \sin \phi^i \sin \theta^i, \quad (3.24)$$

where $k_0 = 2\pi/\lambda$ is the wave number in vacuum and n^i is the refractive index in the medium of the incident wave. Polar angle θ and azimuthal angle ϕ are defined as depicted in Fig. 3.2(b). The wave vector of a diffracted transmitted wave can be expressed as,

$$k_x^d = k_0 n^d \cos \phi^d \sin \theta^d, \quad (3.25)$$

$$k_y^d = k_0 n^d \sin \phi^d \sin \theta^d, \quad (3.26)$$

where n^d is the refractive index of the medium into which the light is partially transmitted and ϕ^d and θ^d are diffraction angles. The components of the wave vector of the m^{th} transmitted order and the components of the incident wave vector are related by:

$$k_{x,m}^d = k_x^i + \frac{2\pi \cdot m}{p}, \quad (3.27)$$

$$k_y^d = k_y^i, \quad (3.28)$$

where p is the period or pitch of the grating. The angular directions of the m^{th} diffracted order can be calculated for varying incident angles θ^i and ϕ^i from the following equations:

$$n^d \cos \phi^d \sin \theta^d = n^i \cos \phi^i \sin \theta^i + \frac{m\lambda}{p}, \quad (3.29)$$

$$n^d \sin \phi^d \sin \theta^d = n^i \sin \phi^i \sin \theta^i. \quad (3.30)$$

When the right-hand side of either (3.29) or (3.30) or both has absolute value larger than n^d , the order is not propagating but evanescent. In our application n^i is the refractive index of polycarbonate (see Table 3.1) and $n^d = 1$, the refractive index of air.

In our grating the pitch $p = 400$ nm. It can then be shown from Eqs. (3.29) and (3.30) that for wavelength $\lambda > 520$ nm and for incident angles $48^\circ \leq \theta^i \leq 90^\circ$ and $0^\circ \leq \phi^i \leq 90^\circ$ only the -1^{st} order is propagating in air. For blue light with short wavelength the -2^{nd} transmitted order is also propagating, but only for very large incident angles.

3.4 Simulations and discussions

3.4.1 Normal incidence

As we discussed in Section 3.3, when the principal axis of the anisotropic material is along the grooves and the plane of incidence is perpendicular to the grooves, we can distinguish s- and p-polarized waves. In this case, the polarization is conserved. When the incident field is s-polarized, then one transmitted diffracted order propagates which is s-polarized as well. According to our definition of angles, $\phi^d = \phi^i = 0$ in this case. We will call this case *normal incidence*.

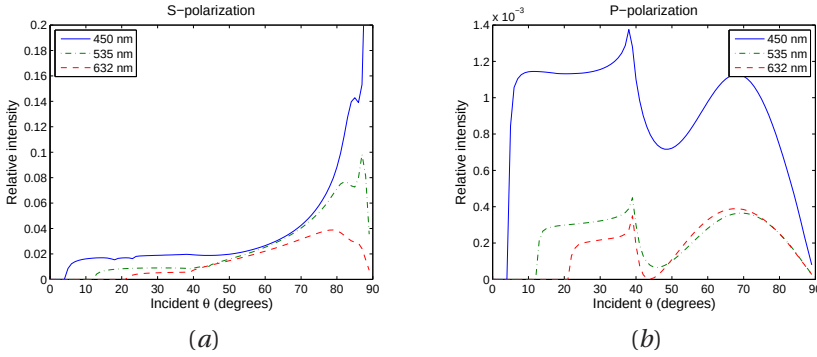


Figure 3.3: Relative intensity of the -1^{st} diffracted transmitted order as function of θ^i for $\phi^i = 0$ and for three wavelengths namely 450, 535 and 632 nm for (a) s-polarization and (b) p-polarization.

The dependence of the intensity of the -1^{st} diffracted transmitted order as function of incident angle θ^i is shown in Fig. 3.3 for both s- and p-polarization. The transmission of s-polarization is much larger than that of p-polarization. The small nonzero intensity of the transmitted order for p-polarization is due to the small mismatch between the refractive index of the polycarbonate and the ordinary refractive index of the LC. It can be seen that the first transmitted order appears above a certain incident angle. The 0^{th} transmitted orders become evanescent above 40° , and the intensity of the -1^{st} transmitted order then changes abruptly. The field increases as the angle becomes larger. For a side-lit lightguide system, the main rays have incident angles θ larger than 48° . For the angles of interest, the transmission of s-polarization is high compared to that of p-polarization. But the absolute transmission for s-polarization is still fairly small. This is not a problem in our application, because the lightguide is very long and therefore the light that is reflected back into the lightguide is recycled many times.

3.4.2 General incidence

In a more general case, when the plane of incidence is not perpendicular to the grooves, the diffracted transmitted field of the s- and p-polarized incident fields couple. Hence the transmitted field does not have the same polarization as the incident field anymore. We did simulations for both s- and p-polarized incident fields. The diffracted transmitted field is decomposed into two linear polarized components. One is along the direction of the grooves, which we call *s-polarization*. The other is perpendicular to the grooves, which is called *p-polarization*. Please notice that the definition of the transmitted s- and p-polarizations differs from that of the incident polarizations. The incident s- and p-polarizations are defined with respect to the plane of incidence through the wave vector $\mathbf{k}^i$ and the z-axis. The definition of the transmitted polarization agrees with what is customary [42].

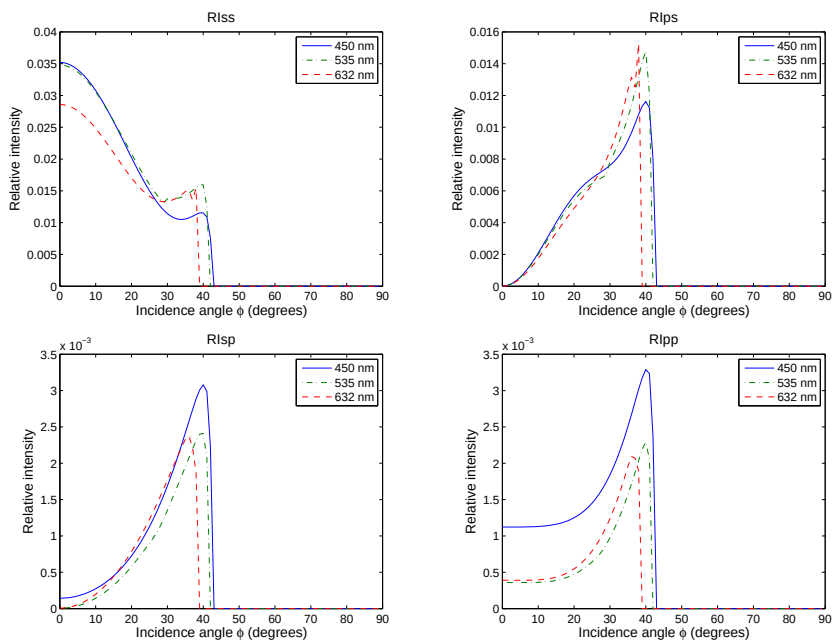


Figure 3.4: Relative intensity of the -1^{st} diffracted transmitted order as function of incident angle ϕ^i for incident angle $\theta^i = 67^\circ$ and for the three wavelengths 450, 535 and 632 nm. RI in the titles is an abbreviation of relative intensity, and the first subscript denotes the polarization of the diffracted transmitted order whereas the second subscript indicates the polarization of the incident field.

In a backlight configuration as that of Fig. 3.1 light is present with a multitude of angles (the source is Lambertian). Typically, the polar angles θ^i range from 48° to 90° . The distribution in diffracted angles θ^d has a typical width of 20° . The maximum of the distribution for diffracted green light occurs close to $\theta^i = 67^\circ$. With fixed polar

angle $\theta^i = 67^\circ$, the azimuthal angle ϕ^i was varied from 0° to 90° . The calculated relative intensities of the -1^{st} transmitted order (i.e. the intensity of the incident wave is unity) are shown in Fig. 3.4 for the three wavelengths 450, 535 and 632 nm. The

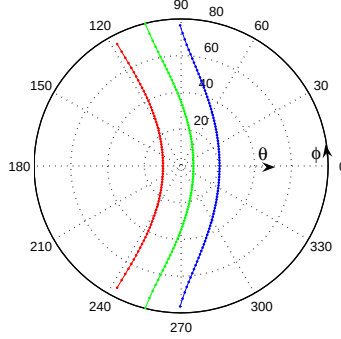


Figure 3.5: Angular distribution (θ^d, ϕ^d) of the -1^{st} diffracted order for different colors, blue (450 nm), green (535 nm) and red (632 nm) for $\theta^i = 67^\circ$ and for varying ϕ^i ($-90^\circ \leq \phi^i \leq 90^\circ$).

angular distribution of the wave components of the diffracted transmitted field was calculated according to the Eqs. (3.29) and (3.30), and is illustrated in Fig. 3.5. To give insight in the angular distribution of the diffracted light, the relative intensity of the -1^{st} diffracted transmitted order from Fig. 3.4 is plotted in Fig. 3.6 as function of the diffraction angle θ^d (instead of angle ϕ^i as is done in Fig. 3.4).

From those figures we can see that for $\theta^i = 67^\circ$ the diffracted order becomes evanescent when the incident angle ϕ^i is larger than approximately 40° . The transmitted field strongly depends on the angle. The transmission of s-polarization is relatively high at small incident angle ϕ^i , whereas the transmission of p-polarization is low.

The transmission of s-polarization is much higher than that of p-polarization. For s-polarized incident light, especially at small diffraction angle θ^d , the large intensity difference between transmitted s-component and p-component results in a high contrast ratio, as is shown in Fig. 3.7(a). The contrast ratio for unpolarized light is somewhat lower, as shown in Fig. 3.7(b), due to the diffracted orders of p-polarization. By using materials with better matching refractive indices for the p-polarization, a higher contrast ratio can be achieved. Our measurements of the angular distribution of extracted s- and p-polarized light [42], repeated in Fig. 3.8, are in agreement with these simulated results. In practice the contrast ratio is not higher than 23, presumably due to non-perfect alignment of the liquid crystal in the grating.

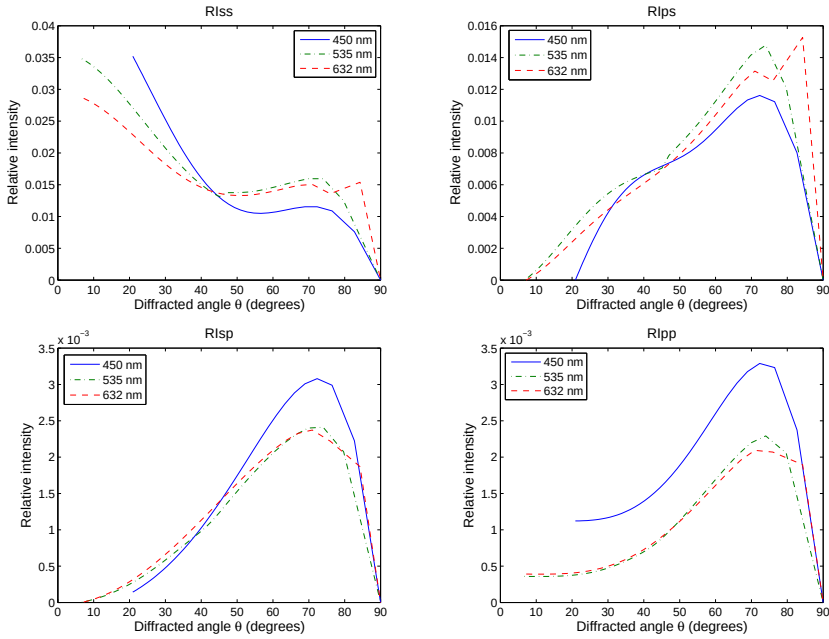


Figure 3.6: Relative intensity of the -1^{st} diffracted transmitted order as function of diffraction angle θ^d , for incident angle $\theta^i = 67^\circ$, and for varying $0^\circ \leq \phi^i \leq 90^\circ$, for the three wavelengths 450, 535 and 632 nm. RI in the titles is an abbreviation for relative intensity, and the first subscript denotes the polarization of the transmitted field whereas the second subscript indicates the polarization of the incident field.

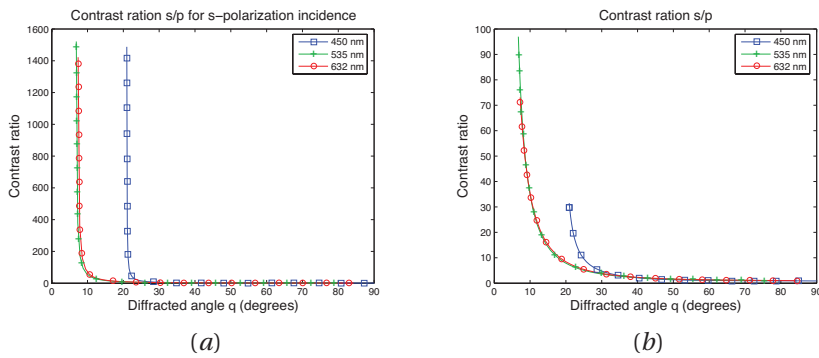


Figure 3.7: Contrast ratio between calculated intensities of the s- and p-polarized components of the -1^{st} transmitted order as function of diffraction angle θ^d for $\theta^i = 67^\circ$, and for varying $0^\circ \leq \phi^i \leq 90^\circ$, (a) for s-polarized incident wave, and (b) for unpolarized incident field.

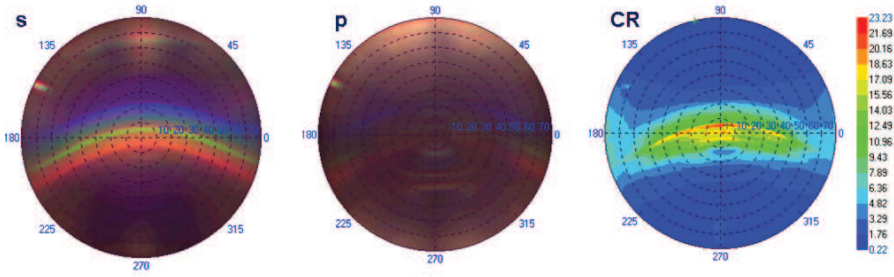


Figure 3.8: Measured angular distribution of color-separated luminance for s-polarized (left) and p-polarized (center) light and angular distribution of s/p contrast ratio (right) for color-separating polarized backlight structure with TL 213 (refractive indices of Table 1) as birefringent material. (Note that the azimuthal angles are shifted by 90° with respect to those used before.)

3.5 Conclusions

Diffraction outcoupling can be used to make efficient backlights. A diffraction grating containing birefringent material was studied with a rigorous model. The results of the simulations show that the diffracted transmitted intensity strongly depends on incident angles but nevertheless has a high contrast ratio for all angles of interest. The simulation results are in good agreement with experiments. In the case presented here, the efficiencies are not very high (around 2%). Note that these efficiencies refer to one interaction with the grating. For a long lightguide the total extraction efficiency will be much higher. Depending on the application, with a long or short lightguide, a higher extraction efficiency may be desired. This can be achieved by the use of a higher index contrast or by adapting the grating depth [42]. If not all light is extracted before the other end of the lightguide is reached, it is desirable to use a diffusing mirror at that side to recycle the light. To avoid mixing of different colors, it was proposed recently [45] to use a grating with a smaller period resulting in the blue light being diffracted along the normal and green and red around it. If combined with a suitable pixel lay-out and a proper lens array that directs the light to the desired pixels, an efficient display system can be achieved.

The rigorous model can be used as a tool to optimize gratings for certain applications. In general it is suitable for simulations of subwavelength structures consisting of anisotropic materials for all kinds of applications, such as, switchable holographic grating made of liquid-crystal films and so-called holographic polymer dispersed liquid crystals (H-PDLC).

Chapter 4

Characterization of POLICRYPS gratings

4.1 Introduction

The realization of holographic diffraction gratings in composite materials has been widely investigated in the past 20 years [46, 47]. It has been shown that devices based on holographic polymer dispersed liquid crystals (H-PDLC) have good diffraction efficiency. However those devices have some intrinsic drawbacks [48] because of the particular droplet structure of the nematic liquid crystal (NLC) formed in such gratings, which can be seen in the polarization microscopic image of an PDLC grating shown in Fig. 4.1(a). Some years ago, a new kind of holographic diffraction grating made of polymer slices alternated with homogeneous films of NLC was realized, named Polymer Liquid Crystal Polymer Slices (POLICRYPS) [49]. The standard POLICRYPS grating writing process was performed on a stabilized setup [50]. A sharp and uniform morphology was obtained in such gratings and no droplet structure was observed [51]. To avoid the formation of the NLC droplets during the curing process, the syrup of photo-initiator-monomer-NLC mixture has to be heated above the nematic-isotropic transition temperature of the NLC. With the same initial mixture, the obtained POLICRYPS grating is shown in Fig. 4.1(b).

The POLICRYPS grating can be applied to high resolution display systems considering its high efficiency and low cost [52]. In this kind of applications, we are interested in the diffraction efficiency of the grating not only at normal incidence, i.e. the plane of incidence is perpendicular to the grating grooves, but also at the conical incidence mount. In the latter case, the direction of the beam has to be specified not only by the polar angle θ with the normal but also by the azimuthal angle ϕ (see Fig. 3.2(b)). The azimuthal angle ϕ is the angle with the x -axis in Fig. 3.2(b).

Therefore in our work the numerical program based on FEM method as described in Chapter 2 is employed to model the POLICRYPS gratings. In this chapter we present

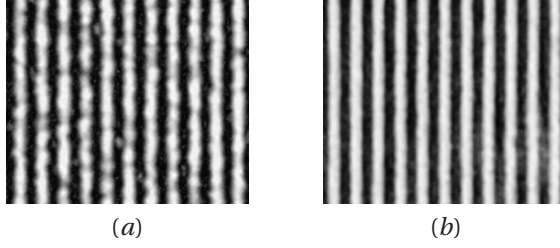


Figure 4.1: Polarization microscopic image of (a) PDLC; (b)POLICRYPS.

both simulation results and experimental measurements for POLICRYPS gratings at normal and conical incidence. The Bragg transmission effect is also studied.

4.2 POLICRYPS gratings

The standard curing process to obtain a POLICRYPS grating has been discussed in Ref. [49], and we re-present the procedure briefly here. A Cartesian coordinate system (x, y, z) as illustrated in Fig. 3.2(b) is used. An Ar^+ laser ($\lambda = 351$ nm) is used to obtain a UV-curing interference pattern. Both recording beams are TE polarized to maximize the intensity modulation. The recording angle θ_{record} is approximately 8.3° to achieve a grating pitch Λ of 1220 nm. The NLC weight concentration is 26% of the initial mixture, obtained by diluting NLC BL-001 (by Merck) with the pre-polymer system Norland Optical Adhesive NOA-61 [53]. In the curing process, first, a polymerization-driven redistribution of monomers and nematic components over the fringes of the interference pattern takes place. Second, a separation of the nematic phase follows which is in the dark regions of the interference pattern. Of the formed grating, the average director of NLC is perpendicular to the walls of polymer slices, i.e. the average director is parallel to the x -axis, and the geometry can qualitatively sketched as in Fig. 1.7(b).

The dielectric permittivity modulations obtained for such a POLICRYPS grating is calculated accurately by a chemical-diffusion model [54]. In this model the concentration profile depends on two parameters G and B . The parameter G is proportional to the diffusion constant of the monomer and the other parameter B is proportional to the intensity of the curing pattern. To make POLICRYPS gratings, it is required that $G \ll 1$ and $B > 1$. The concentration of the polymer and the NLC are periodic functions of position in space, $v(\xi)$ and $\sigma(\xi)$, respectively, with $\xi = (2\pi/\Lambda)x$. With the obtained concentration profiles, the dielectric permittivity modulation profile in the x direction can be calculated by the following equation:

$$\epsilon_x(\xi) = [\epsilon_{\parallel} + (\epsilon_{eff} - \epsilon_{\parallel})\overline{v}(\xi)]\sigma(\xi) + \epsilon_{pol}v(\xi), \quad (4.1)$$

where $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are the dielectric permittivity constants parallel and perpendicular to the director of the pure unaligned NLC, $\epsilon_{eff} = (\epsilon_{\parallel} + 2\epsilon_{\perp})/3$ represents the effective dielectric constant of the LC and ϵ_{pol} is the dielectric permittivity constant of the

polymer. In the above equations $\bar{v}(\xi) = v(\xi)/v_{max}$ represents the normalized polymer concentration. The obtained profile of the x-component is displayed in Fig. 4.2(a). Eq. (4.1) can be understood by considering some extreme cases for the concentrations of the involved chemical species. In those areas of the grating where only LC is present ($v(x) = 0$), we assume that LC molecules are oriented along the x-axis and TM-polarized probe light then experiences the dielectric constant:

$$\epsilon_x(\xi) = \epsilon_{\parallel} \sigma(\xi). \quad (4.2)$$

Another possibility is given when only polymer is present ($\sigma(\xi) = 0$). From Eq. (4.1) we see that:

$$\epsilon_x(\xi) = \epsilon_{pol} v(\xi). \quad (4.3)$$

In points where the polymer concentration reaches a peak value ($\bar{v}(\xi) = 1$), we assume that some liquid crystal is dispersed in the polymer but its concentration is very low and hence its dielectric constant corresponds to the effective one ϵ_{eff} of the LC:

$$\epsilon_x(\xi) = \epsilon_{eff} \sigma(\xi) + \epsilon_{pol} v(\xi). \quad (4.4)$$

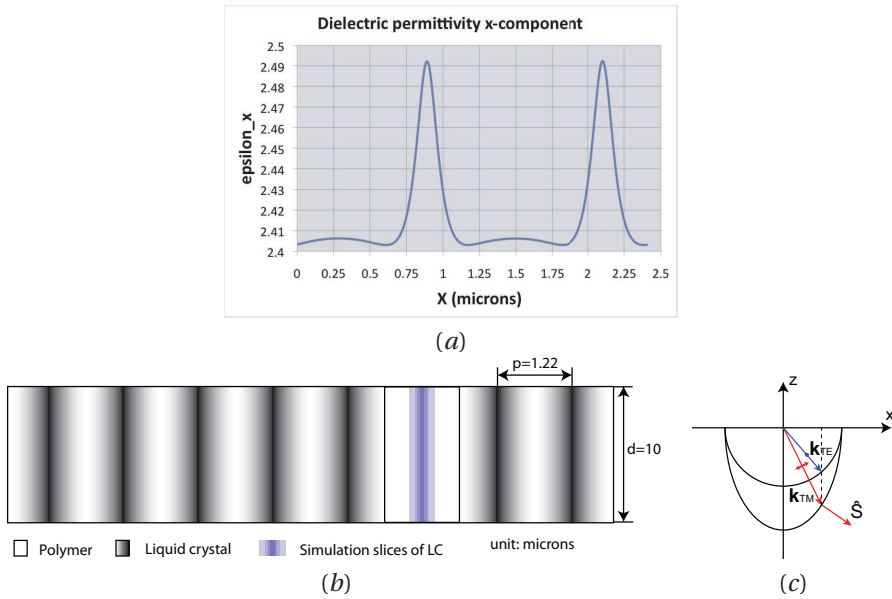


Figure 4.2: (a) Calculated x-component of the dielectric modulation; (b) cross-section of the grating geometry; (c) wave propagation direction of TM polarization. The figure is not in scale. Respectively, $\mathbf{k}_{TM}$ or $\mathbf{k}_{TE}$ indicates the wave vector of the TM or TE polarization and $\hat{\mathbf{S}}$ is the unit vector along the Poynting vector of the TM polarized wave.

In the macro scale, the average director of the NLC is perpendicular to the walls of polymer slices, i.e. the optic axis of the NLC is along the x-axis (as in the defined coordinate system see Fig. 3.2(b)). Therefore at normal incidence, the structure is

functioning as a transmission grating for TM polarization of which the electric field can be expressed as:

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = A \begin{pmatrix} \cos\theta^i \\ 0 \\ \sin\theta^i \end{pmatrix}, \quad TM \text{ polarization}; \quad (4.5)$$

where A is the amplitude of the incident wave with direction specified by the angle θ^i, ϕ^i . If the incident light is TE polarized, the incident electric field is:

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad TE \text{ polarization.} \quad (4.6)$$

This polarization will not see the grating structure because of the match between the ordinary refractive index of the NLC ($n_o = \sqrt{\epsilon_{\perp}}$) and the refractive index of the polymer ($n_{pol} = \sqrt{\epsilon_{pol}}$) and will propagate through without being diffracted. At conical incidence, we define polarization states TM and TE with respect to the plane containing the wave vector $\mathbf{k}^i$ and the grating vector which is along the the direction of grating periodicity in the x -direction with magnitude of $2\pi/\Lambda$. We call this new reference plane, the *grating reference plane*. For conical incidence cases, i.e. $\phi^i \neq 0^\circ$, the electric fields of two polarizations, namely TM and TE can be defined by solving the trigonometric relation between the grating reference plane and the plane of incidence. Thus TM and TE polarizations are defined by:

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = A \begin{pmatrix} \cos\rho \\ 0 \\ \sin\rho \end{pmatrix}, \quad TM \text{ polarization} \quad (4.7)$$

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = A \begin{pmatrix} 0 \\ \cos\tau \\ \sin\tau \end{pmatrix}, \quad TE \text{ polarization} \quad (4.8)$$

with two intermediate angular parameters ρ and σ given by:

$$\begin{aligned} \rho &= \arctan(\tan\theta \cos\phi), \\ \tau &= \arctan(\tan\theta \sin\phi). \end{aligned}$$

This re-defined terminology reveals the true polarization states in our experimental set-up at conical incidence. At normal incidence, i.e. $\phi = 0$, we have $\tau = 0$ and $\rho = \theta$, Eqs. (4.7), (4.8) are the same as Eqs. (4.5), (4.6).

For normal incidence, the wavefronts of the propagating waves in NLC are shown qualitatively in Fig. 4.2(c). The semi-circle is the wavefront of the TE polarization which senses the isotropic refractive index of NLC and the semi-ellipse is the wavefront of the TM polarization which sees the extraordinary refractive index of NLC. For TM polarization, the Poynting vector (energy flow) $\hat{S}$ is not parallel to the direction of the wave vector, instead it is perpendicular to the surface of the wavefront as indicated in Fig. 4.2(c). Because the optic axis of the NLC is along the x -axis, the z -component of the Poynting vector always has the same sign as the wave vector. This

means that the TM polarization is transmitted through the grating. The TE and TM polarizations propagate out of the grating through the same surface. This illustrates the POLICRYPS as transmission gratings.

The geometry of the grating is shown in Fig. 4.2(b). The thickness of the grating is 10 μm , The grating is sandwiched in a glass cell. In our simulation, a slice-structure is adapted to make a similar dielectric permittivity distribution as the real modulation profile shown in Fig. 4.2(b). From the center of the NLC concentration region to the interface with the polymer, 9 slices with gradually changing dielectric permittivities are defined and used. The dielectric permittivities and thicknesses of all slices are listed in Table 4.1. The refractive index of the polymer is about 1.515 for the visible light, so that the dielectric constant is 2.2952. For simplicity we assume that there is only modulation in the x-component of the dielectric permittivity tensor and we take $\epsilon_y = \epsilon_z = 2.2952$ everywhere.

	Thickness(nm)	ϵ_x
LC_1	46	2.492
LC_2	17	2.48
LC_3	12	2.47
LC_4	11	2.46
LC_5	18	2.45
LC_6	17	2.44
LC_7	23	2.43
LC_8	29	2.42
LC_9	215	2.41

Table 4.1: The dielectric permittivity of the vertical slices which parallel to the z-axis, into which the NLC has been divided to approximate the calculated profile shown in Fig. 4.2(b).

4.3 Methodology

4.3.1 Periodic gratings consisting of anisotropic material

We have studied anisotropic media that are translation invariant with respect to one direction in Section 3.3 of the previous chapter. It has been shown there that for a configuration consisting of anisotropic materials of which the dielectric permittivity $\underline{\epsilon}$ is invariant with respect to the y -coordinate, the Maxwell's equations can be reformulated as two coupled partial differential equations for the y -component of the electromagnetic field E_y and H_y only. In the POLICRYPS grating studied in the present chapter, the optic axis of the NLC is perpendicular to the polymer walls and thus parallel to the x -axis. The dielectric permittivity tensor of the NLC can be written

as,

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}, \quad (4.9)$$

with $\epsilon_{xx} = \epsilon_x$ (given by Eq. 4.1) and $\epsilon_{yy} = \epsilon_{zz} = n_o^2 = \epsilon_{\perp}$. Thus the equations in Section 3.3 can be reformulated as,

$$-i\omega\mu_0 H_y = i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q}^T \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega_0 \mu_0 H_y \end{pmatrix} \right] + i\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{M} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} \right], \quad (4.10)$$

$$\begin{aligned} i\omega\epsilon_0\epsilon_{yy}E_y &= -i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{N} \frac{\partial}{\partial x} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} \right] - i\frac{k_y}{\omega\mu_0}\tilde{\nabla} \cdot \left[\frac{1}{D} \mathcal{Q} \frac{\partial}{\partial z} \begin{pmatrix} k_y E_y \\ \omega \mu_0 H_y \end{pmatrix} \right] \\ &\quad - \frac{i}{\omega\mu_0} \Delta E_y, \end{aligned} \quad (4.11)$$

where, as before, $\tilde{\nabla}$ is defined by Eq. (3.11), and

$$D = (\omega^2\epsilon_0\mu_0\epsilon_{xx} - k_y^2)(\omega^2\epsilon_0\mu_0\epsilon_{zz} - k_y^2), \quad (4.12)$$

$$\mathcal{N} = \begin{pmatrix} \omega^2\epsilon_0\mu_0\epsilon_{zz} - k_y^2 & 0 \\ 0 & \omega^2\epsilon_0\mu_0\epsilon_{xx} - k_y^2 \end{pmatrix}, \quad (4.13)$$

$$\mathcal{Q} = \mathcal{N} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -(\omega^2\epsilon_0\mu_0\epsilon_{zz} - k_y^2) \\ \omega^2\epsilon_0\mu_0\epsilon_{xx} - k_y^2 & 0 \end{pmatrix}. \quad (4.14)$$

$$\mathcal{M} = D\mathcal{N}^{-1} = \begin{pmatrix} \omega^2\epsilon_0\mu_0\epsilon_{xx} - k_y^2 & 0 \\ 0 & \omega^2\epsilon_0\mu_0\epsilon_{zz} - k_y^2 \end{pmatrix}, \quad (4.15)$$

Note that D is the determinant of matrix $\mathcal{Q}$. It is assumed here that $D \neq 0$. For conical incidence i.e. $\phi^i \neq 0$, the two polarization states TM and TE as defined in Section 4.2 are coupled. This means when the incident light is purely TM or TE polarized, the diffracted light will be elliptically polarized instead of preserving the linear polarization of the incident field. For $k_y = 0$ the plane of incidence is perpendicular to the grating structure, and then the TE and TM cases are uncoupled because for TE: $H_y = 0$ while for TM: $E_y = 0$.

4.3.2 Bragg transmission grating

Consider a plane wave with wave vector $\mathbf{k}$ incident on a periodic grating with pitch Λ , of which the grating vector is along the x -axis. The incident wave vector is given by,

$$\mathbf{k}^i = \begin{pmatrix} k_x^i \\ k_y^i \\ k_z^i \end{pmatrix} = k_o n^i \begin{pmatrix} \sin\theta^i \cos\phi^i \\ \sin\theta^i \sin\phi^i \\ \cos\theta^i \end{pmatrix}, \quad (4.16)$$

with $k_o = 2\pi/\lambda$, n^i is the refractive index of the incident medium and (θ^i, ϕ^i) are the angles of incidence. When the x -component of the wave vector satisfies the Bragg condition (see Fig. 4.3(a)),

$$-2k_x^i \cdot \Lambda = 2\pi \cdot m \quad (4.17)$$

where m is an integer, constructive interference occurs, due to which a very high diffraction efficiency can be obtained in the m^{th} diffracted order. By substituting Eq. (4.16) in Eq. (4.17), we get

$$\sin\theta^i \cos\phi^i = -m \cdot \frac{\lambda}{2n^i \Lambda}. \quad (4.18)$$

If $m = -1$,

$$\sin\theta_B = \frac{\lambda}{2n^i \Lambda}, \quad \text{for } \phi^i = 0^\circ, \quad (4.19)$$

$$\sin\theta_B = \frac{\lambda}{2n^i \Lambda \cos\phi^i}, \quad \text{for } \phi^i \neq 0^\circ. \quad (4.20)$$

According to Eq. (4.20), the wave vectors of the reading beam with incident angles (θ_B, ϕ^i) , satisfying Eq. (4.20) and $0 \leq \theta_B \leq \pi/2$, $-\pi \leq \phi^i \leq \pi$, form the lower half of the red cone shown in Fig. 4.3(b)

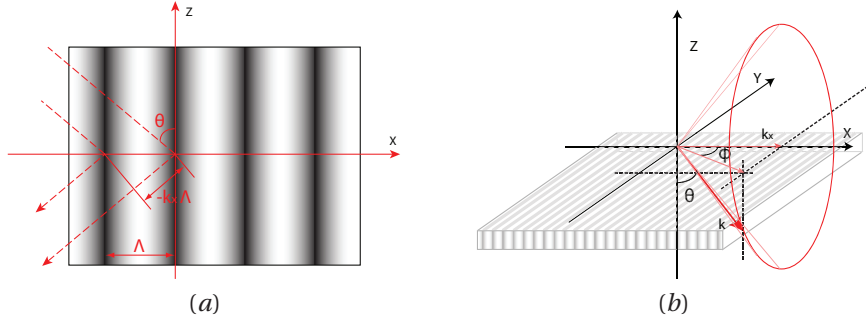


Figure 4.3: (a) Bragg condition in the (x,z) -plane; (b) Bragg condition in three-dimensions.

A schematic diagram of the experimental set-up is shown in Fig. 4.4. Collimated laser light was used as light source. A polarizer was used to control the polarization of the reading beam. Two lasers were used in our experiment. One was a He-Ne Laser with wavelength of 633 nm and the other one, a diode laser with wavelength 532 nm. The intensity of the transmitted diffracted beams is measured by a photo-detector. A translation stage was designed such that the mounted grating sample can be tilted and rotated in three dimensions with any arbitrary angle (θ, ϕ) with high accuracy (in 0.1°) and stability. There are two operating angles α and β as shown in Fig. 4.4, which are different from the conventional definition of the combination of the polar angle θ and the azimuthal angle ϕ as shown in Fig. 4.2. The correlation between them can be expressed as,

$$\tan \alpha = \tan \theta \sin \phi, \quad (4.21)$$

$$\sin \beta = \sin \theta \cos \phi. \quad (4.22)$$

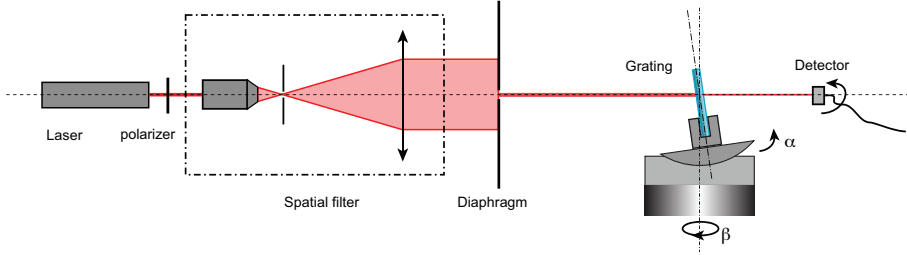


Figure 4.4: *Experimental set-up for both normal and conical incidence.*

4.4 Results and discussion

4.4.1 Normal incidence

As has been discussed in Section 4.3.1, when the plane of incidence is perpendicular to the grating grooves, i.e. $\phi^i = 0^\circ$, the TE and TM polarization are uncoupled. The diffraction efficiency of the transmitted field for TM polarization was calculated and measured in this normal incidence mount. The efficiencies are plotted as function of the angle of incidence θ^i in Fig. 4.5. When the angle of incidence θ^i changes from 0° to 30° , there are three dominant orders for red light (633 nm), namely the 0^{th} order and the $\pm 1^{st}$ orders (indicated as -1T and 1T in Fig. 4.5(a); similar notations are used for the other orders in the plots and we omit explaining each one when it occurs in the following text). For green light (532 nm), there are four dominant diffracted orders, namely the -2^{nd} , $\pm 1^{st}$ and 0^{th} orders. The sum of the intensity of the transmitted orders is defined as the total transmitted intensity I_{total} . The relative intensity of a transmitted diffracted order is defined as the intensity of that order divided by the total intensity, $RI = I_{order} / I_{total}$.

The diffraction efficiencies are strongly dependent on the angle of incidence θ^i . The pitch of the POLICRYPS grating sample is 1220 nm. For red light with $\lambda = 633$ nm, the Bragg angle for the -1^{st} diffracted order is calculated with Eq. (4.19) and is equal to 15° , while for green light with $\lambda = 532$ nm, the Bragg angle for the -1^{st} diffracted order is 12.6° . For TM polarization a very high diffraction efficiency was observed when the angle of incidence is equal to Bragg angle (θ_B). The high diffraction efficiency in the -1^{st} diffracted order at the Bragg angle is verified by the experimental measurements shown in Fig. 4.5. The narrow peak of the -1^{st} order shows that the POLICRYPS grating has a good angular selectivity. For the 633 nm reading beam, a very good agreement is obtained between the simulations and the measurements, by using the dielectric permittivity modulation profile shown in Fig. 4.2(b). For the green wave with $\lambda = 532$ nm, the refractive indices should be modified because of dispersion, but this could not be done accurately because we are dealing with a mixture of materials. This is the reason for the discrepancy at small and large angles of incidence for the green reading beam between the simulated results and the measurements.

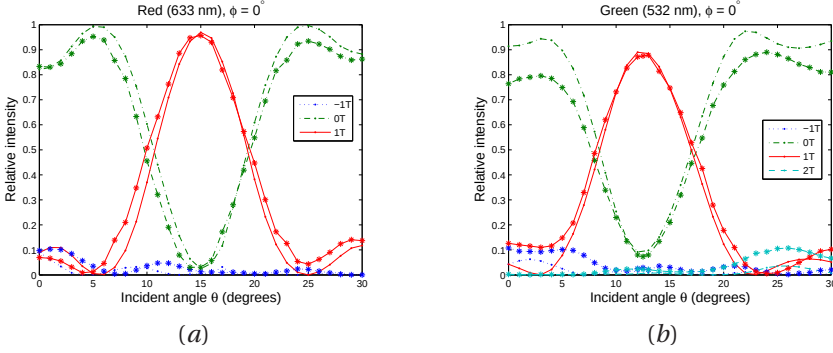


Figure 4.5: The diffraction efficiency of TM polarization at normal incidence, i.e. $\phi^i = 0^\circ$ and for $0^\circ \leq \theta^i \leq 30^\circ$, for (a) red light (633 nm) and (b) green light (532 nm). The lines marked with asterisk are experimental measurements and the lines without marker are calculations. In the legend, $-1T$ denotes the -1^{st} diffracted order of the transmitted field, etc.

4.4.2 Conical incidence

Conical incidence is of interest for the application to display systems [55, 56] because in those applications a range of conical angles of incidence occur. We have studied the diffraction efficiency of the POLICRYPS grating at conical incidence i.e. when the plane of incidence is not perpendicular to the grating, i.e. $\phi \neq 0^\circ$ as shown in Fig. 4.6. The polarization of the reading beam is TM (as given in Eq. (4.7)). For fixed θ^i and varying ϕ^i in the range $0^\circ \leq \phi^i \leq 70^\circ$, the diffraction efficiency of the transmitted field was simulated and measured. The results are shown in Fig. 4.7. For red light (633 nm), $\theta^i = 15^\circ$ and for green light (532 nm), $\theta^i = 13^\circ$. When the plane of incidence is moving away from the perpendicular position to the grating structure, the Bragg condition is not fulfilled anymore. The diffraction efficiency of the -1^{st} diffracted order decreases when ϕ^i is getting larger. This is predicted by the simulations shown in Fig. 4.7 (dot-solid lines in the figure) and confirmed by the experimental measurements (star-dashed lines). From the figure we can see that up to 20° the diffraction efficiency of the -1^{st} order is still quite high. This means that the POLICRYPS grating has a good tolerance range for the conical mount.

In Section 4.3.1, it has been shown that in a conical incidence mount, i.e. $k_y \neq 0$, the system of Eq. (4.10) and Eq. (4.11) are coupled and thus polarization states of the diffracted waves are different from that of the incidence wave. To analyze the polarization states of each diffracted order, the electric field vector of each order is decomposed in the two linear polarizations, TM and TE. The relative intensity for each linear polarization component is calculated based on the simulation results and is plotted in Fig. 4.8. The coupling between the two linear polarizations is easy to infer from the plots. When the azimuthal angle ϕ^i is small and the plane of incidence deviates from the normal incidence mount with a relatively small angle, the TM polarization dominates the diffracted field. With the angle ϕ^i getting larger, TE polarization couples out

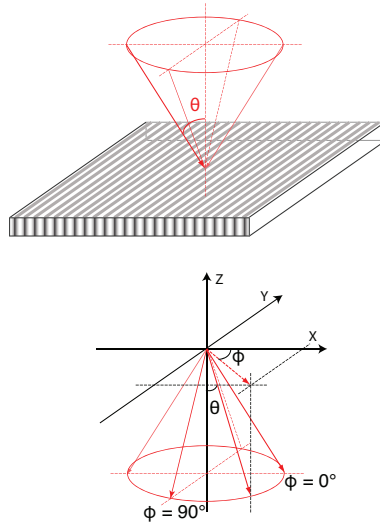


Figure 4.6: Conical incidence configuration: the rays of incidence form a cone with respect to the normal of the POLICRYPS grating.

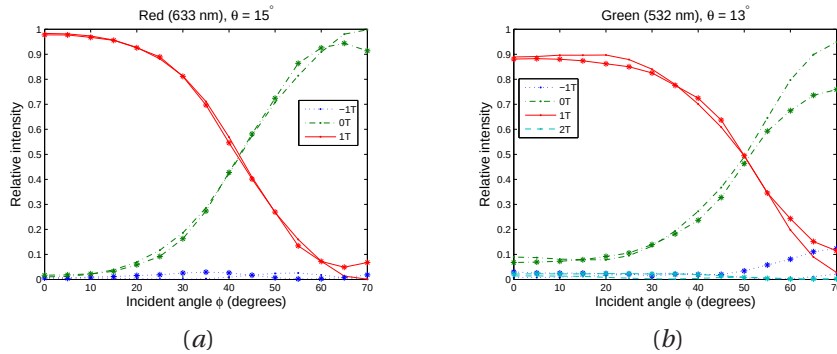


Figure 4.7: The diffraction efficiency of TM polarization at conical incidence, for $0^\circ \leq \phi^i \leq 70^\circ$ and $\theta^i \doteq \theta_B$, for (a) red light (633 nm) $\theta^i = 15^\circ$ and for (b) green light (532 nm) $\theta^i = 13^\circ$. The lines marked with asterisk are experimental measurements and the lines without marker are calculations. In the legend, -1T denotes the -1^{st} diffracted order of the transmitted field, etc.

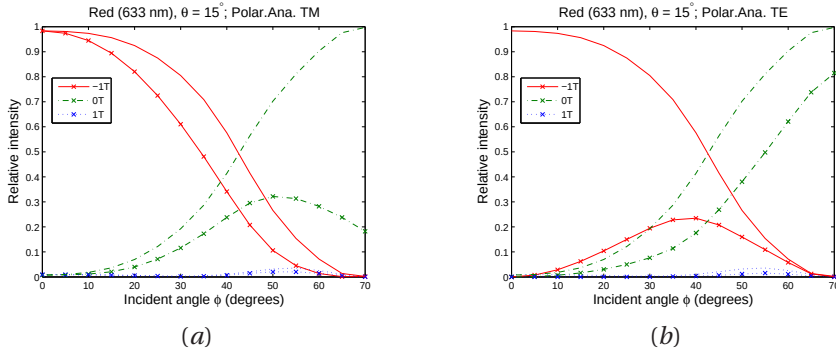


Figure 4.8: Polarization analysis (based on simulation results) of the diffracted orders at the conical mount for 633 nm red light with (a) TM polarization analyzer and (b) TE polarization analyzer. The lines without marker are the same plots of the simulations results as in Fig. 4.7(a) and are included here as a reference for the two decomposed linear polarization in each order. In the legend, $-1T$ denotes the -1^{st} diffracted order of the transmitted field, etc.

in the diffracted orders. For some polarization sensitive applications, a polarization analyzer is necessary to be used with the POLICRYPS grating to remove the undesired polarization component in the transmission field.

4.4.3 3D Bragg condition

As we discussed in Section 4.3.2, when the plane of incidence is not perpendicular to the grating structure, i.e. $\phi^i \neq 0^\circ$, the Bragg angle θ_B is a function of ϕ^i . If the Bragg condition for the incident field is satisfied, a maximum diffraction efficiency can be obtained. To test this effect, TM-polarized light with wavelength 633 nm is used as reading beam. For the -1^{st} order, the Bragg angle θ_B is calculated for varying ϕ^i from 0° to 60° according to Eq. (4.20). For each ϕ^i , the incident angle θ^i is changed from $(\theta_B - 2^\circ)$ to $(\theta_B + 2^\circ)$. For each (θ^i, ϕ^i) the diffraction efficiency was calculated and measured. Then the angular distribution of the m^{th} diffracted order can be calculated for varying incident angles θ^i and ϕ^i from the following equations:

$$n^d \cos \phi^d \sin \theta^d = n^i \cos \phi^i \sin \theta^i + \frac{m\lambda}{\Lambda}, \quad (4.23)$$

$$n^d \sin \phi^d \sin \theta^d = n^i \sin \phi^i \sin \theta^i. \quad (4.24)$$

The angular distribution of the diffraction efficiency of the transmitted field for the -1^{st} order is shown in Fig. 4.9. The transmitted field of the -1^{st} order spans a ribbon region. As is shown in the figure, the diffraction efficiency has a maximum at the Bragg angle of incidence (indicated by the dotted line in the center of the ribbon). The diffraction efficiencies at (θ_B, ϕ^i) (along the center dotted line in the ribbon of Fig. 4.9(a)) and at $(\theta_B \pm 2^\circ)$ (along the edges of the ribbon) are shown as a function

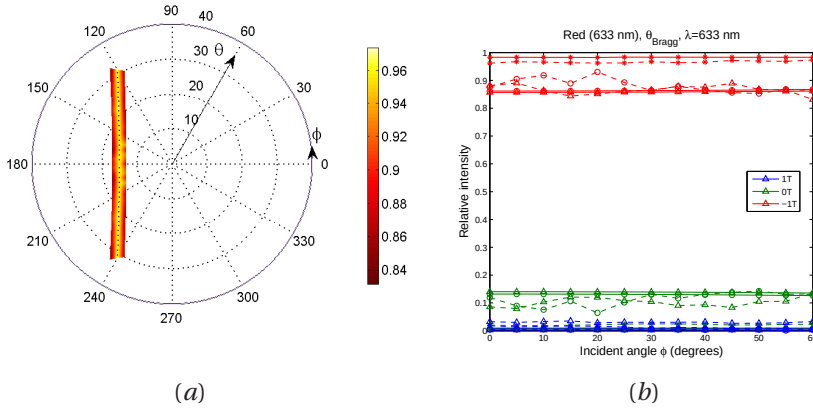


Figure 4.9: (a) Measured angular distribution (θ^d, ϕ^i) of the diffraction efficiency of the -1^{st} diffracted order for the reading beam of 633 nm when the incident angles (θ^i, ϕ^i) follow the Bragg condition as discussed in Sec. 4.3.2. (b) Diffraction efficiency along the Bragg angle (marked with stars) and for angles $\theta_B - 2^\circ$ (triangles) and angles $\theta_B + 2^\circ$ (circles). The solid lines are simulated results, and the dashed lines are measurements.

of the incident angle ϕ^i in Fig. 4.9(b). The curves show a uniform distribution of the diffraction efficiency at the corresponding Bragg incidence angles (θ_B, ϕ^i).

4.5 Conclusions

POLICRYPS gratings offer very good optical properties because of their sharp and uniform morphology. A sample grating has been studied by using a rigorous model, and by experiments. A very good agreement is obtained between calculations and measurements. An experimental check of the grating efficiency at normal incidence shows a strong angular selectivity around the Bragg angle. By using a 633 nm reading beam, a maximum diffraction efficiency at the Bragg angle of approximately 95% is observed. When the reading beam is switched to 532 nm green light, the diffraction efficiency is slightly less and this shows that the diffraction by the POLICRYPS is wavelength dependent. Results of a study of diffraction efficiencies for conical angles of incidence are presented for the first time. The results show a good tolerance range of conical tilting. The grating is suitable for applications in e.g. display systems. Furthermore, a uniform high diffraction efficiency can be achieved by varying the incident angles along the cone of the Bragg angle.

Chapter 5

Recording of photoanisotropic polarization gratings

5.1 Introduction

Photoanisotropic polarization gratings are interesting optical components for many light controlling applications. They can be manufactured with polarization holographic recording. Depending on the polarization states of the two recording beams and the recording angle, periodic modulation of the polarization states and the intensity can be achieved. Materials which are sensitive to polarization can be used to record the interference pattern formed by the two beams. Many theoretical work has been done on the recording and the reading process of photoanisotropic polarization gratings. Kakichashvili [11, 57] started initial research in the field and has built the basic scheme for obtaining the dielectric permittivity changes due to photo-induced anisotropy in materials. Todorov and Nikolova [58, 59, 60, 61, 62, 63] have carried out extensive theoretical and experimental work on the recording and reading processes of polarization gratings. They use Jones Matrix method to study the diffraction from polarization gratings. Coupled mode analysis have been applied to study polarization grating by Huang and Wagner [64, 65]. Interference of two arbitrary polarizations have been studied by Van Heesch [66]. Recently Kilosanidze has summarized formulations for the recording of linear and circular photoanisotropy [67]. Many other publications which are not listed here contain similar theoretical treatments as those mention above. In all those papers only paraxial situations are studied, i.e. the recording angle of the two recording beams is small. Our research objective is to go beyond this approximation regime and develop formulations for polarization holography with arbitrary polarizations and large recording angles, e.g. for angles larger than 10° .

5.2 Polarization holography

Polarization holography is performed by using a photo-anisotropic material that records not only the intensity modulation of the object wavefront, but also information on the state of polarization. Kakichashvili [68, 69, 11, 57] considered that interference of two plane waves and assumed that the angle between the two recording beams was very small, so that paraxial theory can be applied. Under that assumption, he has shown that when two orthogonally linearly polarized waves interfere, the intensity modulation is absent and a periodic modulation of the polarization state is generated instead.

Under the small angle assumption, the electric fields of the two recording beams can be considered to be in one plane. This drastically simplifies calculations and is a sufficiently accurate starting point to study the basic physics of polarization holography. However it also constrains calculations within a small regime and the results become inaccurate for larger recording angles. A larger recording angle results in a smaller modulation pitch with the same recording wavelength. Smaller pitch is a potential advantage for optical components design. Thus, formulations of polarization holography beyond the small angle approximation is desirable.

5.2.1 Interference of two plane waves

In this chapter we present a general theory for the recording of the interference pattern of two plane waves into a thin layer of photoanisotropic material, without using the small angle approximation. In the present section we study the interference of two plane waves in general.

Consider two monochromatic plane waves (see Fig. 5.1) propagating in an (initially) isotropic medium with real refractive index n , having wave vectors in the $y = 0$ plane:

$$\mathbf{k}^+ = (k_x, 0, k_z), \quad (5.1)$$

$$\mathbf{k}^- = (-k_x, 0, k_z), \quad (5.2)$$

where $k_z = (k_0^2 n^2 - k_x^2)^{1/2} > 0$ and $k_x > 0$ with k_0 the wave number in vacuum. Without restricting generality, the electric vectors of the two plane waves are written in the form:

$$\mathbf{E}^+(\mathbf{r}) = \left[a_S^+ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + a_P^+ \frac{1}{k_0 n} \begin{pmatrix} k_z \\ 0 \\ -k_x \end{pmatrix} \right] e^{i\mathbf{k}^+ \cdot \mathbf{r}}, \quad (5.3)$$

$$\mathbf{E}^-(\mathbf{r}) = \left[a_S^- \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + a_P^- \frac{1}{k_0 n} \begin{pmatrix} k_z \\ 0 \\ k_x \end{pmatrix} \right] e^{i\mathbf{k}^- \cdot \mathbf{r}}, \quad (5.4)$$

where $a_S^\pm$ and $a_P^\pm$ are complex numbers. The electric field vector of each wave is thus decomposed into two orthogonal linear polarizations, namely the s - and p -polarization, where 'p' means that the electric field is parallel to the (x, z) -plane in

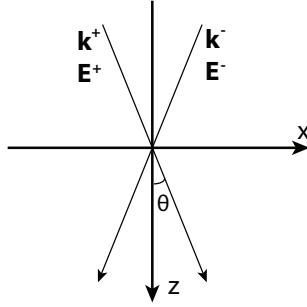


Figure 5.1: Ray arrangement of the interference between two plane waves, of which the one propagating in the positive x -direction has wave vector $\mathbf{k}^+$ and complex electric field $\mathbf{E}^+$, and the other has wave vector $\mathbf{k}^-$ and complex electric field $\mathbf{E}^-$.

which the wave vectors are, and 's' means that the electric field is perpendicular to that plane. The '+' sign indicates that the wave propagates in the positive x -direction with wave vector $\mathbf{k}^+$, while the '-' sign has analogous meaning. The two waves $\mathbf{E}^+$ and $\mathbf{E}^-$ are superposed at every point $\mathbf{r}$ in space. The total electric field at $\mathbf{r}$ is thus the sum of the two waves:

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \mathbf{E}^+(\mathbf{r}) + \mathbf{E}^-(\mathbf{r}) \\ &= \begin{pmatrix} (a_p^- e^{-ik_x x} + a_p^+ e^{ik_x x}) \frac{k_z}{k_0 n} \\ a_s^- e^{-ik_x x} + a_s^+ e^{ik_x x} \\ (a_p^- e^{-ik_x x} - a_p^+ e^{ik_x x}) \frac{k_x}{k_0 n} \end{pmatrix} e^{ik_z z} \end{aligned} \quad (5.5)$$

The physical electric field is thus:

$$\begin{aligned} \mathcal{E}(\mathbf{r}, t) &= \text{Re} \left[\mathbf{E}(\mathbf{r}) e^{-i\omega t} \right] \\ &= \mathbf{p}(\mathbf{r}) \cos(\omega t) + \mathbf{q}(\mathbf{r}) \sin(\omega t), \end{aligned} \quad (5.6)$$

with

$$\mathbf{p}(\mathbf{r}) = \begin{pmatrix} |a_p^-| \frac{k_z}{k_0 n} \cos[\text{Arg}(a_p^-) - k_x x + k_z z] + |a_p^+| \frac{k_z}{k_0 n} \cos[\text{Arg}(a_p^+) + k_x x + k_z z] \\ |a_s^-| \cos[\text{Arg}(a_s^-) - k_x x + k_z z] + |a_s^+| \cos[\text{Arg}(a_s^+) + k_x x + k_z z] \\ |a_p^-| \frac{k_x}{k_0 n} \cos[\text{Arg}(a_p^-) - k_x x + k_z z] - |a_p^+| \frac{k_x}{k_0 n} \cos[\text{Arg}(a_p^+) + k_x x + k_z z] \end{pmatrix}, \quad (5.7)$$

$$\mathbf{q}(\mathbf{r}) = \begin{pmatrix} |a_p^-| \frac{k_z}{k_0 n} \sin[\text{Arg}(a_p^-) - k_x x + k_z z] + |a_p^+| \frac{k_z}{k_0 n} \sin[\text{Arg}(a_p^+) + k_x x + k_z z] \\ |a_s^-| \sin[\text{Arg}(a_s^-) - k_x x + k_z z] + |a_s^+| \sin[\text{Arg}(a_s^+) + k_x x + k_z z] \\ |a_p^-| \frac{k_x}{k_0 n} \sin[\text{Arg}(a_p^-) - k_x x + k_z z] - |a_p^+| \frac{k_x}{k_0 n} \sin[\text{Arg}(a_p^+) + k_x x + k_z z] \end{pmatrix}. \quad (5.8)$$

Eq. (5.6) represents an ellipse, which is called the polarization ellipse. The electric field vector can always be written as the sum of two vectors parallel to the vector $\mathbf{a}$

and $\mathbf{b}$ along the axes of the polarization ellipse [9] (see Section 1.3). The two vectors $\mathbf{a}, \mathbf{b}$ are orthogonal and follow from $\mathbf{p}$ and $\mathbf{q}$ as in Eqs. (1.13), (1.12)

$$\mathbf{a}(\mathbf{r}) = \mathbf{p}(\mathbf{r}) \cos \psi - \mathbf{q}(\mathbf{r}) \sin \psi, \quad (5.9)$$

$$\mathbf{b}(\mathbf{r}) = \mathbf{p}(\mathbf{r}) \sin \psi + \mathbf{q}(\mathbf{r}) \cos \psi, \quad (5.10)$$

where ψ can be calculated by using Eq. (A.12). From these equations we have

$$\operatorname{Re} \mathbf{E}(\mathbf{r}) = \mathbf{p}(\mathbf{r}) = \mathbf{a}(\mathbf{r}) \cos \psi + \mathbf{b}(\mathbf{r}) \sin \psi, \quad (5.11)$$

$$\operatorname{Im} \mathbf{E}(\mathbf{r}) = \mathbf{q}(\mathbf{r}) = -\mathbf{a}(\mathbf{r}) \sin \psi + \mathbf{b}(\mathbf{r}) \cos \psi. \quad (5.12)$$

Thus,

$$\mathbf{E}(\mathbf{r}) \cdot \mathbf{a}(\mathbf{r}) = |\mathbf{a}(\mathbf{r})|^2 (\cos \psi - i \sin \psi) = |\mathbf{a}(\mathbf{r})|^2 e^{-i\psi}, \quad (5.13)$$

$$\mathbf{E}(\mathbf{r}) \cdot \mathbf{b}(\mathbf{r}) = |\mathbf{b}(\mathbf{r})|^2 (\sin \psi + i \cos \psi) = |\mathbf{b}(\mathbf{r})|^2 e^{i(\frac{\pi}{2} - \psi)}. \quad (5.14)$$

The local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$, defined as in Section 1.3, is given by:

$$\hat{\mathbf{e}}_1(\mathbf{r}) = \frac{\mathbf{a}(\mathbf{r})}{|\mathbf{a}(\mathbf{r})|}, \quad \hat{\mathbf{e}}_2(\mathbf{r}) = \frac{\mathbf{b}(\mathbf{r})}{|\mathbf{b}(\mathbf{r})|}, \quad \text{if } |\mathbf{a}| \geq |\mathbf{b}|, \quad (5.15)$$

$$\hat{\mathbf{e}}_1(\mathbf{r}) = \frac{\mathbf{b}(\mathbf{r})}{|\mathbf{b}(\mathbf{r})|}, \quad \hat{\mathbf{e}}_2(\mathbf{r}) = -\frac{\mathbf{a}(\mathbf{r})}{|\mathbf{a}(\mathbf{r})|}, \quad \text{if } |\mathbf{a}| < |\mathbf{b}|, \quad (5.16)$$

and $\hat{\mathbf{e}}_3(\mathbf{r})$ is a unit vector chosen perpendicular to $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ and such that $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ is a positively oriented orthonormal basis. On the local basis, the electric field vector has components,

$$\left. \begin{aligned} E_1(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{a}(\mathbf{r})| e^{-i\psi}, \\ E_2(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = |\mathbf{b}(\mathbf{r})| e^{i(\frac{\pi}{2} - \psi)}, \\ E_3(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_3(\mathbf{r}) = 0. \end{aligned} \right\} \quad \text{if } |\mathbf{a}| \geq |\mathbf{b}|, \quad (5.17)$$

$$\left. \begin{aligned} E_1(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_1(\mathbf{r}) = |\mathbf{b}(\mathbf{r})| e^{i(\frac{\pi}{2} - \psi)}, \\ E_2(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_2(\mathbf{r}) = |\mathbf{a}(\mathbf{r})| e^{i(\pi - \psi)}, \\ E_3(\mathbf{r}) &= \mathbf{E}(\mathbf{r}) \cdot \hat{\mathbf{e}}_3(\mathbf{r}) = 0. \end{aligned} \right\} \quad \text{if } |\mathbf{a}| < |\mathbf{b}|. \quad (5.18)$$

With respect to the global basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$, a transformation matrix V can be defined by

$$V(\mathbf{r}) = (\hat{\mathbf{e}}_1(\mathbf{r}) \ \hat{\mathbf{e}}_2(\mathbf{r}) \ \hat{\mathbf{e}}_3(\mathbf{r})) = \begin{pmatrix} e_{1,x}(\mathbf{r}) & e_{2,x}(\mathbf{r}) & e_{3,x}(\mathbf{r}) \\ e_{1,y}(\mathbf{r}) & e_{2,y}(\mathbf{r}) & e_{3,y}(\mathbf{r}) \\ e_{1,z}(\mathbf{r}) & e_{2,z}(\mathbf{r}) & e_{3,z}(\mathbf{r}) \end{pmatrix}. \quad (5.19)$$

The inverse of $V(\mathbf{r})$ is its hermitian conjugate:

$$V^{-1}(\mathbf{r}) = V^*(\mathbf{r}) = \begin{pmatrix} e_{1,x}(\mathbf{r}) & e_{1,y}(\mathbf{r}) & e_{1,z}(\mathbf{r}) \\ e_{2,x}(\mathbf{r}) & e_{2,y}(\mathbf{r}) & e_{2,z}(\mathbf{r}) \\ e_{3,x}(\mathbf{r}) & e_{3,y}(\mathbf{r}) & e_{3,z}(\mathbf{r}) \end{pmatrix}. \quad (5.20)$$

We can use either the global basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ or the local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ to expand the electric field vector $\mathbf{E}$

$$\mathbf{E}(\mathbf{r}) = E_x(\mathbf{r})\hat{\mathbf{x}} + E_y(\mathbf{r})\hat{\mathbf{y}} + E_z(\mathbf{r})\hat{\mathbf{z}} \quad (5.21a)$$

$$= E_1(\mathbf{r})\hat{\mathbf{e}}_1(\mathbf{r}) + E_2(\mathbf{r})\hat{\mathbf{e}}_2(\mathbf{r}). \quad (5.21b)$$

The transformation between the components of $\mathbf{E}$ on the two bases is given by the matrices $V(\mathbf{r})$ and $V^*(\mathbf{r})$:

$$\begin{pmatrix} E_1(\mathbf{r}) \\ E_2(\mathbf{r}) \\ 0 \end{pmatrix} = V^*(\mathbf{r}) \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \quad (5.22a)$$

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = V(\mathbf{r}) \begin{pmatrix} E_1(\mathbf{r}) \\ E_2(\mathbf{r}) \\ 0 \end{pmatrix}. \quad (5.22b)$$

The ellipticity of the state of the polarization is defined by

$$e = \frac{|E_2|}{|E_1|}. \quad (5.23)$$

If we omit the factor $\frac{1}{2}\epsilon_0 n^2$, the intensity of the local field is the square modules of the complex amplitudes,

$$I = |E_1|^2 + |E_2|^2. \quad (5.24)$$

The state of polarization seems to depend on both x and z , but in fact it depends only on x and it is independent of z . To show this we first remark that Eqs. (5.7) and (5.8) imply

$$\frac{\partial \mathbf{p}}{\partial z} = -k_z \mathbf{q}, \quad (5.25)$$

$$\frac{\partial \mathbf{q}}{\partial z} = k_z \mathbf{p}. \quad (5.26)$$

From Eqs. (5.9) and (5.10):

$$\begin{aligned} \frac{\partial \mathbf{a}}{\partial z} &= \frac{\partial \mathbf{p}}{\partial z} \cos \psi - \frac{\partial \mathbf{q}}{\partial z} \sin \psi - (\mathbf{p} \sin \psi + \mathbf{q} \cos \psi) \frac{\partial \psi}{\partial z} \\ &= -(k_z + \frac{\partial \psi}{\partial z}) \mathbf{b}, \end{aligned} \quad (5.27)$$

$$\begin{aligned} \frac{\partial \mathbf{b}}{\partial z} &= \frac{\partial \mathbf{p}}{\partial z} \sin \psi + \frac{\partial \mathbf{q}}{\partial z} \cos \psi + (\mathbf{p} \cos \psi - \mathbf{q} \sin \psi) \frac{\partial \psi}{\partial z} \\ &= (k_z + \frac{\partial \psi}{\partial z}) \mathbf{a}. \end{aligned} \quad (5.28)$$

Hence,

$$\frac{\partial}{\partial z} (\mathbf{a} \cdot \mathbf{b}) = \frac{\partial \mathbf{a}}{\partial z} \cdot \mathbf{b} + \mathbf{a} \cdot \frac{\partial \mathbf{b}}{\partial z} = (k_z + \frac{\partial \psi}{\partial z}) (\mathbf{a} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b}). \quad (5.29)$$

But ψ is chosen such that $\mathbf{a} \cdot \mathbf{b} = 0$, hence Eq. (5.29) implies that

$$\frac{\partial \psi}{\partial z} = -k_z \quad \text{or} \quad \mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b}. \quad (5.30)$$

Now $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b}$ is equivalent to circular polarization, hence if the state of polarization is not circular we have, $\frac{\partial \psi}{\partial z} = -k_z$. Then Eqs. (5.27) and (5.28) imply that the vectors

a, **b** and hence the polarization ellipse is independent of z . Therefore we will always write $\mathbf{a}(x)$, $\mathbf{b}(x)$ instead of $\mathbf{a}(\mathbf{r})$ and $\mathbf{b}(\mathbf{r})$. Thus the local basis is also independent of z , i.e. we can write $\{\hat{\mathbf{e}}_1(x), \hat{\mathbf{e}}_2(x), \hat{\mathbf{e}}_3(x)\}$.

The angle of recording $\theta_{\text{record.}}$ is the angle between the wave vector of one of the two recording beams and the z -axis. It satisfies: $\sin\theta_{\text{record.}} = k_x/(k_0 n)$ in a medium with refractive index n . The angle $\theta_{\text{record.}}$ determines the pitch of the modulation Λ , which can be expressed as,

$$\Lambda = \frac{\lambda_{\text{record.}}}{2n \sin\theta_{\text{record.}}}, \quad (5.31)$$

where $\lambda_{\text{record.}}$ is the wavelength of the recording beam in vacuum. For a recording wavelength of 351 nm the pitch Λ as a function of the recording angle in air ($n = 1$) is shown in Fig. 5.2.

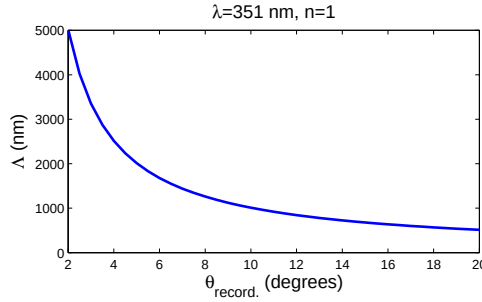


Figure 5.2: Pitch Λ of interference of two plane waves with respect to recording angles $\theta_{\text{record.}}$.

5.2.2 Photo-induced anisotropy in photoanisotropic materials

A photoactive medium, which initially is isotropic with relative permittivity ϵ^i , becomes anisotropic due to the local electric field. Suppose that the recording medium is so thin that throughout the thickness of the medium we may assume that the photoreaction is the same and that there is no depth dependence of the photo-induced anisotropy. When both linear and circular photoanisotropy are present, the relative dielectric permittivity tensor in response to the *local* electric field becomes on the local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ [11, 65],

$$\underline{\underline{\epsilon}}(\mathbf{r}) = \begin{pmatrix} \epsilon^i + \kappa_{\parallel}|E_1(\mathbf{r})|^2 + \kappa_{\perp}|E_2(\mathbf{r})|^2 & -i\kappa_c 2|E_1(\mathbf{r})||E_2(\mathbf{r})| & 0 \\ i\kappa_c 2|E_1(\mathbf{r})||E_2(\mathbf{r})| & \epsilon^i + \kappa_{\perp}|E_1(\mathbf{r})|^2 + \kappa_{\parallel}|E_2(\mathbf{r})|^2 & 0 \\ 0 & 0 & \epsilon^i + \kappa_{\perp}(|E_1(\mathbf{r})|^2 + |E_2(\mathbf{r})|^2) \end{pmatrix}, \quad (5.32)$$

where $E_1(\mathbf{r})$ and $E_2(\mathbf{r})$ are the electric field components on the basis along the long and short semi-axes of the polarization ellipse respectively, and $\kappa_{\parallel}$, κ_{perp} and κ_c characterize the sensitivity of the photoanisotropic medium. The quantities $\kappa_{\parallel}$ and

$\kappa_{\perp}$ are called the sensitivities of the linear photoanisotropy parallel and perpendicular to the principal molecular axis. The quantity κ_c in contrast is called the sensitivity of the circular photoanisotropy. The 'tilde' in $\tilde{\underline{\epsilon}}(\mathbf{r})$ is (and will be) used to emphasize that the permittivity tensor is written on the local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$. Eq. (5.32) can be reformulated in terms of the locally defined Stokes parameters,

$$\tilde{\underline{\epsilon}}(\mathbf{r}) = \begin{pmatrix} \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0(\mathbf{r}) + \frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) & -i\kappa_c S_3(\mathbf{r}) & 0 \\ i\kappa_c S_3(\mathbf{r}) & \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0(\mathbf{r}) - \frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) & 0 \\ 0 & 0 & \epsilon^i + \kappa_{\perp} S_0(\mathbf{r}) \end{pmatrix}, \quad (5.33)$$

with the Stokes parameters given by Eq. (1.22).

The expression of the photo-induced anisotropy in terms of the Stokes parameters gives an intuitive insight in the physics of the photoanisotropic process. We can see from Eq. (5.33) that linear polarization only introduces linear anisotropy and the changes occur on the diagonal elements of the tensor ($\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1$). Similarly, circular polarization only introduces circular anisotropy, as seen in the off-diagonal element $i\kappa_c S_3$.

In general $\tilde{\underline{\epsilon}}(\mathbf{r})$ is hermitian. Hence, for every $\mathbf{r}$, there exists an orthonormal basis of $\mathbb{C}^3$ consisting of eigenvectors of the tensor $\tilde{\underline{\epsilon}}(\mathbf{r})$. The eigenvalues are real and given by:

$$\epsilon_{\xi}(\mathbf{r}) = \epsilon^i + \kappa_{\perp} S_0(\mathbf{r}), \quad (5.34a)$$

$$\epsilon_{\eta}(\mathbf{r}) = \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0(\mathbf{r}) - \sqrt{\frac{(\kappa_{\parallel} - \kappa_{\perp})^2}{4} S_1^2(\mathbf{r}) + \kappa_c^2 S_3^2(\mathbf{r})}, \quad (5.34b)$$

$$\epsilon_{\zeta}(\mathbf{r}) = \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0(\mathbf{r}) + \sqrt{\frac{(\kappa_{\parallel} - \kappa_{\perp})^2}{4} S_1^2(\mathbf{r}) + \kappa_c^2 S_3^2(\mathbf{r})}. \quad (5.34c)$$

It is assumed here that $\kappa_{\parallel} > \kappa_{\perp}$ and that the expression under the square root is so small that

$$\epsilon_{\xi}(\mathbf{r}) < \epsilon_{\eta}(\mathbf{r}). \quad (5.35)$$

The corresponding normalized eigenvectors are, when expressed on the local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$, given by:

$$\hat{\xi}(\mathbf{r}) = \hat{\mathbf{e}}_3(\mathbf{r}), \quad (5.36a)$$

$$\hat{\eta}(\mathbf{r}) = \frac{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right] \hat{\mathbf{e}}_1(\mathbf{r}) + i\kappa_c S_3(\mathbf{r}) \hat{\mathbf{e}}_2(\mathbf{r})}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3^2(\mathbf{r})}}, \quad (5.36b)$$

$$\hat{\zeta}(\mathbf{r}) = \frac{i\kappa_c S_3(\mathbf{r}) \hat{\mathbf{e}}_1(\mathbf{r}) + \left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right] \hat{\mathbf{e}}_2(\mathbf{r})}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3^2(\mathbf{r})}}. \quad (5.36c)$$

This implies that

$$\hat{\mathbf{e}}_1(\mathbf{r}) = \frac{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right] \hat{\boldsymbol{\eta}}(\mathbf{r}) - i\kappa_c S_3(\mathbf{r}) \hat{\boldsymbol{\zeta}}(\mathbf{r})}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3(\mathbf{r})}}, \quad (5.37a)$$

$$\hat{\mathbf{e}}_2(\mathbf{r}) = \frac{-i\kappa_c S_3(\mathbf{r}) \hat{\boldsymbol{\zeta}}(\mathbf{r}) + \left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right] \hat{\boldsymbol{\eta}}(\mathbf{r})}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3(\mathbf{r})}}, \quad (5.37b)$$

$$\hat{\mathbf{e}}_3(\mathbf{r}) = \hat{\boldsymbol{\xi}}(\mathbf{r}). \quad (5.37c)$$

Any complex vector $\mathbf{a} \in \mathbb{C}^3$ can be written on both the local basis and the basis of eigenvectors:

$$\mathbf{a} = a_1 \hat{\mathbf{e}}_1(\mathbf{r}) + a_2 \hat{\mathbf{e}}_2(\mathbf{r}) + a_3 \hat{\mathbf{e}}_3(\mathbf{r}) \quad (5.38)$$

$$= a_{\xi} \hat{\boldsymbol{\xi}}(\mathbf{r}) + a_{\eta} \hat{\boldsymbol{\eta}}(\mathbf{r}) + a_{\zeta} \hat{\boldsymbol{\zeta}}(\mathbf{r}). \quad (5.39)$$

The transformation between the components of $\mathbf{a}$ on the two basis is given by the matrices $W(\mathbf{r})$ and $W^{-1}(\mathbf{r}) = W^*(\mathbf{r})$:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = W(\mathbf{r}) \begin{pmatrix} a_{\xi} \\ a_{\eta} \\ a_{\zeta} \end{pmatrix}, \quad \begin{pmatrix} a_{\xi} \\ a_{\eta} \\ a_{\zeta} \end{pmatrix} = W^*(\mathbf{r}) \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad (5.40)$$

where

$$W(\mathbf{r}) = \begin{pmatrix} 0 & w_{1\eta}(\mathbf{r}) & w_{1\zeta}(\mathbf{r}) \\ 0 & w_{2\eta}(\mathbf{r}) & w_{2\zeta}(\mathbf{r}) \\ 1 & 0 & 0 \end{pmatrix}, \quad (5.41)$$

$$W^*(\mathbf{r}) = \begin{pmatrix} 0 & 0 & 1 \\ w_{1\eta}^*(\mathbf{r}) & w_{2\eta}^*(\mathbf{r}) & 0 \\ w_{1\zeta}^*(\mathbf{r}) & w_{2\zeta}^*(\mathbf{r}) & 0 \end{pmatrix}, \quad (5.42)$$

with

$$w_{1\eta}(\mathbf{r}) = w_{2\zeta}(\mathbf{r}) = \frac{\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2}}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3(\mathbf{r})}}, \quad (5.43)$$

$$w_{2\eta}(\mathbf{r}) = w_{1\zeta}(\mathbf{r}) = \frac{i\kappa_c S_3(\mathbf{r})}{\sqrt{\left[\frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1(\mathbf{r}) - \frac{\epsilon_{\zeta}(\mathbf{r}) - \epsilon_{\eta}(\mathbf{r})}{2} \right]^2 + \kappa_c^2 S_3(\mathbf{r})}}. \quad (5.44)$$

The vector $\mathbf{a}$ can of course also be written on the global Cartesian basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$:

$$\mathbf{a} = a_x \hat{\mathbf{x}} + a_y \hat{\mathbf{y}} + a_z \hat{\mathbf{z}}. \quad (5.45)$$

In Section 2.3 the transformation between the component of $\mathbf{a}$ on the basis of eigenvectors of $\underline{\underline{\epsilon}}(\mathbf{r})$ and the Cartesian global basis was written as Eq. (2.25) with a transformation matrix U . It follows from Eqs. (5.19), (5.20), (5.41) and (5.42) that we have

$$U(\mathbf{r}) = V(\mathbf{r})W(\mathbf{r}), \quad (5.46)$$

$$U^*(\mathbf{r}) = W^*(\mathbf{r})V^*(\mathbf{r}). \quad (5.47)$$

With respect to the *global* basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ (i.e. an orthogonal basis that is independent of position) we have:

$$\underline{\underline{\epsilon}}(\mathbf{r}) = V(\mathbf{r})\underline{\underline{\epsilon}}(\mathbf{r})V^*(\mathbf{r}), \quad (5.48)$$

where V is the transform matrix given by Eq. (5.19). V transforms the components of a vector on the local basis $\{\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3\}$ into the components on the global basis $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$. Since the local basis depends on position, the matrix V depends on $\mathbf{r}$ as well.

We can make the following observations.

- (a) The permittivity tensor $\underline{\underline{\epsilon}}$ is hermitian and therefore, its eigenvalues ϵ_ξ , ϵ_η and ϵ_ζ are all real. This means that the medium does not absorb electromagnetic energy.
- (b) The $\hat{\mathbf{e}}_3(\mathbf{r})$ -direction, which is perpendicular to the local plane of polarization of the recording field, is always principal axis of the tensor $\underline{\underline{\epsilon}}(\mathbf{r})$.
- (c) If $\kappa_c = 0$, the tensor $\underline{\underline{\epsilon}}(\mathbf{r})$ is diagonal. The orthonormal basis of eigenvectors $\{\hat{\xi}(\mathbf{r}), \hat{\eta}(\mathbf{r}), \hat{\zeta}(\mathbf{r})\}$ can then everywhere be chosen identical to the local basis $\{\hat{\mathbf{e}}_1(\mathbf{r}), \hat{\mathbf{e}}_2(\mathbf{r}), \hat{\mathbf{e}}_3(\mathbf{r})\}$ defined by the local state of polarization of the recording beam.

5.3 Special cases

5.3.1 Recording of two orthogonal circular polarizations

Interference of two orthogonally circularly polarized beams has been studied by many researchers and is the most well established case in polarization recording, especially for small recording angles between the two recording beams. Here we discuss a more general formalism for this classical case without using the small angle approximation.

Suppose that the two incident waves are orthogonally circularly polarized with the same amplitude. Then in Eqs. (5.3), (5.4):

$$\left. \begin{aligned} a_S^- &= a_S^+ = a_S, \\ a_P^- &= -i a_S, a_P^+ = i a_S, \end{aligned} \right\} \quad (5.49)$$

with a_S real. Substitution into Eq. (5.5) for the total field of the two waves gives:

$$\mathbf{E}(\mathbf{r}) = 2a_S \begin{pmatrix} -\sin(k_x x) \frac{k_z}{k_0 n} \\ \cos(k_x x) \\ -i \cos(k_x x) \frac{k_x}{k_0 n} \end{pmatrix} e^{ik_z z}, \quad (5.50)$$

and

$$\mathcal{E}(\mathbf{r}, t) = \text{Re} \left[\begin{pmatrix} -2a_S \sin(k_x x) \frac{k_z}{k_0 n} \\ 2a_S \cos(k_x x) \\ -i2a_S \cos(k_x x) \frac{k_x}{k_0 n} \end{pmatrix} e^{i(k_z z - \omega t)} \right]. \quad (5.51)$$

Eqs. (5.7) and (5.8) become:

$$\mathbf{p}(\mathbf{r}) = 2|a_S| \begin{pmatrix} -\frac{k_z}{k_0 n} \sin(k_x x) \cos(k_z z) \\ \cos(k_x x) \cos(k_z z) \\ \frac{k_x}{k_0 n} \cos(k_x x) \sin(k_z z) \end{pmatrix}, \quad (5.52)$$

$$\mathbf{q}(\mathbf{r}) = 2|a_S| \begin{pmatrix} -\frac{k_z}{k_0 n} \sin(k_x x) \sin(k_z z) \\ \cos(k_x x) \sin(k_z z) \\ -\frac{k_x}{k_0 n} \cos(k_x x) \cos(k_z z) \end{pmatrix}. \quad (5.53)$$

As has been shown in Section 5.2.1, the state of polarization is independent of z . We may therefore choose $z = 0$. Then

$$\mathbf{p}(x, 0) = 2|a_S| \begin{pmatrix} -\frac{k_z}{k_0 n} \sin(k_x x) \\ \cos(k_x x) \\ 0 \end{pmatrix}, \quad \mathbf{q}(x, 0) = 2|a_S| \begin{pmatrix} 0 \\ 0 \\ -\frac{k_x}{k_0 n} \cos(k_x x) \end{pmatrix}. \quad (5.54)$$

Hence

$$\mathbf{p}(x, 0) \cdot \mathbf{q}(x, 0) = 0, \quad (5.55)$$

and

$$\mathbf{p}(x, 0) \cdot \mathbf{p}(x, 0) = 4|a_S|^2 \left[\cos^2(k_x x) + \frac{k_z^2}{k_0^2 n^2} \sin^2(k_x x) \right], \quad (5.56)$$

$$\mathbf{q}(x, 0) \cdot \mathbf{q}(x, 0) = 4|a_S|^2 \frac{k_x^2}{k_0^2 n^2} \cos^2(k_x x). \quad (5.57)$$

Hence

$$\tan 2\psi = -\frac{2\mathbf{p}(x, 0) \cdot \mathbf{q}(x, 0)}{\mathbf{p}(x, 0) \cdot \mathbf{p}(x, 0) - \mathbf{q}(x, 0) \cdot \mathbf{q}(x, 0)} = 0. \quad (5.58)$$

and thus:

$$\psi = 0, \quad (5.59)$$

$$\mathbf{a}(x) = \mathbf{p}(x, 0), \quad \mathbf{b}(x) = \mathbf{q}(x, 0). \quad (5.60)$$

Then the local basis $\{\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3\}$ is

$$\hat{\mathbf{e}}_1(x) = \begin{pmatrix} \frac{-k_z \sin(k_x x)}{\sqrt{k_z^2 + \cos^2(k_x x) k_x^2}} \\ \frac{k_0 n \cos(k_x x)}{\sqrt{k_z^2 + \cos^2(k_x x) k_x^2}} \\ 0 \end{pmatrix}, \quad (5.61)$$

$$\hat{\mathbf{e}}_2(x) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \quad (5.62)$$

and

$$\begin{aligned} \hat{\mathbf{e}}_3(x) &= \hat{\mathbf{e}}_1(x) \times \hat{\mathbf{e}}_2(x) \\ &= \begin{pmatrix} \frac{-k_0 n \cos(k_x x)}{\sqrt{k_z^2 + \cos^2(k_x x) k_x^2}} \\ \frac{-k_z \sin(k_x x)}{\sqrt{k_z^2 + \cos^2(k_x x) k_x^2}} \\ 0 \end{pmatrix}. \end{aligned} \quad (5.63)$$

Obviously

$$|E_1(x, 0)| = |\mathbf{E}(x) \cdot \hat{\mathbf{e}}_1(x, 0)| = 2|a_S| \sqrt{\cos^2(k_x x) + \frac{k_z^2}{k_0^2 n^2} \sin^2(k_x x)} = |\mathbf{a}(x)| = |\mathbf{p}(x, 0)| \quad (5.64)$$

$$|E_2(x, 0)| = |\mathbf{E}(x) \cdot \hat{\mathbf{e}}_2(x, 0)| = 2|a_S| \cos(k_x x) \frac{k_x}{k_0 n} = |\mathbf{b}(x)| = |\mathbf{q}(x, 0)|. \quad (5.65)$$

Furthermore

$$|E_1(\mathbf{r})|^2 = 4|a_S|^2 \left[\sin^2(k_x x) \frac{k_z^2}{k_0^2 n^2} + \cos^2(k_x x) \right], \quad (5.66)$$

$$|E_2(\mathbf{r})|^2 = 4|a_S|^2 \cos^2(k_x x) \frac{k_x^2}{k_0^2 n^2}, \quad (5.67)$$

$$|E_1(\mathbf{r})||E_2(\mathbf{r})| = 4|a_S|^2 |\cos(k_x x)| \frac{k_x}{k_0 n} \sqrt{\sin^2(k_x x) \frac{k_z^2}{k_0^2 n^2} + \cos^2(k_x x)}. \quad (5.68)$$

Substitution of the above quantities into Eq. (1.22) gives the Stokes parameters for this specific case:

$$S_0(x) = 4|a_S|^2 \left[\left(1 + \frac{k_x^2}{k_0^2 n^2}\right) \cos^2(k_x x) + \frac{k_z^2}{k_0^2 n^2} \sin^2(k_x x) \right], \quad (5.69)$$

$$S_1(x) = 4|a_S|^2 \frac{k_z^2}{k_0^2 n^2}, \quad (5.70)$$

$$S_3(x) = 8|a_S|^2 |\cos(k_x x)| \frac{k_x}{k_0 n} \sqrt{\sin^2(k_x x) \frac{k_z^2}{k_0^2 n^2} + \cos^2(k_x x)}. \quad (5.71)$$

It is seen that when $k_x x = \pi/2 + l\pi$, l integer, $S_3(x) = 0$, so that the state of polarization is then linear along the x -direction because in this case $\hat{\mathbf{e}}_1(x) = \hat{\mathbf{x}}$. Furthermore, S_1 is independent of x . The ellipticity is given by:

$$e(x) = \frac{|\cos(k_x x)| \frac{k_x}{k_0 n}}{\sqrt{\cos^2(k_x x) + \frac{k_z^2}{k_0^2 n^2} \sin^2(k_x x)}}. \quad (5.72)$$

The polarization modulation over one period of the interference pattern is shown in Fig. 5.3(a), (c) and (e), respectively for small recording angle (2.5°) and large recording angle (10° and 15°). For each case the intensity modulation and the ellipticity of the polarization are plotted as function of position along the x -axis over one period in Fig. 5.3(b), (d) and (f).

For small angles, $k_x \ll k_0 n$ and hence the ellipticity is small. This means that the ellipse is strongly elongated in the direction of $\hat{\mathbf{e}}_1(x)$. Since for small angle: $k_z \approx k_0 n$, it follows from the expression for $\hat{\mathbf{e}}_1(x)$, that $\hat{\mathbf{e}}_1(x)$ rotates over 2π in the (x, y) -plane around the z -axis, when x varies over one period $p = 2\pi/k_x$. Since the material is only sensitive to the intensity of the light, the period of the formed grating is $P = p/2 = \pi/k_x$. For small recording angle, $S_0(x) \approx 4a_s^2$, hence the intensity is almost independent of x . These results are confirmed by Fig. 5.3(a), (b).

In the case of $\theta_{\text{record.}} = 2.5^\circ$, the polarization state (see Fig. 5.3(a)) seems linear at each point over one modulation period. On the other hand, the ellipticity curve in Fig. 5.3(b) shows that in fact the field is elliptically polarized in most of the points. However, the ellipticity is so small, $e < 0.05$, that an approximation that polarization is linear everywhere is justified. The modulation follows the change of orientation of the local basis. It is seen that over one period, the local basis rotates over 180° around the z -axis. The intensity is almost constant. Notice that in Fig. 5.3(b), (d) and (f) the scales are the same. Thus it is easy to see that when the angle of recording becomes bigger, the amplitude of modulation in both intensity and polarization gets larger. It can also be seen from (c) and (e), that at many points in one modulation period, the polarization state is clearly elliptic.

The photo-induced dielectric permittivity tensor on the local basis is obtained by substituting the electric field into Eq. (5.32). When the recording angle is small, we use the approximation: $k_x \approx 0$ and $k_z \approx k_0 n$. Then, $E_2 \approx 0$ which means that the polarization is approximately linear and remains in the (x, y) -plane. The interference pattern of the two orthogonal circular polarizations is linearly polarized everywhere with the vibration direction of the electric field rotating over 180° over a period. Since $E_1 E_2 \approx 0$, the circular birefringence is equal to zero and the off diagonal elements vanish. The dielectric permittivity on the local basis is thus simplified to,

$$\underline{\underline{\tilde{\epsilon}}}(x) = \begin{pmatrix} \epsilon^i + \kappa_{\parallel} |E_1(x, 0)|^2 & 0 & 0 \\ 0 & \epsilon^i + \kappa_{\perp} |E_1(x, 0)|^2 & 0 \\ 0 & 0 & \epsilon^i + \kappa_{\perp} |E_1(x, 0)|^2 \end{pmatrix} \quad (5.73)$$

The material is then inhomogeneously uniaxial with optic axis parallel to the local $\hat{\mathbf{e}}_1$ -axis, i.e. the optic axis is rotating in the (x, y) -plane. For larger recording angles this

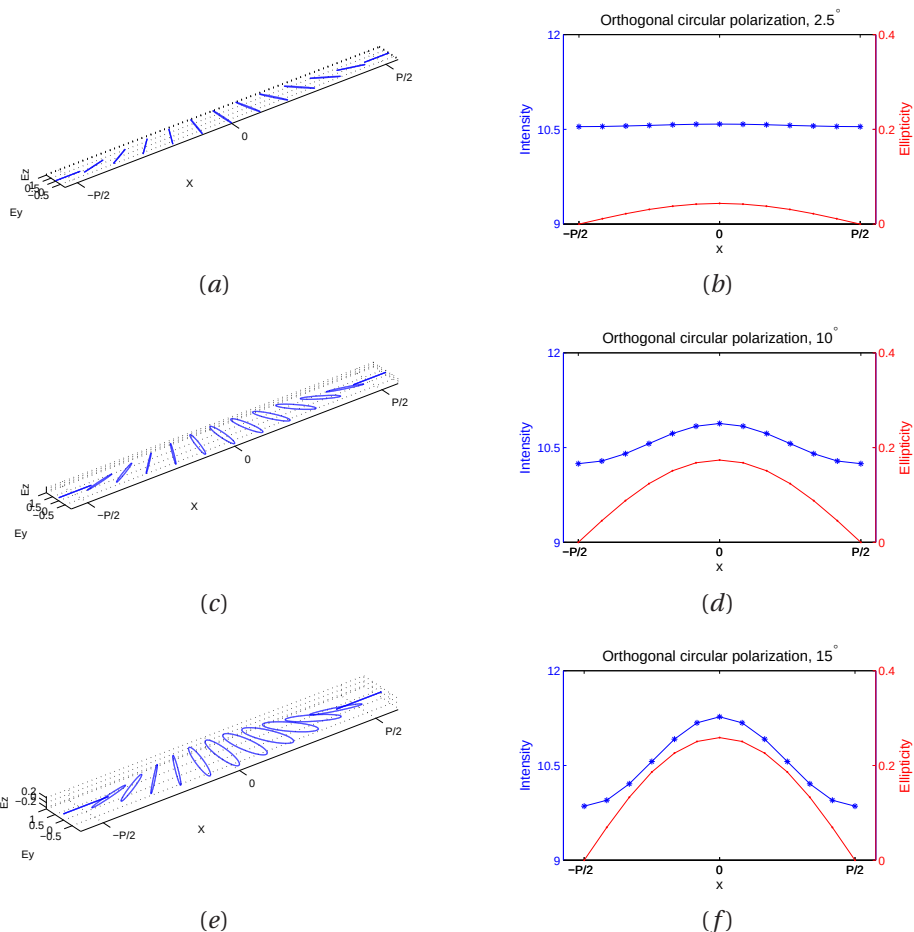


Figure 5.3: (a) Polarization states of interference of two orthogonally circularly polarized plane waves with recording angle 2.5° ; (b) Polarization ellipticity and total intensity modulation over one period of the pattern in (a); (c) Polarization states of interference of two orthogonally circularly polarized plane waves with recording angle 10° ; (d) Polarization ellipticity and total intensity modulation over one period of the pattern in (c); (e) Polarization states of interference of two orthogonally circularly polarized plane waves with recording angle 15° ; (f) Polarization ellipticity and total intensity modulation over one period of the pattern in (e).

does not hold and the permittivity tensor $\underline{\epsilon}(\mathbf{r})$ follows by substituting the total electric field into Eq. (5.32). In the next chapter we shall study the diffraction efficiencies of the thus obtained anisotropic gratings.

5.3.2 Recording of two orthogonal linear polarizations

For two orthogonally linearly polarized waves, we assume that one is s- and the other is p-polarized with equal intensity:

$$\left. \begin{aligned} a_S^+ &= a, & a_P^+ &= 0; \\ a_S^- &= 0, & a_P^- &= a. \end{aligned} \right\} \quad (5.74)$$

The total field of the two waves is

$$\mathbf{E}(\mathbf{r}) = a \begin{pmatrix} \frac{k_z}{k_0 n} e^{-ik_x x} \\ e^{ik_x x} \\ \frac{k_x}{k_0 n} e^{-ik_x x} \end{pmatrix} e^{ik_z z}. \quad (5.75)$$

Eqs. (5.7) and (5.8) become

$$\mathbf{p}(\mathbf{r}) = a \begin{pmatrix} \frac{k_z}{k_0 n} \cos(-k_x x + k_z z) \\ \cos(k_x x + k_z z) \\ \frac{k_x}{k_0 n} \cos(-k_x x + k_z z) \end{pmatrix}, \quad (5.76)$$

$$\mathbf{q}(\mathbf{r}) = a \begin{pmatrix} \frac{k_z}{k_0 n} \sin(-k_x x + k_z z) \\ \sin(k_x x + k_z z) \\ \frac{k_x}{k_0 n} \sin(-k_x x + k_z z) \end{pmatrix}. \quad (5.77)$$

In particular, when $z = 0$:

$$\mathbf{p}(x, 0) = a \begin{pmatrix} \frac{k_z}{k_0 n} \\ 1 \\ \frac{k_x}{k_0 n} \end{pmatrix} \cos(k_x x), \quad \mathbf{q}(x, 0) = a \begin{pmatrix} -\frac{k_z}{k_0 n} \\ 1 \\ -\frac{k_x}{k_0 n} \end{pmatrix} \sin(k_x x), \quad (5.78)$$

and

$$\tan 2\psi = -\frac{2\mathbf{p}(x, 0) \cdot \mathbf{q}(x, 0)}{\mathbf{p}(x, 0) \cdot \mathbf{p}(x, 0) - \mathbf{q}(x, 0) \cdot \mathbf{q}(x, 0)} = 0. \quad (5.79)$$

So that $\psi = 0$. Hence

$$\mathbf{a}(x) = \mathbf{p}(x, 0), \quad \mathbf{b}(x) = \mathbf{q}(x, 0). \quad (5.80)$$

The local basis is thus obtained as well,

$$\left. \begin{aligned} \hat{\mathbf{e}}_1(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{k_z}{k_0 n} \\ 1 \\ \frac{k_x}{k_0 n} \end{pmatrix}, \\ \hat{\mathbf{e}}_2(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{k_z}{k_0 n} \\ 1 \\ -\frac{k_x}{k_0 n} \end{pmatrix}, \\ \hat{\mathbf{e}}_3(x) &= \hat{\mathbf{e}}_1(x) \times \hat{\mathbf{e}}_2(x) = \begin{pmatrix} -\frac{k_x}{k_0 n} \\ 0 \\ \frac{k_z}{k_0 n} \end{pmatrix}, \end{aligned} \right\} \quad \text{if } \cos^2(k_x x) \geq \sin^2(k_x x), \quad (5.81a)$$

$$\left. \begin{aligned} \hat{\mathbf{e}}_1(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{k_z}{k_0 n} \\ 1 \\ -\frac{k_x}{k_0 n} \end{pmatrix}, \\ \hat{\mathbf{e}}_2(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{k_z}{k_0 n} \\ -1 \\ -\frac{k_x}{k_0 n} \end{pmatrix}, \\ \hat{\mathbf{e}}_3(x) &= \hat{\mathbf{e}}_1(x) \times \hat{\mathbf{e}}_2(x) = \begin{pmatrix} -\frac{k_x}{k_0 n} \\ 0 \\ \frac{k_z}{k_0 n} \end{pmatrix}, \end{aligned} \right\} \quad \text{if } \cos^2(k_x x) < \sin^2(k_x x). \quad (5.81b)$$

Then the electric field components on the local basis satisfy:

$$\left. \begin{aligned} |E_1(x, 0)| &= |\mathbf{E}(x, 0) \cdot \hat{\mathbf{e}}_1(x)| = \sqrt{2}a |\cos(k_x x)|, \\ |E_2(x, 0)| &= |\mathbf{E}(x, 0) \cdot \hat{\mathbf{e}}_2(x)| = \sqrt{2}a |\sin(k_x x)|, \end{aligned} \right\} \quad \text{if } \cos^2(k_x x) \geq \sin^2(k_x x), \quad (5.82)$$

$$\left. \begin{aligned} |E_1(x, 0)| &= |\mathbf{E}(x, 0) \cdot \hat{\mathbf{e}}_1(x)| = \sqrt{2}a |\sin(k_x x)|, \\ |E_2(x, 0)| &= |\mathbf{E}(x, 0) \cdot \hat{\mathbf{e}}_2(x)| = \sqrt{2}a |\cos(k_x x)|, \end{aligned} \right\} \quad \text{if } \cos^2(k_x x) < \sin^2(k_x x). \quad (5.83)$$

Substitution the above quantities into Eqs. (1.22) results in the Stokes parameters for this specific case:

$$S_0(x) = |E_1(x, 0)|^2 + |E_2(x, 0)|^2 = 2a^2, \quad (5.84a)$$

$$S_1(x) = |E_1(x, 0)|^2 - |E_2(x, 0)|^2 = 2a^2 \cos(2k_x x), \quad (5.84b)$$

$$S_3(x) = 2|E_1(x, 0)||E_2(x, 0)| = 2a^2 \sin(2k_x x). \quad (5.84c)$$

The ellipticity is

$$e(x) = \begin{cases} |\tan(k_x x)| & \text{if } |\tan(k_x x)| \leq 1, \\ \left| \frac{1}{\tan(k_x x)} \right| & \text{if } |\tan(k_x x)| > 1. \end{cases} \quad (5.85)$$

So the state of polarization changes periodically from linear to circular, with all possible values of ellipticity occur during one period of x . If $\sin(k_x x) = 0$, we have linear polarization, if $\cos(2k_x x) = 0$, the state of polarization is circular. Because $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are independent of x (except that they flip when $|\cos(k_x x)| = |\sin(k_x x)|$), the axes of

the polarization ellipse are the same for all x . Furthermore, the intensity is independent of x and does not depend on the recording angle. This is confirmed in Fig. 5.4(b), (d) and (f). This means that the intensity modulation is totally absent in the interference of two orthogonally linearly polarized waves. Notice that this holds for the most general case of which the recording angle can be arbitrarily large.

We calculated the ellipticity of the polarization modulation for three recording angles 2.5° , 10° and 15° and the results are plotted in Fig. 5.4. Fig. 5.4(a), (c) and (e) reveal that the plane of polarization does not lie in x, y -plane for large recording angles. This indicates that the local basis rotates as is most clearly illustrated in Fig. 5.4(e) for the angle of 15° . The interference patterns formed by two orthogonal linear polarizations are thus more complex for larger recording angles.

With the obtained local electric field, we can express the photo-induced birefringence on the local basis as,

$$\underline{\tilde{\epsilon}}(\mathbf{r}) = \begin{pmatrix} \tilde{\epsilon}_{11} & \tilde{\epsilon}_{12} & 0 \\ \tilde{\epsilon}_{21} & \tilde{\epsilon}_{22} & 0 \\ 0 & 0 & \tilde{\epsilon}_{33} \end{pmatrix}, \quad (5.86)$$

with

$$\begin{aligned} \tilde{\epsilon}_{11} &= \epsilon^i + \kappa_{\parallel} 2a^2 \cos^2(k_x x) + \kappa_{\perp} 2a^2 \sin^2(k_x x), \\ \tilde{\epsilon}_{12} &= -i\kappa_c 2a^2 \sin(2k_x x), \\ \tilde{\epsilon}_{21} &= \tilde{\epsilon}_{12}^* = i\kappa_c 2a^2 \sin(2k_x x), \\ \tilde{\epsilon}_{22} &= \epsilon^i + \kappa_{\perp} 2a^2 \cos^2(k_x x) + \kappa_{\parallel} 2a^2 \sin^2(k_x x) \\ \tilde{\epsilon}_{33} &= \epsilon^i + \kappa_{\perp} 2a^2. \end{aligned}$$

When the recording angle θ is small ($\theta \leq 5^\circ$), k_x is very small and then the local basis becomes approximately,

$$\hat{\mathbf{e}}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{\mathbf{e}}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{\mathbf{e}}_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (5.87)$$

When the recording angle θ is small, the plane of polarization coincides in good approximation with the (x, y) -plane.

5.4 Representations of states of polarization

The state of polarization of an arbitrary electric field is fixed by 5 parameters:

- *two angles* to fix the direction of the unit normal $\hat{\mathbf{e}}_3$ of the polarization ellipse, i.e. the direction of rotation of the electric field vector;
- *one angle* to fix the direction of one of the two axes of the polarization ellipse, e.g. $\hat{\mathbf{e}}_1$ which is parallel to the long axis of the polarization ellipse;

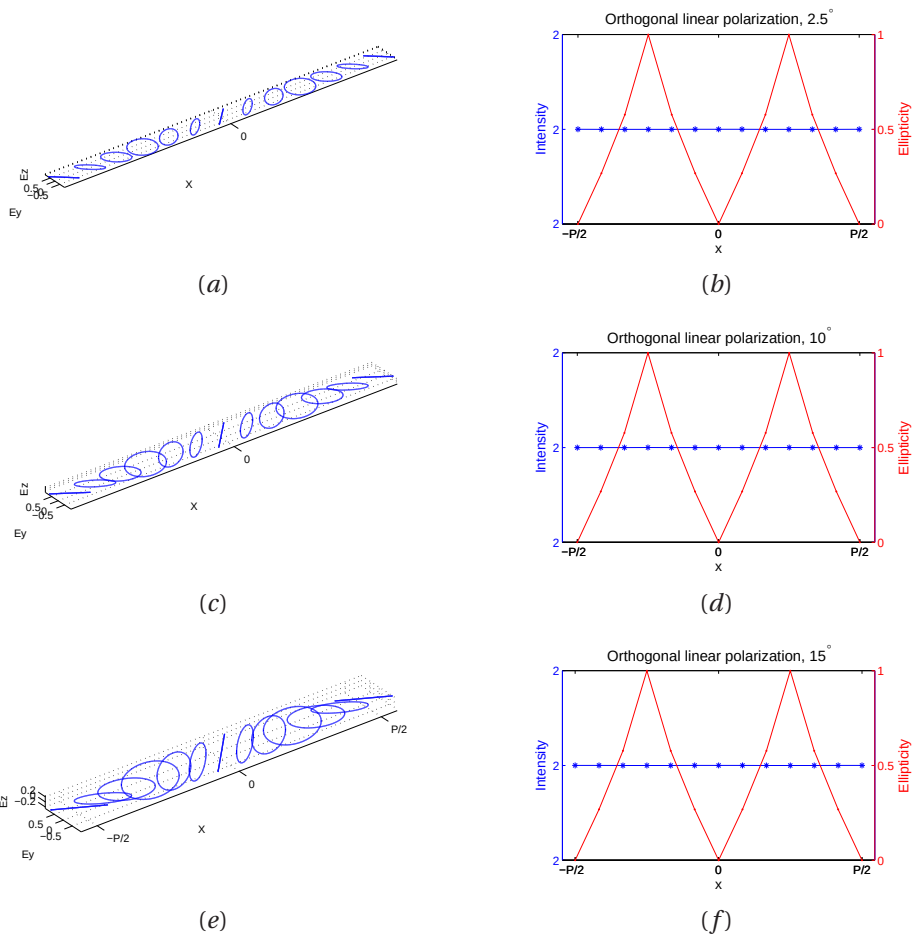


Figure 5.4: (a) Polarization states of interference of two orthogonally linearly polarized plane waves with recording angle 2.5° ; (b) Polarization ellipticity and total intensity modulation over one period of the pattern in (a); (c) Polarization states of interference of two orthogonally linearly polarized plane waves with recording angle 10° ; (d) Polarization ellipticity and total intensity modulation over one period of the pattern in (c); (e) Polarization states of interference of two orthogonally linearly polarized plane waves with recording angle 15° ; (f) Polarization ellipticity and total intensity modulation over one period of the pattern in (e).

- *ellipticity*;
- *intensity*.

The interference pattern of two plane waves is a function of x as we discussed before. To illustrate the whole polarization modulation over one period, we need a 5-dimensional space to describe each polarization state in space. The conventional representation method, such as the Poincaré sphere, illustrates a polarization state in three dimensional space by using 3 parameters, and it requires that two of the five parameters are negligible or that two parameters are the same for all the polarization states. In a general case where the angle of recording is arbitrarily large, it is impossible to fulfill these criteria. However, it is possible to plot all the polarization states that occur during one cycle of x in the Poincaré sphere when the recording angle θ_{record} is small, as was shown by Van Heesch in his Ph.D thesis [66].

5.4.1 Small recording angles and the Poincaré sphere

Suppose the angle of recording is small, so that $k_x \approx 0$ and $k_z \approx k_0 n$. Then Eqs. (5.7), (5.8) become approximately:

$$\mathbf{p}(\mathbf{r}) \approx \begin{pmatrix} |a_p^-| \cos[\text{Arg}(a_p^-) - k_x x] + |a_p^+| \cos[\text{Arg}(a_p^+) + k_x x] \\ |a_s^-| \cos[\text{Arg}(a_s^-) - k_x x] + |a_s^+| \cos[\text{Arg}(a_s^+) + k_x x] \\ 0 \end{pmatrix}; \quad (5.88)$$

$$\mathbf{q}(\mathbf{r}) \approx \begin{pmatrix} |a_p^-| \sin[\text{Arg}(a_p^-) - k_x x] + |a_p^+| \sin[\text{Arg}(a_p^+) + k_x x] \\ |a_s^-| \sin[\text{Arg}(a_s^-) - k_x x] + |a_s^+| \sin[\text{Arg}(a_s^+) + k_x x] \\ 0 \end{pmatrix}. \quad (5.89)$$

The z -components are zero in both equations. The electric field can now be written as:

$$\mathbf{E}(x) = E_x(x)\hat{\mathbf{x}} + E_y(x)\hat{\mathbf{y}}. \quad (5.90)$$

This means that in good approximation the electric field in all points does not have a z -component. If the phase difference between E_x and E_y is δ , the Stokes parameters with respect to the global $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$ basis are given by:

$$S_0(x) = |E_x(x)|^2 + |E_y(x)|^2, \quad (5.91a)$$

$$S_1(x) = |E_x(x)|^2 - |E_y(x)|^2, \quad (5.91b)$$

$$S_2(x) = 2|E_x(x)| |E_y(x)| \cos \delta(x), \quad (5.91c)$$

$$S_3(x) = 2|E_x(x)| |E_y(x)| \sin \delta(x). \quad (5.91d)$$

Only three of them are independent since they are related by

$$S_0^2 = S_1^2 + S_2^2 + S_3^2. \quad (5.92)$$

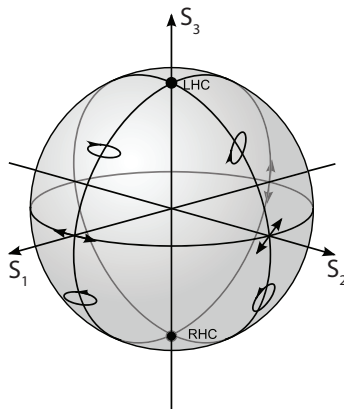


Figure 5.5: Poincaré representation of the state of polarization of a monochromatic wave. (The Poincaré sphere)

Then the *Poincaré sphere* is defined on the Cartesian basis $\{\hat{x}, \hat{y}, \hat{z}\}$ as shown in Fig. 5.5.

On the Poincaré sphere, the north pole indicates the left-handed circular polarization (LHC), and the south pole indicates the right-handed circular polarization (RHC). The states of polarization on the equator are linear and rotate by the azimuthal angle π when completing a whole circle. At the cross-point with the positive S_1 -axis, the linear polarization is along the x -axis and at the cross-point with the positive S_2 -axis, the linear polarization is oblique to the x -axis of $\pi/4$. The states of polarization at all the other points of the sphere are elliptic.

The interference patterns of two orthogonally polarized beams at small angle thus can be represented on the Poincaré sphere using the global basis $\{\hat{x}, \hat{y}, \hat{z}\}$. For the two special cases, namely, the interference of two orthogonally circularly and linear polarizations at small angles, the states of polarization over one modulation period are depicted on the Poincaré sphere in Fig. 5.6.

5.4.2 Large recording angles

In order to use the Poincaré sphere, all the polarization states have to be defined on the same basis of which the plane of polarization is parallel to the plane formed by the coordinates, e.g. the (x, y) -plane. When the recording angle is large, the Poincaré sphere is not applicable, since the plane of polarization rotates in 3-D space due to the presence of the z -component of the electric field. In this case, the local basis $\{\hat{e}_1(x), \hat{e}_2(x), \hat{e}_3(x)\}$ is defined of which $\hat{e}_1(x)$ and $\hat{e}_2(x)$ are parallel to the major and minor axes, respectively, of the polarization ellipse. This local basis is different for different points in space. We take a most general case to illustrate this, for instance,

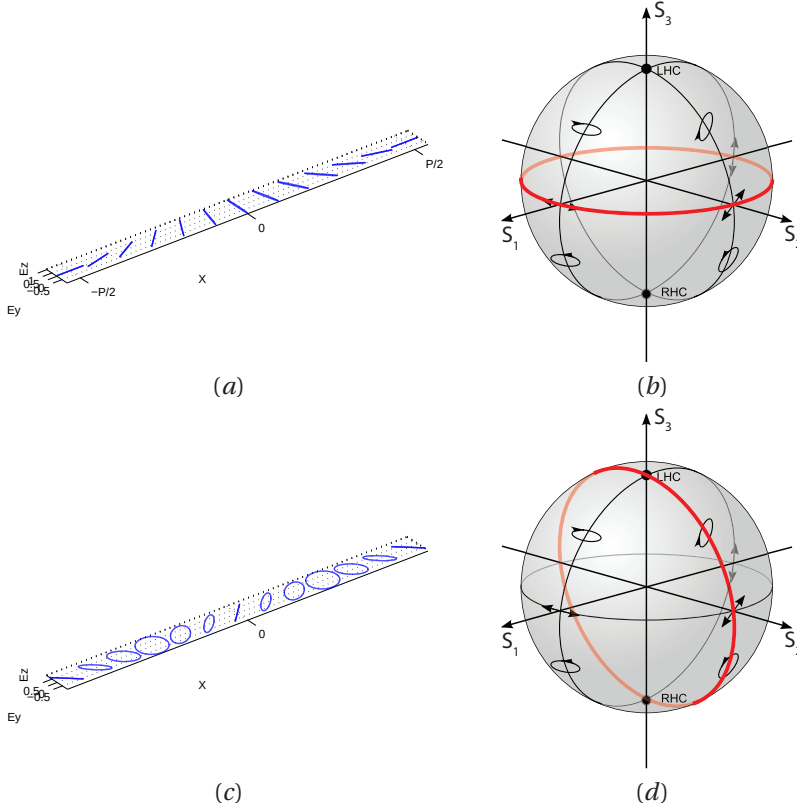


Figure 5.6: The states of polarization of interference of two orthogonal polarized waves over one period are represented by complete circles (the thick lines) on the Poincaré sphere. Top row: interference of two orthogonal circular polarizations; lower row: interference of two orthogonal linear polarizations.

the interference of two elliptically polarized beams. Suppose the two beams are:

$$\begin{aligned} a_S^+ &= 1, & a_P^+ &= i; \\ a_S^- &= 1.5, & a_P^- &= -2i. \end{aligned} \quad (5.93)$$

The interference pattern formed by these two beams is shown in Fig. 5.7 right bottom panel. From the plot we can see that over one period, the states of polarization are elliptic everywhere. The ellipticity is shown in the figure on the upper right. In this figure the intensity is also plotted. Besides the two parameters, ellipticity and intensity, the other three parameters namely, *two angles* to fix the direction of $\hat{\mathbf{e}}_3$ and *one angle* to fix the direction of $\hat{\mathbf{e}}_1$ are illustrated on the unit sphere shown in Fig. 5.7 at the left. For each state of polarization plotted in the right bottom panel, the $\hat{\mathbf{e}}_1$ major axis of the polarization ellipse is plotted as a unit vector in red color on the Cartesian basis, and $\hat{\mathbf{e}}_3$, which is normal to the plane of polarization, is plotted as a blue unit

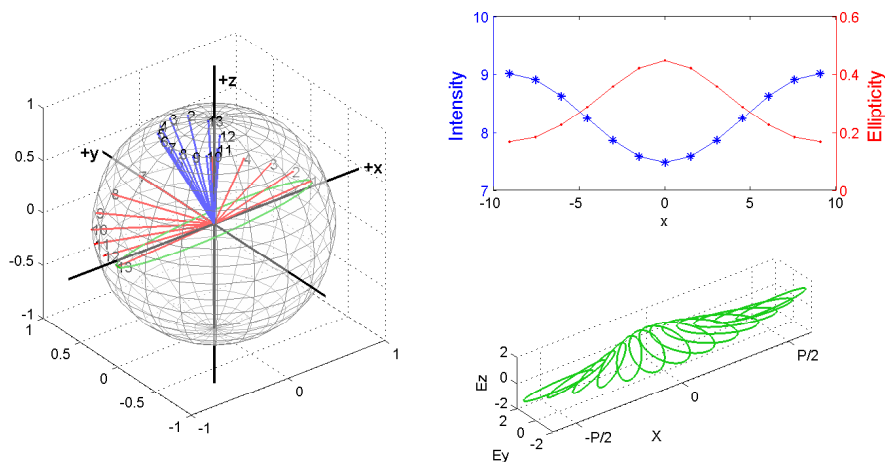


Figure 5.7: Polarization representation of interference pattern of two elliptically polarized waves.

vector. All of the vectors are on the unit sphere centered in the origin.

The figures show clearly, that the major axis of the polarization ellipse rotates 180° over one period of modulation, while the plane of polarization also rotates, as is clear from the blue vectors. Thus the five parameters to determine the state of polarization of the vector field are all displayed using the combination of the left and upper right figures.

For two orthogonally circularly and linearly polarized beams at recording angle of 10° , the states of polarization of interference are shown in Fig. 5.8(a) and (c) respectively. The two unit vectors $\hat{e}_1$ and $\hat{e}_3$ are plotted in Fig. 5.8(b) and (d) for each state of polarization plotted in (a) and (c) accordingly.

Together with the plots of intensity and ellipticity in Fig. 5.3(d) and Fig. 5.4(d), the polarization representations are complete for the two special cases.

From Fig. 5.8(a), we can see that when the recording angle is large, the states of polarization of the interference of two OC beams are elliptic at some points in space. The major axis of the polarization ellipse which is plotted as the red vectors in the sphere in Fig. 5.8(b), rotates in the (x, y) -plane. And the normal to the polarization plane also rotates in the same (x, y) -plane. Thus the two vectors overlap at some points in the plot. The interference pattern of the two OL beams contains all the variation from linear polarization to circular polarization. As indicated in the sphere in Fig. 5.8(d), the normal to the plane of polarization (blue lines) is always parallel to one direction deviating from the z -direction. The direction of $\hat{e}_3$ flips when the state of polarization is linear. The major axis of the polarization ellipse indicated by the red unit vectors is fixed in a few position in the space.

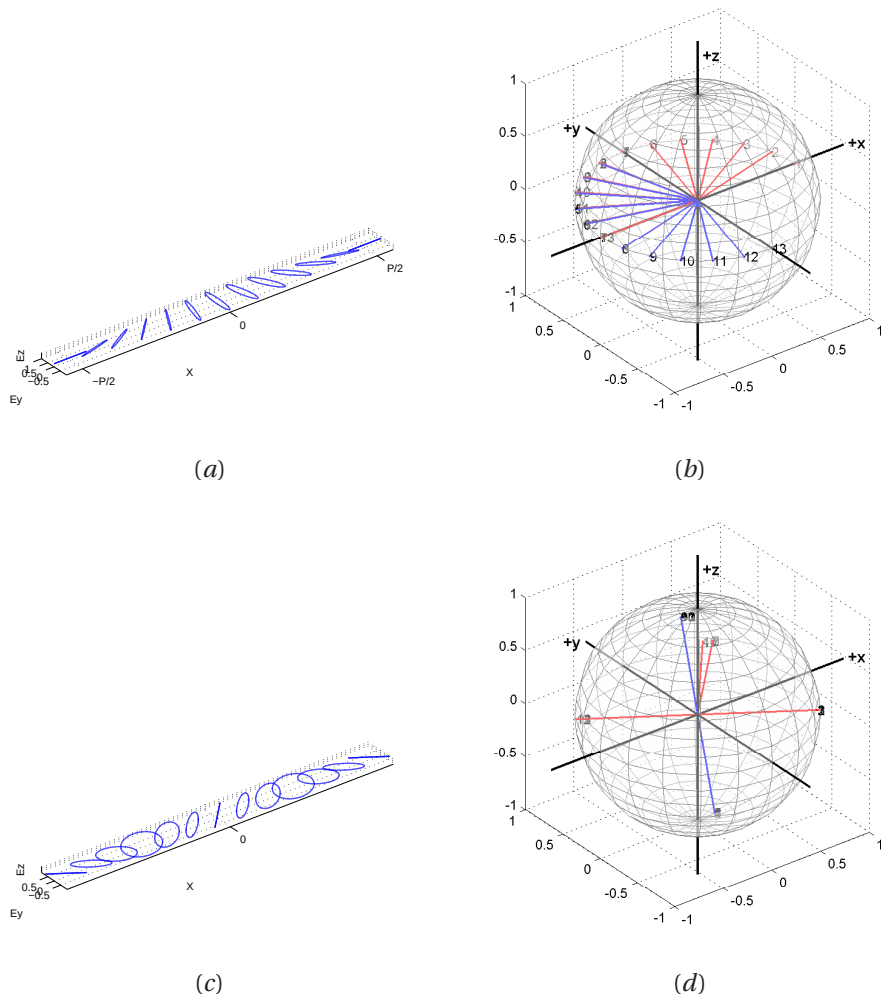


Figure 5.8: (a) The interference pattern of two orthogonally circularly polarized beams for recording angle of 10° ; (b) the major axis of the polarization ellipse $\hat{\mathbf{e}}_1$ (red unit vectors) and the normal $\hat{\mathbf{e}}_3$ to the plane of polarization of the interference pattern in (a). (c) The interference pattern of two orthogonally linearly polarized beams for recording angle of 10° ; (d) the major axis of the polarization ellipse $\hat{\mathbf{e}}_1$ (red unit vectors) and the normal $\hat{\mathbf{e}}_3$ to the plane of polarization of the interference pattern in (c).

5.5 Photoanisotropic materials and recording procedure

5.5.1 Azobenzene polymers

If materials which are sensitive to variation in intensity and polarization are used to record the interference patterns formed by the two beams, information of the modulation of both the intensity and polarization can be stored. A typical material of which the linear photoanisotropic properties have been demonstrated experimentally is azobenzene polymer [70, 62, 71, 72]. When the material is irradiated by linearly polarized light, linear anisotropy is introduced. It has been reported [62] that the effect is caused by trans-cis isomerization (see Fig. 5.9) and reorientation of the azobenzene moieties and parts of the polymer chains. Depending on the photoisomerization, the azobenzene group tends to line up in a direction perpendicular to the direction of polarization of the irradiation. The photo-induced linear birefrin-

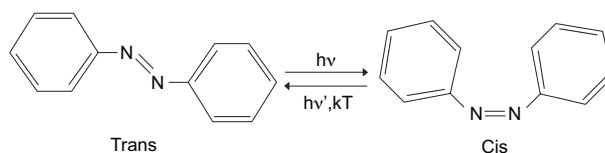


Figure 5.9: Isomers of azobenzene polymer.

gence is approximately $\Delta n_{lin} = 0.025$ [58, 73, 63]. Circular anisotropy has also been reported for polarization grating recorded in films of side-chain liquid-crystalline azobenzene polyester [62, 74, 73]. The reported circular birefringence Δn_{cir} is about 0.004.

Recording polarization gratings in azobenzene polymer thin film is a rather simple process. Interference patterns can be written directly into the polymer film. Many research groups have successfully fabricated polarization grating with this material [62, 75]. However it is still challenging to record a uniform polarization grating structure because of the volume thickness of the material. It has been reported by Nikolova [76] that when elliptically polarized light propagates through photobirefringent material (and these are the materials used for polarization holography), its polarization azimuth (the direction of the major axis of the polarization ellipse) is rotated along the depth of the material, i.e. as function of z . This rotation is proportional to the photo-induced birefringence Δn and is significant in materials with large value of Δn , in which efficient polarization gratings can be written. Therefore, the expression for the Stokes parameters of the interference of two recording plane waves which have been computed above are valid only for the front surface of the recording material. The Stokes parameters change significantly along z -axis. Therefore, the tensor that describes the induced anisotropy in the material Eq. (5.33) is also dependent on z . While the objective of our work is to understand the physical limitation and to initiate research on large angle recording, we stress here that in our analysis there is no

z -dependence. As follows from the assumptions made in Section 5.2.2, we consider only thin layer recording. This may cause discrepancies between results that we discuss in this thesis and the experimental measurements of polarization grating consisting of azobenzene polymer materials in practice. Our model is however accurate for another kind of material, namely, liquid crystals.

5.5.2 Liquid crystals

The birefringence of NLC comes from its rod- or dish-like shaped molecules (see Fig. 5.10). To align those molecules several methods can be used, such as, an orientation layer, an external electric field or a flow. In the case of polarization holography, the LC directors should rotate in a short period of μm . To achieve this, only an orientation layer can be used.

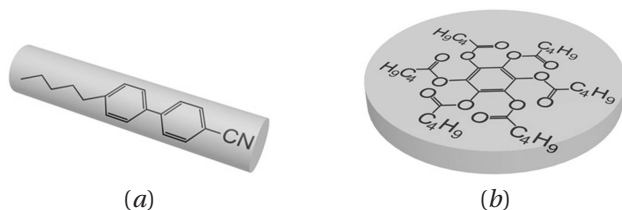


Figure 5.10: (a) 5CB (4-pentyl-4'-cyanobiphenyl): rod-like LC molecule; (b) benzene-hexa-n-alkanoate derivatives: dish-like LC molecule. The illustrations are taken from [77].

Linear photo polymer (LPP) is well known for its UV dichroism and optical anisotropy. Its ability to record linearly polarized light made it possible to obtain planar, high-resolution LC director patterns without external mechanical stress. As a photoalignment film, LPP is sensitive to linear polarization [78, 79, 80, 81, 82]. Before the recording, LPP is first spin-coated on a piece of thin glass. Upon irradiation with interference pattern, linear polarization is recorded in LPP layer locally. When the irradiation is elliptic polarization, the order parameter of the LPP is assumed to be proportional to the square of the long axis of the polarisation ellipse [83]. This order parameter is transferred to the LC, whose birefringence is proportional to it. To realize the basic concept of this anisotropically photoreactive polymer system, there are two ways for manufacture [84]. One is using a glass cell formed by two pieces of LPP coated glass, the other using just one piece of glass coated with LPP.

We first consider the assembly process of a glass cell. A glass cell is made with two pieces of glass coated with LPP with the LPP coatings facing each other. Spacers can be used to fix the thickness of the gap. To avoid misalignment of recorded patterns in the two LPP layers, polarization holography recording is performed after assembling the cell. Usually the spacer between the two pieces of glass is smaller than $10\ \mu\text{m}$. This is small enough that the recorded patterns in both LPP layers can be considered the same. After the gap is filled with LC, the LC molecules follow the patterns of

both the upper and lower LPP layers perfectly provided that the thickness of the LC bulk is relative small. However for a thick cell, the LC molecules loses stability and deformation of the LC will happen [82]. The reason is that, the tendency of a homogeneous director distribution in the center of the LC cell reduces the elastic energy of the surface anchoring. To ensure that the transition through the LC bulk is stable and uniform in the direction normal to the surface, the ratio between the thickness of the LC and the period of the modulation patterns has to be equal to or smaller than 0.4, as was reported by Sarkissian et al.

The advantage of the first method is its easiness to handle LC, and the good alignment of LC molecules because of the strength of two LPP layers, but it is not easy to control the thickness of the grating and not suited for all applications. Therefore a second method to obtain a polarization grating with an anisotropically photoreactive polymer system has been developed. First, a piece of glass with LPP spin-coated on it is prepared. Then UV polarization holographic recording takes place, and the interference patterns is recorded in the LPP. Third, on top of the LPP layer, a thin layer of LC is spin-coated with a thickness of about 200 nm. The patterns in the LPP are then automatically copied by the LC molecules. Fourth, the LC layer is polymerized to keep the recorded pattern firm and stable. One then just needs to repeat steps 3 and 4 several times to get the desired thickness. With this method, the grating thickness is adjustable but the alignment of LC molecules after several layers is more difficult than in the previous glass-cell process. Depending on application, one should choose the most suitable and convenient manufacturing method. Most commonly, the ordinary refractive index of LC, $n_o = 1.5$ and the extraordinary one $n_e = 1.7$. Thus the introduced linear birefringence is about $\Delta n_{lin} = 0.2$ which is much larger than can be realized in Azobenzene polymers.

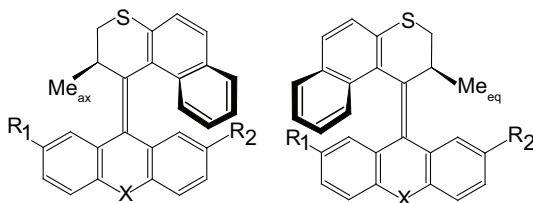


Figure 5.11: Right-handed and left-handed isomer of a molecular rotor.

Chiral dopants are well known circular birefringent materials [85, 86] which give rise to a circular birefringence in the order of 10^{-3} . Circular birefringence can be achieved by mixing light-driven rotor [87] materials into LCs. The molecular rotor switches from the initially left-handed circular birefringent isomer to the right-handed isomer upon UV-exposure. The molecular structure of the molecular rotor is shown in Fig. 5.11. The right-handed circular birefringent isomer can be switched back (isomerized) by heating. The experimental value of circular birefringence is about 0.1 which is also much higher than in azobenzene polymers.

Additionally, a photocrosslinkable polymer liquid crystal without the use of LPP has been reported to be suitable for forming linear polarization holographic gratings [88,

89].

Chapter 6

Reading of photoanisotropic polarization gratings

After the formation of the photoanisotropic polarization grating, we study the reading process by considering the diffraction of an incident plane wave. For the gratings recorded at small recording angles approximate analytic solutions for the transmission coefficients are derived. The two special gratings recorded with orthogonal circular and linear polarizations are studied and explicit formulas are provided in this chapter. To verify the approximations and to study more general cases for which the approximate formulas are not valid, a numerical program based on Finite Element Method (FEM) (as described in Section 2.6.1) is used in the simulations.

6.1 Derivation of the transmission matrix

Consider a plane wave $\mathbf{E}^i$ that is perpendicularly incident from the upper half space onto the polarization grating and is partially transmitted by it. Then in the lower half space, we have the transmitted field $\mathbf{E}^t$. Here we consider a relatively thin grating structure and for simplicity we neglect the reflection at the interfaces (see Fig. 6.1). The relation between the transmitted field and the incident field is assumed to be expressible by a local *transmission matrix* $\underline{\underline{\mathcal{T}}}$ which depends on x ,

$$\mathbf{E}^t(x) = \underline{\underline{\mathcal{T}}}(x)\mathbf{E}^i(x). \quad (6.1)$$

In this section, we assume that the sensitivity of the photoanisotropic material is perfect for the purpose of understanding the physical limit of the maximum diffraction efficiencies that can be obtained theoretically. We first derive a transmission matrix for a recorded polarization grating of which the polarization states of the recording beams are arbitrary and the recording angle is arbitrarily large. Then the transmis-

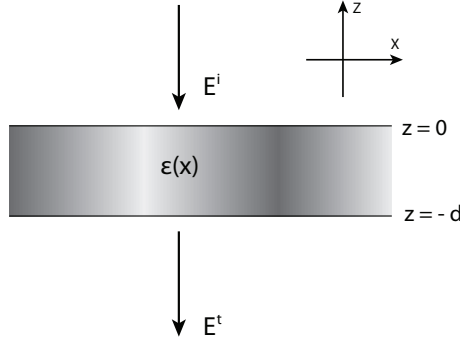


Figure 6.1: Reading process for a polarization grating.

sion matrix for small recording angle cases are studied applying the analytic approximation, which is followed by two specific polarization gratings, namely polarization gratings formed by two orthogonally circularly polarized beams (OC polarization gratings) and formed by two orthogonally linearly polarized beams (OL polarization gratings).

6.1.1 Cases of arbitrarily large recording angles

Consider now an incident plane wave that propagates in the negative z -direction. Its wave vector is given by

$$\mathbf{k} = -k_0 n_i \hat{\mathbf{z}}, \quad (6.2)$$

where n_i is the refractive index of the isotropic half space above the grating. We now assume that for every x the field in the grating may be approximated using a vertical propagation model. This means that for every x the wave in the grating is the same as in a three-layer system that is invariant in the x -direction and of which the middle layer is anisotropic with permittivity tensor everywhere equal to $\underline{\underline{\epsilon}}(x)$. If in addition we neglect reflection at the interfaces, the wave in the middle layer propagates in the negative z -direction. There are two possible values for the refractive index, so that the wave vectors in the middle layer are

$$\mathbf{k}_1 = -k_0 n_1 \hat{\mathbf{z}}, \quad (6.3a)$$

$$\mathbf{k}_2 = -k_0 n_2 \hat{\mathbf{z}}. \quad (6.3b)$$

The values of n_1 and n_2 can be determined from Fresnel's equation (2.34). To solve this equation, we have to determine the components of the wave vector on the orthonormal basis $\{\hat{\xi}(x), \hat{\eta}(x), \hat{\zeta}(x)\}$ of eigenvectors of the permittivity tensor $\underline{\underline{\epsilon}}(x)$. To obtain these we use the matrix U^* defined in Eq. (2.24). Write

$$\mathbf{k} = -k_0 n \hat{\mathbf{z}}. \quad (6.4)$$

Then, with Eq. (5.20):

$$V^*(\mathbf{r})\mathbf{k} = -k_0 n \begin{pmatrix} e_{1,z}(\mathbf{r}) \\ e_{2,z}(\mathbf{r}) \\ e_{3,z}(\mathbf{r}) \end{pmatrix}, \quad (6.5)$$

and

$$\begin{pmatrix} k_\xi \\ k_\eta \\ k_\zeta \end{pmatrix} = U^*(\mathbf{r})\mathbf{k} = W^*(\mathbf{r})V^*(\mathbf{r})\mathbf{k} = -k_0 n W^*(\mathbf{r}) \begin{pmatrix} e_{1,z}(\mathbf{r}) \\ e_{2,z}(\mathbf{r}) \\ e_{3,z}(\mathbf{r}) \end{pmatrix}, \quad (6.6)$$

with

$$W^*(\mathbf{r}) = \begin{pmatrix} \xi_1^*(\mathbf{r}) & \xi_2^*(\mathbf{r}) & \xi_3^*(\mathbf{r}) \\ \eta_1^*(\mathbf{r}) & \eta_2^*(\mathbf{r}) & \eta_3^*(\mathbf{r}) \\ \zeta_1^*(\mathbf{r}) & \zeta_2^*(\mathbf{r}) & \zeta_3^*(\mathbf{r}) \end{pmatrix}. \quad (6.7)$$

We could be more explicit by substituting Eqs. (5.42)-(5.44) into Eq. (6.7), but in fact, the procedure to obtain the transmission matrix is easier to understand by using the notation Eq. (6.7). Thus we can write:

$$k_\xi = -k_0 n \sum_{j=1}^3 e_{j,z}(\mathbf{r}) \xi_j^*(\mathbf{r}), \quad (6.8a)$$

$$k_\eta = -k_0 n \sum_{j=1}^3 e_{j,z}(\mathbf{r}) \eta_j^*(\mathbf{r}), \quad (6.8b)$$

$$k_\zeta = -k_0 n \sum_{j=1}^3 e_{j,z}(\mathbf{r}) \zeta_j^*(\mathbf{r}). \quad (6.8c)$$

Then n follows by substitution of k_ξ , k_η , k_ζ into Eq. (2.34). There are two solutions $n_1(x)$, $n_2(x)$ and the corresponding complex electric vectors are follow from Eq. (2.39):

$$\mathbf{A}_1(x) = \frac{-1}{c_1(x)} \left[\frac{k_\xi}{k_0^2 n_1^2(x) - k_0^2 \epsilon_\xi(x)} \hat{\xi}(x) + \frac{k_\eta}{k_0^2 n_1^2(x) - k_0^2 \epsilon_\eta(x)} \hat{\eta}(x) + \frac{k_\zeta}{k_0^2 n_1^2(x) - k_0^2 \epsilon_\zeta(x)} \hat{\zeta}(x) \right], \quad (6.9a)$$

$$\mathbf{A}_2(x) = \frac{-1}{c_2(x)} \left[\frac{k_\xi}{k_0^2 n_2^2(x) - k_0^2 \epsilon_\xi(x)} \hat{\xi}(x) + \frac{k_\eta}{k_0^2 n_2^2(x) - k_0^2 \epsilon_\eta(x)} \hat{\eta}(x) + \frac{k_\zeta}{k_0^2 n_2^2(x) - k_0^2 \epsilon_\zeta(x)} \hat{\zeta}(x) \right], \quad (6.9b)$$

where $c_1(x)$, $c_2(x)$ are scalars which will be defined below. In the above equation ϵ_ξ , ϵ_η , ϵ_ζ are the real eigenvalues of the permittivity tensor and are given by Eq. (5.34). The electric field in $-d < z < 0$ can then be written as

$$\mathbf{E}(x, z) = E_1(x) \mathbf{A}_1(x) e^{-ik_0 n_1(x)z} + E_2(x) \mathbf{A}_2(x) e^{-ik_0 n_2(x)z}, \quad (6.10)$$

with $E_1(x)$ and $E_2(x)$ still to be determined. We are primarily interested in the tangential components of the electric field:

$$\hat{\mathbf{z}} \times \mathbf{E}(x, z) = E_1(x) \hat{\mathbf{z}} \times \mathbf{A}_1(x) e^{-ik_0 n_1(x)z} + E_2(x) \hat{\mathbf{z}} \times \mathbf{A}_2(x) e^{-ik_0 n_2(x)z}. \quad (6.11)$$

We now choose $c_1(x)$ and $c_2(x)$ such that

$$|\hat{\mathbf{z}} \times \mathbf{A}_1(x)|_{\mathbb{C}^3} = |\hat{\mathbf{z}} \times \mathbf{A}_2(x)|_{\mathbb{C}^3} = 1. \quad (6.12)$$

Note that the vectors $\hat{\mathbf{z}} \times \mathbf{A}_1$ and $\hat{\mathbf{z}} \times \mathbf{A}_2$ are in general not orthogonal. We therefore define the vectors $\mathbf{B}_1(x)$, $\mathbf{B}_2(x)$ in the (x, y) -plane, such that

$$\langle \hat{\mathbf{z}} \times \mathbf{A}_1(x), \mathbf{B}_1(x) \rangle_{\mathbb{C}^3} = 1, \quad \langle \hat{\mathbf{z}} \times \mathbf{A}_2(x), \mathbf{B}_2(x) \rangle_{\mathbb{C}^3} = 1, \quad (6.13a)$$

$$\langle \hat{\mathbf{z}} \times \mathbf{A}_1(x), \mathbf{B}_2(x) \rangle_{\mathbb{C}^3} = 0, \quad \langle \hat{\mathbf{z}} \times \mathbf{A}_2(x), \mathbf{B}_1(x) \rangle_{\mathbb{C}^3} = 0. \quad (6.13b)$$

Then

$$E_1(x) = \langle \mathbf{E}(x, 0), \mathbf{B}_1(x) \rangle_{\mathbb{C}^3} = E_x(x, 0) B_{1,x}^*(x) + E_y(x, 0) B_{1,y}^*(x), \quad (6.14a)$$

$$E_2(x) = \langle \mathbf{E}(x, 0), \mathbf{B}_2(x) \rangle_{\mathbb{C}^3} = E_x(x, 0) B_{2,x}^*(x) + E_y(x, 0) B_{2,y}^*(x). \quad (6.14b)$$

The tangential components of the electric field at $z = -d$ are then

$$\begin{aligned} E_x(x, -d) &= E_1(x) e^{ik_0 n_1(x)d} A_{1,x}(x) + E_2(x) e^{ik_0 n_2(x)d} A_{2,x}(x) \\ &= \left[B_{1,x}^*(x) A_{1,x}(x) e^{ik_0 n_1(x)d} + B_{2,x}^*(x) A_{2,x}(x) e^{ik_0 n_2(x)d} \right] E_x(x, 0) \\ &\quad + \left[B_{1,y}^*(x) A_{1,x}(x) e^{ik_0 n_1(x)d} + B_{2,y}^*(x) A_{2,x}(x) e^{ik_0 n_2(x)d} \right] E_y(x, 0), \end{aligned} \quad (6.15a)$$

$$\begin{aligned} E_y(x, -d) &= E_1(x) e^{ik_0 n_1(x)d} A_{1,y}(x) + E_2(x) e^{ik_0 n_2(x)d} A_{2,y}(x) \\ &= \left[B_{1,x}^*(x) A_{1,y}(x) e^{ik_0 n_1(x)d} + B_{2,x}^*(x) A_{2,y}(x) e^{ik_0 n_2(x)d} \right] E_x(x, 0) \\ &\quad + \left[B_{1,y}^*(x) A_{1,y}(x) e^{ik_0 n_1(x)d} + B_{2,y}^*(x) A_{2,y}(x) e^{ik_0 n_2(x)d} \right] E_y(x, 0). \end{aligned} \quad (6.15b)$$

Hence

$$\begin{pmatrix} E_x(x, -d) \\ E_y(x, -d) \end{pmatrix} = \underline{\underline{\mathcal{T}}}(x) \begin{pmatrix} E_x(x, 0) \\ E_y(x, 0) \end{pmatrix}, \quad (6.16)$$

with

$$\underline{\underline{\mathcal{T}}}(x) = \begin{pmatrix} t_{11}(x) & t_{12}(x) \\ t_{21}(x) & t_{22}(x) \end{pmatrix}, \quad (6.17)$$

and

$$\begin{aligned} t_{11}(x) &= B_{1,x}^*(x) A_{1,x}(x) e^{ik_0 n_1(x)d} + B_{2,x}^*(x) A_{2,x}(x) e^{ik_0 n_2(x)d}, \\ t_{12}(x) &= B_{1,y}^*(x) A_{1,x}(x) e^{ik_0 n_1(x)d} + B_{2,y}^*(x) A_{2,x}(x) e^{ik_0 n_2(x)d}, \\ t_{21}(x) &= B_{1,x}^*(x) A_{1,y}(x) e^{ik_0 n_1(x)d} + B_{2,x}^*(x) A_{2,y}(x) e^{ik_0 n_2(x)d}, \\ t_{22}(x) &= B_{1,y}^*(x) A_{1,y}(x) e^{ik_0 n_1(x)d} + B_{2,y}^*(x) A_{2,y}(x) e^{ik_0 n_2(x)d}. \end{aligned}$$

Remarks:

(a) For an oblique incident plane wave with wave vector

$$\mathbf{k} = k_x^i \hat{\mathbf{x}} + k_z^i \hat{\mathbf{z}}, \quad k_z^i < 0. \quad (6.18)$$

the wave in $z < 0$ consists of two plane waves with wave vectors:

$$\mathbf{k}_1 = k_x \hat{\mathbf{x}} + k_{1,z} \hat{\mathbf{z}}, \quad (6.19)$$

$$\mathbf{k}_2 = k_x \hat{\mathbf{x}} + k_{2,z} \hat{\mathbf{z}}. \quad (6.20)$$

The values of $k_{1,z}, k_{2,z}$ follow from the solution of the equation obtained by setting the determinant of system (2.45) equal to zero. The components of the corresponding electric field $\mathbf{A}_1, \mathbf{A}_2$ on the eigenvector basis $\{\hat{\xi}(x), \hat{\eta}(x), \hat{\zeta}(x)\}$ follow from (2.47). we omit the details.

- (b) The analysis can be easily extended to incorporate reflections at the interfaces $z = 0$ and $z = -d$. When reflections are taken into account, there are in addition two upward propagating waves in the region $0 < z < d$.

6.1.2 Special cases of small recording angles

As we know, when the recording angle is small ($\theta_{record} < 5^\circ$), the approximation that the two recording beams both propagate in the z -direction can be made. In this subsection we derive explicit formulas for the transmission matrix in this paraxial regime for a perpendicularly incident plane wave.

Suppose that with small recording angles, the major and minor axes of the polarization ellipses of the interference patterns are always in the (x, y) -plane. Then the photo-induced photoanisotropy in the medium is also restrained in the (x, y) -plane, and thus the dielectric permittivity tensor becomes:

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}. \quad (6.21)$$

For a perpendicular incident plane wave with the wave vector $\mathbf{k} = (0, 0, k_z)^T$, Eq. (2.16) then becomes,

$$\begin{pmatrix} \epsilon_{xx} - \frac{k_z^2}{k_0^2} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} - \frac{k_z^2}{k_0^2} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = 0. \quad (6.22)$$

It is obvious that always $A_z = 0$. Thus the top left 2×2 submatrix of the 3×3 tensor, which contains only the x and y components of the tensor, is enough for solving the light propagation problem. Therefore the permittivity tensor with the photo-induced anisotropy of the stated typed can be effectively reduced to a 2×2 tensor which can be written as:

$$\begin{aligned} \underline{\underline{\epsilon}} &= V \underline{\underline{\tilde{\epsilon}}} V^{-1} \\ &= V \begin{pmatrix} \epsilon^i + \kappa_{\parallel} |E_1|^2 + \kappa_{\perp} |E_2|^2 & i\kappa_c (-2|E_1||E_2|) \\ -i\kappa_c (-2|E_1||E_2|) & \epsilon^i + \kappa_{\perp} |E_1|^2 + \kappa_{\parallel} |E_2|^2 \end{pmatrix} V^{-1}, \end{aligned} \quad (6.23)$$

where V is the transformation matrix which transforms a vector on the local basis to that on the global basis. Note that in the present section and the following Sections 6.1.3 and 6.1.4 the matrices V and W should be understood to be (2,2)-matrices obtained by omitting the components that are related to the z -axis. On the local basis, the dielectric permittivity tensor with photo-induced anisotropy in terms of the Stokes parameters is given by:

$$\begin{aligned}\tilde{\underline{\epsilon}} &= \begin{pmatrix} \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0 + \frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1 & -i\kappa_c S_3 \\ i\kappa_c S_3 & \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0 - \frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1 \end{pmatrix} \\ &= \begin{pmatrix} \epsilon'_0 & 0 \\ 0 & \epsilon'_0 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{12}^* & -a_{11} \end{pmatrix},\end{aligned}\quad (6.24)$$

with

$$\begin{aligned}\epsilon'_0 &= \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} S_0, \\ a_{11} &= \frac{\kappa_{\parallel} - \kappa_{\perp}}{2} S_1, \\ a_{12} &= -i\kappa_c S_3.\end{aligned}$$

The star in the equation represents complex conjugate. Notice that $\tilde{\underline{\epsilon}}$ is a complex hermitian matrix. The eigenvalues of a hermitian matrix are real numbers with in general complex eigenvectors. The two real eigenvalues are the square of the two real refractive indices.

The eigenvalues λ' and eigenvectors $\mathbf{v}$ of the second part of the right side of Eq. (6.24) can be calculated and are given by

$$\lambda'_+ = \sqrt{a_{11}^2 + |a_{12}|^2}, \quad (6.25)$$

$$\lambda'_- = -\lambda'_+; \quad (6.26)$$

and the corresponding (normalized) eigenvectors are on the local basis:

$$\mathbf{v}_+ = \frac{1}{\sqrt{(\lambda'_+ + a_{11})^2 + |a_{12}|^2}} \begin{pmatrix} \lambda'_+ + a_{11} \\ a_{12}^* \end{pmatrix}, \quad (6.27)$$

$$\begin{aligned}\mathbf{v}_- &= \frac{1}{\sqrt{(\lambda'_- - a_{11})^2 + |a_{12}|^2}} \begin{pmatrix} a_{12} \\ \lambda'_- - a_{11} \end{pmatrix} \\ &= \frac{1}{\sqrt{(\lambda'_+ + a_{11})^2 + |a_{12}|^2}} \begin{pmatrix} a_{12} \\ -\lambda'_+ - a_{11} \end{pmatrix}.\end{aligned}\quad (6.28)$$

$\{\mathbf{v}_+, \mathbf{v}_-\}$ is an orthogonal normalized basis of eigenvectors. Then the transformation matrix W is given by:

$$W = \frac{1}{\sqrt{(\lambda'_+ + a_{11})^2 + |a_{12}|^2}} \begin{pmatrix} \lambda'_+ + a_{11} & a_{12} \\ a_{12}^* & -\lambda'_+ - a_{11} \end{pmatrix}. \quad (6.29)$$

Thus the transformation from the basis of the eigenvectors to the local basis is expressed as:

$$\underline{\underline{\tilde{\epsilon}}} = W \begin{pmatrix} \epsilon'_0 + \lambda' & 0 \\ 0 & \epsilon'_0 - \lambda' \end{pmatrix} W^{-1} \quad (6.30)$$

On the basis expanded by $\mathbf{v}_+, \mathbf{v}_-$, the refractive index of the media can be written as,

$$n_1 = \sqrt{\epsilon'_0 + \lambda'}, \quad (6.31)$$

$$n_2 = \sqrt{\epsilon'_0 - \lambda'}. \quad (6.32)$$

Then the transmission matrix for a general case is obtained and can be expressed as

$$\underline{\underline{\mathcal{T}}} = V W \begin{pmatrix} \exp(ik_0 n_1 z) & 0 \\ 0 & \exp(ik_0 n_2 z) \end{pmatrix} W^{-1} V^{-1}. \quad (6.33)$$

6.1.3 Transmission matrix for OC polarization gratings

We have discussed the interference pattern formed by two orthogonal circular (OC) polarization in Section 5.3.1. With small angle approximation, i.e. $k_x \approx 0$ and $k_z/(k_0 n) \approx 1$, the interference pattern of the two waves is a sequence of linear polarizations with the major axis rotating in the (x, y) -plane (as shown in Fig. 5.3(a)). Substitution of the small angle assumption $k_x \approx 0$ into Eq. (5.61), (5.62), (5.63) and considering that the modification takes place in x, y -plane, we obtain the transform matrix V as follows,

$$V(x) = \begin{pmatrix} -\sin(k_x x) & -\cos(k_x x) \\ \cos(k_x x) & -\sin(k_x x) \end{pmatrix} = \begin{pmatrix} -\sin(\frac{\pi}{\Lambda} x) & -\cos(\frac{\pi}{\Lambda} x) \\ \cos(\frac{\pi}{\Lambda} x) & -\sin(\frac{\pi}{\Lambda} x) \end{pmatrix}, \quad (6.34)$$

with $\Lambda = \pi/k_x$ being the period of interference pattern. On the local basis, the relative dielectric permittivity tensor is

$$\underline{\underline{\tilde{\epsilon}}}(x) = \begin{pmatrix} \epsilon^i + \kappa_{\parallel} |E_1|^2 & 0 \\ 0 & \epsilon^i + \kappa_{\perp} |E_1|^2 \end{pmatrix}. \quad (6.35)$$

The tensor is already diagonal, thus we can take

$$n_1 = \sqrt{\epsilon^i + \kappa_{\parallel} |E_1|^2}; \quad (6.36)$$

$$n_2 = \sqrt{\epsilon^i + \kappa_{\perp} |E_1|^2}. \quad (6.37)$$

Then the transmission matrix is obtained

$$\begin{aligned} \underline{\underline{\mathcal{T}}}(x) &= V(x) \begin{pmatrix} \exp(ik_0 n_1 z) & 0 \\ 0 & \exp(ik_0 n_2 z) \end{pmatrix} V(x)^{-1} \\ &= \begin{pmatrix} \sin^2(\frac{\pi}{\Lambda} x) e^{ik_0 n_1 z} + \cos^2(\frac{\pi}{\Lambda} x) e^{ik_0 n_2 z} & -\sin(\frac{\pi}{\Lambda} x) \cos(\frac{\pi}{\Lambda} x) (e^{ik_0 n_1 z} - e^{ik_0 n_2 z}) \\ -\sin(\frac{\pi}{\Lambda} x) \cos(\frac{\pi}{\Lambda} x) (e^{ik_0 n_1 z} - e^{ik_0 n_2 z}) & \cos^2(\frac{\pi}{\Lambda} x) e^{ik_0 n_1 z} + \sin^2(\frac{\pi}{\Lambda} x) e^{ik_0 n_2 z} \end{pmatrix}. \end{aligned} \quad (6.38)$$

The transmitted field at plane $z = -d$ (see Fig. 6.1) is equal to

$$\begin{pmatrix} E_x^t \\ E_y^t \end{pmatrix} = \underline{\underline{\mathcal{T}}}(x, z = -d) \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix}. \quad (6.39)$$

From Eq. (6.38), it is easy to see that there are only 3 diffracted orders in the transmitted field, namely,

- 0^{th} order:

$$\mathbf{E}_0 = \frac{1}{2}(e^{ik_0 n_1 d} + e^{ik_0 n_2 d}) \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix}; \quad (6.40)$$

- -1^{st} order with $k_x = -2\pi/\Lambda$:

$$\mathbf{E}_{-1} = -\frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix}; \quad (6.41)$$

- $+1^{st}$ order with $k_x = +2\pi/\Lambda$:

$$\mathbf{E}_{+1} = \frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} -1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix}; \quad (6.42)$$

The equations reveal the polarization states of the diffracted orders. The polarization of the 0^{th} order is always the same as that of the incident field. The relationship between the polarization states of the first orders and that of the incident field is listed in Table. 6.1. In the table, a horizontal line '—' represents p-polarization with $E_x = 1, E_y = 0$; a vertical line '|' represents s-polarization with $E_x = 0, E_y = 1$; a clockwise circle '↻' represents left-hand circular polarization with $E_x = 1, E_y = i$; a counterclockwise circle '↺' represents right-hand circular polarization with $E_x = 1, E_y = -i$ [90]. These agree with results reported by many other researchers, [66].

$\hat{\mathbf{E}}^i$	—		↻	↺
$\hat{\mathbf{E}}_{-1}$	↻	↻	0	↻
$\hat{\mathbf{E}}_{+1}$	↺	↺	↺	0

Table 6.1: Relations between the polarization of the incidence and polarizations of the diffracted 1^{st} orders. In the table $\hat{\mathbf{E}}$ represents the unit vector along the electric field vector. — represents p-polarization; | represents s-polarization; ↻ represents left-hand circular polarization; ↺ represents right-hand circular polarization.

From the table we can see that, when the incident field is linear polarized, both -1^{st} and $+1^{st}$ orders propagate, and that the polarizations are left-handed and right-handed respectively. When the incident field is circular polarized, depending on the handedness of the polarization, one of the first orders is present and the other one is absent. If the incident field is right-handed circular, there will be 0^{th} and $+1^{st}$ orders

and the polarization of the $+1^{st}$ order is left-hand circular. Analogously, when the incident field is left-handed, in the transmitted 0^{th} and -1^{st} orders propagate. The polarization of the -1^{st} order is right-handed. In summary, the polarization grating formed by two orthogonal circular polarized waves behaves as a circular polarization converter which switches the handedness of the circular polarization, from right-handed to left-handed and vice versa.

Suppose we have an OC polarization grating which is recorded with the recording condition mentioned in Section 5.2.1, i.e. the wavelength is 351 nm UV and the recording angle θ_{record} of 2.5° . The period of such formed grating is 4023 nm. The electric field vectors have the form of Eq. (5.49) with $a_S = 1.25$. For the photoanisotropic material we assume $\kappa_{\parallel} = 0.2, \kappa_{\perp} = 0$. Thus the photo-induced birefringence is 0.2 approximately (with local refractive indices $n_1 = 1.6952, n_2 = 1.5$). We use the wavelength 633 nm as reading beam. The diffraction efficiency η_m of the m^{th} diffraction order is calculated as the ratio between the intensity in the order and the intensity of the incidence:

• 0^{th} order:

$$\begin{aligned}\eta_0 &= \frac{\left| \frac{1}{2}(e^{ik_0 n_1 d} + e^{ik_0 n_2 d}) \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{2} + \frac{1}{2} \cos[k_0(n_1 - n_2)d];\end{aligned}\quad (6.43)$$

• -1^{st} order:

$$\begin{aligned}\eta_{-1} &= \frac{\left| -\frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{4} \{1 - \cos[k_0(n_1 - n_2)d]\} \left(1 - \frac{S_3^i}{S_0^i}\right);\end{aligned}\quad (6.44)$$

• $+1^{st}$ order:

$$\begin{aligned}\eta_{+1} &= \frac{\left| \frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} -1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{4} \{1 - \cos[k_0(n_1 - n_2)d]\} \left(1 + \frac{S_3^i}{S_0^i}\right).\end{aligned}\quad (6.45)$$

In the above equations, S_0^i and S_3^i are the Stokes parameters of the incident reading field which are defined in Appendix D. The efficiencies are plotted as function of the

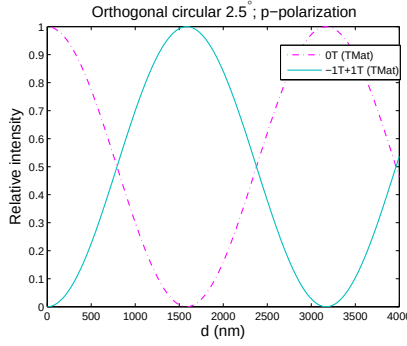


Figure 6.2: Diffraction efficiency for a perpendicular p-polarized incident beam as function of the thickness of the photoanisotropic polarization grating. Relative intensity is defined as the ratio of intensities between the transmission and incidence. $-1T$ in the legend represents the -1^{st} order in the transmitted field. Analogous conventions apply to $0T$ and $1T$.

thickness of the polarization grating in Fig. 6.2. We can see from the plot that the intensities of the 0^{th} and $\pm 1^{st}$ orders oscillate as function of the thickness of the grating. 100% diffraction efficiency is achieved for the first orders at a thickness of 1600 nm, while the 0^{th} order is absent. Thus, if the incident beam is linearly polarized and the thickness is appropriately chosen, two equally distributed first orders are observed, and if the incident beam is circular polarization, light is diffracted into one of the first orders with 100% diffraction efficiency. For this thickness the electric near field is shown in Fig. 6.3 for a circular polarized reading beam with $E_x^i = 1, E_y^i = i$. Fig 6.3(a) and (b) the amplitude of the x- and y-component of the electric field are shown. In Fig. 6.3(c) and (d) the phase of E_x and E_y are shown. The phase singularities in Fig. 6.3(c) and (d), correspond to points where the amplitude is zero. The asymmetry of the near field distribution explains the difference between orders $+1^{st}$ and -1^{st} in the diffracted far field.

6.1.4 Transmission matrix for OL polarization gratings

Another typical polarization grating is formed by two orthogonal linear polarized (OL) fields. The interference of this case has been discussed in Section 5.3.2. With the small angle approximation, $k_x \approx 0$, the transform matrix V is found by substitution of this condition into Eqs. (5.81). Considering that the modification of the electric field can be approximately regarded to be in the (x, y) -plane, V is expressed as a 2×2 matrix as:

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \quad (6.46)$$

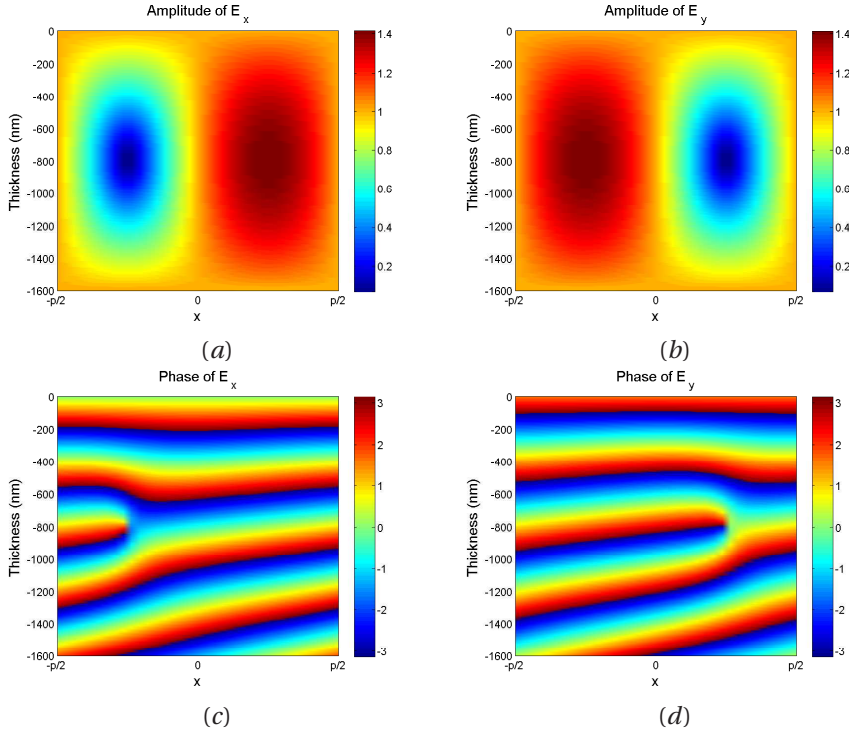


Figure 6.3: The electric field inside the OC polarization grating. Amplitude (a) and (b) and phase (c) and (d) of tangential components of electric field vector in the polarization grating for one period for a perpendicular incident polarized beam ($E_x^i = 1, E_y^i = i$). The thickness of the grating is 1600 nm.

On the local basis, the Stokes parameters are simplified to,

$$S_0 = |E_1(\mathbf{r})|^2 + |E_2(\mathbf{r})|^2 = 2a^2, \quad (6.47)$$

$$S_1 = |E_1(\mathbf{r})|^2 - |E_2(\mathbf{r})|^2 = 2a^2 \cos(2\frac{\pi}{\Lambda}x), \quad (6.48)$$

$$S_3 = 2|E_1(\mathbf{r})||E_2(\mathbf{r})| = 2a^2 \sin(2\frac{\pi}{\Lambda}x), \quad (6.49)$$

where $\Lambda = \pi/k_x$. Substitution of the obtained Stokes parameters in Eq. (6.24) results in the local dielectric tensor. Analogous to the frame work in Section 6.1.2, the two refractive indices are obtained,

$$n_1 = \sqrt{\epsilon'_0 + \lambda'}, \quad (6.50)$$

$$n_2 = \sqrt{\epsilon'_0 - \lambda'}, \quad (6.51)$$

with

$$\epsilon'_0 = \epsilon^i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} 2a^2, \quad (6.52)$$

$$\lambda' = 2a^2 \sqrt{\frac{1}{4}(\kappa_{\parallel} - \kappa_{\perp})^2 \cos^2 2\frac{\pi}{\Lambda}x + \kappa_c^2 \sin^2 2\frac{\pi}{\Lambda}x}. \quad (6.53)$$

The transformation matrix W is given by Eq. (6.29). Finally the transmission matrix is obtained by using Eq. (6.33).

If $\kappa_c = \frac{\kappa_{\parallel} - \kappa_{\perp}}{2}$, the equations can be simplified,

$$\epsilon'_0 = \epsilon_i + \frac{\kappa_{\parallel} + \kappa_{\perp}}{2} 2a^2, \quad (6.54)$$

$$\lambda' = 2a^2 \frac{\kappa_{\parallel} - \kappa_{\perp}}{2}. \quad (6.55)$$

$$(6.56)$$

Thus n_1 and n_2 are given by,

$$n_1 = \sqrt{\epsilon^i + \kappa_{\parallel} 2a^2}, \quad (6.57)$$

$$n_2 = \sqrt{\epsilon^i + \kappa_{\perp} 2a^2}, \quad (6.58)$$

which are independent of x . And the corresponding transformation matrix W is

$$W = \frac{1}{2 \cos \frac{\pi}{\Lambda}x} \begin{pmatrix} 1 + \cos 2\frac{\pi}{\Lambda}x & -i \sin 2\frac{\pi}{\Lambda}x \\ i \sin 2\frac{\pi}{\Lambda}x & -1 - \cos 2\frac{\pi}{\Lambda}x \end{pmatrix} \quad (6.59)$$

$$= \begin{pmatrix} \cos \frac{\pi}{\Lambda}x & -i \sin \frac{\pi}{\Lambda}x \\ i \sin \frac{\pi}{\Lambda}x & -\cos \frac{\pi}{\Lambda}x \end{pmatrix}. \quad (6.60)$$

The inverse of this matrix is:

$$W^{-1} = \begin{pmatrix} \cos \frac{\pi}{\Lambda}x & -i \sin \frac{\pi}{\Lambda}x \\ i \sin \frac{\pi}{\Lambda}x & -\cos \frac{\pi}{\Lambda}x \end{pmatrix}. \quad (6.61)$$

Then the transmission matrix is obtained:

$$\begin{aligned} \underline{\underline{\mathcal{T}}} &= VW \begin{pmatrix} e^{ik_0 n_1 z} & 0 \\ 0 & e^{ik_0 n_2 z} \end{pmatrix} W^{-1} V^{-1} \\ &= \frac{1}{2} \begin{bmatrix} e^{ik_0 n_1 z} + e^{ik_0 n_2 z} & (e^{ik_0 n_1 z} - e^{ik_0 n_2 z})e^{-i2\frac{\pi}{\Lambda}x} \\ (e^{ik_0 n_1 z} - e^{ik_0 n_2 z})e^{i2\frac{\pi}{\Lambda}x} & e^{ik_0 n_1 z} + e^{ik_0 n_2 z} \end{bmatrix}. \end{aligned} \quad (6.62)$$

Then the transmitted field is given by

$$\mathbf{E}^t = \underline{\underline{\mathcal{T}}} \mathbf{E}^i. \quad (6.63)$$

From the above equation it can be seen that three orders propagate and the efficiency η_m of each order m can be calculated as,

• 0^{th} order:

$$\begin{aligned}\eta_0 &= \frac{\left| \frac{1}{2}(e^{ik_0 n_1 d} + e^{ik_0 n_2 d}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{2} + \frac{1}{2} \cos[k_0(n_1 - n_2)d];\end{aligned}\quad (6.64)$$

• -1^{st} order with $k_x = -2\pi/\Lambda$:

$$\begin{aligned}\eta_{-1} &= \frac{\left| \frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{4} \{1 - \cos[k_0(n_1 - n_2)d]\} \left(1 - \frac{S_1^i}{S_0^i}\right);\end{aligned}\quad (6.65)$$

• $+1^{st}$ order with $k_x = +2\pi/\Lambda$:

$$\begin{aligned}\eta_{+1} &= \frac{\left| \frac{1}{4}(e^{ik_0 n_1 d} - e^{ik_0 n_2 d}) \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2}{\left| \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} \right|^2} \\ &= \frac{1}{4} \{1 - \cos[k_0(n_1 - n_2)d]\} \left(1 + \frac{S_1^i}{S_0^i}\right).\end{aligned}\quad (6.66)$$

In the above equations, S_0^i and S_1^i are the Stokes parameters of the incident reading field which are defined in Appendix D.

We give an example of a OL polarization grating. Suppose that the recording beam is UV light. Two orthogonal linear polarizations, s- and p-, with an amplitude of $a = 1.25$ interfere at a recording angle of 2.5° . The photoanisotropy sensitivity of the material is $\kappa_{\parallel} = 0.2, \kappa_{\perp} = 0$ and $\kappa_c = 0.1$. Thus the two real refractive indices on the basis of the eigenvectors are $n_1 = 1.6955, n_2 = 1.5$. For the recorded grating, the diffraction efficiency expressed as relative intensity ($RI = I_{order}/I_{in}$) with respect to the thickness of the grating is calculated and plotted in Fig. 6.4 for a linear s-polarized reading beam with $\lambda = 633$ nm. The trend of curves in the figure looks similar to dependence of the efficiencies of the transmitted orders of the OC polarization grating (see Fig. 6.2). For an s-polarized reading beam, the transmitted far field consists of only -1^{st} and 0^{th} propagating orders. The polarization selectivity of the first orders is listed in Table 6.2.

S-polarization of the reading beam is converted to p-polarization and diffracted into the -1^{st} order. The 0^{th} order always maintains the same polarization state as the incident polarization. With an s-polarized reading beam, the $+1^{st}$ order is absent.

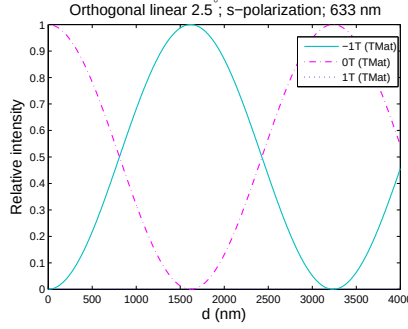


Figure 6.4: Diffraction efficiency for an *s*-polarized perpendicular incident beam, as function of the thickness of the photoanisotropic polarization grating. Relative intensity is defined as the ratio of intensities between the transmission and incidence. *-1T* in the legend represents the -1^{st} order in the transmitted field. Analogous conventions apply to *0T* and *1T*.

$\hat{\mathbf{E}}^i$	—		⊙	⊗
$\hat{\mathbf{E}}_{-1}$	0	—	—	—
$\hat{\mathbf{E}}_{+1}$		0		

Table 6.2: Relations between the polarization of the incidence and polarizations of the diffracted 1^{st} orders for OL polarization grating. In the table $\hat{\mathbf{E}}$ represents the unit vector along the electric field vector. — represents *p*-polarization; | represents *s*-polarization; ⊙ represents left-hand circular polarization; ⊗ represents right-hand circular polarization.

It is clearly shown in the table that independent of the incident polarization state, the polarization of the -1^{st} diffracted order is always *p*-polarization while for the $+1^{st}$ order it is always *s*. Notice that an OL polarization grating with specific condition ($\kappa_{\perp} \approx 0, \kappa_c = \kappa_{\parallel}/2$) converts *s*-polarization into *p*-polarization and vice versa. For a certain thickness of which 0^{th} order disappears, light is diffracted at 100% into the first order. It can be concluded that, an OL polarization grating functions as linear polarization converter.

The near field is illustrated in Fig. 6.5 for a 4000 nm thick OL grating. All the parameters for both recording and reading are the same as above. From the phase distribution we can see that, E_y goes through the grating without being modulated. E_y therefore contributes only to the transmitted 0^{th} order. On the other hand E_x is diffracted and the phase velocity has an angle with the *z*-axis, which is the diffracted angle of the first order and thus E_x contributes to the 1^{st} diffracted order. Combining information with the far field as shown in Fig. 6.4, we observe that for a thickness of 1600

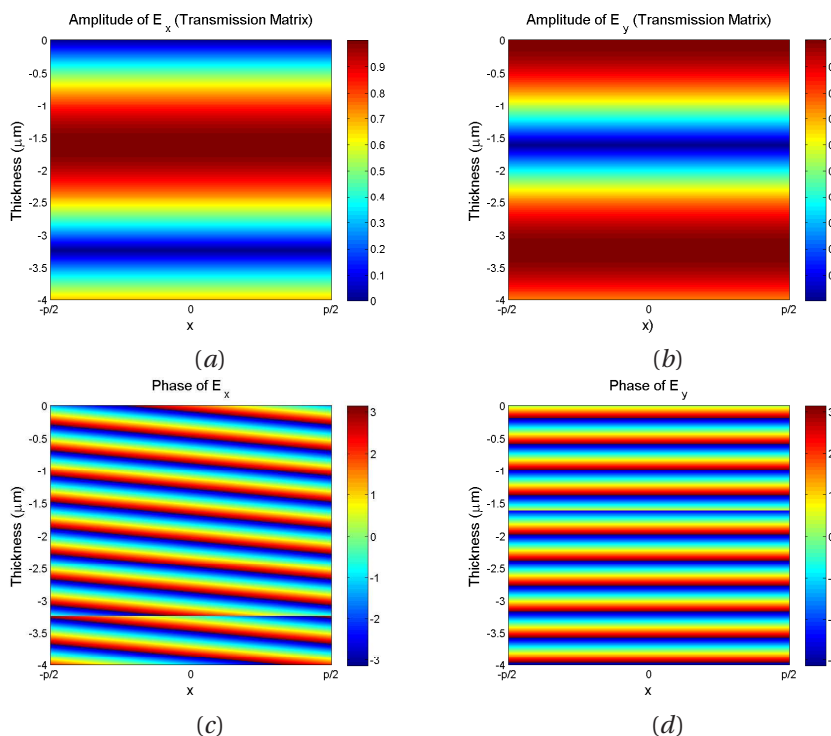


Figure 6.5: The electric field inside the OL polarization grating. Amplitude (a) and (b) and phase (c) and (d) of the tangential components of the electric field vector in the OL polarization grating for an s -polarized incident beam. The thickness of the grating is 4000 nm.

nm, in the near field plot E_x has a maximum in amplitude whereas E_y has a minimum. Thus for this thickness, in the far field, the -1^{st} order reach a 100% efficiency and the 0^{th} order is absent. As the grating grows thicker, both orders propagate. Until the thickness increases up to 3250 nm, where in the near field plot, the amplitude of E_x reaches a minimum and E_y is maximum. Then there is only the 0^{th} order in the far field. Through the OL grating, linear polarization is being rotated.

6.2 Simulations and discussions

The transmission matrix method is an approximation that is useful to study the diffraction from a polarization grating. It assumes that the recording angle is small enough so that the electric field vector of the recording beam is in good approximation in the plane perpendicular to the normal of the layer. But when the angle of recording is so large that k_x can not be approximated by zero any more, the electric field of the recording beam is not any more in one plane. In this case, the photo-

induced anisotropy of the medium is represented accurately by Eq. (5.48). In general the tensor is a full 3×3 matrix. When the material is an active medium, i.e. $\kappa_c \neq 0$ and in the same time the interference is not purely linear polarized everywhere ($S_3 \neq 0$), the tensor becomes complex hermitian. To calculate the diffraction efficiency from such a grating, a numerical approach becomes necessary. The program Cyclop3D [33, 34, 26] which is based on the Finite Element Method (FEM) is therefore used to simulate the transmission accurately for an anisotropic medium with full hermitian dielectric permittivity tensor.

The dielectric tensor is a periodic function of the x coordinate. To specify the tensor as given by Eq. (5.48), a subfunction has to be programmed as an additional Fortran routine for Cyclop3D. The FEM code has been modified so that the complex hermitian type of matrices is handled properly. Simulations with the FEM program have been performed and the results are shown in this section. Different types of polarization gratings will be studied in this section, especially the two typical ones, orthogonal circular (OC) polarization gratings and orthogonal linear (OL) polarization gratings.

6.2.1 OC polarization gratings

Thickness Dependence and comparison between the rigorous FEM code and the Transmission Matrix Method

It is known that for an OC polarization grating, the diffraction efficiency of the grating depends on the thickness of the structure. This is confirmed by the transmission matrix method discussed in Section 6.1.3. Here we test the FEM program by simulating this classic case.

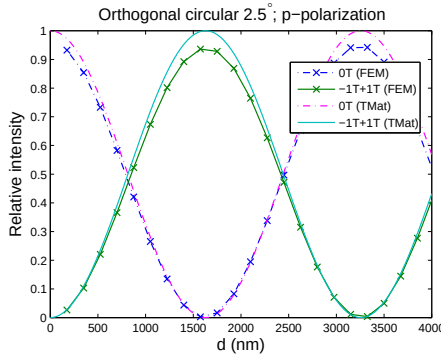


Figure 6.6: Thickness dependence of the diffraction efficiencies of OC polarization grating which is recorded at $\theta_{\text{record}} = 2.5^\circ$. The reading beam has wavelength 633 nm and is p-polarized. The lines without marker are analytically calculated with the transmission matrix method and the lines with cross markers are FEM simulation results. -1T in the legend represents the -1^{st} order in the transmitted field. Analogous conventions apply to OT and 1T.

Suppose we have the same grating as described in Section 6.1.3. Hence, $\lambda_{record.} = 351$ nm, $\theta_{record.} = 2.5^\circ$, $\kappa_{\parallel} = 0.2$, $\kappa_{\perp} = \kappa_c = 0$; the pitch $\Lambda = 4023$ nm, and the local refractive indices are $n_1 = 1.6925$, and $n_2 = 1.5$. The reading beam has wavelength of 633 nm and is a p-polarized plane wave. For a p-polarized incident beam we show in Fig. 6.6 the efficiencies of the transmitted diffracted orders as function of the thickness of the grating, calculated rigorously and by the transmission matrix method.

The difference between the two methods, i.e. transmission matrix method and the rigorous FEM, is small. The analytic transmission matrix method neglects reflection in the calculation, thus the diffraction efficiency predicted by this method reaches 100%.

Angular dependence

In the previous sections, we have developed and discussed the analytic transmission matrix method. This method is limited to the case of perpendicular incidence, i.e. for which the incident wave vector of incidence is perpendicular to the grating surface with $\theta^i = 0^\circ$, $\phi^i = 0^\circ$ (see Fig. 3.2(b) for the definition of the angles). It has been verified that the FEM simulations give reliable results. We now discuss the results of rigorous simulations that are performed to study the diffraction efficiencies of OC polarization gratings as function of the angles of incidence. From Fig. 6.6, we see that when the grating has a thickness of 1575 nm, light is 100% diffracted into the first orders. Therefore we choose this optimized thickness for a 2.5° OC grating in the following discussion.

First, we study the incidence mount of which the plane of incidence is the x, z -plane with $\phi^i = 0^\circ$. We calculated the relative intensities, i.e. diffraction efficiencies, which are defined as the intensity of each diffracted order normalized to the intensity of the incident field. The polar incident angle θ^i varies from 0° to 85° . For a 633 nm p-polarized incident field, the diffraction efficiencies for both transmitted and reflected fields are calculated and are shown in Fig. 6.7(a). Then the reading beam is switched from p-polarization to s, and the simulations are repeated for varying θ^i . The results are shown in Fig. 6.7(b). Comparing these two figures, we see that the diffraction efficiency is slightly different for the two linear polarizations. When θ^i increases, some light goes into the 2^{nd} orders in transmission which is not shown in the plots. This is the reason why the total intensity of the plotted orders is not equal to 1 for both cases. When $\theta^i \leq 20^\circ$, more than 90% of the incident light goes into the first orders. As θ^i rises to 40° the OC grating still gives rather high performance for both polarizations, where diffraction efficiency in first orders is as high as 80%.

Second, the incidence mount is rotated to $\phi^i = 90^\circ$, i.e. the plane of incidence is now the y, z -plane. For varying θ^i , the OC polarization grating are simulated for both s- and p-polarization. Results are shown in Fig. 6.7(c) and (d), respectively. As compared to frame (a) and (b), we find that the diffraction efficiency of the first orders at this incidence mount is higher than for the other mount at larger incident angle θ^i . For p-polarization, up to $\theta^i = 65^\circ$, the intensity in the first order remains higher than 90% of the total. For both polarizations at large angle, light intensity in the transmitted

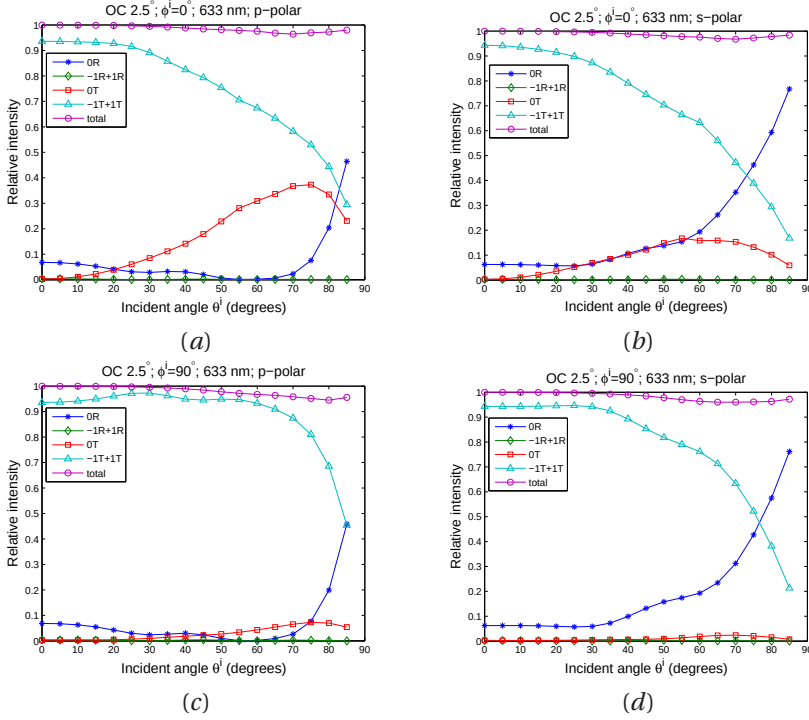


Figure 6.7: Diffraction efficiency for OC polarization grating recorded at 2.5° as function of the incident angle. The reading beam is 633 nm, with (a) and (b) incident angles $\phi^i = 0^\circ$ and (c) and (d) $\phi^i = 90^\circ$. For both incidence mounts, θ^i varies from 0° to 85° . For (a) and (c), the incident wave is p-polarized and for (b) and (d), it is s-polarized.

0^{th} order is quite low as compare to the previous case ($\phi^i = 0^\circ$).

For both linear polarizations, at large θ^i , more light goes into the 0^{th} order in both transmission and reflection. As for a plane surface, s-polarization is more reflected at large angle than p. This can be understood qualitatively by considering the *Brewster angle*. The Brewster angle is the angle of incidence at which p-polarization is transmitted without reflection, it is therefore also known as the *polarization angle*. At the Brewster angle, s-polarization can be both reflected and refracted. At the Brewster angle, the sum of the angle of incidence and the angle of refraction equals 90° (see Fig. 6.8). Thus the Brewster angle can be calculated by:

$$\theta_B = \arctan \frac{n_2}{n_1}. \quad (6.67)$$

The average refractive index of the polarization grating is between 1.5 and 1.7. By using these two indices, an estimate range for the Brewster angle is obtained which is between 56° and 60° . The calculation agrees with Fig. 6.7(a) and (c) nicely, where for p-polarization incidence, the reflection is totally absent. In contrast, the reflection of s-polarization continually increases as the angle of incidence increases.

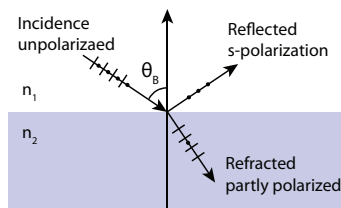


Figure 6.8: Light is incident on an surface at the Brewster angle θ_B .

To summarize, when the angle of incidence θ^i is smaller than 30° , the diffraction from a OC polarization grating is hardly sensitive to the angle of incidence. In this region, more than 90% light is diffracted into 1^{st} orders. As θ^i gets larger, light gradually goes into reflected and transmitted 0^{th} orders. At the same time, a small fraction of light leaks into transmitted 2^{nd} orders. The insensitivity to the angle of incidence of the OL grating makes it a potential candidate for applications that require large conical angle tolerance, for instance, backlights for flat panel display systems.

Beyond small writing angles

So far, only the small angle OC polarization gratings have been studied. As we have discussed in Section 5.2.1, with larger recording angles, smaller period of interference patterns can be formed and subsequently a grating with smaller pitch can be made. However, the complexity of the interference patterns formed at large angles makes it impossible to achieve analytic solutions of light propagation in the obtained gratings. Therefore, the rigorous numerical program becomes necessary for research of more general cases. In this section, simulations are performed to explore OC polarization gratings recorded at large angles.

From Fig. 5.2 we can read off the period of the formed interference patterns when the recording angle is varied. We choose 10° and 15° as examples. The angles and respective pitches are listed in Table 6.3.

$\theta_{record.}$	Λ (nm)
2.5°	4023
10°	1011
15°	678

Table 6.3: A few specific recording angles and the corresponding obtained pitches.

Just as in the study of the dependence on the thickness of the grating, we use again the two OC polarization gratings recorded at 10° and 15° . To describe the photo-induced anisotropy we use the general full matrix expression for the permittivity tensor as given by Eq. (5.48) with local tensor given by Eq. (5.33) and the transformation matrix for the local basis described in Eqs. (5.61), (5.62) and (5.63). We assume that the circular birefringence sensitivity κ_c is zero. The reading light has a wavelength of 633 nm

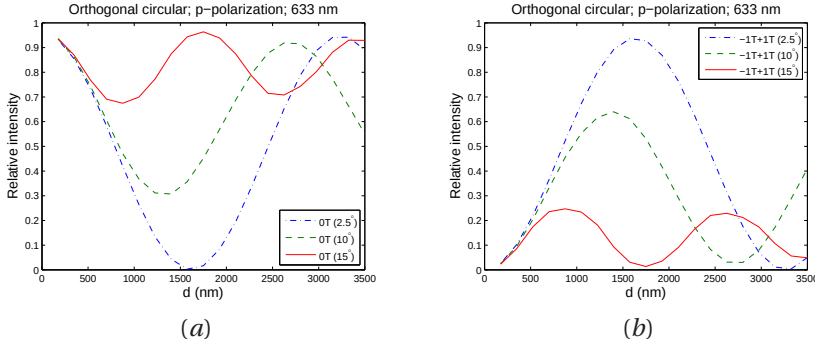


Figure 6.9: Comparison of the diffraction efficiency for a OC polarization gratings recorded at angles $\theta_{\text{record}} = 2.5^\circ$ (dash-dot lines), 10° (dashed lines) and 15° (solid lines). Panel (a) shows the diffraction efficiencies of the 0^{th} orders of the transmitted field, and (b) shows the efficiencies of the transmitted 1^{st} orders.

and is a p-polarized perpendicular incident plane wave ($\theta^i = 0^\circ, \phi^i = 0^\circ$). For varying thickness of the grating, the relative intensity is plotted separately for 0^{th} and 1^{st} orders in Fig. 6.9(a) and (b). In the figure, the blue curves are for OC grating recorded at 2.5° , the green curves for 10° and the red curves for 15° . The curves for the 2.5° OC grating are reproduced from Fig. 6.6 for comparison.

From Fig. 6.9(b), we can see that, although the shape of the curves are similar, for smaller pitch, the diffraction efficiencies of the 1^{st} orders drop considerably for the reading wavelength 633 nm. Correspondingly, more light goes into the 0^{th} order in transmission as shown in Fig. 6.9(a). At the optimum thickness for the 1^{st} orders, for the 10° OC grating, the relative intensity in the 1^{st} orders is lowered by 30%, and 70% the 15° as compared to the small recording angle 2.5° .

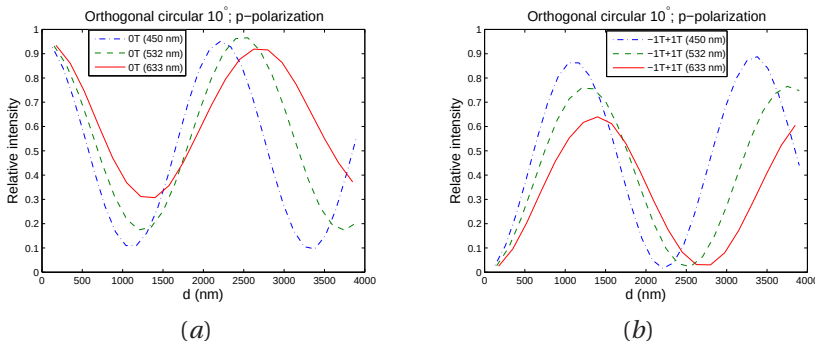


Figure 6.10: Wavelength dependence of the diffraction efficiency of an OC polarization grating recorded at 10° . In the figure, dash-dot, dashed, and solid lines indicate 450 nm, 532 nm and 633 nm respectively. The incident wave is p-polarized and at perpendicular incidence, i.e. with $\theta^i = 0^\circ, \phi^i = 0^\circ$. Panel (a) shows relative intensity for the 0^{th} order in transmission and (b) for the 1^{st} transmitted orders.

The big drop in diffraction efficiency is due to the smaller size of the pitch of the grating. This is proved by simulating the large angle grating with shorter wavelength reading beams. The thickness dependence of the diffraction efficiencies of light of shorter wavelengths, 532 nm and 450 nm, are compared to the already known data for 633 nm for OC polarization grating recorded at 10° in Fig. 6.10. The pitch of this grating is approximately $1\ \mu\text{m}$ (see Table 6.3). Light is incident perpendicularly with $\theta^i = 0^\circ, \phi^i = 0^\circ$. For the smaller reading wavelengths, the diffraction efficiency of the 1^{st} orders is much higher. For 450 nm light, 90% light goes into the 1^{st} orders. It also follows from the figure, that to optimize the performance of the grating for shorter wavelength, the grating has to be even thinner. When a liquid crystal would be used to make the grating, making these structures is challenging.

6.2.2 OL polarization gratings

Thickness Dependence and comparison between the rigorous FEM code and the Transmission Matrix Method

Until now we have been using the numerical program to study the OC polarization gratings. For OC gratings, we assumed there is no circular anisotropy in the material, i.e. $\kappa_c = 0$. Thus the dielectric permittivity is a real symmetric tensor of rank 2. For OL gratings, the circular polarization state is a very important character of the interference of the recording beams and thus of the obtained gratings. From the discussion in Section 6.1.4, we know that in the recording medium $\kappa_c = \kappa_{\parallel}/2$, the obtained grating gives only three propagating transmitted orders. For the numerical simulations the same recording condition as in Section 6.1.4 is assumed. To remind the reader, important parameters are repeated here. The recording beam consists of two orthogonal linear polarized beams, namely an s -polarized and a p -polarized beam, with amplitude $a = 1.25$ and with recording angle 2.5° . The photoanisotropy sensitivity of the material is given by $\kappa_{\parallel} = 0.2$, $\kappa_{\perp} = 0$ and $\kappa_c = 0.1$. The two real refractive indices on the basis of the eigenvectors are $n_1 = 1.6955$, $n_2 = 1.5$. To be clear, those parameters are not directly related to a real material. We simulate this ideal case to gain insight.

For simulations, the dielectric permittivity of the grating is described by Eq. (5.48) with local basis Eq. (5.81) and local dielectric tensor Eq. (5.86). In this case, the full matrix is complex hermitian. The diffraction efficiency is plotted as function of the thickness of the grating. The reading beam used in the simulations is s -polarized 633 nm red light. The results are shown in Fig. 6.11. We observe the same kind of discrepancy as in Fig. 6.6. Again, the 100% diffraction efficiency obtained with the transmission matrix method is due to the fact that the reflection is neglected. The trend of the curves agree well, and both methods predict the same thickness for maximum and minimum. This illustrates that the transmission matrix method is accurate enough to study OL polarization grating recorded with small angles for a perpendicular incidence reading and also that the rigorous FEM code is reliable.

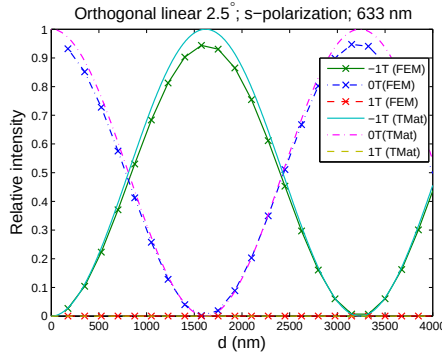


Figure 6.11: Diffraction efficiency of an OL polarization grating recorded at $\theta_{\text{record.}} = 2.5^\circ$ as function of the thickness of the grating. The reading beam has wavelength of 633 nm and is an s-polarized perpendicular incident plane wave. The solid lines are obtained by analytic calculation with the transmission matrix method and the lines with cross markers are rigorous FEM simulation results. -1T in the legend represents the -1^{st} order in the transmitted field. Analogous conventions apply to 0T and 1T.

Angular dependence

As we mentioned before, the transmission matrix method was developed only for perpendicular incidence. To investigate the performance of the grating for other angles of incidence we have performed simulations. The same grating profile as in the previous case is used. For the expected optimum diffraction efficiency for the 1^{st} orders, the thickness of the grating is set to 1575 nm. Two incidence mounts are used for simulations, the same as described in Section 6.2.1. For one the plane of incidence is the x, z -plane, i.e. $\phi^i = 0^\circ$; for the other the plane of incidence is the y, z -plane, i.e. $\phi^i = 90^\circ$. For each incidence mount, simulations are performed for s- and p-polarized reading beams separately. For both mounts, the polar angle θ^i varies from 0° to 85° . The results are displayed in Fig. 6.12.

The total intensity in the plotted orders, (namely, the transmitted 0^{th} , $\pm 1^{\text{st}}$ and the reflected 0^{th} and $\pm 1^{\text{st}}$ orders), is almost equal to 1 for the entire range of incident angles. At large incidence angle, the total is slightly lower than 1, which indicates that a small amount of light leaks into the 2^{nd} orders. As follows from all the four figures, the intensity of the reflected 1^{st} orders is zero, which means that light goes into only into the 0^{th} order in the reflected field. The insensitivity to angle of incidence for $\theta^i \leq 30^\circ$ also holds here for OL polarization gratings (as for OC polarization gratings).

More into details, let us now compare the two figures, Fig. ??(a) and ??(b). For the same incident mount $\phi^i = 0^\circ$, the polarization state of the reading beam is switched from p to s from Fig. ??(a) to Fig. ??(b). An OL polarization grating functions as a linear polarization converter, and depending on the linear polarization state of the reading beam, only one of the $\pm 1^{\text{st}}$ orders propagates in the transmitted field. That is the reason why with p-polarized reading beam we see the 1^{st} order in (a), while with

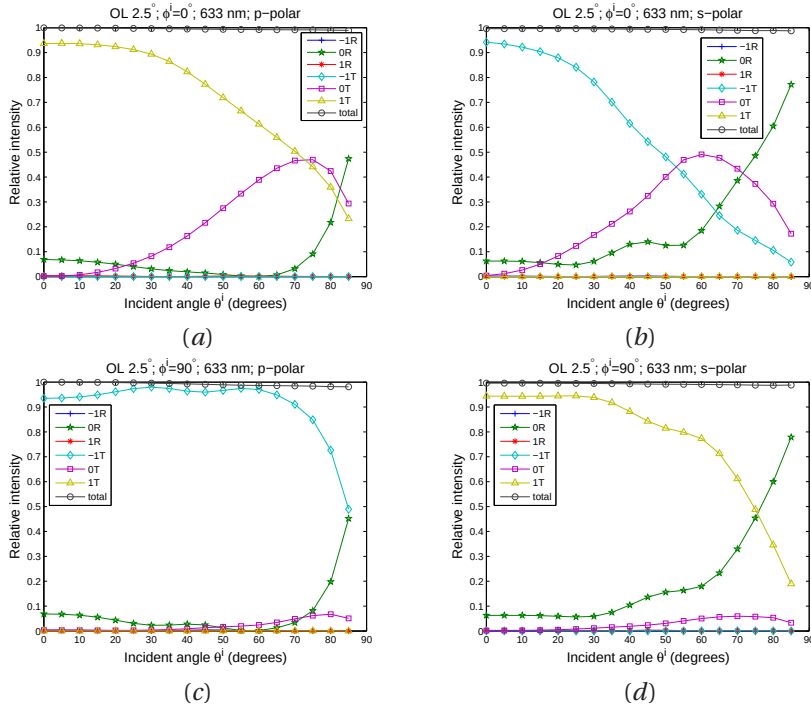


Figure 6.12: Diffraction efficiency of an OL polarization grating recorded at 2.5° as function of the incident angle. The reading beam is 633 nm, with (a) and (b) incident angles $\phi^i = 0^\circ$ and (c) and (d) $\phi^i = 90^\circ$. For both incidence mounts, θ^i varies from 0° to 85° . For (a) and (c), the incident wave is p-polarized and for (b) and (d), it is s-polarized. -1T in the legend represents the -1^{st} order in the transmitted field. Analogous conventions apply to 0T and 1T. R represents the reflected field.

s-polarized reading beam, we observe the -1^{st} order in (b).

As the incidence mount rotates to $\phi^i = 90^\circ$, with s-polarization incidence the $+1^{st}$ order is present instead of the -1^{st} as in the other mount. This can be explained by examining the electric vector of the reading beam. For $\phi^i = 0^\circ$ incidence mount, the electric field vector of s-polarization is $E_x = 0, E_y = 1, E_z = 0$ which is along the y-axis. When $\phi^i = 90^\circ$, the s-polarization is $E_x = 1, E_y = 0, E_z = 0$ which is along the x-axis. The electric field vector of s-polarization actually is rotated 90° as the incidence mount switches. The same argument applies to p-polarization.

Beyond small writing angles

Another advantage of numerical simulation is its ability to handle general polarization gratings, which are recorded with large recording angles. Here we go beyond the small angle regime, and simulate two OL polarization gratings recorded at large angles 10° and 15° . The corresponding pitch for each recording angle can be found in

Table 6.3. For three different recording angles which include the previously discussed small angle of 2.5° , the diffraction efficiencies, i.e. the relative intensities (normalized to the intensity of the incident beam) are plotted as function of thickness of the grating in Fig. 6.13 for s-polarization. Fig. 6.13(a) shows the relative intensity values for the 0^{th} transmitted order, and Fig. 6.13(b) shows the values for the 1^{st} transmitted order.

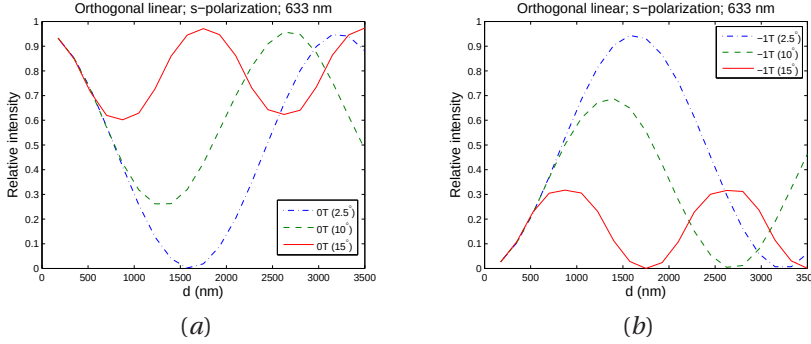


Figure 6.13: Comparison of diffraction efficiency of OL polarization gratings recorded at angles $\theta_{record} = 2.5^\circ$ (dash-dot lines), 10° (dashed lines) and 15° (solid lines). Subfigure (a) shows the diffraction efficiency of the 0^{th} transmitted order, and subfigure (b) shows the efficiency of the transmitted 1^{st} orders.

We conclude from the figure that with larger recording angles, gratings with smaller pitches are obtained, at the cost of lower diffraction efficiencies. This result is similar to what we observed for OC polarization gratings in Section 6.2.1. Analogous to the large angle OC grating, an improved diffraction efficiency can be obtained with a shorter wavelength reading beam.

Imperfect OL polarization gratings – influence of κ_c

A key point to make a *perfect* OL polarization grating is that the photoanisotropic material must be sensitive to circular polarization and the circular anisotropy sensitivity κ_c is half the linear anisotropy sensitivity. What if the material is not an active medium, i.e. $\kappa_c = 0$ or the circular anisotropy sensitivity is not big enough to satisfy the perfect condition? To answer those questions a few simulations have been conducted. For this topic, we assume the recording angle is small, 2.5° .

First, suppose the recording medium is not active with $\kappa_c = 0$. All the other parameters are kept the same as in the thickness dependence study. As function of the thickness of the grating, the relative intensity of each propagating order is plotted in Fig. 6.14. Because many orders propagate in the transmitted field, we plot the total intensity of the transmitted field and the 0^{th} order in Fig. 6.14(a), and the other higher diffracted orders in Fig. 6.14(b). First we notice that the total intensity is not 1. It is because some light is reflected into the 0^{th} order and the total amount is small. In

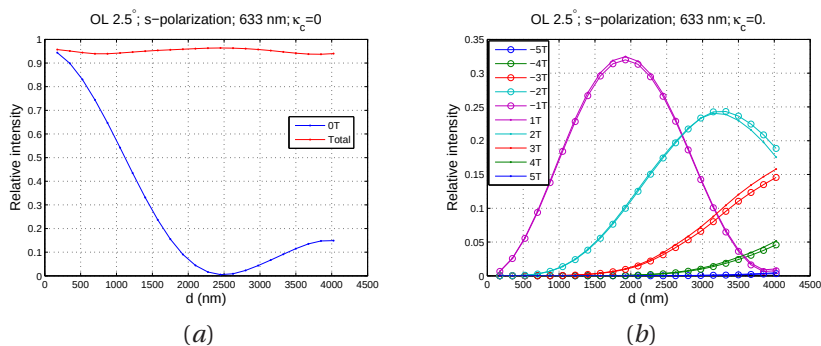


Figure 6.14: OL polarization grating with $\kappa_c = 0$. Panel (a) displays the relative intensity for 0th order and total intensity in transmission; (b) displays higher diffracted orders in transmission.

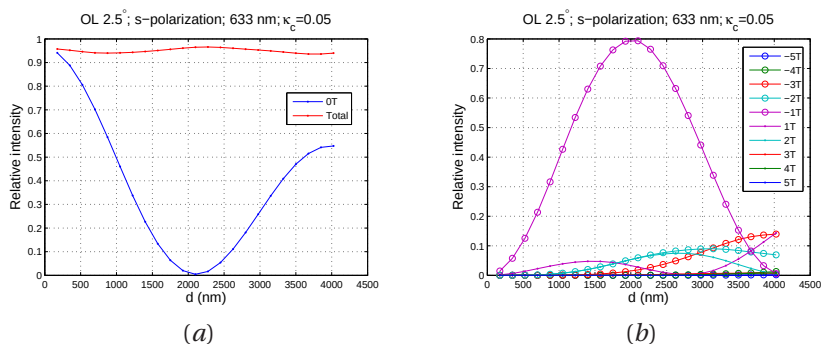


Figure 6.15: OL polarization grating with $\kappa_c = 0.05$. Panel (a) displays the relative intensity for the 0th order and total intensity in transmission; (b) displays higher diffracted orders in transmission.

the transmitted field, there are nine propagating orders of which the intensity is not zero. As the grating grows thicker, more diffracted orders emerge out of the grating in transmission. Instead of having only the -1^{st} order as in the 'perfect' case, now there are the $\pm 1^{st}$ orders with almost equal intensity. For the each pair of $\pm 2^{nd}$, $\pm 3^{rd}$ and even $\pm 4^{th}$ orders, light is almost equally distributed. In the absence of the circular sensitivity, the polarization states of each order is of a complicated elliptic nature.

Suppose that a medium has circular sensitivity, but rather weak with $\kappa_c = 0.05$ (which is a quarter of the linear one). It is not difficult to predict the diffraction efficiency. It would behave in between the perfect OL grating and the one without circular sensitivity. This is proved by the simulations shown in Fig. 6.15. With κ_c present in the material, circular polarization of the interference patterns is partly recorded. It results in that the -1^{st} transmitted order dominates the transmitted field with a maximum of 80%. The optimum thickness of the grating with respect to the maximum diffraction of the -1^{st} order is 2000 nm, which is somewhat thicker than the 'perfect' case

(1575 nm).

To conclude, the performance of OL polarization grating is very sensitive to κ_c . A small variation of it affects both the diffraction efficiency and optimal thickness of the grating. In practice, complicates the manufacture of a grating considerably.

6.3 Applications

In many modern applications, polarization management and measurement of light play an essential role. Such applications include flat panel displays, polarimetry, optical fiber communications, quantum computing, microscopy etc.. With polarization gratings, we are able to split light into orthogonal polarization states. To be specific, an OC polarization grating splits unpolarized light or linear polarized light into orthogonal circular polarization in the first orders, while an OL grating separate unpolarized light or circular polarized light into orthogonal linear polarization in the first orders. Compared to a conventional glass beam splitter with an undesirable large cube size, the polarization gratings can be used in many scaled down assemblies due to their thin film configuration.

Because almost all the light can be diffracted into the two first orders, polarization gratings can be used in a flat panel display system to create high contrast black/white states. This is illustrated in Fig. 6.16. A similar scheme has been shown and discussed in more details by Sarkissian [82] and Escuti [80].

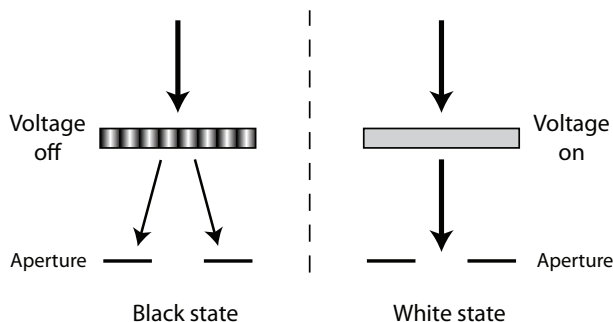


Figure 6.16: A polarization grating is used in a design of a single pixel for flat panel display systems. When the external voltage is off as on the left side, the grating diffracts light into two first orders which are blocked by the aperture, thus a black state is created. When the voltage is on as on the right, it reorients the LC molecules along the direction of the electric field and the grating structure disappears. Light propagates straight without diffraction resulting in a white state. The high diffraction efficiency (almost 100%) guarantees a high contrast ratio.

In the late 90's, azo polymers have been studied as recording media for both non-binary digital optical storage and holographic storage [91, 92]. Many studies [93, 74,

94, 95] on polarization holographic optical data storage have been carried out until nowadays.

6.4 Conclusions

In the preceeding Chapter 5 and the current Chapter 6 photoanisotropic polarization gratings have been studied extensively.

In Chapter 5, the recording of photoanisotropic polarization gratings was investigated beyond the small recording angle regime. Formulas for the interference of two plane waves have been derived for arbitrary polarizations and for arbitrarily large recording angles. When the recording angle between the two recording beams is large, the interference pattern of two orthogonally polarized beams consists of periodic modulations of polarization states and intensity with the electric field not remaining in one plane. The spatial varying plane of the polarization ellipse of the interference pattern rotates with respect to the 3D Cartesian basis. Thus we define for each position over one period of modulation a local basis with its coordinates coinciding with the major and minor axes of the polarization ellipse. With the aids of the local basis, photo-induced anisotropy of photoanisotropic materials has been explained. If the material is photoactive and also the ellipticity of the polarization is not zero, a complex hermitian type of dielectric permittivity tensor on the local basis is formed. For this kind of tensor there exists a basis of eigenvectors which are complex in general, with three eigenvalues which are real numbers.

In Chapter 6, the reading processes from photoanisotropic polarization gratings are studied by analytic formulations and also by numerical simulations. A derivation of the transmission matrix method for polarization gratings has been carried out in general form. By using the result from Chapter 5, one can find two refractive indices for the two propagating wave vectors locally for each point over one period of modulation. The two indices are real and are defined on the basis of the eigenvector of the dielectric permittivity tensor. Neglecting the reflection at interfaces, the transmission matrix is obtained by calculating the phase shift in the propagating directions with the two real refractive indices. After applying basis rotation by using the transform matrices, the transmission matrix on the global Cartesian basis is finally achieved. In this thesis, the transmission matrix is only developed for perpendicular incidence, but it is not difficult to extend the method to oblique incidence.

In this thesis, two special photoanisotropic polarization gratings are analyzed in detail, namely the polarization gratings formed by two orthogonally circularly (OC) polarized beams and those formed by two orthogonally linearly (OL) polarized beams. The interference of the two special cases have been studied in Chapter 5 for both small and large recording angles. In Chapter 6, the transmission matrices have been derived for small recording angles. For polarization gratings formed at large recording angles, numerical simulations have been performed by using the program Cyclops which is based on the finite element method. The OC polarization grating functions as a circular polarization converter which transfers the incident right-handed circular

polarization to left-handed and vice versa. The OL polarization grating functions as a linear polarization converter which transfers the incident linear polarization state to an orthogonal state of linear polarization in the transmitted field. Simulation results for polarization gratings recorded at larger angles have been presented for the first time.

We summarize the basic properties of the OC and the OL polarization gratings formed with small recording angles.

- (a) OC polarization gratings
 - The permittivity on the local basis $\underline{\underline{\epsilon}}$ is diagonal and independent of x , thus eigenvalues and refractive index are independent of x .
 - The local basis depends on x , hence $\underline{\underline{\epsilon}} = V\underline{\underline{\epsilon}}V^{-1}$ depends on x .
 - The eigenvectors of $\underline{\underline{\epsilon}}$ are independent of x , but on the global cartesian basis they vary with x .
- (b) OL polarization gratings
 - The permittivity on the local basis $\underline{\underline{\epsilon}}$ depends on x , hence refractive indices depend on x .
 - The local basis depends on x , hence eigenvectors depend on x .
 - However, if $\kappa_c = \frac{\kappa_{||} - \kappa_{\perp}}{2}$, then it happens that the refractive indices do not depend on x . The eigenvectors still depend on x .

Simulation results have shown that with polarization gratings very high diffraction efficiencies can be achieved, higher than 95% for appropriate thicknesses of the grating and only for three diffraction orders in the transmitted field, namely the $\pm 1^{st}$ and 0^{th} orders. The diffraction efficiencies do not have strong dependence on the incident angles of the reading beams. Within a range of 20° of incident angles, the diffraction efficiencies remain higher than 90% for both OC and OL polarization gratings. Larger recording angles result in smaller pitches at the cost of lower diffraction efficiency for the same wavelength of the reading beam. In this case, reading beams with shorter wavelength perform better. For OL polarization grating, the circular polarization sensitivity of the recording material is critical for a high diffraction efficiency. The influence of the parameter has been studied by simulations.

Some possible applications of photoanisotropic polarization gratings have been presented briefly. It is meant to evoke other novel invention or creation of products by adapting this promising optical component.

For the future research, using recording fields that consist of more than two interfering plane waves should be considered.

Appendix A

Orthogonal condition for **a** and **b**

By substituting (1.8) and (1.9) into (1.12) and (1.13), we find:

$$\mathbf{a}(\mathbf{r}) = \begin{pmatrix} A_x(\mathbf{r}) \cos[\phi_x(\mathbf{r}) + \psi(\mathbf{r})] \\ A_y(\mathbf{r}) \cos[\phi_y(\mathbf{r}) + \psi(\mathbf{r})] \\ A_z(\mathbf{r}) \cos[\phi_z(\mathbf{r}) + \psi(\mathbf{r})] \end{pmatrix}, \quad (\text{A.1})$$

$$\mathbf{b}(\mathbf{r}) = \begin{pmatrix} A_x(\mathbf{r}) \sin[\phi_x(\mathbf{r}) + \psi(\mathbf{r})] \\ A_y(\mathbf{r}) \sin[\phi_y(\mathbf{r}) + \psi(\mathbf{r})] \\ A_z(\mathbf{r}) \sin[\phi_z(\mathbf{r}) + \psi(\mathbf{r})] \end{pmatrix}. \quad (\text{A.2})$$

For **a** and **b** to be orthogonal, i.e. $\mathbf{a} \cdot \mathbf{b} = 0$, ψ must satisfy,

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= A_x^2 \cos(\phi_x + \psi) \sin(\phi_x + \psi) + A_y^2 \cos(\phi_y + \psi) \sin(\phi_y + \psi) \\ &\quad + A_z^2 \cos(\phi_z + \psi) \sin(\phi_z + \psi) \\ &= 0. \end{aligned} \quad (\text{A.3})$$

By using,

$$\begin{aligned} \cos(\phi_x + \psi) \sin(\phi_x + \psi) &= \frac{1}{2} \sin(2\phi_x + 2\psi) \\ &= \frac{1}{2} \sin 2\phi_x \cos 2\psi + \frac{1}{2} \cos 2\phi_x \sin 2\psi, \end{aligned} \quad (\text{A.4})$$

we find,

$$\begin{aligned}
 \mathbf{a} \cdot \mathbf{b} &= \frac{1}{2} A_x^2 [\sin(2\phi_x) \cos(2\psi) + \cos(2\phi_x) \sin(2\psi)] \\
 &\quad + \frac{1}{2} A_y^2 [\sin(2\phi_y) \cos(2\psi) + \cos(2\phi_y) \sin(2\psi)] \\
 &\quad + \frac{1}{2} A_z^2 [\sin(2\phi_z) \cos(2\psi) + \cos(2\phi_z) \sin(2\psi)] \\
 &= \frac{1}{2} \left[A_x^2 \sin(2\phi_x) + A_y^2 \sin(2\phi_y) + A_z^2 \sin(2\phi_z) \right] \cos(2\psi) \\
 &\quad + \frac{1}{2} \left[A_x^2 \cos(2\phi_x) + A_y^2 \cos(2\phi_y) + A_z^2 \cos(2\phi_z) \right] \sin(2\psi). \tag{A.5}
 \end{aligned}$$

Let

$$c = A_x^2 \cos(2\phi_x) + A_y^2 \cos(2\phi_y) + A_z^2 \cos(2\phi_z), \tag{A.6}$$

$$s = A_x^2 \sin(2\phi_x) + A_y^2 \sin(2\phi_y) + A_z^2 \sin(2\phi_z), \tag{A.7}$$

Hence $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if

$$\tan 2\psi = -\frac{s}{c}. \tag{A.8}$$

Without restricting the generality, we may choose ψ such that $0 \leq \psi \leq \pi/2$.

We can also express ψ in terms of the vectors $\mathbf{p}$ and $\mathbf{q}$.

$$\mathbf{a} = \mathbf{p} \cos \psi - \mathbf{q} \sin \psi, \tag{A.9}$$

$$\mathbf{b} = \mathbf{p} \sin \psi + \mathbf{q} \cos \psi. \tag{A.10}$$

Then

$$\begin{aligned}
 \mathbf{a} \cdot \mathbf{b} &= (\mathbf{p} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q}) \cos \psi \sin \psi + \mathbf{p} \cdot \mathbf{q} (\cos^2 \psi - \sin^2 \psi) \\
 &= \frac{1}{2} (\mathbf{p} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q}) \sin(2\psi) + \mathbf{p} \cdot \mathbf{q} \cos(2\psi). \tag{A.11}
 \end{aligned}$$

Hence

$$\mathbf{a} \cdot \mathbf{b} = 0 \quad \Rightarrow \quad \tan(2\psi) = -\frac{2\mathbf{p} \cdot \mathbf{q}}{\mathbf{p} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q}}. \tag{A.12}$$

Furthermore, if $\mathbf{p} \cdot \mathbf{q} = 0$, then we choose $\psi = 0$ if $|\mathbf{p}| \geq |\mathbf{q}|$, and $\psi = \pi/2$ if $|\mathbf{p}| < |\mathbf{q}|$. Suppose that $\mathbf{p} \cdot \mathbf{q} \neq 0$, then we consider the case of $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b}$.

$$\begin{aligned}
 \mathbf{a} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b} &= (\mathbf{p} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q}) (\cos^2 \psi - \sin^2 \psi) - 2\mathbf{p} \cdot \mathbf{q} \cos \psi \sin \psi \\
 &= \frac{(\mathbf{p} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q})^2 + 2(\mathbf{p} \cdot \mathbf{q})^2}{\mathbf{q} \cdot \mathbf{p} - \mathbf{q} \cdot \mathbf{q}} \cos 2\psi. \tag{A.13}
 \end{aligned}$$

Hence $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b}$ if and only if $\cos 2\psi = 0$ or $\mathbf{p} \cdot \mathbf{p} = \mathbf{q} \cdot \mathbf{q}$ and $\mathbf{p} \cdot \mathbf{q} = 0$. In the later case, we have defined $\psi = 0$. Thus, suppose that $\mathbf{p} \cdot \mathbf{q} \neq 0$, then $\mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b}$ if and only if $\psi = \pi/4$.

Appendix B

Circular birefringence and circular dichroism

In physical chemistry [96], an *optically active* substance is a substance that rotates the direction of linear polarized light. A linear polarized plane wave can be decomposed into two orthogonally circularly polarized components, namely left- and right-handed circular polarizations. On entering the medium, the two components propagate with different speeds. Analogous to a linear birefringent medium who has two refractive indices for two orthogonal linear polarizations, the optically active medium possesses two refractive indices for two orthogonal circular polarizations, namely n_R for right-handed circular polarization and n_L for left-handed circular polarization. The difference between the two $\Delta n = |n_R - n_L|$ is defined as *circular birefringence*. The phase difference between the two components when they emerge from the medium is $\Delta\theta = (n_R - n_L)2\pi d/\lambda$, where, d is the thickness of the medium, and λ is the wavelength of the light in vacuum. Depending on the travel distance or the thickness of the medium, when the two components leave the medium their superposition can give a linear polarization which has an angle $\Delta\theta$ relative to that of the incident linear polarization.

From the point of view of chemistry, why the refractive indices depend on the handedness of the light can be explained by the handedness dependence of the polarizabilities of the materials. An applied electric field can distort a molecule as well as align its permanent electric dipole moment. The *induced dipole moment*, μ^* , is proportional to the field strength E , and can be written as $\mu^* = \alpha E$, where the constant α is the so-called polarizability of the molecule. The greater the polarizability, the larger is the induced dipole moment. If a molecule is helical (such as a polypeptide α -helix) or a crystal has molecules in a helical arrangement (as in a cholesteric liquid crystal), its polarizability depends on whether or not the electric field of the incident light rotates in the same sense as the helix.

If the medium has a difference in absorption intensities for right- and left-handed

circularly polarized light, the property of the material is known as *circular dichroism*. Circular dichroism (CD) spectroscopy measures differences in the absorption of left-handed polarized light versus right-handed polarized light. CD spectroscopy is a supplement to visible and UV spectroscopy. For some materials, the CD spectra are distinctly different, whereas there is little difference between their absorption spectra.

In general, the dielectric permittivity of the optical active medium is a full complex tensor. When absorption losses can be neglected, the tensor becomes hermitian as discussed in Chapter 2. In general, in such a medium, the eigenstates of the electric field vector are elliptically polarized. The case when the eigenstates are orthogonal circular polarizations is a special one. In order to have the property, i.e. circular birefringence, the permittivity tensor of the medium has to satisfy the conditions that derived in Ref. [13].

Appendix C

Orthonormal basis on $\mathbb{C}^3$

$\mathbb{C}^3$ is the set of complex vectors $\mathbf{a} = (a_1, a_2, a_3)$ with $a_i \in \mathbb{C}$. The standard scalar product on $\mathbb{C}^3$ is defined by:

$$\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3} = a_1 b_1^* + a_2 b_2^* + a_3 b_3^*. \quad (\text{C.1})$$

Hence,

$$\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3} = \mathbf{a} \cdot \mathbf{b}^* \quad (\text{C.2})$$

where the dot-product is without complex conjugation:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad (\text{C.3})$$

Note that

$$\langle \mathbf{b}, \mathbf{a} \rangle_{\mathbb{C}^3} = \langle \mathbf{a}, \mathbf{b} \rangle_{\mathbb{C}^3}^*. \quad (\text{C.4})$$

An orthonormal basis of $\mathbb{C}^3$ is a set of three complex vectors $\hat{\xi}, \hat{\eta}, \hat{\zeta} \in \mathbb{C}^3$ such that

$$\left. \begin{aligned} \langle \hat{\xi}, \hat{\xi} \rangle_{\mathbb{C}^3} &= \langle \hat{\eta}, \hat{\eta} \rangle_{\mathbb{C}^3} = \langle \hat{\zeta}, \hat{\zeta} \rangle_{\mathbb{C}^3} = 1; \\ \langle \hat{\xi}, \hat{\eta} \rangle_{\mathbb{C}^3} &= \langle \hat{\xi}, \hat{\zeta} \rangle_{\mathbb{C}^3} = \langle \hat{\eta}, \hat{\zeta} \rangle_{\mathbb{C}^3} = 0. \end{aligned} \right\} \quad (\text{C.5})$$

Any $\mathbf{a} \in \mathbb{C}^3$ can be written on this basis as:

$$\mathbf{a} = a_{\xi} \hat{\xi} + a_{\eta} \hat{\eta} + a_{\zeta} \hat{\zeta} \quad (\text{C.6})$$

where

$$\left. \begin{aligned} a_{\xi} &= \langle \mathbf{a}, \hat{\xi} \rangle_{\mathbb{C}^3} = \mathbf{a} \cdot \hat{\xi}^* \\ a_{\eta} &= \langle \mathbf{a}, \hat{\eta} \rangle_{\mathbb{C}^3} = \mathbf{a} \cdot \hat{\eta}^* \\ a_{\zeta} &= \langle \mathbf{a}, \hat{\zeta} \rangle_{\mathbb{C}^3} = \mathbf{a} \cdot \hat{\zeta}^* \end{aligned} \right\} \quad (\text{C.7})$$

Let $\underline{\epsilon}$ be a complex matrix: $\underline{\epsilon} = \epsilon_{ij}$, $\epsilon_{ij} \in \mathbb{C}$. $\underline{\epsilon}$ is called Hermitian if

$$\epsilon_{ij} = \epsilon_{ji}^* \quad \text{for all } i, j \quad (\text{C.8})$$

For a Hermitian matrix there exists an orthonormal basis $\hat{\xi}, \hat{\eta}, \hat{\zeta}$ on which it is diagonal, i.e. such that if $\mathbf{a} \in \mathbb{C}^3$ and $\mathbf{a} = a_{\xi} \hat{\xi} + a_{\eta} \hat{\eta} + a_{\zeta} \hat{\zeta}$, then

$$\underline{\underline{\epsilon}} \mathbf{a} = \epsilon_{\xi} a_{\xi} \hat{\xi} + \epsilon_{\eta} a_{\eta} \hat{\eta} + \epsilon_{\zeta} a_{\zeta} \hat{\zeta} \quad (\text{C.9})$$

where $\epsilon_{\xi}, \epsilon_{\eta}, \epsilon_{\zeta}$ are the eigenvalues (which are real!)

Appendix D

Stokes parameters

Consider a time-harmonic plane wave as described in Section 1.3. If the wave propagates in the z -direction on the cartesian basis, only the x - and y -components of $\mathbf{E}$ are different from zero. The Stokes parameters of this field are defined as

$$S_0 = E_x E_x^* + E_y E_y^* = |E_x|^2 + |E_y|^2, \quad (\text{D.1a})$$

$$S_1 = E_x E_x^* - E_y E_y^* = |E_x|^2 - |E_y|^2, \quad (\text{D.1b})$$

$$S_2 = E_x E_y^* + E_x^* E_y = 2|E_x||E_y|\cos\delta, \quad (\text{D.1c})$$

$$S_3 = iE_x E_y^* - iE_x^* E_y = 2|E_x||E_y|\sin\delta, \quad (\text{D.1d})$$

where δ is the phase difference between E_y and E_x . The parameter S_0 is proportional to the total intensity of the field. The ratio of S_1/S_0 is the degree of linear polarization along the x -axis. The ratio of S_2/S_0 describes the extend to which the field is linear polarized along direction a angles 45° to the x -axis. The ratio of S_3/S_0 represents the degree of circular polarization [9, 90].

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Delft, February 2009

About the author

Man Xu was born in Zhengzhou, China on July 28, 1979. Between 1998 and 2002, she did her Bachelor study in Harbin University of Science and Technology, China. In September 2002, she received a scholarship from Delft University of Technology, and started her Master study in The Netherlands. She completed the study in 2004 in the Optics group under the supervision of Prof.Dr. H.P. Urbach. The subject of the graduation project was *Diffraction Gratings Applied to Liquid Crystal on Silicon – wire-grid gratings*. This project was mainly carried out as an internship in Philips Research Laboratory in Eindhoven, The Netherlands.

At the end of 2004, Man started her Ph.D research in the Optics group at Delft University of Technology under the supervision of Prof.Dr. H.P. Urbach. The topic of the research was *Diffraction Optics Applied to Flat Panel Display Systems*. The main results of the research are presented in this thesis, entitled: *Diffraction Optics of Anisotropic Polarization Gratings*.