Stability and Robustness Analysis of Acyclic Timetable

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Abstract

To satisfy the growing passenger transportation demands and improve the service quality in railway system, a more stable and robust timetable needs to be designed while considering highly utilized capacity. Acyclic timetable is extensively applied in large railway networks. In order to acquire the quality of timetable, analytical timetable stability analysis software PETER (Performance Evaluation of Timed Events in Railways) is used to analyse timetable stability and robustness with delay impact, delay sensitivity and delay propagation. The method has been applied to the Yangtze River Delta of the Chinese railway network.

Keywords

Acyclic timetable, Stability, Robustness

1 Introduction

The railway timetabling problem is a critical issue in railway operation. With the development of railway systems, this problem has become more complex and difficult to solve. After years of research, two main types of timetables came into practice: cyclic timetables which show the same pattern of train services in each period, and acyclic timetables which do not include line regularity but instead departure times are defined mostly based on varying travel demand. Due to the diversity of railway networks and passenger travel behaviour, which timetable is suitable for a certain network needs to be discussed. With the development of Chinese railway network, plenty of new high-speed rail lines occur, and which type of timetable should be applied needs to be studied. Therefore, this paper mainly focuses on timetable stability and robustness of an existing high-speed railway networks, aiming to show the advantage and disadvantage of acyclic timetable.

The railway timetabling problem can be defined as matching train lines with given frequencies to the available infrastructure by allocating to each train line a feasible schedule of arrival and departure times at the consecutive served stations while considering safety and operational constraints. As for acyclic timetables, many literatures considers mixed integer linear programming formulations in which the arrival and departure times are expressed by continuous variables, and train orders are represented by binary variables. Branch-and-bound techniques was used for solving this model by Jovanovic and Harker (1991), and heuristic techniques of local search, genetic algorithms, tabu search, and hybrid algorithms were compared for finding a feasible solution by Higgings and Kozan (1997). Caprara et al. (2002, 2006) designed integer linear programming models based on a graph representation, which discretized time into minutes and used Lagrangian relaxation to derive bounds for the optimal solution. In general, both types of timetables are already intensively studied. For cyclic timetables, the Periodic

Event Scheduling Problem (PESP) is frequently used for macroscopic scheduling. PESP was first proposed by Serafini and Ukovich(1989), and later it was extensively studied by many railway researchers, like Nachtigall (1996), Lindner (2000), Peeters (2003), and Liebchen (2006). Based on this model, the robustness and stability of cyclic timetables were studied by Goverde (2007) and Kroon et al. (2007). Goverde (2007) also developed the analytical timetable stability analysis software PETER (Performance Evaluation of Timed Events in Railways). However, the research always was done in a separated way, except in Caimi's (2009) work about partial cyclic patterns, so that the comparison of cyclic and acyclic timetables has not been discussed in literature. And when mentioned about timetable stability, it is always along with cyclic timetable.

From the literature, this timetabling problem has been well-studied for both cyclic and acyclic patterns. The cyclic pattern is mainly used in Europe for passenger trains, especially in railway networks with plenty of commuting passengers. The acyclic pattern is mainly used for freight trains and some other big networks of passenger services, like heavy-traffic corridors or long-distance trains. As mentioned above, the cyclic timetable has the same pattern in each period for a certain line, which provides a convenient service of regular stop patterns and consistent transfer connections for passengers. However, the acyclic timetable could provide a flexible service with direct trains for passengers of long distances and ODs (origins and destinations) of limited passenger flows. However, for big rail network, the advantage and disadvantage is not that straightforward. This paper studies the performance of acyclic timetable on stability and robustness, which gives a basic analysis of the characteristic of acyclic timetable.

This paper is organised as follows. Chapter 2 describes the mathematical model for timetable and the max-plus algebra representation for the analytical approach. Chapter 3 analyses the stability and robustness of acyclic timetable considering delay sensitivity, delay impact and delay propagation. Chapter 4 provides the final conclusions of the of the timetable performance.

2 Analytical approach

In railway systems, ex-ante timetable analysis includes stability, feasibility, robustness, resilience and efficiency (Goverde and Hansen, 2013). Stability could be expressed as the ability to return to the scheduled timetable after disruptions. Feasibility means that trains must be able to run as scheduled without any conflicts, and robustness means that trains must be able to deviate a bit from schedule. Timetable stability and robustness depends on the distribution of time supplements and buffer times. The stability and robustness analysis could test network dependencies and therefore is essential for reliable operations.

With the purpose of analysing timetable stability and robustness, the analytical software PETER has been introduced. It is developed by Goverde (2005) based on max-plus algebra, where the railway traffic network was modelled as a discrete event dynamic system, and in particular as a timed-event graph. It is a macroscopic railway traffic model including periodic variables of event times (arrivals, departures and passage times) and precedence constraints. For example, the actual departure time is the maximum of the scheduled departure time, arrival time plus dwell time, arrival time of feeder trains plus transfer times, and the event times of conflicting trains plus minimum headway times.

In this approach, max-plus spectral analysis and critical path algorithms are used for evaluating the timetable and robustness (Goverde, 2007). Moreover, the recursive equations of the max-plus model and bucket-based graph algorithms are used to compute delay propagation effectively (Goverde, 2010). Based on this methodology, the three complementary functionalities of critical circuit analysis, recovery time analysis and delay propagation analysis are implemented in PETER, aiming to identify and quantify the critical cycles in the network with maximal mean cycle time or minimal mean slack, the minimal total slack time between any pair of train events, and the propagation over time and space with different initial delays, respectively.

Input data is a fundamental part to analyse the stability and robustness of timetable. Table 1 shows the format of the input data required by PETER. Most of these parameters are self-explaining. Stations are all timetable nodes where events take place and include actual stations and junctions (e.g. merging and crossing railway lines). The model is built up from line segments consisting of a train line number and the successive segments between consecutive timetable nodes on the line. The minimum running times are obtained from the scheduled running time plus the run deviation, i.e., a running time supplement corresponds to a negative run deviation, and likewise for the dwell time. Dwell types are train runs (no stop), stops and turns. Line types are e.g. local, intercity, high-speed and freight trains. Connection types are passenger or rolling stock connections.

Table 1: Input data format of PETER

Basic parameters	Cycle time, recovery threshold
Stations	Station name (abbreviation), coordinate, type
Train lines	Line number, segment, origin, destination, scheduled departure time, scheduled running time, run deviation, scheduled dwell time, dwell deviation, dwell type, line type
Connections	Feeder line, feeder segment, connecting line, connecting segment, connection time, connection type
Headway times	Line1, segment1, line2, segment2, minimum headway, event type1, event type 2

3 Acyclic timetable

3.1 Data gathering

Acyclic timetables have been used in the Chinese railway network for a long period, and are as popular as cyclic timetables in the Netherlands. In order to make a comparison of the advantages and disadvantages of the two types of timetables, the existing timetable from the Chinese high-speed railway network in the Yangtze River Delta has been derived from the database as a case study of an acyclic pattern (24 hours' data for a regular day, regardless of festival, holiday, and weekends). In this network, there are three corridors: Shanghai - Nanjing (SH-NJ) with distance of 301 km, Shanghai - Hangzhou (SH-HZ) of 159 km, and Nanjing - Hangzhou (NJ-HZ) of 256 km.

In total 444 trains of both directions and 41 stations were selected, with different train operational speeds and station types. As this timetable has an acyclic pattern, every pair of trains (both directions) is treated as one train line. Table 2 shows the number of trains running in the three corridors. High-speed trains and bullet trains are two different train types with maximum speed of 300 km/h and 250km/h, respectively. We distinguish between *inner* line train and *crossing* line trains based on whether the train line operates only on these high-speed railway lines or exceeding to external railway lines. Thus, an inner train mainly serves passengers inside the corridor, whereas a crossing train serves especially for connections between different corridors. In this timetable, almost all bullet trains are crossing line trains with a longer travel distance and slower speed.

Table 2: Number of trains operating in the existing timetable

Corridor		**SH-NJ**	**SH-HZ**	**NJ-HZ**	**SH-NJ &SH-HZ**	**Total**
High-speed train	On line train	131	37	11	2	181
	Crossing line train	15	33	65	6	119
Bullet train	On line train	0	2	2	0	4
	Crossing line train	37	78	10	15	140
Total		183	150	88	23	444

The original timetable is passenger-oriented and specifies only departure time, arrival time and dwell time of stopping stations for each train line. Therefore the passage times at non-stopping stations need to be calculated. The running time between successive stations is obtained from the train lines having stops on both stations, with an additional estimation of the time losses due to acceleration and deceleration. Then the passage time is computed, which may have a slight error due to the approximate treatment. Since the actual running time supplements are unknown, 4% of scheduled running time is assumed as the time supplement. Finally, conflicts in headway constraints at stations are detected from forbidden overtakings on the open tracks. Figure1 and Figure 2 show the time-distance graph of the macroscopic conflict-free timetable on the three corridors.

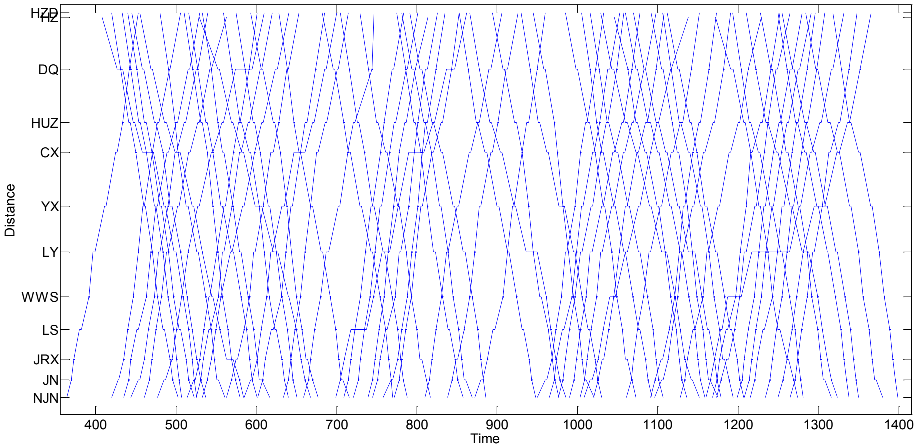

Figure 1: Time- distance graph of NJ-HZ corridor

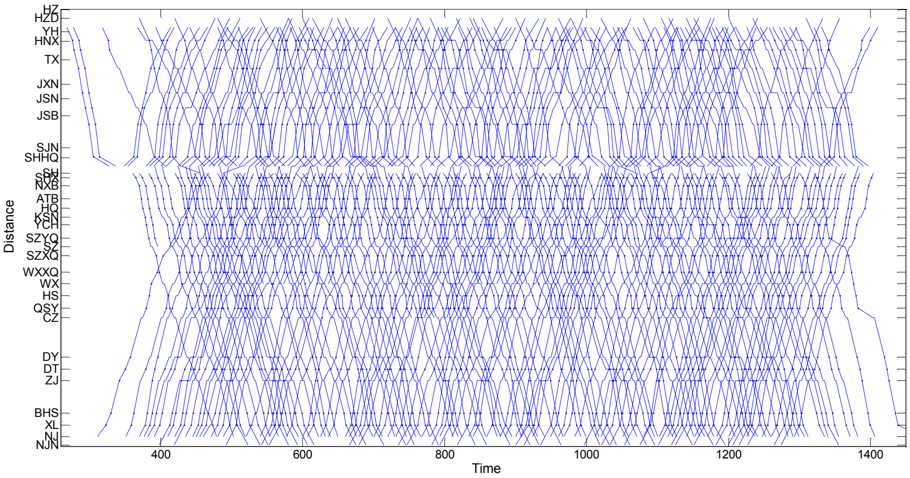

Figure 2: Time- distance graph of SH-NJ corridor and SH-HZ corridor

In order to analyse the existing timetable, the input should be transformed into PETER's generic input format. To meet the model's requirement of a cyclic timetable, 1440 minutes is set as the cycle time of a period. Hence, an acyclic timetable is viewed as a one day period. In addition, the maintenance time for high-speed railway infrastructure should be more than 240 minutes which is always held between 00:00 to 5:00. This means that the real cycle time is 1140 minutes after subtracting 300 minutes. Furthermore, according to the Chinese high-speed railway operation norms, the layover time of turning trains should be at least 15 minutes, and the headway between successive trains should be at least 3 minutes, except for some overtaking with passing trains with a minimal arrival-through headway and through-departure headway of 2 minutes. Table 3 summarizes the data input in PETER.

Table 3: Input data of PETER

Timetable points	41	stations, stops
Train line segments	6208	1796 stops, 21 turns, 3968 runs, 423 ends
Connections	192	rolling stock connections
Headway constraints	24648	arrivals, departures, in/outbound, overtaking

3.2 Timetable stability analysis

A circuit is a closed sequence of events and processes with the first event equal to the last event of a later period. The cycle mean of a circuit is computed as the total process time divided by the number of related periods and a circuit with maximum cycle mean over all circuits in the network is considered as a critical circuit. To test the timetable stability, the maximum cycle mean is used to compare with the timetable period (cycle time of a period). If it is smaller, then the timetable is structurally stable.

The critical circuit analysis shows that only one critical circuit exists with critical cycle mean of 834 minutes and 12 seconds, which implies that the timetable is stable considering the definition of structurally stable timetable. Figure 1(a) depicts the critical circuit in red colour where nodes represent stations and arcs are line segments connecting stations. One critical circuit infers there is only one component in the network, and Figure1(b) illustrates the component (in black) of the critical circuit, showing that all nodes are reachable from the critical circuit. To be specific, the network is entirely connected, and any delay of a certain line could propagate to other lines. Especially, if a serious delay occurs in the first critical line, secondary delays will be generated throughout the whole network. The critical circuit consists of 112 events with a mixed sequence of 24 train lines on the corridors SH-HZ and SH-NJ, including 3 rolling stock connections, 37 stops and 28 headway constraints. Table 4 shows some events in the critical circuit with description of line segment, event time, event type, process time and process type. From a macro level, the network capacity utilization equals the critical cycle mean divided by timetable period length. Hence, the timetable is stable with a capacity utilization of 73.2%, resulting in enough slack time to compensate delays.

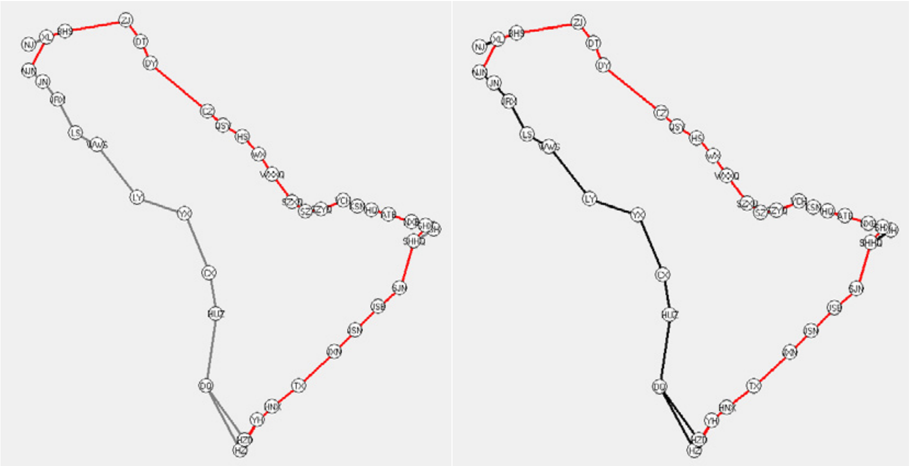

Figure 3: (a) Critical circuit and (b) the component of critical circuit

Table 4: The critical circuit

Line segment	Event time	Event type	Process time	Process type
103-00102 SJN-SJN	476:00	Departure	3:00	Headway
147-00102 SJN-JSB	481:00	Through	8:38	Run
147-00102 JSB-JSB	490:00	Arrival	10:00	Stop
147-00103 JSB-JSN	500:00	Departure	8:38	Run
147-00104 JSN-JSN	509:00	Through	2:00	Headway
155-00104 JSN-JSN	511:00	Through	3:00	Headway
149-00104 JSN-JXN	515:00	Through	8:38	Run
…	…	…	…	…
126-10108 SHHQ-SHHQ	1378:00	Arrival	3:00	Headway
107-10108 SHHQ-SHHQ	1390:00	Arrival	15:00	Coupling
107-00101 SHHQ-SHHQ	425:00	Departure	3:00	Headway
170-00101 SHHQ-SJN	430:00	Departure	14:24	Run
170-00101 SJN-SJN	445:00	Arrival	2:00	Stop
170-00102 SJN-SJN	447:00	Departure	3:00	Headway
153-00102 SJN-SJN	454:00	Through	3:00	Headway
168-00102 SJN-SJN	468:00	Through	3:00	Headway
103-00101 SJN-SJN	475:00	Arrival	1:00	Stop

3.3 Timetable robustness analysis

Recovery time analysis and delay propagation analysis are proposed to evaluate the robustness for a deterministic timetable. The recovery time between two events is the minimal slack time from a certain event to another event, or equivalently the maximal delay of this event to still ensure a punctual event time of the other event. The delay propagation tests how initial delays influence other events in the network.

Recovery time analysis

Delay sensitivity and delay impact are two aspects of recovery time analysis (Goverde, 2007). Delay sensitivity refers to the recovery times from all preceding events to some event, while delay impact refers to the recovery times from a certain event to all other reachable events. We focus on departure events.

For example, take train line segment 165-00104 ZJ-DY from the critical circuit. The visualizations of delay sensitivity and delay impact are shown in Figure 4, with darker colours meaning less recovery time. Since this event departs from ZJ to DY, the punctuality of this departure has a direct impact on the departure events from DY as well as some stations beyond, which is observed from the dark colours in Figure 4(b). Futhermore, it can be observed that 15 minutes delay would have a huge influence to the whole network.

Ten events are within the impact area of two minutes, while ten events are within the sensitivity area of one minute (see Table 5). In this table the events are indicated by their line segment and event type (Arrival, Departure or Through event). It can be seen that all of the impact events are from the same train line within this one minutes impact area, and two preceding events have zero recovery time to segment 165-00104 ZJ-DY. It can be concluded the time supplements for this line are quite small, and moreover that this segment is connected closely to others, which could lead to a disruption for the whole network.

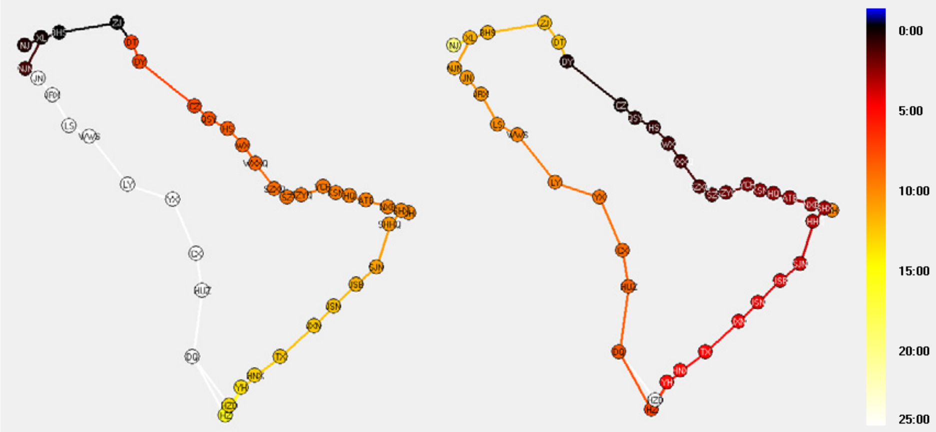

Figure 4: (a) Delay sensitivity and (b) delay impact of 165-00104 ZJ-DY

Table 5: Recovery time of delay sensitivity and delay impact

Delay impact		**Delay sensitivity**	
Event	**Recovery time**	**Event**	**Recovery time**
165-00105 DT-DY T	0:17	165-00103 BHS-ZJ A	0:00
165-00105 DT-DY A	0:31	344-00104 ZJ-DT T	0:00
165-00106 DY-CZ D	0:31	344-00103 BHS-ZJ T	0:17
165-00106 DY-CZ A	1:07	344-00102 XL-BHS T	0:24
165-00107 CZ-WX D	1:07	165-00103 BHS-ZJ T	0:29
165-00108 QSY-HS T	1:19	324-00103 BHS-ZJ T	0:29
165-00109 HS-WX T	1:29	165-00102 XL-BHS T	0:36
165-00109 HS-WX A	1:43	324-00102 XL-BHS T	0:36
165-00110 WX-SZ D	1:43	344-00101 NJ-DY D	0:43
165-00111 WXXQ-SZXQ T	1:57	324-00101 NJ-ZJ D	0:55

For the events outside the critical circuit, line segment 171-00102 JN-WWS has the least circuit recovery time of 306:25 to itself, and 503-00102 JN-LY has the maximal circuit recovery time of 1134 minutes. Figure 5 displays the delay sensitivity and delay impact of two other events.

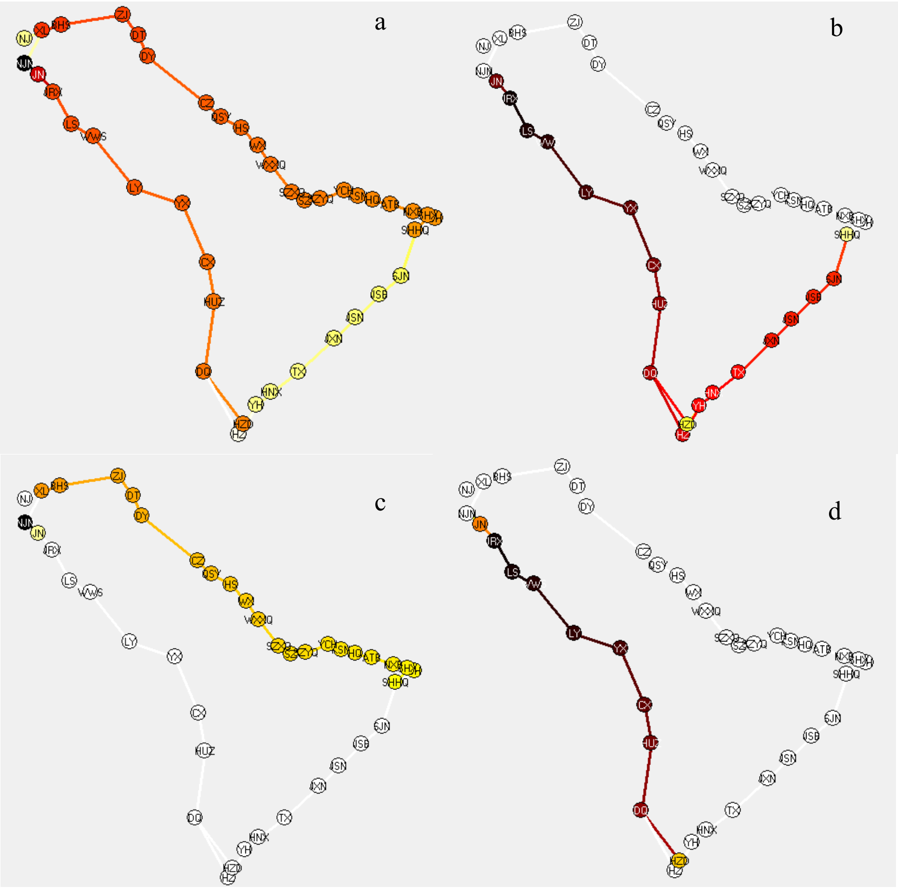

Figure 5: Delay sensitivity and delay impact of 171-00102 JN-WWS (a,b) and 503-00102 JN-LY(c,d)

With 5 minutes delay, 171-00102 JN-WWS could affect 29 events and 503-00102 JN-LY affects 14 events from its own line. Both events could be influenced by events from other corridors. Train line 171 has a rolling stock connection with another line from the HZ-SH corridor on a shared station and some headway constraints on NJN which lead to the large network impact. The arrival-departure headway on NJN results in the large sensitivity area. The infrastructure constraints on the shared stations of different corridors contribute a lot to the delay impact and delay sensitivity on a network level.

Delay propagation analysis

Two types of delays could be generated from initial delays in the timetable. A delayed train may keep delayed on consecutive arrival, departure, and through events on consecutive line segments. These are called consecutive delays, which could be eliminated only by running time supplement and dwell buffer times. On the other hand, secondary delays are the propagated delays to other trains by infrastructure or connection constraints, which are influenced by the buffer times between the train paths in the timetable design. This section considers several initial delay scenarios to evaluate the timetable.

In order to study the critical circuit in this network, an initial delays of 10 minutes and one of 20 minutes are given to 165-00104 ZJ-DY. Figure 6 shows a visual performance for the resulting delay propagations. The 10 minutes initial delay causes 19 consecutive delays and 24 secondary delays, while the 20 minutes initial delay causes 78 consecutive delays and 79 secondary delays with an impact to almost the entire network. Both delays are settled within period zero (during the day). Table 6 gives the statistics of the delay propagations for three different events with 10 and 20 minute initial delays. Event 165-00104 ZJ-DY from the critical circuit causes the biggest secondary and consecutive delay. An initial delay of 10 minutes of 503-00102 JN-LY propagates to 4 other trains and reaches 11 stations with total secondary and consecutive delay of 161 minutes and 6 seconds. Comparing the influence of 10 minutes initial delay to 20 minutes, the secondary and consecutive delay increases dramatically.

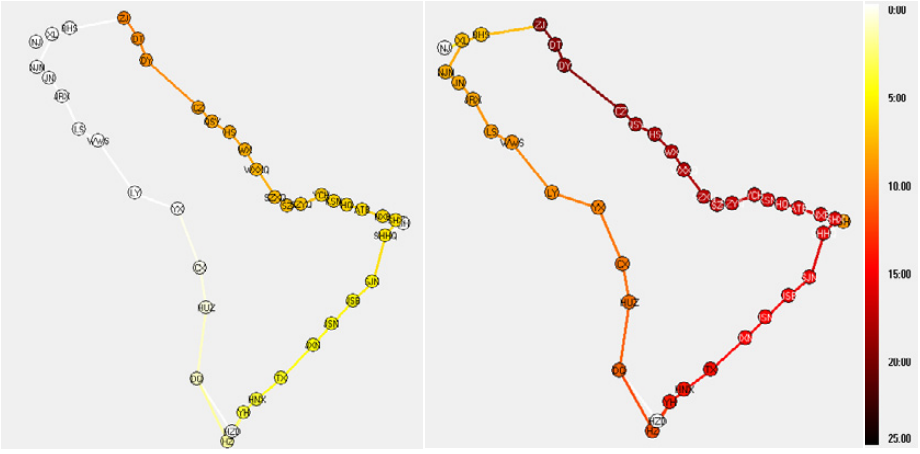

Figure 6: Delay propagation of initial delay 165-00104 ZJ-DY by (a) 10 min, (b) 20 min

Table 6: Delay propagation of three events

Line segment	**165-00104 ZJ-DY**		**171-00102 JN-WWS**		**503-00102 JN-LY**	
Initial delay	10	20	10	20	10	20
Secondary delay	86:32	592:54	29:38	197:06	11:32	107:13
Total delay	462:29	2873:26	284:06	1004:32	161:06	579:21
Average secondary delay	3:36	7:30	3:17	7:18	2:53	7:08
Secondary delayed trains	24	79	9	27	4	15
Number of delayed stations	29	39	17	18	11	12

When turns are discarded, the critical circuit and delay impact are the same as when they were included. In other words the layover time has little effect on timetable stability and robustness in this timetable. Table 7 demostrates the delay propagation on different events when discarding coupling constraints and infrastructure constraints, respectively. Compared with the original performance, it can be concluded that most of the delay propagation comes from the infrastructure constraints. The rolling stock coupling constraints play a considerable role on this timetable stability and robustness as the moderate change of total delays.

Table 7: Delay propagation disregarding

Variant	**Maximum cycle mean [mm:ss]**	**Event**	**Initial delays [min]**	**Total delays [mm:ss]**	**Secondary delays [mm:ss]**	**Trains**	**Stations**
No coupling	189:09	165-00104 ZJ-DY	20	1553:43	433:52	48	26
		171-00102 JN-WWS	20	480:59	32:41	2	9
		503-00102 JN-LY	20	443:36	76:17	10	9
No infra	168:39	165-00104 ZJ-DY	20	588:37	0:00	0	25
		171-00102 JN-WWS	20	409:22	15:47	1	17
		503-00102 JN-LY	20	271:51	2:43	1	10

In addition, from analysis result that all of the delays could be settled in the first period no matter the initial delay is 10, 20 or 100 minutes. Generally the delay will be propagated to the next period if the delay exceeds the corresponding circuit recovery time. Because this is a non-cyclic timetable considering one day as a period, there exists long maintenance time which consumed all of the delays. In this case, the delay should not propagate to maintenance time period for the sake of safety. If delays occurs before maintenance time, the common way is to cancel the related train.

4 Conclusions

In general, this acyclic timetable is structurally stable. However, from the analysis of delay sensitivity, delay impact and delay propagation, 4% running time supplement is not enough to absorb delays, and headway constrains have a prime effect on delay propagation. With respect to recovery time, most delays less than 100 minutes could be settled in period zero (during the day), which shows a high robustness to deviate from the scheduled timetable. The number of *crossing* line trains and the infrustucture constrains for trains from different corridor on shared stations have a big effect on the delay propagation.

Acknowledgements

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