

A ZONAL CHARACTERISTIC BOUNDARY CONDITION FOR NUMERICAL SIMULATIONS OF AERODYNAMIC SOUND (ECCOMAS CFD 2006)

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Abstract. *This paper presents a non-reflecting boundary condition that significantly reduces the spurious pressure oscillations that are produced when vortical structures in a compressible flow cross the computational farfield boundaries. The method is based on commonly used characteristic boundary conditions. Here, incoming characteristics are ramped to zero in a buffer region as opposed to merely setting them to zero at the boundary. One of the key features of the approach is that it is free of coefficients requiring calibration. The equivalence of two formulations of the zonal boundary condition is verified with a model problem. Direct numerical simulations of trailing edges at different Mach numbers are conducted to demonstrate the effectiveness of the novel approach.*

1 INTRODUCTION

With the dramatic increase in available computational power over the last decade, direct numerical simulations (DNS) and large eddy simulations (LES) have become viable tools for the computation of aerodynamically generated sound. Solution of the full unsteady Navier–Stokes equations (or the filtered equations in LES) is required to accurately predict the near field hydrodynamics and thereby determine relevant sources of sound. Provided the computational domain is sufficiently large, the far field is computed at the same time, and physical mechanisms of sound generation can be investigated or acoustic theories can be evaluated. However, due to the large ratio between the energy of the hydrodynamic and the acoustic fields, special care has to be applied to the boundary conditions to avoid reflections that can potentially render the simulation useless. This applies in particular to far field boundaries that are crossed by nonlinear disturbances, such as high-energy vortical structures in wakes or jets.

Nonreflecting boundary conditions have been extensively studied. From characteristic wave analysis, local characteristic boundary conditions (CBC) were derived by Thompson¹.

This type of boundary condition has been scrutinized extensively in the literature^{2–5}. The common conclusion is that the local CBC works satisfactorily for what it is derived for: the attenuation of small amplitude waves that are normal to the respective boundary^{1,6,7}. However, for flows where nonlinear disturbances cross the outflow boundary, considerable acoustic reflections are generated by the local CBC.

For most simulations of aerodynamically generated sound, where it is essential that high-energy structures exit the computational domain with minimal acoustic reflections, previous studies have deemed it necessary to add nonphysical buffer zones to the physical domain. These zones may contain (i) grid stretching, (ii) low-pass filtering, (iii) the addition of convection and/or dissipation terms to the governing equations, or (iv) a combination of some or all of these. However, such approaches typically involve coefficients that are highly flow dependent and assume prior knowledge of the size of the structures to be damped. To avoid this major drawback, a highly effective zonal CBC that does not include coefficients which require calibration was developed⁸. In the present paper, the ability of this method to reduce spurious pressure oscillations that are produced when vortical structures in compressible flows cross the far field boundaries will be evaluated by conducting DNS of trailing edges.

2 ZONAL CHARACTERISTIC BOUNDARY CONDITION

The zonal CBC is based on the local CBC derived by Thompson¹. Linearizing the compressible Navier–Stokes equations and neglecting viscous terms, a wave equation is obtained which describes waves propagating in the direction normal to a boundary. Using characteristic analysis, eigenvalues with corresponding eigenvectors can be computed. The eigenvalues λ_i , $i = 1 \dots 5$, correspond to the characteristic velocities of acoustic, vorticity, or entropy waves, with the sign determining whether the waves are incoming or outgoing. The associated eigenvectors \mathcal{L}_i characterize the rate of change of the amplitude of the characteristic wave.

For example, at a subsonic outflow, four characteristics are outgoing (i.e. $\lambda_i \geq 0$), and the characteristic associated with $\lambda_1 = u - c < 0$ is incoming, where u is the streamwise velocity component and c is the speed of sound. To avoid reflections from the outflow boundary, in the local CBC, \mathcal{L}_1 at the boundary is either set to zero as originally proposed by Thompson¹, or set to $\mathcal{L}_1 = K(p - p_\infty)$ to ensure well-posedness as suggested by Poinot and Lele⁵, adding an additional constant K . However, in the current zonal CBC approach, the treatment of \mathcal{L}_1 is not restricted to the boundary itself, but is applied within a zone. The eigenvalues λ_i and the associated eigenvectors \mathcal{L}_i are computed within a buffer zone which extends from x_s to the outflow boundary located at x_{out} . Within this interval, the eigenvectors \mathcal{L}_i are modified as

$$\tilde{\mathcal{L}}_i = \begin{cases} g(x) \cdot \mathcal{L}_i & : \lambda_i < 0, \text{ with } g(x) = 0.5 \left\{ 1 + \cos \left[\frac{\pi(x-x_s)}{(x_{out}-x_s)} \right] \right\} \\ \mathcal{L}_i & : \lambda_i \geq 0. \end{cases} \quad (1)$$

Thus, the incoming characteristics are gradually ramped to zero at the outflow.

This approach can be easily extended to local one-dimensional inviscid relations (LODI) which are commonly used when solving the full Navier–Stokes equations. The modified wave amplitude variations $\tilde{\mathcal{L}}_i$ are used to assemble the gradients normal to the boundaries. Although only the application to a subsonic outflow boundary was discussed, this approach can be adopted for all other boundaries.

2.1 Quantitative analysis of alternative formulation

In the present paper an alternative formulation of the CBC is tested. In the original zonal CBC approach, the sign of the eigenvalues λ_i at each streamwise point determines whether \mathcal{L}_i need to be ramped to zero. Alternatively, the sign of λ_i at the outflow boundary only can be used to decide whether \mathcal{L}_i is treated or left unchanged, and \mathcal{L}_i is smoothly ramped to zero over the entire buffer region. For a quantitative evaluation of the alternative formulation of the zonal CBC, the same model problem was chosen as in Sandberg and Sandham⁸. A two-dimensional compressible vortex in a uniform flow crosses the outflow boundary and the norm of the residual disturbance pressure

$$L_p = \left[\frac{1}{N_x N_y} \sum_{x,y}^{N_x, N_y} (p(x, y) - p_0)^2 \right]^{1/2} \quad (2)$$

is used to quantitatively assess the magnitude of the reflected pressure waves into the physical domain. The residual pressure is computed within the physical domain $-50 \leq x, y \leq 50$ which is discretized with $N_x = N_y = 101$ uniform grid points in each direction. At the outflow, a buffer zone extending from $x_s = 50$ to $x_{out} = 90$ with 40 uniformly spaced grid points is specified. Figure 1 shows the result obtained from the alternative formulation, the original zonal CBC, and a local CBC. Initially, L_p rises to approximately 2×10^{-3} and slowly decays in time due to viscous effects while the vortex moves towards

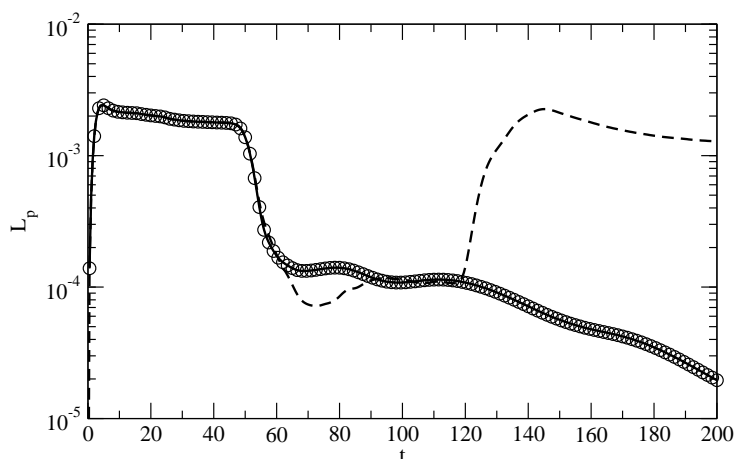


Figure 1: Residual pressure, L_p , in physical domain; (---) local CBC at x_{out} , (—) zonal CBC, (○) alternative formulation of zonal CBC.

the outflow boundary. For the case of a local CBC specified at the outflow boundary, L_p decays rapidly once the vortex leaves the physical domain. However, considerable reflections are caused when the vortex reaches the outflow boundary ($t = 90$) which propagate upstream and lead to an increase in L_p once they have reached the physical domain ($t > 120$). In fact, L_p due to the spurious pressure reflections rises to a level even higher than the residual pressure caused by the original vortex. Using the zonal CBC results in a continuous decay of L_p for $t > 50$, illustrating the effectiveness of the approach. Only a local penalty in performance can be observed ($t \approx 58$) which can be attributed to the fact that any kind of treatment causes reflections by itself within the buffer zone. These can be minimized by choosing a smooth ramping function, e.g. equation 1. The result obtained when employing the alternative formulation of the zonal CBC is identical to the original zonal CBC in the current model problem. Thus, both versions of the zonal CBC appear to be suitable for attenuating spurious pressure reflections caused by vortical structures crossing a boundary.

3 RESULTS

3.1 Trailing edge simulations

Direct numerical simulation of compressible flow over an infinitely thin plate with a trailing edge (TE) were conducted. Downstream of the trailing edge a wake develops that contains energetic structures. The computational domain has the dimensions $3 \leq x \leq 11$ and $-5 \leq y \leq 5$ and the singular plate at $y = 0$ extends from the inflow boundary to $x = 6$. At the TE, the top and bottom boundary layers have a displacement thickness of $\delta^* = 0.0192$, resulting in $Re_{\delta^*} = 960$ and the Mach number is specified at $M = 0.6$. 1800 and 400 non-equidistantly spaced points are used for the x - and y -direction, respectively, with the finest resolution at the TE. Two simulations were conducted, one with a traditional sponge approach employing low-pass filtering along with strong grid-stretching³, and another employing the zonal CBC. In both cases, the buffer extended

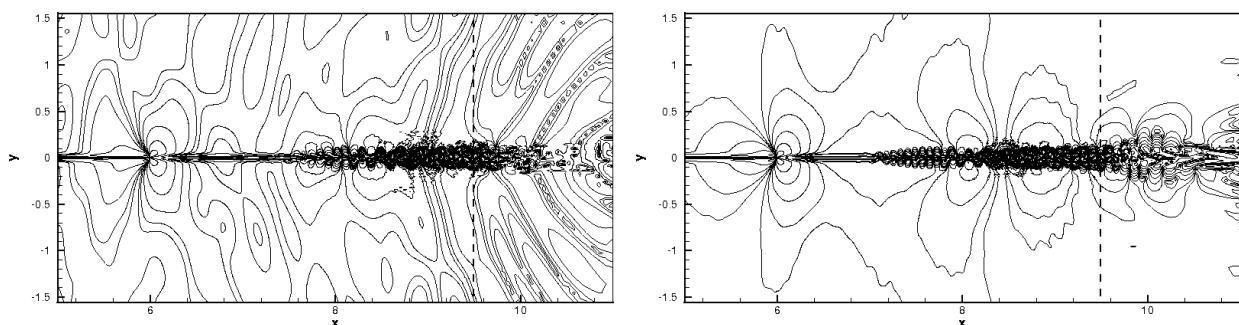


Figure 2: Instantaneous contours of magnitude of dilatation for unforced TE simulations using local CBC combined with grid-stretching and low-pass filtering (left) and zonal CBC (right), eight exponentially distributed contour levels for $[0.0005; 0.064]$; $Re = 50,000$, $M = 0.6$.

from $x_s = 9.5$ (denoted by the dashed line) to the outflow boundary, requiring merely 5.5% of the streamwise points. In figure 2, iso-contours of magnitude of dilatation rate obtained from both DNS are shown of the flow field after 9 flow-through-times.

When using the standard sponge technique, a considerable reduction of vorticity magnitude of the structures is achieved. Nevertheless, when using this buffer technique in conjunction with the local CBC, severe acoustic reflections from the outflow boundary can be observed which propagate upstream. In contrast, when employing the zonal CBC, the results are dramatically improved. No reflections from the outflow boundary can be observed. The computational cost of using the zonal CBC over the local CBC with a sponge approach increased by roughly 6% in the above case.

3.2 Airfoil simulations

One of the main advantages of the zonal CBC is the avoidance of coefficients to be calibrated. Flow over a NACA0012 airfoil at zero incidence was simulated at $M = 0.2$ to demonstrate that the zonal CBC performs well for low Mach numbers. The Reynolds number based on chord and freestream velocity was chosen large enough ($Re_C = 10,000$) for an unsteady wake to develop⁹. The airfoil simulation is conducted on a C-type grid with 1180×258 non-equidistantly spaced points in the directions tangential and perpendicular to the airfoil, respectively. 490 points are used in the streamwise direction in order to adequately resolve the unsteady wake. The dimensions of the domain are $-6 \leq x \leq 6$ and $-5 \leq y \leq 5$. The zonal CBC was applied to a buffer extending from $x_s = 5.3$ to the outflow boundary using 31 points, thus requiring only 5.25% of the streamwise points. For comparison, a DNS was performed adding a dissipation term and a convection term

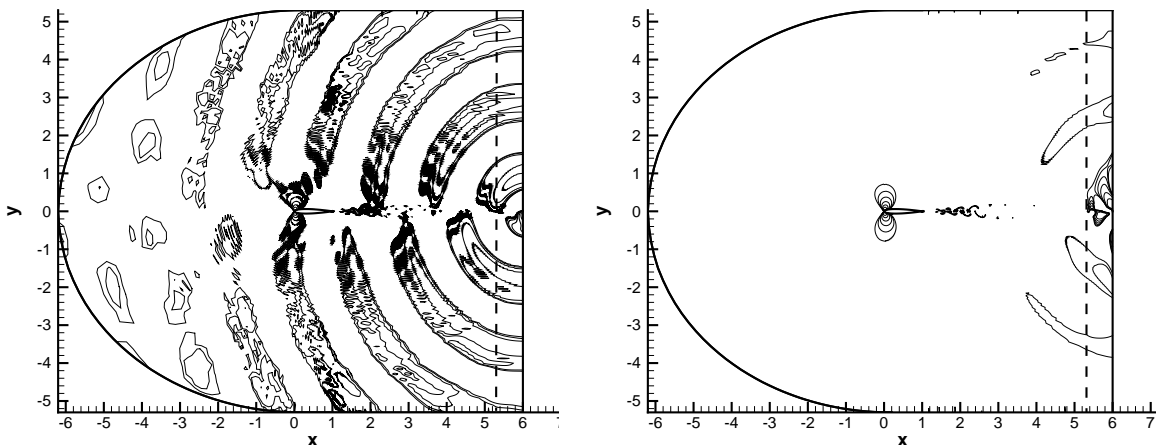


Figure 3: Instantaneous contours of magnitude of dilatation for airfoil simulations using local CBC combined with dissipation and convection terms in buffer (left) and zonal CBC (right), six exponentially distributed contour levels for $[0.002; 0.064]$; $Re_C = 10,000$, $M = 0.2$.

to the right-hand-side of the governing equations⁶, combined with a local CBC at the outflow boundary. This type of sponge was found to yield the best results for the flow under consideration when setting the required coefficients to $\sigma(x) = 0.05[1 - g(x)]$ and $U_c(x) = 0.5[1 - g(x)]$, with $g(x)$ from (1).

Figure 3 shows instantaneous iso-contours of dilatation rate magnitude obtained from both DNS. When using the dissipation and convection terms in conjunction with a local CBC, significant acoustic waves can be observed originating from the outflow boundary. These waves propagate upstream and affect airfoil data, such as time dependent lift coefficient etc. The strength of the reflected waves could certainly be reduced by choosing a larger buffer region. However, when employing the zonal CBC using the same (short) buffer region, the spurious pressure reflections are considerably reduced in amplitude. This demonstrates that for various Mach numbers, the zonal CBC outperforms standard sponge techniques for a given buffer size. Being able to choose a fairly small buffer region reduces the computational overhead, making this method an inexpensive and coefficient-free approach for drastically reducing spurious pressure oscillations.

4 CONCLUSIONS

A nonreflecting zonal characteristic boundary condition has been presented. Since this zonal CBC is based on the commonly-used local CBC schemes, only marginal modifications to existing codes are required. Moreover, in contrast to most other zonal approaches, the method is free of coefficients that require calibration. An alternative formulation of the zonal CBC was shown to yield identical results than the original zonal CBC when applied to a model problem. In addition, direct numerical simulations of compressible flow past trailing edges at different Mach numbers were performed. The simulations demonstrated that the zonal CBC approach is a highly effective and inexpensive method for reducing spurious pressure oscillations generated by energetic vortical structures crossing the far field boundaries.

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