Numerical modelling of forces, stresses and breakages of concrete armour units

Latham, John Paul; Xiang, Jiansheng; Anastasaki, Eleni; Guo, Liwei; Karantzoulis, Nikolaos; Viré, Axelle; Pain, Christopher

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

NUMERICAL MODELLING OF FORCES, STRESSES AND BREAKAGES OF CONCRETE ARMOUR UNITS

John-Paul Latham[1], Jiansheng Xiang[1], Eleni Anastasaki[1], Liwei Guo[1] Nikolaos Karantzoulis[1]
Axelle Viré[2] and Christopher Pain[1]

Numerical modelling has the potential to probe the complexity of the interacting physics of rubble mound armour systems. Through forward modelling of armour unit packs, stochastic variables such as unit displacement and maximum contact force per unit during an external oscillatory disturbance can be predicted. The combined finite-discrete element method (FEMDEM) is a multi-body method ideally suited to model the behaviour of the armour layer system and the stresses generated within complex shape units. In this paper we highlight the latest developments made with the application of FEMDEM technology to breakwater modelling including realistic rock underlayer and concrete unit layer topologies, maximum contact force distributions, internal unit stresses, fracture and unit breakages. Finally, fully coupled wave and multi-body armour unit motion with internal dynamic stress generation is illustrated.

Keywords: rubble-mound breakwater, concrete armour units, FEMDEM, combined finite-discrete element method, fracture, modelling, wave-structure interaction

INTRODUCTION

Today, physical wave tank modelling is used to assess breakwater design stability but this lacks correct material strength representation at full scale, provides limited detailed information and is expensive for design optimisation. The goal of the Applied Modelling Computational Group's (AMCG's) work on coastal structure simulation is to provide industry with virtual breakwater design tools (VBDT) which have the potential to replace costly physical models. We use FEMDEM modelling technology to investigate breakwaters at a representative scale where the solid material is performing as a gigantic granular system and where detailed individual unit interactions, deformations and discrete motions can also be captured by the multi-body mechanics solver. Construction of such a 'virtual breakwater' and the use of accompanying tools for the analysis of different armour unit designs has been successfully demonstrated for 'dry' static load conditions (Latham et al., 2013). The next objective was to capitalise on the dynamic capabilities of FEMDEM by modelling the unit motion response to an applied oscillatory disturbance of the foundation underlayer and toe. The simulated response of breakwater armour layer structures during an externally driven earthquake-type ground vibration was then used to highlight relative differences in the performance of different armour layer designs, e.g. systems of armour units that start out with different packing densities. It is recognized that these 'dry' models and vibrating boundary condition fall short of the design needs of coastal engineers who are concerned with resilience (i.e. minimal movement and damage) to hydraulic loads under storm wave action and reliability and structural robustness to resist dynamic and static stresses.

To improve the capability of the FEMDEM tools for coastal engineering design, a 'Wave Proxy' method has recently been invented and demonstrated to proof-of-concept (Xiang et al., 2013). In addition to the 'dry solid mechanics' considerations (i.e. contact, gravity and inertial forces, displacements that bring the units into a state of rest under equilibrium of forces, internal e.g. mainly elastic deformation within units), the Wave Proxy method takes all the dry problem forces and introduces hydraulic load interactions through additional oscillatory drag, torque, lift and buoyancy force terms. The armour layer units can then be subjected to a combination of most of the significant forces acting upon the units during the wave action. The structure's response to a design storm condition can therefore be simulated. Provided Wave Proxy parameters have been adjusted to represent wave actions for a given design storm, the motion and stability, forces and stresses, together with stochastic variations are then available for analysis by the engineer. For example, the influence of different designs, placement patterns and placement methods, armour unit placement densities and armour unit shapes can be examined. The Wave Proxy method is a stepping stone simulation method that is a form of one-way solid-fluid coupling where the fluid forces which are derived from imposed oscillatory water particle velocity histories, act independently of the motion of the solid armour units

and there is no simulation of fluid flow. More typically, coastal engineering wave-structure interaction research is focused on run-up and overtopping where a different form of numerical one-way coupling model is employed that focusses on Numerical Wave Tank capability designed around a preferred Navier-Stokes CFD solver (e.g. Dentale et al., 2009). For such cases, discrete armour unit packs are represented as granular armour layer geometries over which the wave breaking simulations can run.

In future the two-way coupled processes of wave forces driving the armour unit motions will be modelled with both discrete motions of units and turbulent flow fluids. Such two-way coupling, together with the Virtual Breakwater features described above is illustrated in this paper.

Another phenomenon associated with full-scale structures under on-site wave loading is the damage to the concrete. The material behavior of concrete units near contacts and under high stress concentrations is both elastic and in-elastic. In further developments of the core FEMDEM technology, consideration has been given to plastic and brittle-fracture expressions of the response of units to loading. The extension of FEMDEM constitutive models to simulate such inelastic behavior has been the focus of further recent work, which is also briefly reported here.

FEMDEM SIMULATION

The FEMDEM method was pioneered by Munjiza whose first working 2D FEMDEM code was developed in 1990 (Munjiza 2004). FEMDEM has proven its efficiency and reliability as a computational tool to solve problems involving transient dynamics of systems in which deformation and fracturing play an important role. In 2009, an efficient 10-noded quadratic element was developed in a format suitable for the FEMDEM by Xiang (Xiang et al. 2009). The 3D FEMDEM code (Y3D) which was developed by Xiang and Munjiza in 2009 has great potential to be applied in the field of coastal structures. In FEMDEM, a penalty function method is employed to calculate the normal contact force when the two particles are in contact. The penalty function method in its classical form assumes that two particles penetrate each other. The elemental contact force is directly related to the overlapping volume of the finite element in contact. The distributed contact force approach takes into account the shape and the size of the overlap volume in order to be distributed among the surrounding nodes. It is worth commenting that a number of FEMDEM codes exist with names similar to Y3D. The current 3D FEMDEM development in AMCG is a multi-body transient dynamic solids solver that also runs in parallel. The plasticity and fracture model capability, at the time of writing, are not fully parallelized.

CONSTRUCTION OF ARMOUR LAYERS

Full scale or laboratory scale armour layers with realistic packing density, neighbour contacts, orientation distribution and underlayer roughness are constructed using the FEMDEM code, and a row-by-row placement method is employed, as illustrated in Fig. 1.
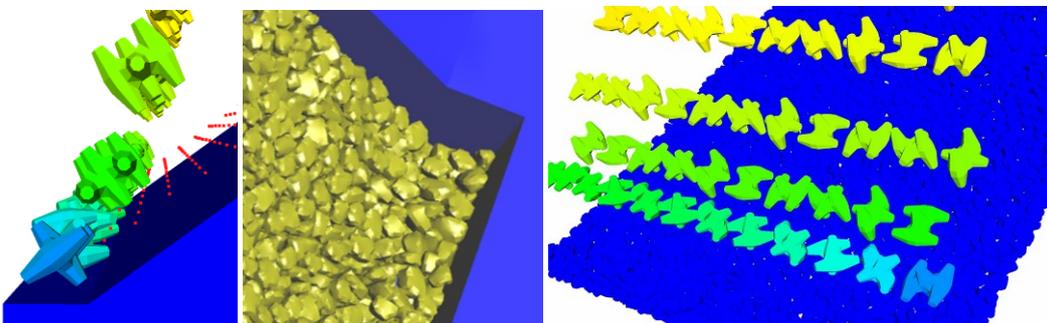
Figure 1. FEMDEM model and construction procedure. Left: row-by-row entry of 8m³ CORE-LOC™ units with target grid. Centre: toe detail of correctly scaled rough underlayer of ~3,700 rocks. Right: placement on rough underlayer, note 'family of four' neighbour orientation pattern suited to high interlock requirements.

Single layer concrete armour unit systems when placed on site according to unit designers' guidance exhibit a characteristic row and column lozenge structure. Such structures can be realistically created although to achieve realism and especially at relatively high packing densities recommended by designers, careful algorithmic procedures have been developed. The maximum contact force associated with each unit can reveal the existence of force chains and heavily loaded units near the toe (Fig. 2). Furthermore, the distribution of such maximum contact forces was found to change as the packing density of the initially constructed layer was increased (Fig. 3), see Latham et al. (2013).
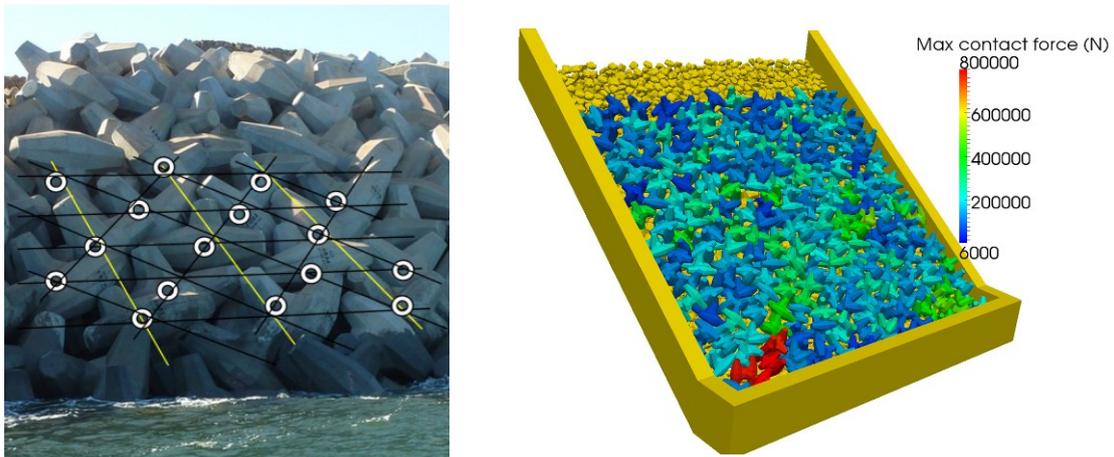
Figure 2. Left: breakwater trunk section of CORE-LOC[TM] concrete units, built to a designated packing density with rules adopted to achieve good interlocking. Right: model of 242 Core-Loc units of 8m[3] volume with colour key indicating maximum contact forces on each unit. These are generally higher in the toe rows and in the example shown, force chains can be observed.
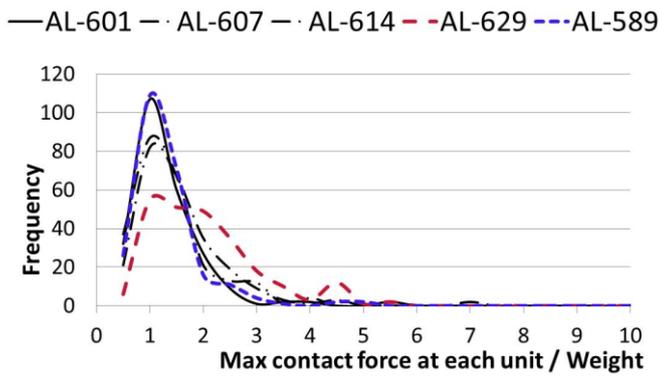

Figure 3. FEMDEM full-scale CORE-LOC[TM] simulation and resulting distribution of maximum contact force normalised to the weight of each unit, for 5 different initial dimensionless packing densities, from 0.589 to 0.629, (Latham et al., 2013).

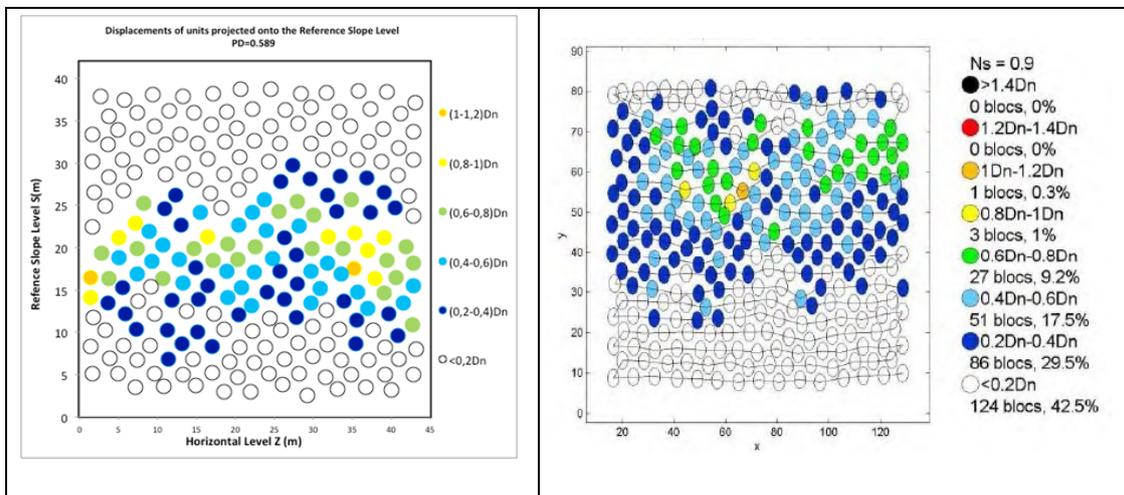**Figure 4. Displacement maps of centroids of units after a 'storm' view projected normal to slope and displacement colour coded and represented as a proportion of the nominal diameter of the concrete Core-Loc unit. Left: Numerical model results after short sequence of Wave Proxy disturbance, Right: e.g. of hydraulics laboratory test results deduced from video analysis, (method discussed in Garcia et al., 2013).**

Numerical analysis of centroid positions in the original placed layer and their displacement history when later subjected to disturbance provides a forensic tool to investigate damage, for example when applying a vibration or 'wave proxy' disturbance (Fig. 4). Garcia et al. (2013) emphasise the importance the client and designer attach to quantitative as well as qualitative results from laboratory tests to evaluate stability (extractions, rocking, progressive movements etc.). It is abundantly clear that this form of quantitative data is readily available from the FEMDEM numerical models although currently, the storm process realism is less convincing than in hydraulics laboratory tests, due to the need for further research into representing wave loadings.

STRESS DETERMINATION WITHIN UNITS

Stress capture in static packs at full scale and during disturbance by vibration has been presented in preliminary work with fewer units (Xiang et al., 2011). Stress levels for a realistic pack are shown here in Fig. 5. In this work we apply a simple friction law based on Coulomb friction in dry conditions. In this respect and considering other simplifying assumptions, it is important the reader understands that the results do not represent the real behaviour of units and that this is a model with various assumptions for which we are reporting a work in progress. The simulation is expensive to run in terms of CPU when adopting the deformable code throughout the dynamic disturbance. The simulation results shown here are for static stresses within the elements of all units when at rest e.g. at the end of a sequence.
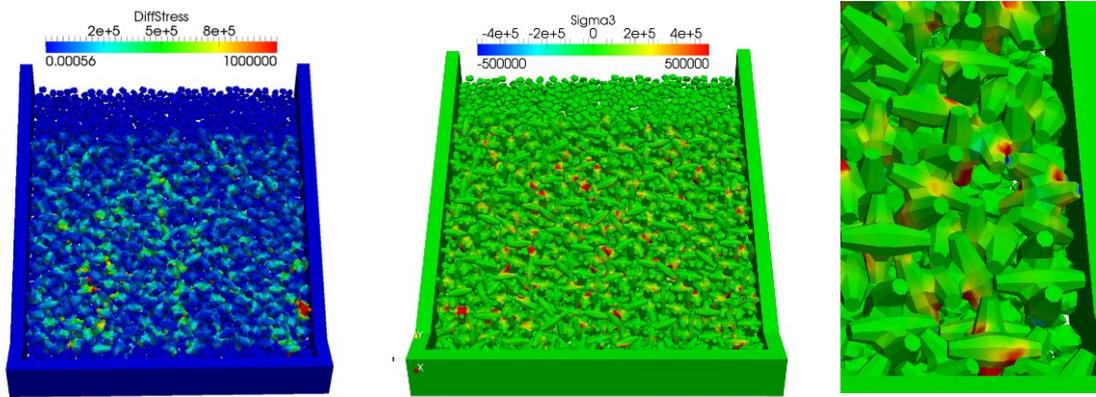

Figure. 5 Static Stress variations determined at the finite element level, tensile stress shown positive. Left: differential stress, Centre: tensile stress, Right: tensile stresses close up view.

STORM SIMULATION BY PROOF-OF-CONCEPT WAVE PROXY METHOD

It is quite common to simplify wave impacts on structures as cyclic wave loading in offshore geotechnical engineering. Physical modeling of cyclic loading is carried out using regular or random wave loading. When a regular wave strikes a sloping coastal structure, the potential for destabilizing the units is greatly affected by the wave run-up and run-down along the slope. In the proof-of-concept first illustration of the wave proxy, the run-up and run-down velocities are greatly simplified and are assumed to approximate a sinusoidal function and applied as a velocity controlled boundary condition. For the time history of water particle velocities imposed on the breakwater, we can accept velocity data from any sources, e.g. derived from theory, measured from experiments, or from a CFD wave simulator. Using the difference of fluid and solid velocities, we then assume a drag coefficient (such values will later be calibrated e.g. from numerical simulations), calculate drag forces and torques and apply them in addition to the summed solid forces acting on the armour units. According to the effects of wave action and water moving up and down the breakwater slope, we can divide the domain into three zones (see Fig. 6), note that for this illustration a highly simplified approximation is made to obtain run-up and run-down from an assumed wave height. Zone A: Units are always submerged under water, and not influenced by waves, but water resistance is considered by setting water velocity as zero. Buoyancy forces and lubrication effect (in the form of a reduced friction coefficient) are taken into account. Zone B: in this zone, wave run-up and run-down take place. Buoyancy forces and lubrication are therefore applied along the water surface to units in this zone when below the moving water surface. The drag forces are calculated based on the difference between water velocity and unit velocity, at the same time

torque due to drag forces is also calculated; Zone C: in this zone, units are always above the water surface, so the units are not influenced by wave disturbance.
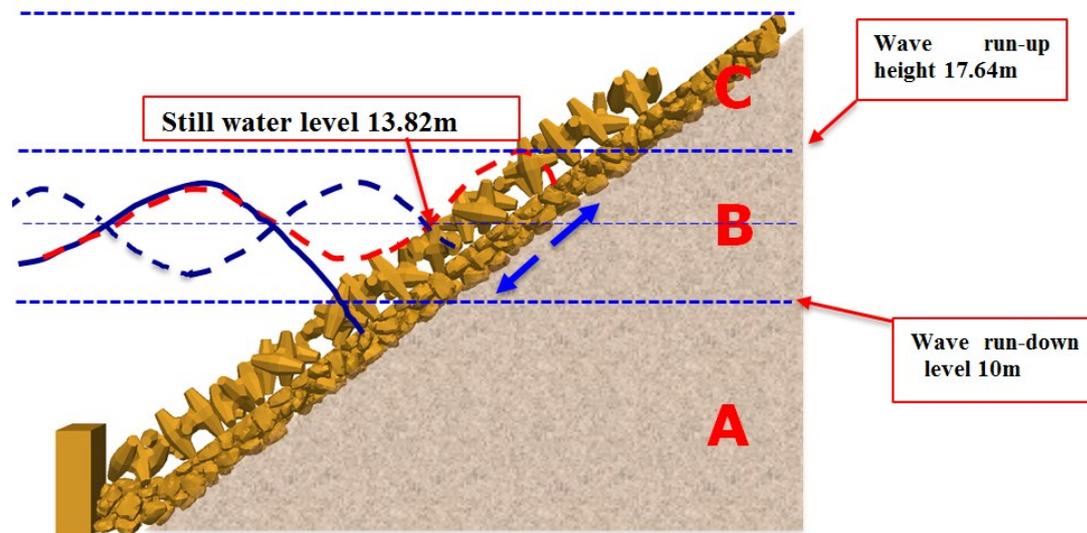

Figure 6. Cross-section of a numerical rubble mound breakwater and assumed simplified run-up and run-down (Xiang et al., 2013)

Results for proof-of-concept wave proxy disturbance on two packs

Using POSITIT/Y3D_R a series of 242 8m^3 CORE-LOC^{TM} armour unit layers with different packing density (PD) from 0.589 to 0.629 was created as described in Latham et al. (2013). In this research, two amour unit layers are chosen for wave proxy analysis, the loosest layer (PD=0.589) and the tightest one (PD=0.629). The same 6 cycle oscillatory velocities were applied to these two layers. Figs. 7 and 8 clearly show the tight pack is more stable than the loose pack. In the loose pack (PD=0.589), there are visible gaps found around still water level as the packing density of 0.589 is far below the designed value of 0.619. In contrast to the loose pack, for the tight pack it is hard to tell the difference between initial and final packs. This trend is also in good agreement with experimental observations, however the nature of the unit movements in detail is likely to be made significantly more realistic as further hydraulic force terms and more realistic velocity histories are introduced to the Wave Proxy method. In order to analyse unit response to wave disturbance, 6 units highlighted in Figs. 7 and 8 are selected from toe berm to last (top) row of the layer around the middle line of the test section. Maximum contact forces of these six units are recorded and shown in Fig. 9. It is found in the loose pack, units 7 and 41 have higher maximum contact forces, reaching 12 times the dry weight of units when the layer collapsed under 'wave disturbance'. In the tight pack, unit 7 has higher contact force mainly because it is at the toe and supports many units in the rows above. In this work we apply a simple friction law based on Coulomb. In this respect and considering other simplifying assumptions, it is very important the reader understands that the results do not represent the real behaviour of units under wave loads and that this is a model with various assumptions and we are reporting a work in progress.
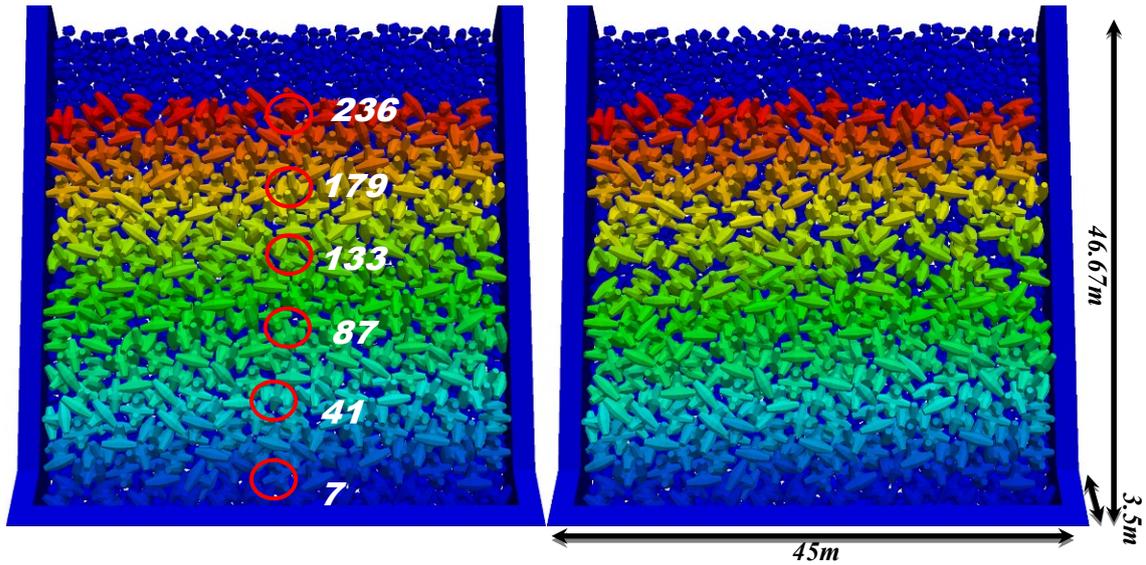
Figure 7 Placement of 242 CORE-LOC^TM armour loose pack (PD=0.589) left: initial packing; right: packing after wave action (Xiang et al., 2013)
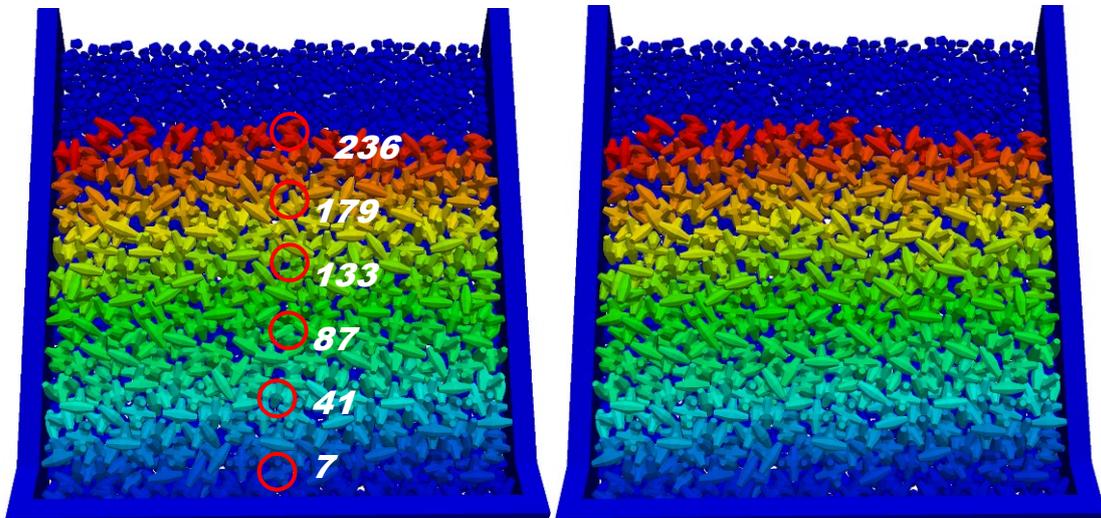

Figure 8 Placement of 242 CORE-LOC^TM armour tight pack (PD=0.629) left: initial packing; right: packing after wave action
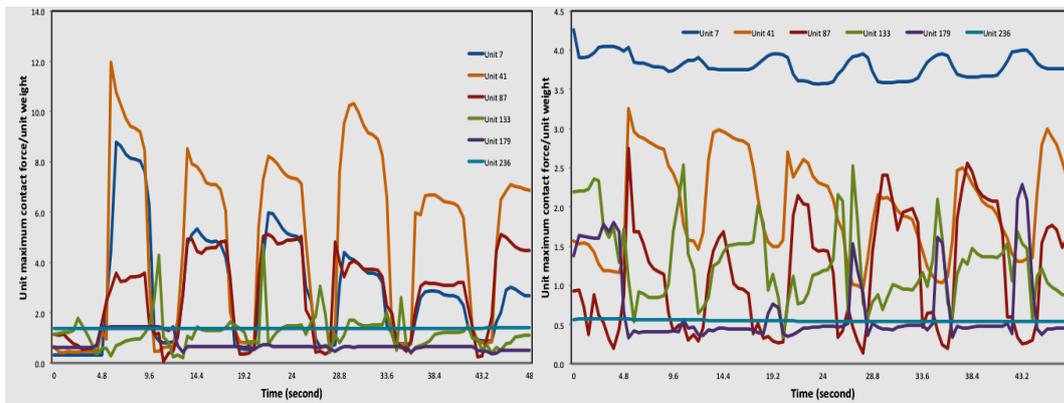

Figure 9 History of maximum contact forces of six selected units for six waves. Left: loose pack; Right: tight pack. Note change in force scale. See also the disclaimer in the text above (Xiang et al., 2013).

IN-ELASTIC BEHAVIOUR OF UNITS

Plasticity

The in-elastic response at unit contacts under compression, rather than being expressed as localized fracture, may sometimes be better described as energy losses caused by a diffuse micro-cracking that only damages the concrete locally near the collision or small area contact regions or under intense shearing of contacts. Compared with an idealized very strong concrete behaving purely elastically, this diffuse crushing can have the effect of reducing an otherwise very high peak in the tensile stress wave that would be experienced when concrete units collide.. To fully appreciate the extent to which a more plastic and lower strength concrete might reduce the peak tensile stresses encountered in units, the FEMDEM code has been extended beyond Neo-Hookian visco-elasticity and is now able to model plasticity, including large strain plasticity and with a range of plastic constitutive laws including those often preferred to describe concrete crushing such as the Drucker Prager model, (Karantzoulis et al., 2013). Such plastic behaviour is illustrated in Fig. 10.
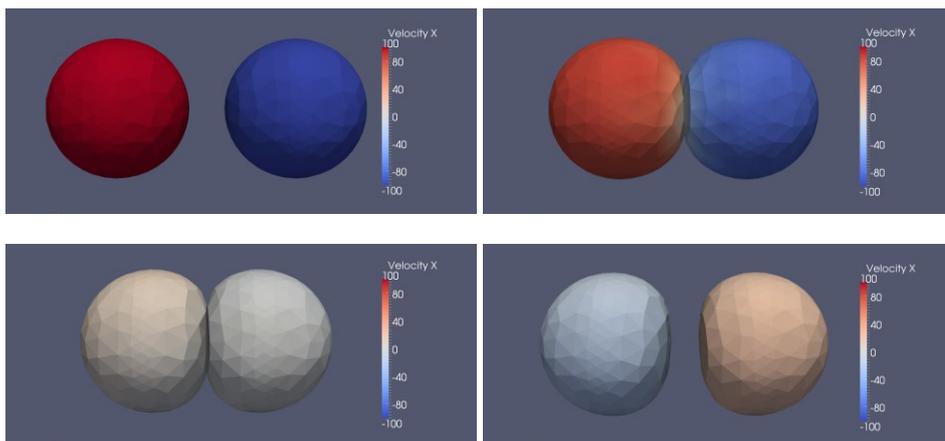

Figure 10 High speed collision of two plastic spherical bodies (Karantzoulis et al., 2013)

Fracture of Units

Recently, a 3D fracture model has been developed in the context of the combined finite-discrete element method (Guo et al., 2014) which was applied to investigate the structural integrity of concrete armour units under dynamic and extreme loading conditions. Two types of concrete armour units were studied. Dolosse units are simulated in drop tests and pendulum tests, and the numerical results are compared with physical experiments of Burcharth (1981).
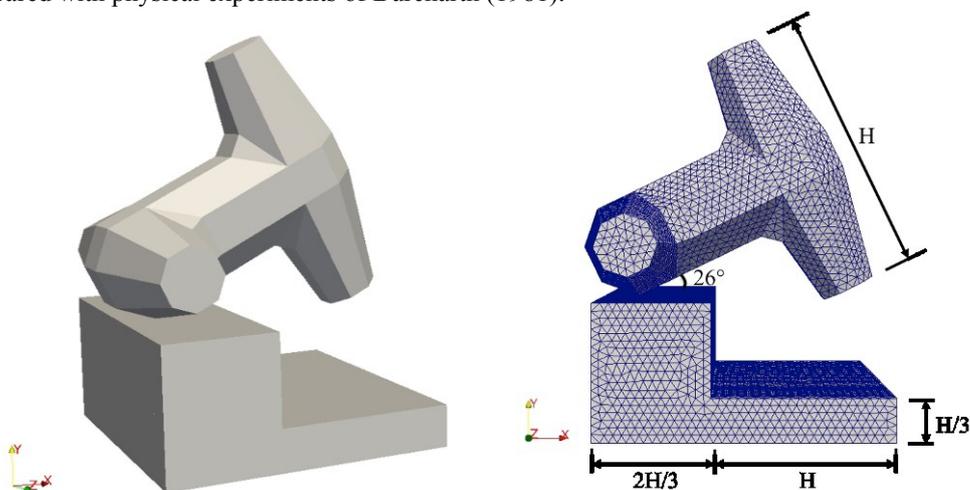

Figure 11 Drop test setup for dolos fracture simulation (Guo et al., 2014)

a. *t* = 0.05 ms.

b. *t* = 0.45 ms.

c. *t* = 0.90 ms.
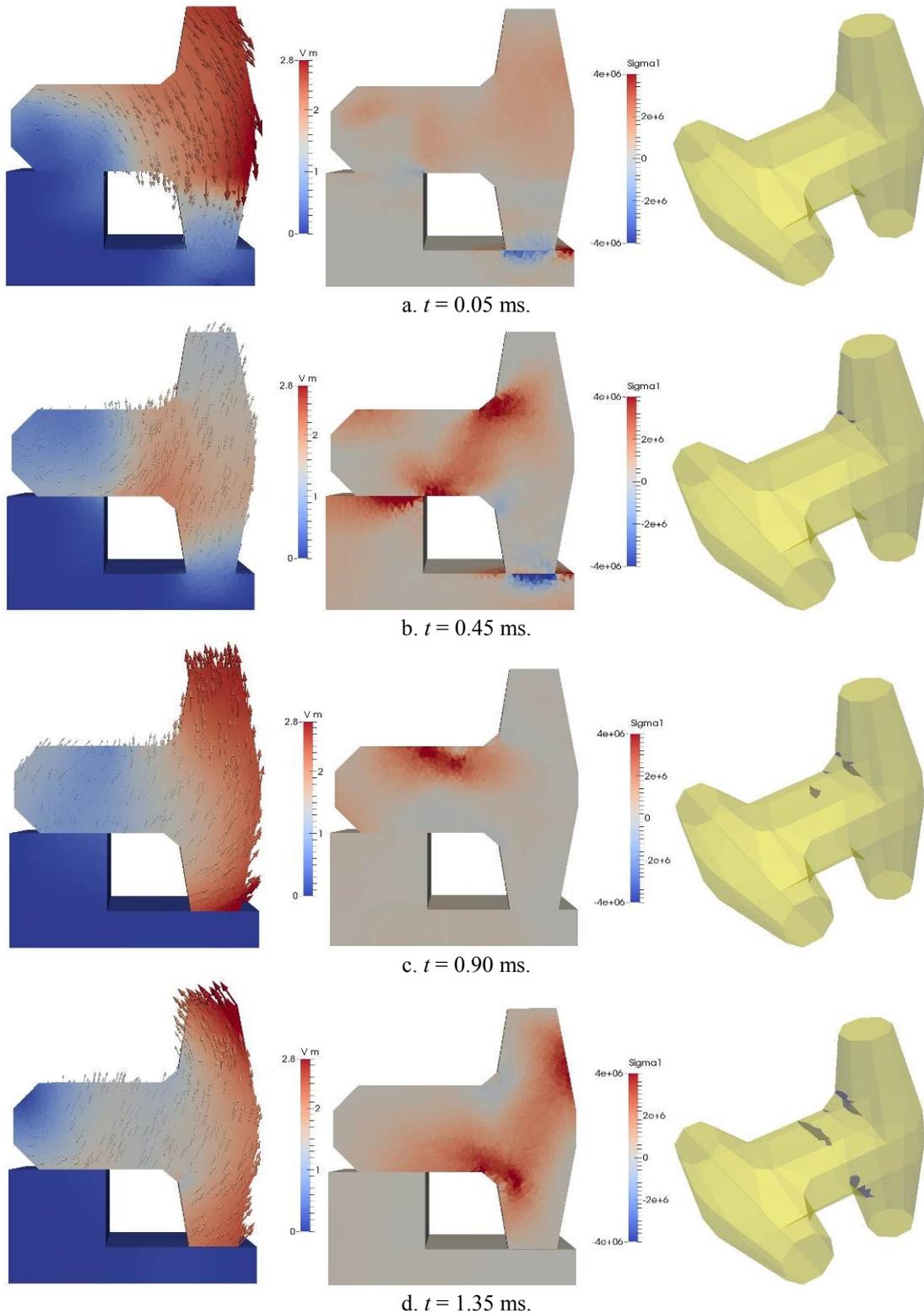

d. *t* = 1.35 ms.

Figure 12 Numerical simulation results of the drop test of a Dolos unit arranged in time sequence. Note that the time starts when the lower end of the right fluke hits the base. Left: velocity vector and magnitude in the cut plane perpendicular to the z-direction, in ms^{-1}. Centre: engineering stress convention; maximum principal stress σ_1 in the cut plane perpendicular to the z-direction, where tensile stress is positive, and compressive stress is negative, and the unit of stress is Pa. Right: the 3D fracture development in the Dolos unit, where the yellow colour represents the surfaces of the Dolos unit and the blue colour represents fracture surfaces, (Guo et al., 2015).
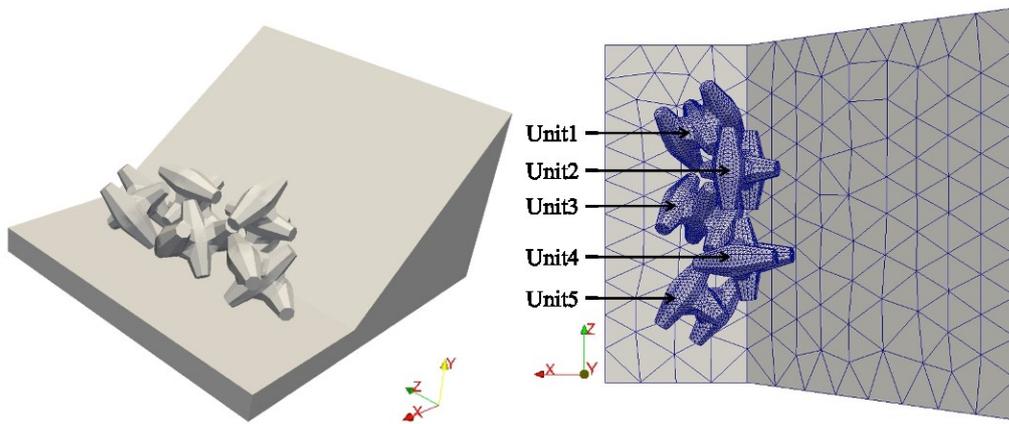
Figure 13 Model setup and mesh of the simulation of multi-body CORE-LOC^{TM} **units.**
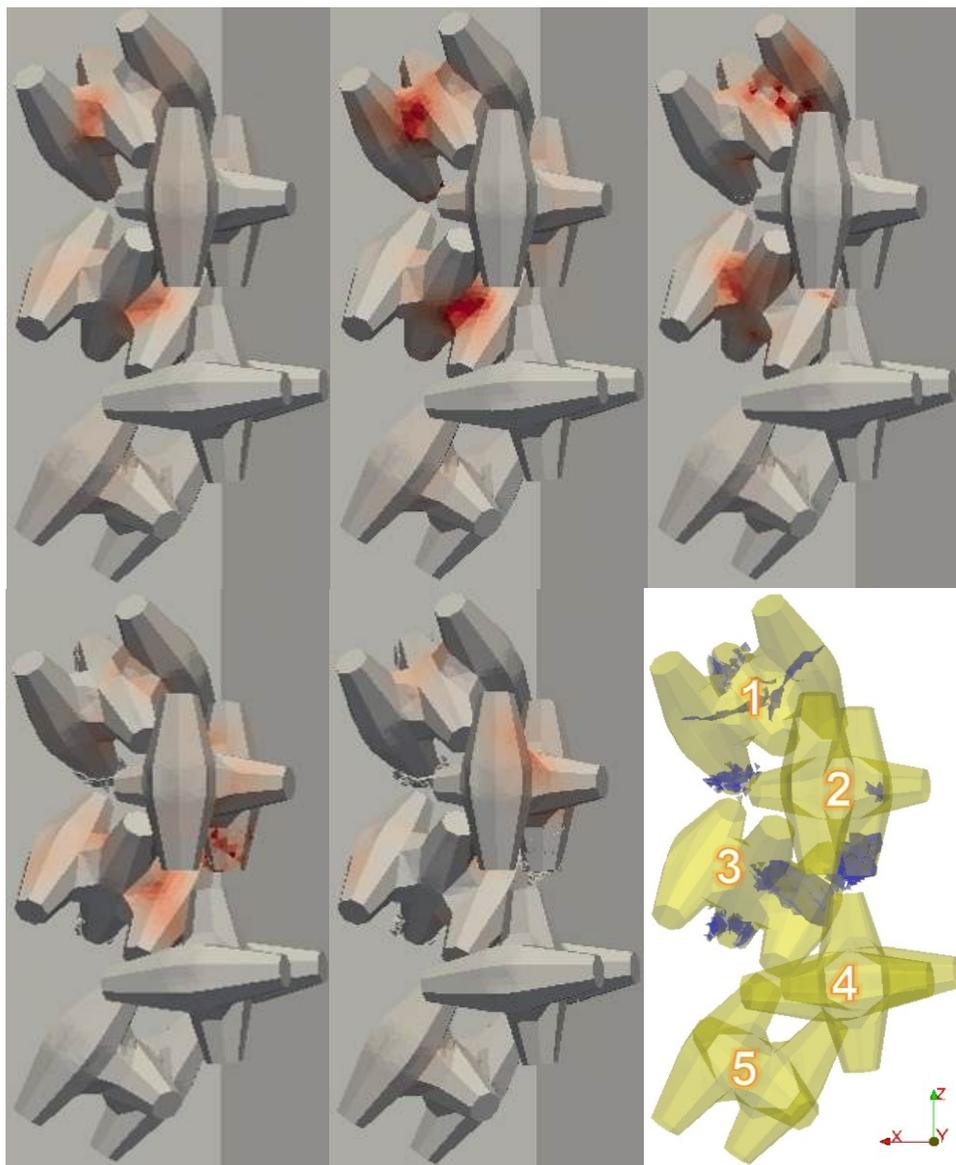

Figure 14 Numerical simulation results of interaction between CORE-LOC^{TM} **units under gravity on a sloping base, sequence at t=0.061, 0.063, 0.067, 0.084, and 0.01 seconds, showing maximum principal stress** σ_1 **where tensile stress is positive, (engineering stress convention). The final image shows 3D fractures.**

The dynamic stress and fracture development for both the drop test shown here and the pendulum test are in good agreement with the fracturing observed by Burcharth (1981) and what is considered physically realistic transient behavior (Guo et al., 2015).

CORE-LOC^{TM} units of prototype scale were also simulated under an imaginary extreme loading condition, which represents a close but not touching configuration of interlocking units that are lifted, and then dropped from slightly above a horizontal and sloping base. The whole structural response of concrete armour units, including multi-body interaction, rigid-body motion, continuum deformation, fracture initiation and propagation, and post-fracturing interaction between discrete fracture surfaces, is accurately captured by 3D numerical simulations.

Note that unit 4 does not collide at all, whereas unit 5 collides but does not break. Different failure modes are generated by the complex stress field in this simulation. Transient failure modes of Core-Loc units 1, 2 and 3 at the end of the simulation are shown in Figure 14 and specifically those of Unit 3 in Fig. 15. Several typical modes of breakage were generated by these extreme conditions including the discrete shear fracture causing the loss of one leg and more widespread crushing fragmentation of the unit nose.
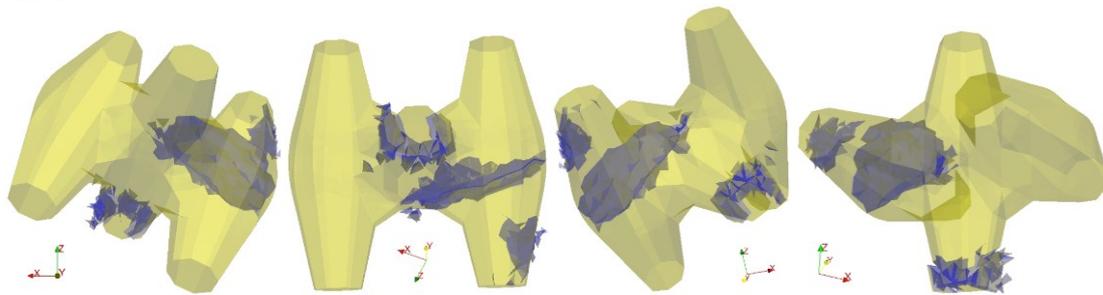
Figure 15 3D views of transient failure pattern of Core-Loc unit 3 at t = 0.1 s from different angles. The first one from the left is the original orientation as shown in Figure 13.

WAVE-STRUCTURE INTERACTION USING AN IMMERSED BODY METHOD

To simulate wave-structure interaction, an immersed body method for FSI problems is used. Our approach is to project relevant quantities between the two meshes, one for each phase, via a supermesh (see Farrell et al., 2011). In the proposed approach the two codes, 'Y3D' based on FEMDEM and the multi-purpose CFD code, 'Fluidity', are coupled to model fluid-structure interactions. Y3D captures nonlinear material properties using a finite strain formulation and also has multi-body and granular media capability. Fluidity is a multi-phase CFD code based on arbitrary unstructured finite element meshes with the capability of anisotropic dynamic mesh optimisation. Among the most important optimization for our approach was the adaptive tetrahedral mesh optimization that was developed and implemented, as seen in Pain et al. (2001). The capability of concentrating resolution where it is needed is a key element in describing the solid-fluid interface, and reducing the computational cost.

To meet the request of solving realistic-scale engineering problem, we also propose a simple approach for wave-structure interaction. In this approach, a rigid FEMDEM version - Y3D_R is employed to simulate structure-structure interaction and wave motions are treated as cyclic loading. Drag forces are calculated based on the difference between solid and fluid velocity at local element level. Then these forces are integrated and applied to the unit. Buoyancy and lubrication effects are considered when units are submerged in water.

Fluidity is a three dimensional Finite Element code for solving the Navier Stokes equations. It is a powerful generic multi-phase CFD code that has been further developed by the authors to handle fluid-structure interaction. The combination of Fluidity coupled together with Y3D forms the basis of our multi-physics modelling capability. Fluidity employs a wide variety of solving and optimization techniques.

A core concern with modelling relatively large moving particles (i.e. not sub-grid sized particles) is the computational expense of fluid re-meshing. Our approach is based on solving Navier-Stokes equations using a Petrov-Galerkin Finite Element method with an adaptive unstructured mesh (Pain et al., 2001), together with recently enhanced interface tracking. This meshing technology allows the fluid to refine around the complex (possibly moving) geometries, or seek-out the complex topology of void space in porous granular media, starting with an arbitrary coarse mesh. This step also reduces the effort

of meshing around solid bodies within a fluid and thus drastically decreases the time taken to set up fluid-structure models.

Coupling of *Fluidity* with *Y3D*

The immersed body method employs two meshes, one for each phase. Relevant quantities, such as solid position, solid and fluid velocities, and drag forces, are projected conservatively between the two meshes using supermesh technology (see Farrell et al., 2011). The detailed description of the immersed body method can be found in the full paper (Vire et al., 2012). In summary, *Y3D/Fluidity* presents a novel and full two-way coupling of fluid-structure mathematical models. It has the capability of simulating not only interaction between waves and maritime structures whether emergent or submerged, but also multi-body structure-structure interactions, as illustrated below in Fig. 16.
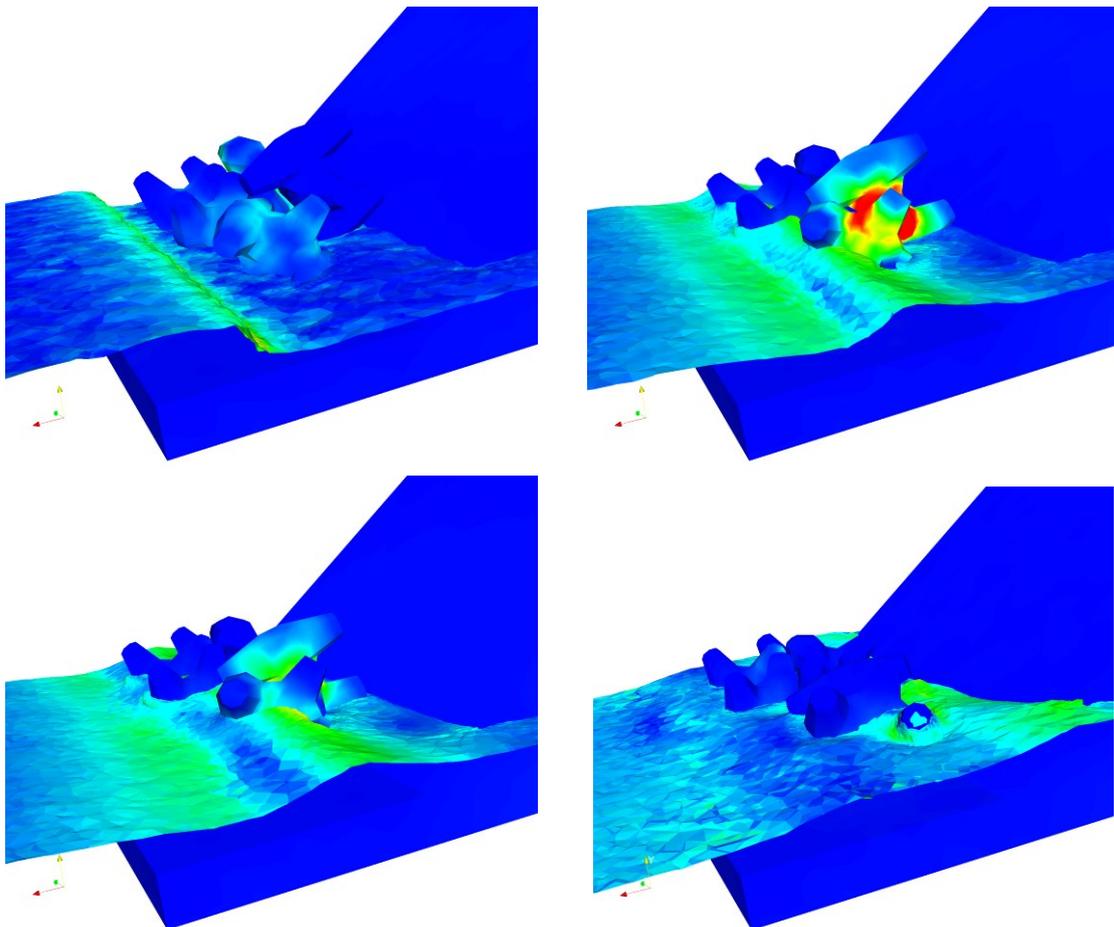
Figure 16 Two-way coupled motion of five CORE-LOC^{TM} concrete units falling through water, with gravitational acceleration, modelled with Y3D/Fluidity (Xiang et al., 2013).

Numerical test of five CORE-LOC^{TM} units sliding into water

In this example, a single wave is generated by water column collapse and five CORE-LOC^{TM} shape armour units are loosely packed and are free to slide into water with no friction being applied for this simple case. The wave quickly approaches the armour units, surging through and further destabilizing them. Figure 16 also shows the fluids flowing around the armour units while stress waves propagate in the armour units when they collide with each other and with the slope. The results appear quite realistic and are in qualitative agreement with the expected flows.

CONCLUDING REMARKS

Several unique modelling achievements applied to the field of single layer coastal armour unit systems have been demonstrated. A great deal has been learned and the foundations laid for a method

of placing rock and unit layers numerically. Modelling different full scale systems under intense vibrations can show us the relative merits of higher initial packing densities. But this type of oscillatory disturbance is unlike wave action which introduces lift and drag forces. With some relatively simple improvements to the current 'proof of concept' version of the wave proxy model reported here, such as calibration of water particle velocity time histories for storm wave sequences using CFD, the coastal engineer will be able to address special features of a design's resilience to wave loads. For example, ideal target packing density, best use (e.g. pattern or random placement), of a particular existing or new unit geometry, the stability near the toe or settlements around the still water level, as well as potential fragility of units if the construction conditions for ensuring excellent interlocking are not favourable.

An important benefit is that the results are presented throughout the domain for inspection at specific locations and also stochastically for a whole representative domain in a form convenient for probabilistic design approaches. As well as quantifying the heterogeneity of a host of parameters, these heterogeneous effects such as stress chains can be visualized.

The developments that have now incorporated plasticity models may provide a research method to study the relative merits of higher strength or more easily yielding concretes. The latter have been associated with a suspected damping of high tensile stress waves and this process can be easily seen in these FEMDEM models. Furthermore, it is rare to have a 3D fracture model incorporated in a multi-body simulator and with more computational power, any tendencies towards greater breakages could be highlighted. Indeed, behavior in the stocking yard and during handling and construction is well within scope for the existing research methodologies.

It should be recognized that all these simulation capabilities, depending on problem set-up, are prone to impractical simulation runtimes, especially if the deformability and dynamic stress state is the focus of the study. Therefore, progress with code parallelization that scales well with the number of processors and other speed-up strategies have to be integral to the research effort in this field in order to deliver on the promise afforded by such tools. A key challenge for the next phase is to provide a suitable technology from which the compromise between accurate fully coupled wave-structure interaction with a granular armour layer and computational resources available can be set according to the investigator's or client's constraints.

REFERENCES

Burcharth, H. F. 1981. Full-scale dynamic testing of Dolosse to destruction. *Coastal Engineering*, 4:229-251.

Dentale, F., Russo, S. D., Carratelli, E. 2009. Innovative numerical simulation to study the fluid motion within rubble mound breakwaters and the armour stability.
http://www.flow3d.com/pdfs/tp/wat_env_tp/FloSci-Bib01-10.pdf. Published online.

Farrell P.E., Maddison J.R., 2011. Conservative interpolation between volume meshes by local Galerkin projection, Computer Methods in Applied Mechanics and Engineering, 200:89-100.

Garcia, N., Richardson, S., and Rigden, T. 2014. Physical Model Testing of the Hydraulic Stability of Single-layer Armour Units. In: From Sea to Shore - Meeting the Challenges of the Sea, Thomas Telford. p 122-130.

Guo, L., Latham, J.-P., and Xiang, J. 2015. Numerical simulation of breakages of concrete armour units using a three-dimensional fracture model in the context of the combined finite-discrete element method. *Computers & Structures*, 146:117-142.

Karantzoulis, N., Xiang, J., Izzuddin, B. and Latham, J.-P. 2013. Numerical Implementation of plasticity material models in the combined finite-discrete element method and verification tests. DEM6: International Conference on DEMs, CSM, p 247-251. ISBN 0-918062-20-9

Latham, J-P., Anastasaki, E. and Xiang, J. 2013. New modelling and analysis methods for concrete armour unit systems using FEMDEM, *Coastal Engineering*, 77:151-166.

Munjiza, A., 2004. The Combined Finite-Discrete Element Method, John Wiley & Sons.

Pain, C. C., Mansoorzadeh, S. and de Oliveira, C.R.E. and Goddard, A.J.H., 2001, Numerical modelling of gas-solid fluidised beds using the two-fluid approach, *Int. J. Numer. Method. Fluids*, 36:91-124.

Viré, A., Xiang, J., Milthaler, F., Farrell, P. E., Piggott, M. D., Latham, J.-P., Pavlidis, D. and Pain, C. C., 2012. Modelling of fluid-solid interactions using an adaptive mesh fluid model coupled with a combined finite-discrete element model. *Ocean Dynamics*, 62:1487-1501.

Xiang, J., Munjiza, A. and Latham, J-P., 2009. Finite strain, finite rotation quadratic tetrahedral element for the combined finite-discrete element method. *Journal for Numerical Methods in Engineering*, 79: 946-978.

Xiang, J., Latham, J.-P., Zimmer, D., Baird, W.F. and Fons, M. 2011. Modelling breakwater armour layers and the dynamic response of armour units. Proceedings of International Conference on Coastal Structures, Yokohama, Japan. September 6-8, 2011.

Xiang, J., Latham, J-P., Viré, A., Anastasaki, E., Pain, C.C. 2013. Numerical modelling of wave – rubble mound breakwater interaction. DEM6: International Conference on DEMs, CSM, p426-431, ISBN 0-918062-20-9.