# The effect of particle image blur on the correlation map and velocity measurement in PIV

Gerrit E. Elsinga, Bas W. van Oudheusden, Fulvio Scarano

Delft University of Technology, Dept. of Aerospace Eng. PO Box 5058, 2600 GB Delft, The Netherlands phone: +31 15 2786386 fax: +31 15 2787077 email: G.E.Elsinga@lr.tudelft.nl B.W.vanOudheusden@lr.tudelft.nl F.Scarano@lr.tudelft.nl

#### ABSTRACT

In PIV particle image blur is usually observed near fluid optical interfaces, i.e. shock waves, and thin flow structure with large density variations, e.g. shear layers and boundary layers. In such an environment the particle image is not only subject to blur, but is also displaced from its actual position due to refraction, which is denoted as optical displacement. In this study particle image blur near a shock wave is investigated in relation to the auto- and cross-correlation map, measurement accuracy and confidence level. The results from a numerical study are supported by PIV measurements of a shock wave in a supersonic wind tunnel. It is demonstrated that particle images are blurred in the direction of lower refractive index (directional blurring). The particle images are also skewed. Therefore particle image blur not only causes correlation peak broadening due to the fact that the particle images increase in size, but more importantly can introduce an asymmetry in the correlation peak and in turn introduce a small bias error in the measured velocity. However, experimental results indicate that particle image blur itself is not the main cause for the increase in measurement uncertainty near shock waves, but that the reduced accuracy can be attributed to the optical displacement. The observation of particle image blur can be used as a detection criterion for a qualitative assessment of the optical displacement. Certain combinations of experimental parameters (viewing angle, f/# and interrogation window size) yield significant errors in the measured velocity. Under certain circumstances optical distortion can become so strong to introduce an unphysical acceleration within the shock wave, visualized as an inflection point with positive slope in the velocity profile across the shock. The study provides some practical suggestions to limit the effect of aero-optical distortion on the velocity measurement.

Keywords: Aerodynamics, Particle Image Velocimetry, Particle Image Blur, Shock wave

## 1. INTRODUCTION

In recent years Particle Image Velocimetry (PIV) has become a commonly employed measurement technique in compressible flows ranging from the transonic (Raffel and Kost 1998) and supersonic (Urban and Mungal 2001, Haertig et al. 2002, Scarano and van Oudheusden 2003) to the hypersonic regime (Schrijer et al. 2005). The fluid medium becomes optically inhomogeneous due to compressibility effects. For many years these flows were visualized and studied using experimental techniques based on light transmittance (e.g. shadowgraphy, schlieren methods and interferometry, Merzkirch 1974), which in fact rely on light beam distortion due to the variation of the refractive index in the medium. Under these conditions particle imaging can be far from trivial. Many PIV studies report particle image blur (Raffel and Kost 1998, Abart et al. 2004 and Elsinga et al. 2005a), often at optical interfaces such as shock waves (Fig. 5) and shear layers. Although the causes of particle image blur are well known, its effect on the PIV cross-correlation signal and therefore the accuracy of the local velocity measurement have not been explored in depth although some possible explanations have been proposed. The simplest observation is that blur results in correlation

Optical Diagnostics, edited by Leonard M. Hanssen, Patrick V. Farrell, Proceedings of SPIE Vol. 5880 (SPIE, Bellingham, WA, 2005) · 0277-786X/05/\$15 · doi: 10.1117/12.616572 peak broadening with consequent drop in signal to noise ratio and an increase in cross-correlation uncertainty (accuracy).

Since particle image blur is usually observed near small flow structures and shock waves, the discussion in this paper will be limited to planar shock waves being a typical example and the strongest cause of blur. First the theory related to aero-optical distortion and particle image blur will be discussed in the next section. Then from a numerical simulation the typical shape of blurred particle images is retrieved and is put in relation to the auto- and cross-correlation peak shapes returned from synthetic and real PIV images. The amount of blur observed in the PIV recordings for optical interfaces is investigated in relation to the velocity bias error and the noise level in the velocity measurement due to loss of correlation. Special attention is given to the influence of the observation angle with respect to the shock wave  $\theta$ , the numerical aperture (or f/#) and the spatial resolution with the interrogation window size Ws as problem governing parameters.



Figure 1: Experimental arrangement for the numerical simulation of particle imaging through shock waves.

#### **2. THEORY**

Figure 1 shows the geometrical configuration of the investigated problem. The planar shock is at an angle  $\theta$  with the viewing direction and terminates at the tunnel window. Part of the light scattered by the tracer particles in the light sheet is collected by the imaging optics to form the particle image. In case a light ray intersects with the shock Snell's law applies and given the intersection angle and shock strength the deflection angle can be calculated. Because a one-to-one relation exist between the plane of focus POF (the light sheet) and the image plane, the light ray position in the image is determined by a linear backward extension of the light ray from the tunnel window (or edge of the refractive index field) to the POF, thus the magnification can be ignored. The optical displacement  $\xi$  is then defined as the distance in the POF between the actual position of the light ray (the actual particle position) and its location in the image (the particle image position). The optical displacement is a function of the particle position in the POF and its gradient is the

relative velocity error due to refraction  $\left(\frac{\Delta u}{u} = \frac{\partial \xi_x}{\partial x}\right)$ , Elsinga et al 2005a). It is seen from figure 1 that in the presence of

a shock not all light rays originating from the same particle are imaged onto a single point, which results in defocusing of the particle or particle image blur. The blur length  $\xi_{blur}$  is defined as the distance in the image between the two limiting light rays of the captured light cone starting at the particle. Both the blur length and the optical displacement of the particle image are linear with the distance between particle and shock in the viewing direction, hence distance in the measurement plane, because the incidence angles of all light rays with the shock and consequently the deflection angles do not change with the position of the particle (Fig. 1). So for planar shock waves, and more in general for flow related optical interfaces,  $\xi_{blur_x} = \Delta \xi_x = c \xi_x$ , where *c* is a constant depending on the angle between the shock, the viewing direction and the shock strength. In that case the gradient of the blur length is related to the direct velocity error by:

$$\frac{\partial \xi_{blur_x}}{\partial x} = \frac{\partial \Delta \xi_x}{\partial x} = \frac{\partial (c\xi_x)}{\partial x} = c \frac{\Delta u}{u}$$
(1)

This relation can be of practical use when performing a qualitative assessment of the aero-optical errors near interfaces, which is not easily achieved otherwise. Furthermore particle image blur is usually only observed (or at least is most visible) near those optical interfaces.

In real applications shocks are generally curved surfaces with varying orientation and strength, hence c is not constant but varies with the position in the image. It is therefore more useful to be able to estimate the order of magnitude of such effects rather than attempting their exact determination.

#### **3. SIMULATION**

A numerical simulation of the particle imaging (Elsinga et al. 2005b) is performed to investigate the optical distortion effects. From the ray trace data the geometrical shape of the particle image is determined and after convolution with the diffraction spot the final particle image is obtained. Simulations are carried out for  $\theta$  ranging from -2 to 2 degrees and f/# from 8 to 22 and all physical dimensions are that of the TST-27 wind tunnel used in the experiments (section 4). The density increase over the shock is 0.3 kg/m<sup>3</sup>.

Based on the simulation results the particle imaging can be categorized into three basic types (Elsinga et al. 2005b), which occurrence depends primarily on the viewing angle  $\theta$  and the position of the particle relative to the shock (Fig. 2). The first type is the undistorted particle image, which is formed when none of the light rays intersect the shock. The second type results when all rays cross the shock. The distorted particle image is characterized by an asymmetrical stretching in the direction normal to the shock (skewed directional blurring). The last type occurs when part of the rays intersect the shock and are in that respect end-effect. Surprisingly, particle doublets can be observed in that case.



Figure 2: Particle images near a shock wave in the first (blue) and second frame (red) showing different types of blur. The shock is oriented vertically with high density on the right and is imaged at a negative viewing angle.

Distortion of particle images in a PIV recording can affect the measurement through a change in the shape of the cross-correlation map and its peak. Since doublets are local (end-) effects, the directional and skewed blurring of

particle images generally dominates. They cause cross-correlation peak broadening in the direction of the particle image blur with an expected consequent drop in signal to noise ratio and cross-correlation accuracy. Whether the correlation peak is skewed depends on the particle displacement between the recordings. If the particle is displaced from the undisturbed to the blurred region, the cross-correlation map will be skewed resulting in a bias error expected to be of sub-pixel order. However when the particle is blurred by the same amount in both recordings the map is symmetrical and no bias is expected. Particle image doublets can further decrease the signal to noise ratio, because they are imaged as traveling at speeds different from the actual particles (Fig. 2).

To determine the particle image blur length from PIV recordings, the particle image recordings auto-correlation peak width is evaluated. In this case the peak is assumed to be of elliptical Gaussian shape expressed by

$$R(r,s) = e^{-\frac{1}{2}\left(\left(\frac{r}{\sigma_r}\right)^2 + \left(\frac{s}{\sigma_s}\right)^2\right)}$$
(2)

A 3×3 kernel is adopted to fit the expression to the auto-correlation maximum neighborhood, where the coordinates r and s are taken along the principal axes of the ellipse, which are determined from the eigenvectors of the Hessian matrix of the auto-correlation map. The standard deviation  $\sigma$  is the measure for the peak width and the particle image blur length. The procedure is assessed making use of computer generated PIV recordings of a shock wave using the particle images from the numerical simulation above. The image resolution is 40 pixels/mm. An auto-correlation window size of 21×21 pixels with 90% overlap is used in order to minimize effects associated to coarse sampling. The correlation maps are averaged over 120 realizations. Figure 3 shows for the simulation conditions  $\theta = -2$  deg the optical displacement of a particle (left) and the estimate of the particle image blur length (right). The displacement error is a linear function of the position and independent of f/# apart from some edge effects near x = -5 mm. The directional stretching of the auto-correlation peak also depends on the position, but shows dependence on f/#. For small f/# light is captured from a relatively large solid angle, which makes the effect of the shock on the particle image blurring with respect to undisturbed particle image size is smaller. Apart from smoothing the profile for the optical displacement scales with the estimate of the particle image f/# less smoothing the profile for the optical displacement scales with the estimate of the particle image blur length. This is the underlying assumption for Eq. 1, so that this estimate for the blur length can be used to indicate the direct velocity error near shock waves.

Figure 4-left shows the normalized velocity error returned from the synthetic PIV images with uniform particle displacement of 10 pixels. The velocity error is weakly dependent on f/# although some smoothing is observed for small f/# as before. A pin-hole imaging configuration would solely introduce optical particle displacement and no particle blur, returning the same velocity error. Therefore the main cause for error is the optical displacement rather than blur. Figure 4-right shows the gradient of the auto-correlation peak width derived from figure 3-right. For f/22 the gradient does not exceed the noise level. When comparing the velocity error with its prediction by the gradient of the auto-correlation peak width at location results from a very local change in peak width smoothed by the effect of a finite size interrogation window and by the finite solid angle to capture light (related to f/#) and therefore it is not seen as clearly in the velocity measurement. Between x = -4 and 0 mm, however, the gradient of the auto-correlation peak width is more constant over a larger region and therefore the prediction is in agreement with the measured velocity error. The simulation shows that the auto-correlation peak width can be used to indicate velocity errors resulting from aero-optical distortion, although some care needs to taken near peaks in the velocity error prediction.



Figure 3: Left: displacement error due to optical distortion across a shock wave. Right: auto-correlation peak width in PIV recordings of the same shock.



Figure 4: Left: velocity error due to optical distortion across a shock wave. Right: gradient of the auto-correlation peak width in PIV recordings of the same shock.

## 4. EXPERIMENTS

The PIV experiments presented in this section are performed in the TST-27 transonic-supersonic wind tunnel of Delft University of Technology in the Aerospace Engineering Department. The experimental setup is that of the numerical simulation (Fig. 1). The bow-shock from the 2D wedge-plate model (Scarano and Van Oudheusden 2003) was measured in a Mach 1.96 free stream flow expanded from 1.94 bars stagnation pressure at ambient temperature, yielding a free-stream velocity of 500 m/s and density of  $0.56 \text{ kg/m}^3$ . The model spans the width of the test section (280 mm) and consists of a wedge with sharp leading edge imposing a flow deflection of 11.3 degrees. The resulting oblique shock wave has an angle of 41 degrees with respect to the free stream. The flow velocity is decreased to 440 m/s and the density increases to  $0.86 \text{ kg/m}^3$ . The wedge is followed by a plate 50 mm long and 20 mm thick truncated with a sharp base. The flow is seeded with 50 nm TiO<sub>2</sub> particles, which are illuminated by a double pulse Nd:YAG laser (400 mJ per pulse) in a 1 mm thick light sheet. The model spans the width of the test section (280 mm) and consists of a wedge imposing a flow deflection of 11.3 degrees at 1376×432 pixels resolution corresponding to a field of view of 35x11 mm<sup>2</sup>. The camera can be translated and rotated in the horizontal plane to

control the viewing direction. Therefore instead of changing the flow, i.e. the orientation of the shock, as done in the numerical simulation, the imaging optics are moved, which is equivalent for small viewing angles. The viewing angle with respect to the shock is obtained from the goniometric relation  $\sin\theta = \sin\sigma\sin\theta_x$ , in which  $\sigma = 41^\circ$  is the shock angle with the horizontal direction and  $\theta_x$  is the angle of rotation of the camera. The time separation is set at 0.6 µs yielding a 12 pixel particle displacement in the free stream. For each test case the dataset consists of 300 recordings, which are analyzed by cross- and auto-correlation using an interrogation window size of 31x31 pixels with 50% overlap. In the following discussion only the average velocity and correlation maps are considered.

Figure 5 shows a PIV recording of the shock wave with corresponding instantaneous velocity field (f/16 and  $\theta = -1.3$  deg.). Details of the image at location A and B are presented in figure 6-left. In the undisturbed region A the particle images are circular, while in the vicinity of the shock (region B) the particle images are blurred in the direction normal to the shock. Figure 6-right shows an example of the average cross-correlation map at locations A and B (applying a window offset of 10 pixels in the horizontal direction). As expected the correlation peak is circular in the undisturbed region A and directional and skewed at location B near the shock resembling the shape of the blurred particle image found in the numerical simulation (Fig. 2).





Figure 5: Upper: PIV recording of a shock wave. Lower: corresponding instantaneous velocity field.

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Figure 6: Left: 100x100 pixel details of PIV recording in undistorted region (A) and near the shock (B). Right: normalized cross-correlation maps in regions A and B.

#### 4.1. Effect of a varying viewing angle $\theta$

The profiles for the measured average velocity in the direction normal to the shock *s* for viewing directions ranging from -3.3 to 3.3 degrees with f/16 are given in figure 7. The velocity is normalized with the normal component of the free stream velocity (340 m/s). The profiles are compared with the reference profile obtained with the viewing direction aligned with the shock, in which case the area affected by aero-optical distortion is so small that its influence on the velocity is negligible due to insufficient spatial resolution. The affected area shifts from the pre-shock region (left) to the post-shock region (right) when going from negative viewing angles (looking from the high density side) to positive angles and it increases with the viewing angle as expected. The velocity error increases with decreasing angle until the affected area becomes so small that it goes below the resolution capability of the imaging system making the error vanish (angles below 0.65 deg). In some cases aero-optical distortion is so strong to introduce an unphysical measurement of the particle behavior downstream of the shock wave. The known exponential decay law for the velocity relaxation is then corrupted by a wiggle, which in some cases assumes a positive slope at the inflection point suggesting a local particle acceleration downstream of the shock. From the figure it is concluded that the most critical angles for the present velocity measurement are between -1.3 and -2.0 degrees, where velocity errors up to 10% are returned. The error for positive viewing angles (looking from the low density side) generally tends to be smaller and is estimated at 4% of the local velocity.

The viewing angle below which the distortion has little influence can be related to the interrogation window size Ws and the refractive field width W. The velocity error is considered to be a sinusoid with a wavelength  $\lambda$  equal to the

affected area ( $\lambda = W \tan \theta$ ), which must be sampled by 4 interrogation windows in order to be detected without large amplitude modulation. Then the critical angle is given by:

$$\theta > a \tan\left(\frac{4ws}{W}\right) = 1.3^{\circ} \tag{3}$$

which is in agreement with the experimental results.



Figure 7: The effect of the viewing direction on the average normalized velocity profiles across a shock wave.



Figure 8: The effect of the viewing direction on the normalized auto-correlation peak width across a shock wave.

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Figure 8 shows the average auto-correlation map peak width for the same set of experiments. According to Eq. 1 these results indicate that wiggles in the velocity are predicted (Fig. 9). For positive viewing angles the wiggle shape and location of the velocity error is well predicted (with c<0 in Eq. 1). Furthermore it is seen from a comparison of Figs. 7 and 9 that the constant c depends on the viewing angle. For negative viewing angles obtaining information from the correlation peak width is more complicated. Figure 8 shows a *camel* shape profile for the peak width (e.g.  $\theta = -2.0^{\circ}$ ), which results in a double wiggle in the prediction of the velocity error (Fig. 9) that is not observed in the measured velocity profiles (Fig. 7). The origin for the camel shape is presently unknown and may be related to additional optical distortion effects as a result of shock wave boundary layer interaction occurring at the wind tunnel side window.



Figure 9: The gradient of the normalized auto-correlation peak width across a shock wave for varying viewing angle.

## 4.2. Effect of a varying numerical aperture or *f*/#

Figure 10 presents the average velocity profile for  $\theta = -1.3^{\circ}$  and varying f/#. Decreasing f/# (optically) smoothes the velocity profiles reducing the error, which is most clearly seen at s = -2 mm. However at s = -1.5 mm the opposite is true. At that location the peak in the velocity profile becomes so sharp with increasing f/# that cross-correlation is unable to resolve it, as also indicated by the increasing RMS of the *u*-component of velocity (Fig. 11). The RMS error in the present case is dominated by the effect of spurious peak detection or outliers, which is the consequence of low signal-to-noise and results in a poor confidence level. The normalized correlation peak width (Fig. 12) also increases with decreasing f/#. The double peak (or camel shape) observed for f/11 and f/16 has disappeared for f/8. For f/22 the particle diameter, hence the reference auto-correlation peak width has become so large that blur increases the peak width by only 26% and details (i.e. the double peak) in the profile are lost. The f/# determines the optical smoothing of the error profile, which diameter  $d_s$  is estimated by (the calculated value is for f/16):

$$d_s = \frac{W}{\left(\frac{M+1}{M}\right)f_{\#}} = 1.8mm\tag{4}$$

The convolution kernel has a parabolic distribution since the cross-section of the captured light cone is circular.



Figure 10: The effect of f/# on the average normalized velocity profile.



Figure 11: The effect of f/# on the RMS of the *u*-component of velocity.

The RMS profiles of the u-component of velocity show a peak near s = -1.5 mm (Fig. 11), which correspond to the location of the highest measured velocity gradient (Fig. 10). The RMS profiles do not match the profiles for correlation peak broadening (Fig. 12). For f/22 the relative peak broadening is smallest while the RMS and velocity gradient are highest, because reduced optical smoothing results in larger velocity error and gradients and consequently RMS error. Furthermore the RMS fluctuations are much larger than the sub-pixel noise that is expected from the correlation peak broadening by particle image blur alone. Therefore it is concluded that blur itself does not increase the measurement noise and accuracy, but that it is mainly related to the optical particle displacement and the velocity gradients introduced by it.

In conclusion: it is favorable to use a relatively small f/# to measure shockwaves and thin flow structures in the presence of optical distortion and blur, because of the optical smoothing effect on the optical displacement. Even though it increases the normalized correlation peak width.



Figure 12: The effect of f/# on the normalized auto-correlation peak width.



Figure 13: The effect of the integration window size on the average normalized velocity profile.

#### 4.3. Effect of finite spatial resolution

The measurement resolution is determined by the interrogation window size *Ws*, as in general the spatio-temporal averaging due to the particle images finite displacement is a smaller effect. Figure 13 presents the average velocity profiles of the case  $\theta = -1.3^{\circ}$  f/16 analyzed at windows sizes of  $21^2$ ,  $31^2$  and  $61^2$  pixels. From Eq. 3 a clear effect of the window size is expected with strong amplitude modulation for *ws* = 61 pixels, which is confirmed by the experiments. Using even a larger window size will cause the peak near *s* = -1.5 mm to disappear completely, leading to a smooth velocity profile across the shock wave. However such a low spatial resolution is not adequate to describe the sharp flow variations occurring across a shock wave, especially when analyzing the particle dynamical response over the shock to determine its time relaxation constant.

#### 5. CONCLUSIONS

The investigation of particle image blur caused by a planar shock wave demonstrates that blur itself is not a major source for error in PIV. Although cross-correlation peak skewing and broadening related to blur are observed, which degrade the signal with consequent (theoretical) drop in measurement accuracy, the error introduced by the optical displacement, which adds to the particle displacement related to the flow velocity, clearly dominates both in numerical simulations as in real experiments and ranges up to 10% in the present experiments. Moreover experimental results showed that the regions of high measurement noise (large RMS error) do not necessary coincide with regions of particle image blur, but are linked to large velocity gradients or even discontinuities in the velocity introduce by the optical displacement near optical interfaces (Fig. 3-left).

Furthermore a measure for particle image blur was established based on the width of the auto-correlation peak. For optical interfaces, or thin structures, the particle image blur can in principle be used to predict the particle pattern distortion and the velocity error, or at least the location of the distortion. However the quantitative interpretation of the predicted error still requires some knowledge of the imaging system and the flow field.

From the practical side it is concluded that increasing integration window size and decreasing f/# decrease the velocity error due to aero-optical optical distortion, as they both have a smoothening effect. The experiments also show that a positive viewing angle with respect to the shock plane (looking from the low density side) is to be preferred since it returns lower percentage error. For the present experiments the critical viewing angle returning the largest errors is found to be between -1.3 and -2.0 degrees. For other experimental arrangements it can be estimated using Eq. 3.

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